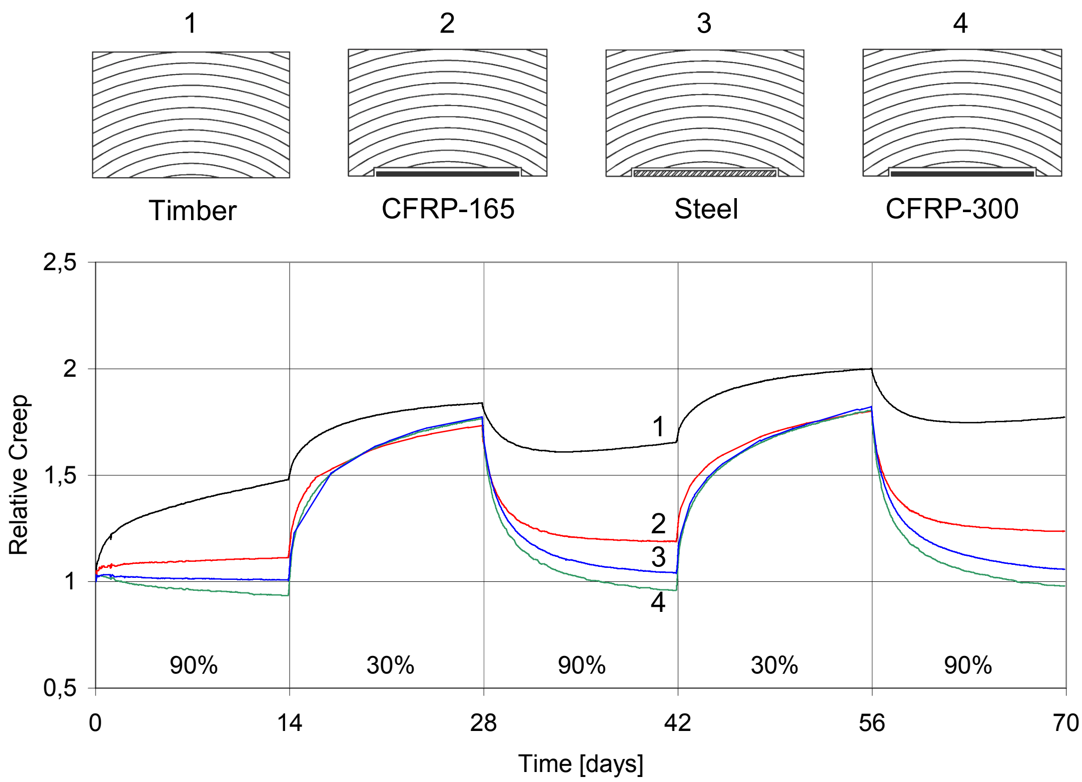
CHALMERS



Moisture-related creep of reinforced timber

Theoretical studies and Laboratory tests

Master's Thesis in the Programme Structural Engineering

SIMON HANSSON

KRISTOFFER KARLSSON

Department of Civil and Environmental Engineering
Division of Structural Engineering
Steel and Timber Structures
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2007
Master's Thesis 2007:123

MASTER’S THESIS 2007:123

Moisture-related creep of reinforced timber

Theoretical studies and Laboratory tests

Master’s Thesis in the Programme Structural Engineering

SIMON HANSSON

KRISTOFFER KARLSSON

Department of Civil and Environmental Engineering
Division of Structural Engineering
Steel and Timber Structures
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2007

Moisture-related creep of reinforced timber
Theoretical studies and Laboratory tests
Master's Thesis in the Programme Structural Engineering

SIMON HANSSON

KRISTOFFER KARLSSON

Master's Thesis 2007:123
Department of Civil and Environmental Engineering
Division of Structural Engineering
Steel and Timber Structures
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

Cover: Mean value of test series of the relative creep.

Chalmers Reproservice / Department of Civil and Environmental Engineering
Göteborg, Sweden 2007

Moisture-related creep of reinforced timber
Theoretical studies and Laboratory tests
Master's Thesis in the Programme Structural Engineering
SIMON HANSSON
KRISTOFFER KARLSSON
Department of Civil and Environmental Engineering
Division of Structural Engineering
Steel and Timber Structures
Chalmers University of Technology

ABSTRACT

Wood is a natural material which results in great variability in material properties. Timber used for structural purposes has to be designed with several material parameters that take large spread in properties into account. Designing in the serviceability limit state requires a reduction of Young's modulus due to load duration and climate class. This gives horizontal timber members somewhat poor performance in terms of deflection and vibration. Another major problem is that timber increases its deformation with time. Load duration, stress level, temperature and moisture content are all factors influencing creep in timber.

Carbon fibre reinforced polymers are being used more often as a reinforcing material on concrete structures that need repairing. Laminates are popular thanks to their favourable application method and unidirectional strength and stiffness. There are already timber structures that have been repaired and reinforced with these carbon fibre reinforced polymer laminates.

This study mainly concerns timber reinforced with both carbon fibre reinforced polymers and steel. The material structure and material parameters are first explained individually and then after having been combined as a composite material. A description of the production of the test specimens follows, together with a description of the test setup. Finally, the results of creep tests are shown and discussed. A simple calculation example of a reinforced timber floor joist has also been produced.

Dynamic measurement of timber is a non-destructive testing method to determine Young's modulus based the first eigen frequency of a material. This is a quick, simple way of obtaining the stiffness of any material. In this study, the results show that this testing method is also valid for composite materials, such as reinforced timber.

The results from the test show that reinforcement reduces the initial deformation. For a carbon fibre reinforced specimen with a reinforcement level of 2% of the cross-section, the increase in stiffness could be as much as 85%. The reinforcement also reduces the mechano-sorptive creep to be as low as a fourth of the mechano-sorptive creep in un-reinforced timber

Key words: Mechano-sorptive creep, mechano-sorptive testing of timber beams, FRP laminates, reinforced timber, dynamic measurement, carbon fibre

Fuktrelaterad krypning av förstärkt trä
Teoretiska studier och Laboratorieförsök
Examensarbete inom Konstruktionsteknik
SIMON HANSSON
KRISTOFFER KARLSSON
Institutionen för bygg- och miljöteknik
Avdelningen för Konstruktionsteknik
Stål- och träbyggnad
Chalmers tekniska högskola

SAMMANFATTNING

Trä är ett naturligt växande material vilket resulterar i en stor variation i materialegenskaper. Trä som används i syfte för byggnation måste dimensioneras med flera materialparametrar, som inkluderar stor spridning. Dimensionering i bruksgränstillstånd kräver en reduktion av elasticitetsmodulen på grund av klimatklass och lasters varaktighet. Reduktionen ger horisontella trädelar dålig prestanda vid nedböjning och vibration. Att deformationer i trä ökar med tiden är också av stort intresse. Lasters varaktighet, spänningsnivåer, temperatur och fukthalt är alla faktorer som påverkar krypningen i trä.

Kolfiberarmerat plastlaminat används allt oftare som förstärkningsmaterial på betong- och stålkonstruktioner som kräver reparationer. Laminat är populära på grund av dess fördelaktiga appliceringsmetoder och deras enkelriktade hållfasthet och styvhet. Det finns även vissa träkonstruktioner som har blivit reparerade och förstärkta med dessa kolfiberlaminat.

Denna rapport berör huvudsakligen trä förstärkt med kolfiber eller stål. Litteraturstudien inkluderar; materialstrukturer och materialparametrar individuellt och kombinerat som ett kompositmaterial. En beskrivning av tillverkningen av provbalkarna följer tillsammans med en beskrivning av provningskonfiguration. Slutligen visas och diskuteras resultaten från krypförsöken. Ett räkneexempel på ett förstärkt golvbjälklag i trä har också genomförts.

Dynamisk mätning på trä är en ickedestruktiv testmetod för att bestämma elasticitetsmodulen. Metoden baseras på den första egenfrekvensen hos ett material. Detta är ett snabbt och enkelt sätt att bestämma styvheten hos många material. I denna rapport visar resultaten att denna testmetod även fungerar för kompositmaterial såsom för förstärkt trä.

Resultaten från testerna visar att förstärkningen minskar den initiala nedböjningen. För ett kolfiberförstärkt provexemplar med en armeringsmängd på 2 % av tvärsnittsarean, kunde styvheten ökas upp till 85 %. Förstärkningarna förhindrar även den ökande mekanosorptiva krypningen i trä ner till en fjärdedel av den mekanosorptiva krypningen i oförstärkt trä.

Nyckelord: Mekanosorptiv krypning, mekanosorptiv testning av träbalkar, FRP laminat, förstärkt trä, dynamisk mätning, kolfiber

Contents

Preface

This master thesis has been carried out at Chalmers university of Technology, Department of Civil and Environmental Engineering, Division Structural Engineering, Steel and Timber Structures. It has been done in co-operation with VBK, Moelven Töreboda AB and Johns Bygg och Fasad AB.

In the study tests of moisture-related creep of reinforced timber has been done. The tests started in September 2007 and will end in January 2008. Results in this thesis are based on data from the period 11th of September to 20th of November 2007.

We would like to give a big thank to Lars Wahlström for all his help and support with the creep tests. From the Division of Structural Engineering, Steel and Timber structures we would like to thank our examiner Professor Robert Kliger and Dr. Marie Johansson and Dr. Mohammad Al-Emrani which all have contributed to this thesis in one way or another.

For the sponsoring and providing of material for the tests we specially would like to thank Martin Reinholdsson engineer at VBK and Dr. Roberto Crocetti at Moelven Töreboda AB and Simon Dahlberg at Johns Bygg och Fasad AB.

To our opponents Rickard Caster and Gustav Deuschl we send big thanks for their interest and comments on the thesis.

Göteborg November 2007

Simon Hansson & Kristoffer Karlsson

Notations

Roman upper case letters

$A_{adhesive}$	Area of adhesive in timber beam cross-section
A_{beam}	Cross-section area of the timber beam
A_{cut}	Area of cut in timber beam cross-section
A_{rein}	Cross-section area of the reinforcement
E	Young's modulus
$E_{adhesive}$	Young's modulus of the adhesive
$E_{analytic.composite}$	Compensated Young's modulus for the equivalent cross-section [MPa]
$E_{compensated}$	Compensated Young's modulus
$E_{dyn,initial}$	Measured dynamic Young's modulus after cutting
$E_{dyn,conditioned}$	Measured dynamic Young's modulus after conditioned to 65% RH
$E_{dyn,composite}$	Measured dynamic Young's modulus of composite beam
E_{dyn}^{long}	Measured dynamic longitudinal Young's modulus
E_{rein}	Young's modulus of the reinforcement
L	Length of beam
V	Volume
V_0	Volume at zero moisture content
V_ω	Volume at specific moisture content

Roman lower case letters

a	Lever arm
f_1	First resonance frequency
f_2	Second resonance frequency
k_{def}	Modification factor considering creep and moisture
m	Mass
m_0	Mass at zero moisture content
m_ω	Mass at specific moisture content
r	Correlation coefficient
t	Time
u	Displacement
$u_{analytic}$	Analytical value of instantaneous deflection
u_{inst}	Instantaneous deflection

Greek letters

α	Ratio of Young's modulus between two materials
β_ω	Volumetric swelling per change in moisture content
Δ	Tolerance in finger joints
κ	Curvature
κ_s	Reduction factor for load duration and climate class
ρ	Density
ρ_0	Density at zero moisture content
ρ_i	Initial density
ρ_ω	Density at specific moisture content
σ_{1-5}	Stress level
ω	Specific moisture content
Ψ	Relative humidity

Abbreviations

AFRP	Aramid fibre reinforced polymer
AR – glass	Alkali resistant-glass
C – glass	Corrosion-glass
CFRP	Carbon fibre reinforced polymer
D – glass	Dielectric-glass
E – glass	Electrical-glass
EWP	Engineered wood product
FFT	Fast Fourier transforms
FRP	Fibre reinforced polymer
GFRP	Glass fibre reinforced polymer
Glulam	Glue laminated timber
HM	High modulus
HS	High strength
HT	High tension
IM	Intermediate modulus
LVDT	Linear variable differential transducer
MC	Moisture content
RC	Relative creep
RH	Relative humidity
S – glass	Strength-glass
SFRP	Steel fibre reinforced polymer

PAN	Polyacrylonitrile
PITCH	Synthetic pitch
UHM	Ultra high modulus
UV	Ultra violet

1 Introduction

1.1 Background

Even though wood has been used as a building material for ages there are still several uncertainties when it comes to design of wood structures. Wood is a natural material with natural imperfections such as knots and varying growing conditions that makes it more difficult to decide the strength of the material. As a result of these imperfections a grading system was introduced to predict the material properties. A further attempt to improve the quality of wood products was done by combining wood from several trees of different qualities. This new 'Engineered wood product', EWP, has an improved quality due to the spreading of imperfections over a larger area or over several products. The most common EWP today is the glued laminated timber, glulam beams. Thanks to its variety in length and shape this product is today widely used.

Timber has good properties in both tension and compression of the fibres when compared to concrete for instance. Concrete has a low tensile strength that would cause it to fail when loaded in bending if reinforcement was missing on the tension side. The use of steel reinforcement bars in the concrete improves the tension capacity of the reinforced concrete. Wood has the disadvantage of a low stiffness which cause timber beams to a rather high deflection when loaded. This is often the limitation when using timber beams as load bearing members. Another problem is that the deflection of timber members increases with time.

For the last decade several efforts to reduce the decrease of stiffness has been made. Presence of moisture and especially moisture variation causes the increase of creep deflection. This can be controlled with several methods of treatment of wood. The magnitude of deflection with time is probably directly related to the initial deflection directly after load is applied. By reinforcing timber with material that has superior qualities when it comes to short- and long-term stiffness, the behaviour after long time could be improved and deflection could be reduced. It has been documented by Jobin, Olga (2007) that reinforced timber could very well be improved in ultimate limit state. The question is if these improvements could last over the entire service life of the structure and if the long-term creep which effects timber could be reduced. Another question that is of great importance for the future use of timber reinforcements is the economic aspect of reinforced timber.

1.2 Objectives and methodology

The aim of the project is to study the long-term effects of reinforced solid timber beams in the serviceability limit state. The specific objectives are:

- To measure Young's modulus of composite beams
- To investigate the behaviour of the long-term effect due to creep in timber reinforced with steel and fibre reinforced polymers
- To evaluate the cost of reinforcement compared to the increased performance, and investigates if the reinforcement is justified

To reach these objectives the following methodologies are used:

- To find and study similar projects
- To compare structures where timber beams could have been substituted with reinforced timber
- To perform long-term creep tests with varying moisture content on timber beams reinforced with steel and fibre reinforced polymers

1.3 Limitations

The master's thesis is limited with the following points:

- Only the behaviour in the serviceability limit state is investigated
- The tests are made with standardised solid timber beams with reinforcement of carbon fibre and steel
- The solid timber beams have the dimensions of 45x70x1100 mm^3
- The bond between timber and the reinforcement is assumed to have full interaction
- The tests are performed for 10 weeks
- Not enough specimens are used to produce a population for statically evaluation
- Same adhesive is used in all tests to neglect the creep influence of it

2 Literature studies

2.1 Wood

Wood is a natural anisotropic material with variations in material parameters in different directions. To be able to understand the behaviour of wood it is of great importance to understand the composition of the material.

Wood can generally be divided in to hardwood and softwood. Examples of hardwood are: Oak, Ash, Cherry and Walnut. Hardwood trees are generally angiosperms, plants that produce seeds with some kind of covering. Softwood on the other hand is often gymnosperms (conifers), with seeds without covering that falls down to the ground. Typical softwoods are: pines, firs and spruces.

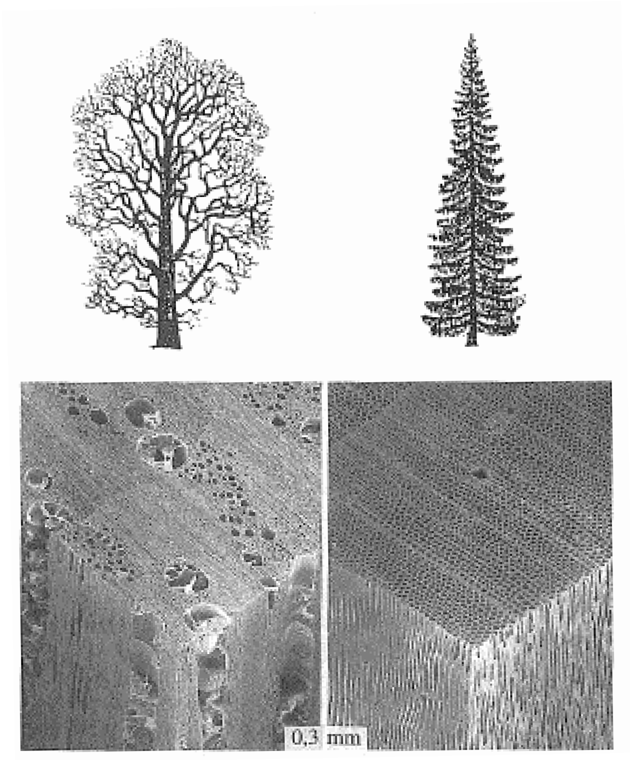


Figure 2.1 Timber from hardwood (left) and softwood (right), adopted from Blass et al. Ed., (1995)

2.1.1 Microstructure

Every year trees has an annual growth in both the longitudinal and the radial direction. In the radial direction the growth of new cells expands the diameter of the tree.

For softwood the growth of new cells can be divided in two types of new cell depending on the growth time of the year. Earlywood grows during spring and early summer when both the temperature and the moisture content are high and the conditions for growing are good. These fast growing cells has a thin cell wall and a large area of lumen (air) which makes the earlywood cells pale coloured and large. Latewood grows during late summer and fall until the climate is to cold for any growth to take place. Due to less favourable growing conditions the latewood cells have a thicker cell wall which makes the latewood denser and dark coloured. Earlywood and latewood together makes one anural ring.

Hardwood is more varied and complicated in its anatomy than softwood. Most of the structural concepts are analogue with softwood except that hardwood has a denser structure of libriform fibres. Within this tissue there are long pores, often with large lumina. Hardwood has thicker cell walls and smaller area of lumen then softwood generally has. The difference between earlywood and latewood is not as extreme as for softwood. For hardwood it is the diameter of the lumen that varies with the growing season. (Blass et al. Ed., 1995)

2.1.2 Ultra structure

Almost all species of wood have the same features of wood cells. Elementary fibrils consist of cellulose formed into larger units. Several elementary fibrils together form thread like entries called microfibrils. Microfibrils contain an estimated number of 100 to 2000 cellulose chains embedded in a matrix of hemicelluloses and enveloped by lignin. (Blass et al. Ed., 1995)

Between all individual cells in wood there are a middle lamella (ML) which contains lignin and pectic substances. The most outer layer of the cell is called the primary wall (P). This layer consists of cellulose microfibrils which are arranged in an irregular pattern. After the primary wall there is a secondary wall, this normally consists of three layers S1, S2 and S3, Figure 2.2. In the very thin first layer (S1) the angle of microfibrils has an average of 50-70°. In the second layer, which is rather thick compared to the other two, the slope of microfibrils is about 5-20°. In tension of fibres this layer is the most important one since most of the tensile force has to be taken by this layer. Closest to the lumen core, layer (S3) has microfibrils with a smaller slope but not in a defined order. In compression the S2 layer will act as a column. To prevent the layer from buckling the S1 and S3 layer acts as reinforcement since the microfibrils has a larger slope then the middle layer. (Blass et al. Ed., 1995)

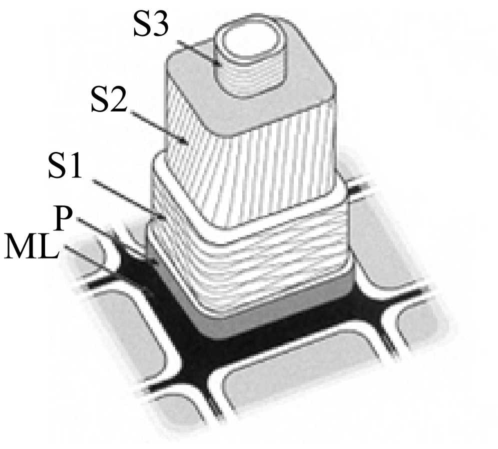


Figure 2.2 Cell structure for wood, adopted from Nakano (2003). ML-Middle lamella; P-Primary wall; S1, S2 and S3- layers of the secondary wall

2.1.3 Sapwood and heartwood

There are two types of wood tissue, sapwood and heartwood. Sapwood is the living part that supports the trees with nutrition and water upwards. Heartwood has no influence on the physiology of the tree but both sapwood and heartwood has influence on the mechanical properties. (Bengtsson, 1997)

In most species a darker colour appears in the heartwood as an effect of incrustation with organic extractives. This also results in a better resistance against decay and wood boring insects. A loss in moisture content also takes place which leads to plugged vessels in the wood. Heartwood looses most of its permeability and can not be chemical preserved in the same manner as sapwood. (Blass et al. Ed., 1995)

2.1.4 Juvenile and reaction wood

Juvenile wood is formed in the first 5-20 years of a trees life. Mechanical properties of juvenile wood can differ a lot from normal wood. Especially the forming of short, thin walled tracheids in the S2 layer makes the juvenile wood experience much greater longitudinal shrinkage but also reduced strength and stiffness.

External forces acting on trees over long time will result in formation of reaction wood. For softwood trees compression wood will be formed, while hardwood trees instead will develop tension wood. Tension wood seldom causes any problems in timber engineering but compression wood is of much greater concern. When growing in compression the microfibrils in the S2 layer will develop a large slope of the micro fibrils, sometime as large as 45°. Also the inner S3 layer will be missing entirely. This will cause great longitudinal shrinkage and a greater density of the compression wood. This is not to be confused with the improved mechanical properties the high density brings. High density in compression wood will not improve the mechanical properties. Instead compression wood has a tendency to break in a brittle manner when dried out. (Blass et al. Ed., 1995; Bengtsson, 1997)

2.2 Material properties of timber

Material properties of wood are determined by several factors, e.g. density and moisture content of the material. All properties are determined by the growing condition and what type of tree that is used. Also the appearance of knots and defects will influence the material properties.

2.2.1 Density

Density is the single most important factor that determines the mechanical behaviour. Several mechanical properties are positive correlated to density; see Figure 2.3. (Hansson & Gross, 1991)

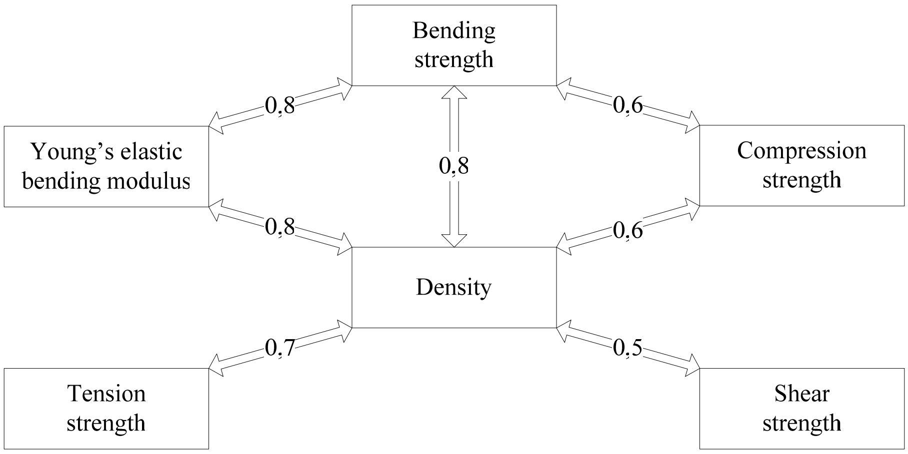


Figure 2.3 Relation between density and other mechanical properties and rate of influence, adopted from Hansson & Gross (1991)

Density is defined as

$$\rho = \frac{m}{V} \tag{2.1}$$

where m is the mass (kg) and V is the volume (m³). This can also be written as density at specific moisture content ω (%).

$$\rho_\omega = \frac{m_\omega}{V_\omega} = \frac{m_0(1+0{,}01\omega)}{V_0(1+0{,}01\omega)} = \rho_0 \frac{1+0{,}01\omega}{1+0{,}01\beta_V\omega} \tag{2.2}$$

Where m_0, V_0 and ρ_0 are mass, volume and density at zero moisture content. β_ω is a coefficient defining the volumetric swelling with unit of percent of swelling per percent of moisture change, Blass et al. Ed. (1995).

Density of pure timber (only the cell wall and not the lumen from the cell) is about 1500 kg/m³. This density is not relevant when relating to timber for structural purpose. The most common definition of density is the weight of the timber at 0 or 12 percent moisture content. In Eurocode 5 (1993) density is given as the mass and volume at equilibrium at 20°C and a relative humidity of 65%.

Density is often compared to the width of one growth ring. This relationship is not really clear and there is a great scatter in experiments with density and growth ring width. There are several other factors influencing the density of wood. According to Hansson & Gross (1991) these factors are:

- *Temperature:* warmer climate gives wood a higher density at a certain growth ring width

- *Moisture:* dryer climate results in wood with a lower concentration latewood, and therefore a lower density

- *Stand concentration:* stands with larger amount trees per area will grow slower then culled stands. Culling results in a higher growth rate and therefore increased density

- *Position in tree:* wood at the same growth ring width has a higher density near the root of the tree then at the top. Trees develop wood with higher density in parts with high strain due to wind loads

- *Fertilise:* trees growing in soil with low amount of nitrogen can be fertilised to increase the growth rate without decreasing the density

- *Genetically properties:* trees from same stand under same growing conditions but with different genetically properties can have a great variety in density. Such a large variety as 25% has been recorded

2.2.2 Moisture content

Wood has very good water transportation properties because it needs water to grow. After the tree is cut down and sawn in to timber many of these water transportation properties remain. Wood is a hygroscopic material which means that it absorbs and desorbs moisture from the surrounding air. The moisture content (MC) in wood is therefore dependent on the relative humidity (RH) of the surrounding air. Moisture in wood can either be found as moisture in the cell wall or as free water inside the lumens. Increased MC in the cell walls will decrease the mechanical properties of wood. This is due to water penetrating the cell wall which will weaken the hydrogen bonds that hold the cell wall together. Wood has a fibre saturation point of approximately 30% MC. In Figure 2.4 an example of how the MC varies with the RH for both absorption and desorption. There are different curves depending of the type of wood and temperature of the surrounding air. It takes a long time for wood to adapt its MC from the surrounding RH. Studies like Bäckström (2006) show that for a timber stud with cross section dimensions 45x70 mm^2 it will take up to 60 days to change the MC from 19% to 9%. After approximately two weeks the surface has reached 9% in MC while the middle of the cross section is still at 15-16% MC. The change in MC throughout the cross section can be seen in Figure 2.5.

