

CHALMERS



Design of arch bridges using non-linear analysis

Master's Thesis in the International Master's Programme Structural Engineering

EDINA SMLATIC AND MARCELL TENGELIN

Department of Civil and Environmental Engineering

Division of Structural Engineering

Concrete Structures

CHALMERS UNIVERSITY OF TECHNOLOGY

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Cover:
The Munkedal Bridge

Reproservice / Department of Civil and Environmental Engineering
Göteborg, Sweden 2005

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ABSTRACT

During the process of concrete structures, linear analysis is used to obtain the cross-sectional forces and moments. This method gives, in some cases, an overestimation of the amount of material needed, such as the reinforcement amount. In order to optimize the design of the structure, non-linear structural analysis can be used.

The purpose of the thesis was to show the economical potential of using non-linear analysis as a design method for bridge design. The benefit can, in most cases, be seen for slender and compressed concrete structures. In this Master's project the arch of the Munkedal Bridge was used as an example.

The study was performed in two steps: linear analysis and non-linear analysis. The amount of reinforcement needed was first calculated using linear analysis according to the Boverket (2004), Engström (2001) and Handboken Bygg (1985). The reinforcement amount obtained was then reduced using non-linear analysis by iteratively updating of the cross-sectional constants I_{ekv} , A_{ekv} and x_{ip} . The original cross-section of the arch was redesigned from a box girder section to a solid beam, in order to get a cross-section that would crack and, consequently, require bending reinforcement.

The results from linear analysis and non-linear analysis were compared in order to determine if the economical profit was obtained. It was observed that with use of non-linear analysis, the amount of reinforcement could be reduced with at least 20 % in the cross-sections with more than minimum reinforcement. The overall reduction for the whole arch was estimated to be about 17 %. Since the material dimensions were reduced so does the economical cost, in this case, the cost of reinforcement, decreases.

It was concluded that the use of non-linear analysis in the design process is economical for slender and compressed concrete structures that has a need for reinforcement, if the reinforcement amount is large.

Key words: *linear analysis, non-linear analysis, arch bridge design, reinforced concrete.*

Dimensionering av bågbroar med icke-linjär analys
Examensarbete inom Internationella Masters Programmet Structural Engineering
EDINA SMLATIC OCH MARCELL TENGELIN
Institutionen för bygg- och miljöteknik
Avdelningen för Konstruktionsteknik
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SAMMANFATTNING

Vid dimensionering av betongkonstruktioner används linjär analys för att bestämma snittkrafter och -moment. Denna metod kan, i vissa fall, leda till överdimensionering, t ex av armeringsmängden. För att optimera konstruktionens dimensioner, kan icke-linjär analys användas vid systemberäkning.

Syftet med detta examensarbete var att visa att användning av icke-linjär analys som dimensioneringsmetod kan leda till ekonomiska besparingar. Vinsten med metoden framkommer som regel för slanka och tryckta betongkonstruktioner. I detta examensarbete användes Munkedalsbrons båge som exempel.

Studien genomfördes i två steg: linjär analys och icke-linjär analys. Behovet av armeringen beräknades först med linjär analys i enlighet med Boverket (2004), Engström (2001) and Handboken Bygg (1985). Den beräknade armeringsmängden reducerades sedan med användning av icke-linjär analys. Detta genomfördes genom uppdateringar av tvärsnittskonstanterna I_{ekv} , A_{ekv} och x_{lp} genom iteration. Bågens ursprungliga tvärsektion konstruerades om från lådbalksektion till ett homogent rektangulärt tvärsnitt, eftersom lådbalksektionen inte sprack och ingen armering utöver minimiarmering krävdes här.

Resultaten från de båda analyserna jämfördes för att se om det var möjligt att åstadkomma några besparingar av armeringsmängden. Det observerades att med användning av icke-linjär analys kunde armeringsmängden reduceras med åtminstone 20 % för de tvärsnitt som krävde mer än minimiarmering. Den totala minskningen i hela bågen uppskattades till ca 17 %. Eftersom armeringsmängden reducerades, reduceras följaktligen även kostnaden för projektet.

Slutsatsen drogs att användning av icke-linjär analys under dimensioneringsskedet är ekonomisk för slanka och tryckta betongkonstruktioner som har behov av armering, om den erforderade armeringsmängden är stor.

Nyckelord: *linjär analys, icke-linjär analys, bågbroar, armerad betong, betongkonstruktioner.*

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Preface

This Master's Thesis was carried out from October 2004 to March 2005 at ELU Konsult AB. The thesis has been developed based on the initiation of ELU Konsult AB and the support from Division of Structural Engineering, Concrete Structures, Chalmers University of Technology, Göteborg, Sweden.

The thesis has been carried out with Per Olof Johansson as a supervisor at ELU Konsult AB and Mario Plos as an examiner at Division of Structural Engineering, Concrete Structures, Chalmers University of Technology. Our sincere gratitude has to be given to both of them for their guidance and support throughout the duration of this thesis.

An extension of our gratitude has to be made to all the staff at ELU Konsult AB for their support, advices and making us feel as part of their group.

Finally, it should be noted that we are forever grateful for the support and patience of our closest ones throughout this project. We would also like to thank all those who have been directly or indirectly related to the successful accomplishment of this project.

Göteborg, March 2005

Edina Smlatic

Marcell Tengelin

Notations

Roman upper case letters

A	Axle load on the bridge deck
A_{ekv}	Equivalent area of the transformed concrete section
A_s	Steel area in tension
A_{st}	Steel area in compression
A_{s1}	Steel area
E_c	Design modulus of elasticity of concrete
E_{ck}	Characteristic modulus of elasticity of concrete
E_s	Design modulus of elasticity of steel
E_{sk}	Characteristic modulus of elasticity of steel
F_c	Compressive concrete force
F_{cs}	Shrinkage force
F_s	Tensile steel force
F_{st}	Compressive steel force
I_c	Moment of inertia of the compressive zone
I_{ekv}	Equivalent moment of inertia of the transformed concrete section
M_d	Bending moment capacity
M_s	Combined bending moment and normal force
M_{sd}	Design bending moment
M_l	Part of M_s taken by the concrete cross-section and the remaining tension reinforcement

M_{II}	Part of M_s taken by the compressive reinforcement including the corresponding part of the tension reinforcement
N_{sd}	Design normal force
T^+	Mean positive temperature
T^-	Mean negative temperature
T_{\max}	Maximum temperature
T_{\min}	Minimum temperature
ΔT^+	Positive temperature difference
ΔT^-	Negative temperature difference

Roman lower case letters

b	Width of the cross-section
d	Distance of the tension reinforcement from the top of the cross-section
d_t	Distance of the compression reinforcement from the top of the cross-section
e	Eccentricity of the normal force
f_{ck}	Characteristic compressive strength of concrete
f_{cc}	Design compressive strength of concrete
f_{sd}	Design tensile strength of the reinforcement
f_{st}	Tensile strength of the reinforcement
h	Height of the cross-section
m_r	Relative moment
m_{bal}	Balanced moment
p	Evenly distributed load on the bridge deck
x	Depth of the compressive zone

x_{ip}	Gravity centre of the cross-section
z	Local coordinate and starts from the equivalent concrete cross-section's gravity centre

Greek lower case letters

α	Stress block factor
α_{ef}	Effective ratio between the modulus of elasticity of steel and concrete
α_l	Length expansion coefficient for steel and concrete
β	Stress block factor for the location of the stress resultant
ε_{cs}	Shrinkage strain for concrete
ε_{cu}	Concrete strain
ε_s	Steel strain, tension
ε_{st}	Steel strain, compression
ε_{sy}	Steel strain, yielding
ϕ	Reinforcement bar diameter
γ_n	Partial safety factor for safety class
γ_m	Partial safety factor for strength
η	Partial safety factor for the material
σ_s	Steel stress in tension
σ_{st}	Steel stress in compression
ω	Mechanical reinforcement content
ω_{bal}	Balanced mechanical reinforcement content
ψ	Creep factor

1 Introduction

1.1 Background

During the design process of concrete structures, linear analysis is used to obtain the dimensions for the structure. This method gives, in some cases, an overestimation of the amount of material needed, for example the reinforcement content. To optimize the structure, non-linear analysis can be used. Generally, it is not practical to use non-linear analysis in the design process since it is time consuming and since the superposition principle is then not applicable. The method is mostly used for evaluation of the response and behaviour of existing structures. A simplified method for non-linear analysis as a design method has been used at ELU Konsult AB to design an arch bridge, but it appeared to be both time consuming and expensive.

1.2 Aim and implementation

The main aim of the Master's project 'Design of arch bridges with non-linear analysis' is to show the economical potential of using non-linear analysis for the bridge design. The profit can, in most cases, best be seen for slender and compressed concrete structures like the arch of the bridge in this Master's project.

The Master's project was carried out in two steps. The first step was to obtain the preliminary design by the use of linear analysis. The second step was to optimize the design with the use of non-linear analysis. The results were then compared to see how much there is to save by using non-linear analysis in the design process.

1.3 Limitations

The Master's Thesis was based on a concrete arch that is a part of a bridge consisting of a bridge deck that is connected to the arch with concrete columns. The arch and the bridge deck were considered as two separate structures, where the bridge deck loads were transferred to the arch through the columns as reaction forces obtained by the structural analysis program, Strip Step 2. In this Master's project, only the design of the arch was of interest and that is why no calculations were made for the design of the bridge deck and columns. In the analysis of the bridge deck subjected to traffic load, it was assumed that the bridge deck was resting on a stiff arch. The calculations of the reinforcement needed in the arch were done under the assumption that the cross-sections are sufficient enough to resist the shear forces acting on it. In the design, some loads were not taken into account, such as wind load since the design was limited to two dimensions only and the effects of differential settlements since they have minor influence in this case. Concerning the design of the arch cross-section, only the height of the arch could be altered since the width is fixed to 13 m in order to support the columns from the bridge deck.

2 The Munkedal Bridge

2.1 Description

The Munkedal Bridge is to be built over the river Örekilsälven which is situated north of Uddevalla. The main purpose for the Munkedal Bridge is to improve the accessibility of the highway route E6. The bridge is designed as an arch, with the deck on top connected to the arch with columns, see Figure 2.1. The span of the Munkedal Bridge is 225 m with 3 % inclination and the maximum height is about 39 m from the ground.

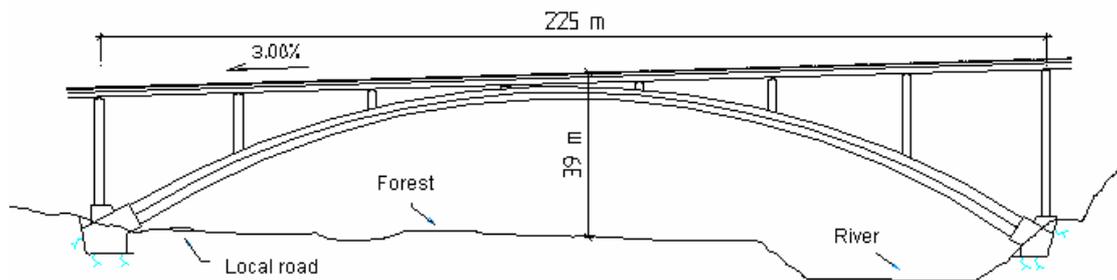


Figure 2.1 The Munkedal Bridge.

The bridge deck is made of concrete and is supported by two steel box girders, see Figure 2.2. It is 23.30 m wide with 2.5 % inclination at both sides and has four lanes of traffic, two in each direction. The arch and the columns are made of concrete.

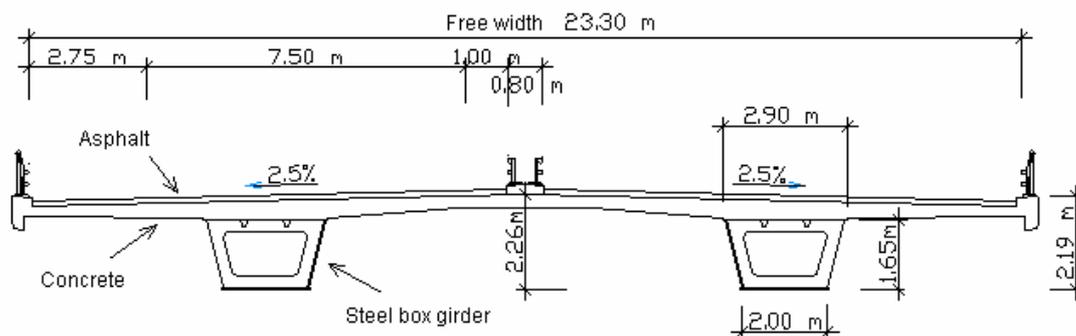


Figure 2.2 The bridge deck profile.

The boundary conditions and the model of the bridge created for the structural analysis program are described in detail in Chapter 4.

2.2 The Loads

The main loads on the arch were the gravity loads due to self-weight of the structure and that of moving traffic. All the loads and the load coefficients were taken from Vägverket (2004).

2.2.1 Permanent Loads

Permanent loads are defined as dead loads from the self-weight of the structure which remain essentially unchanged during the life time of the bridge. The self-weight of the bridge deck and the columns were placed as point loads on the arch. The material properties are given in Table 2.1. Included in the permanent loads acting on the arch were also the loads imposed due to shrinkage and creep. For the detailed permanent load calculation, see Appendix A.

