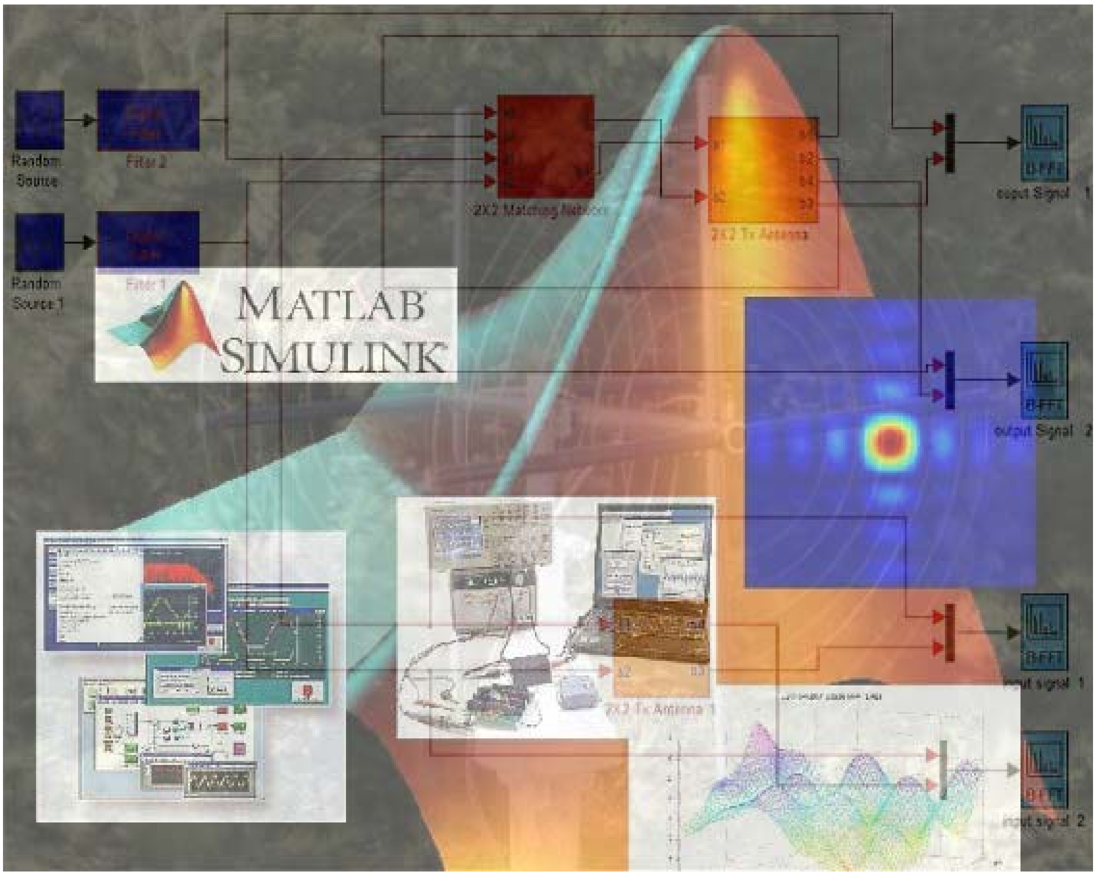

Modelling and Simulation of Matching Networks for Multi-Antenna Communication Systems

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EX022/2008

ABSTRACT

Future cellular systems such as MIMO (Multiple Input Multiple Output) will use multiple antennas, both at the transmitter and receiver ends of the communication links to boost the performance of the communication system. When antennas are close to each other (in terms of wave length), mutual coupling will arise having a negative effect on the performance of the antenna system. In addition, a mismatch between the source and transmit antenna from one side, and the load and the receive antenna from the other side causes an undesirable power loss.

This master thesis assesses the impact of a matching network on the mutual coupling effects between two adjacent antennas. A method to describe a network in terms of scattering parameters is used to represent both of the matching network and the antenna system. This allows for a physical model where RF Blockset is used in the implementation. The front end of the MIMO system is implemented in another master thesis [1] and a mathematical model of the channel is available. The three models can be connected to form the overall MIMO system chain.

Key words: Antenna arrays, Multiple Input Multiple Output (MIMO), mutual coupling, network theory, S-parameters, impedance matching.

ACKNOWLEDGMENT

This master thesis finalizes our post graduate studies at Chalmers University of Technology. The work was done under the supervision of professor Mats Viberg at the department of Signals and Systems. We would like to thank professor Mats Viberg for his guidance and help throughout the course of this thesis work. We would also like to thank the Charmant WP 1 participants for their valuable presentations during the Charmant WP 1 meetings. Special thank also goes out to Patrik Persson and Ulf Carlberg for their support, answering questions, and providing input data to this master thesis.

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Antenna Theory

1.1 Motivation

In the past few decades, technology has developed at a rapid rate in the field of mobile communications due to the introduction of new mobile communication networks. The number of subscribers worldwide has drastically risen up. The requirements on the antennas needed for the ever expanding networks are becoming continually higher. The radiation patterns of the antennas need to be strictly defined to allow accurate network planning. In conclusion, the antenna becomes one of the major components that need to be efficiently utilized to compromise the rapid expansion in the communication networks and enhance the performance of the overall system.

In this chapter we will give a general insight into antenna theory providing a definition to the main parameters that describe the performance of an antenna. We will also introduce antenna arrays and focus on the mutual coupling that occurs when the antennas in the array are close to each other (in terms of wavelength).

1.2 Basic principles of the antenna

An antenna is a transducer designed to transmit or receive electromagnetic waves. It transforms wire propagated waves into space propagated waves. It receives electromagnetic waves and passes them onto a receiver or transmits electromagnetic waves which have been produced by a transmitter.

Antennas are reciprocal that is, all of the antenna parameters that are expressed in terms of a transmission antenna can be applied to a receiving antenna. However, impedance is not applied in an obvious way; the impedance at the load (where the power is consumed) is most critical. For a transmitting antenna, this is the antenna itself. For a receiving antenna, this is at the (radio) receiver rather than at the antenna.

1.3 Definitions

1.3.1 Radiation Pattern

An antenna radiation pattern or antenna pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. In most cases, the radiation pattern is determined in the far-field region and represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity phase or polarization [2].

1.3.2 Radiation Intensity

Radiation intensity in a given direction is defined as the power radiated from an antenna per unit solid angle [2].

1.3.3 Directivity

The directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied [2].

1.3.4 Gain

Another useful measure describing the performance of an antenna is the gain. Although the gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities.

Absolute gain of an antenna (in a given direction) is defined as the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π [2].

In most cases we deal with relative gain, which is defined as the ratio of the power gain in a given direction to the power gain of a reference antenna in its reference direction [2]. The power input must be the same for both antennas. The reference antenna is usually a dipole, horn or any other antenna whose gain can be calculated or is known.

1.3.5 Antenna efficiency

The total antenna efficiency is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due to

1- Reflections because of the mismatch between the transmission line and the antenna.

2- I^2R Losses (conduction and dielectric).

1.3.6 Input impedance

Input impedance is defined as the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point [2].

1.4 Antenna Arrays

Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna.

Enlarging the dimensions of single elements often leads to more directive characteristics. Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multielements is referred to as an array.

An important application where antenna arrays are useful is MIMO systems, where antenna arrays are used to explore the spatial properties of the channel and enhance the diversity order of the system. In most cases, the elements of an array are identical. This is not necessary, but it is often convenient, simpler, and more practical. The individual elements of an array may be of any form (wires, apertures, etc.).

The total field of the array is determined by the vector addition of the fields radiated by the individual elements. This assumes that the current in each element is the same as that of the isolated element. This is usually not the case and depends on the separation between the elements. To provide very directive patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space. Ideally, this can be accomplished, but practically it is only approached. In an array of identical elements, there are five controls that can be used to shape the overall pattern of the antenna. These are

1- The geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
2- The relative displacement between the elements.
3- The excitation amplitude of the individual elements.
4- The excitation phase of the individual elements.
5- The relative pattern of the individual elements.

1.5 Mutual coupling

When the antennas in an array are placed close to each other, they will affect each other through mutual coupling. Mutual coupling describes the electromagnetic interaction between antenna elements [3]; it arises from the fact that when one antenna is transmitting (or receiving) some of the radiated energy will induce currents on the other antenna. The figure below illustrates the mutual coupling between two antennas functioning in a receiving mode

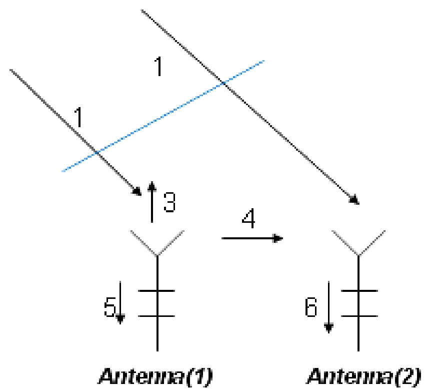


Figure 1.1 Receiving mode coupling paths between antennas 1 and 2.

In the above figure an incident plane wave (1) strikes antenna (1) first where it causes current flow (5). Part of the incident wave will be re-scattered into space as (3). Some part of the scattering (3) will be directed towards antenna (2) as (4) which will add with the incident wave (1) and then be received by antenna(2) as (6). Antenna (1) therefore transfers some of its received energy to antenna (2). In a similar way, antenna (2) transfers some of its received energy to antenna (1). This phenomenon is called mutual coupling.

Mutual coupling can be described with impedance and scattering matrices. An example of how mutual coupling between two antennas can be described with an impedance matrix is shown below

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} * \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \quad (1.1)$$

where

V_1 and V_2 are the voltages at antenna 1 and 2 respectively

I_1 and I_2 are the currents at antenna 1 and 2 respectively

Z_{11} and Z_{22} are the self impedances on antenna 1 and 2 respectively

Z_{21} and Z_{12} are the mutual impedances. They describe how the current on one antenna gives rise to voltage on the other antenna.

Working out the above matrix equation we get

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned} \tag{1.2}$$

The main effects of mutual coupling are

1. Distortion of the effective radiation pattern of each antenna. An example of how the mutual coupling distorts the radiation pattern for two half wave dipoles in free space is shown in Figures 1.2 and 1.3 [4]. Figure 1.2 shows the radiation pattern when there is one isolated half wave dipole in free space, while figure 1.3 shows the radiation pattern when there are two half wave dipoles that are in close presence of each other.

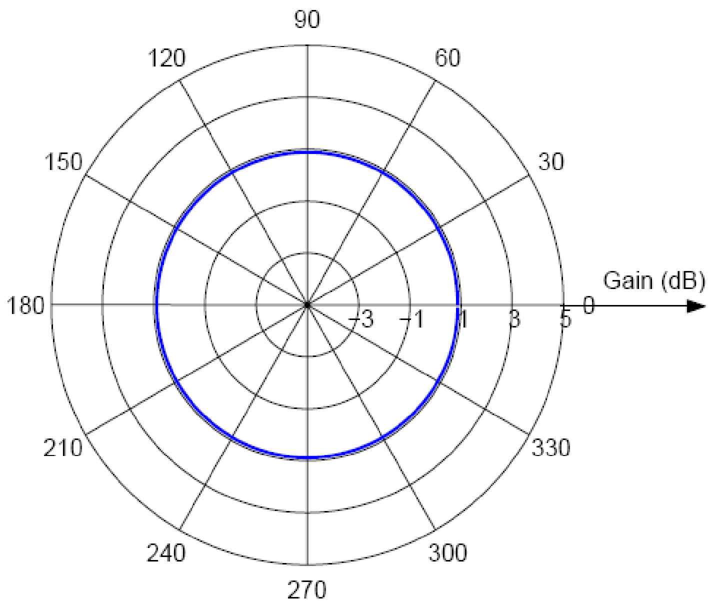


Figure 1.2 Far field radiation pattern for one isolated half wave dipole.

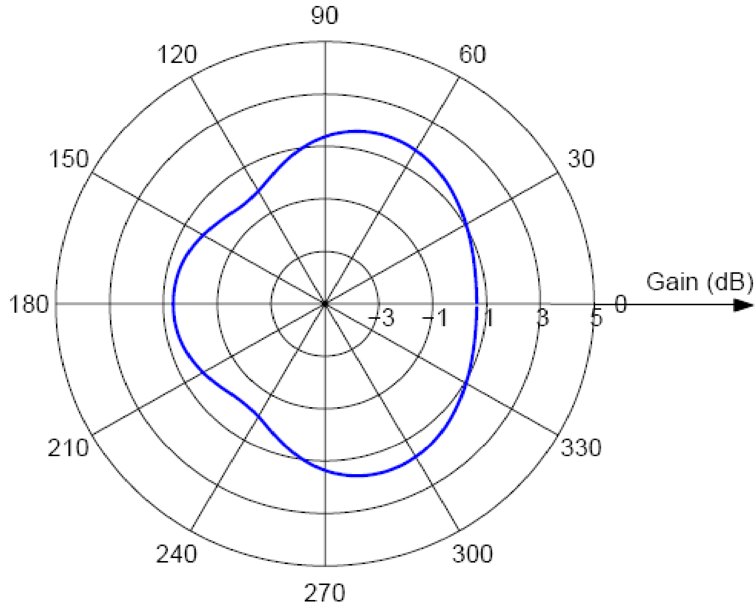


Figure 1.3 Far field radiation pattern for a half wave dipole when there are two half wave dipoles placed side-by-side with distance $\frac{\lambda}{2}$ *apart.*

2. The mutual coupling between antennas also changes the correlation of the signal levels at each antenna. The signal correlation describes how much the signals at the antennas depend on each other. The more correlated the more dependent.

3. The radiation efficiency of the antenna is also affected by the mutual coupling. Radiation efficiency of an antenna is the ratio of the power radiated to the total power supplied to the antenna at a given frequency.

2 Describing N-port network by means of scattering Parameters

In this chapter, we begin with a short statement of the transmission line theory. Then we give a brief review about the traditional methods of network characterization (H, Y, and Z-parameters) to emerge into the desired S-parameters characterization.

2.1 Transmission line theory

A transmission line is the material medium or structure that forms all or part of a path from one place to another by directing the transmission of energy, such as electromagnetic waves or acoustic waves as well as electric power transmission [5].

In many electrical circuits, the length of the wires connecting the components can for the most part be ignored. That is, the voltage on the wire at a given time can be assumed to be the same at all points. However, when the voltage changes in a time interval comparable to the time it takes for the signal to travel down the wire, the length becomes important and the wire must be treated as a transmission line. Stated another way, the length of the wire is important when the signal includes frequency components with corresponding wave lengths comparable to or less than the length of the wire.

For the purpose of analysis, an electric transmission line can be modelled as a two port network as shown in the figure below

Figure 2.1 Electrical transmission line modelled as a two port network.

In the simplest case, the network is assumed to be linear (i.e. the complex voltage across either port is proportional to the complex current flowing into it when there are no reflections), and the two ports are assumed to be interchangeable. If the transmission line is uniform along its length, then its behaviour is largely described by a single parameter called the characteristic impedance (Z_0). This is the ratio of the complex voltage of a given wave to the complex current of the same wave at any point on the line.

When sending power down a transmission line, it is usually desirable that as much power as possible will be absorbed by the load and as little as possible will be reflected back to the source. This can be ensured by making the source and load impedances equal to Z_0, in which case the transmission line is said to be matched.

2.2 Network Characterization

Networks are usually characterized by their H, Y or Z-parameters. However, at high frequencies another method of characterization (S-parameters) is useful. We will first describe the traditional methods of characterization then we will see how a network can be characterized in terms of its S-parameters.

Consider the network in the figure below

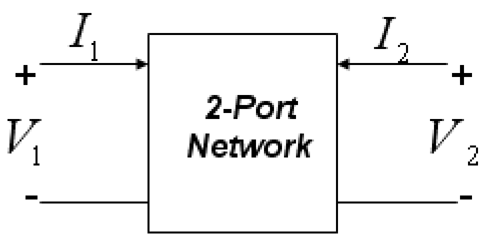


Figure 2.2 Two port network.

This two port network can be described by a number of parameter sets (Z-parameter, Y-parameter and H-parameter). All of these network parameters relate total voltages and total currents at each of the two ports in the following way

$$\begin{aligned} &Hybrid\ (H - Parameters) \\ &V_1 = H_{11}I_1 + H_{12}V_2 \\ &I_2 = H_{21}I_1 + H_{22}V_2 \end{aligned} \tag{2.1}$$

$$\begin{aligned} &Admittance\ (Y - Parameters) \\ &I_1 = Y_{11}V_1 + Y_{12}V_2 \\ &I_2 = Y_{21}V_1 + Y_{22}V_2 \end{aligned} \tag{2.2}$$

$$
\begin{aligned}
&Impedance\ (Z - Parameters)\\
&V_1 = Z_{11}I_1 + Z_{12}I_2\\
&V_2 = Z_{21}I_1 + Z_{22}I_2
\end{aligned}
\tag{2.3}
$$

These parameters can be measured by applying open circuit or short circuit to one of the ports of the network and measuring the ratio between voltage and current on the other port. For example, H_{11} can be measure by setting V_2 to zero (applying a short circuit to the output port of the network). H_{11} is then the ration of V_1 to I_1 (the input impedance of the resulting network). Further, H_{12} is determined by measuring the ratio of V_1 to V_2 (the reverse voltage gain with the input port open circuited). All other parameters can be measured in similar way.

Moving to higher and higher frequencies some problems arise:

1. Equipment is not readily available to measure total voltage and total current at the ports of the network.

2. Short and open circuits are difficult to achieve over a broad band of frequencies.

3. Active devices, such as transistors and tunnel diodes, will very often not be short or open circuit stable.

Some method of characterization is necessary to overcome these problems. The logical variables to use at high frequencies are travelling waves rather than voltages and currents, and the network is described in terms of S-parameters rather than Z, H, or Y-parameters.

High frequency systems have a source of power. A portion of this power is delivered to a load by means of a transmission line. The physical components of such a system is shown in the figure below

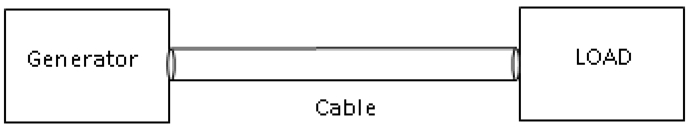


Figure 2.3 Physical components of a high frequency system.

Figure 2.4 below shows the electric circuit corresponding to figure 2.3

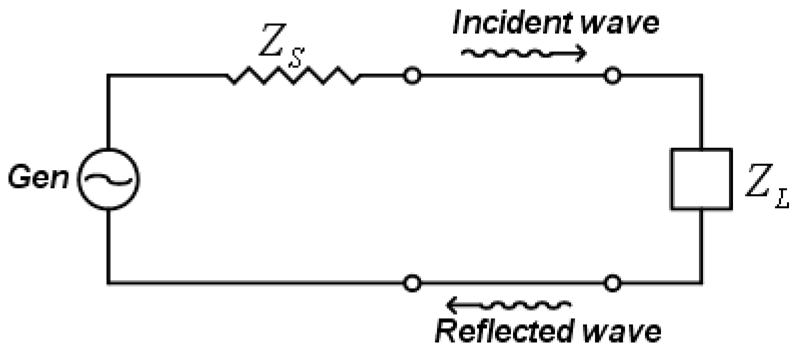


Figure 2.4 Electric circuit corresponding to the physical components in Fig. 2.3

Voltage, current and power can be considered to be in the form of waves travelling in both directions along this transmission line. A portion of the wave incident on the load will be reflected. It then becomes incident on the source, and in turn re-reflects from the source (if $Z_S \neq Z_0$), resulting in a standing wave on the line.

The value of this total voltage at a given point along the length of the transmission line is the sum of the incident and reflected waves at that point.