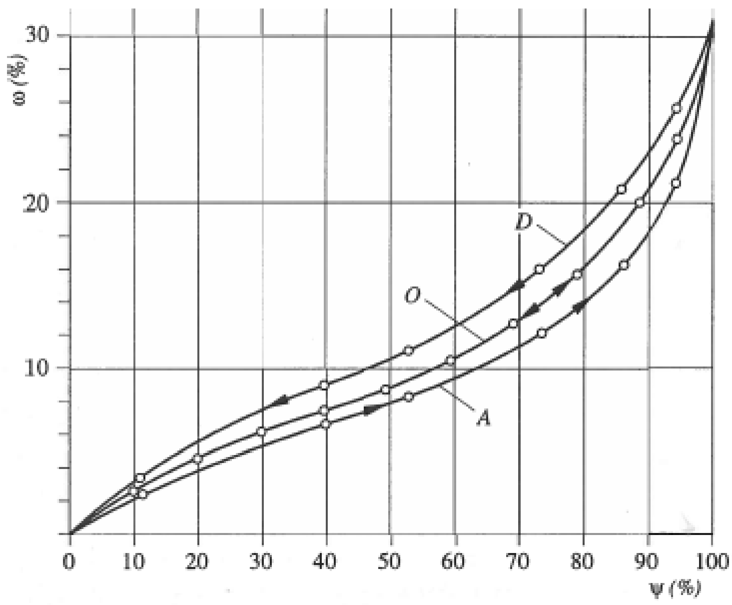


Figure 2.4 *Sorption curves for spruce at 20°C. D-desorption; A-adsorption; O-oscillating, adopted from Blass et al. Ed. (1995).*

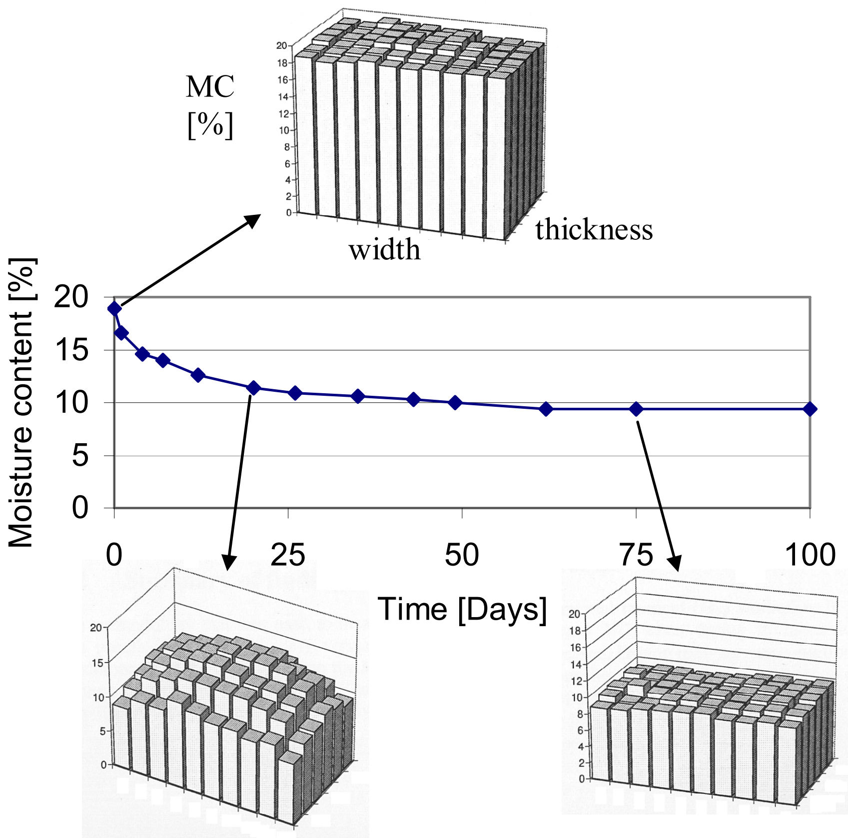


Figure 2.5 *Variation of moisture content of a cross-section 45x70 mm², adopted from Bäckström (2006).*

Variations of mechanical properties can be said to have a linear relationship for clear wood in the range of an MC between 8% and 20%. In Table 2.1 the change in mechanical properties per change in percent MC is shown. Some changes of the material properties will not be as important for timber as it would be for clear wood. Several experiments have been carried out to find out how mechanical properties varies with MC. Results from these experiments show that the tensile strength of low quality wood is independent of the MC while both bending and compression strength is highly dependent of the MC. (Blass et al. Ed., 1995)

Table 2.1 Approximate change of clear wood properties. Change in property per percentage lower moisture content. (Blass et al. Ed., 1995)

Property	Change (%)
Compression strength parallel to the grain	5
Compression strength perpendicular to the grain	5
Bending strength parallel to the grain	4
Tension strength parallel to the grain	2,5
Tension strength perpendicular to the grain	2
Shear strength parallel to the grain	3
Impact bending strength parallel to the grain	0,5
Modulus of elasticity parallel to the grain	1,5

To ensure sufficient strength of the material there are three service classes defined in Eurocode 5 (1993) depending on the RH of the surrounding where the timber is placed:

- Service class 1: is characterised by moisture content in the material corresponding to a temperature of 20°C and the relative humidity of the surrounding air only to exceeding 65% for a few weeks per year

- Service class 2: is characterised by moisture content in the material corresponding to a temperature of 20°C and the relative humidity of the surrounding air only to exceeding 85% for a few weeks per year

- Service class 3: climatic conditions leading to higher moisture content then in service class 2

2.2.3 Shrinkage and swelling

Moisture in air has such similarities as the substance in the cell wall that it can even penetrate the almost non-porous wood material. Moisture finds its way into the wood cell which pushes the microfibrils apart. When wood cells swells the volume of the lumens stays constant. The volumetric swelling of the cell has the same volume as the moisture absorbed. When moisture is removed from the cells shrinkage occurs in the opposite manner as swelling. The shrinkage and swelling of timber are called movements.

The size of movement in timber is mainly dependent on the microfibrillar angle in the S2 layer. For normal timber the layer S2 has a rather small angle which causes small movements in the longitudinal direction but greater movements in the transversal direction. For juvenile and compression wood the transversal and longitudinal movements are equal. Normally the movement in radial direction is ten times as large as the movement in the longitudinal direction. The tangential movement is twice the magnitude of radial movements.

Shrinkage and swelling can differ on both sides in sawn timber. This causes distortions. This variation between the top and the bottom layer of a beam causes a curvature on the beam, usually called spring or bow. (Kliger et al., 1994)

2.2.4 Duration of load

Wood loaded under a long period of time will experience an instant deformation right after load is applied. With time creep deformations will develop in the loaded specimen. Part of the deformation will be elastic and disappear right after the load is removed. The other part is a plastic deformation that is due to viscous flow within the molecules that leaves a permanent deformation.

Several studies have shown that with a stress level beneath the failure stress, long-term effects can cause failure of wood, Figure 2.6. It has also been shown that long-term loading does not affect short-term strength and elasticity if the long-term load is kept underneath the proportional limits. For decreasing rate of creep failure due to creep will not occur. (Hoffmeyer, 1990)

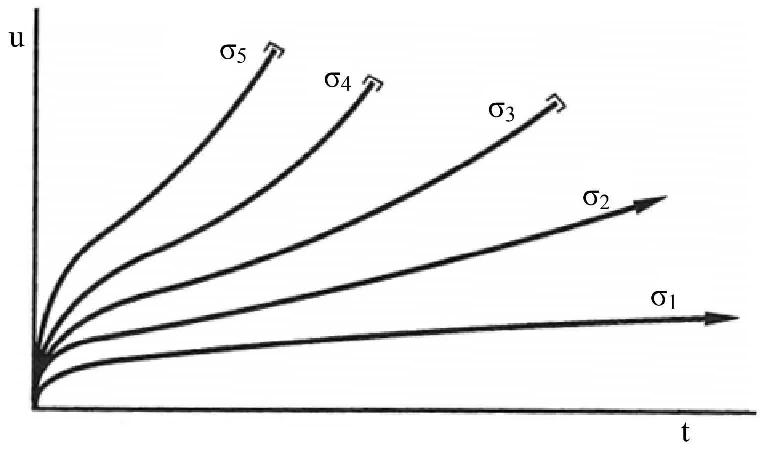


Figure 2.6 Influence of stress levels on creep, $\sigma_1<\sigma_2<\sigma_3<\sigma_4<\sigma_5$, adopted from Blass et al. Ed. Hout (1995).

The most important of the early studies of long-term behaviour of wood is carried out by Madsen in 1947 and 1951. He performed long-term bending test under constant load with 1x1x22" test specimens and could show that the relation between stress level until failure and logarithmic time has a linear relationship. These curves are called "The Madison Curve" and are still used today to define the duration of load capacity. (Hoffmeyer, 1990)

Duration of load is taken into concern in Eurocode 5 (1993) by dividing the load time in to 5 categories:

- Permanent load has an accumulated duration of load for more then 10 years, for example self weight
- Long-term load is defined as 6 months to 10 years, for example storage
- Medium-term load is defined as 1 week to 6 months, for example imposed loads
- Short-term load is duration less then one week. Wind, temporary load and snow load for some countries
- Instantaneous load is usually accidental loads

2.2.5 Mechano sorption

The first recordings of accelerated creep due to varying humidity conditions were described in the 1960. These recordings were later verified by performing bending tests on small wood specimens in both cyclic and constant humidity. The results from the experiments show a great difference in deflection where the specimens subjected to constant humidity show an almost constant deflection. For cyclic humidity the deflections varies with the drying and wetting cycles, but the total deflection increases for every cycle. For higher stress rates, the rate of deflection increases prominent for cyclic humidity. In the 1970's the phenomena received the name mechano sorption. (Bengtsson, 1999) and (Mårtensson, 1992)

According to Mollier (1994) following variables are involved in mechano-sorptive creep:

- Wood characteristics (see Chapters 2.1.2, 2.2.1, 2.2.3 and 2.2.4)
- Time (see Chapter 2.2.5)
- Stress (see Chapter 2.2.5)
- Stress history
- Moisture content (see Chapter 2.2.2)
- Moisture content change
- Moisture content history
- Temperature
- Temperature history

When performing test of mechano-sorptive creep it is difficult to measure the phenomena itself. Since there are so many variables involved there would have to be two exactly similar test specimens subjected to cyclic and constant humidity. It is known that it would be impossible to perform a test like that since wood has great variations both between different trees but also within the tree itself. (Bengtsson, 1999)

According to Epmeier et al. (2007) creep behaviour in cyclic climate and with constant loading can behave in three different ways:

- Type A: increasing deformation during drying (desorption) and decreasing deformation during wetting (adsorption)
- Type B: decreasing deformation during drying and increasing deformation during wetting
- Type C: increasing deformation during both adsorption and desorption or indecisive.

In Figure 2.7 the three creep curves is shown with a relative humidity cycling between 90 and 30%. The most common creep behaviour is Type A.

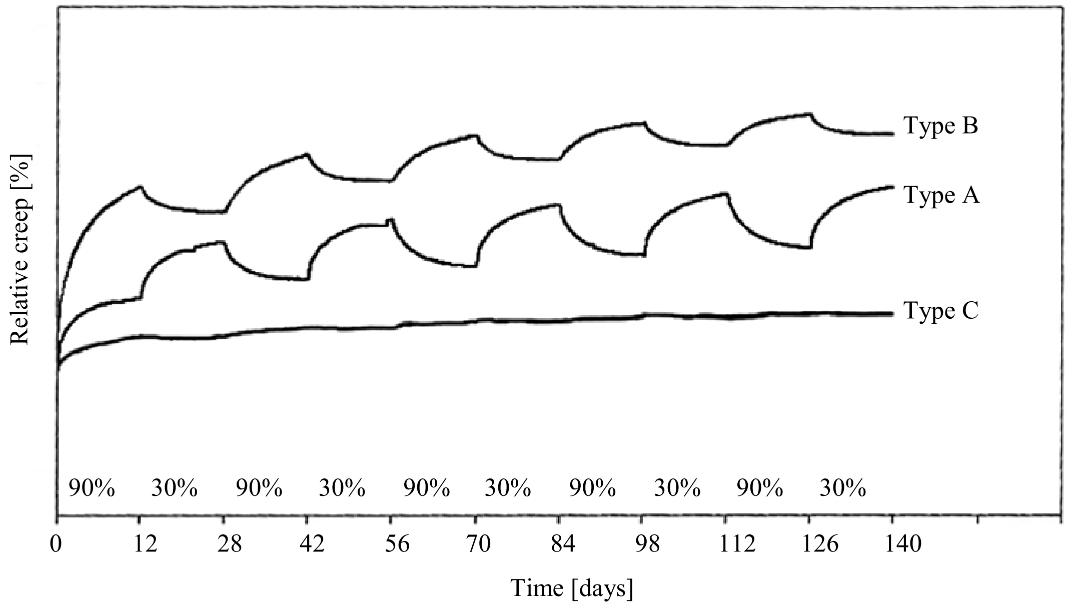


Figure 2.7 Typical creep curves A, B and C, due to cycling of relative humidity between 90% and 30%, adopted from Epmeier et al. (2007).

At low stresses (about 10% of ultimate load) the mechano sorption is calculated as linear. At higher stress rates the mechano sorption is nonlinear. Another problem that makes the mechano sorption even more complex is the difference in compression and tension. In compression the strain increases fast and is irrecoverable due to buckling of cell wall. The final magnitude of them both appears to be similar thou.

2.3 Fibre reinforced polymers

Fibre Reinforced Polymers (FRP) is becoming more interesting alternative as a construction material during the last few decades. FRP has many different applications such as sports equipment, aircraft industry, spacecraft industry and building material. FRP is a composite usually made of either glass (GFRP), carbon (CFPR) or aramid (AFRP) fibres bonded together with a resin based of polyester, epoxy or vinylester. This gives FRP many advantages such as high strength, high stiffness, low weight and good resistance of corrosion, Triantafillou (1997). When combining materials it is useful to compare the most important characteristics to get the final composite as wanted. Figure 2.8 and Figure 2.9 shows the tensile strength and Young's modulus of commonly used building materials compared with fibre composites.

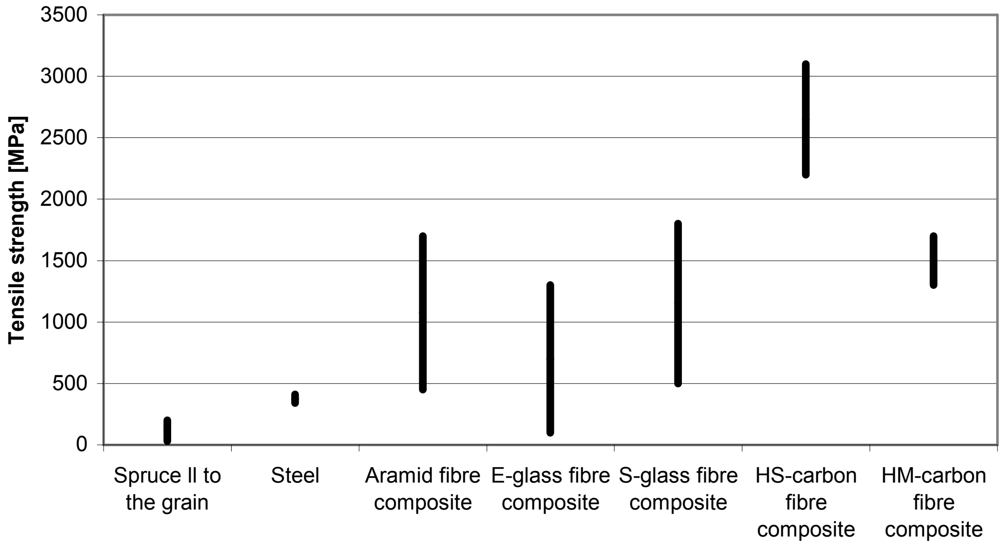


Figure 2.8 Comparing tensile strength between ordinary building materials and fibre reinforced polymers (FRP).

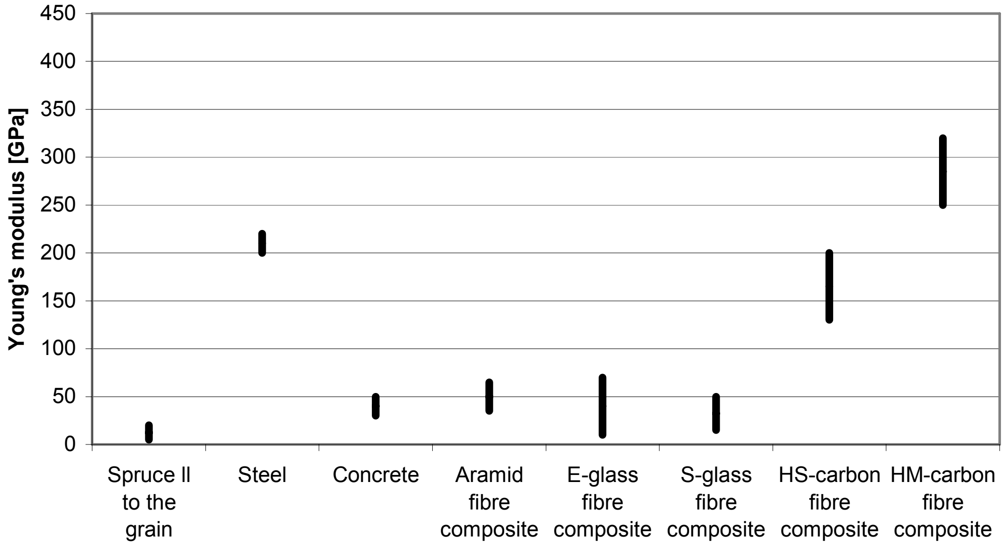


Figure 2.9 Comparing Young's modulus between ordinary building materials and fibre reinforced polymer (FRP).

2.3.1 Fibres

The property of the composite depends mainly on the choice of the fibre. The most important properties that differ between the fibres are stiffness and tensile strength, which make one more suitable then the other for different purposes. All fibres have generally higher stress limits than ordinary steel and are linear elastic until failure. (Carolin, 2003)

The quality of fibres determined which strength and what overall behaviour the final composite will have. The orientation of the fibres in the composite can be discontinuous “short fibres” or continuous “endless fibres” and randomly oriented or aligned, see Figure 2.10. This affects the properties in the composite in the three directions. The binder material only keeps the fibres together and does not affect the strength properties, Åström (2002). The fact below about different kinds of fibres is taken from Dejke (2001).

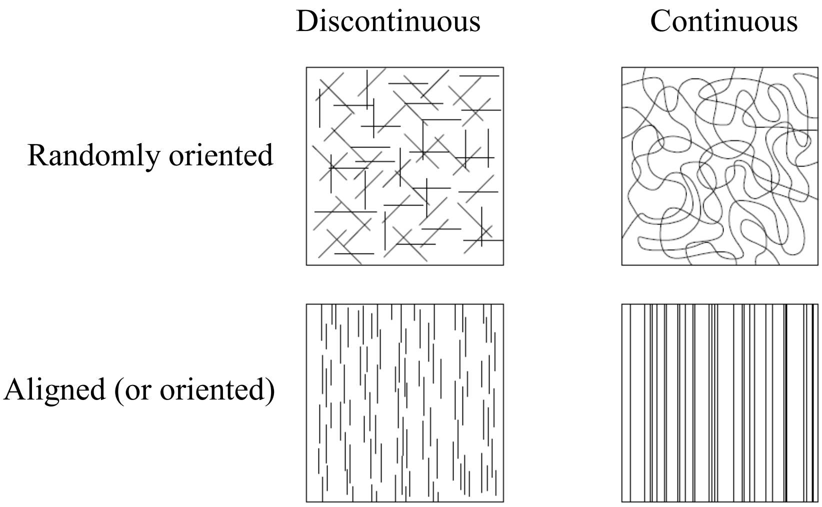


Figure 2.10 Different fibre configurations.

Glass fibre

Glass is an isotropic material (it has the same properties in all directions) which is based on silica (SiO_2) with additions of different oxides. Glass fibres are made by drawing molten glass trough an aperture until the fibre diameter is 3-20 µm. Glass fibres are more susceptible to corrosion than carbon and aramid fibres.

Glass fibre has the following general properties: (Komposithandboken, 2001)

- High tension strength
- Attractive thermal properties
- Corrosion resistant
- Good electrical isolator
- Low price

There are several different compositions of glass. Electrical-glass (E-glass) is the most common glass used in composite reinforcement. E-glass has good mechanical properties to a relatively low price. Other kinds of glass fibre are Strength-glass (S-glass), Alkali Resistant-glass (AR-glass), Dielectric-glass (D-glass) and Corrosion-glass (C-Glass). Table 2.2 shows properties for commonly used glass fibre.

Table 2.2 Glass fibre properties. (Komposithandboken, 2001)

Properties	E-Glass	S-Glass	C-Glass	D-Glass
Tension strength (MPa)	2800	3700	2300	2000
Young's modulus (MPa)	74000	85000	70000	52000
Density (kg/m^3)	2540	2490	2490	2160
Thermal expansion coefficient (10^{-6}/°K)	5,0	3-5,0	7,0	3,0

Carbon fibre

Carbon fibre is made from polyacrylonitrile (PAN), petroleum, coal or synthetic pitch (PITCH). Both can be made of either high tension (HT) fibres which have high tension strength and low modulus of elasticity or high modulus (HM) fibres, which have higher modulus of elasticity but lower tension strength. Carbon fibre is resistant to many chemical solutions and does not absorb water. Table 2.3 shows properties for different carbon fibres. (Dejke, 2001)

The following properties are significant for carbon fibre: (Komposithandboken, 2001)

- Very high modulus of elasticity
- Very high tension strength
- High thermal and electrical conductivity
- Good fatigue properties
- Low thermal expansion coefficient
- High price

Table 2.3 Carbon fibre properties. (KompositHandboken, 2001)

Properties	PAN HT	HS	IM	HM	UHM	PITCH P55	P75	P100
Tension strength (MPa)	3500	4500	4200	2250	1860	1900	2050	2250
Young's Modulus (GPa)	230	245	295	395	515	380	515	690
Density (kg/m^3)	1760	1800	1740	1810	1960	2020	2060	2150
Fibre diameter (µm)	7	6	6	6,5	8,4	10	10	11
Thermal expansion coefficient (10^{-6}/°K)	-0,5			-1,5	-1,5	-0,9	-1,2	-1,6

(HT=high tension, HS=high strength, IM=intermediate modulus, HM/UHM=high/ultrahigh modulus, P=pitch grade)

Aramid fibre

Aramid is short for aromatic polyamid. The fibre is made of polymer and has an outstanding toughness and damage tolerance but is susceptible to moisture. The most known aramid is Kevlar which are used in bullet-proof vests and in helmets. Table 2.4 shows properties of aramid fibres. (Åström, 2002)

Properties that are distinguish for aramid fibres: (Kompositthandboken, 2001)

- High tension strength
- High Young's modulus
- High ductility
- Good fatigue properties
- Corrosion resistant
- Very high price
- Fibre is highly damping, epoxy with Kevlar have 5x higher damping capacity compared to glass fibre/epoxy

Table 2.4 Properties of aramid fibres used in Kevlar. (Chalmers, 2007)

Tension strength (MPa)	2400-3000
Young's modulus (GPa)	65-135
Strain failure (%)	2,5-4,4
Density (kg/m^3)	1470
Fibre diameter (mm)	12-15
Thermal expansion coefficient (10^{-6}/°K)	-2

2.3.2 Matrices

The matrix in a composite has several different functions; it holds the reinforcement (fibres) in place, it transfers external load to the reinforcement and protects the reinforcement from environmental effects. It also redistributes load to surrounding fibres when a particular fibre fractures, Åström (2002). When selecting polymer matrix material there are several considerations like manufacturing aspects, size of the composite component, environmental issues but the main consideration is the service temperature. The matrix material for fibre reinforced polymers is divided into two parts, thermoplastics and thermosets. In the market thermosets is the dominating material. Common used matrix material when manufacturing glass and carbon fibre reinforced polymers is epoxy. This because of its high service temperature, good mechanical properties and that it has an excellent adhesion to other materials, Komposithandboken (2001). Table 2.5 shows typical properties of epoxy.

Table 2.5 Typical properties of epoxy. (Komposithandboken, 2001)

Tension strength (MPa)	55-110
Young's modulus (GPa)	2.8-4.2
Density (kg/m^3)	1.2-1.3
Strain failure (%)	3-6
Bending strength (MPa)	125
Thermal expansion coefficient (10^{-6}/°K)	50-80
Poisson's ratio	0.25-0.33

2.3.3 Durability

CFRP is a very durable material in most aspects and the problems that can occur are mostly connected to the matrix. The biggest advantage of CFRP is that the composite do not corrode, providing that the matrix is intact when carbon fibre comes in contact with steel.