Table 2.1 *Material properties.*

<i>Material</i>	<i>Load [kN/m³]</i>
Concrete	25
Asphalt	23
Steel	1,6

The drying of concrete due to evaporation of absorbed water causes shrinkage. If deformation of the structure is prevented, the shrinkage will lead to a constant shrinkage force. Creep is a long term effect leading to increased deformations with time of a loaded structure. The creep modifies the effect of shrinkage. This can be accounted for by reducing the modulus of elasticity of concrete. The shrinkage and creep characteristics of concrete induce internal stresses and deformations in the arch.

$$\text{Shrinkage force: } F_c = E_s \cdot A_s \cdot \varepsilon_{cs} \quad (2.1)$$

$$\text{Effects of creep: } \alpha_{ef} = \alpha \cdot (1 + \psi) \quad (2.2)$$

$$\text{where: } \alpha = \frac{E_s}{E_c} \quad (2.3)$$

2.2.2 Variable loads

Variable loads are all loads other than the permanent loads, and have a varying duration. The variable loads acting on the bridge are the traffic load, the braking load and the temperature load.

The variable loads that were not taken into account in this Master's project are fatigue, side force, snow, wind and different types of vehicle loads (emergency services vehicles, working vehicles etc). These loads have minor influence on the structure as compared to the traffic, braking and temperature loads.

2.2.2.1 Traffic load

According to Vägverket (2004), there are two types of traffic load that can be critical on this bridge: equivalent load type 1 and equivalent load type 5, see Figure 2.3. Equivalent load type 1 consists of an evenly distributed load p [kN/m] and one load group with three concentrated axle loads A [kN] with minimum longitudinal axle distances of 1.5 m and 6 m. Equivalent load type 5 consists of an evenly distributed load p [kN/m] and two load groups with three concentrated axle loads A [kN] with minimum longitudinal axle distances according to Figure 2.3. The values for the traffic loads A and p are given in Table 2.2.

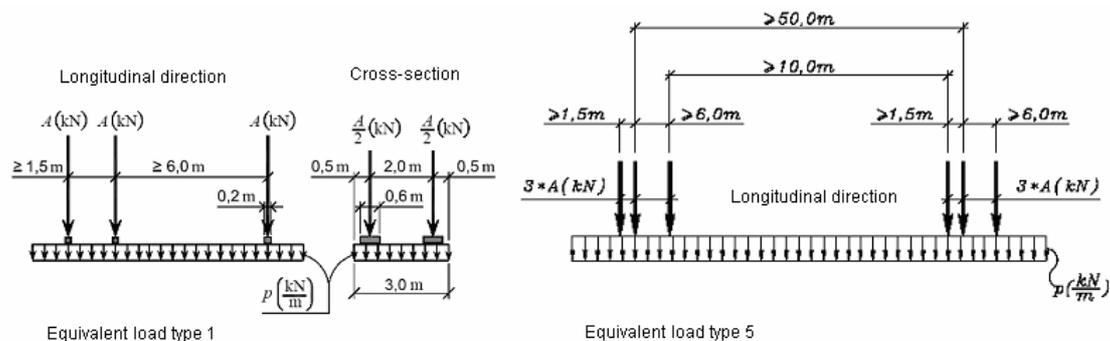


Figure 2.3 Equivalent load types, adapted from Vägverket (2004).

Table 2.2 The magnitude of the traffic loads according to Vägverket (2004).

A [kN]	P [kN/m]	Lane
250	12	1
170	9	2
	6	3

In the design, the bridge deck was loaded with six lanes of traffic, even though the bridge has four lanes under normal traffic conditions. In this Master's project, equivalent load type 5 was considered as the most critical and the bridge was designed for this load type. An example of the total traffic load in a cross section can be seen in Figure 2.4.

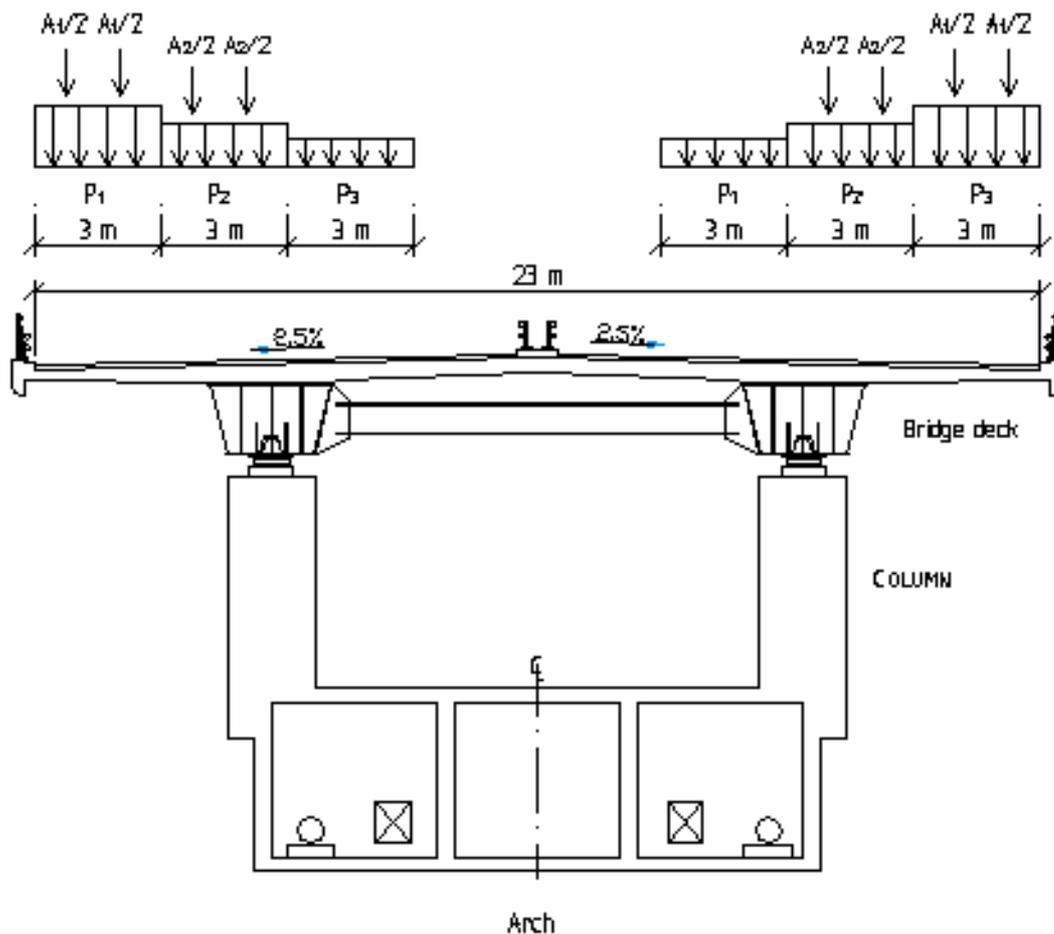


Figure 2.4 The total traffic load in a cross-section.

2.2.2.2 Braking load

According to Vägverket (2004), the braking load acting on the bridge is 800 kN as the bridge length is greater than 170 m. The horizontal braking force was applied on the bridge deck at the section where it is rigidly connected to the arch, through the shortest column.

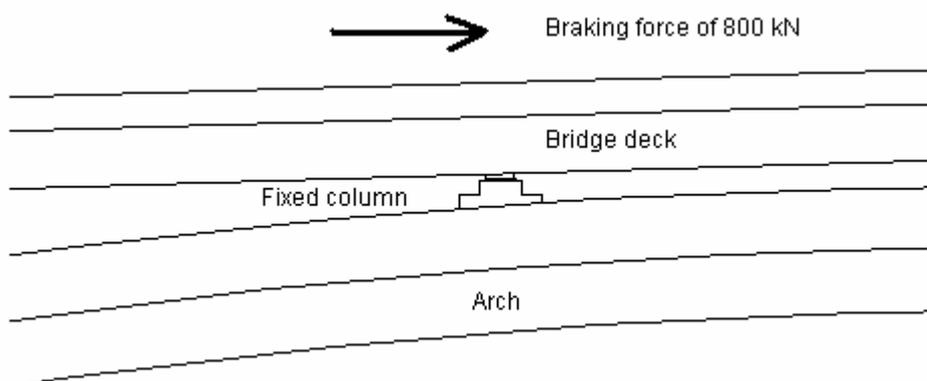


Figure 2.5 The braking force acting on the arch.

2.2.2.3 Temperature load

The temperature of the arch and its environment changes on a daily and seasonal basis. This influences both the overall movement of the structure and the stresses within it. The daily effects give rise to a temperature variation within the arch, which varies depending on cooling or heating. The idealized linear temperature gradient to be expected for a certain structure when heating or cooling can be seen in Table 2.3.

Table 2.3 The idealized linear temperature gradient according to Vägverket (2004).

Structure type	Mean temperature °C in the structure		Temperature difference °C	
	T^+	T^-	ΔT^+	ΔT^-
1. Steel or aluminium bridge deck on box girder or I-beam of steel	$T_{\max} + 15$	$T_{\min} - 5$	+20	-5
2. Concrete or timber bridge deck on box girder or I-beam of steel	$T_{\max} + 5$	$T_{\min} + 5$	+10	-5
3. Concrete bridge deck on box girder or T-beam of concrete	T_{\max}	$T_{\min} + 10$	+10	-5
4. Timber bridge deck on timber beams	$T_{\max} - 5$	$T_{\min} + 10$	+5	-5

Values for T_{\max} and T_{\min} depend on the geographical location and are given in Figure 2.6.

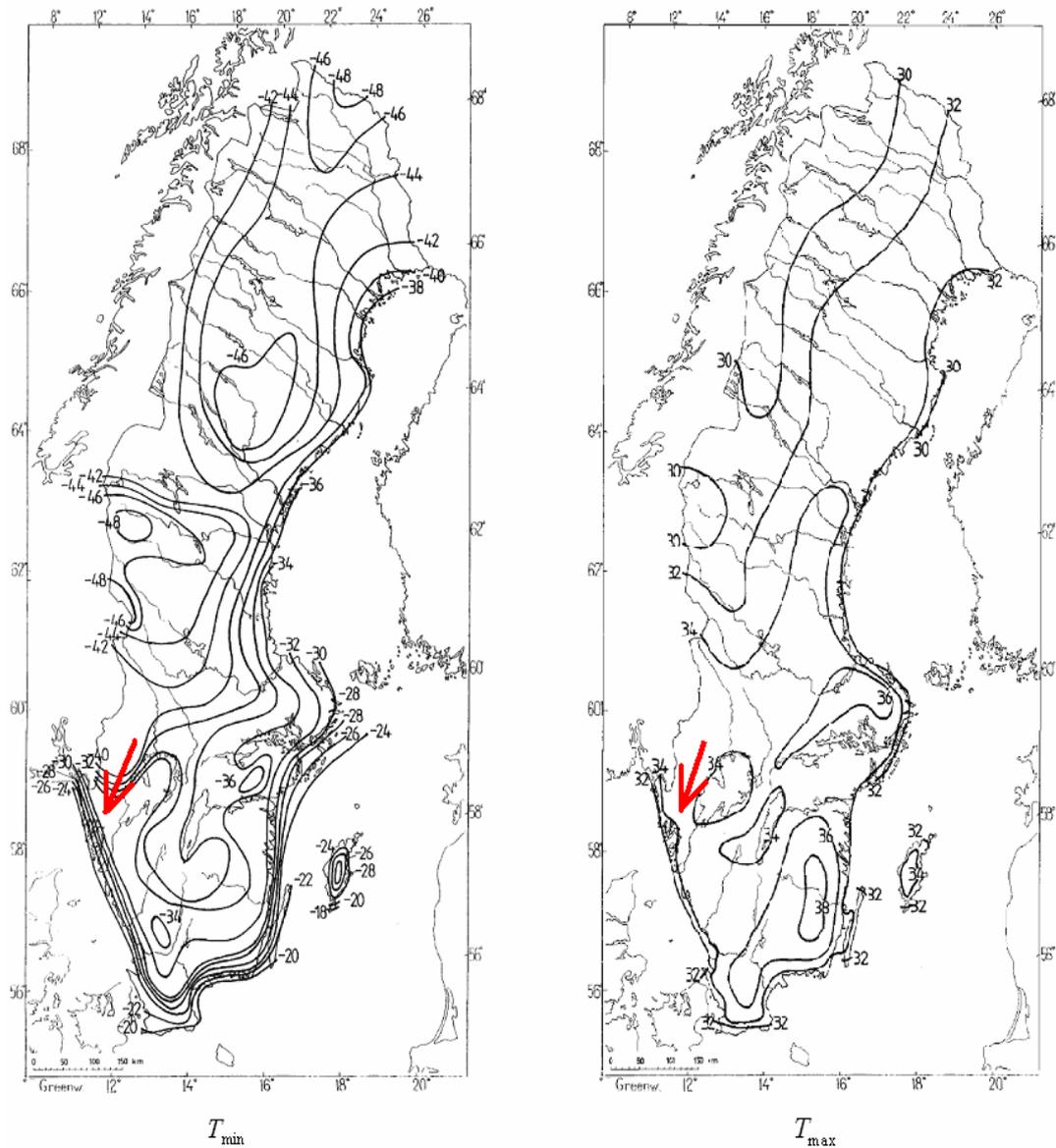


Figure 2.6 Values for T_{max} and T_{min} , adapted from Vägverket (2004).

Structure type 3 was chosen as it fits the description of the bridge in this Master's project. The values of the temperatures acting on the structure can be seen in Table 2.4.

Table 2.4 Temperatures acting on the structure.