The total current on the line is the difference between the incident and reflected voltage waves divided by the characteristic impedance of the line. The voltage and current equations are given below

$$V_t = V^+ + V^- \tag{2.4}$$

$$I_t = \frac{V^+ + V^-}{Z_0} \tag{2.5}$$

2.3 S-parameters representation

To introduce the S-parameter representation we insert a two-port network into the transmission line in Fig. 2.4:

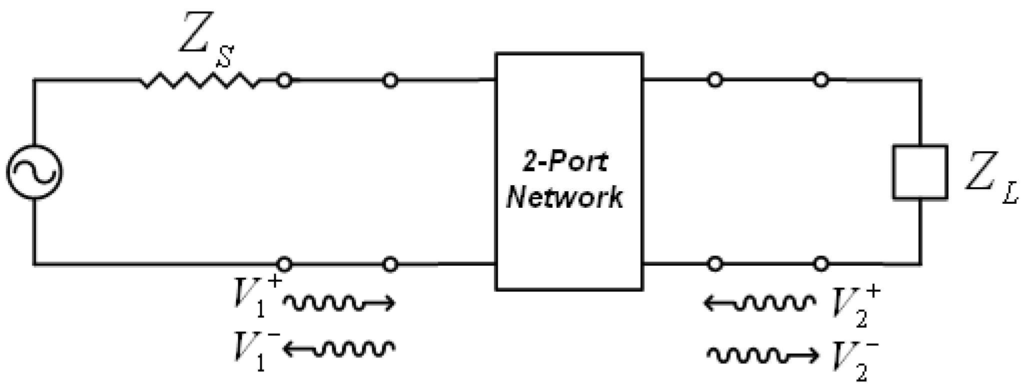


Figure 2.5 Two port network connected into a source and a load through a transmission line. Additional travelling waves arise at the transmission line due to the insertion of the two port network.

We now have additional travelling waves that are interrelated. Looking at V_2^-, we see that it is made up of that portion of V_2^+ reflected from the output port of the network as well as that portion of V_1^+ that is transmitted through the network. Each of the other waves is similarly made up of a combination of two waves.

The voltages and currents at the two ports are given in the equations below

$$V_1 = V_1^+ + V_1^- \tag{2.6}$$

$$V_2 = V_2^+ + V_2^- \tag{2.7}$$

$$I_1 = \frac{V_1^+ - V_1^-}{Z_0} \tag{2.8}$$

$$I_2 = \frac{V_2^+ - V_2^-}{Z_0} \tag{2.9}$$

It is possible to relate these four travelling waves by some parameter set. While the derivation of this parameter set will be made for two-port networks, it is applicable for N-ports as well. Let's start with the H-parameter set

$$
\begin{aligned}
&H - Parameters \\
&V_1 = H_{11} I_1 + H_{12} V_2 \\
&I_2 = H_{21} I_1 + H_{22} V_2
\end{aligned}
\tag{2.10}
$$

By substituting the expressions for total voltage and total current on a transmission line into this parameter set, we can rearrange these equations such that the incident travelling voltage waves are the independent variables; and the reflected travelling voltage waves are the dependent variables

$$
\begin{aligned}
V_1^- &= F_{11}(H)V_1^+ + F_{12}(H)V_2^+ \\
V_2^- &= F_{21}(H)V_1^+ + F_{22}(H)V_2^+
\end{aligned}
\tag{2.11}
$$

The functions F_{11}, F_{21} and F_{12}, F_{22} represent a new set of network parameters relating travelling voltage waves rather than total voltages and total currents. In this case these functions are expressed in terms of H-parameters. They could have been derived from any other parameter set.

It is appropriate to call this new parameter set "scattering parameters" since they relate those waves scattered or reflected from the network to those waves incident upon the network. These scattering parameters are referred to as S-parameters. If we divide both sides of these equations by $\sqrt{Z_0}$, the characteristic impedance of the transmission line, the relationship will not change. It will, however, give us a change in variables. The new variables are defined in the equations below

$$
a_1 = \frac{V_1^+}{\sqrt{Z_0}} \tag{2.12}
$$

$$
a_2 = \frac{V_2^+}{\sqrt{Z_0}} \tag{2.13}
$$

$$
b_1 = \frac{V_1^-}{\sqrt{Z_0}} \tag{2.14}
$$

$$
b_2 = \frac{V_2^-}{\sqrt{Z_0}} \tag{2.15}
$$

Notice that the square of the magnitude of these new variables has the dimension of power. Therefore, $|a_1|^2$ can be thought of as the incident power on port one and $|b_1|^2$ as power reflected from port one. These new waves are called travelling power waves rather than travelling voltage waves. Using (2.11-2.15) and doing appropriate substitutions, we see that the S-parameters relate these four waves in this fashion

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned} \qquad (2.16)$$

Equation 2.16 can be written in the form of a matrix equation as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad (2.17)$$

2.3.1 S-parameters measurement

We have seen how a network is described in terms of its scattering parameters; now let's see how these scattering parameters are measured. Figure 2.6 shows a two port network connected to a source and a load through a transmission line where, S_{11} represents the input reflection coefficient of the network, S_{22} represents the output reflection coefficient, S_{21} represents the transmission from port 1 to port 2 and S_{12} represents the transmission from port 2 to port 1. Furthermore, a_1 and a_2 are the input travelling power waves, while b_1 and b_2 are the output travelling power waves. Finally, Z_0 is the characteristic impedance of the transmission line, while Z_L and Z_S are the impedances of the load and source respectively. The output travelling power waves can be written in terms of the input travelling power waves and the scattering parameters as shown in (2.16).

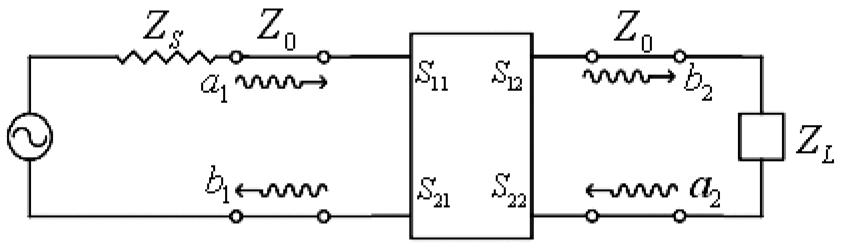


Figure 2.6 Two port network connected into a source and a load through a transmission line. The two port network is described in terms of its scattering parameters.

If we terminate the output port of the network in an impedance equal to the characteristic impedance of the transmission line ($Z_L = Z_0$), all the power incident to the load will be totally absorbed. This is equivalent to setting $a_2 = 0$. Going back to (2.16), S_{11} is then measured as the ratio between b_1 and a_1 , while S_{21} is measured as the ratio between b_2 and a_1. To measure S_{22} and S_{12} we terminate the input port of the network by a matched load ($a_1 = 0$), and then S_{22} is measured as the ratio between b_2 and a_2, while S_{12} is the ratio between b_1 and a_2.

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a2=0} \tag{2.18}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a2=0} \tag{2.19}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a1=0} \tag{2.20}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a1=0} \tag{2.21}$$

2.3.2 Multi-port Networks

So far we have just discussed two-port networks. These concepts can be expanded to multiple-port networks. The three port network in figure 2.7, for example, can be characterized by the following equation

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2.22}$$

The above matrix equation can be written out to give

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \\ b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \\ b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3 \end{aligned} \tag{2.23}$$

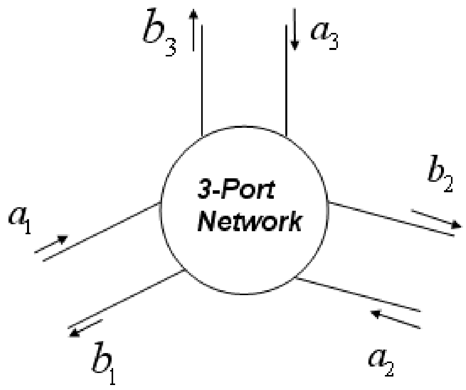


Figure 2.7 Three port network.

In a similar manner as in two port networks, S_{11} is measured by terminating the second and third ports with a matched load. This again ensures that $a_2 = a_3 = 0$. S_{11} is then given by (2.24). We could go through the remaining S-parameters and measure them in a similar way, once the other two ports are properly terminated.

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a2=a3=0} \tag{2.24}$$

What is true for two and three port networks is similarly true for N-port networks.

2.4 Describing N-port networks as a 2-port network

Consider the N-port network in the figure below

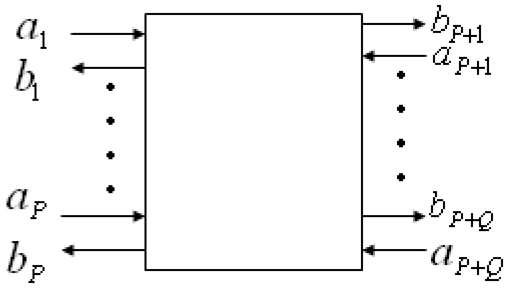


Figure 2.8 N port network.

This network has P ports from one side and Q ports form the other side where P+Q=N. The scattering matrix of this network is defined by the relation

$$\begin{bmatrix} b_1 \\ b_2 \\ \bullet \\ \bullet \\ \bullet \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \bullet & \bullet & \bullet & S_{1N} \\ S_{21} & S_{22} & \bullet & \bullet & \bullet & S_{2N} \\ \bullet & & & & & \bullet \\ \bullet & & & & & \bullet \\ \bullet & & & & & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & \bullet & S_{NN} \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ \bullet \\ \bullet \\ \bullet \\ a_N \end{bmatrix}$$

If we group together the ports at each side of the network, we can look at the network as a 2-port network with the following scattering matrix equation

$$\begin{pmatrix} b_1^{P\times 1} \\ b_2^{Q\times 1} \end{pmatrix} = \begin{pmatrix} S_{11}^{P\times P} & S_{12}^{P\times Q} \\ S_{21}^{Q\times P} & S_{22}^{Q\times Q} \end{pmatrix} * \begin{pmatrix} a_1^{P\times 1} \\ a_2^{Q\times 1} \end{pmatrix} \tag{2.25}$$

Each of the scattering parameters in (2.25) is a matrix. The dimension of the matrix is given by the superscript of the corresponding scattering parameter. For example, $S_{12}^{P\times Q}$ is a matrix with P rows and Q columns.

The parameters a_1^{Px1} and a_2^{Qx1} are the inward propagating wave vectors of port group 1 (P ports) and 2 (Q ports) respectively, while b_1^{Px1} and b_2^{Qx1} are the outward propagating wave vectors. The sub matrices S_{ii} represent the reflection of the wave vector a_i to b_i, whereas $S_{ij}, i \neq j$ represents the transmission from a_j to b_i. The voltages at the port group K is given by

$$V_K = \sqrt{Z_0} * (a_K + b_K) \tag{2.26}$$

where

Z_0 represents the diagonal characteristic impedance matrix of port group K.

Equation 2.26 is called the heavyside transformation.

From now on we will use the above representation for any network.

3 Complete Chain of MIMO system using S-parameters

3.1 Introduction

The main components of a MIMO system are: the signal source, the transmit antenna, the channel, the receive antenna and the signal drain. All these components are described in terms of their scattering parameters. They are connected together to form the complete MIMO system chain as shown in the figure below

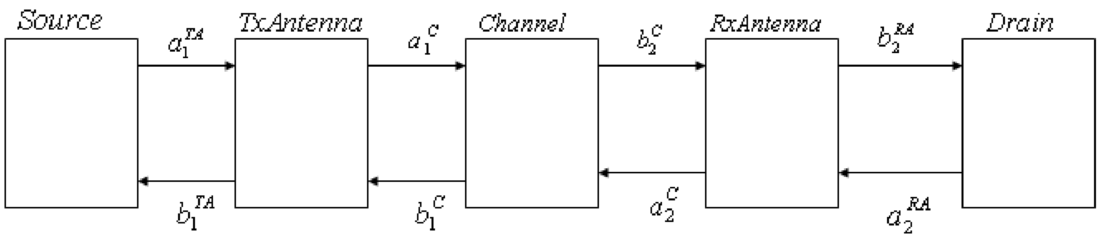


Figure 3.1 System model of the complete RF transmission chain. All elements are described by scattering matrices.

We will first describe each of these components alone, then we will see how the system components are merged and the overall channel matrix is produced.

3.2 System Components

3.2.1 Signal Source

The signal source is depicted in the figure below

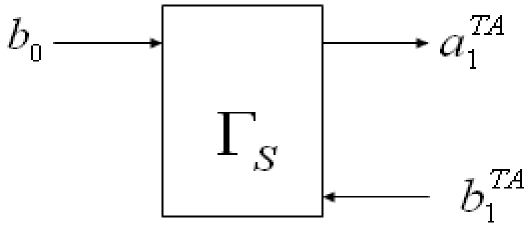


Figure 3.2 Signal source, stand for the amplifier in our model where Γ_S corresponds to S_{22} of the amplifier.

The signal source is the beginning of the transmission chain and determines the transmit power distribution among the transmit antennas. It has the same number of ports as the transmit antenna. The output impedances of the signal source are characterized by the reflection coefficient Γs. The output propagating wave vector is given as

$$a_1^{TA} = b_0 + \Gamma s * b_1^{TA} \tag{3.1}$$

3.2.2 Transmit Antenna

The antenna array plays a critical role in MIMO systems. Since the antennas will be closely spaced they will couple and interact, and they can't be considered as separate elements. The main effects which have to be taken into account are

1. The shape of the radiation pattern of the single antennas changes due to the effect of the other elements.
2. Since the energy radiated by one antenna may be directly absorbed by another antenna due to the close separation, the gain of the antenna reduces. Thus the gain of the antenna is a property of each individual antenna in the array and depends on the array topology and termination of the antenna.

These effects are included in the general scattering parameter description of the antenna.

An antenna array of M elements can be seen as 2M-port network. By grouping the ports at the two sides of the network, the antenna array can be represented by 2-port network as shown in the figure below

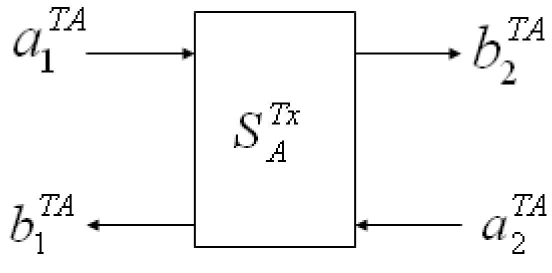


Figure 3.3 Transmit antenna described as a two port network.

where

$$S_{A}^{Tx} = \begin{bmatrix} S_{11}^{TA} & S_{12}^{TA} \\ S_{21}^{TA} & S_{22}^{TA} \end{bmatrix} \tag{3.2}$$

Ports group 1 represent the excitation ports of the antennas in the array while ports group 2 represents the far field properties.

S_{11}^{TA} describes the excitation ports of the antennas and contains the scattering parameters that represent the self (diagonal elements) and mutual coupling (off diagonal elements) effects of the array.

S_{21}^{TA} and S_{12}^{TA} describe the transmission of the signals from the excitation side to the far field and vice versa, thus they contain information on the pattern and gain of the antenna.

S_{22}^{TA} contains the structural antenna scattering of the array with the excitation ports of the antennas terminated in matched loads.

The elements of S_{21}^{TA}, S_{12}^{TA} and S_{22}^{TA} are directional, which means that they are function of the geometry and change for different angles of arrival and departure for different paths.

3.2.3 Channel

The communication channel is also modelled as a 2-port network as shown in the figure below

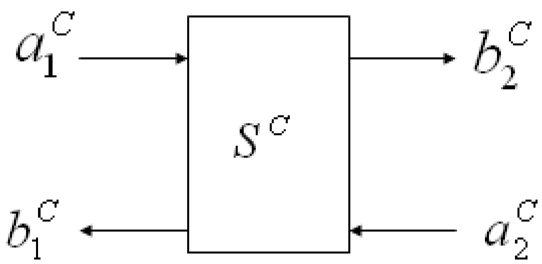


Figure 3.4 Communication channel described as a two port network.

Port 1 represent the far field ports of the transmitting antenna array while port 2 represents the far field ports of the receiving antenna array. The relationship between these ports is given by the scattering matrices in (3.3)

$$\begin{pmatrix} b_1^C \\ b_2^C \end{pmatrix} = \begin{pmatrix} S_{11}^C & S_{12}^C \\ S_{21}^C & S_{22}^C \end{pmatrix} * \begin{pmatrix} a_1^C \\ a_2^C \end{pmatrix} \tag{3.3}$$

Assuming that only far-field effects are of interest, the coupling disappears by definition. Note that the coupling between single antennas is included in the scattering matrices for the antennas. Additionally, it is assumed that there is no reflection from the far fields. Thus, the submatrices S_{11}^C and S_{22}^C can be set to zero.

Moreover, the back transmission of signals through the physical channel is subjected to the channel attenuation; thus, the power reradiated by the receiver and received by the transmitter is twice as strongly attenuated as the signals at the receiver. Therefore, it is justified to neglect the back transmission and set S_{12}^C to zero.

In summary, the communication channel is modelled by S_{21}^C alone, and (3.3) can be written as $b_1^C = 0$, $b_2^C = S_{21}^C a_1^C$

3.2.4 Receive antennas

The receive antennas can be modelled as

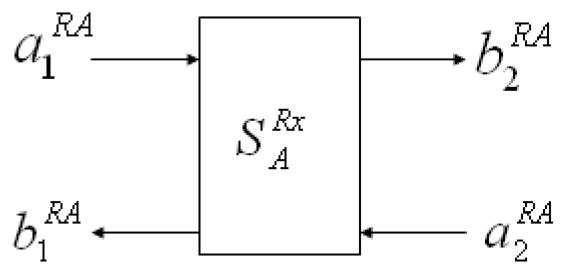


Figure 3.5 Receive antenna described as a two port network.

where

$$S_A^{Rx} = \begin{bmatrix} S_{11}^{RA} & S_{12}^{RA} \\ S_{21}^{RA} & S_{22}^{RA} \end{bmatrix} \tag{3.4}$$

S_{11}^{RA} contains the structural antenna scattering of the array with the excitation ports of the antennas terminated in matched loads.

S_{22}^{RA} describes the excitation ports of the antennas and contains the scattering parameters that represent the self (diagonal elements) and mutual coupling (off diagonal elements) effects of the array.

S_{12}^{RA} and S_{21}^{RA} describe the transmission of the signals from the excitation side to the far field and vice versa.

3.2.5 Signal Drain

The signal drain is similar to the signal source and is described by the reflection coefficient Γ_D.

3.3 Finding the overall channel matrix

Figure 3.1 shows the five system components attached together. The overall channel matrix of the system in figure 3.1 is found in two steps as shown in sections 3.3.1 and 3.3.2.

3.3.1 Merging the inner components

The transmit antenna, channel and receive antenna are merged into one network as shown in figure 3.6. The scattering parameters of the new network are function of the scattering parameters of the 3 merged components.

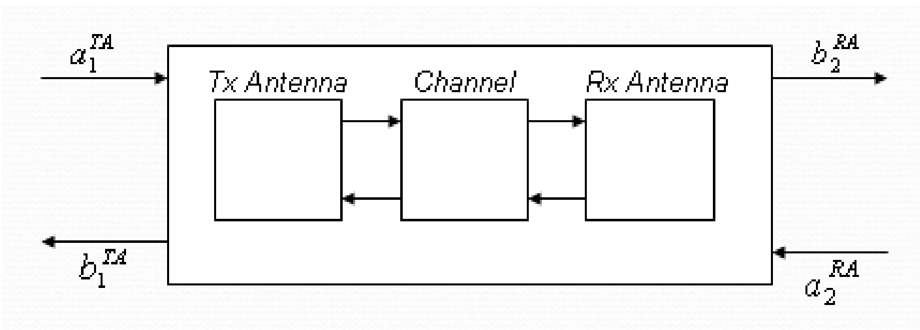


Figure 3.6 Transmit antenna, communication channel and receive antenna are merged into one network.