The knowledge of durability in FRP is very limited. Although according to test data a comparison between steel and FRP is made. The FRP is made of the same matrix material but with different fibre reinforcement, Table 2.6. (Täljsten, 2002)

Table 2.6 Comparison between different FRP and steel influenced by different factors: + indicates a good; = indicates average; - indicates a bad behaviour. (Täljsten, 2002)

Criteria	Steel	AFRP	CFRP	GFRP
Relaxation and creep	+	-	+	=
Moisture resistance	=	=	+	-
Alkali resistance	+	=	+	-
Thermal stability	=	=	+	-
Chloride resistance	-	=	+	=
Fatigue	=	=	+	-
Long-time	+	-	+	=

Epoxy is a polymer that resists deterioration fairly well. The matrix is sensitive to ultra violet (UV) radiation and high temperatures. The composite can be protected from UV with special additives and if there is risk of fire the composite should be protected by special arrangement. "Plate bonding with composites withstands fire a longer time than plate bonding with steel plates due to the lower thermal conductivity of the composites". (Carolin, 2003)

2.3.4 Manufacturing

There are several ways of producing fibre composites. The most common and cost-efficient technique for mass producing composites is pultrusion. This technique is limited in the way that components will have constant cross-section. The different types of manufacturing processes are wet layup, spray layup, prepreg layup, pultrusion, liquid moulding, compression moulding and filament winding. (Åström, 2002)

Pultrusion

Pultrusion is a technique where continuous fibre reinforcement is impregnated with a resin and then formed into a solid composite profile. There are several different ways to achieve impregnation and consolidation, Figure 2.11 illustrates the different steps in a basic pultrusion process. The reinforcement is pulled from the supply and is then gradually brought together into the resin bath, where the reinforcement is impregnated. From the resin bath the reinforcement entering the heated die. The die has a constant cross-section except for the tapered entrance, where excess resin is squeezed out of the reinforcement. The mixture of reinforcement and resin which now is a composite exits the die as a solid. The composite cool off before the pulling-mechanism pulls it to the saw where desired lengths are cut. (Åström, 2002)

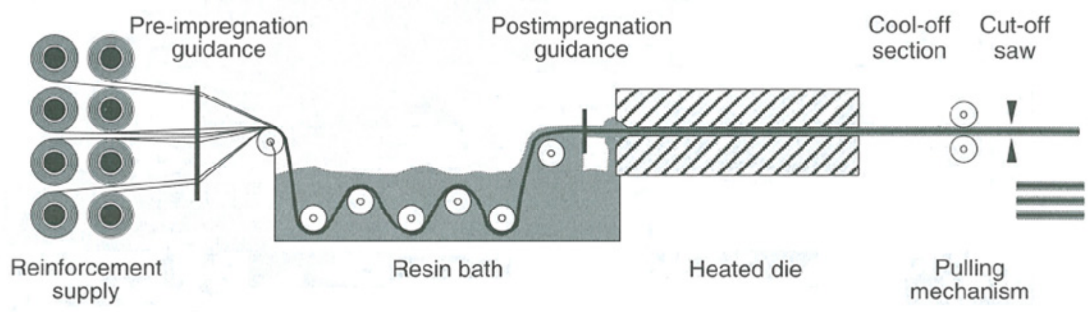


Figure 2.11 Pultrusion process, adopted from Åström (2002).

Wet layup

The wet layup technique is a process where the resin is applied in liquid form and the fibres is impregnated as part of the layup. Wet layup can be performed by hand (hand layup) or in a partially automated procedure (spray-up).

Hand layup is the most simple and versatile procedure to produce a fibre composite. The mould is treated by a mould-release agent. On this a neat resin layer (gelcoat) are sprayed or sometimes painted. This makes the reinforcement smooth and hides the reinforcement. On the gelcoat an appropriate amount of resin is applied and then the lays dry reinforcement on top of the gelcoat. A hand-held roller compact the laminate and the resin is worked upwards through the reinforcement. Now are one layer finished and this procedure continuous until intended laminate thickness is reached. (Åström, 2002)

2.3.5 Adhesive

To join composites and to join composite to other materials adhesives are used. The adhesive should have sufficient shear strength to manage to transfer the loads between the components. The most common high-performance adhesives are made of polymers and are of the epoxy family, meaning that the polymer hardens when mixed with a catalysing agent or "hardener", Åström (2002). Table 2.7 shows different adhesives used in construction.

Table 2.7 Different adhesives and manufacturer.

Name of Product	Type	Manufacturer
Sikadur-30	Epoxy	Sika Chemie GmbH
Ispo Concretin SK 41	Epoxy	Ispo GmbH
S&P Resin 220	Epoxy	S&P Reinforcement
Collano Purbond HB 110	Polyurethane	Ebnöther AG
Dynosol S-199 with H-629	Resorcin	Dyno Industries

2.3.6 Steel fibre reinforced polymers (SFRP)

SFRP is a new composite made from ultra high strength steel fibres. The fibres are made of twisted steel wires of fine diameter 0.2-0.35 mm. This steel used in SFRP is 11 times stronger than typical reinforcing bars. The fibres can be impregnated with thermoset, thermoplastics or with cementitious resin systems.

SFRP provides a tensile strength up to 1170 MPa and a Young's modulus up to 80 GPa. These values depend on how the fibres are wrapped together and which twist angles they have, see Figure 2.12. Composites made from SFRP are up to 70% thinner and 25% lighter than composites made with glass fibres with the same properties. SFRP is priced like GFRP and yet performs like CFRP to a much lower cost. A comparison between CFRP and SFRP gives similar results in terms of ultimate tensile strength and ultimate deflection, this if the same reinforcing percentage and bonding agent (epoxy) is used. (Hardwire LLC, 2007)

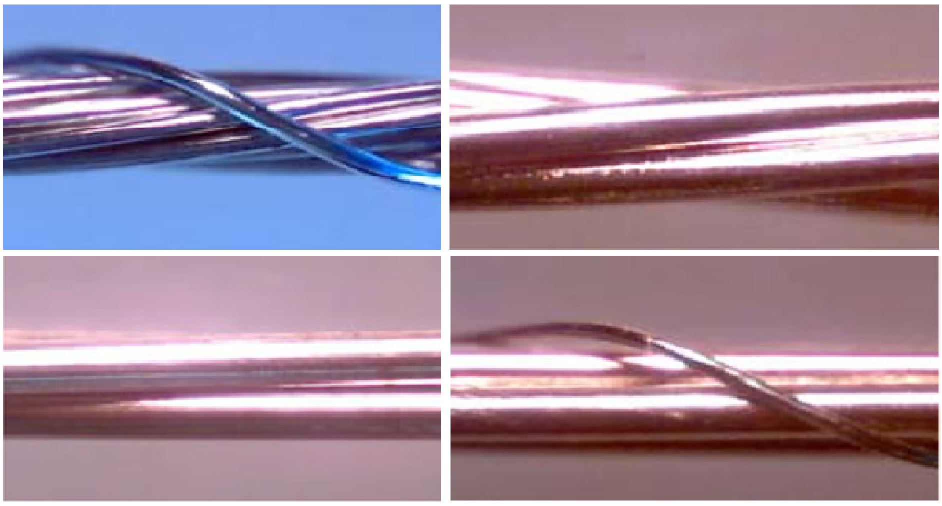


Figure 2.12 Different kinds of steel fibres with different wrapping and twist angle, adopted from Hardwire LLC (2007).

2.4 Timber beams and floor structures

In Sweden very few of all timber houses are higher than two floors. One reason is that historically it was not allowed to build higher than two stories due to the risk of fire. In 1994 this regulation was removed and now the regulation is instead based on function. This means that the structure must fulfil some functions and then the material can be chosen free. The fire separating structure between two apartments must resist fire for one hour, for example. Still very few multi-storey timber houses were built. Perhaps one of the reasons is that timber beams can not fulfil the same design requirements as steel and concrete.

A floor structure is a horizontal load-bearing component that mainly carries vertical loads. There are different kinds of floor structures depending on its position in the building. For example loft ceiling structure, intermediate floor and crawl-space floor structure are some of the different floor types, see Figure 2.13.

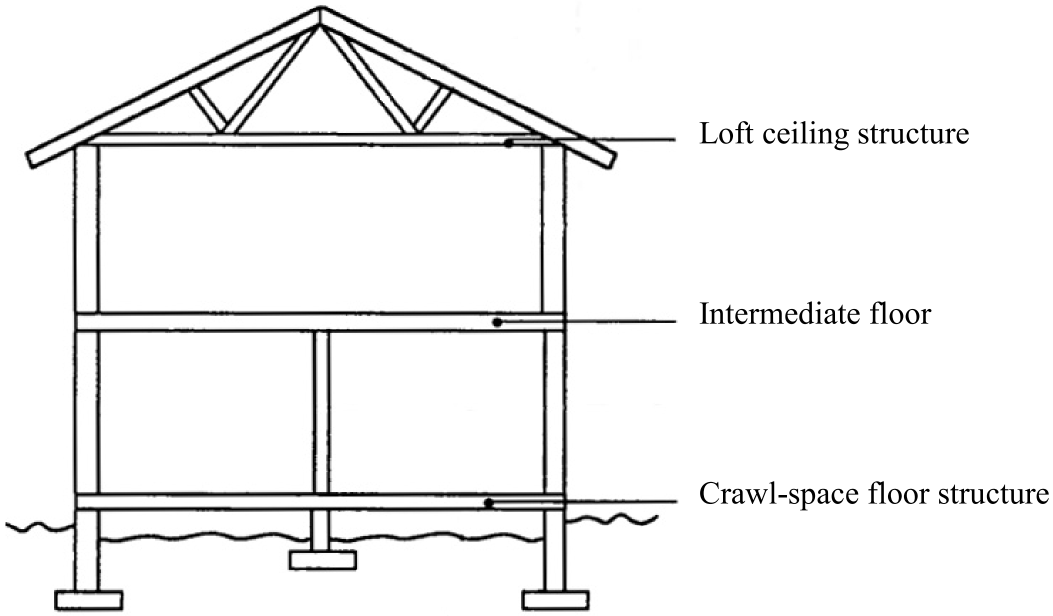


Figure 2.13 Different types of floor structure in common timber structures, adopted from Kliger et al. (1994).

A timber floor is made of solid timber beams, usually with a spacing of 600 mm, with ceiling cover on one side and the flooring on the other. In Sweden the most common flooring is 22 mm thick chipboard. Between the flooring and the ceiling cover there must be enough space to get the required amount of insulation. Figure 2.14 shows a typical floor structure. The floor beams are the part of the floor that carries load. Different loads that may appear on different floor types are specified in BKR and Eurocode. (Kliger et al. 1994)

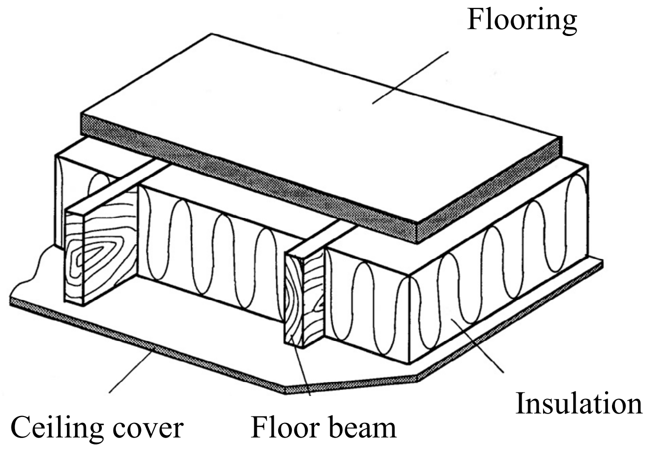


Figure 2.14 Parts in timber floor structure, adopted from Kliger et al. (1994).

2.4.1 Design of timber floor joist

A timber floor joist is affected by different loads; mainly it is self-weight and imposed loads. The self-weight for an ordinary timber floor joist is roughly about 0.5 kN/m^2. Imposed load varies due to the use of the building. Load duration and moisture content affect the load capacity of a floor joist. This is considered in the design of a structure with a service class and a load duration class. The decisive criterion when designing timber floor beams are deflection and vibrations and not the load capacity which always has to be fulfilled.

A floor structure shall be designed not to cause disturbing vibrations to people in the building. To manage this, beams must have sufficient stiffness for example. The height of timber beams is usually based on the stiffness requirements, but also the required insulation can have the decisive effect. One criterion is that the floor joist has enough stiffness so that the deflection in the midpoint of a single floor beam shall not exceed 1.5 mm during the influence of a 1 kN static point load. Another criterion is that the deflection of evenly distributed loads should be checked. The deflection may not exceed 1/500 of the free span length, Kliger et al. (1994).

There are several different ways to increase the stiffness of a floor joist. The height of floor beams can be increased or the spacing between floor beams can be decreased. Another way is to create a load transferring interaction between the floor beams and the sheeting. The most common way is to screw and glue the sheeting to the beams so the sheeting acts with full interaction with the beams. It is not always possible to make any of theses adjustments. This can be due to lack of space or that it is an existing structure. Then reinforcing the floor beams with FRP or steel laminate can be an option.

To be able to use sawn timber of standard dimensions in floor structures the span may not be over 5.5 meters. This due to that the Swedish timber manufacturers do not produce beams longer than 5.5 meters without joints. If the span is over six meters finger joints has to be used to create a continuous beam. If finger joints are used in the beam, different heights cause differences. The acceptable tolerance (Δ) between beams is $\pm$ 1 mm, see Figure 2.15. If the difference is larger than the tolerance it could cause disturbance in interaction between the flooring and floor beams.

When floor beams are delivered it is important that these will be conditioned to match the climate to which the structure is expected to be in. If the climate differs to much from the precondition climate this can cause distortions of the beams e.g. cup, Figure 2.15. Cup results in incomplete contact between the flooring and the beam. This can lead to problems in glued joint and to creaking in the floor. When designing a timber floor, considerations of sound- and heat isolation, air closeness and moisture protection needs to be done to fulfil all requirements. (Kliger et al. 1994)

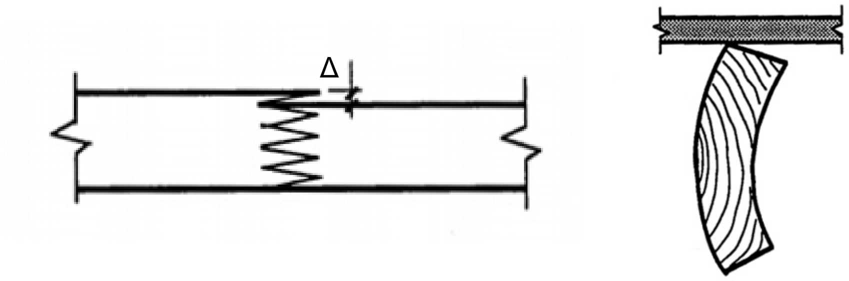


Figure 2.15 Differences in finger joint of floor beams (left), reduced floor contact as a result of cup (right), adopted from Kliger et al. (1994).

3 Materials and method

In this chapter the different materials and their properties that were used in the tests are described. Also how the dynamic testing of Young's modulus was done is explained more thoroughly. Chapter 3.3 and 3.4 explains how the manufacturing was done and the composite specification. Finally, the bending creep tests and the test setup are explained.

The test was supposed to include test series of timber & epoxy (series 1), GFRP (series 2), CFRP-165 (series 3), CFRP-300 (series 4) and steel (series5). GFRP had to be excluded from the test due to lack of GFRP laminates on the Swedish market. The test was carried out with test series 1, 3, 4 and 5.

3.1 Test material

3.1.1 Wood

The wood material used in the test is pine sapwood that has been stored in the lab at Chalmers. In the study 24 specimens are used with the timber dimensions 45x70x1100 mm^3. The 24 specimens are taken from six trees from two different stands. Three of the trees are from a fast growing stand and the other three are from a slow growing stand. All specimens come from the same part of the tree in the length direction. In the radial direction the centre beam from the log is divided into six parts, part three and four are excluded due to juvenile wood that give large movements see Figure 3.1. The material is eight years old and come from old study conducted during the 90's. The origin of the timber is well documented and can be read about in Perstorper et al. (1994).

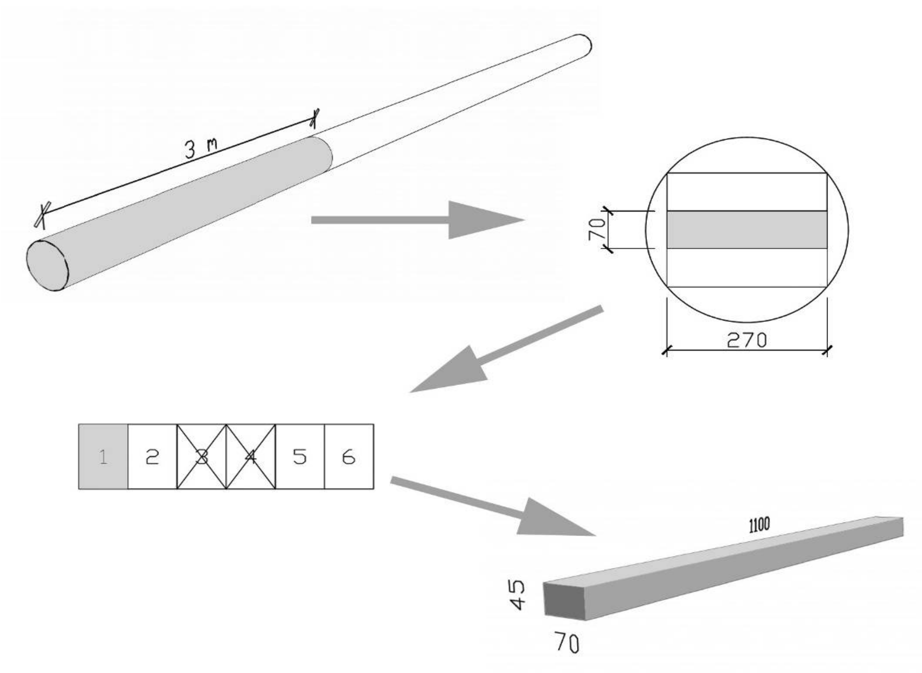


Figure 3.1 Position of the specimens in a stem and sawing pattern.

Young's modulus on each specimen is calculated from obtained eigen frequencies. A detailed description of how the eigen frequency is determined is given in Chapter 3.2. Based on Young's modulus the specimens are divided into four groups with six pieces in each group. The dividing also considers spreading by trees and radial placement. The mean value of Young's modulus in each group is approximate the same (~13 GPa). For information of the spread of specimens see Chapter 3.4. The annual ring width has been measured by counting the annual rings in the radial direction of the specimens and dividing them with the thickness of the specimen. The measured and obtained data for the specimens are shown in Appendix C.

3.1.2 CFRP

In the test two types of CFRP laminate are used, S&P Laminate CFK 150/2000 and Sika CarboDur H514. The laminates are designed to reinforce load-bearing elements made of concrete, wood, steel and natural stone. Properties for the two types of CFRP are shown in Table 3.1 and Table 3.2.

Table 3.1 Technical data of S&P Laminate CFK 150/2000 (provided by S&P).

Property	Value
Cross section (50mm x 1.4mm)	70 mm²
Modulus of elasticity	165000 MPa
Tensile strength in fibre direction	2310 MPa
Elongation at break	1.4%

Table 3.2 Technical data of Sika CarboDur H514 (provided by Sika).

Property	Value
Cross section (50mm x 1.4mm)	70 mm²
Modulus of elasticity	300000 MPa
Tensile strength in fibre direction	1350 MPa
Elongation at break	0.45%

3.1.3 Steel

In the test steel laminates from hot-rolled sheet metal is used. The properties of the steel are shown in Table 3.3.

Table 3.3 Technical data of standard construction steel.

Property	Value
Cross section (50mm x 2mm)	100 mm^2
Modulus of elasticity	210000 MPa
Tensile strength	235-355 MPa

3.1.4 Adhesive

The adhesive used is S&P Resin 220 epoxy which is a solvent-free, thixotropic two-component epoxy from S&P Reinforcement. This epoxy has been specially developed for bonding CFRP-laminates. To achieve maximum performance according to manufacturer the material that will be reinforced needs a tensile strength of at least 1.5 MPa. Both the surfaces where the adhesive is to be applied must be free from substances that may impair adhesion. The surfaces must also be dust-free, clean and more or less dry. The adhesive was used for both CFRP and steel reinforcement. The properties of the adhesive can be seen in Table 3.4. (S&P Reinforcement 2007)

Table 3.4 Technical data of S&P Resin 220 epoxy (provided by S&P).

Property	Value
Form (Comp. A, Comp. B)	Paste
Colour (Comp. A)	Light Grey
Colour (Comp. B)	Black
Colour (Comp. A + Comp. B)	Grey
Density (Comp. A)	1750 kg/m^3
Density (Comp. B)	1750 kg/m^3
Density (Comp. A + Comp. B)	1750 kg/m^3
Mixing Ratio (A:B)	4:1, (parts by weight and volume)
Pot life	> 60 minutes (at +20 °C)

Bending tensile strength	> 30 MPa
Compression strength	> 90 MPa
Adhesive strength	> 3 MPa
Application temperature	+10 °C to +35 °C

3.2 Dynamic measurement of Young's modulus

The 24 specimens were tested dynamically to obtain Young's modulus. The specimens were weighted with a scale to receive the bulk density of each specimen. The accuracy of the scale was 0.01 g. The specimens were simply supported at the ends with flexible foam rubber supports. These supports were used to simulate the free-free support condition. Resonance frequencies were determined from induced vibrations in the fibre direction (longitudinal) generated by impact at one end of the specimen with a steel hammer. The sound pressure was registered in the opposite end of the impact by a microphone connected to a computer-based data acquisition system Pico Scope ver. 1.12 and 2.03. From the sound pressure the frequency spectrum was established with Fast Fourier Transforms (FFT). Figure 3.2 shows the first and second mode shapes corresponding to the first two eigen frequencies of a simply supported beam. The dynamic longitudinal elastic modulus was calculated from the standard solution of the wave equation for longitudinal vibrations of a slender rod with free-free support conditions; using equation 3.1. (Olsson Perstorper, 1991)

$$E_{dyn}^{long} = 4 \cdot L^2 \cdot f_1^2 \cdot \rho \tag{3.1}$$

E_{dyn}^{long}	Measured dynamic longitudinal Young's modulus [Pa]
L	Length of beam [m]
f_1	First resonance frequency [Hz]
ρ	Density [kg/m^3]

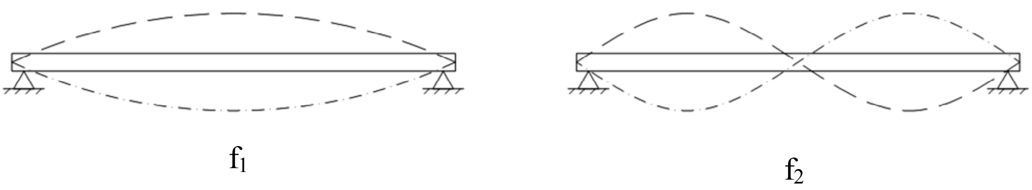


Figure 3.2 The two first mode shapes corresponding to the first two eigen frequencies of a simple supported beam.

3.3 Manufacturing of composite beams

The original timber beams were cut perpendicular to the tension face with a varying depth of 1-4 mm. The reason for the cut is to achieve a levelled surface on the tension side of the specimens, which can be seen in Figure 3.3.

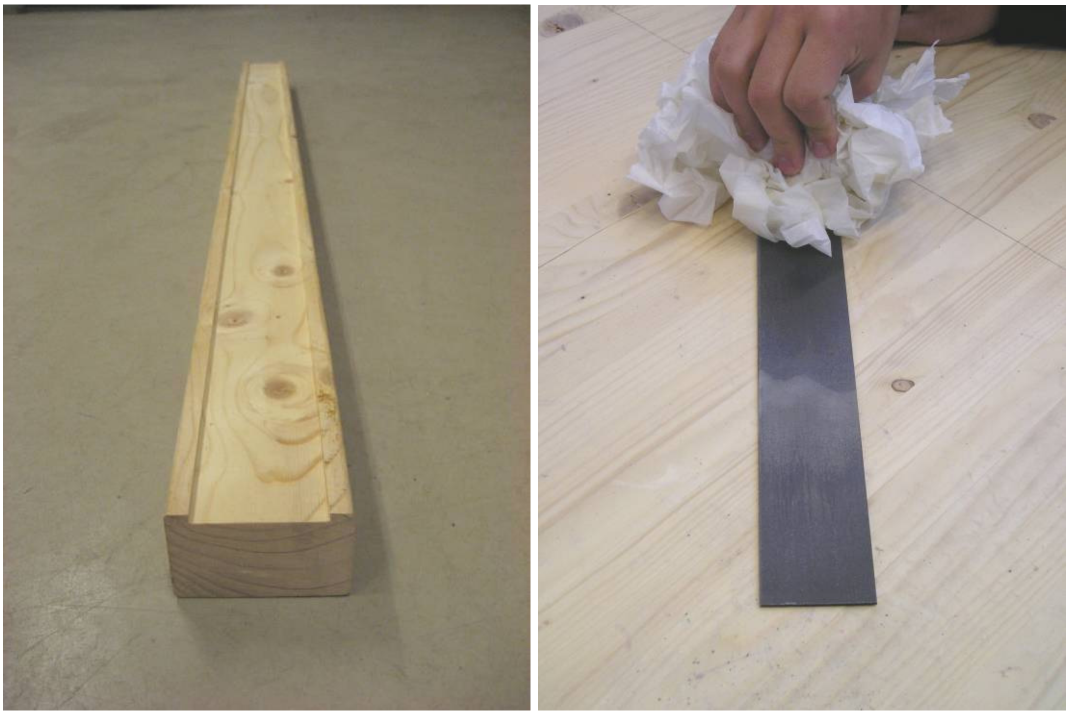


Figure 3.3 *Cut in test specimens for reinforcement (Left), cleaning the CFRP before application (Right).*

The carbon fibre was cut with a hacksaw into specimens with a length of 1100 mm. The ends of the CFRP specimens were duct taped to prevent rupture of the fibres when cutting.

Before the reinforcement could be glued on to the timber beams both materials had to be prepared to ensure sufficient bonding of the epoxy. The steel lamellas were blasted to gain a rough surface. After the blasting the surface was cleaned with acetone. The CFRP lamellas were cleaned with S&P Cleaner to remove dust and grease, Figure 3.3.

The adhesive components were mixed according to the ratio provided by the manufacturer, Figure 3.4. After approximately five minutes of mixing the two components were fully mixed and had a uniformly grey colour.

Figure 3.4 Mixing the epoxy (left), applying the adhesive to the laminates (right).

A layer of adhesive was applied with a spatula onto the beam to a thickness of approximately 1 mm. On the reinforcement a 2 mm roof-shaped layer was applied, seen in Figure 3.4. The two surfaces were pressed together until the adhesive is pressed out on both sides of the joint. Applying reinforcement in this way minimises the risk of air-bubbles trapped inside the adhesive. The adhesive will gain its final strength after curing for seven days.

3.4 Composites specification

The timber specimens are modified in four different ways. Two groups are reinforced with CFRP one with intermediate modulus (IM) of 165 GPa and one with ultra high modulus (UHM) of 300 GPa. One is reinforced with ordinary steel with Young's modulus of 210 GPa. In the last group one half of the specimens are modified with epoxy and the other part is not modified at all. The un-reinforced group is to refer the results from the reinforced specimens. The reason why it is modified with epoxy is to see the difference in creep when one side is sealed. All specimens are sealed with silicone at the ends to ensure a two-dimensional moisture transport.

Table 3.5 shows the different specimens and their modifications. Also the specimen number and the number of origin are shown. In Figure 3.5 the number of origin is explained more detailed.