Mean temperature [°C]		Temperature difference [°C]	
T^+	T^-	ΔT^+	ΔT^-
39	-27	10	-5

The length expansion coefficient for steel and concrete is $\alpha_l = 1 \cdot 10^{-5} [1/^\circ\text{C}]$ and was used for calculation of the deformation due to the temperature variation.

3 Structure Analysis Program, Strip Step 2

3.1 Introduction

The structural analysis program used for the design of the arch is called Strip Step 2. This program was developed in the 60's and was modified to work with today's computers. Strip Step 2 is one of the programs that can be used in bridge design, according to the Swedish Road Administration. This program is well established and efficient for the initiated user, and still widely used in Sweden even if it is old.

The program is intended for calculations of structures that can be represented by elements with linear extension, such as frames and trusses. It allows for curved elements and even cross-section variation along the element. The load cases can consist of evenly distributed loads, point loads, temperature loads, traffic loads, pre-stressing loads and support displacement loads. The load cases can then be combined to find the maximum cross-sectional forces and moments acting on the structure and the influence lines of the applied vertical loads. The calculations for creep and shrinkage are done through gradual iteration.

3.2 Assumptions

The calculations in the structural analysis program, Strip Step 2, are based on the theory of elasticity which states that the stress-strain relationship is linear. Also, the plane cross-section remains plane after deformation. The calculations are performed according to the 1st order (linear) theory. The calculations can also be performed assuming the 2nd order (non-linear) theory with respect to the deformations (not the material). Since the arch element function for the arched elements in the program was not working, the arch elements were modelled as plain beam elements.

3.3 Structure

The structure of the input data is not that difficult to understand even if it is in a DOS environment. Every input has a special four digit code that has a specific function, for instance, 2050 is the code describing the load on the structure. The program starts by defining the structure type and the material constants such as Young's modulus and the Poissons ratio. The geometry of the elements and their cross-section parameters, such as height, area, gravity centre and moment of inertia are then given. The next step is to connect the elements and to specify their degrees of freedom. After this is done, the loads are then defined, combined and applied on the structure. Once the simulation is completed, the cross-sectional forces, stresses and influence-lines are obtained in a result file.

4 The Bridge Model

The bridge was modelled as two separate structures, the bridge deck and the arch. The bridge deck was modelled as a beam with eight supports. The arch was modelled using beam elements connected to each other and with fixed supports at the abutments. The division was based on the assumption that the arch is stiffer than the bridge deck. This means that the arch has no displacements under the loading of the bridge deck.

4.1 The bridge deck model

4.1.1 Geometry and boundary conditions

The function of the bridge deck model was to acquire the reaction force influence lines of the bridge deck caused by the traffic load acting on it. The bridge deck was modelled as a continuous beam with columns acting as supports. The bridge deck is a part of the highway route continuing on both sides of the bridge. In this project, the part of the bridge deck that is situated above the arch and that is connected to the arch through columns is taken into account. The rest of the bridge deck is supported by columns to the ground, and is not included in the model since loads on these parts of the deck have a very small influence on the maximum reaction forces transferred to the arch.

The bridge deck is divided in seven elements between the eight columns supporting it. The supports B1, B5 and B8, were assumed partly fixed; that is no displacements were allowed and only rotations about the x-axis, see Figure 4.1. The rest of the supports were assumed simply supported, allowing for rotations about the x-axis and for displacements along the y-axis, see Figure 4.1. The bridge deck has an inclination of 3% which was taken into account when defining the geometry of the bridge deck.

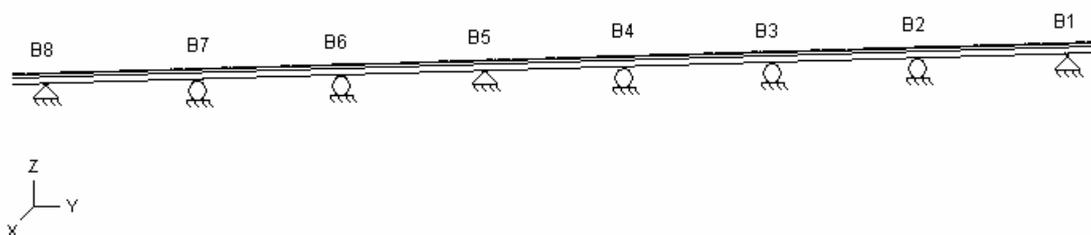


Figure 4.1 Boundary condition of the bridge deck.

4.1.2 The analysis sequence of the bridge deck model

The analysis sequence starts by dividing the bridge deck into elements and defining their degrees of freedom. The dead load of the bridge was introduced as an evenly distributed load. The braking force on the bridge deck was introduced at section B5, see Figure 4.1 and Figure 2.5, where the shortest support column is located. In this

way, the most of the braking force is transferred to the arch through the reaction force. At sections B1 and B8 the braking force was not applied since the supporting columns are long and very little of the braking force is transferred to the arch, see Figure 4.2. A distributed traffic load and two traffic load axle groups were introduced with different axle distances. Combination of the loads into load cases was made and influence lines of the reaction forces were calculated.

4.1.3 Influence lines

Bridge decks on arches should support both fixed and moving loads. Each element of a bridge must be designed for the most severe conditions that can possibly occur in that member. Live loads should be placed at the position where they will produce critical conditions in the member studied. The critical position for the live loads will not be the same for every member. A useful method of determining the most severe condition of loading is by using influence lines.

An influence line for a particular response, such as the reaction force, is defined as a diagram, see Figure 4.2. Influence lines describe how, for example, the force in a given part of the structure varies as the applied unity load moves along the structure. Influence lines are primarily used to determine the critical positions of the live loads.

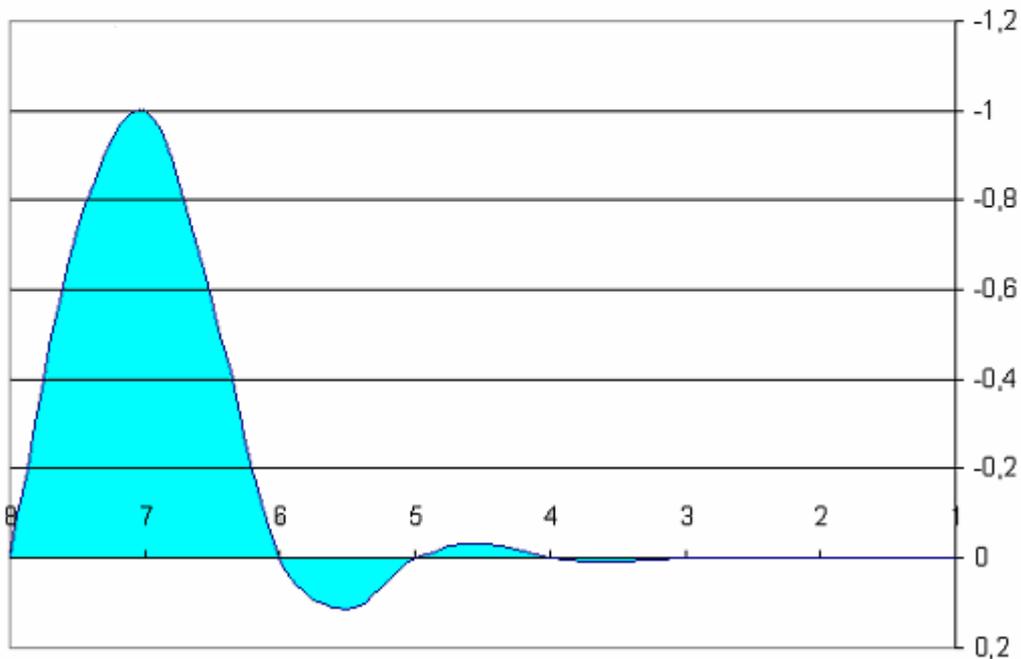
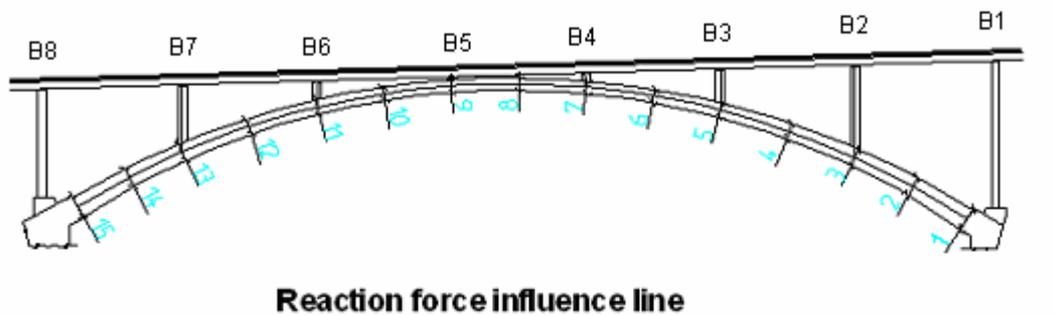


Figure 4.2 Example of an influence line for the reactions force at support 7 for the bridge deck.

The influence lines were used to calculate the highest force that can act on the columns. For each traffic load position, giving a maximum column force, also the forces in the other columns were calculated. The calculated loads acting on the columns were then used as input traffic load on the arch model. From the bridge deck model, seven different load positions on the bridge deck gives seven different load cases, each one with the maximum force in one of the columns, see Appendix B.

4.2 The arch model

4.2.1 Geometry and boundary conditions

The function of the arch model was to obtain the cross-sectional bending moments and the normal forces in the arch. The effects of creep and shrinkage were included in the simulation.

The arch was modelled as an arched structure with two abutments. The abutments were assumed fixed, that is no displacements and rotations were allowed. The arch was divided into fifteen sections creating fourteen elements that were coupled together through common degrees of freedom, see Figure 4.3. Each element's cross-sectional constants and geometry were defined in the structural analysis program. The coordinates and the degrees of freedom of each point are located at the gravity centre of each cross-section.

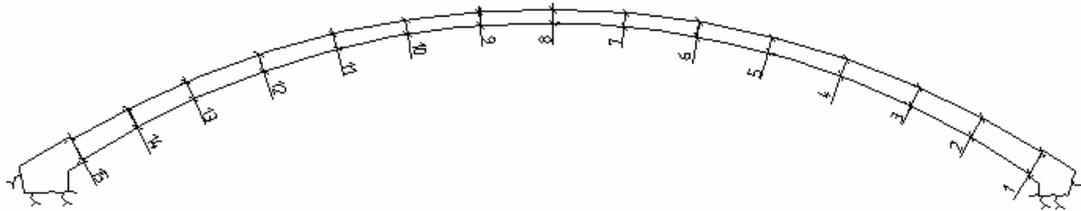


Figure 4.3 The concrete arch divided into fifteen sections.

In addition to the dead weight of the structure and the traffic load acting on it, temperature load caused by temperature difference across the cross-section of the arch was also taken into account as well as the braking force.

4.2.2 The analysis sequence of the arch model

The analysis sequence starts by defining the arch cross-sectional constants in each section. The dead load of the arch was applied as evenly distributed load, while the dead loads from columns and the bridge deck were applied as concentrated loads. The reaction force and the bending moment, obtained from the analysis of the bridge deck, from the braking load were applied on the arch as point loads. The different traffic load cases and the temperature load were introduced and the various load cases were combined. The maximum bending moments and the corresponding normal forces at each section were obtained in the result file.

There were seven different traffic load combinations, resulting in seven different analyses. Each analysis gave different bending moments and normal forces in each cross-section of the arch. The results from each analysis were compared and the maximum value of the bending moment with the normal force from the same load case for each section were then selected and used for calculating the reinforcement area needed for each section.

5 Linear Analysis

5.1 Introduction

A linear analysis is often carried out in a simplified way, using the uncracked gross concrete sections and ignoring the reinforcement. It is generally assumed that the flexural rigidities along the structure are constant during the simulations. Linear analysis is only valid as long as the arch is uncracked.

Cross-sectional normal forces and bending moments were calculated under the assumption of linear elasticity. The cross-section was then designed in the ultimate limit state with the concrete compression failure strain $\varepsilon_{cu} = 3.5 \text{ ‰}$ as failure criteria. In the cross-sectional analysis, the strains were assumed to vary linearly across the cross-section and plane cross-sections were assumed to remain plane after deformation.

In this project, the objective was to study a structure where the non-linear response has a significant influence on the required amount of reinforcement. Considering this, an arch with cross-sections that crack was needed for this Master's project and once it was obtained, non-linear analysis would be required.

It is possible that the original cross-section is strong enough to carry the bending moments and normal forces without the need for reinforcement. If this is the case, the cross-section constants such as height and thicknesses of the slabs will be reduced until the need for reinforcement is obtained, since the purpose of this thesis is to show that the amount of reinforcement can be minimized using non-linear analysis.

5.2 Calculation of required reinforcement

The calculations were carried out according to Engström (2004) in case of box girder cross-sections and according to Handboken Bygg (1985) in case of solid beam cross-section. When using the method according to Engström (2004), the box girder cross-section was simplified into an I-beam cross-section. First, this method was also applied to the solid beam cross-section design, and an over reinforced cross-section was obtained. In this case, the amount of compression reinforcement was guessed and the calculations were made iteratively until the required reinforcement was obtained. To avoid this long process, the method according to Handboken Bygg (1985) was used instead. Here the required amount of compression reinforcement was calculated first, assuming balanced reinforcement. From that amount, the needed tension reinforcement was calculated. Both methods are based on the method with the simplified compressive stress block:

$$F_c = f_{cc} \cdot b \cdot 0.8 \cdot x \quad (5.1)$$

The bending moments and the normal forces for each cross-section were obtained from the linear structural analysis performed with the structural analysis program for the arch.