The transmit antenna is first connected with the communication channel as shown in figure 3.7

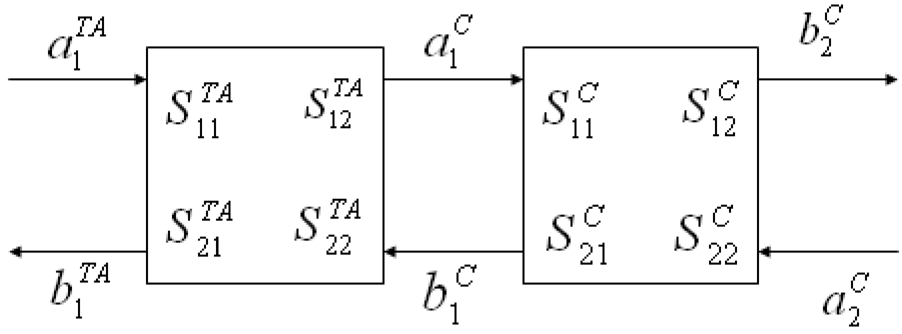


Figure 3.7 Transmit antenna connected to the communication channel.

Following the same procedure as in Appendix C, the above two networks can be merged into one network with new scattering parameters as shown in the below

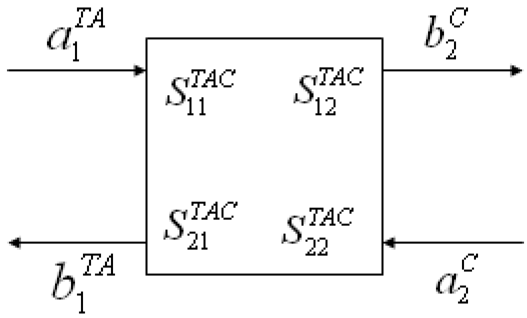


Figure 3.8 Network resulted from merging the Transmit antenna and the Communication channel.

Using equations (C-6 – C-9) in Appendix C we get

$$S_{11}^{TAC} = S_{11}^{TA} + S_{12}^{TA} * \left(I - S_{11}^{C} S_{22}^{TA}\right)^{-1} * S_{11}^{C} S_{21}^{TA} \tag{3.5}$$

$$S_{12}^{TAC} = S_{12}^{TA} * \left(I - S_{11}^{C} S_{22}^{TA}\right)^{-1} * S_{12}^{C} \tag{3.6}$$

$$S_{21}^{TAC} = S_{21}^{C} * \left(I - S_{22}^{TA} S_{11}^{C}\right)^{-1} * S_{21}^{TA} \tag{3.7}$$

$$S_{22}^{TAC} = S_{22}^{C} + S_{21}^{C} * \left(I - S_{22}^{TA} S_{11}^{C}\right)^{-1} * S_{22}^{TA} S_{12}^{C} \tag{3.8}$$

Recalling that from section 3.2.3 $S_{11}^{C} = S_{22}^{C} = 0$ and substituting in the above equations we get

$$S_{11}^{TAC} = S_{11}^{TA} \tag{3.9}$$

$$S_{12}^{TAC} = S_{12}^{TA} S_{12}^{C} \tag{3.10}$$

$$S_{21}^{TAC} = S_{21}^{C} S_{21}^{TA} \tag{3.11}$$

$$S_{22}^{TAC} = S_{21}^{C} S_{22}^{TA} S_{12}^{C} \tag{3.12}$$

The resulting network is then connected with the receive antennas as shown in figure below

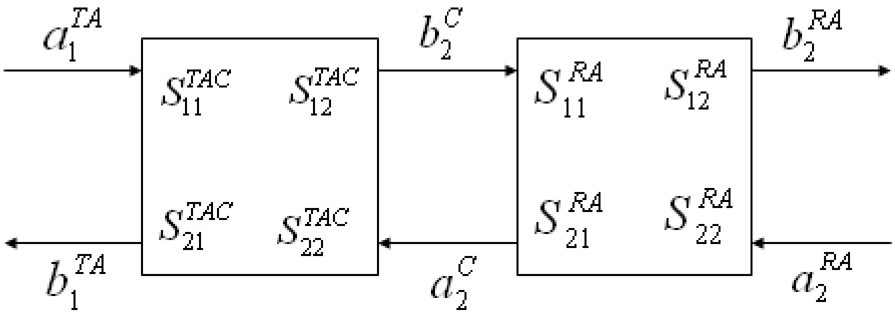


Figure 3.9 Receive antenna connected to the network resulted from merging the transmit antenna and the communication channel.

The above two networks can also be merged into one new network as shown in the figure below

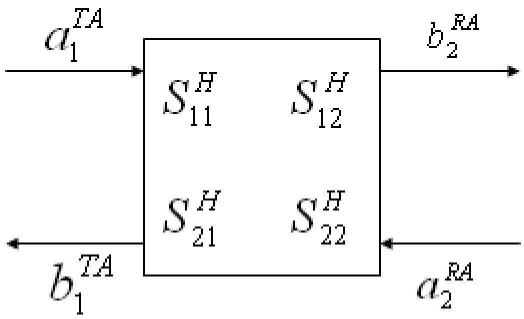


Figure 3.10 Network resulted from merging the Transmit antenna, communication channel and the Receive antenna.

Using (C-6 – C-9) in Appendix C we get

$$\begin{aligned} S_{11}^{H} &= S_{11}^{TAC} + S_{12}^{TAC} * \left(I - S_{11}^{RA} S_{22}^{TAC}\right)^{-1} * S_{11}^{RA} S_{21}^{TAC} \\ &= S_{11}^{TA} + S_{12}^{TA} S_{12}^{C} * \left(I - S_{11}^{RA} S_{21}^{C} S_{22}^{TA} S_{12}^{C}\right)^{-1} * S_{11}^{RA} S_{21}^{C} S_{21}^{TA} \end{aligned} \tag{3.13}$$

$$\begin{aligned} S_{12}^{H} &= S_{12}^{TAC} * \left(I - S_{11}^{RA} S_{22}^{TAC}\right)^{-1} * S_{12}^{RA} \\ &= S_{12}^{TA} S_{12}^{C} * \left(I - S_{11}^{RA} S_{21}^{C} S_{22}^{TA} S_{12}^{C}\right)^{-1} * S_{12}^{RA} \end{aligned} \tag{3.14}$$

$$\begin{aligned} S_{21}^{H} &= S_{21}^{RA} * \left(I - S_{22}^{TAC} S_{11}^{RA}\right)^{-1} * S_{21}^{TAC} \\ &= S_{21}^{RA} * \left(I - S_{21}^{C} S_{22}^{TA} S_{12}^{C} S_{11}^{RA}\right)^{-1} * S_{21}^{C} S_{21}^{TA} \end{aligned} \tag{3.15}$$

$$\begin{aligned} S_{22}^{H} &= S_{22}^{RA} + S_{21}^{RA} * \left(I - S_{22}^{TAC} S_{11}^{RA}\right)^{-1} * S_{22}^{TAC} S_{12}^{RA} \\ &= S_{22}^{RA} + S_{21}^{RA} * \left(I - S_{21}^{C} S_{22}^{TA} S_{12}^{C} S_{11}^{RA}\right)^{-1} * S_{21}^{C} S_{22}^{TA} S_{12}^{C} S_{12}^{RA} \end{aligned} \tag{3.16}$$

Recalling that from section 2.3.3 $S_{12}^{C} = 0$ and substituting it in the above equations we get

$$S_{11}^{H} = S_{11}^{TA} \tag{3.17}$$

$$S_{12}^{H} = 0 \tag{3.18}$$

$$S_{21}^{H} = S_{21}^{RA} S_{21}^{C} S_{21}^{TA} \tag{3.19}$$

$$S_{22}^{H} = S_{22}^{RA} \tag{3.20}$$

The scattering parameters matrix of the resulting network is then given by

$$S^{H} = \begin{pmatrix} S_{11}^{TA} & 0 \\ S_{21}^{RA} S_{21}^{C} S_{21}^{TA} & S_{22}^{RA} \end{pmatrix} \tag{3.21}$$

3.3.2 Termination with Source and Drain

The source and the Drain are connected to the network in figure 3.10 as shown in the figure below

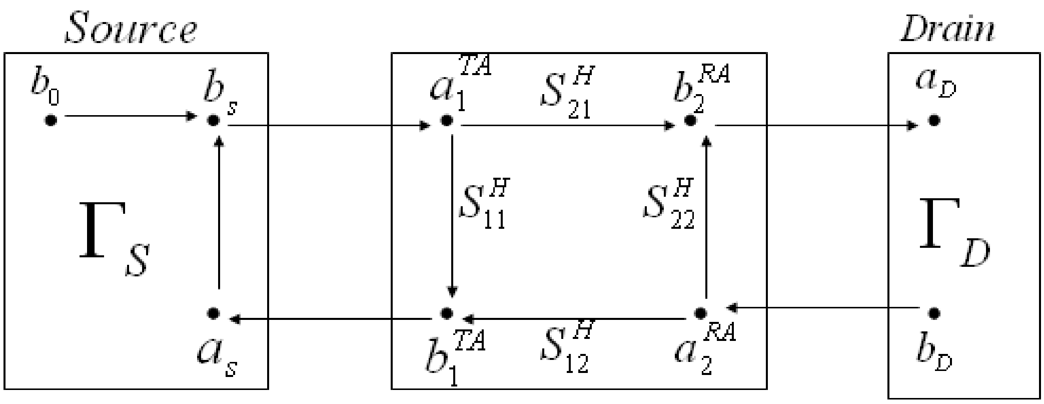


Figure 3.11 Transmit antenna, communication channel and Receive antenna are merged into one network. Source and Drain terminate the network.

Applying equations (C-14 – C-17) in Appendix C we get

$$a_1^{TA} = \left(I - \Gamma_s S_{11}^H\right)^{-1} * b_{0,s} \tag{3.22}$$

$$a_2^{RA} = \left(I - \Gamma_D S_{22}^H\right)^{-1} * \Gamma_D S_{21}^H * \left(I - \Gamma_s S_{11}^H\right)^{-1} * b_{0,s} \tag{3.23}$$

$$b_1^{TA} = \left(I - S_{11}^H \Gamma_s\right)^{-1} * S_{11}^H b_{0,s} \tag{3.24}$$

$$b_2^{RA} = \left(I - S_{22}^H \Gamma_D\right)^{-1} * \left(S_{21}^H + S_{21}^H \Gamma_s * \left(I - S_{11}^H \Gamma_s\right)^{-1} * S_{11}^H\right) * b_{0,s} \tag{3.25}$$

The overall channel matrix expresses the ratio of the voltages at the receive antennas to the voltages at the transmit antennas as

$$V_{RA} = H^{ext} * V_{TA} \tag{3.26}$$

where V is given by the heavyside transformation as

$$V = \sqrt{Z_0}\left(a + b\right) \tag{3.27}$$

Substituting in (3.26) we get

$$\sqrt{Z_{0,RA}} * \left(a_2^{RA} + b_2^{RA}\right) = H^{ext} * \left(a_1^{TA} + b_1^{TA}\right)\sqrt{Z_{0,TA}} \tag{3.28}$$

Using (3.22-3.25) and performing appropriate substitutions, the overall channel matrix is found as

$$H^{ext} = \sqrt{Z_{0,RA}} * \left(I + \Gamma_D\right) * \left(I - S_{22}^H \Gamma_D\right)^{-1} * S_{21}^H * \left(I + S_{11}^H\right)^{-1} * \sqrt{Z_{0,TA}} \tag{3.29}$$

The term $S_{21}^H = S_{21}^{RA} S_{21}^C S_{21}^{TA}$ describes the transmission of the signal from the input ports of the transmit antennas to the output ports of the receive antennas. Using the heavyside transformation it can be expressed as the ratio of the voltages V at the m[th] transmit antenna and nth receive antenna as

$$\left. S_{21}^H \right|_{nm} = \left. \sqrt{\frac{Z_{0,m}}{Z_{0,n}} \frac{V_n}{V_m}} \right|_{ak=0} \qquad K \neq m \tag{3.30}$$

where Z_0 is the characteristic impedance of the scattering parameters.

The ratio $\frac{V_n}{V_m}$ can take different forms depending on the transmission channel representation. In all cases it is mainly a function of the gain and pattern of the antennas in use.

$S_{11}^H = S_{11}^{TA}$ and $S_{22}^H = S_{22}^{RA}$ contain the self and mutual coupling effects of the transmit and receive antennas, while Γ_D is the reflection coefficient of the signal drain.

In this master thesis work, it was decided that the effects of S_{22}^H, S_{11}^H and Γ_D can be modelled physically using the RF toolbox in Simulink, while S_{21}^H can be conveniently represented by a mathematical model. The availability of

Simulink boxes that serve as an interface between a physical and a mathematical model makes our decision feasible.

Since mutual coupling depends on the termination of transmit and receive antennas, a matching network can be inserted between the signal source and transmit antenna from one side and the receive antenna and the signal drain from the other side.

The overall system will then look like

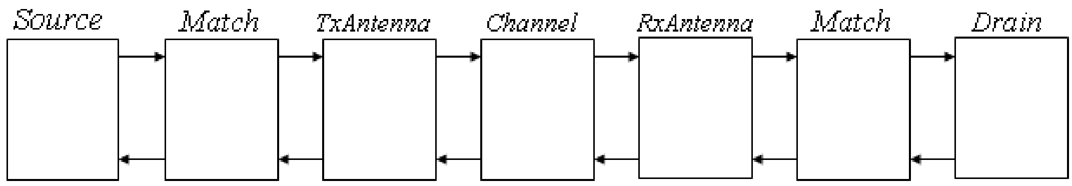


Figure 3.12 Complete RF transmission chain with matching networks inserted between the source and transmit antenna from one side and the receive antenna and the load from the other side.

The matching network effects are described in the next chapter.

4 Matching networks

4.1 Introduction

The Matching network becomes one of the main components of a multiple input multiple output (MIMO) system for many reasons. A MIMO system makes use of multiple antennas at both the transmitter and receiver ends to exploit the spatial channel for increasing the capacity. Correlation of the signals at the different antenna elements can considerably decrease the capacity of a MIMO system [6]. Such correlation occurs particularly when the separation between the antennas is small [7]. In addition, for a small separation, the effect of mutual coupling between the antennas becomes important. Mutual coupling distorts antenna patterns and therefore modifies the correlation results [8], [9]. The change in input impedances of the antennas is another consequence of mutual coupling, and it results in greater mismatch between the antennas and their corresponding source and load impedances [10].

The matching network has a significant influence on the performance of multiple antenna system when the antennas are close to each other. The purpose of the matching network is to minimize the effect of mutual coupling and improve the efficiency of the antennas by maximizing the power transfer to the load.

The use of S-parameter representation to model an entire Multi input Multi output (MIMO) communication system was proposed in the previous chapter. Using this approach, a more diverse range of matching networks can be studied.

4.2 Types of matching network

This section introduces four different types of impedance matching networks with their respective S-parameter representations. The S-matrix of the matching network is represented by

$$S^{M} = \begin{pmatrix} S_{11}^{M} & S_{12}^{M} \\ S_{21}^{M} & S_{22}^{M} \end{pmatrix} \tag{4.1}$$

in the following , we will discuss some different options for choosing S^M .

4.2.1 Characteristic impedance match

The characteristic impedance match is the simplest form of impedance matching, where the antennas are terminated with a load of characteristic impedance Z_0, which is usually taken as 50 ohm. In other words, there is no matching network. This can be modelled either by removal of the matching network or by setting $S_{11}^M = S_{22}^M = 0$ and $S_{21}^M = S_{12}^M = I$. The degree of mismatch depends on the difference between the antenna impedances and the characteristic impedance.

4.2.2 Self impedance match

The Self-impedance match refers to terminating each antenna with a load equivalent to the conjugate of the antenna self-impedance that is, $Z_L = \left(Z_{sf}^A\right)^*$.
In S-parameter form this match can be expressed as

$$S_{11}^M = \Im_{Z-S}\left[\left(diag\left(Z_{sf}^A\right)\right)^*\right] \tag{4.2}$$

where $\Im_{Z-S}$ indicates the transformation from Z to S parameter and Z_{sf}^A is the self impedance of a receiving antenna.

For an isolated antenna, the self-impedance match is also known as the complex conjugate match [11]. It facilitates maximum power transfer to the load when there is no mutual coupling that is, the array antennas are infinitely far apart. However at finite antenna separations, the goodness of the match depends on the behavior of the mutual impedance which is not taken into account.

4.2.3 Input impedance match

While the self-impedance match only takes into account the self-impedance of the antenna, the input impedance match also takes into account mutual coupling [12], [13]. The input impedance match attempts to conjugate-match the antenna pair individually (one at a time) that is, there is a separate matching network for each port, and the matrix of the matching network is diagonal.

For a two antenna array, the input impedance of antenna 1 can be expressed as

$$Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \tag{4.3}$$

where Z_L is the load on the antennas. For input impedance match, $Z_L = Z_{in}^*$ or

$$Z_L = \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \right)^* \tag{4.4}$$

The above equation is iterated to find the desired matching impedance Z_L. A closed form solution of the above equation can also be found, using some straightforward algebraic manipulations. For the simple symmetrical two-dipole case ($Z_{11} = Z_{22}, Z_{12} = Z_{21}$), this solution is given by

$$Z_L = \sqrt{a^2 + d^2 - c^2 - \frac{c^2 d^2}{a^2}} + j\left(\frac{cd}{a} - b \right) \tag{4.5}$$

where

$$\begin{aligned} Z_{11} &= a + jb \\ Z_{12} &= c + jd \end{aligned} \tag{4.6}$$

Thus, S_{11} of the matching network can, in the two-antenna case, be written as

$$S_{11}^{M} = \Im_{Z-S}\left[\begin{pmatrix} Z_L & 0 \\ 0 & Z_L \end{pmatrix}\right] \tag{4.7}$$

4.2.4 Multiport conjugate match

Like the input impedance match, the so-called multiport conjugate match [14] (also called Hermitian match) also takes into account the mutual coupling among the antenna ports. However, unlike the input impedance match, it allows the interconnections between all ports on the two sides of the network.

The Hermitian match condition will be discussed in more details in the next section.

4.3 Hermitian match

4.3.1 Introduction

The Hermitian match is a simplified proof of the multiport conjugate match which states that , for an antenna array with mutual couplings terminated by a load, the maximum power can be transferred to the load if $Z_L = Z_A^*$, where Z_L is the impedance matrix of the load and Z_A^* is the conjugate of the impedance matrix of the antenna array [15]-[18]. The Hermitian match makes use of singular value decomposition to provide the proof which is stated in sections 4.3.3.1 and 4.3.3.2.

4.3.2 Singular Value Decomposition

The singular value decomposition (SVD) is an important factorization of a rectangular real or complex matrix, with several applications in signal processing and statistics. The spectral theorem [19] says that normal matrices, which by definition must be square, can be unitarily diagonalized using a basis of eigenvectors. The SVD can be seen as a generalization of the spectral theorem to arbitrary, not necessarily square, matrices.

Suppose M is an m-by-n matrix whose entries come from the field K, which is either the field of real numbers or the field of complex numbers. Then there exists a factorization of the form

$$M = U\Sigma^{\frac{1}{2}}V^{H} \tag{4.8}$$

where

U is an m-by-m unitary matrix over K which contains a set of orthonormal "output" basis vector directions for M

$\Sigma^{\frac{1}{2}}$ is an m-by-n matrix with nonnegative numbers on the diagonal and zeros off the diagonal containing the singular values, which can be though of as scalar "gain controls" by which each corresponding input is multiplied to give a corresponding output.

V^{H} denotes the conjugate transpose of V , an n-by-n unitary matrix over K that contains a set of orthonormal "input" basis vector directions for M. Such a factorization is called a singular-value decomposition of M.