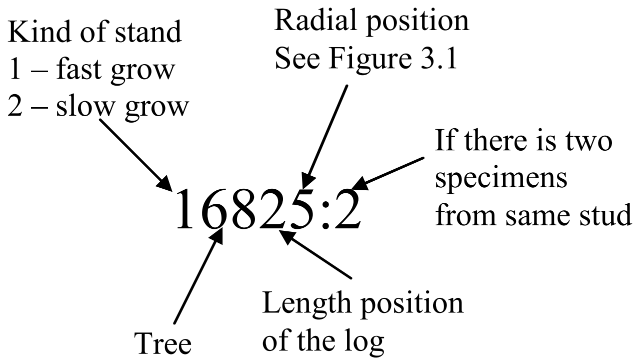
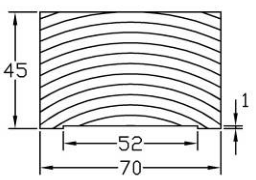
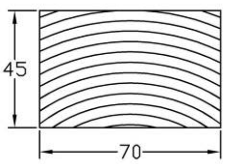
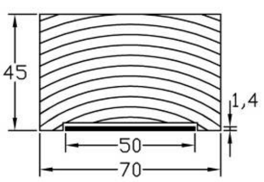
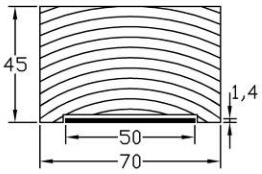
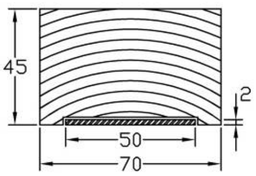


Figure 3.5 *A detailed explanation of the number of origin according to Perstorper et al. (1994).*

Table 3.5 *Data of the specimens and their modifications.*

Test series	Geometry	Modification	Quantity	Specimen	Original Number
1		Epoxy	3	1:1 1:2 1:3	16825 16225:2 20221
1		-	3	1:4 1:5 1:6	16221 16326 21925
3		CFRP-165	6	3:1 3:2 3:3 3:4 3:5 3:6	16821 16222:2 16225 21926 20121 20225

4		CFRP-300	6	4:1 4:2 4:3 4:4 4:5 4:6	16826:2 16822 16221:2 20126 21921 20122
5		Steel	6	5:1 5:2 5:3 5:4 5:5 5:6	16821:2 16826 16322 16325 20266 21922

A composite wood beam can be calculated as any other composite material, e.g. reinforced concrete or timber I-beams. The Young's modulus is calculated as an equivalent timber cross-section where the width of the timber beam is increased with a constant α depending of the ratio of Young's modulus between the two materials. This makes that the cross section is seen as it comes from the same material but with a new shape. The two materials are assumed to be completely interacted which means that no slip will occur in the adhesive. The free shrinkage-swelling movement of timber will now be constrained on one side for all the reinforced beams. In Figure 3.6 an example for equivalent cross section is shown. The example is for the calculation of a timber floor joist with chipboard as floor material and CFRP as reinforcement material.

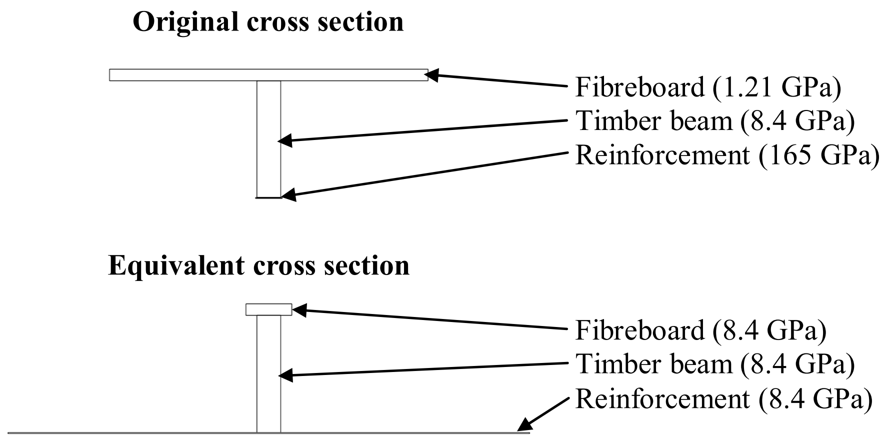


Figure 3.6 Original and equivalent cross sections of a timber floor joist.

3.5 Bending creep test

24 timber specimens were subjected to a four-point bending creep test. The test took place in a climate chamber with the temperature of 23˚C (±1˚C) and a varying RH between 30% and 90% (±2%), see

Figure 3.7. The specimens were exposed to four moisture cycles (16 weeks). Each cycle was four weeks with two weeks in 90% RH and then two weeks in 30% RH. Before the cycles started the specimens were preconditioned to an equilibrium moisture content corresponding to 65% RH with a temperature of approximately 23˚C.

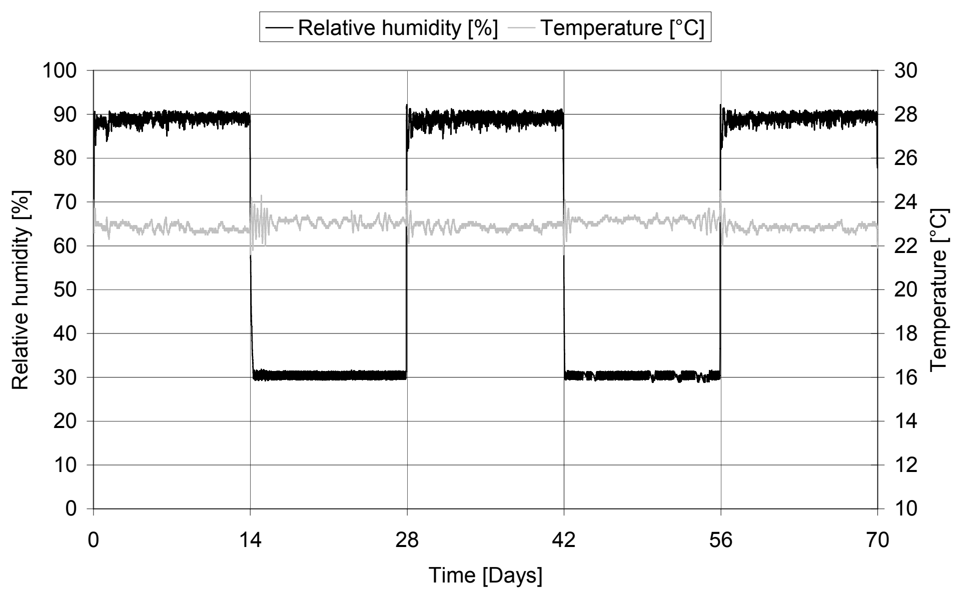


Figure 3.7 Recorded climate variation in the climate chamber during test period.

Six specimens were connected in series to create one set, see Figure 3.8. Each set were loaded flat wise to a bending stress in timber of 8 MPa. This corresponds to an utilisation factor of the bending stress of 54% for the slow growing stand and 87% of the fast growing stand. In Figure 3.9 the steel plates that were attached to each specimen to hold the suspension device in place is shown.

Figure 3.8 Loading arrangement with six specimens in one series (Left). All test specimens loaded (Right).

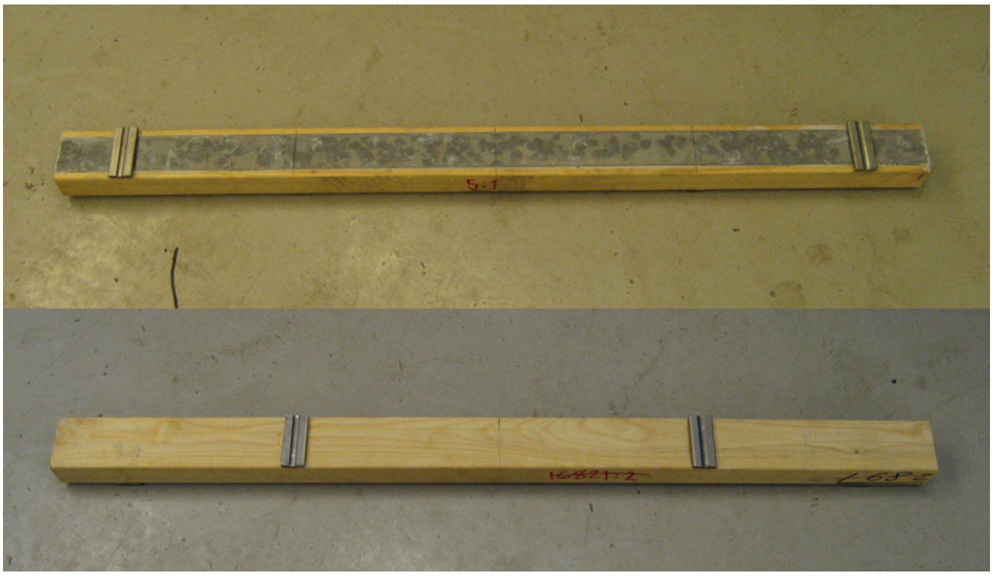


Figure 3.9 Steel plates attached with epoxy on both sides of the beam.

The weights of the specimens, steel plates and the suspension device were taken into account by varying the length of the lever arms (a), shown in Figure 3.10. This is done to obtain a bending stress in timber of 8 MPa on each specimen, see Appendix D for complete calculations. The tension side is the side nearest to the pith. This test setup is adopted from standard EN 408 (Anon. 2003) which describes methods of determine material properties.

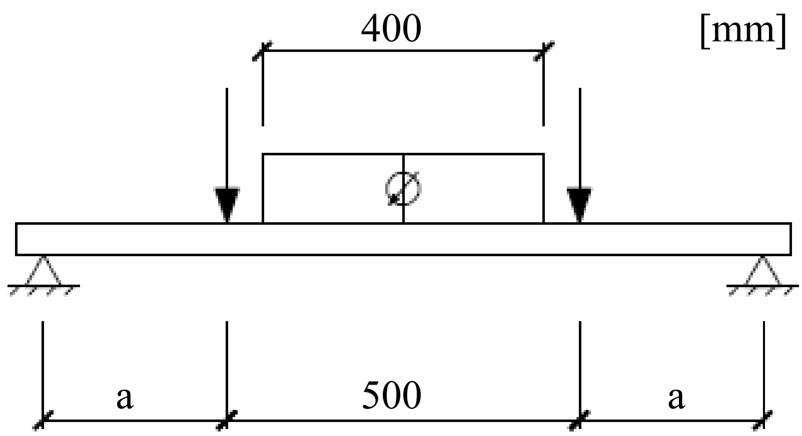


Figure 3.10 Loading arrangement for the creep tests were a is the varying lever arm.

The deflection in the constant moment region was measured using Linear Variable Differential Transducer (LVDT) gauges, with a precision of 0.001 mm, connected to a computer-based data collecting system. During the loading procedure the deflection was recorded every 2 seconds then was the interval 5 seconds for the first hour and finally it was recorded once every hour.

4 Results

4.1 Dynamic measurements of Young's modulus

Young's modulus of the composite beams where all measured with a dynamic testing method that are further explained in Chapter 3.2. When the initial dynamic Young's modulus ($E_{dyn.initial}$) was measured the moisture content in the timber was approximately 10%. After the timber was adapted to an RH of 65% the moisture content has change to 12%. This change in moisture content decreases Young's modulus in the timber with 3% according to Table 2.1. At this moisture content the beams were measured dynamically one more time ($E_{dyn.conditioned}$). The compensated Young's modulus ($E_{compensated}$) is the initial dynamic Young's modulus reduced with 3%. The mean deviation between the $E_{dyn.conditioned}$ and $E_{compensated}$ is approximately 2.3% for series 1. Results are shown in Figure 4.1 and calculations can be found in Appendix D.

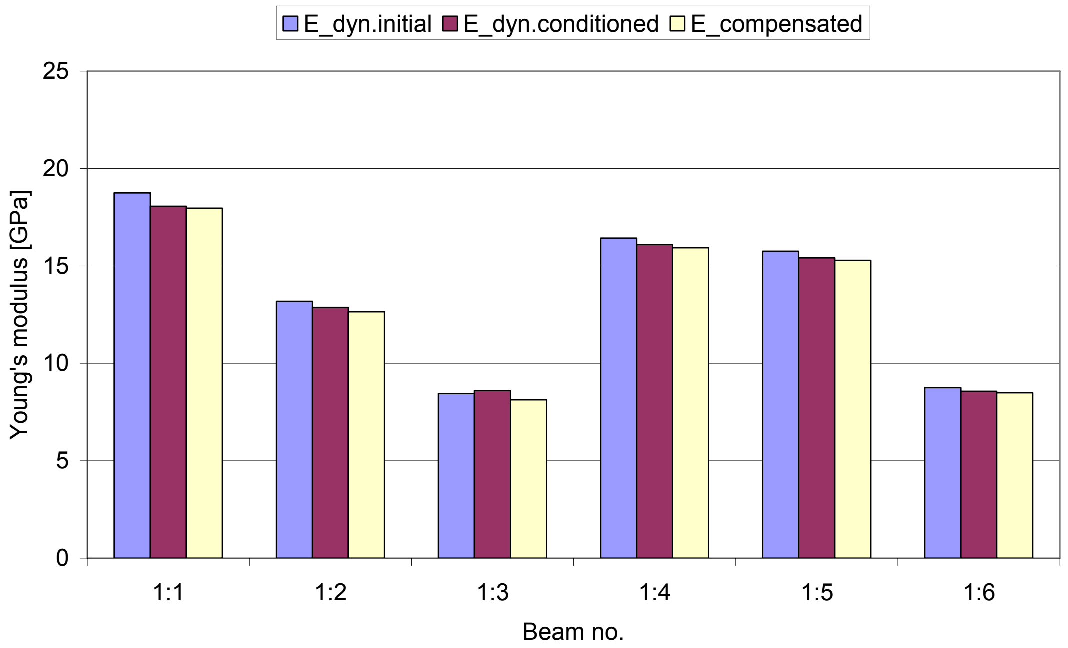


Figure 4.1 Comparison between Young's modulus in series 1 were E_dyn.initial is the first dynamic measurement of Young's modulus. E_dyn.conditioned is the dynamic measurement of Young's modulus after conditioned to 65% RH. E_compensated is the analytical compensated value of E_dyn.initial due to change in moisture content.

The dynamic measured Young's modulus for the composite ($E_{dyn.composite}$) was measured like $E_{dyn.conditioned}$. The analytical Young's modulus for the composite ($E_{analytic.composite}$) was calculated as an equivalent cross-section according to equation 4.1.

$$E_{analyticcomposite} = \frac{0.97 \times E_{dyn.initial} \times (A_{beam} - A_{cut}) + E_{rein} \times A_{rein} + E_{adhesive} \times A_{adhesive}}{(A_{beam} - A_{cut}) + A_{rein} + A_{adhesive}} \quad (4.1)$$

$A_{adhesive}$	Area of adhesive in timber beam cross-section [m²]
A_{beam}	Cross-section area of the timber beam [m²]
A_{cut}	Area of cut in timber beam cross-section [m²]
A_{rein}	Cross-section area of the reinforcement [m²]
$E_{adhesive}$	Young's modulus of the adhesive [MPa]
$E_{analytic.composite}$	Compensated Young's modulus for the equivalent cross-section [MPa]
$E_{dyn.initial}$	Measured dynamic Young's modulus after cutting [MPa]
E_{rein}	Young's modulus of the reinforcement [MPa]

In series 3 which are reinforced with CFRP-165 the average increase in stiffness between $E_{dyn.initial}$ and $E_{dyn.composite}$ is approximately 27%. The beam that has increased the most in the series is the one with the lowest $E_{dyn.initial}$, which has increased with 43%. The difference between $E_{dyn.initial}$ and $E_{analytic.composite}$ are larger then for the difference between $E_{dyn,initial}$ and $E_{compensated}$ in series 1. This is probably due to uncertainties in the thickness of the adhesive layer and the Young's modulus of the adhesive. The Young's modulus of the adhesive is in the calculations estimated to be 4.5 GPa. The average difference in series 3 is 4.5% between $E_{dyn.composite}$ and $E_{analytic.composite}$. This can be seen in Figure 4.2.

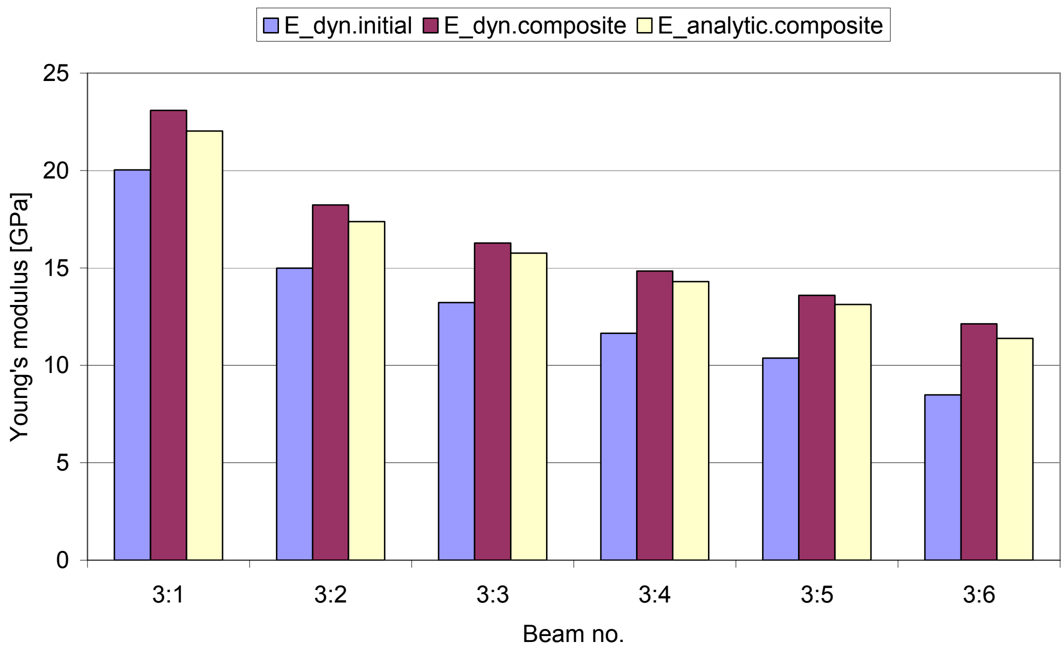


Figure 4.2 Comparison between Young's modulus in series 3 (CFRP-165). E_dyn.initial is the first dynamic measurement of Young's modulus. E_dyn.composite is the dynamic measurement of Young's modulus for the composite beam. E_analytic.composite is the analytical Young's modulus for the equivalent cross-section based on the E_dyn.initial value.

The series reinforced with steel shows similar behaviour as can be seen for all reinforced series. Also in this series the beam with the lowest $E_{dyn.initial}$ ends up with the highest stiffness after reinforcing. The increase for this beam (4:6) is 85% and the average increase between $E_{dyn.initial}$ and $E_{dyn.composite}$ for series 4 is 53%. $E_{analytic.composite}$ and $E_{dyn.composite}$ differ with an average of 3.7%. The results can be seen in Figure 4.3.

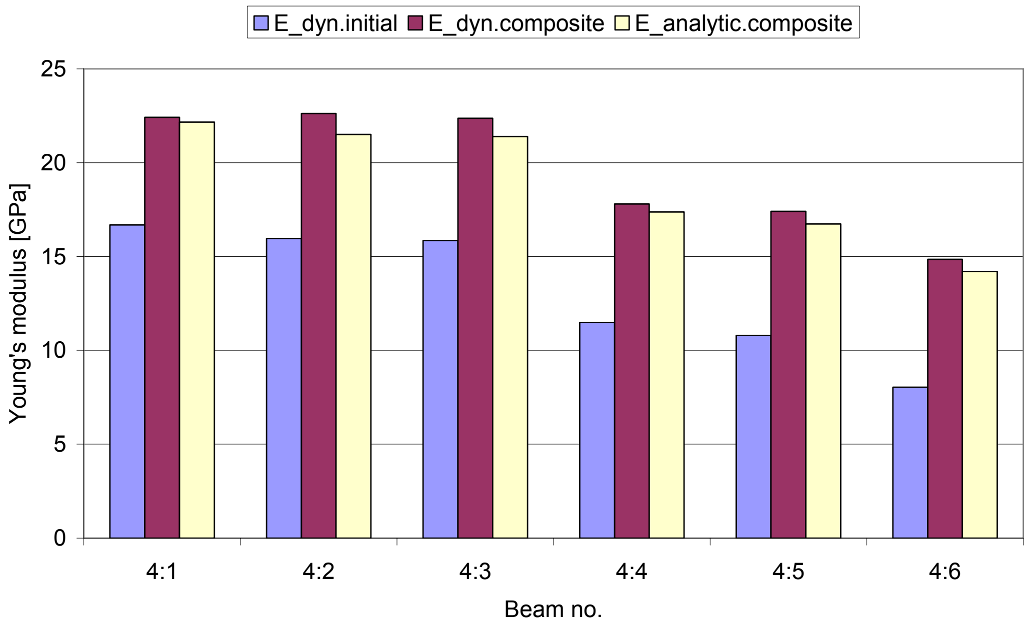


Figure 4.3 *Comparison between Young's modulus in series 4 (Steel). E_dyn.initial is the first dynamic measurement of Young's modulus. E_dyn.composite is the dynamic measurement of Young's modulus for the composite beam. E_analytic.composite is the analytical Young's modulus for the equivalent cross-section based on the E_dyn.initial value.*

In the last series the results from reinforcing shows average value of 48% increase between $E_{dyn.initial}$ and $E_{dyn.composite}$. The difference between $E_{dyn.composite}$ and $E_{analytic.composite}$ has an average of 2.3%. This can be seen in Figure 4.4.

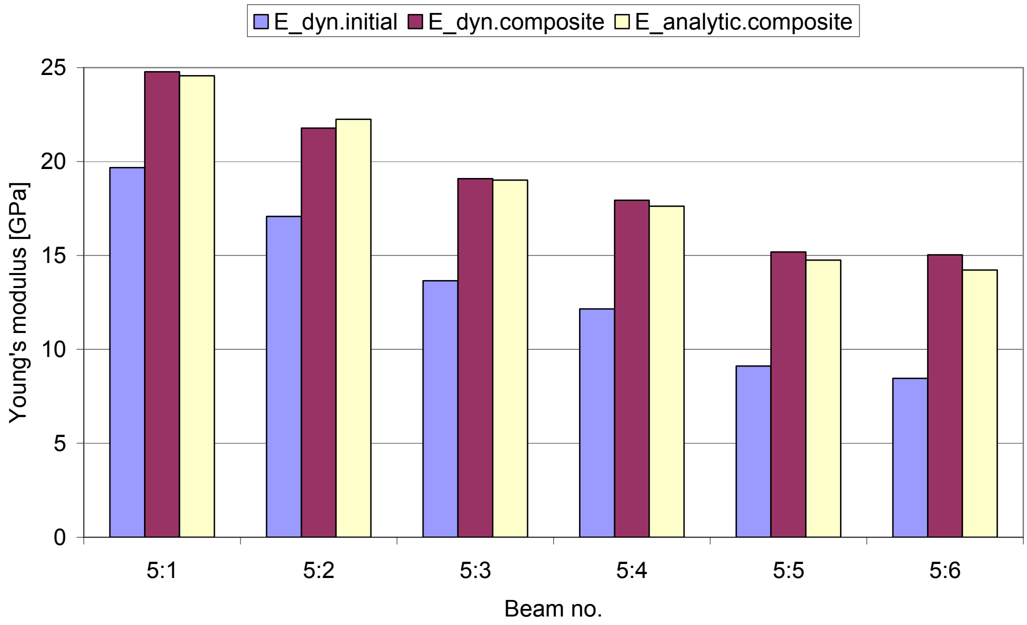


Figure 4.4 *Comparison between Young's modulus in series 5 (CFRP-300). E_dyn.initial is the first dynamic measurement of Young's modulus. E_dyn.composite is the dynamic measurement of Young's modulus for the composite beam. E_analytic.composite is the analytical Young's modulus for the equivalent cross-section based on the E_dyn.initial value.*

4.2 Long-term tests results

In this chapter the results will be shown both for each individual specimen as well as mean values for each test series. In series 3 and 5 there are some data missing in four of the test specimens. In the plots this is shown as broken lines. There were problems with the test equipment which caused the missing data.

From the tests the change in curvature over the constant moment area has been recorded. The relative creep (RC) is the increased deflection compared to the initial value of the deflection. Of the 24 tested specimens 22 behaved according to creep curve A, one like creep curve B and one similar to curve C, see Chapter 2.2.5. All the tested specimens show varying deflection as a result of moisture change. Both mechano-sorptive creep and shrinkage/swelling can be seen in the results.

Mean values of the relative creep from all series are shown in Figure 4.5. The mean values of the movements for the un-reinforced beams are smaller than for the reinforced specimens. Figure 4.6 show trend lines of the relative creep for all series. The creep increases in a higher rate over time than the un-reinforced series. The reinforcement prevents the creep to increase. After the wetting/drying cycle the reinforced beams tend to return to their original value.

The deviation between the analytical and the real values of the initial deflection is small. In Chapter 4.4 the correlation between these values is explained more detailed.

The deviation may be explained with that the analytical initial values are based on calculations from elementary cases for simply-supported beams. The formulas require a perfect curvature throughout the beam. This is often not the case since irregularities in the timber as well as knots can change this curvature and create a larger or smaller curvature at that point. The calculations are also based on values of the initial Young's modulus of the timber that where dynamically measured. A dynamic measurement of Young's modulus gives a mean value of Young's modulus throughout the beam in its length direction. This means that stronger cross sections are neutralised by weaker ones.

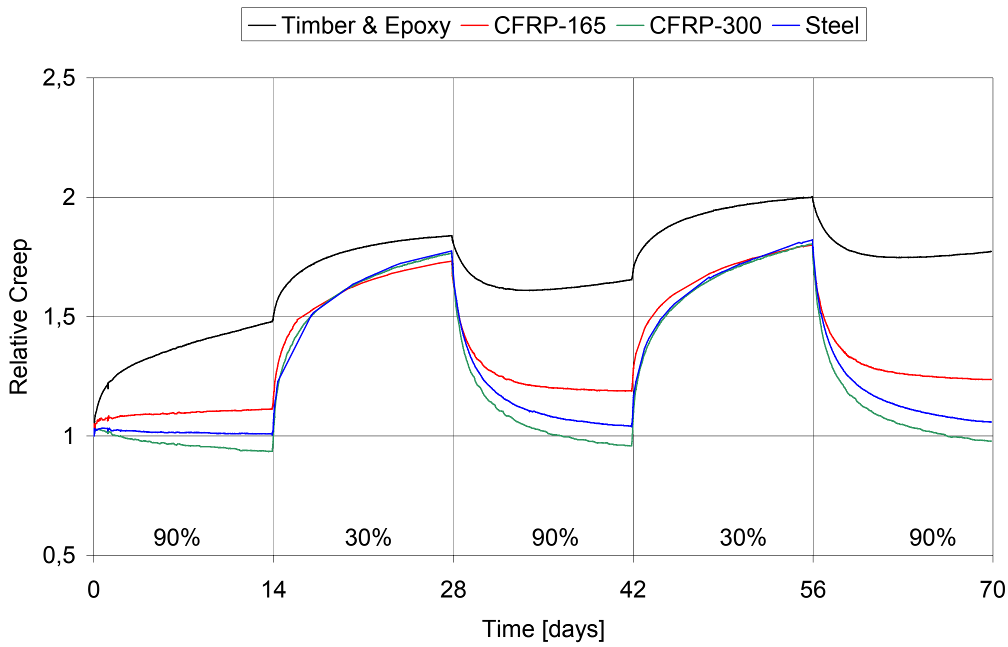


Figure 4.5 Mean value of all series of the relative creep for 30% and 90% RH.