In order to make the calculations simple, the bending moment M_{sd} and the normal force N_{sd} were combined into one moment M_s in both calculation methods, see Figure 5.1.

$$M_s = M_{sd} + N_{sd} \cdot e \quad (5.2)$$

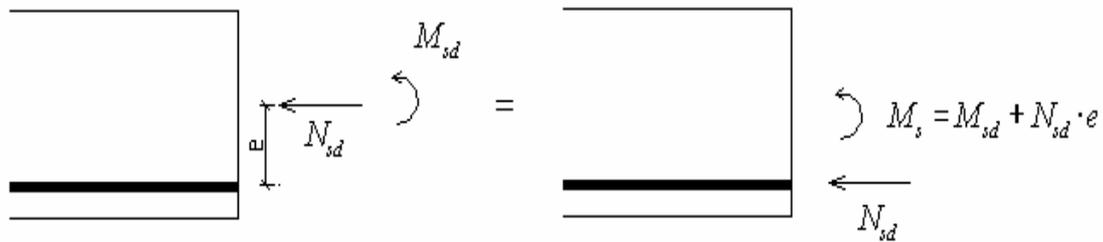


Figure 5.1 Bending moment combined with normal force.

The calculations for both methods were performed under following assumptions:

- The maximum concrete strain is limited to $\varepsilon_{cu} = 3.5 \text{ ‰}$
- The concrete cannot take tensile force in the cracked cross-section
- The concrete compressive stress f_{cc} is constant in the compressive zone
- The reinforcement is hot rolled and not pretensioned.

5.2.1 Calculation method for the box girder cross-section

To simplify the calculations, the box girder cross-section was divided into four equal I-beams, see Figure 5.2.

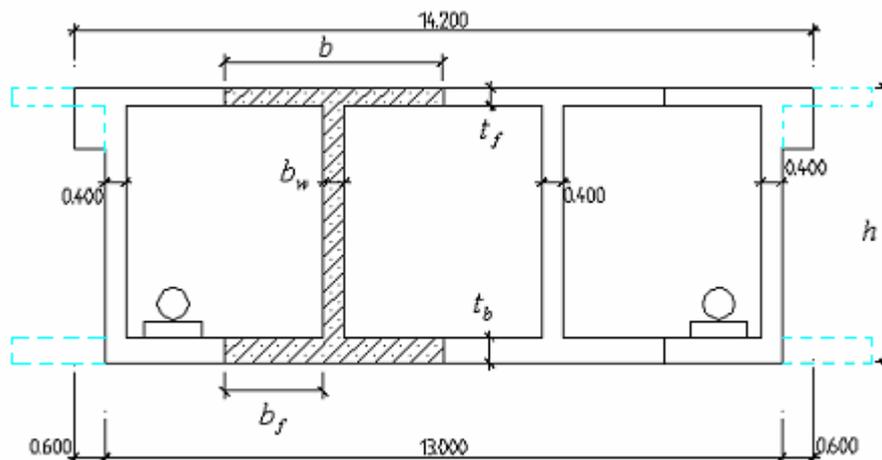


Figure 5.2 The box girder cross-section divided into four I-beam cross-sections.

When calculating for I-beams there were some assumptions that had to be made. The compressive zone, 80% of it, could either be assumed to fit into the flange or not. If it was assumed that the 80% of the compressive zone fit in the flange, see Figure 5.3, the calculations were carried out as for the rectangular cross-sections and the following was valid:

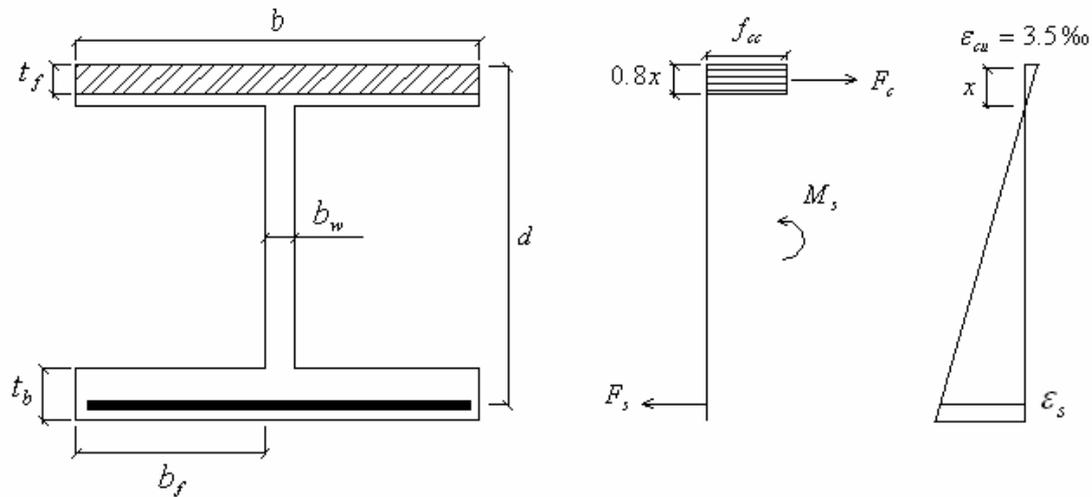


Figure 5.3 The compressive zone fits in the flange.

For $0,8 \cdot x \leq t$

Horizontal equilibrium: $F_c = f_{cc} \cdot b \cdot 0,8 \cdot x \quad F_s = \sigma_s \cdot A_s \quad (5.3)$

$$F_c = F_s - N_{sd} \quad (5.4)$$

Moment equilibrium: $M_s = F_c \cdot (d - 0,4 \cdot x) \quad (5.5)$

Deformation: $\varepsilon_s = \frac{d - x}{x} \cdot \varepsilon_{cu} \quad (5.6)$

Steel stresses: $\sigma_s = E_s \cdot \varepsilon_s \quad \text{if} \quad \varepsilon_s \leq \varepsilon_{sy} \quad (5.7)$

$$\sigma_s = f_{sd} \quad \text{if} \quad \varepsilon_s \geq \varepsilon_{sy} \quad (5.8)$$

If on contrary, the 80% of the compressive zone does not fit into the flange, the shape of the cross-section had to be considered in calculations, see Figure 5.4. The following was valid:

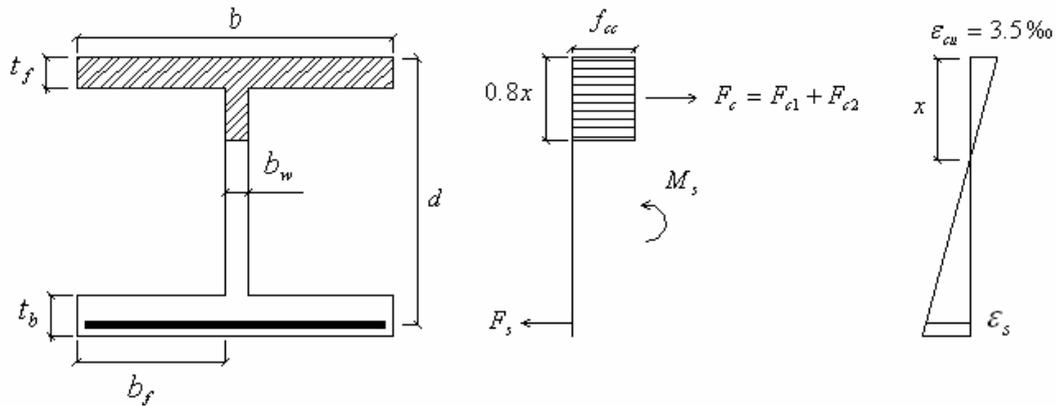


Figure 5.4 The compressive zone does not fit in the flange.

For $0,8 \cdot x > t$

Horizontal equilibrium: $F_{c1} = f_{cc} \cdot b_w \cdot 0,8 \cdot x$ $F_{c2} = f_{cc} \cdot (b - b_w) \cdot t$ (5.9)

$$F_s = \sigma_s \cdot A_s \quad (5.10)$$

$$F_{c1} + F_{c2} = F_s + N_{sd} \quad (5.11)$$

Moment equilibrium: $M_s = F_{c1} \cdot (d - 0,4 \cdot x) + F_{c2} \cdot (d - \frac{t}{2})$ (5.12)

Deformation: $\varepsilon_s = \frac{d - x}{x} \cdot \varepsilon_{cu}$ (5.13)

Steel stresses: $\sigma_s = E_s \cdot \varepsilon_s$ if $\varepsilon_s \leq \varepsilon_{sy}$ (5.14)

$$\sigma_s = f_{sd} \quad \text{if} \quad \varepsilon_s \geq \varepsilon_{sy} \quad (5.15)$$

The needed amount of reinforcement in both cases was calculated from horizontal equilibrium conditions, (5.4) and (5.11).

5.2.2 Calculation method for the solid cross-section

The simple rectangular cross-section was calculated according to the method in Handboken Bygg (1985). For the solid beam cross-section, the equations according to Chapter 5.2.1, when the compressive zone fits into the flange, are valid. This equation system is expressed in a series of equations out of which the needed amount of reinforcement can be solved directly in case of the cross-section being normally reinforced with only tension reinforcement, i.e. the tension reinforcement yields before the concrete compression strain reaches $\varepsilon_{cu} = 3,5 \text{‰}$.

$$m_r = \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad (5.16)$$

$$\omega = 1 - \sqrt{1 - 2 \cdot m_r} \quad (5.17)$$

$$A_{s1} = \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}} \quad (5.18)$$

$$A_s = A_{s1} - \frac{N_{sd}}{f_{sd}} \quad (5.19)$$

In case of A_s being negative, the cross-section does not need reinforcement, since the compressive normal force is large and the cross-section is uncracked.

To check if the cross-section is normally reinforced, calculated values of relative moment m_r and mechanical reinforcement content ω_r were compared with the values of these parameters for balanced reinforcement, see Table 5.1. Balanced reinforcement is obtained when the tensile reinforcement reaches yielding at the same time as the concrete compression strain reaches the failure strain $\varepsilon_{cu} = 3.5 \text{ ‰}$.

Table 5.1 Values for balanced reinforcement, according to Handboken Bygg (1985).

Reinforcement	ω_{bal}	m_{bal}
Ss 22 (s)	0,615	0,426
Ss 26 (s)	0,591	0,416
Ks 22 (s) $\phi < 16$	0,522	0,386
$\phi 20 - 25$	0,532	0,390
$\phi 32$	0,542	0,395
Ps/Ns/Nps/50	0,480	0,365
Ks 60 (s)	0,443	0,345
Bs/Ss/Nps/70	0,412	0,327

If $m_r < m_{bal}$ and $\omega < \omega_{bal}$ a normally reinforced cross-section was obtained. Consequently, the steel area needed was calculated according to the equations (5.18) and (5.19).

In case of relative moment being greater than the balanced moment, $m_r > m_{bal}$, the cross-section will be over-reinforced, i.e. the concrete fails in compression before the reinforcement yields. Measures can be taken in order to prevent over-reinforcement such as to include compression reinforcement or to increase d , b or f_{cc} . Since the cross-section dimensions were already designed in this project, that is the d and b could not be changed and since the concrete strength chosen was already high, the

choice was here to include compression reinforcement. Accordingly, the needed compression reinforcement was calculated in the following way, assuming $\sigma_{st} = f_{sd}$:

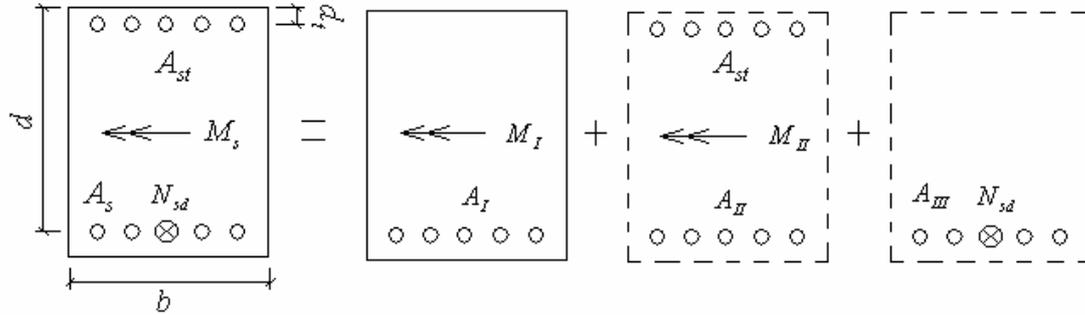


Figure 5.5 Division of the cross-section into a section with compression reinforcement and a section without.

$$A_{st} \geq \frac{M_s - m_{bal} \cdot b \cdot d^2 \cdot f_{cc}}{(d - d_t) \cdot \sigma_{st}} \quad (5.20)$$

$$M_{II} = A_{st} \cdot \sigma_{st} \cdot (d - d_t) \quad (5.21)$$

$$M_I = M_s - M_{II} \quad (5.22)$$

$$m_r = \frac{M_I}{b \cdot d^2 \cdot f_{cc}} \quad (5.23)$$

$$\omega = 1 - \sqrt{1 - 2 \cdot m_r} \quad (5.24)$$

$$\varepsilon_{st} = \varepsilon_{cu} \cdot \left(1 - 0.8 \frac{d_t}{\omega \cdot d}\right) \quad (5.25)$$

Check the assumption: $\varepsilon_{st} > \varepsilon_{sy} \Rightarrow \sigma_{st} = f_{sd}$ (5.26)

If the assumption (5.26) was not satisfied, the calculation had to be redone with $\sigma_{st} = E_s \cdot \varepsilon_{st}$ until convergence was obtained. When the assumption (5.26) was satisfied or convergence has been obtained for non-yielding compression reinforcement, the tension reinforcement was calculated as:

$$A_I = \frac{M_I}{d \cdot \left(1 - \frac{\omega}{2}\right) \cdot f_{sd}} \quad (5.27)$$

$$A_{II} = \frac{M_{II}}{d - d_t} \cdot \frac{1}{f_{sd}} \quad (5.28)$$

$$A_{III} = \frac{N_{sd}}{f_{sd}} \quad (5.29)$$

$$A_{s1} = A_I + A_{II} \quad (5.30)$$

$$A_s = A_{s1} - A_{III} \quad (5.31)$$

5.2.3 Check of the designed cross-section

When the reinforcement needed is known and arranged in the cross-section, the x value of the compressive zone can be solved with the horizontal equilibrium and by assuming the steel strains in each steel level.