4.3.3 Derivation of the S-parameters of the Hermitian matching network

The figure below depicts a receiving condition of a MIMO system, with the matching network inserted between the receiving antennas and the load

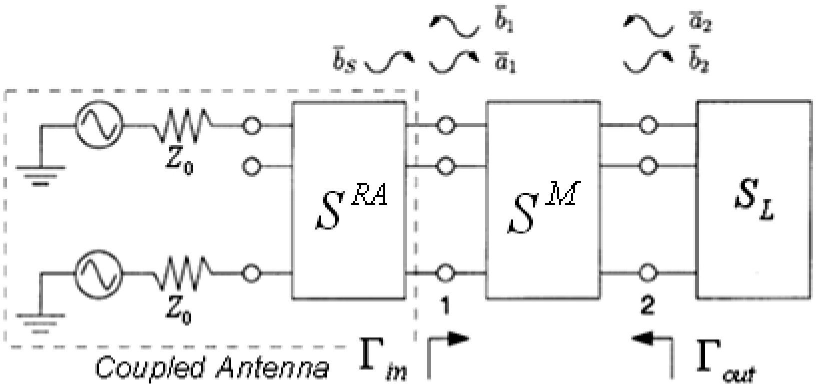


Figure 4.1 Coupled receiving antenna array connected to a multiport matching network and individual loads.

For high-frequency systems such as mutually coupled antenna networks, the S-parameter matrix representation provides a convenient analysis framework.

Each element of the coupled array is characterized by a generator whose signal passes through the matrix S^{RA} with a block representation

$$S^{RA} = \begin{pmatrix} S_{11}^{RA} & S_{12}^{RA} \\ S_{21}^{RA} & S_{22}^{RA} \end{pmatrix} \tag{4.9}$$

where "1" and "2" refer to input and output ports, respectively.

The input and output reflection coefficients in Fig. 4.1 can be expressed as

$$\begin{aligned} \Gamma_{in} &= S_{11}^{M} + S_{12}^{M} * \left(I - S_L S_{22}^{M}\right)^{-1} * S_L S_{21}^{M} \\ &= S_{11}^{M} + S_{12}^{M} S_L * \left(I - S_L S_{22}^{M}\right)^{-1} * S_{21}^{M} \end{aligned} \tag{4.10}$$

$$\Gamma_{out} = S_{22}^{M} + S_{21}^{M} * \left(I - S_{22}^{RA} S_{11}\right)^{-1} * S_{22}^{RA} S_{12}^{M} \tag{4.11}$$

If the matching network is lossless, $\left(S^{M}\right)^{H} S^{M} = I$, which results in

$$\left(S_{11}^{M}\right)^{H} S_{11} + \left(S_{21}^{M}\right)^{H} S_{21} = I \tag{4.12}$$

$$\left(S_{11}^{M}\right)^{H} S_{12} + \left(S_{21}^{M}\right)^{H} S_{22} = 0 \tag{4.13}$$

$$\left(S_{12}^{M}\right)^{H} S_{12} + \left(S_{22}^{M}\right)^{H} S_{22} = I \tag{4.14}$$

Substitution of the SVD of the subblocks $S_{ij} = U_{ij} \Sigma_{ij}^{\frac{1}{2}} V_{ij}^{H}$ into the above equations yields

$$V_{21} \Theta_{21} = V_{11} \tag{4.15}$$

$$V_{12} \Theta_{12} = V_{22} \tag{4.16}$$

$$\Sigma_{21} = I - \Sigma_{11} \tag{4.17}$$

$$\Sigma_{12} = I - \Sigma_{22} \tag{4.18}$$

where Θ_{21} and Θ_{12} are diagonal phase shift matrices with arbitrary complex elements of unit magnitude. This operation also produces the condition

$$\Sigma_{11}^{\frac{1}{2}} U_{11}^{H} U_{12} * \left(I - \Sigma_{22}\right)^{\frac{1}{2}} \Theta_{12} = -\Theta_{21}^{H} * \left(I - \Sigma_{11}\right)^{\frac{1}{2}} * U_{21}^{H} U_{22} \Sigma_{22}^{\frac{1}{2}} \tag{4.19}$$

The SVD proposed above facilitates a simple proof of the fact that multiport conjugate match $S_{11}^{M} = \left(S_{22}^{RA}\right)^{H}$ results in maximum power transfer to the load. In the next section we will provide an abbreviated version of the proof.

4.3.3.1 Load Match

To begin the derivation we remove the source coupling block in Fig. 4.1 to arrive at the network in the figure below

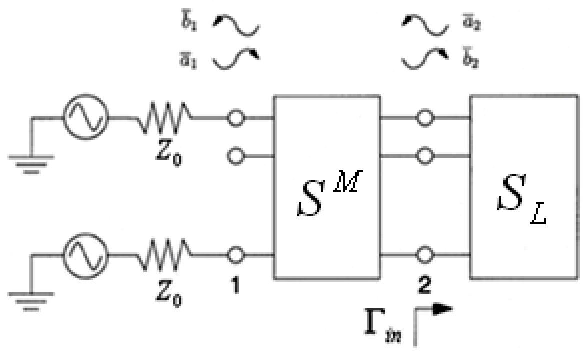


Figure 4.2 Set up for the load matching problem. To ensure maximum power transfer, we must find S^M such that $\Gamma_{in} = 0$.

The power delivered to the load will be maximized for any possible excitation (a_1) if and only if we can find S^M for a lossless network such that $\Gamma_{in} = 0$. We need to show that $S_{22}^{M} = S_{L}^{H}$ is sufficient to get $\Gamma_{in} = 0$ for arbitrary excitation. The below SVD representations for S_{22}^{M} and S_{L}^{H} are used for this purpose

$$S_{22}^{M} = S_{L}^{H} = \left(U_{L} \Sigma_{L}^{\frac{1}{2}} V_{L} \right)^{H} \tag{4.20}$$

$$S_{11}^{M} = U_{11} \Sigma_{11}^{\frac{1}{2}} V_{11}^{H} \tag{4.21}$$

Using the constrains in (4.15-4.18) coupled with (4.10) leads to

$$\Gamma_{in} = \left(U_{11} \Sigma_{11}^{\frac{1}{2}} + U_{12} * \left(I - \Sigma_{L} \right)^{-\frac{1}{2}} * \Sigma_{L}^{\frac{1}{2}} \Theta_{12} V_{L}^{H} U_{21} * \left(I - \Sigma_{11} \right)^{\frac{1}{2}} * \Theta_{21} \right) * V_{11}^{H} \tag{4.22}$$

(4.19) can be re-written for this problem as

$$\left(I - \Sigma_{L} \right)^{-\frac{1}{2}} * \Sigma_{L}^{\frac{1}{2}} \Theta_{12} V_{L}^{H} U_{21} * \left(I - \Sigma_{11} \right)^{\frac{1}{2}} * \Theta_{21} = -U_{12}^{H} U_{11} \Sigma_{11}^{\frac{1}{2}} \tag{4.23}$$

which upon substitution into (4.22) results in

$$\Gamma_{in} = \left(U_{11} \Sigma_{11} - U_{12} U_{12}^{H} U_{11} \Sigma_{11} \right) * V_{11}^{H} = 0 \tag{4.24}$$

Therefore, choosing $S_{22}^{M} = S_{L}^{H}$ is sufficient to ensure that $\Gamma_{in} = 0$ for arbitrary excitation. Next we show that $\Gamma_{in} = 0$ results in $S_{22}^{M} = S_{L}^{H}$, indicating that this later condition is also necessary for maximizing power transfer. Multiplying 4.10 by $\left(S_{12}^{M} \right)^{H}$ assuming $\Gamma_{in} = 0$ and making appropriate substitutions from the lossless conditions leads to

$$\left(-\left(S_{22}^{M} \right)^{H} + \left(I - \left(S_{22}^{M} \right)^{H} S_{22}^{M} \right) * S_{L} * \left(I - S_{22}^{M} S_{L} \right)^{-1} \right) * S_{21}^{M} = 0 \tag{4.25}$$

Let

$$M = \left(-\left(S_{22}^{M} \right)^{H} + \left(I - \left(S_{22}^{M} \right)^{H} S_{22}^{M} \right) * S_{L} * \left(I - S_{22}^{M} S_{L} \right)^{-1} \right) \Rightarrow M * S_{21}^{M} = 0 \tag{4.26}$$

Since for a lossless matching network S_{21}^{M} must be full rank [16], the above equation can be only satisfied if $M = 0$, which after simplification yields to $S_{22}^{M} = S_{L}^{H}$.

4.3.3.2 Source Match

We now consider the problem of matching the arbitrary coupled source element in Fig. 4.1 to a set of uncoupled identical loads of impedance Z_0. In this case $S_L = 0$ such that $a_2 = 0$ and $\Gamma_{in} = S_{11}^{M}$. If we collapse the matching network and load network into a single network block, then our equivalent network is in the form of the figure below

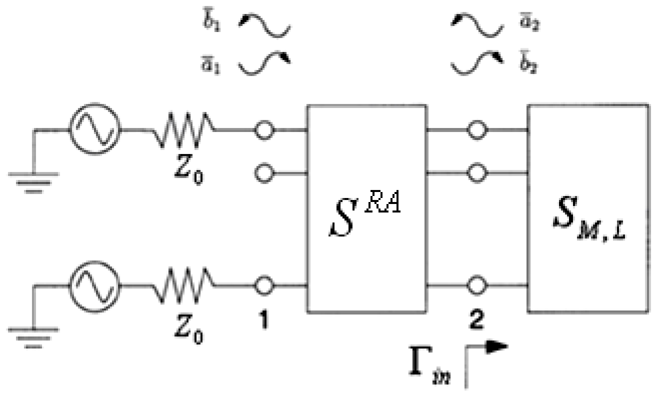


Figure 4.3 Set up for the source matching problem. To ensure maximum power transfer, S_{11}^{M} must equal to $\left(S_{22}^{RA}\right)^{H}$.

Based upon our work above and since the matching network is lossless, we know that all available power will be transferred to the loads if and only if $S_{11}^{M} = \left(S_{22}^{RA}\right)^{H}$ [20].

Knowing S_{11}^{M} and S_{22}^{M}, S_{12}^{M} and S_{21}^{M} can be obtained from (4.12) and (4.14) using singular value decomposition. There are infinitely many solution of S_{12}^{M} and S_{21}^{M} that satisfy (4.12) and (4.14). One possibility is to use the Cholesky factorization [21].

5 Implementation in Simulink

5.1 Simulink and RF Blockset

Simulink is a software for modelling, simulating, and analyzing dynamic systems. With Simulink, we can easily build models from scratch, or modify existing models to meet our needs. Simulink supports linear and nonlinear systems, modelled in continuous time, sampled time, or a hybrid of the two.

The MathWorks provides several additional products that extend the capabilities of Simulink among them are toolboxes for different fields. In our model we used the RF Blockset, which provides a library of blocks for modelling RF systems that include RF filters, transmission lines, amplifiers, and mixers[1]. RF Blockset is used to represent the components of our RF system in a Simulink model. The blockset provides several types of component representations using network parameters (S, Y, Z, ABCD, H, and T format), mathematical descriptions, and physical properties. With RF Blockset we can visualize the network parameters of the blocks using plots and Smith charts. RF Blockset can also be used in conjunction with Real-Time Workshop to automatically generate embeddable C code for real-time execution.

5.2 Introduction to our system model

Recall that from chapter 2 that the overall MIMO system looks like

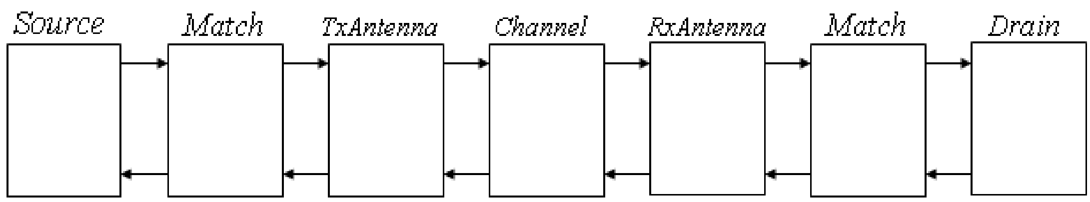


Figure 5.1 Complete RF transmission chain with matching networks inserted between the source and transmit antenna from one side and the receive antenna and the load from the other side.

The “Source” is composed of a series of components (mixer, amplifier, etc...) and is implemented in another master thesis work [1].

In the present master thesis, the implementation of the matching network and the antenna system are presented.

A mathematical model of the channel is also available. The receiving side (Rx Antenna, Match and Drain) is implemented similarly to the transmitting side.

Finally, the above three implementations can be put together to represent the overall system chain.

5.3 System Description

5.3.1 Introduction

Since we are representing the system components (antenna and matching network) by their scattering parameters, a physical model will be put into considered. The physical model is created in Simulink by using the RF Blockset. The blocks available in RF Blockset (S-parameters passive network) can only represent a 2-port network. Since the antenna network represents an array with two elements, we need a block that represents 4-port network.

The main RF blocks that are used in this implementation (input port, output port and S-parameters passive network) are described in Appendix A.

5.3.2 Two port network

A two port network is simply represented by the "S-parameter passive network block" in the RF Blockset. This block takes the scattering parameters of the two port network as an input and it calculates the overall transfer function from the input port (port 1) to the output port (port 2). Refer to Appendix A for more details about how this block functions. For more than two port network, no similar blocks are available and the representation of the network has to be done in a different way. This is done by describing each network graphically by its scattering parameters, and then use the "S-parameters passive network" to represent each of the scattering parameters. The procedure is illustrated in the next section.

5.3.3 Graphical representation of a two port network

For a two port network, the output travelling waves can be given in terms of the scattering parameters of the network and the input travelling waves as in (2.16), which can be represented graphically as shown in figure 5.2

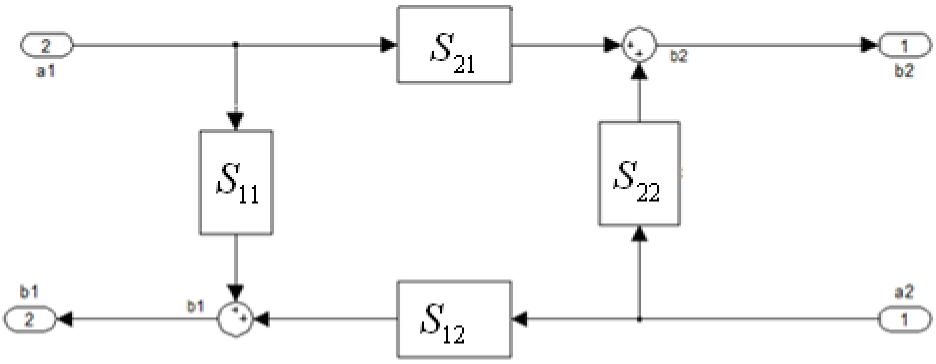


Figure 5.2 Graphical representation of the scattering matrices of a two port network.

where

S_{21} represents the response from point a1 to point b2 (transmission from port 1 to port 2).

S_{11} represents the response from point a1 to point b1 (the reflection from port 1).

S_{12} represents the response from point a2 to point b1 (transmission from port 2 to port1).

S_{22} represents the response from point b2 to point a2 (reflection from port 2).

Recall that from Appendix A that the transfer function of the "S-parameters passive network" is given, at a certain frequency f, by

$$H(f) = \frac{S_{21} * (1 + \Gamma_l) * (1 - \Gamma s)}{2 * (1 - S_{22} * \Gamma_l)(1 - \Gamma_{in} * \Gamma_s)} \tag{5.1}$$

where

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}$$

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$$

$$\Gamma_{in} = S_{11} + \left(S_{12} * S_{21} * \frac{\Gamma_l}{(1 - S_{22} * \Gamma_l)} \right)$$

If we set S_{11}, S_{12} and S_{22} equal to zero, knowing that $Z_L = Z_S = Z_0$, the transfer function of the "S-parameters passive network" will be given by

$$H(f) = \frac{S_{21}}{2} \tag{5.2}$$

Going back to figure 5.2, we can replace each of the boxes by an "S-parameters passive network" with $S_{11} = S_{12} = S_{22} = 0$ and $S_{21} = S_{ij}$. For example S_{11} can be replaced by an "S-parameter passive network" with the following description

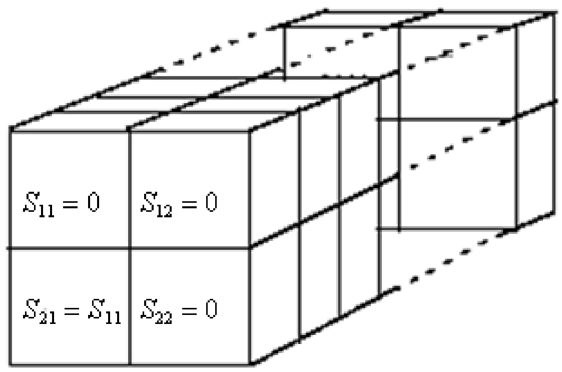


Figure 5.3 S-parameters array. All scattering parameters are zeros except for S_{21}.

An input port/output port is needed before/after the S-parameters passive network to serve as an interface between the mathematical and the physical models (see Appendix A). This interaction introduces a 6dB loss (see Appendix A), thus a gain of 2 is added before the input port to compensate for the loss.

The representation of S_{11} in terms of RF blocks is shown in the figure below

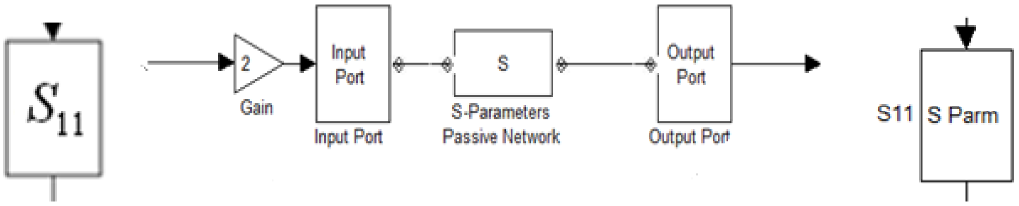


Figure 5.4 Representation of S_{11} by RF blocks.

The overall representation of a 2 port network will then look like

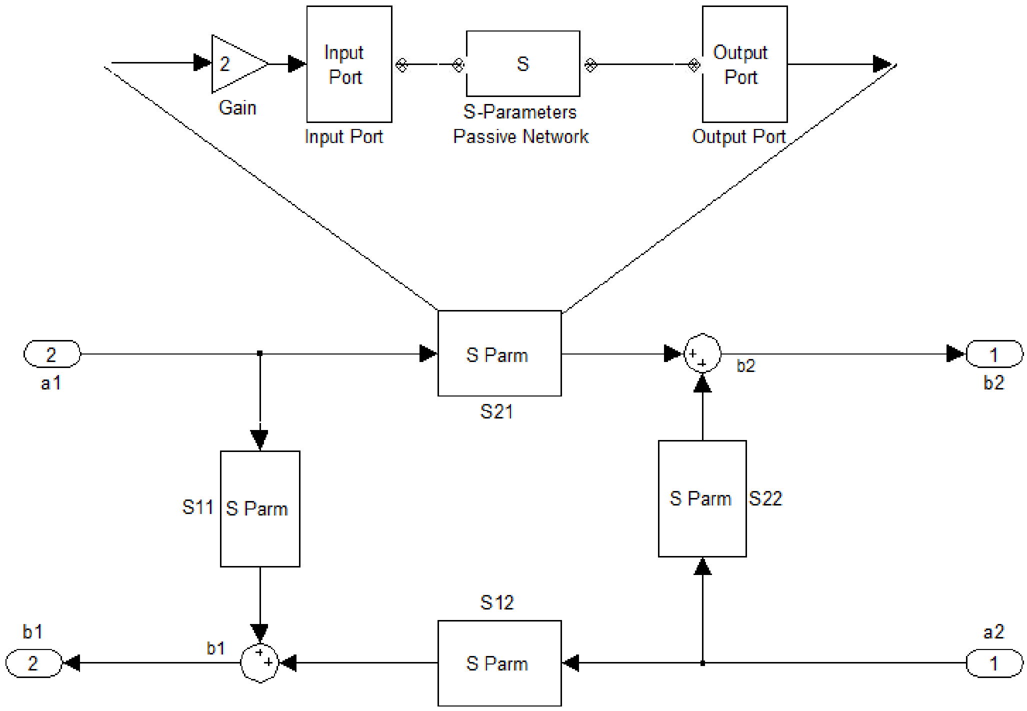


Figure 5.5 Two port network model using RF blocks.