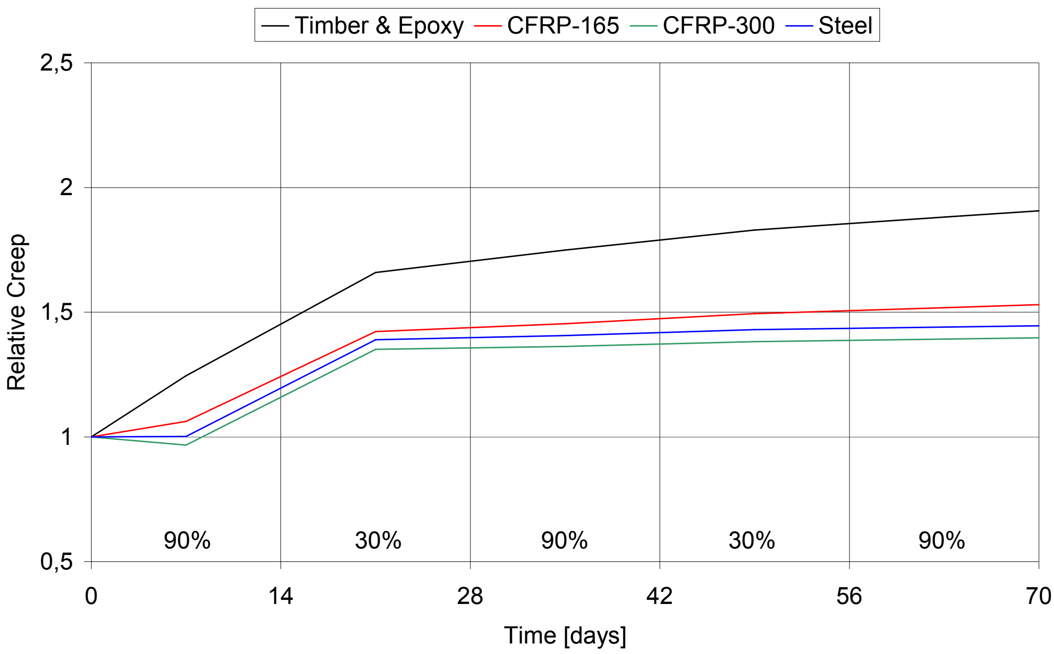


Figure 4.6 Trend lines of all series of the relative creep for 30% and 90% RH.

4.2.1 Series 1: Timber & Epoxy

In series 1 three out of six specimens were modified with an epoxy layer on one of the flat sides. There are no clear results of the effect from the epoxy which would be less creep. In the analysis of the test results the two types of test specimens are not separated.

The average recorded initial deflection of the un-reinforced test specimens is 0.55mm. During the two first weeks (RH=90%) the deflection increases from 0.55mm to 0.9mm which must be considered large since the deflection after the second period of 90% RH is 1.0mm. During one cycle the variation in deflection between the “wet” and “dry” part is about 0.16mm. The mean values of deflection in this series shows an increasing trend. After the first cycle the creep increases with approximately 19%, see Figure 4.8. Specimen 1:3 and 1:1 increases with 30-35%.

Specimens 1:3 and 1:6 have similar material properties yet the deflection behaviour is very different. For test specimen 1:3 the deflection increases at a rate much higher then expected. Specimen 1:6 shows little variation in deflection when the climate changes. Specimen 1:4 is the only beam that decreases its deflection during the drying part of a moisture cycle. This behaviour is represented by creep curve B. In Figure 4.7 the results are shown for the specimens mentioned in the text above.

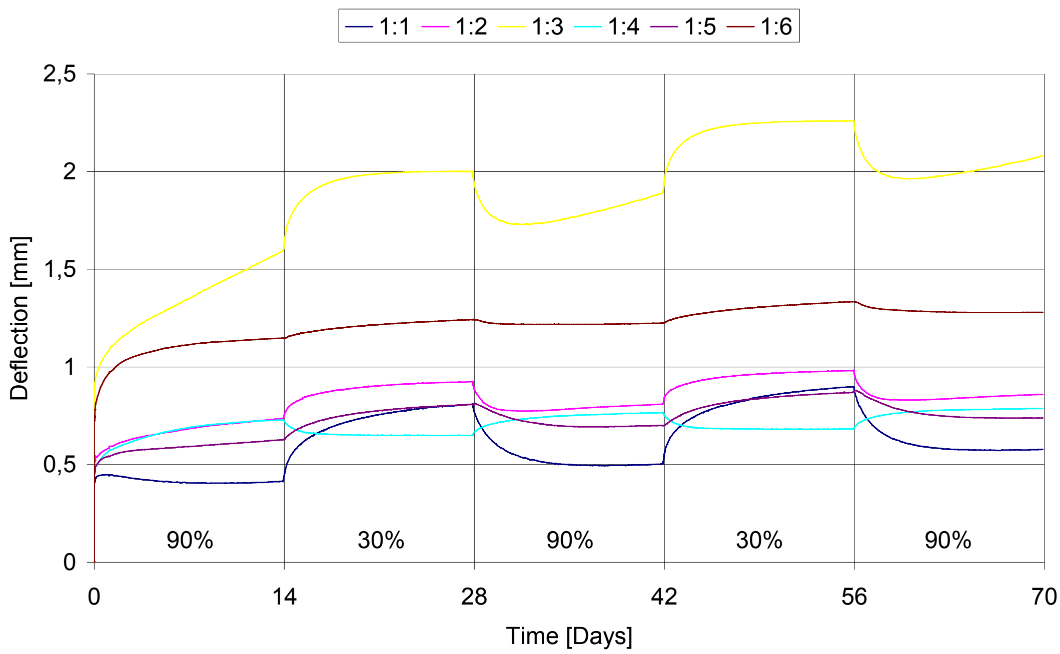


Figure 4.7 Deflection chart of series 1: Timber & Epoxy for 30% and 90% RH.

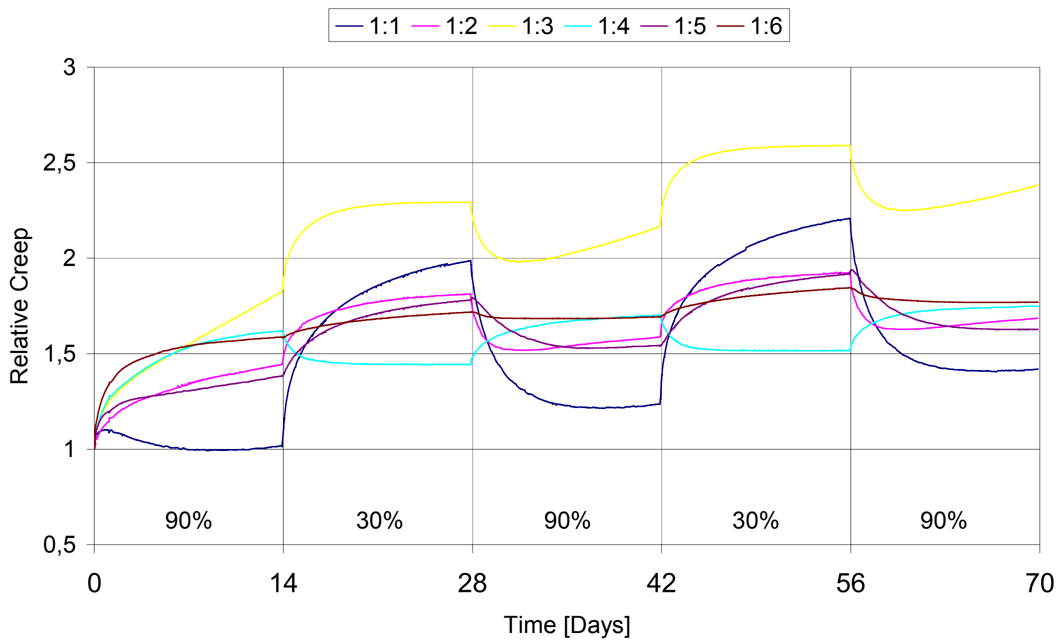


Figure 4.8 Relative creep chart of series 1: Timber & Epoxy for 30% and 90% RH.

4.2.2 Series 3: CFRP-165

The deflection increases with a mean value of 3% for the second moisture cycle in series 3 except for specimen 3:6 which has an increase of about 10% for the same time period. The specimen with the lowest Young's modulus deflect the most and the one with highest deflect the least, see Figure 4.9.

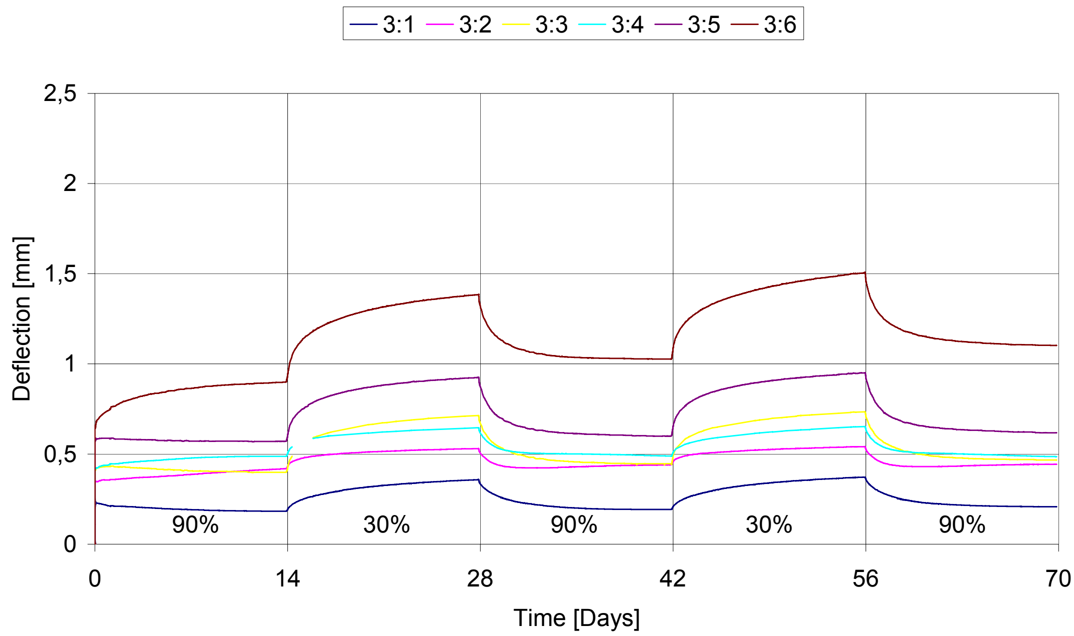


Figure 4.9 Deflection chart of series 3: CFRP-165 for 30% and 90% RH.

In series 3 the movements are rather large, see Figure 4.11. This can be seen in all the reinforced series. This is caused by reinforcement constraining free movements of swelling and shrinkage. When timber absorbs moisture from the air, it wants to expand. Since the reinforcement is placed on the tension side of the beam, the swelling will cause a negative curvature which will counteract the curvature from the load. When the climate changes to a low RH the beams wants to shrink instead. The curvature from shrinkage will now act together with the curvature from the load and the total deformation will be larger, Figure 4.10.

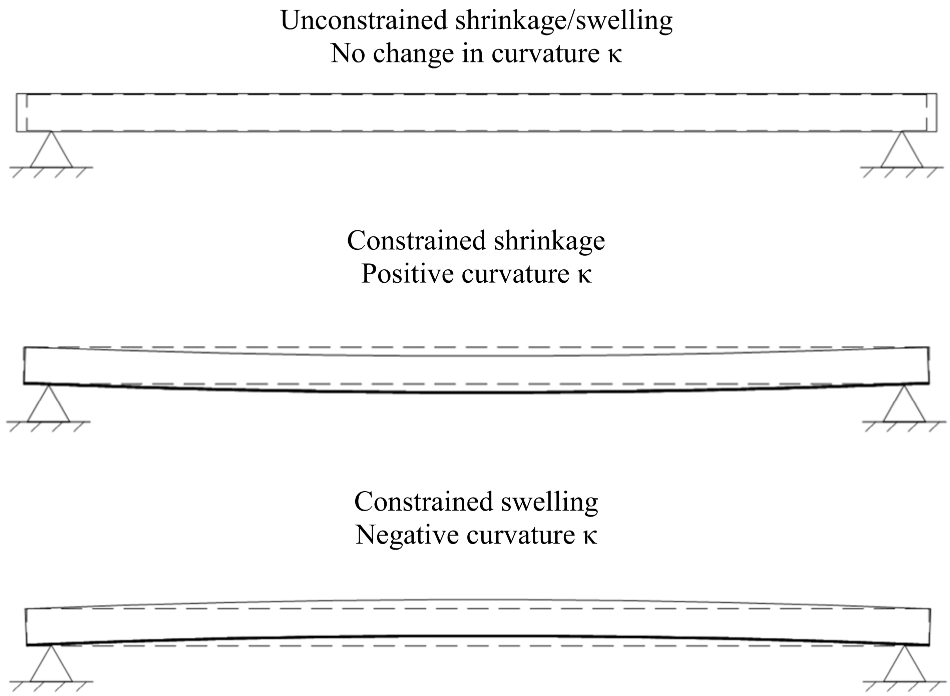


Figure 4.10 Change in curvature of a beam due to constrained edge and varying moisture content.

In specimens 3:3 and 3:4 data has been modified due to problems with the measurement equipment. During the time period were data is missing values that corresponds to curves with similar behaviour have been used to get a representative behaviour of the specimens.

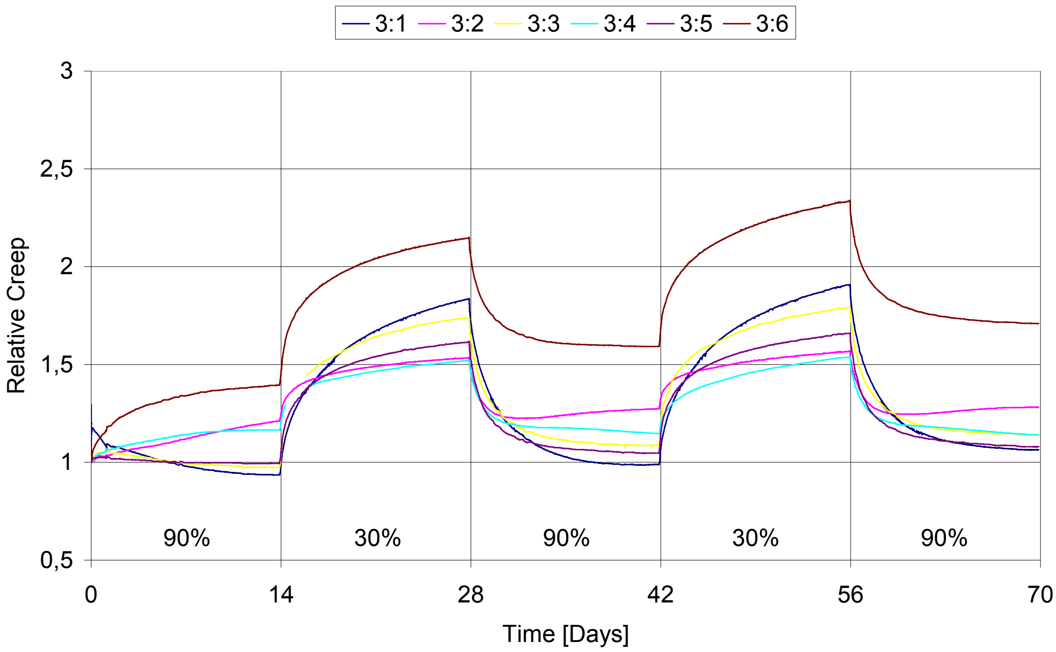


Figure 4.11 Relative creep chart of series 3: CFRP-165 for 30% and 90% RH.

4.2.3 Series 4: CFRP-300

In series 4 which is reinforced with CFRP, which has a stiffness of 300 GPa, the mean creep deflection increases with about 2% for each moisture cycle. In this series the creep deformations are smaller then the swelling. This phenomenon is seen in four of the six tested specimens; see Figure 4.12, which are also seen in the average value of the series, see Figure 4.5.

The deviation of the recorded data is small and all deflection curves are within a measurement of 0.4 – 0.6mm deflection.

The movements due to change in moisture content can be as large as 100% of the initial deformation. This is both from shrinkage/swelling and mechano-sorptive creep. This is especially seen in specimen 4:1 and 4:6. This is hard to explain since the two specimens have very different material properties. Specimen 4:1 has the highest Young's modulus and density; while specimen 4:6 has the lowest value of these parameters, see Figure 4.13. The mean value of the movements for this series is about 81%.

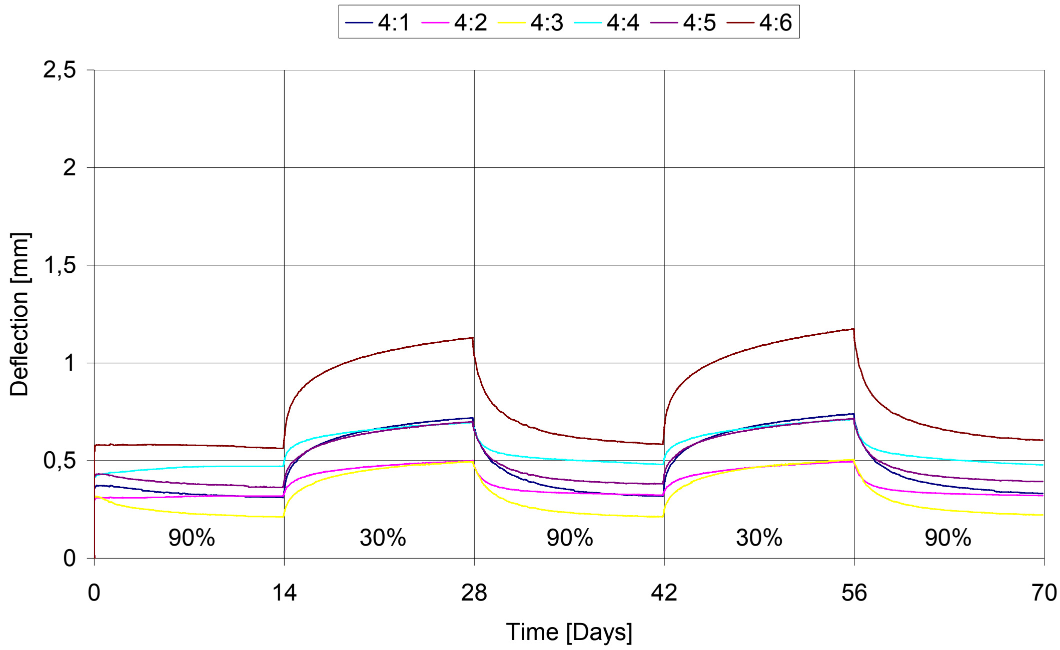


Figure 4.12 Deflection chart of series 4: CFRP-300 for 30% and 90% RH.

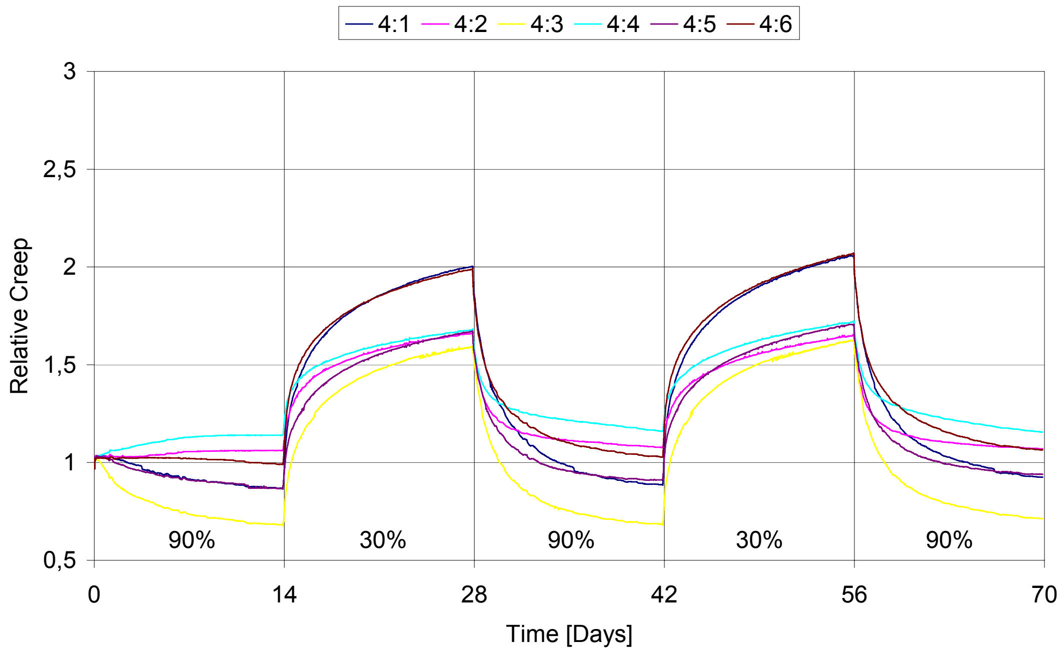


Figure 4.13 Relative creep chart of series 4: CFRP-300 for 30% and 90% RH.

4.2.4 Series 5: Steel

For series 5 the results are very similar to series 4. This series the beams are reinforced with steel instead of CFRP as in series 4. Young's modulus multiplied by the cross section area is the same for these two series. The steel laminates used have a lower Young's modulus then the CFRP-300, but have a thicker cross section. This is the reason for the similar behaviour between series 4 and 5.

Test specimen 5:6 is missing data over a long period of time. This is due to a malfunction in one of the cords between LVDT gauge and the computer that records data. This was discovered after a two week period. The remaining of recorded data from this test specimen has been ignored. For specimen 5:3 the curve has been modified due to modified reference value of the deflection. The deflections correlate to the stiffness well for this series with the exception for specimen 5:2. This specimen has the second highest stiffness but shows the second highest deflection.

Also in this series, as for series 4, the movements are rather large. It is noticeable that an average of series 5 shows that 74% of the movements occur during the initial deflection. Results from series 5 can be seen in Figure 4.14 and Figure 4.15.

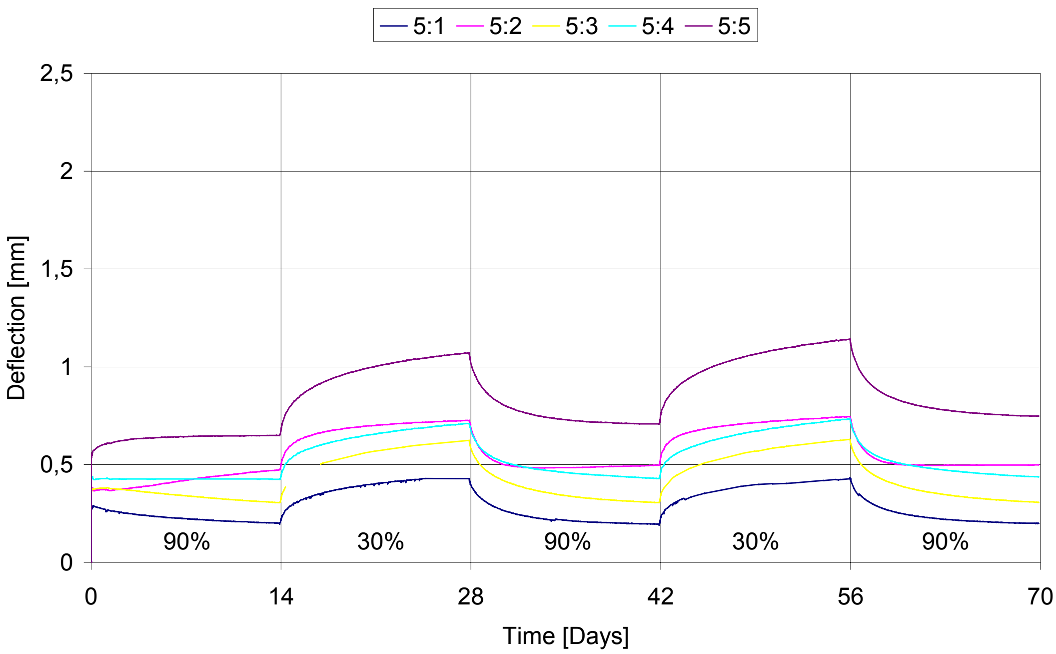


Figure 4.14 Deflection chart of series 5: Steel for 30% and 90% RH.

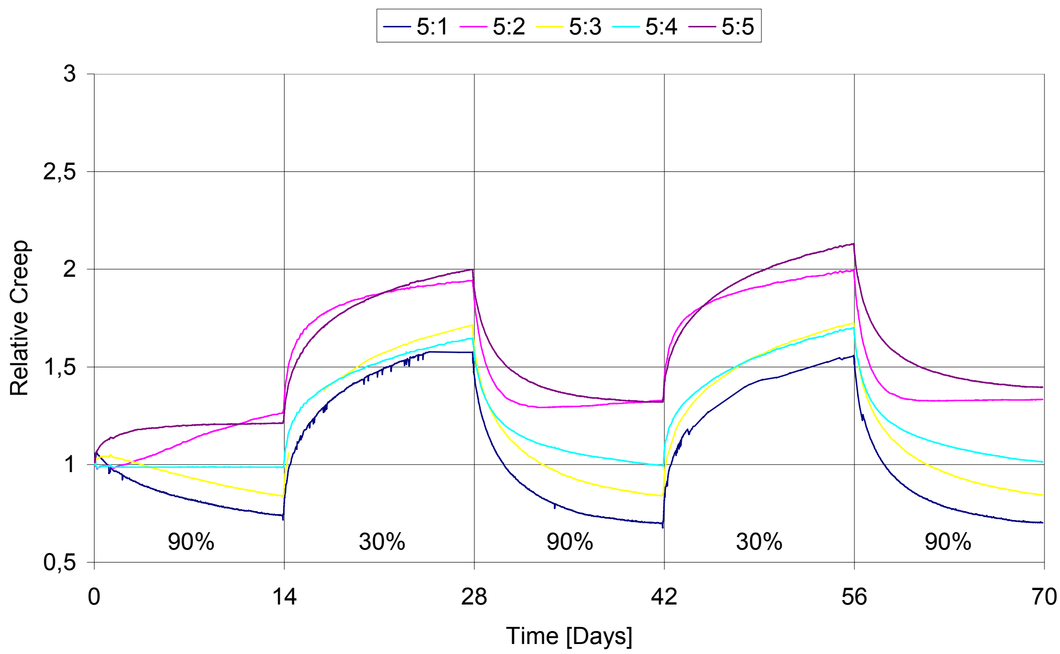


Figure 4.15 Relative creep chart of series 5: Steel for 30% and 90% RH.