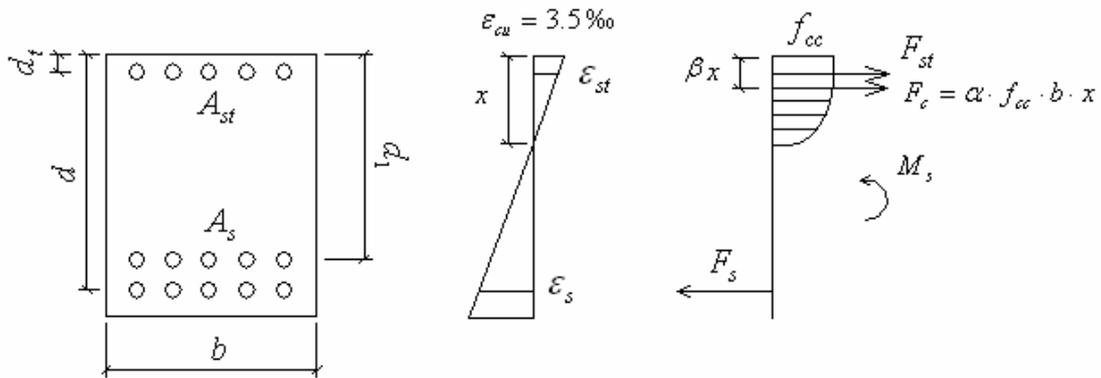


Figure 5.6 Calculation conditions for the cross-section.

Horizontal equilibrium: $F_c + F_{st} = N_{sd} + F_s$ (5.32)

Compressive concrete force: $F_c = \alpha \cdot f_{cc} \cdot b \cdot x$ (5.33)

Tensile steel force: $F_s = \sigma_s \cdot A_s$ (5.34)

Compressive steel force: $F_{st} = \sigma_{st} \cdot A_{st}$ (5.35)

Strains: $\varepsilon_s = \frac{d-x}{x} \cdot \varepsilon_{cu} > \varepsilon_{sy} \Rightarrow \sigma_s = f_{sd}$ (5.36)

$$\varepsilon_s = \frac{d-x}{x} \cdot \varepsilon_{cu} < \varepsilon_{sy} \Rightarrow \sigma_s = E_s \cdot \varepsilon_s \quad (5.37)$$

After the x was solved, the verification of the strain assumptions was done. If they were not satisfied, new strain assumptions had to be made and the new x calculated. In case that the assumptions were satisfied, the right x value was obtained.

The next step was to check the moment capacity of the cross section by taking a moment equation around the lowest tension reinforcement layer.

$$M_d = \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + \sigma_{st} \cdot A_{st} \cdot (d - d_t) - \sigma_s \cdot A_s \cdot (d - d_1) \quad (5.38)$$

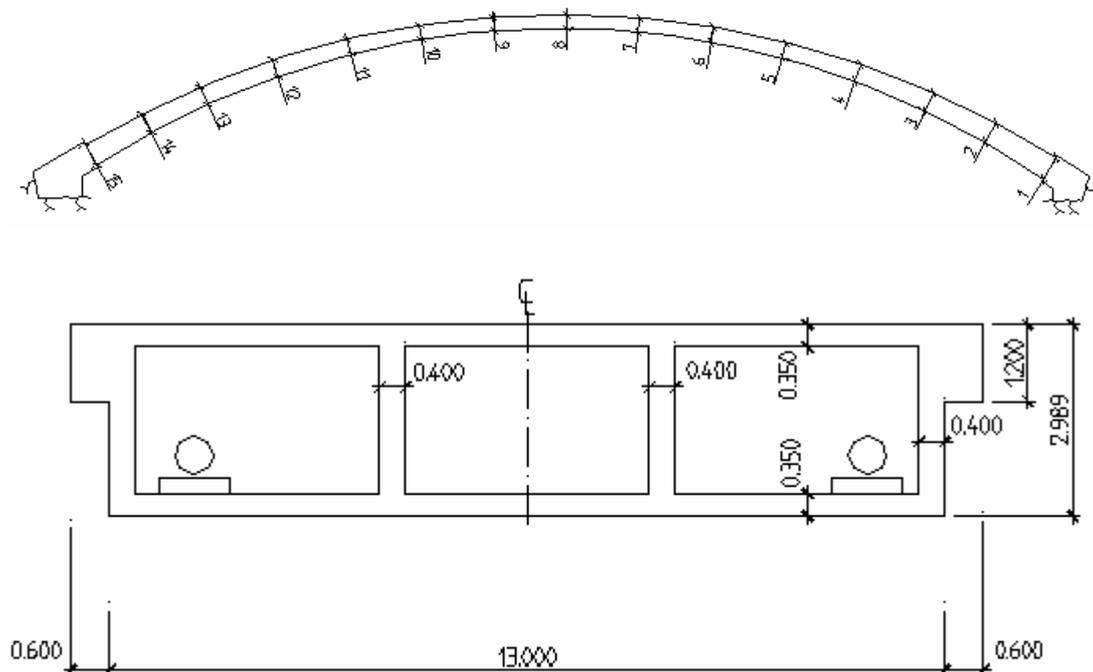
When $M_d > M_s$, the cross-section moment capacity is sufficient.

Once all the assumptions and conditions are satisfied the amount of reinforcement needed is obtained and the cross-section is designed.

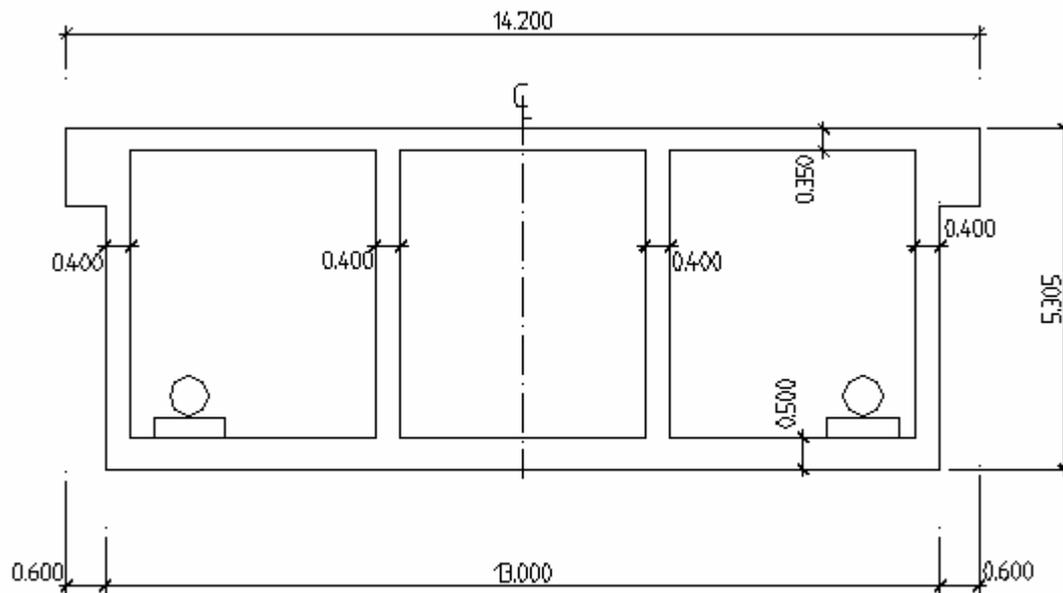
5.3 Results from linear analysis

5.3.1 Original box girder cross-section

The cross section of the bridge is to be constructed as a box girder with two inner walls. The height of the cross-section varies across the arch. The thickness of the inner wall and the top slab is constant across the arch, while the thickness of the bottom slab varies along the arch, see Appendix C1. The dimensions of the cross-section can be seen in Figure 5.7.



Section 8



Section 15

Figure 5.7 Original cross-sections for the arch.

The calculations of the required reinforcement, input data tables and the results from the structural analysis program for the original cross-section are presented in Appendix C1. The bending moments M_{sd} , the normal forces N_{sd} and the amount of reinforcement bars n obtained for the cross-section are presented in the Table 5.2.

Table 5.2 Maximum positive and negative bending moments, normal forces for the same load case and the required amount of tensile reinforcement bars, $\phi = 20\text{ mm}$.

Section	1	3	5	7	9	11	13	15
M_{sd}	113426		44732	107698	113584	84480	30800	
$-N_{sd}$	162957		145297	140175	144533	149071	157897	
n	-127		-126	-65	-64	-102	-154	
$-M_{sd}$	67803	79288	36085			1870	33577	304456
$-N_{sd}$	158312	153890	142493			145771	158884	162780
n	-142	-123	-129			-156	-153	-41

As it can be seen from the Table 5.2, the cross-section is uncracked and does not need reinforcement (negative values for the reinforcement). Unfortunately, this cross-section was not useful for this project since a reinforced arch with cracking cross-section was required. Since the original cross-section was not useful, the height of the cross-section was reduced in order to obtain a cross-section that cracks and that requires reinforcement.

5.3.2 Reduced height of the box girder cross-section, variable height

In order to obtain a cracked cross-section, the height of the original cross-section was reduced. The height of the cross-section still varied along the arch. The thickness of the inner walls was kept the same as the original, but the thickness of the top slab was reduced from 0.35 m to 0.25 m. The thickness of the bottom slab was reduced as well, varying from 0.4 m at the abutments to 0.25 m at the mid section of the arch, for detailed values see Appendix C3.

The bending moments M_{sd} , the normal forces N_{sd} and the amount of reinforcement n obtained for the cross-section are presented in Appendix C3. As can be seen from the results, the cross-sections were still uncracked and do not need any reinforcement.

Seeing that the cross-section is uncracked, several attempts were made to induce cracking and the need for reinforcement. The traffic load coefficient was increased stepwise from 1.5 to 1.7 and 1.9, to see if the cracking would occur. The obtained bending moments, the normal forces and the amount of reinforcement are presented in Appendix C3.

Seeing that the cross-section was still uncracked, a new reduction in height was made to obtain a need for reinforcement and a cracked cross-section.

5.3.3 Reduced height of the box girder cross-section, constant height

In order to obtain cross-sections that need to be reinforced, the height of the original cross-sections were further reduced to 2.5 m and was kept constant along the arch; that is no variation along the arch. This was the minimum height of the box girder cross-section as there must be free height of 2 m inside the box girder. The top and bottom slab thicknesses were reduced to 0.25 m and kept constant along the arch as well. The dimensions of the cross-section are presented in Figure 5.8.

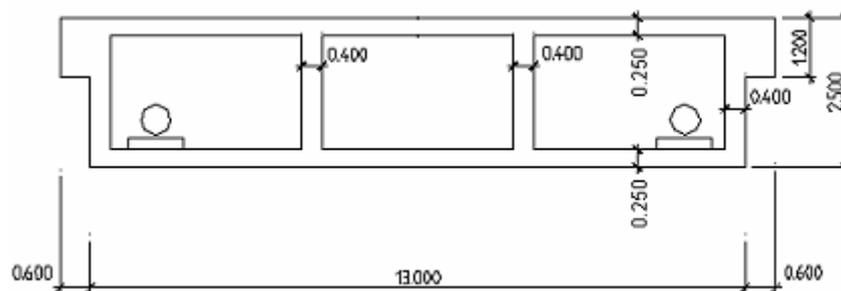


Figure 5.8 Cross-section of the box girder with constant height

The bending moments M_{sd} , the normal forces N_{sd} and the amount of reinforcement n obtained for the cross-section are presented in the Appendix C3. As can be seen, the cross-section started to crack at the supports where reinforcement was needed. However, the need for reinforcement was still quite small, and a larger amount was needed if the advantages with non-linear analysis should be shown. In order to obtain larger reinforcement amounts, it was decided to redesign the cross-section as a solid beam section.

5.3.4 Solid beam section

Redesigning the arch as a solid rectangular cross-section was the final attempt to find a section that cracks and requires reinforcement. The height of the cross-section was chosen to 1.5 m and the width was chosen to 13 m. The cross-section is constant along the arch, and the shape can be seen in Figure 5.9.

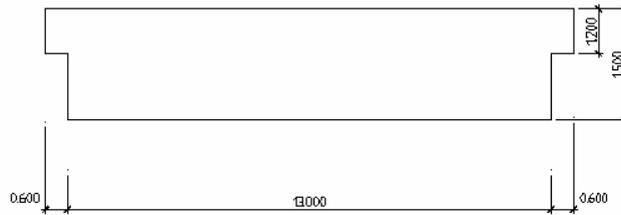


Figure 5.9 Cross-section of the rectangular solid arch

The calculations of the required reinforcement, input data tables and the results from the structural analysis program for the solid cross-section are presented in Appendix C2. The bending moments M_{sd} , the normal forces N_{sd} and the amount of reinforcement bars n obtained for the cross-section are presented in the Table 5.3.