While this representation could be replaced by an S-parameters passive network from the RF blockset, a 4-port network representation could not be replaced by just one block and the above method is used.

5.3.4 Graphical representation of a four port network

For a four port network, the output travelling waves can be given in terms of the scattering parameters of the network and the input travelling waves as

$$
\begin{aligned}
b_1 &= S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 \\
b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4 \\
b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \\
b_4 &= S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4
\end{aligned}
\tag{5.3}
$$

The above equations can be represented graphically, in a similar way as in the two port network; then each of the scattering parameters is replaced by its corresponding representation in terms of RF Blocks as shown in figure 5.4. The resulting 4-port network is shown in the figure below

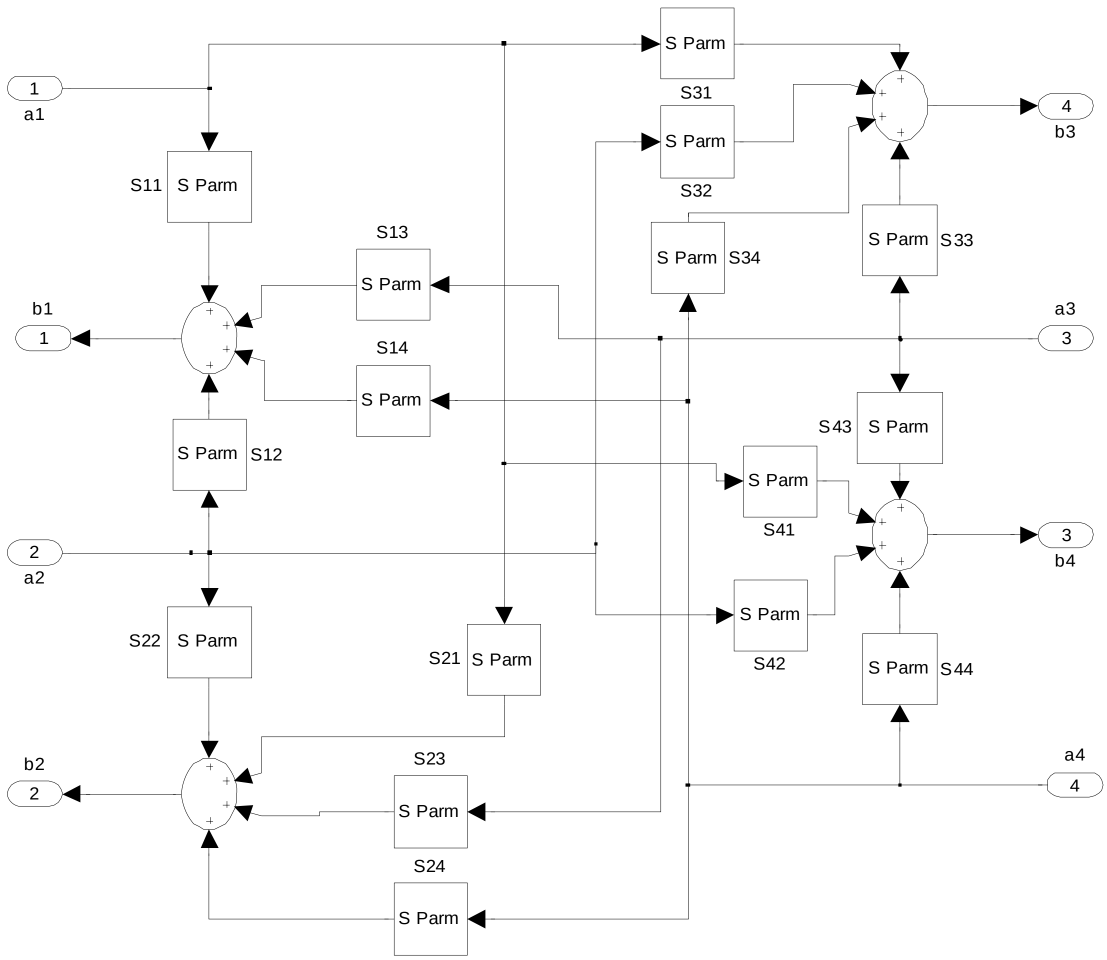


Figure 5.6 Four port network model using RF blocks.

The above figure is a bit complicated, and if we go for 6 or 8 port networks, it will be a complete mess. However, there is an option in Simulink where we can create a subsystem that hides the complication of the original system. Then the subsystem could be masked so that the user can define all the variables of the subsystem from outside without going through the individual blocks in the subsystem [22]. After creating a subsystem for the above 4-port network, the user will see the box in figure 5.7.

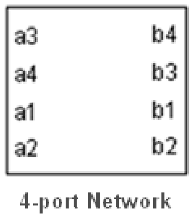


Figure 5.7 Sub-system representing the four port network model.

If you double click on the figure above, a dialog Box will open where you can set all the variables in the subsystem. The box is shown in the figure below

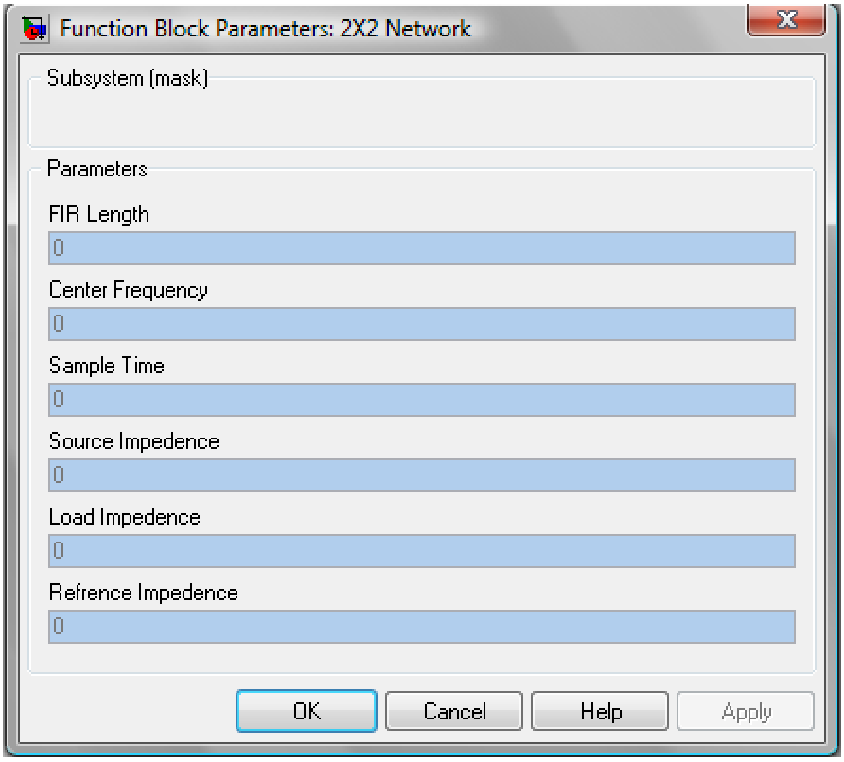


Figure 5.8 Function block parameters of a 4 port network.

Each of the terms in the above dialog box is described below

FIR length

Desired length of the baseband-equivalent impulse response for the physical models.

Center frequency(Hz)

Center of the modeling frequencies.

Sample time(s)

Time interval between consecutive samples of the input signal.

Source impedance (Ohms)

Source impedance of the RF network described in the physical model to which it is connected.

Load Impedance (Ohms)

Load impedance of the RF network described in the physical model to which it is connected.

Reference impedance

Reference impedance of the network.

The above 4-port network can easily be modified to fit our requirements. In the upcoming sections we will see how we use this 4-port network to represent the antenna and the matching network.

5.4 Representing Tx Antenna by a 4-port network

The representation of a Tx antenna in terms of its scattering parameters is given in chapter 2 section 2.3.2 as

$$S_A^{Tx} = \begin{bmatrix} S_{11}^{TA} & S_{12}^{TA} \\ S_{21}^{TA} & S_{22}^{TA} \end{bmatrix} \tag{5.4}$$

For the case of 2 antenna array, the antenna system is a 4-port network. Each of the above scattering matrices is a square matrix. As we mentioned earlier, we are interested in the self and mutual coupling effects (represented by S_{11}^{TA}). The effects of S_{12}^{TA} and S_{21}^{TA} are implicitly contained in the channel model. Thus, the 4-port network antenna system is represented as

$$\begin{pmatrix} \begin{bmatrix} S11 & S12 \\ S21 & S22 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{pmatrix} \tag{5.5}$$

Notice that we have a lot of zeros scattering parameters. Using the 4-port network created in the previous section is a possibility, but an error message (dividing by zero) will appear, and this will result in the block not functioning properly. To get rid of the error message, a special 4-port network is built for the 2 antenna array which removes all the RF blocks that have all zero scattering parameters.

The resulting 4-port antenna is depicted in the figure below

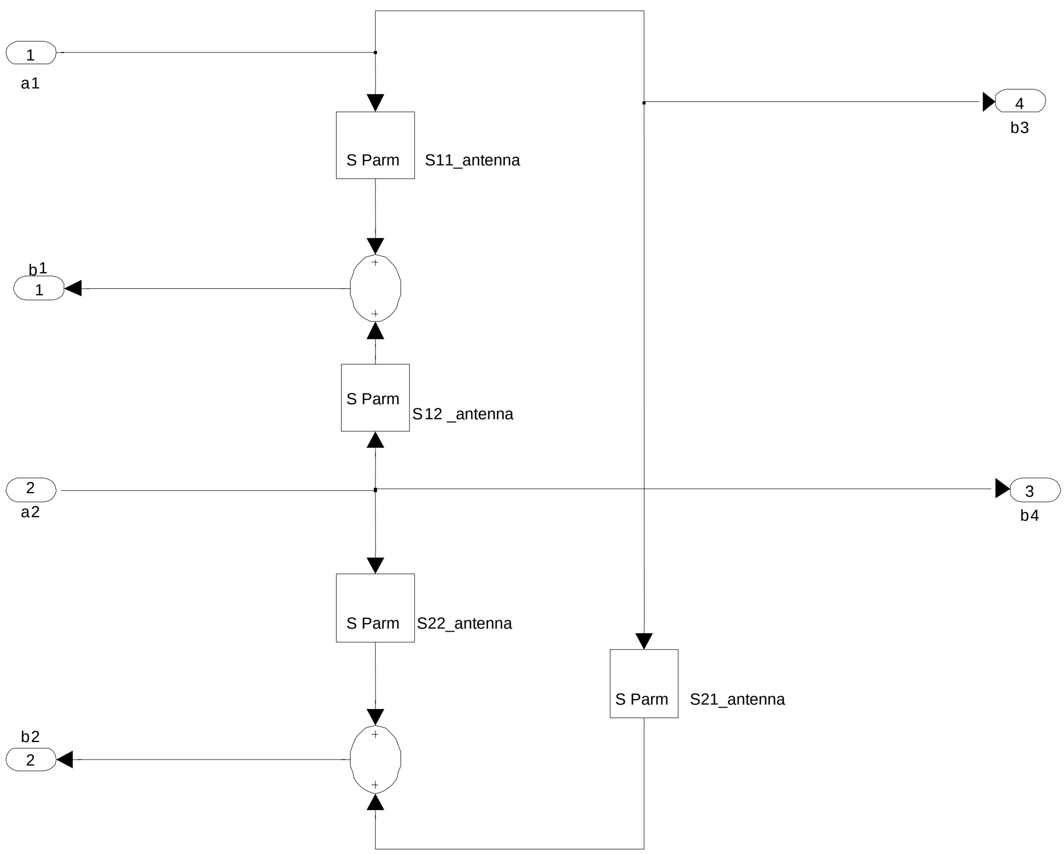


Figure 5.9 4-port antenna network.

5.5 Representing Matching Network with RF Blockset

We have studied different types of matching networks in chapter 4. The network of interest is the one called "Hermitian match". We have seen how the scattering parameters of the Hermitian matching network can be derived from the scattering parameters of the adjacent networks (amplifier and antenna). Knowing the scattering parameters of the matching network, we can represent it by a 4-port network as shown in the figure below

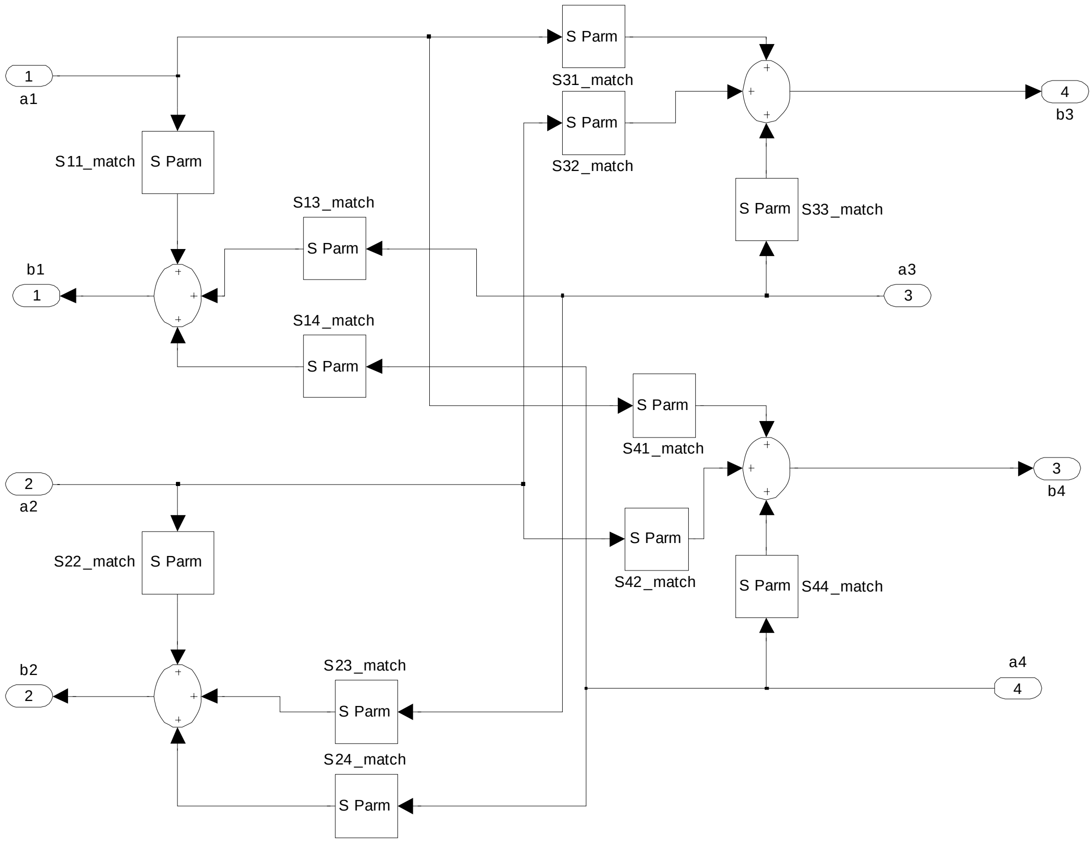


Figure 5.10 4-Port Hermitian Matching Network.

It is noteworthy to mention that there is an ambiguity in finding the scattering parameters of the "Hermitian matching network", namely S_{21} and S_{12} where infinite number of solutions are possible. However, if we consider no interaction between the ports at the two sides of the matching network, which means that we conjugate match each port individually, then S_{21} and S_{12} will equal to zero. Further, S_{11} and S_{22} are determined in the same way as in chapter 4 section 4.3.3. The corresponding matching network is depicted in figure 5.11.

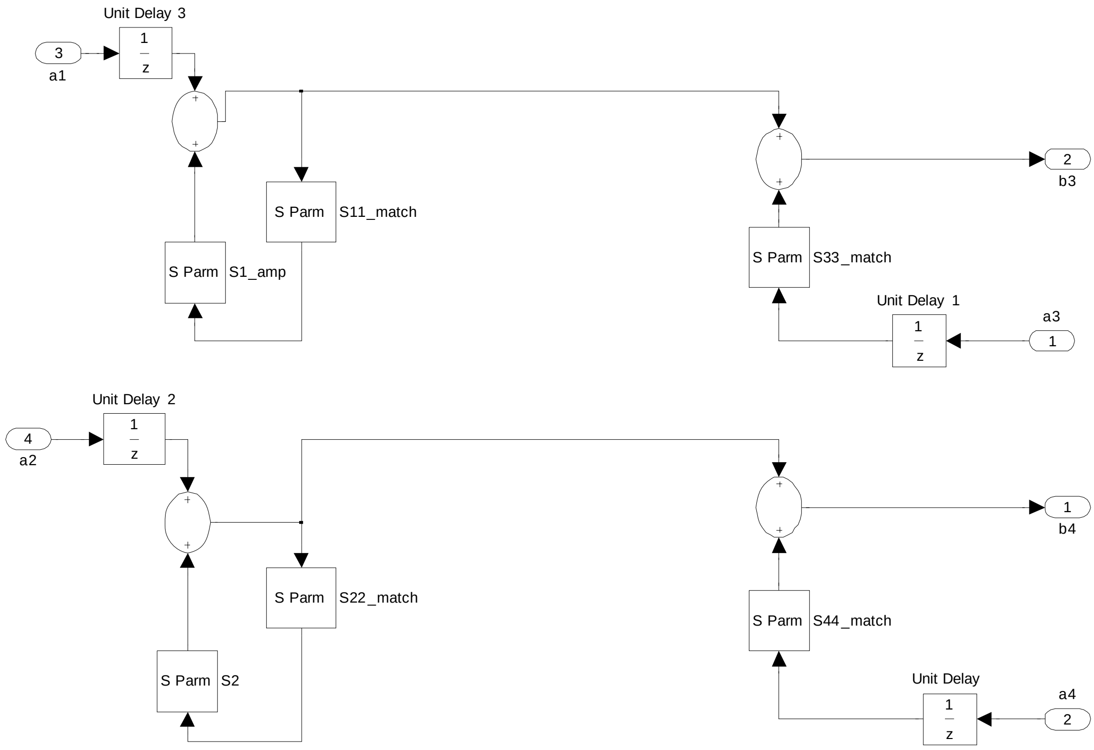


Figure 5.11 4-Port Matching Network (Individual Port Match).

If we take into account that the amplifier is already matched, S_{11} of the matching network is not needed anymore, and we remain with S_{22} of the matching network. This is equivalent to applying the input impedance match which only takes into account the self and mutual coupling effects. The corresponding "matching network" is depicted in figure 5.12.