4.3 Discussion and comparison of test results

When comparing the initial values of the deflection for all the test specimens the decrease in deflection follows the increase in stiffness caused by reinforcement. The composite with the lowest Young's modulus obtain the largest change in its curvature. This relation is also valid for the specimen with the highest Young's modulus which shows the smallest change in curvature.

According to Jobin & Olga (2007) glue laminated (glulam) timber beams could be increased in stiffness with as much as 80-107% depending on the reinforcement type, amount and arrangement. In this study the results show that it is possible to increase the Young's modulus in the range of 15 to 85% for one-sided reinforcement. This great variation in percentage is due to the fact that the increased stiffness depends on many factors. The initial Young's modulus of the timber beam that is supposed to be reinforced is of great importance. A beam with a low initial Young's modulus will obtain a larger increase in stiffness when reinforced, then a beam with high initial stiffness.

For creep deflection related to mechano sorption the connection between composite beams stiffness and increase in deflection is not as obvious. The higher stiffness, the lesser creep is valid for many of the beams but not for all of them. Some beams behave in a way not expected at all. In Series 1 (timber & epoxy), for example, there are three specimens that behave totally different. One specimen has a very high rate of deflection increase for each moisture cycle. One specimen barley varies its curvature when the relative humidity changes. A third example of unexpected behaviour is the specimen that follows creep curve B.

All the specimens that are reinforced have a constrained side which will cause a curvature of the beam when the relative humidity changes. For some of the reinforced beams the force caused by the curvature from the restrained swelling is larger then the force from the load. On the other hand the effect will be opposite when the climate changes and the timber want to shrink instead. Now the force acting on the beam will be “doubled”. This causes large changes in deformation between the two climates. If the beams where longer and with larger dimensions the deformations would increase and the adhesive stresses could be too large.

For the mean values of deflection between the different test series a clear trend is shown. For the un-reinforced timber specimens the creep increases for every moisture cycle. The reinforced series on the other hand may experience great variations in deflection through out the moisture cycle, but will not increase their deflection as much as a result of mechano-sorptive creep.

4.4 Correlation of material parameters

A correlation coefficient (r) shows the strength of a linear relationship between two sets of data. The r-value can be in the range -1 and 1 where 0 represents no relationship and between 0.8 and 1 is a strong relationship. A positive r-value represents a positive correlation when one value increases the correlating value increases. The opposite is a negative correlation when one value increases the correlating value decreases. The measured density of the timber specimens has a strong correlation to the initial Young’s modulus measured dynamically. They have a correlation of 0.89. The correlation between analytical values of the initial deflection compared with the real instantaneous deflection is 0.96. In Table 4.1 the r-values between the material parameters is shown. The analytical deflection values correlate better with the analytic Young’s moduli then the real deflection values does. With an r-value of 0.94 compared to 0.91 for the real deflection must both be considered to have strong correlation between deformation and Young’s modulus. To compare the deflections with the dynamic Young’s modulus of the composite instead is the values 0.94 and 0.92. The dynamic Young’s modulus of the composite correlates almost completely with the analytic Young’s modulus of the composite. The relative creep has an r-value of 0.99 between the first and the second cycle. The instantaneous deflection correlates to the relative creep with 0.58 and 0.59. In Appendix A the correlation matrix is shown in term of all values and their interrelation.

Table 4.1 Pearson correlation matrix.

	$E_{dyn.initial}$	$E_{dyn.composite}$	$E_{analytic.composite}$	ρ_i	$u_{analytic}$	$u_{inst.}$	Creep 1st cycle	Creep 2nd cycle
$E_{dyn.initial}$	1.00							
$E_{dyn.composite}$	0.79	1.00						
$E_{analytic.composite}$	0.81	0.996	1.00					
ρ_i	0.89	0.69	0.73	1.00				
$u_{analytic}$	-0.86	-0.94	-0.94	-0.78	1.00			
u_{inst}.	-0.79	-0.92	-0.91	-0.67	0.96	1.00		
Creep 1st cycle	-0.31	-0.40	-0.38	-0.10	0.50	0.58	1.00	
Creep 2nd cycle	-0.32	-0,42	-0.41	-0.11	0.52	0.59	0.99	1.00

4.5 Case study of possible application of reinforced timber

For a simplified timber floor joist the span length can be increased up to as much as 23%, Figure 4.17. This is done by combining timber of Swedish quality K12 together with CFRP-300. Comparison has been made with timber qualities K12, K18, K24, K30 and K35. These timber qualities have been compared to reinforcement of steel, GFRP, CFRP-165 and CFRP-300.

The calculation is carried out according to the Swedish design regulations (BKR) in both ultimate limit state and serviceability limit state, Appendix B. The climate is assumed to be climate class 1 and the structure is assumed to be in safety class 3. The beams are of dimensions 220x45 mm^2 with a spacing of 600 mm. All calculations are made on a one meter wide strip of the floor. On this 22 mm thick chipboard is placed. The chipboard is assumed to be both glued and screwed into place to achieve full interaction between the two materials. The setup can be seen in Figure 4.16. The loads acting on the floor is the self-weight of the material and the load for residential areas according to BKR. This gives a total load that varies between 1.5 and 1.6 kN per meter length, depending on what timber quality class and what type of reinforcement.

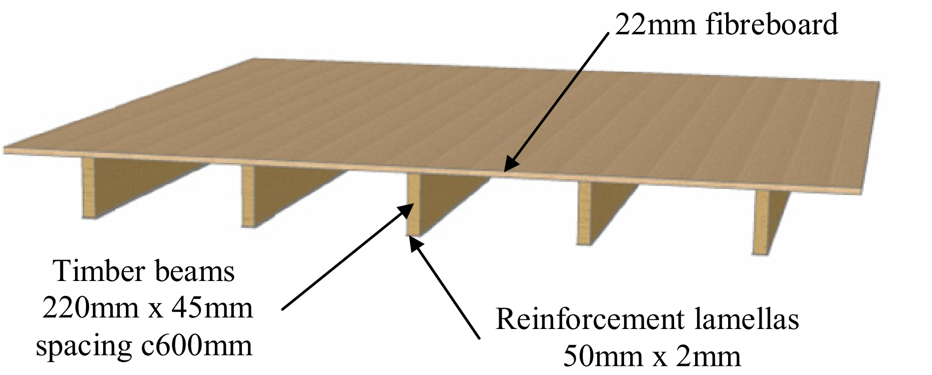


Figure 4.16 The setup of a timber floor joist.

In ultimate limit state the major concern for a floor of this configuration is the particle board. With a compressive strength capacity of 3.67 MPa this is the reason for limiting the span widths. If reinforcement is applied, the utilisation of the tensile capacity is very low (less then 1%). The rolling shear between the different materials is also rather small.

Almost always the vibration properties of the floor are decisive of the span length. For one single beam, the deflection for a point load placed at the worst point can not exceed 1.5mm per 1kN point load according to the requirements. In this test the Young's modulus is not modified for load duration since the point load is applied instantaneously.

The deflection of the reinforced joist is 17 mm when the span is 5.1 m long. This is the maximum deflection allowed to meet the requirement of the length divided by 300. For a timber floor joist of un-reinforced timber beams the span would be 4.1 m long and deflection of 14 mm. These values are valid for a combination of K12 timber and CFRP-300.

4.6 Discussion and comparison results timber floor joist.

Comparing materials with such large difference in stiffness, the outcome is very obvious. The timber floor joist reinforced with CFRP-300 has the largest increase in span length compared to the other reinforcement materials mentioned. Next is steel with a maximum increase of 17%. CFRP-165 follows with an increase of about 14%. Smallest increase is achieved with the GFRP, which has a maximal increase in span with about 8%. All these values are for timber beams of K12 quality. In Figure 4.17 the results for all the timber qualities versus the reinforcement materials is shown.

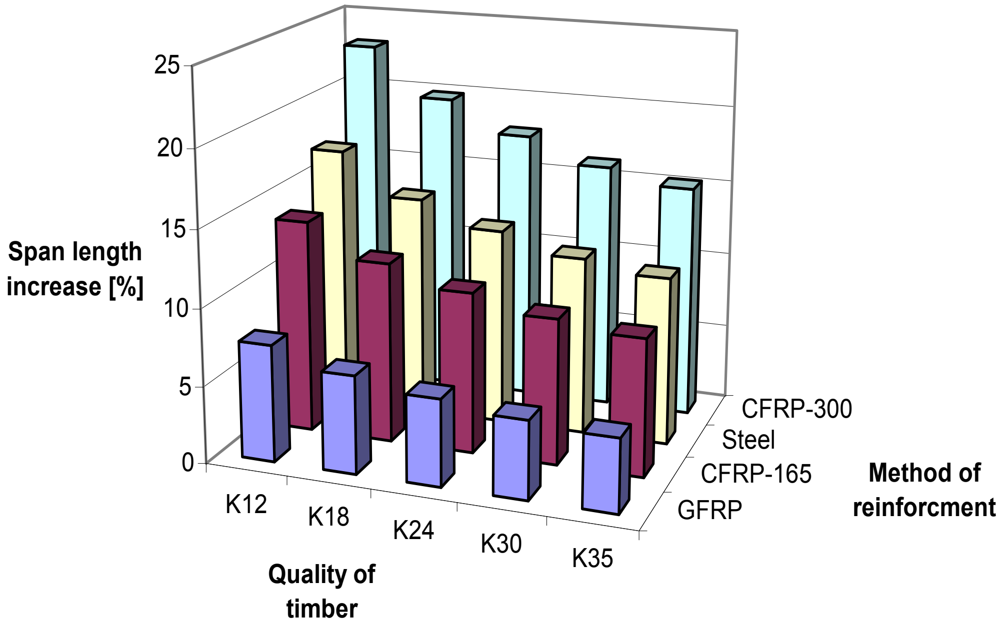


Figure 4.17 Timber qualities versus reinforcement materials.

The results clearly show that it is more sufficient to reinforce timber beams of lower quality for higher increase in span length. The increase in span length has a linear relationship with the Young's modulus of the reinforcement.

GFRP are not mentioned in the comparison of the simple reason that there are no manufacturers of GFRP laminates in Sweden for this type of application. The values used in the comparison of increased span length are theoretical.

When looking at these types of reinforcement from an economic point of view the preferred material is different. Taking into account that CFRP-300 costs about 900 SEK per meter length compared to the second most efficient, reinforcement steel which only cost around 25-35 SEK per meter, the choice is rather easy. Both types of CFRP are still to expensive to be able to compete with steel for a reinforcement of this type. If steel can be used it is impossible to defend a use of CFRP instead. On the other hand if the environmental conditions are such as metals cannot be used, a reinforcement of CFRP is possible, but to a higher cost. The costs only include the material and do not consider cost of labour since the comparison of costs is between different types of reinforcement. It can be assumed that labour required for applying the reinforcement does not differ too much.

5 Conclusion and suggestion for further research

In this chapter conclusions from the results and some suggestions for further research are made. The conclusions are drawn directly from the results that were obtained. In some cases the data, of which the conclusions are based upon, may be insufficient. This is due to the delays that occurred so the tests could not be initiated.

5.1 Conclusion

The results from dynamic measurement of Young's modulus correlate very well with the analytical values of the composites Young's modulus. This implies that the interaction between timber and the reinforcement material may correctly be assumed to fully interact with each other.

Dynamic measurements of the Young's modulus works as well for composite materials as it do for all the materials separately. It does not matter if the sound wave is initiated in only one of the materials. The results of the eigen frequency for the composite will be the same. One problem with this method is that the result that comes out of the testing is a mean value of the composites stiffness.

By reinforcing timber beams the stiffness can be improved easily. Previous studies have shown that both CFRP and steel increases the stiffness of glulam beams significantly. This study shows that this is also valid for timber beams of dimensions 45x70 mm^2.

There were several problems with the tests that were performed. For a few of the test specimens the test equipment failed to read data several times which lead to missing data over several periods of time. One specimen where unconnected for a two week period of time which resulted in data from this specimen was neglected.

The reinforced beams have one disadvantage in this type of test. They are reinforced only on one of the loaded sides. This causes a curvature as an effect of constrained shrinkage and swelling in timber. For the un-reinforced beams the swelling/shrinkage does not have any effect since they are free to move in any direction without affecting the curvature. In the test all reinforced specimens show a larger variation in deflection between the dry and wet part of one moisture cycle. For CFRP-300 and steel reinforced specimens the movement could be as large as 100% of the initial deflection.

The reinforcement prevents the mean value of the deflection to increase. Higher stiffness of the reinforcement material gives smaller mechano-sorptive creep. For the series reinforced with steel and CFRP-300, the mechano-sorptive creep is significantly reduced. The un-reinforced timber shows for a 10 week period a mean value increase of 17% in relative creep due to mechano-sorptive creep.

There is a tendency towards less mechano-sorptive creep for timber beams with higher original stiffness. This goes for both timber beams with or without reinforcement. This could change the parameters κ_s used in BKR or k_{def} used in Eurocode that considers load duration class and climate class.

Reinforcing timber floor joist can give an increase in the span length with as much as 23% compared to the same floor without the reinforcement. A lower quality of timber combined with a higher stiffness in the reinforcement gives the largest increase in the span. In the calculations a floor that consist of 22 mm chipboard, 45x220 mm² K12 timber beams and 2x50 mm² CFRP-300 gives the largest increase as well as the largest span. For several combinations of timber quality and reinforcement type the compressive stresses in the chipboard become too large. This decreases the "allowed" span more then the floor vibration check which is usually the governing parameter. The increase in span could be increased even more by changing the chipboard to plywood for instance.

To compare steel and CFRP as reinforcement in new production is not justified due to the economical aspect since CFRP equivalent to steel in stiffness costs up to 30 times as much. By far, the most economic reinforcement is steel if compared to CFRP. One of the possible applications for CFRP would be in an environment harmful for steel. This could be marine environment for example, where resistance against corrosion is of great importance.

5.2 Suggestion for further research

Dynamic measurement of composite stiffness could be an easy way of ensuring material quality of timber structures. If it is possible to simplify the testing method even more should be investigated. Also performing more tests to get more accurate data on where the difference between a static Young's modulus and a dynamic one lies.

Glass reinforced polymers should be added as a possible reinforcement material if GFRP laminates appears on the market.

Research on a much deeper level has to be done in order to be able to understand the phenomena of mechano-sorptive creep. To be able to find a connection between mechano-sorptive creep and material parameters several more tests has to be done.

Tests should be performed to investigate the durability of the materials. Will cyclic load cause fatigue failure in the adhesive, as well as in the reinforcement material? Test of varying temperature with or without varying relative humidity would also be a interesting idea to investigate the behaviour of reinforced timber.

Investigate with several more tests if the modification factors in both BKR and Eurocode could be change due to less mechano-sorptive creep for timber with higher stiffness.

6 References

Bengtsson, C. (1997): *Creep in sawn spruce exposed to varying humidity*. LicEng Thesis, Department of Structural Engineering, Chalmers University of Technology, Publ. S97:1, Göteborg, Sweden.

Bengtsson, C. (1999): *Mechano-sorptive creep in wood.* Ph.D. Thesis. Department of Structural Engineering, Chalmers University of Technology, Publ. S99:3, Göteborg, Sweden

Blass, H.J. Aune, P. Choo, B.S. Görlacher, R. Griffiths, D.R. Hilson, B.O. Racher, P. Steck, G. (1995): *Timber Engineering STEP 1: Basis of design, material properties, structural components and joints,* STEP/EUROFORTECH, Netherlands

Bäckström, M. (2006): *Moisture-indeced distortion in timber structures – examples based on partition walls.* Ph. D. Thesis. Department of Civil and Environmental Engineering, Division of Structural Engineering, Steel and Timber Structures, Chalmers University of Technology, Publ. New Series:2423, Göteborg, Sweden

Carolin, A. (2003): *Carbon fibre reinforced polymers for strengthening of structural elements*. Doctoral Thesis. Department of Civil and Mining Engineering, Division of Structural Engineering, Luleå University of Technology, Publ. 2003:18, Luleå, Sweden, 193 pp.

Chalmers (2007*)*: *Composity and Nanocomposity Materials – Compendium.* Department of Materials and Manufacturing Technology, Chalmers University of Technology, Göteborg, Sweden

Dejke, V. (2001): *Durability of FRP Reinforcement in Concrete.* Lic. Thesis. Department of Building Materials, Chalmers University of Technology, Göteborg, Sweden

Epmeier, H. Johansson, M. Kliger, R. Westin, M. (2007) Bending creep performance of modified timber. *Holz Roh Werkst.* No. 65, July 2007, pp. 343-351.

European Committee for Standardization (1993): *ENV 1995-1-1 Eurocode 5 – Design of timber structurer – Part 1-1: General rules and rules for buildings*.

European Committee for Standardization (2000): *EN 408 Timber structures - Structural timber and glued laminated timber - Determination of some physical and mechanical properties.*

Hansson, T., Gross, H. (1991): *Träbyggnadshandbok, Band 9, Material* (Handbook on wood construction, Vol. 1 In Swedish). Träinformation och Trätek – Institutet för träteknisk forskning, Malmö, Sweden

Hardwire LLC (2007). *What is Hardwire* [Online]. Available from: <http://www.hardwirellc.com/what_is.html> [Accessed 2007-11-02].

Hoffmeyer, P. (1990): *Failure of wood as influenced by moisture and duration of load.* Ph.D. Thesis. College of Environmental Science and Forestry, State University of New York, Syracuse, New York

Jobin, J. Olga L G B. (2007): *Flexural strengthening of glued laminated timber beams with steel and carbon fibre reinforced polymers.* Master's Thesis. Department of Civil and Environmental Engineering, Division of Structural Engineering, Chalmers University of Technology, Steel and Timber Structures, Publication no. 07:28, Göteborg, Sweden, 152 pp.

Kliger, I.R. Johansson, G. Perstorper, M. Engström, D. (1994): *Formulation of requirements for the quality of wood products used by the construction industry.* EEC Forest Project, Final Report. Department of Structural Engineering, Chalmers University of Technology. Publ. S94:4, Göteborg, Sweden

Morlier, P. (1994): *Creep in timber structures*. E&F Spoon, London, United Kingdom

Mårtensson, A. (1992): *Mechanical behaviour of wood exposed to humidity varations.* Ph.D. Thesis. Department of Structural Engineering, Lund Institute of Technology, Publ. TVBK-1006, Lund, Sweden

Nakano, T. (2003): Effects of cell structure on water sorption for wood. *Walter de Gruyter and Co.,* Vol. 57, No. 2, 2003, pp. 213-218.

Olsson, S. Perstorper, M. (1991): Elastic wood properties from dynamic tests and computer modelling. *ASCE Journal of Structural Engineering,* Vol. 118, No. 10, October 1992

Perstorper, M. Pellicane, P.J. Kliger, I.R. Johansson, G. (1994) Quality of timber products from Norway spruce, *Wood Science and Technology,* No. 29, 1995, pp. 157-170.

Sveriges verkstadsindustrier (2001): *Komposithandboken Polymerbaserade fiberkompositer*, Industrilitteratur AB, Stockholm, Sverige, 447 pp.

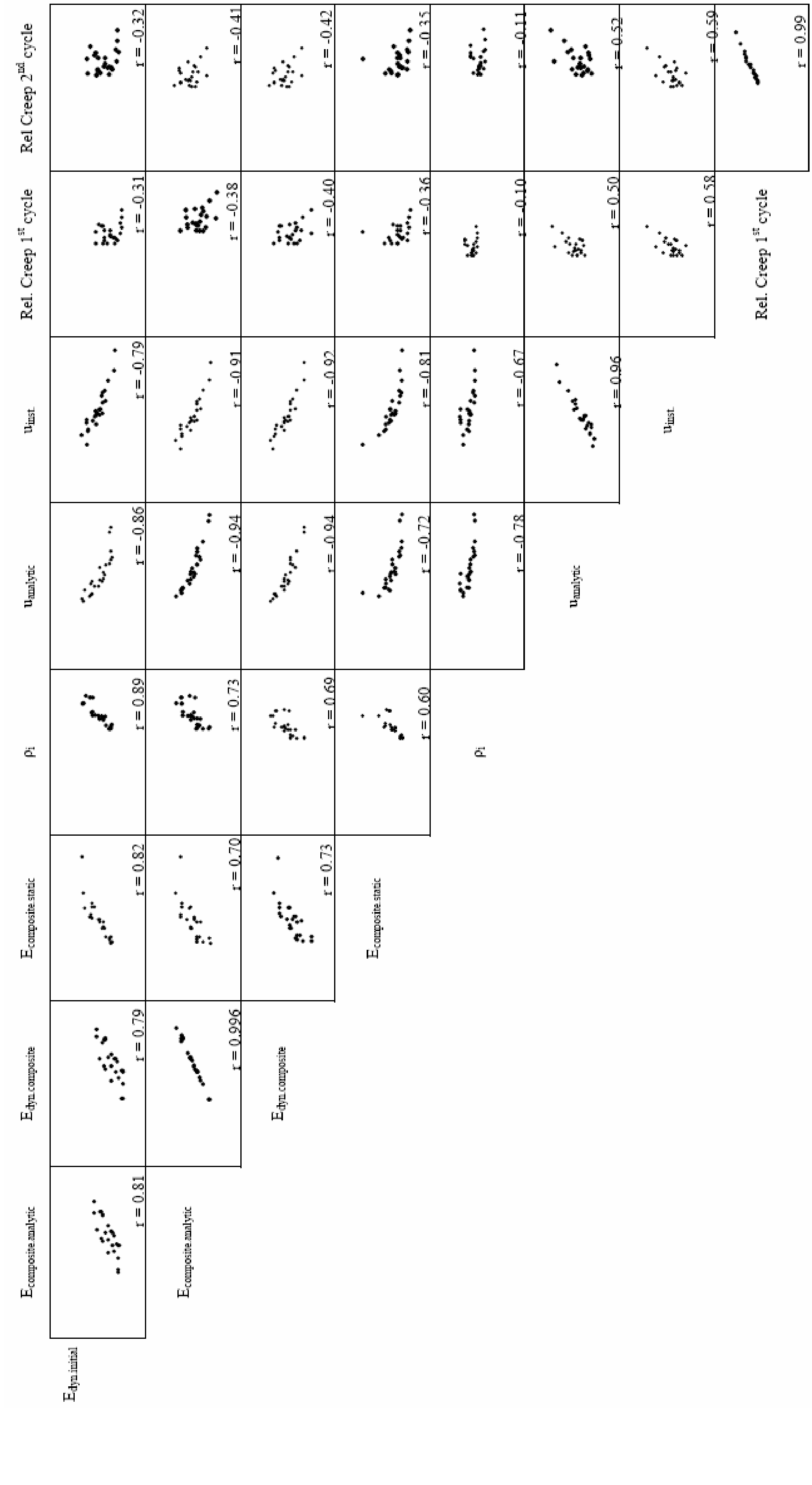
Triantafillou, T. (1997): Shear reinforcement of wood using FRP materials. *ASCE Journal of Materials in Civil Engineering*, Vol. 9, No. 2, May 1997, pp. 65-69.

Täljsten, B. (2002): *FRP Strengthening of Existing Concrete Structures.* Department of Civil Engineering, Luleå, Sweden

Åström, B. T. (2002): *Manufacturing of Polymer Composites*. Department of Aeronautics, Royal Institute of Technology, Stockholm, Sweden

7 APPENDICES

7.1 APPENDIX A: Correlation matrix



7.2 APPENDIX B: Calculation of timber floor joist

1. Definitioner

1.1 Klassindelning

Klimatklass: 1 (BKR 5:21)

Säkerhetsklass: 3 (BKR 2:115)

$\gamma_n := 1.2$

$\gamma_{m.brott\_trä} := 1.25$ i brottgränstillstånd

$\gamma_{m.bruk\_trä} := 1.0$ i brukgränstillstånd

$\gamma_{m.brott\_rf} := 1.0$ i brottgränstillstånd

$\gamma_{m.bruk\_rf} := 1.0$ i brukgränstillstånd

Partialkoeficient för material kan sättas till 1.0 för alla förstärkningsmaterial. (BKR 8:312)
Detta beror på att stålets partialkoeficienter är satta till 1.0 och några partialkoeficienter för kolfiber och glasfiber ej finns att finna.