Table 5.3 Maximum positive and negative bending moments, normal forces for corresponding load case and the required amount of reinforcement bars $\phi = 20$ mm in compression and tension.

Section	1	3	5	7	9	11	13	15
M_{sd}	194902		13148	86438	98991	79401	65396	
$-N_{sd}$	189113		170620	164354	168705	174116	183246	
n compression	750			150	238	64	64	
n tension	1075			64	100	64	64	
$-M_{sd}$		68215	65496			4590		163314
$-N_{sd}$		180944	168304			171084		189220
n compression		64	64					552
n tension		64	64					745

Finally a cracked cross-section with a large amount of reinforcement needed was obtained. With these values for the bending moments, the normal forces and the reinforcement the next phase of the Master's project, designing with non-linear analysis started.

6 Non-linear Analysis

6.1 Introduction

With non-linear analysis, it is possible to follow the real behaviour of a bridge, an arch or a structure. The non-linear analysis is not one unique method, but a range of methods at different levels of accuracy. Common for these approaches is that the non-linear behaviour of the structure is in some way taken into account. In this Master's project, a simplified use of non-linear analysis based on stepwise change of the flexural rigidity of the cross-section, based on a cracked cross-section where both concrete and reinforcing steel have elastic response (state II model), was used.

6.2 Methodology

The preliminary design of the cross-section was done using linear analysis, see Chapter 5. According to this, a preliminary amount of reinforcement needed was obtained. Usually, the amount obtained by linear analysis is an overestimation for structures under compression and in this chapter; the reinforcement amount will be reduced using non-linear analysis for improved design.

The first step was to calculate the cross-sectional constants for the designed cross-section using the moments, the normal forces and the amount of the reinforcement obtained from the linear analysis. The linear analysis process can be seen in Figure 6.1.

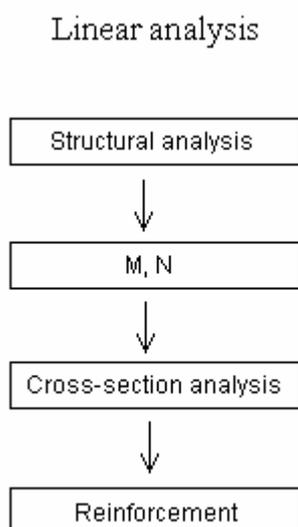


Figure 6.1 The linear analysis process.

The cross-sectional constants were calculated according to the equations for state II cross-sectional modelling for sustained loading, where creep and shrinkage were

taken into account. The equations are presented in the following subchapters. Once the cross-sectional constants were calculated, they were inserted into the structural analysis program, Strip Step 2. The constants that were changed were the centre of gravity x_{tp} , the equivalent moment of inertia I_{ekv} and the equivalent area of the cross-section A_{ekv} . After running the program for the different traffic load cases, new lower moments and normal forces were obtained showing that the reinforcement amount could be reduced.

The next step was to determine the decreased amount of reinforcement required by an iterative process. This started by guessing lower amounts of reinforcement than the ones calculated by linear analysis. With the decreased reinforcement amount and the moments and normal forces obtained in the previous step, new cross-sectional constants were calculated. Special attention was here needed to be given to the sign change of the bending moment, since the cross-sectional constants normally vary depending on whether the top or the bottom is cracked in tension. This is discussed more in detail in Chapter 6.3. The new cross-sectional constants were used in a new Strip Step 2 analysis for the different load cases in order to obtain new moments and normal forces. The sequence was repeated until the bending moments started to converge i.e. the change of the moment approaches zero. Once convergence was reached, the iteration was stopped, and the cross-section was finally designed for the new lower amount of reinforcement. The non-linear iteration process can be seen in Figure 6.2.

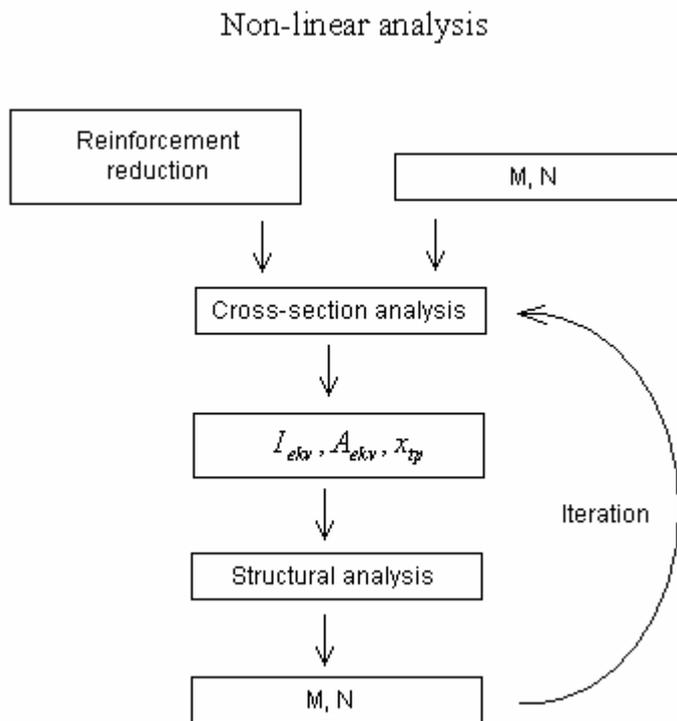


Figure 6.2 Non-linear iteration.

6.3 Approximations

The arch was divided into fifteen sections, see Figure 4.3, but in the calculations only eight of these were taken into account. Sections one and fifteen are the support sections, so they were important to look at as they would carry rather high moments. Sections three, five, seven, nine, eleven and thirteen were the other sections studied. These sections were chosen since they were loaded with concentrated loads transferred from the bridge deck through the columns and into the arch. Since the arch was loaded in these sections, the highest moments and normal forces would arise in these sections. For the sections in between: two, four, six, eight, ten and fourteen, the cross-sectional constants were not changed. This was to ease both the calculation and iteration process.

Another approximation made concerns the bending moments. Some of the sections are exposed to both negative and positive bending moments, with belonging compressive normal forces, for different load cases. The sign change of the bending moment will affect the eccentricity, e , of the normal force. The eccentricity to the tension reinforcement will either increase causing a higher bending moment, or decrease leading to lower contribution to the bending moment.

For the calculation of the cross-sectional constants, both the negative and the positive moments were inserted into equation (6.1). The higher of the two was chosen for the calculation and the cross-sectional constants were calculated accordingly.

$$M_s = M_{sd} + N_{sd} \cdot e \quad (6.1)$$

6.4 Calculation of the cross-sectional constants, state II

The calculations for the cross-sectional constants were done based on the state II model calculation with sustained loading taken into account. State II model is based on a cracked cross-section where the concrete is assumed to carry no tensile stresses, and where both the concrete under compression and the reinforcing steel have elastic response. The long term effect of creep was introduced through the relation between the effective modulus of elasticity for steel and concrete, equation (6.2).

$$\alpha_{ef} = \frac{E_s}{E_c} \cdot (1 + \psi) \quad (6.2)$$

where ψ is the creep factor.

6.4.1 Calculation of the depth of the compressive zone, x

The first step was to calculate the x value, the depth of the compressive zone. This value was obtained by taking moment equilibrium for each section on the arch that was studied, according to:

$$M_d = \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + \sigma_{st} \cdot A_{st} \cdot (d - d_t) - \sigma_s \cdot A_s \cdot (d - d_1) - N_{sd} \cdot e \quad (6.3)$$

$$\text{where } \sigma_s = E_s \cdot \varepsilon_s \quad \text{for} \quad \varepsilon_s \leq \varepsilon_{sy} \quad (6.4)$$

$$\sigma_s = f_{sd} \quad \text{for} \quad \varepsilon_s \geq \varepsilon_{sy} \quad (6.5)$$

The same conditions for the steel stresses were valid for the compressive reinforcement as well. Once this value was attained, the new cross-sectional constants such as x_{tp} , A_{ekv} and I_{ekv} were calculated with the formulas in the following chapters.

It was discovered at the end of the Master's project, that this calculation way was not correct since the depth of the compressive zone was calculated according to state III and not state II. The correct way is to iterate the x value with help of Navier's formula, see equation (6.6).

$$\sigma_c(z) = \frac{N_{sd} + F_{cs} + F_{cst}}{A_{ekv}} + \frac{N_{sd} \cdot e + F_{cs} \cdot e_s + F_{cst} \cdot e_{st} + M_{sd}}{I_{ekv}} \cdot z \quad (6.6)$$

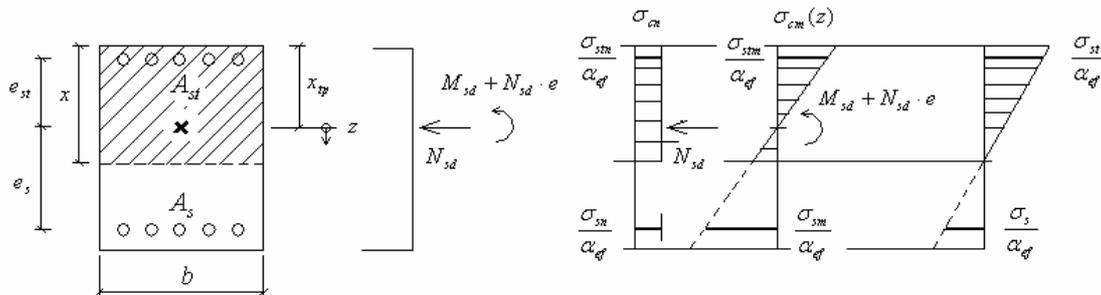


Figure 6.3 Cracked reinforced concrete section exposed to bending moment and compressive normal force.

where: z = sectional co-ordinate from the centre, positive

e = eccentricity of axial force relative to the centre of the transformed section, positive downwards

F_{cs}, F_{cst} = shrinkage restraint forces in the tension respectively compression reinforcement layer

$$e_s = d - x_{tp}$$

$$e_{st} = d_t - x_{tp}$$

The first step is to guess an x value and calculate the cross-sectional constants according to Chapters 6.4.2-6.4.4. The following step is to calculate the stresses at the

level of the guessed compression depth, the neutral zone. If the stress in the neutral zone is approximately zero, the correct x value has been guessed and obtained.

$$\sigma_c(z = x - x_{tp}) \approx 0 \quad \Rightarrow \quad \text{correct } x \text{ value}$$

If the stress in the neutral zone is not zero, a new x value has to be guessed and calculations redone until the stress is close to zero in the neutral zone.

$$\sigma_c(z = x - x_{tp}) \neq 0 \quad \Rightarrow \quad \text{incorrect } x \text{ value, guess a new } x \text{ value}$$

Since this mistake was discovered at the end of the project a check was made to see the deviation in the cross-sectional constants with the two calculation ways. A deviation of approximately 5% was observed and it was decided that the change in cross-sectional constants with the 5%-deviation would not have a major impact on the calculations of bending moments and normal forces. Still, if the method is to be used, the correct way of calculating the x value in state II should be used.

6.4.2 Calculation of the equivalent area, A_{ekv}

The effective area A_{ekv} was calculated according to

$$A_{ekv} = A_{cc} + (\alpha_{ef} - 1) \cdot A_{st} + \alpha_{ef} \cdot A_s \quad (6.7)$$

where $A_{cc} = b \cdot x$, area of compressive zone

$A_{st} =$ total area of compressive reinforcement

$A_s =$ total area of tensile reinforcement.

6.4.3 Calculation of the gravity centre of the transformed section, x_{tp}

The gravity centre of the transformed section x_{tp} was calculated according to

$$x_{tp} = \frac{A_{cc} \cdot \frac{x}{2} + (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t + \alpha_{ef} \cdot A_s \cdot d}{A_{ekv}} \quad (6.8)$$

where $x =$ depth of the compressive zone

$d_t =$ depth of the compressive reinforcement

$d =$ depth of the tensile reinforcement.

6.4.4 Calculation of the moment of inertia of the transformed section, I_{ekv}

The moment of inertia of the transformed section I_{ekv} , was calculated according to

$$I_{ekv} = I_c + A_{cc} \cdot (x_{tp} - x)^2 + (\alpha_{ef} - 1) \cdot A_{st} \cdot (x_{tp} - d_t)^2 + \alpha_{ef} \cdot A_s \cdot (d - x_{tp})^2 \quad (6.9)$$

where $I_c =$ moment of inertia of the compressive zone.

6.5 Iteration results

Presented in this chapter are the maximum positive and negative moments, normal forces for corresponding load cases and calculated cross-sectional constants for the iterations performed. Detailed calculations of the cross-sectional constants and the results from the structural analysis program for iteration one and five are presented in Appendix D1 and Appendix D2 respectively. A compilation of all the iteration results is presented in Appendix D3.

6.5.1 Iteration zero

Iteration zero was the start of the iteration process. The uncracked cross-section with state I sectional constants was analysed with the Strip Step 2 program giving the design moments and normal forces. The cross-section was designed for these loads, and the amount of reinforcement needed, determined in section 5.3.4, is summarised in Table 6.1.

Table 6.1 The amount of reinforcement bars $\phi = 20\text{ mm}$ obtained with linear analysis.

Section	1	3	5	7	9	11	13	15
Top reinforcement bars $\phi = 20\text{ mm}$	1075	64	64	64	100	64	64	745
Bottom reinforcement bars $\phi = 20\text{ mm}$	750	64	64	150	238	64	64	552

In sections three, five and thirteen, calculations showed no need for reinforcement. Since the rest of the sections were reinforced, it was decided to put in minimum

reinforcement in the sections where no reinforcement was required according to the calculations.