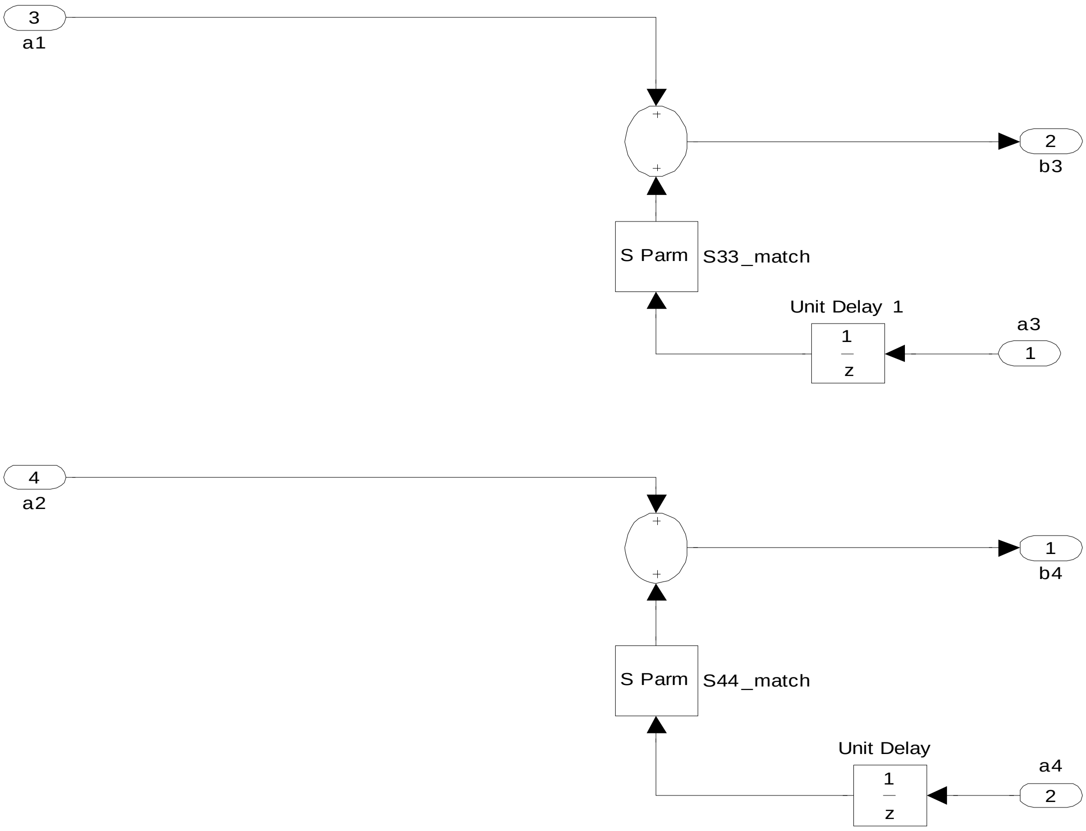


Figure 5.12 Input Impedance Match.

5.6 Combining Antennas and Matching Network

We have described the antenna and the matching network models in the above sections. These models are masked and connected to each other as shown in the figure below

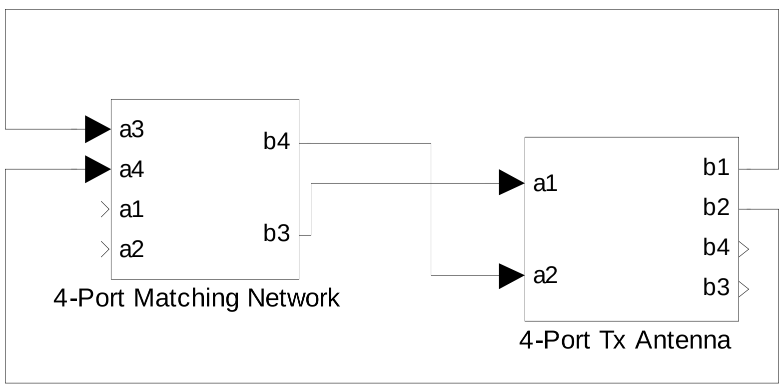


Figure 5.13 Connections between the antenna and the matching network models.

The ports, a1 and a2 of the " 4-Port Matching Network" are the input ports through which the signals from the source is fed to the two transmitting antennas. Further, b4 and b3 of the "4-Port Tx Antenna" are the output signals from the two transmitting antennas to be passed through the channel whereas, b1 and b2 of the (4-port Matching Network) are the reflected signals from the matching network to the amplifier.

If we look under the mask we can see in more detail how all the ports of the two networks are connected. Figure 5.14 shows the connections for the Hermitian match where all the scattering parameters of the network are included. Figure 5.15 shows the connections when we consider no interaction between the ports at the two sides of the matching network. And finally, figure 5.16 shows the connections when the amplifier is already matched.

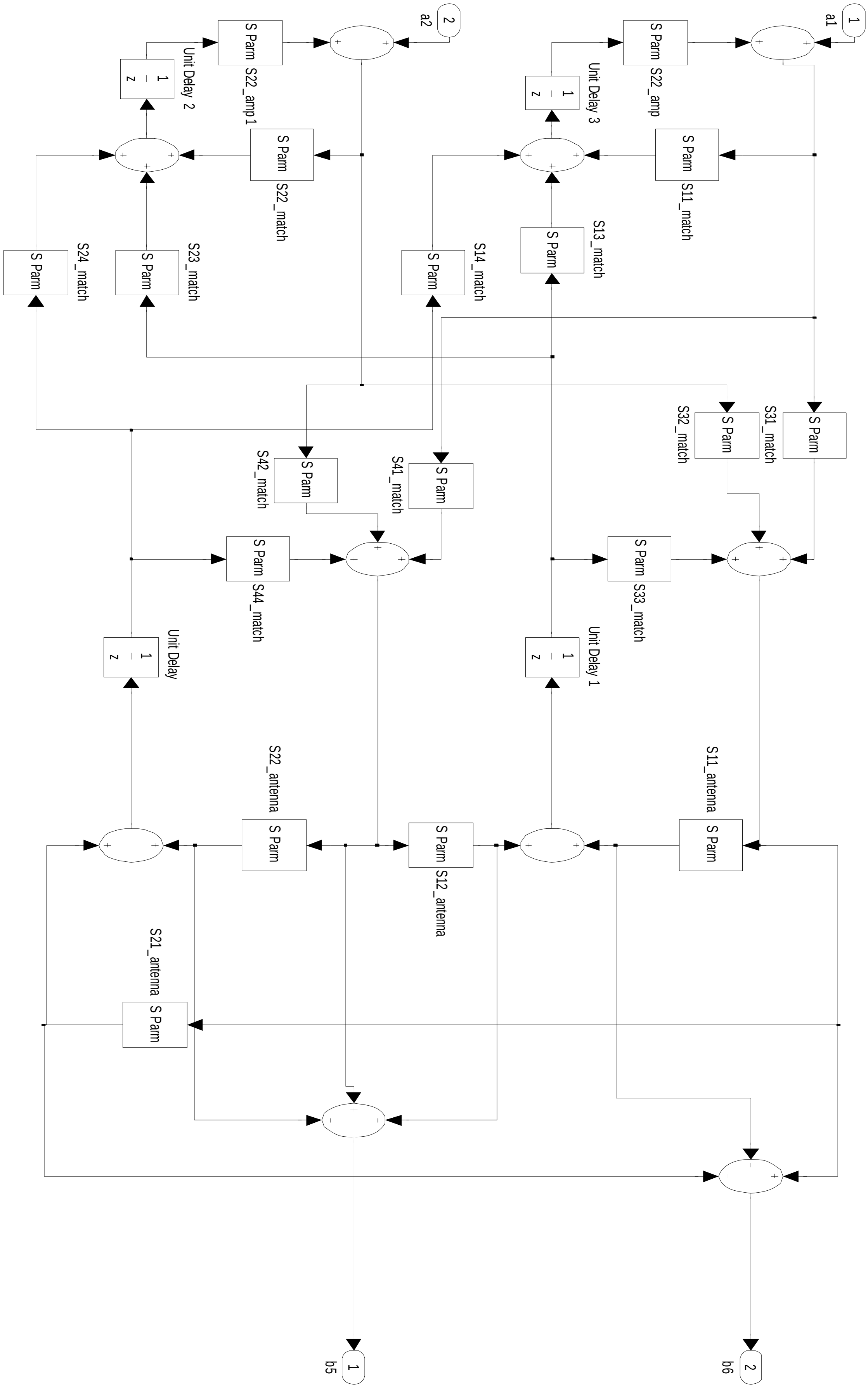


Figure 5.14 Flow chart for the case of a 4-port Hermitian match

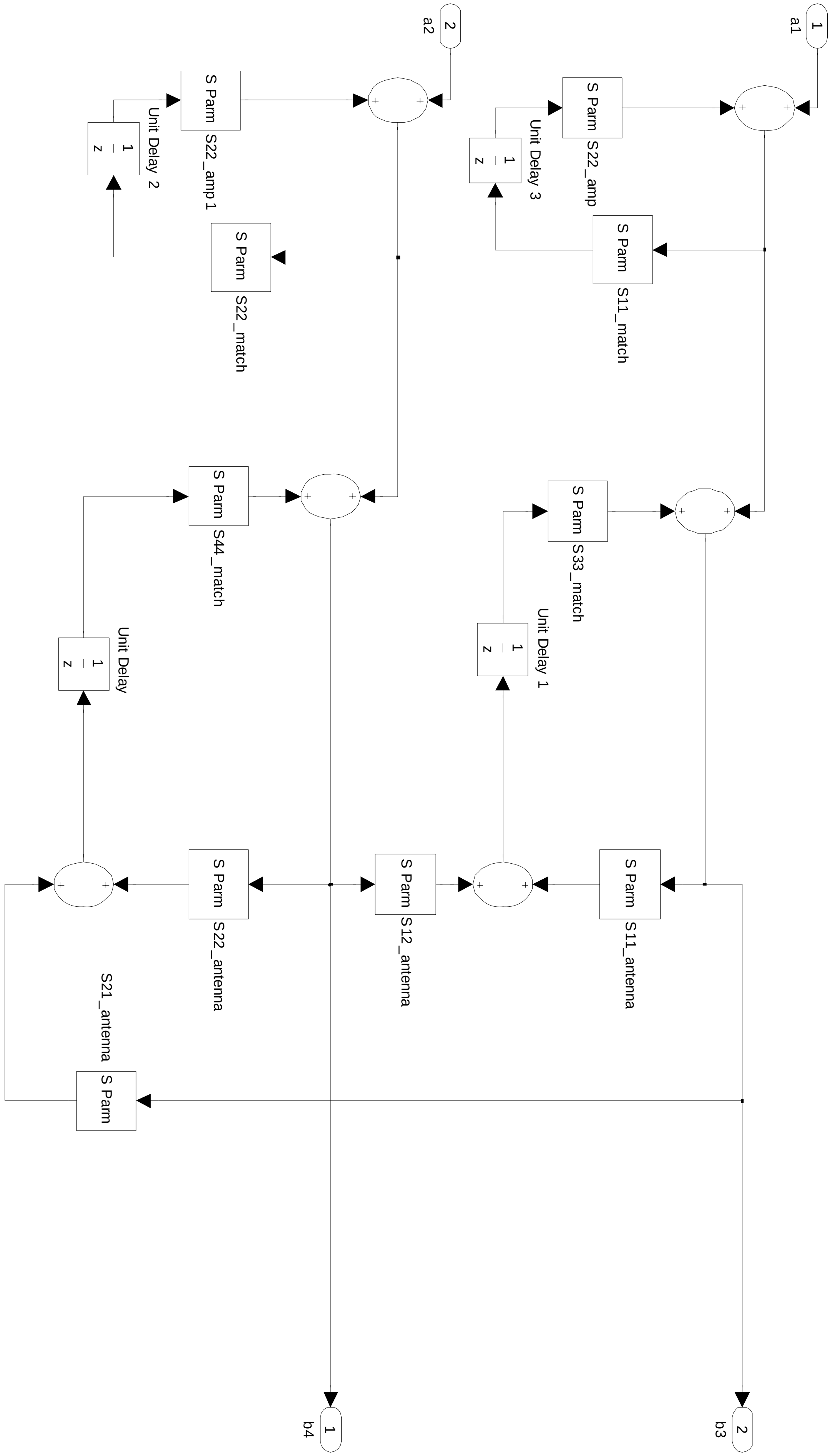


Figure 5.15 Flow chart for the case of individual port match.

The block S22_amp in the above two figures represent the reflection coefficient of the amplifier. We mentioned earlier that this system will be connected to the front end (source) model from the master thesis in [1]. In their model, they consider no backward transmission (reflection). Thus, there is only one input line at the interface with the matching network. However, in our model there are two lines for each port (input to the port, reflection from the port). In the above model, the input port a1 takes the signal from the first amplifier while the reflected signal is fed through the block that represents the reflection coefficient of the amplifier to be added to the input signal. In this way we considered the reflection from the amplifier. The input port a2 takes the signal from the second amplifier.

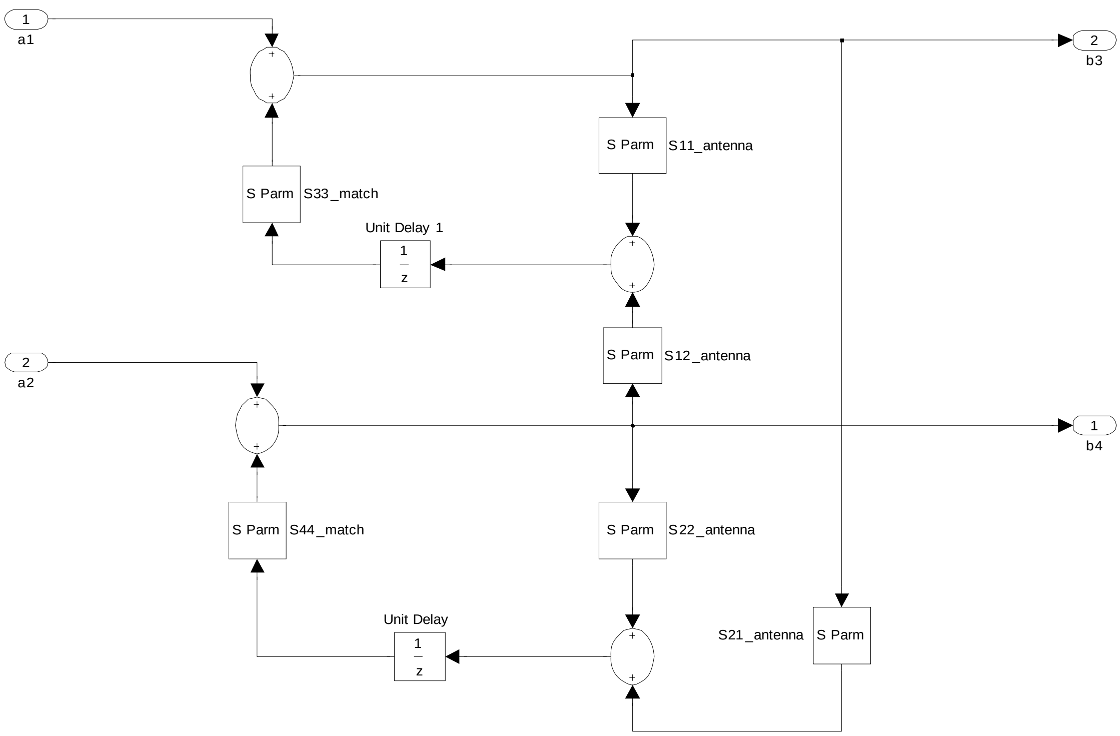


Figure 5.16 Flow chart of the individual port match taking into account that the amplifier is already matched.

In all the above figures, the unit delay box is used to avoid algebraic loops. Refer to Appendix B to learn more about algebraic loops.

5.7 Data initialization

Recall that from section 5.2.4 after creating the subsystem, a box is available where all the variables of the subsystem (center frequency, sampling time …) can be defined. What remains is how to define the scattering parameters of each block. This is done by a Matlab file that takes the scattering parameters of the antenna system (Touchstone s2p format) and the reflection coefficient of the amplifier (S22_amp) as an input, and then it determines the scattering parameters of the corresponding matching network. The Matlab file also changes the format of the scattering parameters from Touchstone s2p to the format that is suitable to be used by the RF blocks. After running the Matlab file, all the scattering parameters in the Simulink boxes will be defined and the user can run the simulation to get the results.

5.8 Results

To test our system, the below configuration is used

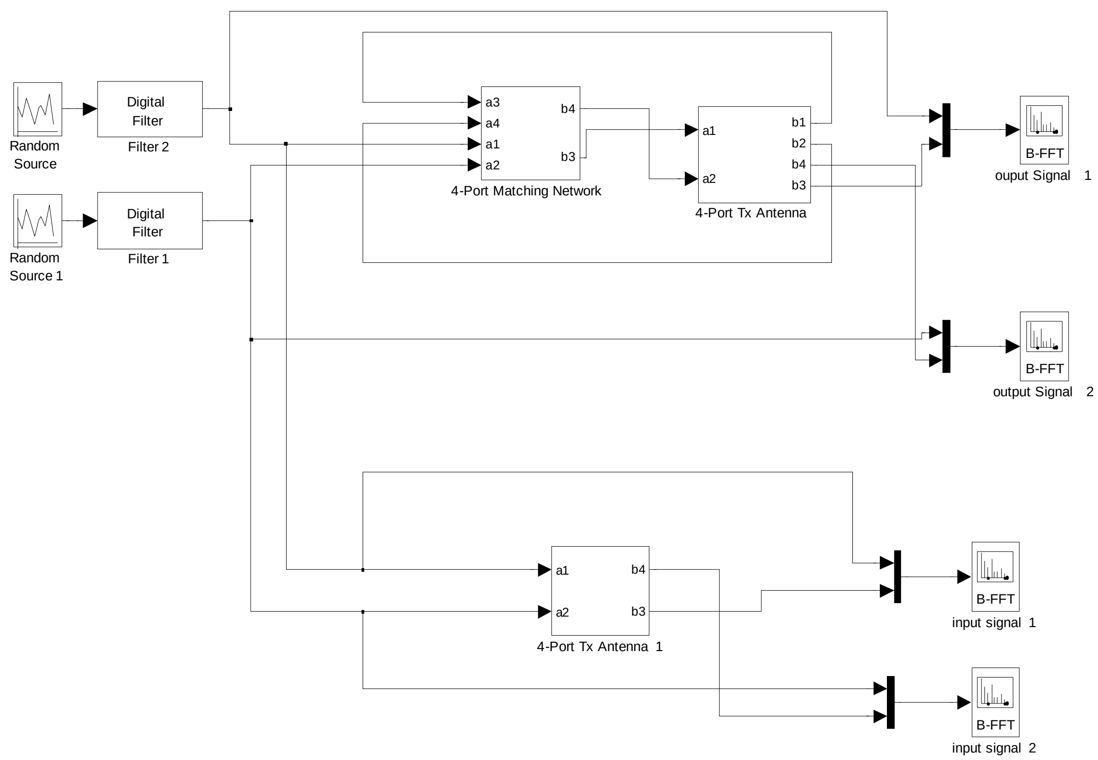


Figure 5.17 Simulation Setup.

The main model is depicted in the upper part of figure 5.17, where we have the matching network connected to the antenna. To see the effect of mutual coupling on the antennas, we created another model where we didn't include the matching network as can be seen in the lower part of figure 5.17. The two systems are driven by a random source connected to a Digital filter which makes the signal band limited. The Spectrum Scope (B-FFT) is used to plot the spectrum of input and output signals verses frequency for each of the two antennas.

The antennas system under test is a two different dipole antenna array that has the following configuration
Length of antenna one: 0.3 λ
Length of antenna two: 0.5 λ
Distance between antennas: 0.4 λ
Radius of the antenna: 0.003 λ
The scattering parameters corresponding to this antenna array are shown in the figure below

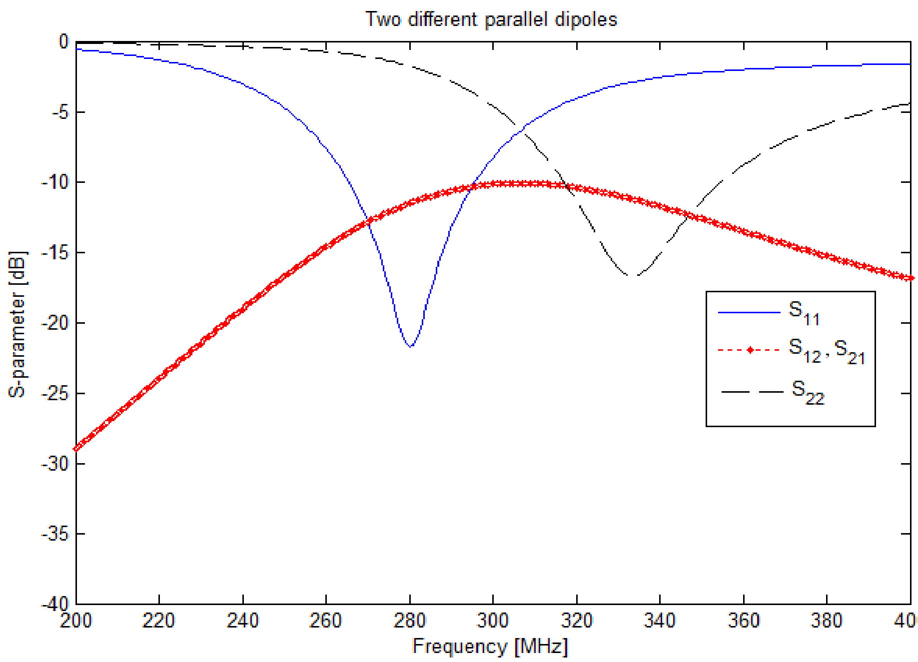


Figure 5.18 S-Parameters plots for 2 Different Length Parallel Dipoles.