1.2 Material

Konstruktionsvirke : ◉ K12 ○ K18 ○ K24 ○ K30 ○ K35 $i = 0$

Spånskiva : K-spånskiva 22mm

Förstärkning : ingen ○ Stål ○ Kolfiber IM ◉ Kolfiber UHM ○ Glasfiber $j = 2$

Hållfasthetsvärden för Konstruktionsvirke (BKR Tabell 5:23a)

Dragning parallellt fibrerna $f_{tk\_trä} := (8 \;\; 11 \;\; 16 \;\; 20 \;\; 21)^T \cdot MPa$

Tryck parallellt fibrerna $f_{ck\_trä} := (14 \;\; 18 \;\; 21 \;\; 23 \;\; 25)^T \cdot MPa$

Böjning parallellt fibrerna $f_{mk\_trä} := (12 \;\; 18 \;\; 24 \;\; 30 \;\; 35)^T \cdot MPa$

Längsskjuvning $f_{vk\_trä} := 3MPa$

Styvhetsvärden för bärförmågeberäkningar för Konstruktionsvirke

Elasticitetsmodul $E_{Rk\_trä} := (4.2 \;\; 5.1 \;\; 6.9 \;\; 8.7 \;\; 9.0)^T \cdot GPa$

Styvhetsvärden för deformationsberäkningar för Konstruktionsvirke

Elasticitetsmodul $E_{k\_trä} := (8 \;\; 9 \;\; 10.5 \;\; 12 \;\; 13)^T \cdot GPa$

Hållfasthetsvärden för K-spånskiva (BKR Tabell 5:23d)

Tryck vinkelrätt fibrerna $f_{ck\_spån} := 10MPa$

Styvhetsvärden för bärförmågeberäkningar för K-spånskiva

Elasticitetsmodul tryckt $E_{cRk\_spån} := 1.5 GPa$

Styvhetsvärden för deformationsberäkningar för K-spånskiva

Elasticitetsmodul tryckt $E_{ck\_spån} := 2.2 GPa$

Hållfasthetsvärden för förstärkningsmaterial (Tillverkar specifikationer)

Dragning parallellt fibrerna $f_{tk\_rf} := (235 \;\; 2310 \;\; 1350 \;\; 600)^T \cdot MPa$

Styvhetsvärden för bärförmåge- samt deformationsberäkningar för förstärkningsmaterial

Elasticitetsmodul $E_{Rk\_rf} := (210 \;\; 165 \;\; 300 \;\; 85)^T \cdot GPa$

1.3 Enheter

Mått	m
Kraft	kN
Moment	kNm
Spänning	MPa
Modul	GPa

1.4 Korrigeringsfaktorer

Med avseende på klimatklass och lasternas varaktighet för Konstruktionsvirke (BKR Tabell 5:3121a)

$$\kappa_{r\_P\_trä} := \begin{cases} 0.6 & \text{if } i = 4 \\ 0.65 & \text{if } i = 3 \\ 0.7 & \text{if } i = 2 \\ 0.75 & \text{otherwise} \end{cases} \qquad \kappa_{r\_B\_trä} := \begin{cases} 0.75 & \text{if } i = 4 \\ 0.80 & \text{if } i = 3 \\ 0.85 & \text{if } i = 2 \\ 0.90 & \text{otherwise} \end{cases}$$

för K35

för K30

för K24

för K18 och K12

$\kappa_{r\_P\_trä} = 0.75$ $\qquad \kappa_{r\_B\_trä} = 0.9$

Bruksgränstillstånd för Konstruktionsvirke (BKR Tabell 5:322a)

$\kappa_{s\_P\_trä} := 0.55$ för klimatklass 1 och lasttyp P

$\kappa_{s\_B\_trä} := 0.8$ för klimatklass 1 och lasttyp B

Med avseende på klimatklass och lasterbas varaktighet för K-spånskiva (BKR Tabell 5:3121c)

$\kappa_{r\_P\_spån} := 0.4$ för klimatklass 1 och lasttyp P

$\kappa_{r\_B\_spån} := 0.55$ för klimatklass 1 och lasttyp B

Bruksgränstillstånd för K-spånskiva (BKR Tabell 5:322c)

$\kappa_{s\_P\_spån} := 0.3$ för klimatklass 1 och lasttyp P

$\kappa_{s\_B\_spån} := 0.55$ för klimatklass 1 och lasttyp B

1.5 Dimensionerande värden

Brottgräns (BKR 5:3121a,c) (BKR 8:312a)

Dimentionerande materialvärden beräknas efter den kortvarigaste lasteni lastkombinationen. I detta fallet är det lastfall B vilket representerar den variabla lasten av inredning och personer. (BKR 3:41)

$$f_{td\_trä} := \frac{\kappa_{r\_B\_trä} \cdot f_{tk\_trä_i}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad f_{td\_trä} = 4.8 \times 10^6\ \mathrm{Pa}$$

$$f_{cd\_trä} := \frac{\kappa_{r\_B\_trä} \cdot f_{ck\_trä_i}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad f_{cd\_trä} = 8.4 \times 10^6\ \mathrm{Pa}$$

$$f_{md\_trä} := \frac{\kappa_{r\_B\_trä} \cdot f_{mk\_trä_i}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad f_{md\_trä} = 7.2 \times 10^6\ \mathrm{Pa}$$

$$E_{Rd\_trä} := \frac{\kappa_{r\_B\_trä} \cdot E_{Rk\_trä_i}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad E_{Rd\_trä} = 2.52 \times 10^9\ \mathrm{Pa}$$

$$f_{cd\_spån} := \frac{\kappa_{r\_B\_spån} \cdot f_{ck\_spån}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad f_{cd\_spån} = 3.667 \times 10^6\ \mathrm{Pa}$$

$$E_{cRd\_spån} := \frac{\kappa_{r\_B\_spån} \cdot E_{cRk\_spån}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad E_{cRd\_spån} = 5.5 \times 10^8\ \mathrm{Pa}$$

$$f_{td\_rf} := \frac{f_{tk\_rf_j}}{\gamma_n \cdot \gamma_{m.brott\_rf}} \qquad f_{td\_rf} = 1.125 \times 10^9\ \mathrm{Pa}$$

$$f_{vd\_trä} := \frac{f_{vk\_trä}}{\gamma_n \cdot \gamma_{m.brott\_trä}} \qquad f_{vd\_trä} = 2\,\mathrm{MPa}$$

Bruksgräns (BKR 5:322a) (BKR 8:312d)

$$E_{d\_trä} := \frac{\kappa_{s\_B\_trä} \cdot E_{k\_trä_i}}{\gamma_{m.bruk\_trä}} \qquad E_{d\_trä} = 6.4 \times 10^9\ \mathrm{Pa}$$

$$E_{cd\_spån} := \frac{\kappa_{s\_B\_spån} \cdot E_{ck\_spån}}{\gamma_{m.bruk\_trä}} \qquad E_{cd\_spån} = 1.21 \times 10^9\ \mathrm{Pa}$$

$$E_{Rd\_rf} := \frac{E_{Rk\_rf_j}}{\gamma_{m.bruk\_rf}} \qquad E_{Rd\_rf} = 3 \times 10^{11}\ Pa$$

$$E_{d\_trä\_svikt} := \frac{E_{k\_trä_i}}{\gamma_{m.bruk\_trä}} \qquad E_{d\_trä\_svikt} = 8 \times 10^{9}\ Pa$$

1.6 Mått

$h_{trä} := 0.220m$ Standard regel 2" x 9" används i detta fallet.

$b_{trä} := 0.045m$

$s_{bjälke} := 0.6m$ Reglarna är utsatta med c-avstånd 600mm.

$t_{spån} := 0.022m$ K-spånskiva 22mm.

$b_{rf} := 0.05m$ Förstärkning av dimensionerna 50x1.4 mm^2

$t_{rf} := 0.0014m$

$h_{total\_rf} := h_{trä} + t_{spån} + t_{rf}$

$h_{total} := h_{trä} + t_{spån}$

$L := 4m$ Godtycklig längd på bjälklaget

1.7 Tvärsnittsdata

Omräkningsfaktor för ekvivalent tvärsnitt med en effektiv area.

$$\alpha_{spån} := \frac{E_{cd\_spån}}{E_{d\_trä}} \qquad \alpha_{spån} = 0.189$$

$$\alpha_{rf} := \frac{E_{Rd\_rf}}{E_{d\_trä}} \qquad \alpha_{rf} = 46.875$$

$$z_{eff\_rf} := \frac{\alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} \cdot \frac{t_{spån}}{2} + b_{trä} \cdot h_{trä} \cdot \left(t_{spån} + \frac{h_{trä}}{2}\right) \dots + \alpha_{rf} \cdot b_{rf} \cdot t_{rf} \cdot \left(t_{spån} + h_{trä} + \frac{t_{rf}}{2}\right)}{\alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} + b_{trä} \cdot h_{trä} + \alpha_{rf} \cdot b_{rf} \cdot t_{rf}}$$

$$z_{eff\_rf} = 0.136\,m$$

$$z_{eff} := \frac{\alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} \cdot \frac{t_{spån}}{2} + b_{trä} \cdot h_{trä} \cdot \left(t_{spån} + \frac{h_{trä}}{2}\right)}{\alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} + b_{trä} \cdot h_{trä}}$$

$$z_{eff} = 0.108\,m$$

$$I_{eff\_rf} := \frac{b_{trä} \cdot h_{trä}^3}{12} + b_{trä} \cdot h_{trä} \cdot \left(t_{spån} + \frac{h_{trä}}{2} - z_{eff} \right)^2 \ldots$$

$$+ \alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} \cdot \left(\frac{t_{spån}}{2} - z_{eff} \right)^2 + \alpha_{rf} \cdot b_{rf} \cdot t_{rf} \cdot \left(t_{spån} + h_{trä} + \frac{t_{rf}}{2} - z_{eff} \right)^2$$

$$I_{eff\_rf} = 1.29 \times 10^{-4}\ m^4$$

$$I_{eff} := \frac{b_{trä} \cdot h_{trä}^3}{12} + b_{trä} \cdot h_{trä} \cdot \left(t_{spån} + \frac{h_{trä}}{2} - z_{eff} \right)^2 + \alpha_{spån} \cdot s_{bjälke} \cdot t_{spån} \cdot \left(\frac{t_{spån}}{2} - z_{eff} \right)^2$$

$$I_{eff} = 6.911 \times 10^{-5}\ m^4$$

2 Laster

2.1 *Egenvikt*

$\rho_{trä} := (290\ \ 320\ \ 350\ \ 380\ \ 400)^T \frac{kg}{m^3}$ Densitet för Konstruktionsvirke

$\rho_{spån} := 600 \frac{kg}{m^3}$ Densitet för K.spånskiva

$\rho_{rf} := (7800\ \ 1600\ \ 1700\ \ 2000)^T \frac{kg}{m^3}$ Densitet för förstärkningsmaterial

Egenvikten beräknas för en "remsa" av golvbjälklaget. Dvs en balk plus 600mm spånskiva.

$G_{k\_rf} := s_{bjälke} \cdot t_{spån} \cdot \rho_{spån} + b_{trä} \cdot h_{trä} \cdot \rho_{trä_i} + b_{rf} \cdot t_{rf} \cdot \rho_{rf_j}$ $G_{k\_rf} = 10.91 \frac{kg}{m}$

$G_k := s_{bjälke} \cdot t_{spån} \cdot \rho_{spån} + b_{trä} \cdot h_{trä} \cdot \rho_{trä_i}$ $G_k = 10.791 \frac{kg}{m}$

2.2 *Variabla laster*

(BKR Tabell 3:41a)
1.Vistelselast

$q_{k.bunden} := 0.5 \frac{kN}{m^2}$ Utbredd last, bunden lastdel

$q_{k.fri} := 1.5 \frac{kN}{m^2}$ Utbredd last, fri lastdel

$\Psi := 0.33$

2.3 Lastkombination

(BKR Tabell 2:322a)

Lastkombination 1 är dimensionerande.

$$Q_{d\_rf} := 1.0 \cdot \left(g \cdot G_{k\_rf} + q_{k.bunden} \cdot 0.6m\right) + 1.3 \cdot \left(q_{k.fri} \cdot 0.6m\right) \qquad Q_{d\_rf} = 1.577 \frac{kN}{m}$$

$$Q_d := 1.0 \cdot \left(g \cdot G_k + q_{k.bunden} \cdot 0.6m\right) + 1.3 \cdot \left(q_{k.fri} \cdot 0.6m\right) \qquad Q_d = 1.576 \frac{kN}{m}$$

2.4 Moment

$$M_{d\_rf} := \frac{Q_{d\_rf} \cdot L^2}{8} \qquad M_{d\_rf} = 3.154 kN \cdot m$$

$$M_d := \frac{Q_d \cdot L^2}{8} \qquad M_d = 3.152 kN \cdot m$$

2.5 Tvärkraft

$$V_{d\_rf} := \frac{Q_{d\_rf} \cdot L}{2} \qquad V_{d\_rf} = 3.154 kN$$

$$V_d := \frac{Q_d \cdot L}{2} \qquad V_d = 3.152 kN$$

3 Kontroll av hållfasthet

3.1 Dragning

Kontroll av dragspänningar i förstärkningsmaterial

$$\sigma_{t\_rf} := \frac{M_{d\_rf}}{I_{eff\_rf}} \cdot \left(h_{total\_rf} - z_{eff\_rf}\right) \qquad \sigma_{t\_rf} = 2.629 MPa$$

$$\sigma_{t\_rf} < f_{td\_rf} = 1 \qquad \frac{\sigma_{t\_rf}}{f_{td\_rf}} = 0.234\%$$ utnyttjandegrad

3.2 Tryck

I denna konrtollen tas ej hänsyn till eventuell instabilitet hos det tryckta materialet.

Kontroll av tryckspänningar i K-spånskiva

$$\sigma_{c\_spån\_rf} := \frac{M_{d\_rf}}{I_{eff\_rf}} \cdot (z_{eff\_rf}) \qquad \sigma_{c\_spån\_rf} = 3.324\text{MPa}$$

$$\sigma_{c\_spån\_rf} < f_{cd\_spån} = 1 \qquad \frac{\sigma_{c\_spån\_rf}}{f_{cd\_spån}} = 90.647\%$$ utnyttjandegrad

$$\sigma_{c\_spån} := \frac{M_d}{I_{eff}} \cdot (z_{eff}) \qquad \sigma_{c\_spån} = 4.909\text{MPa}$$

$$\sigma_{c\_spån} < f_{cd\_spån} = 0 \qquad \frac{\sigma_{c\_spån}}{f_{cd\_spån}} = 133.869\%$$ utnyttjandegrad

3.3 Böjspänningar

Kontroll av böjspänningar i konstruktionsvirke

$$\sigma_{t\_trä\_rf} := \frac{M_{d\_rf}}{I_{eff\_rf}} \cdot (h_{total\_rf} - t_{rf} - z_{eff\_rf}) \qquad \sigma_{t\_trä\_rf} = 2.595\text{MPa}$$

$$\sigma_{t\_trä\_rf} < f_{md\_trä} = 1 \qquad \frac{\sigma_{t\_trä\_rf}}{f_{md\_trä}} = 36.036\%$$ utnyttjandegrad

$$\sigma_{t\_trä} := \frac{M_d}{I_{eff}} \cdot (h_{total} - z_{eff}) \qquad \sigma_{t\_trä} = 6.127\text{MPa}$$

$$\sigma_{t\_trä} < f_{md\_trä} = 1 \qquad \frac{\sigma_{t\_trä}}{f_{md\_trä}} = 85.099\%$$ utnyttjandegrad

$$\sigma_{c\_trä\_rf} := \frac{M_{d\_rf}}{I_{eff\_rf}} \cdot (z_{eff\_rf} - t_{spån}) \qquad \sigma_{c\_trä\_rf} = 2.786\text{MPa}$$

$$\sigma_{c\_trä\_rf} < f_{md\_trä} = 1 \qquad \frac{\sigma_{c\_trä\_rf}}{f_{md\_trä}} = 38.69\%$$ utnyttjandegrad

$$\sigma_{c\_trä} := \frac{M_d}{I_{eff}} \cdot (z_{eff} - t_{spån}) \qquad \sigma_{c\_trä} = 3.905\text{MPa}$$

$$\sigma_{c\_trä} < f_{md\_trä} = 1 \qquad \frac{\sigma_{c\_trä}}{f_{md\_trä}} = 54.24\%$$ utnyttjandegrad

3.4 Skjuvspänningar

Kontroll av skjuvspänningarna i limfogen mellan träreglarna och förstärkningsmaterialet/spånskivan

$b := 0.04m$ Bredd på kontaktyta mellan trä och förstärkningsmaterial/spånskiva

$$S_{rf} := \alpha_{rf} \cdot b_{rf} \cdot t_{rf} \cdot \left(h_{total\_rf} - z_{eff\_rf}\right)$$

$$S_{spån} := \alpha_{spån} \cdot t_{spån} \cdot s_{bjälke} \cdot z_{eff\_rf}$$

$$S := \min\left(S_{rf}, S_{spån}\right)$$

$$\tau_{skjuv\_rf} := \frac{V_{d\_rf} \cdot S}{I_{eff\_rf} \cdot b} \qquad \tau_{skjuv\_rf} = 0.207MPa$$

$$\tau_{skjuv\_rf} < f_{vd\_trä} = 1 \qquad \frac{\tau_{skjuv\_rf}}{f_{vd\_trä}} = 10.368\%$$ utnyttjandegrad

3.4 Kontroll av hållfastheterna

$$ng := \left(\frac{\sigma_{t\_rf}}{f_{td\_rf}} \quad \frac{\sigma_{c\_spån}}{f_{cd\_spån}} \quad \frac{\sigma_{t\_trä}}{f_{md\_trä}} \quad \frac{\sigma_{c\_trä}}{f_{md\_trä}} \quad \frac{\tau_{skjuv\_rf}}{f_{vd\_trä}}\right)^T \qquad ng = \begin{pmatrix} 0.234 \\ 133.869 \\ 85.099 \\ 54.24 \\ 10.368 \end{pmatrix}\%$$

$$fel := \begin{vmatrix} 1 \text{ if } ng_0 > \left(ng_1 \wedge ng_2 \wedge ng_3 \wedge ng_4\right) \\ 2 \text{ if } ng_1 > \left(ng_0 \wedge ng_2 \wedge ng_3 \wedge ng_4\right) \\ 3 \text{ if } ng_2 > \left(ng_0 \wedge ng_1 \wedge ng_3 \wedge ng_4\right) \\ 4 \text{ if } ng_3 > \left(ng_0 \wedge ng_1 \wedge ng_2 \wedge ng_4\right) \\ 5 \text{ if } ng_4 > \left(ng_0 \wedge ng_1 \wedge ng_2 \wedge ng_3\right) \\ 0 \text{ otherwise} \end{vmatrix} \qquad fel = 2$$

$$ny\_L\_rf\_1 := L$$

$$ny\_L\_rf\_1 := \mathrm{root}\left[\frac{\frac{\frac{Q_{d\_rf} \cdot ny\_L\_rf\_1^2}{8}}{I_{eff\_rf}} \cdot \left(z_{eff\_rf}\right)}{f_{cd\_spån}} - 1, ny\_L\_rf\_1\right]$$

$$ny\_L\_rf\_1 = 4.201\,m$$

$$ny\_L\_1 := L$$

$$ny\_L\_1 := \mathrm{root}\left[\frac{\dfrac{\dfrac{Q_d \cdot ny\_L\_1^2}{8}}{I_{eff}} \cdot (z_{eff})}{f_{cd\_spån}} - 1, ny\_L\_1\right]$$

$Q_d = 1.576 \times 10^3 \frac{kg}{s^2}$

$ny\_L\_1 = 3.457\,m$

$I_{eff} = 6.911 \times 10^{-5}\,m^4$

Ökning i spånnvidd

$$\frac{ny\_L\_rf\_1}{ny\_L\_1} - 1 = 21.524\%$$

4 Kontroll av deformationer

4.1 Svikt

Följande förenklade beräkningsmetod kan användas för bostadsbjälklag med massiva träbalkar i huvudbärriktningen. Samverkan mellan bjälklag och golvskiva antas. (BKR 5:323)

$P := 1kN$

$$\delta_{svikt\_rf} := \frac{P \cdot L^3}{48 E_{d\_trä} \cdot I_{eff\_rf}} \qquad \delta_{svikt\_rf} = 1.615mm$$

$$\delta_{svikt} := \frac{P \cdot L^3}{48 E_{d\_trä} \cdot I_{eff}} \qquad \delta_{svikt} = 3.014mm$$

Nedböjning av lasten P får ej vara större en 1,5mm. $\delta_{krav\_svikt} := 1.5mm$

$ny\_L\_rf\_2 := L$

$$ny\_L\_rf\_2 := \mathrm{root}\left(\frac{P \cdot ny\_L\_rf\_2^3}{48 E_{d\_trä} \cdot I_{eff\_rf}} - \delta_{krav\_svikt}, ny\_L\_rf\_2\right)$$

$ny\_L\_rf\_2 = 3.902\,m$

$ny\_L\_2 := L$

$$ny\_L\_2 := \mathrm{root}\left(\frac{P \cdot ny\_L\_2^3}{48 E_{d\_trä} \cdot I_{eff}} - \delta_{krav\_svikt}, ny\_L\_2\right)$$

Ökning i spännvidd

$$\frac{ny\_L\_rf\_2}{ny\_L\_2} - 1 = 23.114\%$$

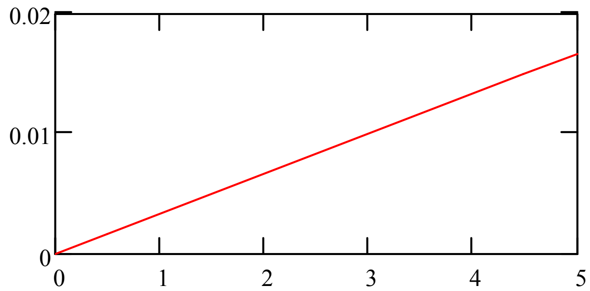
4.2 Deformation av golvet

I BKR finns inget krav på deformationerna hos ett golvbjälklag.
Kravet antas vara L / 300, då det är ett rimligt krav för deformationerna.

$$\delta_{golv\_rf} := \frac{5 \cdot Q_{d\_rf} \cdot L^4}{384 \cdot E_{d\_trä} \cdot I_{eff\_rf}} \qquad \delta_{golv\_rf} = 6.369\text{mm}$$

$$\delta_{golv} := \frac{5 \cdot Q_d \cdot L^4}{384 \cdot E_{d\_trä} \cdot I_{eff}} \qquad \delta_{golv} = 11.876\text{mm}$$

Krav på deformationer mindre än L/300 $\quad l := 0, 0.5\text{m} .. 5\text{m}$



Tillåten deformation ökar med ökad spännvidd

$$ny\_L\_rf\_3 := L$$

$$ny\_L\_rf\_3 := \text{root}\left(\frac{5 \cdot Q_{d\_rf} \cdot ny\_L\_rf\_3^4}{384 \cdot E_{d\_trä} \cdot I_{eff\_rf}} - \frac{ny\_L\_rf\_3}{300}, ny\_L\_rf\_3\right)$$

$$ny\_L\_rf\_3 = 5.117\,\text{m}$$

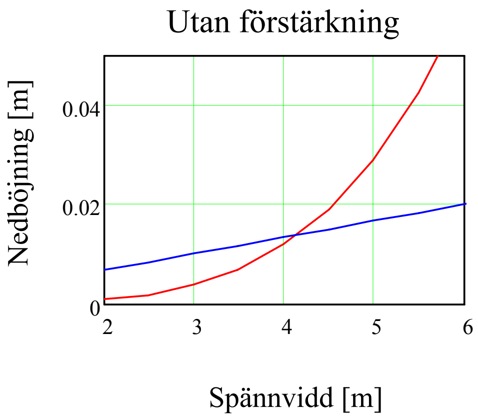
$$ny\_L\_3 := L$$

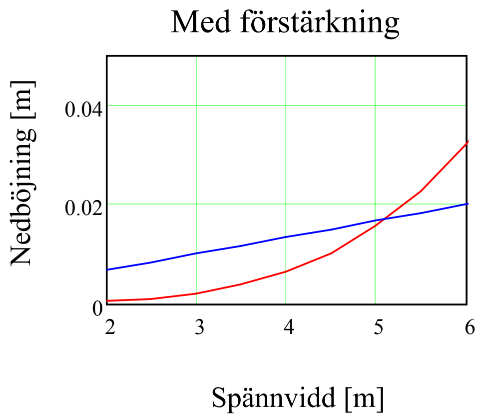
$$ny\_L\_3 := \text{root}\left(\frac{5 \cdot Q_d \cdot ny\_L\_3^4}{384 \cdot E_{d\_trä} \cdot I_{eff}} - \frac{ny\_L\_3}{300}, ny\_L\_3\right)$$

$$ny\_L\_3 = 4.157\,\text{m}$$

Nedan kan man se hur förhållandet mellan spännvidd och nedböjning samt kravet på nedböjning.

$$\text{Length} := 0, 0.5\text{m} .. 10\text{m}$$





Ökning i spännvidd

$$\frac{ny\_L\_rf\_3}{ny\_L\_3} - 1 = 23.084\%$$

4.3 Minsta tillåtna spännvidd

$$L\_rf := \begin{pmatrix} ny\_L\_rf\_1 \\ ny\_L\_rf\_2 \\ ny\_L\_rf\_3 \end{pmatrix} \qquad L\_rf = \begin{pmatrix} 4.201 \\ 3.902 \\ 5.117 \end{pmatrix} m \qquad L\_rf := \min(L\_rf) \qquad L\_rf = 3.902\,m$$

$$L := \begin{pmatrix} ny\_L\_1 \\ ny\_L\_2 \\ ny\_L\_3 \end{pmatrix} \qquad L = \begin{pmatrix} 3.457 \\ 3.17 \\ 4.157 \end{pmatrix} m \qquad L := \min(L) \qquad L = 3.17\,m$$

Maximal ökning i spännvidd:

$$\frac{L\_rf}{L} - 1 = 23.114\%$$

5 Kostnader

Det är svårt att uppskatta kostnaden för utförande av golvbjälklag.
Priserna som gäller då endast materialkostnader per meter bredd av golvet.

$$kostnad := (25 \;\; 300 \;\; 900 \;\; 0)^T$$

$$\frac{L\_rf \cdot kostnad_j}{s_{bjälke}} = 5.854 \times 10^3$$ kronor / breddmeter

7.3 APPENDIX C: Measurement of timber specimens

Test series	Original number	Frequency	Weight	Length	Height	Thickness	Density	Young's modulus	Annual ring
		[Hz]	[kg]	[cm]	[mm]	[mm]	[kg/m^3]	[MPa]	[mm]
1	16825	2681	1,84	109,9	69,9	44,3	540,0	18750	4,4
	16225:2	2539	1,45	110,0	70,3	44,5	422,5	13182	5,6
	20221	2137	1,32	110,0	70,6	44,5	382,4	8451	7,4
	16221	2733	1,56	109,8	70,1	44,5	454,4	16368	4,4
	16326	2478	1,81	109,9	69,8	44,4	531,1	15756	1,7
	21925	2188	1,29	109,9	70,2	44,3	378,5	8755	11,1
3	16821	2881	1,71	110,0	70,1	44,4	499,5	20067	3,2
	16222:2	2656	1,52	110,0	70,4	44,7	439,1	14991	5,0
	16225	2529	1,46	109,9	70,3	44,3	428,0	13226	6,3
	21926	2381	1,46	109,9	70,3	44,5	425,0	11640	11,1
	20121	2351	1,35	109,9	70,6	44,7	388,3	10369	8,9
	20225	2168	1,29	110,0	70,4	44,7	372,9	8484	8,9
4	16826:2	2539	1,81	110,0	69,6	44,3	534,6	16680	3,7
	16822	2733	1,51	110,0	70,2	44,3	441,7	15966	3,7
	16221:2	2691	1,57	110,0	70,7	44,6	453,1	15882	5,0
	20126	2381	1,44	109,9	70,2	44,5	418,6	11464	8,9
	21921	2269	1,49	110,0	70,5	44,5	433,3	10796	7,4
	20122	2061	1,36	110,0	70,5	44,7	391,3	8044	14,9
5	16821:2	2845	1,72	110,0	69,9	44,4	502,3	19677	2,8
	16826	2569	1,81	109,9	69,7	44,1	535,6	17077	3,4
	16322	2529	1,51	110,0	70,0	44,4	440,9	13649	3,4
	16325	2391	1,50	110,0	70,1	44,3	439,1	12150	6,3
	20226	2239	1,29	109,9	70,4	44,4	375,2	9088	8,9
	21922	2132	1,31	109,9	69,4	44,7	385,1	8457	14,9

7.4 APPENDIX D: Calculations for test weights and lever arms

The beams are devided into groups of 6 which can be seen below. The timber specimens are well documented and have the following number of their origin. All matrices in the calculations below have the same structure as the specimen number below.