The reinforcement was then placed into the cross-section and the cross-sectional constants for state II model were calculated for each section using the moments and normal forces obtained from the uncracked cross-sections, see section 5.3.4. The results are seen in Table 6.2.

Table 6.2 Maximum positive and negative bending moments, normal forces for corresponding load case from linear analysis and cross-sectional constants for iteration zero.

Section	1	3	5	7	9	11	13	15
M_{sd} [kNm]	194902		13148	86438	98991	79401	65396	
$-N_{sd}$ [kN]	189113		170620	164354	168705	174116	183246	
$-M_{sd}$ [kNm]		68215	65496			4590		163314
$-N_{sd}$ [kN]		180944	168304			171084		189220
x_{tp} [m]	0,517	0,371	0,348	0,438	0,493	0,384	0,368	0,497
A_{ef} [m ²]	19,033	9,504	8,789	10,462	11,903	9,92	9,426	16,563
I_{ef} [m ⁴]	4,9	0,828	0,77	1,43	2,015	0,868	0,821	3,975

After the sectional constants for state II were calculated, the new values were used in a new Strip Step 2 analysis and new moments and normal forces were obtained. These are presented in the Table 6.3.

Table 6.3 Maximum positive and negative bending moments and normal forces for corresponding load case obtained in iteration zero.

Section	1	3	5	7	9	11	13	15
M_{sd}	125661	24062	72044	38888	69112	29105	6045	31621
$-N_{sd}$	188632	169191	170373	163884	168287	173883	182866	187100
$-M_{sd}$		20204		33530		44391	32930	108697
$-N_{sd}$		180533		166704		170507	184936	188549

The new moments and normal forces indicated that the reinforcement amount could be reduced. A new lower amount of reinforcement was guessed and the iteration process started.

6.5.2 First iteration

For the first iteration, the reinforcement was reduced with 20 % from the original reinforcement amounts, calculated with linear analysis. In the cross-sections where the number of reinforcement bars was 64, no reduction was made since this was the minimum reinforcement. The new reinforcement amounts were fixed to the values according to Table 6.4.

Table 6.4 The reduced amount of reinforcement bars presumed for the non-linear iteration.

Section	1	3	5	7	9	11	13	15
Top reinforcement bars $\phi = 20$ mm	860	64	64	64	80	64	64	596
Bottom reinforcement bars $\phi = 20$ mm	600	64	64	120	190	64	64	442

The new cross-sectional constants were calculated using the moments from Table 6.3 from the iteration zero and the reduced amount of reinforcement from Table 6.4. The new calculated cross-sectional constants are presented in Table 6.5 and the new moments and normal forces due to the new sectional constants in Table 6.6. For the detailed calculations of the cross-sectional constants and results from the structural analysis program, see Appendix D1.

Table 6.5 Cross-sectional constants for the first iteration.

Section	1	3	5	7	9	11	13	15
$x_{tp} [m]$	0,493	0,415	0,264	0,426	0,505	0,414	0,389	0,489
$A_{ef} [m^2]$	16,789	10,869	5,526	10,498	12,687	10,83	10,089	15,848
$I_{ef} [m^4]$	4,226	0,979	0,646	1,26	1,855	0,974	0,886	3,445

Table 6.6 Maximum positive and negative bending moments and normal forces for corresponding load case obtained in the first iteration.

Section	1	3	5	7	9	11	13	15
M_{sd}	147185	48753		65890	92336	43714	1246	4657
$-N_{sd}$	187765	168407		163060	167451	173053	182199	186507
$-M_{sd}$			111469	6224		31864	38436	134550
$-N_{sd}$			166763	165847		169798	184193	187898

6.5.3 Final iteration

The following iterations were made in the same way as the first iteration. The iteration results are summarised in Appendix D3. For the fifth and final iteration, detailed calculations of the cross-sectional constants and results obtained from the structural analysis program are shown in Appendix D2. The cross-sectional constants used are shown in Table 6.7.

Table 6.7 Cross-sectional constants for the final iteration.

Section	1	3	5	7	9	11	13	15
$x_{tp} [m]$	0,644	0,29	0,291	0,429	0,485	0,392	0,371	0,475
$A_{ef} [m^2]$	25,213	6,709	6,748	10,602	11,946	10,18	10,154	14,86
$I_{ef} [m^4]$	5,666	0,668	0,669	1,269	1,776	0,896	0,893	3,396

The new moments and normal forces obtained from the Strip Step 2 analysis after inserting the new sectional constants are presented in Table 6.8.

Table 6.8 Maximum positive and negative bending moments and normal forces for corresponding load case obtained in the final iteration.

Section	1	3	5	7	9	11	13	15
M_{sd}	158447	54396		65679	90351	41726		5772
$-N_{sd}$	187587	168008		162660	167127	172782		186107
$-M_{sd}$			112574	6563		32459	40317	131532
$-N_{sd}$			166528	165530		169388	183928	187636

6.5.4 Comments on the iterations

It was mentioned earlier in Chapter 6.2 that special attention needed to be given to the sign change of the bending moment, since the cross-sectional constants normally vary then as well. This means that when the bending moment is positive, the “bottom” reinforcement is in tension and the cross-sectional constants in state II should be determined accordingly. On the other hand, if the same section is exposed to a negative bending moment, the “top” reinforcement is in tension and the cross-sectional constants will have new values. From linear analysis some of the sections obtained both the negative and the positive bending moments. The moments then used in calculations were calculated according to equation (6.1) in Chapter 6.3 and the sectional constants are calculated consequently.

7 Results and Discussion

The bridge was originally designed with a box girder cross-section. Box girders are very stable for arches, since the distribution of moments and normal forces is good throughout the cross-section. The original cross-section was very stiff and did not need to be reinforced. Due to this, the cross-section was redesigned as a solid cross-section in order to illustrate the aim of this Master's project.

The amounts of reinforcement bars obtained with both linear and non-linear analysis for the solid cross-section are presented and compared in Table 7.1. With respect to that the brake load can act in both directions, and that the arch span of the bridge is almost symmetrical, a symmetric distribution of the required reinforcement has been assumed based on the results in Chapters 5 and 6.

Table 7.1 Number of reinforcement bars, $\phi = 20\text{ mm}$, obtained with linear and non-linear analysis.

Section	1	3	5	7	9	11	13	15
	Linear analysis							
Top reinforcement bars	1075	64	64	100	100	64	64	1075
Bottom reinforcement bars	750	64	64	238	238	64	64	750
	Non-linear analysis							
Top reinforcement bars	860	64	64	80	80	64	64	860
Bottom reinforcement bars	600	64	64	190	190	64	64	600

The reduction of reinforcement was done with 20 % at each section, except for the ones with the minimum amount of reinforcement bars. As can be seen from the Table 7.1, calculating with non-linear analysis, lower amounts of reinforcement bars were obtained. The total reduction of the required reinforcement amount was estimated to be around 17 %. With non-linear analysis the real behaviour of the structure was taken into account resulting in lower bending moments and showing that the use of linear analysis, in this case, leads to overestimation of the amount of reinforcement bars.

The loads on the structure in this Master's project were limited to the vertical loads acting on it and other loads such as wind load were not taken into account. This was done in order to ease the calculations and keep the model in two dimensions only. In order to include the side loads, for example the wind load acting perpendicular to the bridge, a 3D-model should be created where the risk for torsion and buckling is taken into account. In this Master's project this was not taken into account since the calculations get more extensive and a more advanced structural analysis program is required.

In this Master's project, the bridge was modelled as two separate structures, the bridge deck and the arch. This could have influenced the results in a negative way. If the arch is not stiff enough, i.e. the support points for the bridge deck are deflecting; the calculated loads on the arch can be incorrect. A test should be made to check whether the assumption of an infinity stiff arch holds with respect to the reaction forces in the columns. This can be made by including the arch, the bridge deck and the columns in one model. However, this was not made in this study, since this appeared to be too complicated with the analysis program used in this Master's project.

Another thing that could have influenced the results is the approximations made for the sections in between the ones studied. The stiffness of these sections was not updated during the iteration process, resulting in uneven distribution of the centre of gravity as well as of the stiffness along the arch. This might have had effects on the convergence of the bending moments. A test where all the sections are taken into account should be made to see if the stiffness and the centre of gravity would be more even along the arch, leading to a better convergence of the bending moments.

8 Conclusions

The main aim of this Master's project was to show the economical potential of using non-linear analysis as a design method, especially for slender and compressed concrete structures. This was successfully demonstrated in this Master's project. In the study, the Munkedal Bridge, a planned highway bridge with an underlying concrete arch supporting the bridge deck on columns, was used as an example.

The original box girder cross-section of the arch was heavily over-dimensioned and could carry the bridge loads without any need for reinforcement. In this study, the cross-section was gradually reduced to a slender solid beam cross-section in order to obtain cracking and a need for reinforcement in the arch.

The required amount of reinforcement was calculated with both linear and non-linear analysis and it was observed that with non-linear analysis, the total amount of reinforcement could be reduced with at least 20 % in the cross-sections with more than minimum reinforcement. The reduction obtained with non-linear analysis was estimated to be about 17 % for the whole arch.

The iteration process for the non-linear analysis was not as time consuming as expected. The most time consuming process was to find a cross-section that needed to be reinforced, as the original cross-section was over-dimensioned. It can be concluded that for a structure that needs to be reinforced from the start, it is economical to use non-linear analysis in order to reduce the amount of reinforcement obtained through linear analysis. On the other hand, for a structure that does not need to be reinforced from the start, this is not the case since the process in finding a structure that cracks and that needs to be reinforced could be time consuming. Consequently, such a process is economical only if a lower concrete amount or a more rational production can motivate the effort.

A disadvantage with non-linear analysis is in general that the procedure requires a study with summarised loads, i.e. the law of superposition is not valid. However, with the methodology used in this Master's project this has not to be the case. Within each iteration, the calculations are linear (with a reduced stiffness) and superposition can be used. However, this requires that the complete structure, the arch, the columns and the bridge deck, can be modelled as one structure in 3D and that the critical load combination is found within each iteration.

Finally it can be concluded that the use of non-linear analysis in the design process is economical for slender and compressed concrete structures that has a need for reinforcement and if the reinforcement amount is large.

9 References

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APPENDIX A: PERMANENT LOAD CALCULATIONS

Calculations for the permanent loads

The material properties were taken from BBK04, Boverkets handbook in concrete structures.

Material properties

C40/50 $E_{ck} =$ 35 Mpa

Partial safety factor

Safety class 3

$\eta\gamma_m =$ 1,2
 $\gamma_n =$ 1,2

$$E_c = \frac{E_{ck}}{\eta\gamma_n\gamma_m} \qquad E_c = \qquad 24,3 \qquad \text{Mpa}$$

The design value of the modulus of elasticity of concrete C40/50 $E_c = 24,3 \text{ Mpa}$ was used in the Strip Step program.

Material	Load	
Concrete	25	kN/m ³
Asphalt	23	kN/m ³
Steel	1,6	kN/m ²

Cross-section constants of the bridge deck

Deck area	9,1829	m ²
Asphalt area	2,1936	m ²
Cross-section length	24,07	m

Permanent load

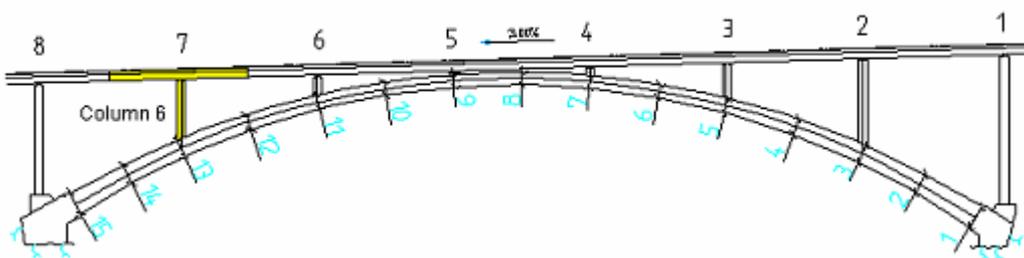
Bridge deck	Load	
Concrete	229,5725	kN/m
Asphalt	50,4528	kN/m
Steel	38,512	kN/m
Total	318,5373	kN/m

Column		
Concrete	25	kN/m ³
Diameter	2	m
Area	3,14	m ²
Weight per length	78,5	kN/m

	Length		Point load (2 columns)	
Column 1	19,858	m	3117,706	kN
Column 2	7,938	m	1246,266	kN
Column 3	1,8	m	282,6	kN
Column 4	0,31	m	48,67	kN
Column 5	4,309	m	676,513	kN
Column 6	13,748	m	2158,436	kN

Load on the arch

The total point load on the arch is the sum of the permanent loads from the bridge deck and the columns. The Figure below illustrates the point load acting on section 13 of the arch.