Since our focus in this master thesis is the effect of the matching network on the self and mutual coupling of antenna arrays, we used the configuration shown in figure 5.12 for the matching network. The scattering parameters of the matching network are derived from the scattering parameters of the two antennas.

For pedagogical reasons, we feed the two antennas by two signals that operate at different frequencies. The spectra of the two input signals are shown in the two figures below

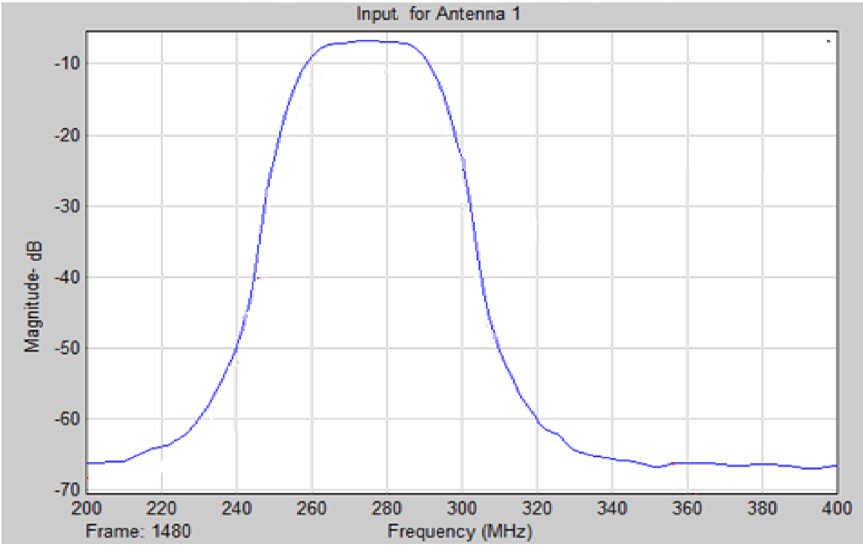


Figure 5.19(a)

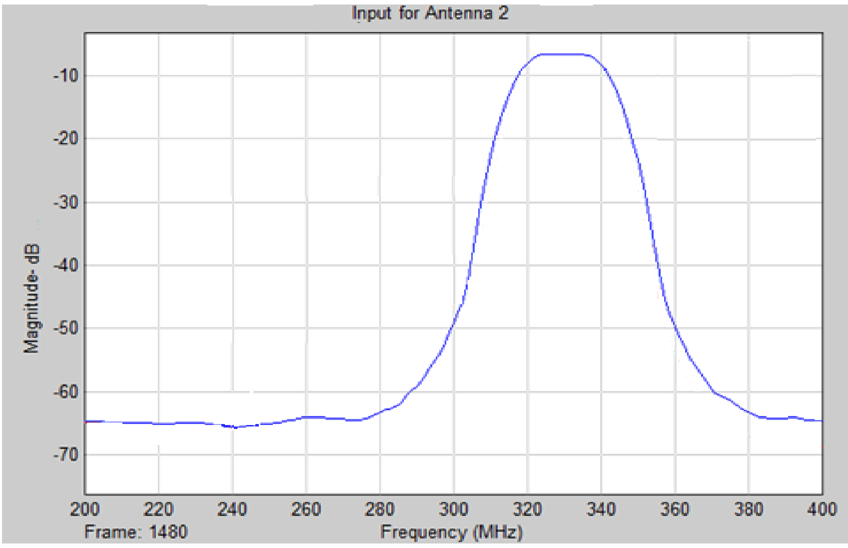


Figure 5.19 (b)

Figure 5.19 Input signal spectra (a) for first antenna and (b) for second antenna.

To see the effect of mutual coupling we have used the lower part of Fig. 5.17 where the spectra of the output signals for each of the two antennas are plotted verses the spectra of the input signals and the results are shown in figures 5.20 (a) and 5.20 (b)

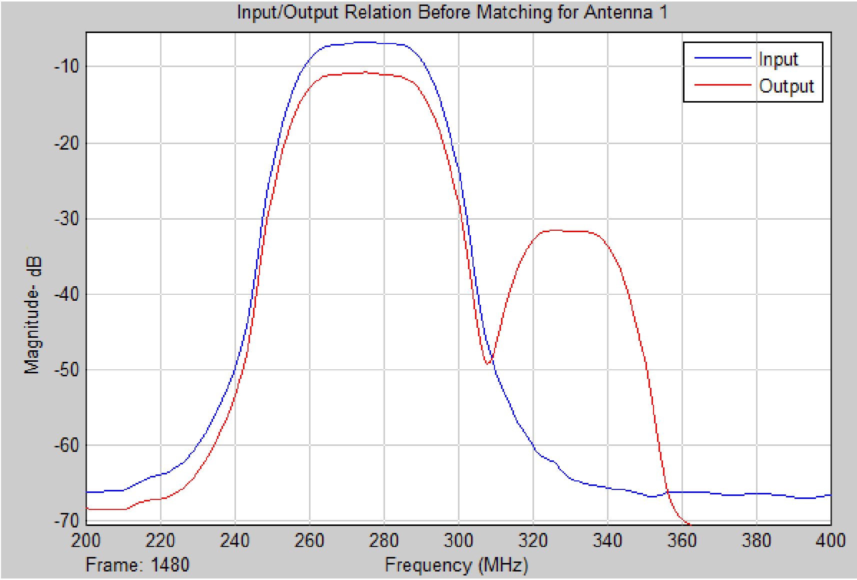


Figure 5.20 (a)

From Figure 5.20 (a), we see that the output signal is composed of two parts. The first part shows the effect of the self impedance on the input signal where we see a drop of around 5 dB. The second part represents the mutual coupling from the second antenna. The same effects can be seen in the output signal of the second antenna.

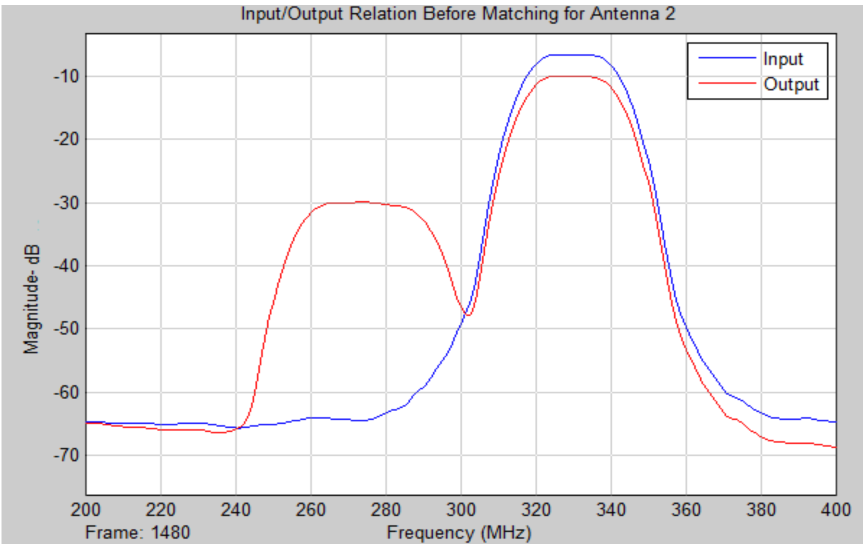


Figure 5.20 (b)

Figure 5.20 Coupling and Reflections effects (a) for First Antenna and (b) for Second Antenna.

Figures 5.21 (a) and 5.21 (b) below show the output signals after connecting the matching network to the transmit antennas

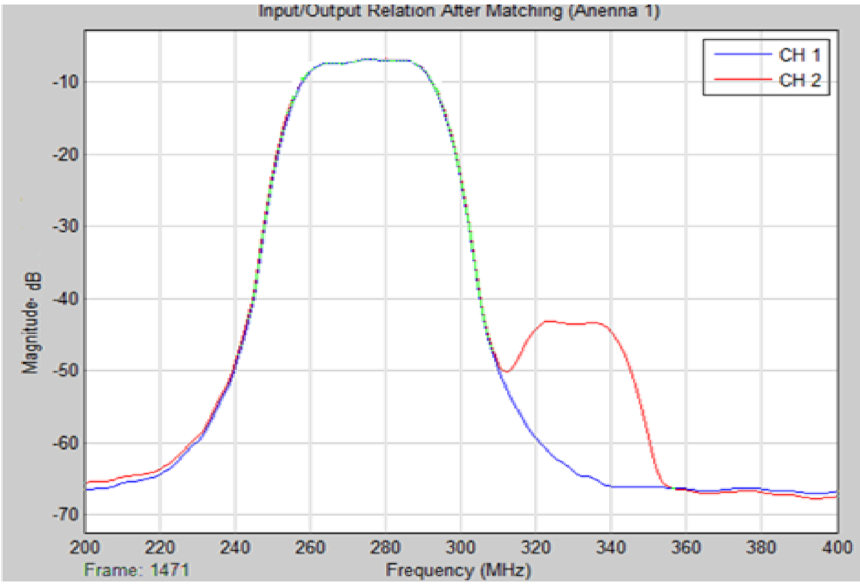


Figure 5.21 (a)

If we compare Fig. 5.20 (a) with Fig. 5.21 (a), we can see that the matching network has reduced the effect of the self impedance, and also significantly reduced the effect of the mutual coupling from the second antenna on the first antenna. The same results are obtained for the second antenna, and are shown in the figure below

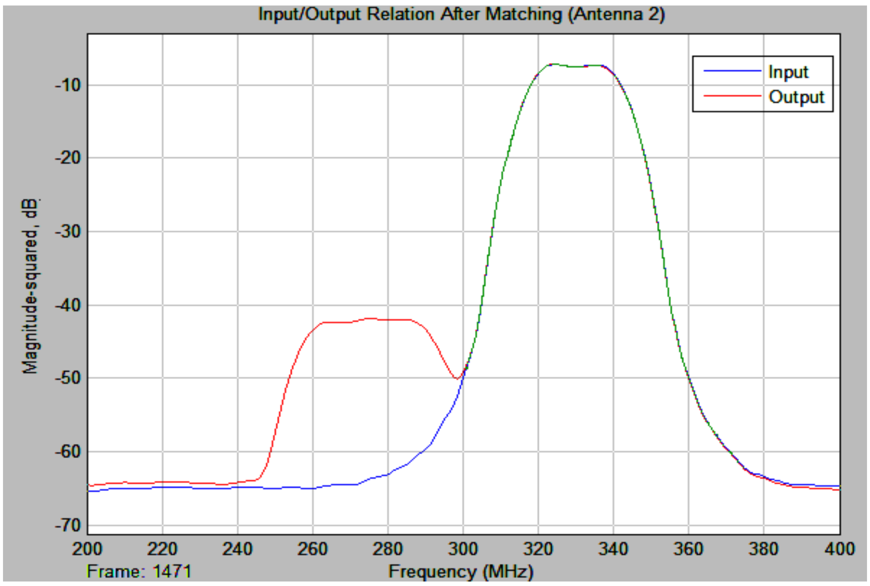


Figure 5.21 (b)

Figure 5.21 Reduction in Coupling and Reflections effects (a) for First Antenna and (b) for Second Antenna

6 Conclusion and future work

In this master thesis, we have shown how to model physical MIMO antennas with matching network connected to the source (RF frontend) from one side and to the antenna array from the other side. We have presented a way to implement this in simulink using the RF Blockset where the scattering parameters representation of a network is considered. The resulting model was used to check the effect of the matching network on the mutual coupling and antenna efficiency. The results show that the matching network has significantly reduced the effect of mutual coupling and thereby increased the antenna efficiency.
The lack of a more than 2-port network block in the RF Blockset imposes limitations for this kind of applications where the networks are represented in terms of their scattering parameters. Due to this limitation, the proposed approach to implement the matching network and antenna model is impractical for MIMO systems bigger than 2x2. For longer systems it is better to compute the resulting scattering parameters in impeded Matlab file, and then implement the all system using just one compound block in Simulink.

As for the future work, the whole system (Source (RF frontend), transmit antenna, channel, receive antenna and Drain) has to be put together. The design of a “full conjuage match” has to be clarified, implemented and tested in Simulink. It is also proposed to implement an interface in Matlab that can handle bigger MIMO systems.

Appendix A

A-1 RF blocks used

A-1.1 S-Parameters Passive Network

The S-parameter Network models a two port passive network in terms of its S-parameters, frequencies and impedances of these S-parameters.
The S-parameters are given in the form of a 2x2xM matrix, where M represents the number of frequencies for which measurements are taken. The following figure shows the correspondence between the S-parameters array and the vector of frequencies

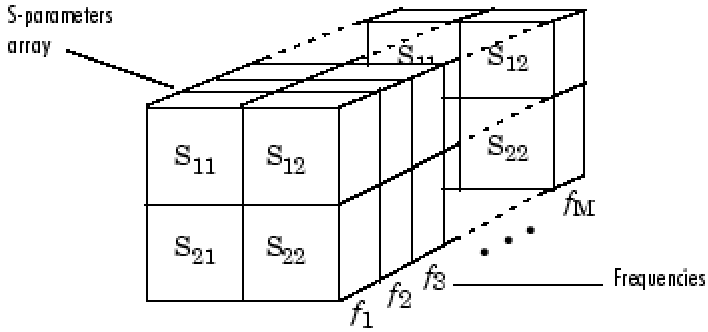


Figure A.1 Correspondence between the S-parameters array and the vector of frequencies.

Each of the terms in the dialog box is described below

S-Parameters

S-parameters for a two-port passive network in a 2-by-2-by-M array. M is the number of frequencies.

Frequency (Hz)

Frequencies of the S-parameters as an M-element vector. All frequencies must be positive.

Reference impedance (Ohms)

Reference impedance of the network.

Interpolation method

The method used to interpolate the network parameters. Table A.1 lists the available methods and the description of each one

Method	Description
Linear (default)	*Linear Interpolation*
Spline	*Cubic Spline Interpolation*
Cubic	*Piecewise Cubic Hermite Interpolation*

Table A.1

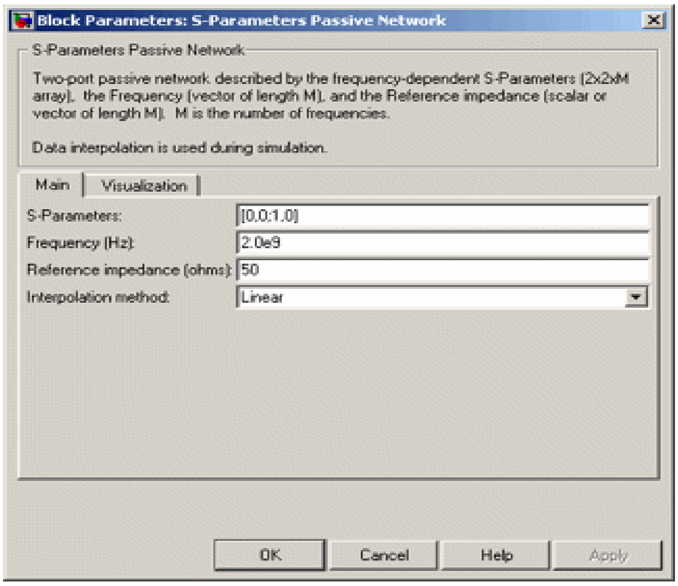


Figure A.2 Dialog Box for S-Parameter Passive Network.

Visualization Tab

It is used to plot Magnitude, Angle, Real or Imaginary parts of the S-Parameters.

A-1.2 Input Port

The input port is used as a connection port from Simulink environment to RF physical blocks. It also calculates modelling frequencies and baseband-equivalent impulse response for the physical subsystem. The input port has the following parameters

FIR length

Desired length of the baseband-equivalent impulse response.

Centre frequency(Hz)

Centre of the modelling frequencies.

Sample time(s)

Time interval between consecutive samples of the input signal.

Source impedance (Ohms)

Source impedance of the RF network described in the physical model to which it is connected.

Add noise

If this parameter is selected, noise data in the RF physical is considered.

Initial seed

You specify initial seed value for random number generator.

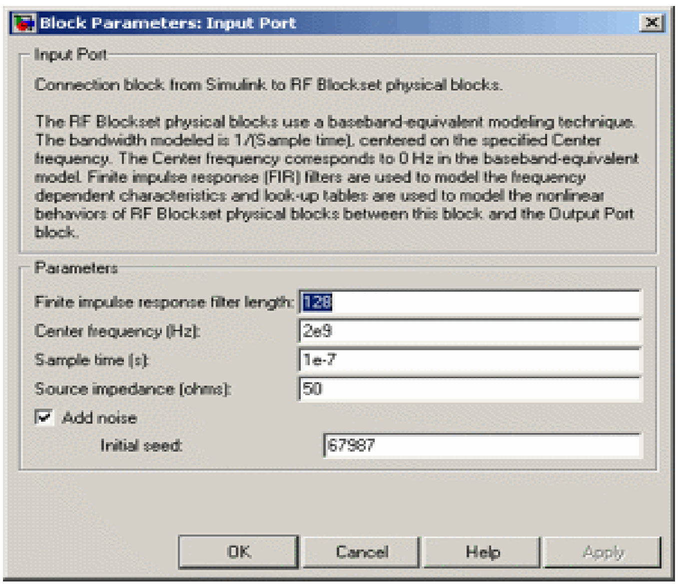


Figure A.3 Dialog Box for Input Port.

A-1.3 Output Port

The Output Port block produces the baseband-equivalent time-domain response of an input signal travelling through a series of RF physical components. It performs the following functions

a. Partitioning of RF physical components into linear and nonlinear subsystems.

b. Extracts the complex impulse response of the linear subsystem for baseband-equivalent modelling of the RF linear system.

c. Extracts the nonlinear AMAM/AMPM modelling for the RF nonlinearity.

A-1.3.1 Linear Subsystem

For a linear subsystem, the output port uses input port parameters and interpolated S-parameters calculated by physical blocks to find the baseband-equivalent impulse response. Specifically, it

a. Determines the modelling frequencies f , which is a function of the centre frequency *fc*, the sample time *ts*, and the finite impulse response filter length *N*, which are given as Input–port parameters.

   The *n*th element of *f*, *fn*, is given by

$$f_n = f_{\min} + \frac{n-1}{t_s N}, n = 1, \ldots, N \tag{A-1}$$

   Where

$$f_{\min} = f_c - \frac{1}{2t_s}$$

b. Calculates the passband transfer function for the frequency range as

$$H(f) = \frac{V_L(f)}{V_S(f)}$$

   Where V_S and V_L are the source and load voltages and f represents the modelling frequencies. More specifically

$$H(f) = \frac{S_{21} * (1 + \Gamma_l) * (1 - \Gamma s)}{2 * (1 - S_{22} * \Gamma_l)(1 - \Gamma_{in} * \Gamma_s)} \tag{A-2}$$

where

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}$$

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$$

$$\Gamma_{in} = S_{11} + \left(S_{12} * S_{21} * \frac{\Gamma_l}{(1 - S_{22} * \Gamma_l)} \right)$$

and

- Z_S, Z_L are source and load impedances.
- *Sij* are the S-parameters of a two-port network.