Specimen nr.	1	2	3	4	5	6
Ref / Epoxy	16825	16225:2	20221	16221	16326	21925
CFRP low	16821	16222:2	16225	21926	20121	20225
CFRP high	16826:2	16822	16221:2	20126	21921	20122
Steel	16821:2	16826	16322	16325	20226	21922

Specimen nr.	1	2	3	4	5	6
Ref / Epoxy	1:1	1:2	1:3	1:4	1:5	1:6
CFRP low	3:1	3:2	3:3	3:4	3:5	3:6
CFRP high	4:1	4:2	4:3	4:4	4:5	4:6
Steel	5:1	5:2	5:3	5:4	5:5	5:6

Dynamically measured values of the initial timber Young's modulus.

$$E_{dyn.initial} := \begin{pmatrix} 18.75 & 13.182 & 8.451 & 16.428 & 15.756 & 8.755 \\ 20.033 & 14.991 & 13.226 & 11.64 & 10.369 & 8.484 \\ 16.68 & 15.966 & 15.852 & 11.488 & 10.796 & 8.044 \\ 19.677 & 17.077 & 13.649 & 12.15 & 9.108 & 8.457 \end{pmatrix} GPa$$

Measured initial weight of the timber beams.

$$w_{initial} := \begin{pmatrix} 1.838 & 1.455 & 1.322 & 1.562 & 1.81 & 1.293 \\ 1.71 & 1.519 & 1.465 & 1.462 & 1.346 & 1.29 \\ 1.812 & 1.511 & 1.571 & 1.437 & 1.494 & 1.356 \\ 1.716 & 1.81 & 1.506 & 1.498 & 1.289 & 1.312 \end{pmatrix} kg$$

Measured weight of the composite beams.

$$w_{comp} := \begin{pmatrix} 1.92 & 1.532 & 1.408 & 1.592 & 1.844 & 1.324 \\ 2.103 & 1.851 & 1.779 & 1.796 & 1.632 & 1.6 \\ 2.126 & 1.884 & 1.844 & 1.793 & 1.794 & 1.649 \\ 2.752 & 2.808 & 2.583 & 2.494 & 2.323 & 2.401 \end{pmatrix} kg$$

Dynamically measured values of the reinforcement Young's modulus.

$$E_{rein} := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 155 & 155 & 155 & 155 & 155 & 155 \\ 300 & 300 & 300 & 300 & 300 & 300 \\ 200 & 200 & 200 & 200 & 200 & 200 \end{pmatrix} GPa$$

Estimated value of adhesive Young's modulus.

$$E_{adhesive} := 4.5GPa$$

Dimensions of the beam and reinforcement.

$h_{beam} := 45mm$ $w_{beam} := 70mm$ $l_{beam} := 1100mm$

$w_{cut} := 52mm$ $w_{rein} := 50mm$

$A_{beam} := h_{beam} \cdot w_{beam}$

$A_{beam} = 3.15 \times 10^3 \, mm^2$

Thickness of the reinforcement lamellas.

$$h_{rein} := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1.4 & 1.4 & 1.4 & 1.4 & 1.4 & 1.4 \\ 1.4 & 1.4 & 1.4 & 1.4 & 1.4 & 1.4 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix} mm$$

$A_{rein} := h_{rein} \cdot w_{rein}$

$$A_{rein} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 70 & 70 & 70 & 70 & 70 & 70 \\ 70 & 70 & 70 & 70 & 70 & 70 \\ 100 & 100 & 100 & 100 & 100 & 100 \end{pmatrix} mm^2$$

Depth of cut in timber beams, in beam 4:5 the cut got 1mm to deep.

$$h_{cut} := \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 4 & 3 \end{pmatrix} mm$$

$A_{cut} := h_{cut} \cdot w_{cut}$

$$A_{cut} = \begin{pmatrix} 52 & 52 & 52 & 0 & 0 & 0 \\ 156 & 156 & 156 & 156 & 156 & 156 \\ 156 & 156 & 156 & 156 & 156 & 156 \\ 156 & 156 & 156 & 156 & 208 & 156 \end{pmatrix} mm^2$$

$h_{adhesive} := \left(h_{cut} - h_{rein}\right)$

Estimated thickness of the adhesive layer.

$$h_{adhesive} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1.6 & 1.6 & 1.6 & 1.6 & 1.6 & 1.6 \\ 1.6 & 1.6 & 1.6 & 1.6 & 1.6 & 1.6 \\ 1 & 1 & 1 & 1 & 2 & 1 \end{pmatrix} mm$$

$$A_{adhesive} := h_{adhesive} \cdot w_{cut}$$

Factor α which is used to calculated the composite as an equivalent timber cross-section.

$$\alpha := \frac{E_{rein}}{E_{dyn.initial}}$$

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 7.737 & 10.34 & 11.719 & 13.316 & 14.948 & 18.27 \\ 17.986 & 18.79 & 18.925 & 26.114 & 27.788 & 37.295 \\ 10.164 & 11.712 & 14.653 & 16.461 & 21.959 & 23.649 \end{pmatrix}$$

Center of rotation for the equivalent cross-section

$$z_{TP} := \frac{A_{beam} \cdot \frac{h_{beam}}{2} - \overrightarrow{\left[A_{cut} \cdot \left(h_{beam} - \frac{h_{cut}}{2}\right)\right]} + \overrightarrow{\left[\alpha \cdot A_{rein} \cdot \left(h_{beam} - h_{cut} + 1mm + \frac{h_{rein}}{2}\right)\right]}}{A_{beam} - A_{cut} + \overrightarrow{(\alpha \cdot A_{rein})}}$$

$$z_{TP} = \begin{pmatrix} 22.131 & 22.131 & 22.131 & 22.5 & 22.5 & 22.5 \\ 24.821 & 25.746 & 26.201 & 26.699 & 27.18 & 28.078 \\ 28.005 & 28.211 & 28.244 & 29.857 & 30.186 & 31.79 \\ 27.132 & 27.759 & 28.83 & 29.421 & 30.432 & 31.377 \end{pmatrix} mm$$

Conditioned due to 2% moisture change => lower Young's modulus with approx. 3%

$$E_{analytic.composite} := \frac{\overrightarrow{\left[(0.97 E_{dyn.initial}) \cdot (A_{beam} - A_{cut})\right]} + \overrightarrow{\left[E_{rein} \cdot (A_{rein})\right]} + \overrightarrow{(E_{adhesive} \cdot A_{adhesive})}}{A_{beam}}$$

$$E_{analytic.composite} = \begin{pmatrix} 17.962 & 12.65 & 8.136 & 15.935 & 15.283 & 8.492 \\ 22.033 & 17.384 & 15.757 & 14.295 & 13.123 & 11.385 \\ 22.164 & 21.506 & 21.4 & 17.377 & 16.739 & 14.202 \\ 24.565 & 22.168 & 19.007 & 17.625 & 14.749 & 14.221 \end{pmatrix} GPa$$

Dynamically measured value of Young's modulus of the final composite beams.

$$E_{dyn.composite} := \begin{pmatrix} 18.06 & 12.866 & 8.599 & 16.103 & 15.423 & 8.565 \\ 23.096 & 18.239 & 16.273 & 14.838 & 13.594 & 12.133 \\ 22.425 & 22.623 & 22.37 & 17.801 & 17.412 & 14.849 \\ 24.781 & 21.772 & 19.086 & 17.935 & 15.181 & 15.031 \end{pmatrix} \cdot GPa$$

Differece between the dynamic value of the composite Young's modulus and the dynamic value of the initial Young's modulus.

$$diff_1 := E_{dyn.composite} - E_{dyn.initial}$$

$$diff_1 = \begin{pmatrix} -0.69 & -0.316 & 0.148 & -0.325 & -0.333 & -0.19 \\ 3.063 & 3.248 & 3.047 & 3.198 & 3.225 & 3.649 \\ 5.745 & 6.657 & 6.518 & 6.313 & 6.616 & 6.805 \\ 5.104 & 4.695 & 5.437 & 5.785 & 6.073 & 6.574 \end{pmatrix} GPa$$

$$diff_{1.procent} := \frac{E_{dyn.composite}}{E_{dyn.initial}} - 1$$

$$diff_{1.procent} = \begin{pmatrix} -3.68 & -2.397 & 1.751 & -1.978 & -2.113 & -2.17 \\ 15.29 & 21.666 & 23.038 & 27.474 & 31.102 & 43.01 \\ 34.442 & 41.695 & 41.118 & 54.953 & 61.282 & 84.597 \\ 25.939 & 27.493 & 39.834 & 47.613 & 66.678 & 77.734 \end{pmatrix} \%$$

Differece between the dynamic value of the composite Young's modulus and the analytic value of the composite Young's modulus.

$$diff_2 := E_{dyn.composite} - E_{analytic.composite}$$

$$diff_2 = \begin{pmatrix} 0.098 & 0.216 & 0.463 & 0.168 & 0.14 & 0.073 \\ 1.063 & 0.855 & 0.516 & 0.543 & 0.471 & 0.748 \\ 0.261 & 1.117 & 0.97 & 0.424 & 0.673 & 0.647 \\ 0.216 & -0.396 & 0.079 & 0.31 & 0.432 & 0.81 \end{pmatrix} GPa$$

$$diff_{2.procent} := \frac{E_{dyn.composite}}{E_{analytic.composite}} - 1$$

$$diff_{2.procent} = \begin{pmatrix} 0.548 & 1.71 & 5.685 & 1.053 & 0.914 & 0.855 \\ 4.825 & 4.916 & 3.274 & 3.799 & 3.588 & 6.568 \\ 1.178 & 5.196 & 4.53 & 2.44 & 4.02 & 4.557 \\ 0.88 & -1.786 & 0.414 & 1.757 & 2.928 & 5.699 \end{pmatrix} \%$$

Stiffness for the beams, calculated as the analytical value of the Young's modulus multiplied with the second moment of inertia for the beams.

$$EI_{beam} := \overrightarrow{\left(E_{dyn.initial} \cdot \frac{w_{beam} \cdot h_{beam}^3}{12}\right)} + \overrightarrow{\left[E_{dyn.initial} \cdot A_{beam} \cdot \left(\frac{h_{beam}}{2} - z_{TP}\right)^2\right]} \ldots$$

$$+ (-1) \cdot \overrightarrow{\left(E_{dyn.initial} \cdot \overrightarrow{\left[A_{cut} \cdot \left(h_{beam} - z_{TP} - \frac{h_{cut}}{2}\right)^2\right]}\right)} \ldots$$

$$+ \overrightarrow{\left(\alpha \cdot E_{dyn.initial} \cdot \overrightarrow{\left[A_{rein} \cdot \left(h_{beam} - z_{TP} - h_{cut} + 1 \cdot mm + \frac{h_{rein}}{2}\right)^2\right]}\right)}$$

$$EI_{beam} = \begin{pmatrix} 9.487 & 6.67 & 4.276 & 8.733 & 8.375 & 4.654 \\ 13.765 & 11.227 & 10.306 & 9.457 & 8.757 & 7.674 \\ 15.007 & 14.583 & 14.515 & 11.756 & 11.284 & 9.269 \\ 16.658 & 15.181 & 13.122 & 12.167 & 9.506 & 9.588 \end{pmatrix} kN\cdot m^2$$

Weights of the different parts used in the test setup.

$$w_{measure} := 0.540 kg$$

$$w_{hanger} := 0.380 kg$$

$$w_{plate} := 0.090 kg$$

$$w_{bottom} := 2.150 kg$$

$$h := 0..5 \qquad v := 0..3$$

Values of the external weights used.

$$P := \begin{pmatrix} 146 \\ 196 \\ 221 \\ 219 \end{pmatrix} kg$$

$$P_{weights_{v,h}} := P_v$$

$$P_{weights} = \begin{pmatrix} 146 & 146 & 146 & 146 & 146 & 146 \\ 196 & 196 & 196 & 196 & 196 & 196 \\ 221 & 221 & 221 & 221 & 221 & 221 \\ 219 & 219 & 219 & 219 & 219 & 219 \end{pmatrix} kg$$

Total weight of the different parts used in the test setup summed up together for each beam.

$$P_{set} := \begin{pmatrix} 10\cdot w_{hanger} + 20\cdot w_{plate} + 4\cdot w_{measure} + w_{bottom} \\ 8\cdot w_{hanger} + 16\cdot w_{plate} + 5\cdot w_{measure} + w_{bottom} \\ 6\cdot w_{hanger} + 12\cdot w_{plate} + 2.w_{measure} + w_{bottom} \\ 4\cdot w_{hanger} + 8\cdot w_{plate} + 3.w_{measure} + w_{bottom} \\ 2\cdot w_{hanger} + 4\cdot w_{plate} + 0.w_{measure} + w_{bottom} \\ 0\cdot w_{hanger} + 0\cdot w_{plate} + 1.w_{measure} + w_{bottom} \end{pmatrix}$$

$$P_{setT} := P_{set}^{T} \qquad P_{setup_{v,h}} := P_{set_h}$$

$$P_{setup} = \begin{pmatrix} 9.91 & 9.33 & 6.59 & 6.01 & 3.27 & 2.69 \\ 9.91 & 9.33 & 6.59 & 6.01 & 3.27 & 2.69 \\ 9.91 & 9.33 & 6.59 & 6.01 & 3.27 & 2.69 \\ 9.91 & 9.33 & 6.59 & 6.01 & 3.27 & 2.69 \end{pmatrix} \text{kg}$$

Summed weight of the composite that affects the beam above it.

$$P_{timber} := \begin{pmatrix} \sum_{a=1}^{5} w_{comp_{0,a}} & \sum_{a=2}^{5} w_{comp_{0,a}} & \sum_{a=3}^{5} w_{comp_{0,a}} & \sum_{a=4}^{5} w_{comp_{0,a}} & \sum_{a=5}^{5} w_{comp_{0,a}} & 0 \\ \sum_{a=1}^{5} w_{comp_{1,a}} & \sum_{a=2}^{5} w_{comp_{1,a}} & \sum_{a=3}^{5} w_{comp_{1,a}} & \sum_{a=4}^{5} w_{comp_{1,a}} & \sum_{a=5}^{5} w_{comp_{1,a}} & 0 \\ \sum_{a=1}^{5} w_{comp_{2,a}} & \sum_{a=2}^{5} w_{comp_{2,a}} & \sum_{a=3}^{5} w_{comp_{2,a}} & \sum_{a=4}^{5} w_{comp_{2,a}} & \sum_{a=5}^{5} w_{comp_{2,a}} & 0 \\ \sum_{a=1}^{5} w_{comp_{3,a}} & \sum_{a=2}^{5} w_{comp_{3,a}} & \sum_{a=3}^{5} w_{comp_{3,a}} & \sum_{a=4}^{5} w_{comp_{3,a}} & \sum_{a=5}^{5} w_{comp_{3,a}} & 0 \end{pmatrix}$$

For the second, fourth and sixth beam which are hanging up-side-down, the weight of the beam and measuring equipment affects the load negative.

$$P_{total} := P_{timber} + P_{setup} + P_{weights} + \overrightarrow{\left[\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \cdot \left(w_{initial} + w_{measure} \right) \right]}$$

Total load acting on each and every beam.

$$P_{total} = \begin{pmatrix} 161.232 & 163.493 & 155.488 & 157.28 & 148.244 & 150.523 \\ 212.318 & 214.196 & 205.613 & 207.244 & 198.984 & 200.52 \\ 237.522 & 239.461 & 230.715 & 232.43 & 223.885 & 225.586 \\ 239.263 & 240.481 & 230.762 & 231.772 & 222.842 & 223.542 \end{pmatrix} \text{kg}$$

Since its a four point bending test the load acts on two points on the beam. The load is also multiplied with the gravity-acceleration-constant.

$$P_{load} := \frac{P_{total} \cdot g}{2}$$

$$P_{load} = \begin{pmatrix} 0.791 & 0.802 & 0.762 & 0.771 & 0.727 & 0.738 \\ 1.041 & 1.05 & 1.008 & 1.016 & 0.976 & 0.983 \\ 1.165 & 1.174 & 1.131 & 1.14 & 1.098 & 1.106 \\ 1.173 & 1.179 & 1.132 & 1.136 & 1.093 & 1.096 \end{pmatrix} \text{kN}$$

$x := 0.2m \qquad d := 0.5m$

To obtain the same bending stress in all beams in one series the length between the two supports can be varied. The maximum span is not allowed to be larger then 1.055 m.

$$length_{v,h} := root\left[\left[\frac{P_{load_{v,h}} \cdot \left(\frac{x - 0.5m}{2}\right) \cdot E_{dyn.initial_{v,h}}}{EI_{beam_{v,h}}} \cdot z_{TP_{v,h}} - 8 \cdot MPa\right], x\right]$$

$$length = \begin{pmatrix} 0.963 & 0.956 & 0.98 & 0.99 & 1.02 & 1.012 \\ 0.925 & 0.943 & 0.972 & 0.979 & 1.01 & 1.024 \\ 0.941 & 0.941 & 0.959 & 0.981 & 1.005 & 1.024 \\ 0.926 & 0.935 & 0.972 & 0.979 & 1.002 & 1.027 \end{pmatrix} m$$

$$\frac{length}{2} = \begin{pmatrix} 0.481 & 0.478 & 0.49 & 0.495 & 0.51 & 0.506 \\ 0.463 & 0.472 & 0.486 & 0.49 & 0.505 & 0.512 \\ 0.471 & 0.471 & 0.479 & 0.491 & 0.502 & 0.512 \\ 0.463 & 0.467 & 0.486 & 0.49 & 0.501 & 0.514 \end{pmatrix} m$$

The compressive stress in the timber at this span length is checked to ensure bending stress of 8 MPa.

$$\sigma_{c_{v,h}} := \frac{P_{load_{v,h}} \cdot \left(\frac{length_{v,h} - 0.5m}{2}\right) \cdot E_{dyn.initial_{v,h}}}{EI_{beam_{v,h}}} \cdot z_{TP_{v,h}}$$

$$\sigma_c = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix} MPa$$

The change in curvature can be measured with how much the beam deflects over the distance of the measuring device. In this case this length is 400 mm. The equation used is adopted from the standard EN-408. Therefore the deflection δ is named standard for these values.

$$a_1 := \frac{length - 0.5m}{2}$$

$$a_1 = \begin{pmatrix} 0.231 & 0.228 & 0.24 & 0.245 & 0.26 & 0.256 \\ 0.213 & 0.222 & 0.236 & 0.24 & 0.255 & 0.262 \\ 0.221 & 0.221 & 0.229 & 0.241 & 0.252 & 0.262 \\ 0.213 & 0.217 & 0.236 & 0.24 & 0.251 & 0.264 \end{pmatrix} m$$

$L := length$

$$L = \begin{pmatrix} 0.963 & 0.956 & 0.98 & 0.99 & 1.02 & 1.012 \\ 0.925 & 0.943 & 0.972 & 0.979 & 1.01 & 1.024 \\ 0.941 & 0.941 & 0.959 & 0.981 & 1.005 & 1.024 \\ 0.926 & 0.935 & 0.972 & 0.979 & 1.002 & 1.027 \end{pmatrix} m$$

$$\delta_{standard} := \overrightarrow{\frac{a_1 \cdot (0.4m)^2 \cdot 2 \cdot P_{load}}{16 \cdot EI_{beam}}}$$

$$\delta_{standard} = \begin{pmatrix} 0.386 & 0.548 & 0.855 & 0.433 & 0.451 & 0.812 \\ 0.322 & 0.415 & 0.462 & 0.515 & 0.568 & 0.672 \\ 0.343 & 0.355 & 0.357 & 0.466 & 0.491 & 0.626 \\ 0.3 & 0.338 & 0.407 & 0.448 & 0.577 & 0.603 \end{pmatrix} mm$$

The actual measured values of the initial deflection can be seen below.

$$\delta_{measured} := \begin{pmatrix} 0.392 & 0.499 & 0.841 & 0.44 & 0.442 & 0.706 \\ 0.2 & 0.339 & 0.404 & 0.415 & 0.567 & 0.635 \\ 0.349 & 0.292 & 0.305 & 0.404 & 0.413 & 0.554 \\ 0.265 & 0.369 & 0.363 & 0.43 & 0.533 & 0.487 \end{pmatrix} mm$$

The difference between the measured initial deflection and the analytical value of the initial deflection.

$$diff_3 := \delta_{measured} - \delta_{standard}$$

$$diff_3 = \begin{pmatrix} 6.412\times 10^{-3} & -0.049 & -0.014 & 7.135\times 10^{-3} & -9.327\times 10^{-3} & -0.106 \\ -0.122 & -0.076 & -0.058 & -0.1 & -7.275\times 10^{-4} & -0.037 \\ 6.484\times 10^{-3} & -0.063 & -0.052 & -0.062 & -0.078 & -0.072 \\ -0.035 & 0.031 & -0.044 & -0.018 & -0.044 & -0.116 \end{pmatrix} mm$$

$$diff_{3.procent} := \frac{\delta_{measured}}{\delta_{standard}} - 1$$

$$diff_{3.procent} = \begin{pmatrix} 1.663 & -9.018 & -1.694 & 1.648 & -2.067 & -13.079 \\ -37.845 & -18.225 & -12.501 & -19.393 & -0.128 & -5.457 \\ 1.893 & -17.8 & -14.651 & -13.392 & -15.881 & -11.456 \\ -11.576 & 9.325 & -10.724 & -3.931 & -7.668 & -19.233 \end{pmatrix} \%$$

From the measured value of the initial deflection a theoretical value of the initial Young's modulus of the timber beam can be calculated.

$x := 1\text{MPa}$

$$E_{static_{v,h}} := \text{root}\left(\overrightarrow{\frac{a_{1_{v,h}}\cdot(0.4m)^2\cdot 2\cdot P_{load_{v,h}}}{16\cdot\left[\left[x\left[\frac{w_{beam}\cdot h_{beam}^3}{12} + A_{beam}\cdot\left(\frac{h_{beam}}{2} - z_{TP_{v,h}}\right)^2 \ldots\right] \ldots + -1\cdot\left[A_{cut_{v,h}}\cdot\left(h_{beam} - z_{TP_{v,h}} - \frac{h_{cut_{v,h}}}{2}\right)^2\right]\right] + \left[E_{rein_{v,h}}\cdot A_{rein_{v,h}}\cdot\left(h_{beam} - z_{TP_{v,h}} - h_{cut_{v,h}} + 1mm + \frac{h_{rein_{v,h}}}{2}\right)^2\right]\right]} + -1\cdot\delta_{measured_{v,h}}}\ldots, x\right)$$

$$E_{static} = \begin{pmatrix} 18.443 & 14.489 & 8.597 & 16.162 & 16.088 & 10.072 \\ 36.996 & 19.844 & 16.015 & 15.83 & 10.389 & 9.232 \\ 16.207 & 21.249 & 20.01 & 14.189 & 13.884 & 9.578 \\ 23.59 & 14.845 & 16.174 & 12.914 & 10.241 & 11.474 \end{pmatrix} \text{GPa}$$

The values of the theoretical values of Young's modulus can be compared to the measured initial values of Young's modulus for the timber beams.

$$diff_4 := E_{static} - E_{dyn.initial}$$

$$diff_4 = \begin{pmatrix} -0.307 & 1.307 & 0.146 & -0.266 & 0.332 & 1.317 \\ 16.963 & 4.853 & 2.789 & 4.19 & 0.02 & 0.748 \\ -0.473 & 5.283 & 4.158 & 2.701 & 3.088 & 1.534 \\ 3.913 & -2.232 & 2.525 & 0.764 & 1.133 & 3.017 \end{pmatrix} \text{GPa}$$

$$diff_{4.procent} := \frac{E_{static}}{E_{dyn.initial}} - 1$$

$$diff_{4.procent} = \begin{pmatrix} -1.636 & 9.911 & 1.723 & -1.622 & 2.11 & 15.047 \\ 84.677 & 32.372 & 21.085 & 35.994 & 0.194 & 8.813 \\ -2.835 & 33.086 & 26.232 & 23.51 & 28.599 & 19.064 \\ 19.885 & -13.073 & 18.501 & 6.288 & 12.438 & 35.67 \end{pmatrix} \%$$

The theoretical deflection over long period of time due to Eurocode can be calculated.
The conditioned assumed is Medium-term load, Service class 3.

$$k_{def.beam} := 0.75$$

$$\delta_{final.1} := \delta_{standard} \cdot \left(1 + k_{def.beam}\right)$$

$$\delta_{final.1} = \begin{pmatrix} 0.675 & 0.96 & 1.497 & 0.758 & 0.79 & 1.421 \\ 0.563 & 0.725 & 0.808 & 0.901 & 0.994 & 1.175 \\ 0.599 & 0.622 & 0.625 & 0.816 & 0.859 & 1.095 \\ 0.524 & 0.591 & 0.712 & 0.783 & 1.01 & 1.055 \end{pmatrix} \text{mm}$$

$$\delta_{final.2} := \delta_{measured} \cdot \left(1 + k_{def.beam}\right)$$

$$\delta_{final.2} = \begin{pmatrix} 0.686 & 0.873 & 1.472 & 0.77 & 0.773 & 1.235 \\ 0.35 & 0.593 & 0.707 & 0.726 & 0.992 & 1.111 \\ 0.611 & 0.511 & 0.534 & 0.707 & 0.723 & 0.97 \\ 0.464 & 0.646 & 0.635 & 0.753 & 0.933 & 0.852 \end{pmatrix} \text{mm}$$