RF Blockset finds the passband transfer function from the Input Port parameters as shown in the following figure

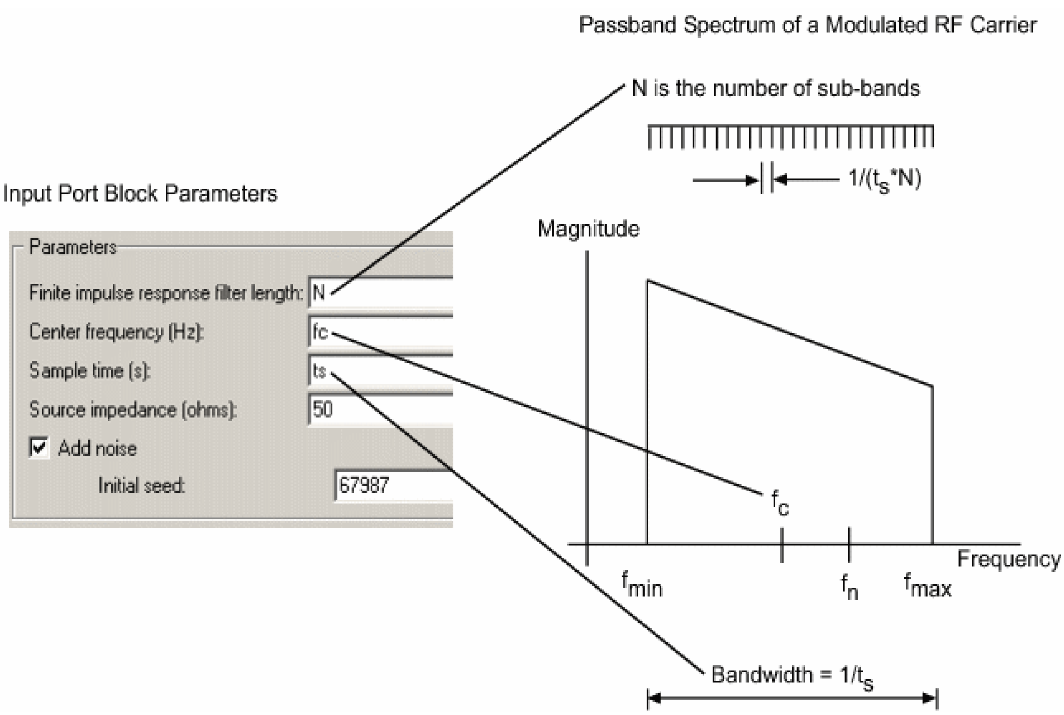


Figure A.4 Representation of Input Port Parameters.

c. Translates the passband transfer function to baseband as $H(f - f_c)$, The baseband transfer function is shown in the following figure

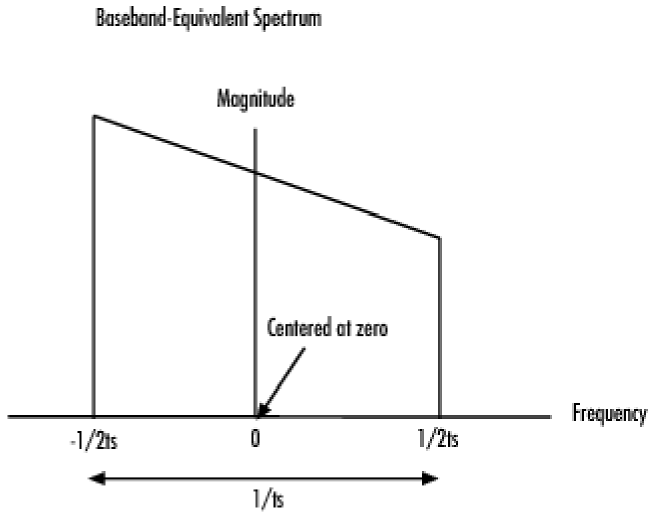


Figure A.5 Representation of Input Port Parameters.

d. Obtains the baseband-equivalent impulse response by calculating the inverse FFT of the baseband transfer function.

Each of the terms in the dialog box is described below

Load Impedance (Ohms)

Load impedance of the RF network described in the physical model to which it is connected.

Visualization Tab

This tab shows parameters for creating plots if you display the Output Port mask after you perform one or more of the following actions

- Run a model with two or more blocks between the Input Port block and the Output Port block.
- Click the Update Diagram button to initialize a model with two or more blocks between the Input Port block and the Output Port block.

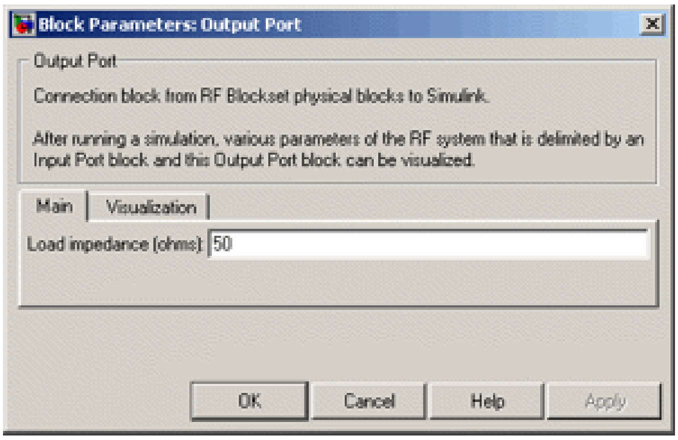


Figure A.6 Dialog Box for Output Port.

A-2 Converting to and from Simulink Signals

When we simulate an RF model, RF Blockset must convert the mathematical Simulink signal to and from the RF Blockset physical modelling environment. To do this, RF Blockset interprets the Simulink signal that enters the Input Port block, *S*in, as the voltage V_S across the source impedance Z_S .

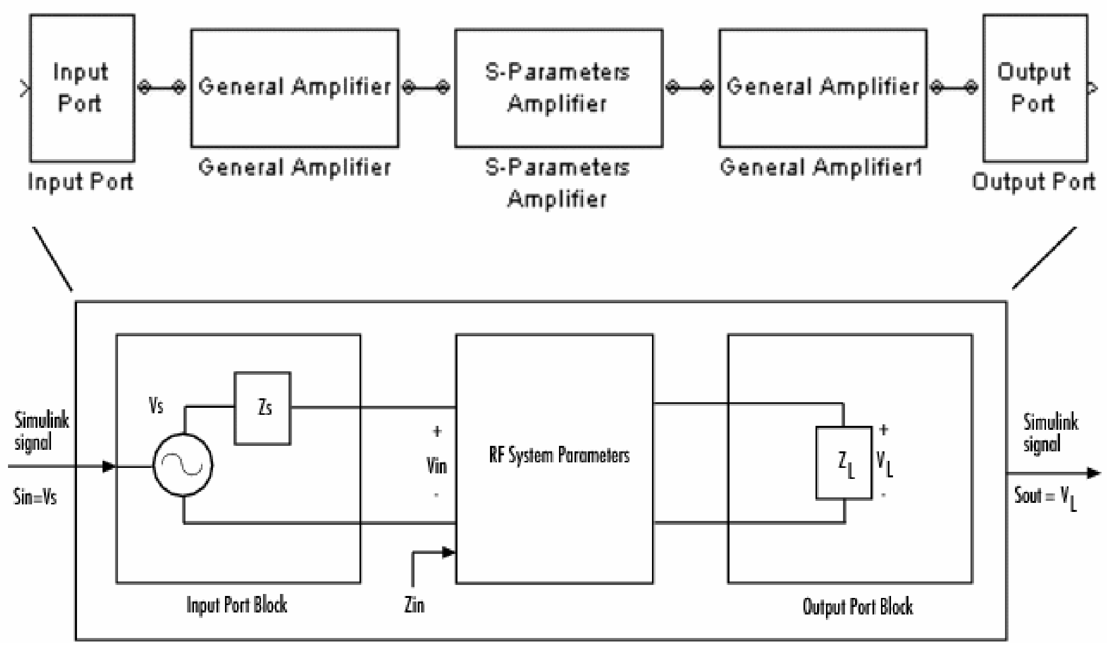
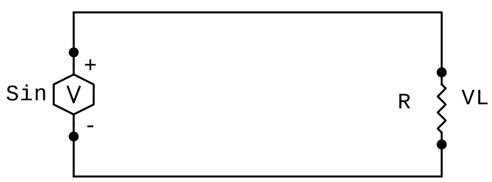


Figure A.7 RF Blockset interpretation of Input and Output signals. The input signal is represented as a voltage across the source impedance. The output signal is represented as a voltage over the load Impedance.

RF Blockset interprets the output Simulink signal as the voltage V_L over the load impedance Z_L . The RF Blockset interpretation of the input Simulink signal as the source voltage, V_S , produces different results than the interpretation that the input Simulink signal is the input voltage, Vin. When the source and load impedances are the same, the former interpretation produces 6 dB of loss compared to the latter.

The following figure shows the equivalent circuit model when the input Simulink signal, S_{in} , is taken to be the input voltage shown in the previous diagram



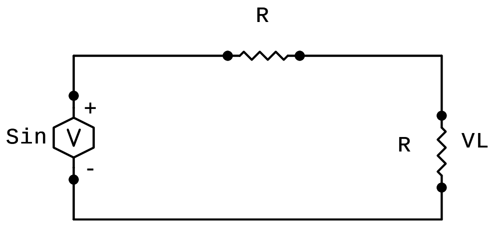
The load power is

$$P_L = \frac{V_L^2}{R} = \frac{S_{in}^2}{R} \tag{A-3}$$

In decibels, the load power is

$$10\log(P_L) = 10\log\left(\frac{S_{in}^2}{R}\right) \tag{A-4}$$

The following figure shows the equivalent circuit model when the input Simulink signal, Sin, is taken to be the source voltage shown in the previous diagram



The load power is

$$P_L = \frac{V_L^2}{R} = \frac{\left(S_{in}/2\right)^2}{R} \tag{A-5}$$

In decibels, the load power is

$$10\log(P_L) = 10\log\left(\frac{S_{in}^2}{4R}\right) = 10\log\left(\frac{S_{in}^2}{R}\right) - 6.02 \tag{A-6}$$

Appendix B

B-1 Algebraic Loops

Some blocks have direct feedback, meaning that the output cannot be computed without knowing the values entering the block at input port. When an input port with direct feedback is driven by the output of the same block or by a feedback path from other blocks algebraic loops occur.

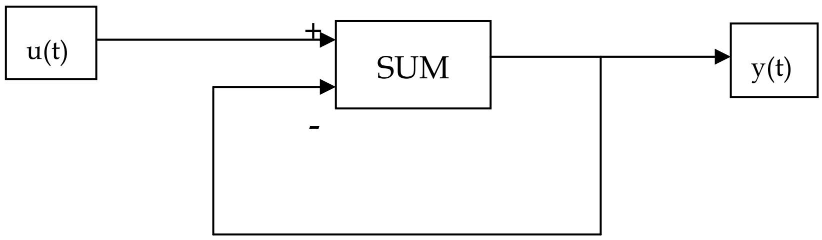


Figure B.1 $y(t) = u(t) - y(t)$.

Figure B-1 shows a simple example [23] where a block is summing two inputs. Feeding the output $y(t)$ as input to the SUM block creates an algebraic loop. On detection of this problem, Simulink uses iterative loops and ends up solving it correctly i.e. $y(t) = u(t)/2$. Simulink was able to solve this simple loop but there are situations where the internal solver in Simulink does not work. For these situations, Simulink provide another solution.

Solution

Simulink has an “Algebraic Constraint block” which is a convenient way to model algebraic equations and specify initial guesses. The Algebraic Constraint block constrains its input signal to zero and outputs an algebraic state. This block outputs the value necessary to produce a zero at the input. The output must affect the input through some feedback path. Initial guess can be provided for the algebraic state value in the block's dialog box to improve algebraic loop solver efficiency.

Another way to get rid of these loops is to introduce a small delay [23] in your model at its highest level. You have to be careful where you put this unit delay as this can change your output considerably. We have introduced unit delay in our matching network and antenna to solve algebraic loops.

Appendix C

C-1 Merging Two Consecutive Networks

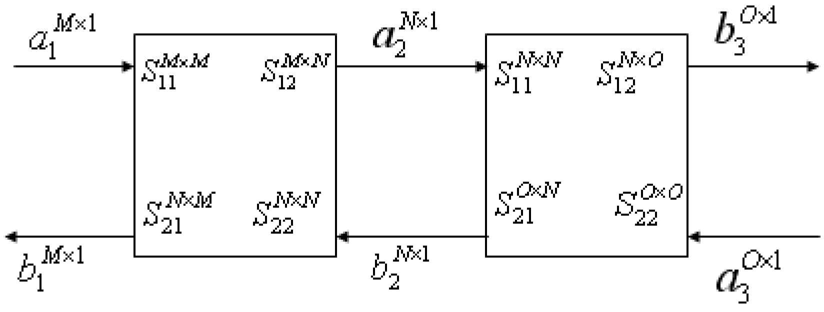


Figure C-1 Two connected multiport networks with different numbers of outward- and inward-propagating waves.

The figure above shows two multiport networks with different numbers of outward- and inward propagating waves. The two networks above are described by their scattering matrices. The first network has M and N ports; the second has N and O ports. For both networks the following equations yield

$$b_1^{M\times1} = S_{11}^{M\times M} a_1^{M\times1} + S_{12}^{M\times N} b_2^{N\times1} \tag{C-1}$$

$$a_2^{N\times1} = S_{21}^{N\times M} a_1^{M\times1} + S_{22}^{N\times N} b_2^{N\times1} \tag{C-2}$$

$$b_2^{N\times1} = S_{11}^{N\times N} a_2^{N\times1} + S_{12}^{N\times O} a_3^{O\times1} \tag{C-3}$$

$$b_3^{O\times1} = S_{21}^{O\times N} a_2^{N\times1} + S_{22}^{O\times O} a_3^{O\times1} \tag{C-4}$$

By solving the equations for b_1^{Mx1} and b_3^{Ox1} as a function of a_1^{Mx1} and a_3^{Ox1}, the following equation is obtained

$$\begin{pmatrix} b_1^{M\times 1} \\ b_3^{O\times 1} \end{pmatrix} = \begin{pmatrix} \breve{S}_{11}^{M\times M} & \breve{S}_{12}^{M\times O} \\ \breve{S}_{21}^{O\times M} & \breve{S}_{22}^{O\times O} \end{pmatrix} * \begin{pmatrix} a_1^{M\times 1} \\ a_3^{O\times 1} \end{pmatrix} \tag{C-5}$$

where

$$\breve{S}_{11}^{M\times M} = S_{11}^{M\times M} + S_{12}^{M\times N} * \left(I - S_{11}^{N\times N} S_{22}^{N\times N}\right)^{-1} * S_{11}^{N\times N} S_{21}^{N\times M} \tag{C-6}$$

$$\breve{S}_{12}^{M\times O} = S_{12}^{M\times N} * \left(I - S_{11}^{N\times N} S_{22}^{N\times N}\right)^{-1} * S_{12}^{N\times O} \tag{C-7}$$

$$\breve{S}_{21}^{O\times M} = S_{21}^{O\times N} * \left(I - S_{22}^{N\times N} S_{11}^{N\times N}\right)^{-1} * S_{21}^{N\times M} \tag{C-8}$$

$$\breve{S}_{22}^{O\times O} = S_{22}^{O\times O} + S_{21}^{O\times N} \left(I - S_{22}^{N\times N} S_{11}^{N\times N}\right)^{-1} * S_{22}^{N\times N} S_{12}^{N\times O} \tag{C-9}$$

The resultant network which is equivalent to the two networks shown in figure C-1 is depicted in the figure below

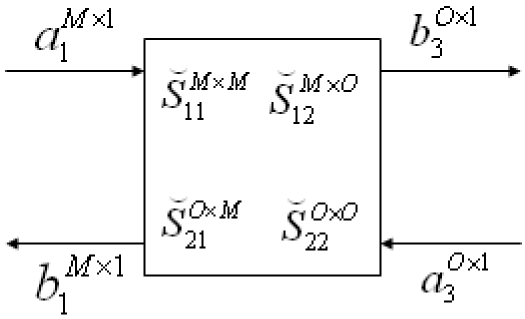


Figure C-2 Network resulting from merging the two networks in Fig.C-1.

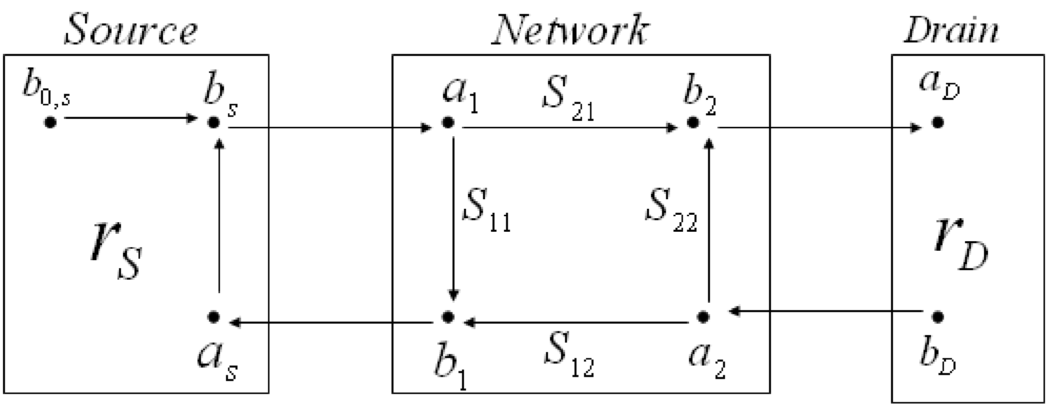


Figure C-3 Multiport network terminated with source and drain.

The above figure shows a network with source and drain termination. The wave vectors a1, a2, b1 and b2 are described as

$$a_1^{M\times 1} = b_{0,s}^{M\times 1} + r_s^{M\times M}\, b_1^{M\times 1} \tag{C-10}$$

$$b_1^{M\times 1} = S_{11}^{M\times M} a_1^{M\times 1} + S_{12}^{M\times N} a_2^{N\times 1} \tag{C-11}$$

$$a_2^{N\times 1} = r_D^{N\times N} b_2^{N\times 1} \tag{C-12}$$

$$b_2^{N\times 1} = S_{21}^{N\times M} a_1^{M\times 1} + S_{22}^{N\times N} a_2^{N\times 1} \tag{C-13}$$

After some algebraic transformations, the equations can be solved for a1, a2, b1 and b2 as a function of $b_{0,s}$

$$a_1 = \left(I - r_s S_{11} - r_s S_{12} * \left(I - r_D S_{22}\right)^{-1} * r_D S_{21}\right)^{-1} * b_{0,s} \tag{C-14}$$

$$a_2 = \left(I - r_D S_{22} - r_D S_{21} * \left(I - r_s S_{11}\right)^{-1} r_s S_{12}\right)^{-1} * r_D S_{21} * \left(I - r_s S_{11}\right)^{-1} * b_{0,s} \tag{C-15}$$

$$b_1 = \left(I - S_{11} r_s - S_{12} r_D * \left(I - S_{22} r_D\right)^{-1} * S_{21} r_s\right)^{-1} * \left(S_{11} + S_{12} r_D * \left(I - S_{22} r_D\right)^{-1} * S_{21}\right) * b_{0,s} \tag{C-16}$$

$$b_2 = \left(I - S_{22} r_D - S_{21} r_s * \left(I - S_{11} r_s\right)^{-1} * S_{12} r_D\right)^{-1} * \left(S_{21} + S_{21} r_s * \left(I - S_{11} r_s\right)^{-1} * S_{11}\right) * b_{0,s} \tag{C-17}$$

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