Section	length [m]	Load [kN]	Total	
3	32,44	10333,35	13451,056	kN
5	31,75	10113,56	11359,8253	kN
7	31,75	10113,56	10396,1593	kN
9	31,75	10113,56	10162,2293	kN
11	31,75	10113,56	10790,0723	kN
13	32,44	10333,35	12491,786	kN

Self-weight of the arch

Cross-section: Original

Section	Ac [m ²]	Concrete [kN/m ³]	Load [kN/m]
1	19,674	25	491,85
2	18,211	25	455,28
3	16,825	25	420,63
4	15,500	25	387,5
5	14,914	25	372,86
6	14,514	25	362,86
7	14,278	25	356,94
8	14,202	25	355,06
9	14,287	25	357,18
10	14,532	25	363,3
11	14,938	25	373,46
12	15,529	25	388,22
13	16,857	25	421,43
14	18,181	25	454,52
15	19,618	25	490,45

Self-weight of the arch**Cross-section:** Box girder with reduced height

Section	Ac [m ²]	Concrete [kN/m ³]	Load [kN/m]
1	15,250	25	381,25
2	14,080	25	352
3	12,856	25	321,4
4	11,940	25	298,5
5	11,735	25	293,375
6	11,401	25	285,025
7	11,202	25	280,05
8	11,140	25	278,5
9	11,210	25	280,25
10	11,415	25	285,375
11	11,756	25	293,9
12	12,249	25	306,225
13	13,454	25	336,35
14	14,654	25	366,35
15	15,949	25	398,725

Self-weight of the arch**Cross-section:** Box girder with constant cross-section

Section	Ac [m ²]	Concrete [kN/m ³]	Load [kN/m]
1 to 15	11,140	25	278,5

Self-weight of the arch**Cross-section:** Solid beam with constant cross-section

Section	Ac [m ²]	Concrete [kN/m ³]	Load [kN/m]
1 to 15	19,500	25	487,5

APPENDIX B: TRAFFIC LOAD CALCULATIONS

$N := \text{newton}$ $kN := 1000 \text{ N}$ $GPa := 10^9 \cdot Pa$
 $MPa := 10^6 Pa$ $kNm := 1000 \text{ N}\cdot\text{m}$

Calculation for traffic load acting on the arch

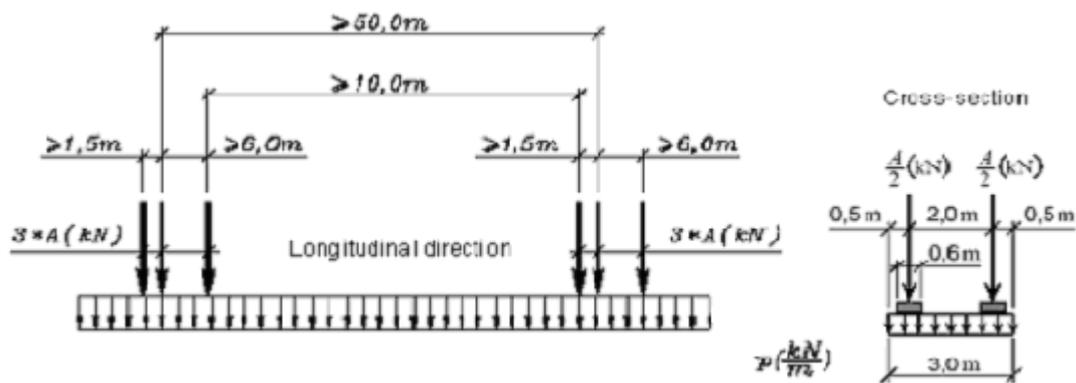


Figure 1. Load type 5.

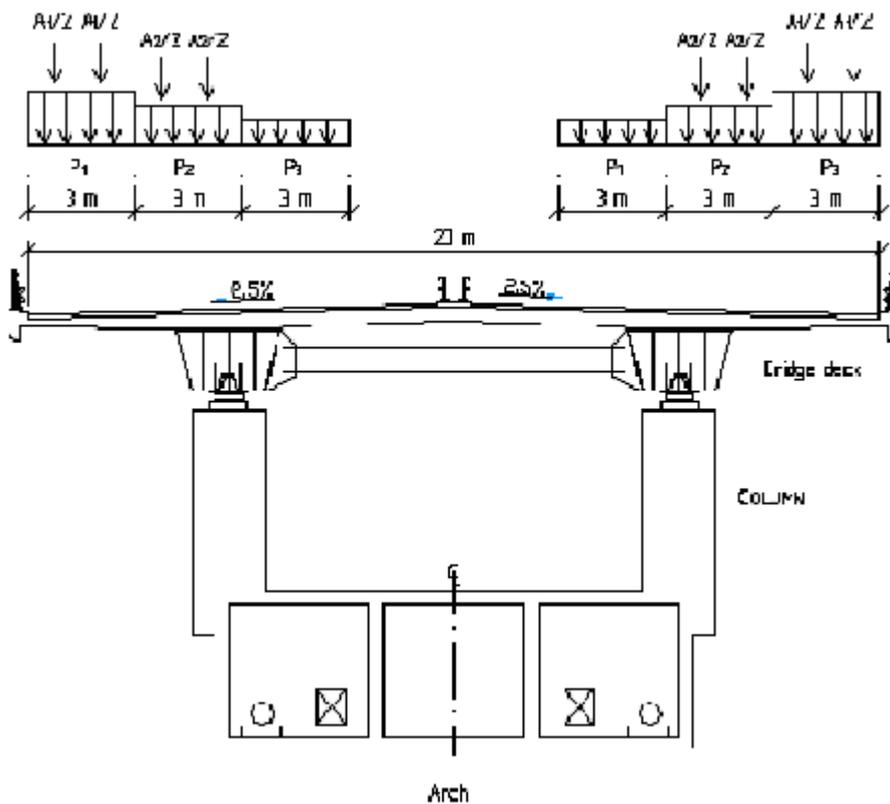
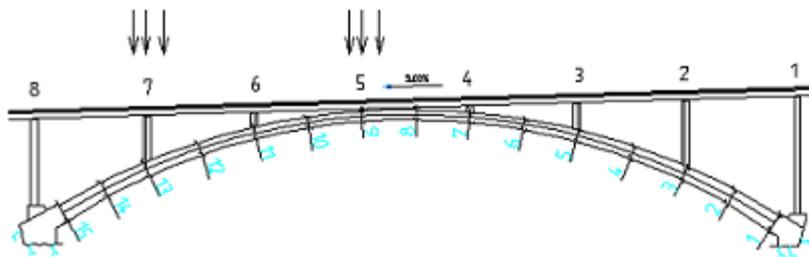


Figure 2. Traffic load in a cross-section.

Axle load

	Quantity		
$A_1 := 250 \text{ kN}$	$a_1 := 2$	$A_{1t} := A_1 \cdot a_1$	$A_{1t} = 5 \times 10^5 \text{ N}$
$A_2 := 170 \text{ kN}$	$a_2 := 2$	$A_{2t} := A_2 \cdot a_2$	$A_{2t} = 3.4 \times 10^5 \text{ N}$
Total axle load	$A_{\text{tot}} := A_{1t} + A_{2t}$		$A_{\text{tot}} = 840 \text{ kN}$



Reaction force influence line from FARBANA

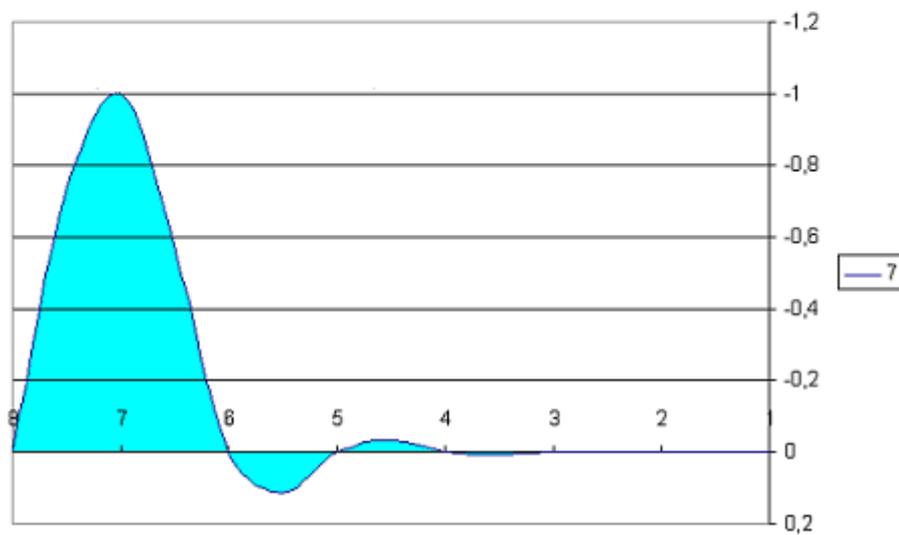


Figure 3. Influence line of section 7 of the bridge deck due to the load position.

Formula: $R_{zn} = A_{\text{tot}} (\text{Influence line values})$

Section 3

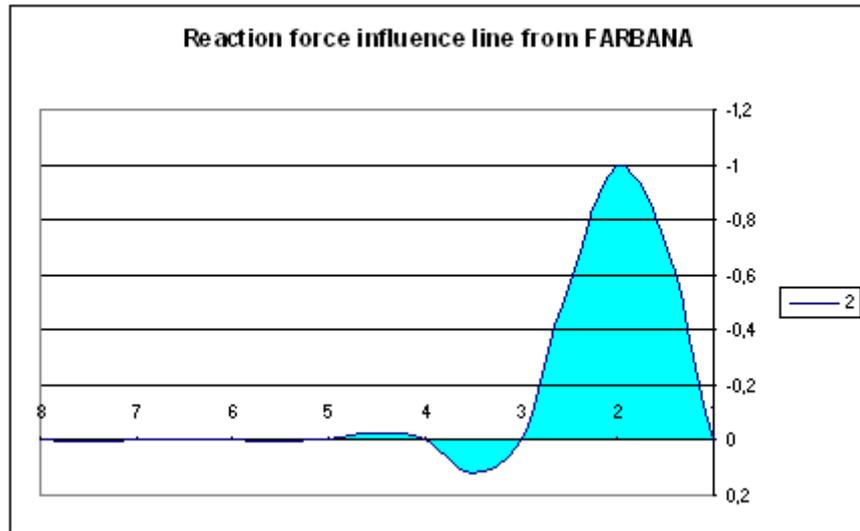


Figure 4. Influence line of section 2 of the bridge deck due to the load position.

$$\max R_{z2} := A_{\text{tot}} \cdot (-0.98 - 1 - 0.96) \quad R_{z2} = -2.47 \times 10^6 \text{ N}$$

$$\text{till } R_{z23} := A_{\text{tot}} \cdot (-0.05 + 0.1) \quad R_{z23} = 4.2 \times 10^4 \text{ N}$$

$$\text{till } R_{z24} := A_{\text{tot}} \cdot (-0.04 + 0.02) \quad R_{z24} = -1.68 \times 10^4 \text{ N}$$

$$\text{till } R_{z25} := A_{\text{tot}} \cdot (-0.01 + 0.02) \quad R_{z25} = 8.4 \times 10^3 \text{ N}$$

$$\text{till } R_{z26} := A_{\text{tot}} \cdot (0) \quad R_{z26} = 0 \text{ N}$$

$$\text{till } R_{z27} := A_{\text{tot}} \cdot (0) \quad R_{z27} = 0 \text{ N}$$

Section 5

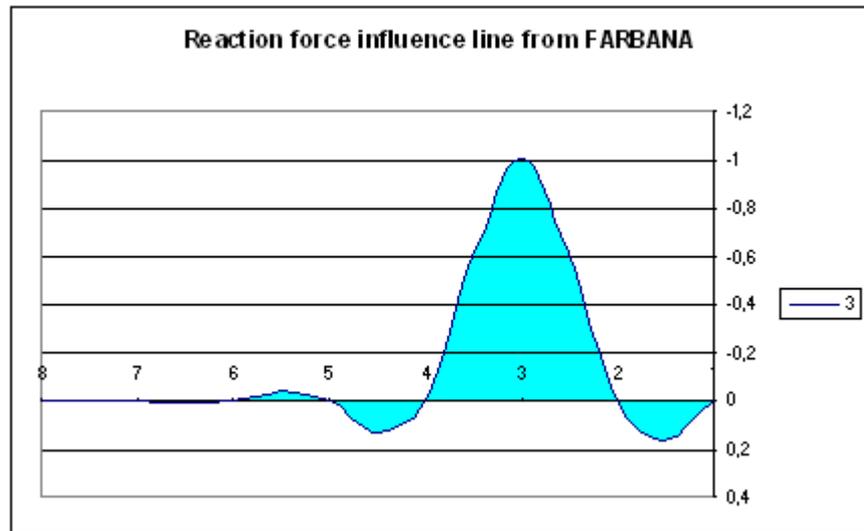


Figure 5. Influence line of section 3 of the bridge deck due to the load position.

max	$R_{Z3} := A_{\text{tot}} \cdot (-0.894 - 1 - 0.99 - 0.031 - 0.025 - 0.025)$	$R_{Z3} = -2.491 \times 10^6 \text{ N}$
till	$R_{Z32} := A_{\text{tot}} \cdot (0.067 - 0.033)$	$R_{Z32} = 2.856 \times 10^4 \text{ N}$
till	$R_{Z34} := A_{\text{tot}} \cdot (0.122 + 0.089 + 0.0722 - 0.2 - 0.022)$	$R_{Z34} = 5.141 \times 10^4 \text{ N}$
till	$R_{Z35} := A_{\text{tot}} \cdot (-0.655 - 0.8325 - 0.878 + 0.05)$	$R_{Z35} = -1.945 \times 10^6 \text{ N}$
till	$R_{Z36} := A_{\text{tot}} \cdot (-0.5 - 0.275 - 0.219 - 0.0125)$	$R_{Z36} = -8.455 \times 10^5 \text{ N}$
till	$R_{Z37} := A_{\text{tot}} \cdot (0.11 + 0.055 + 0.044)$	$R_{Z37} = 1.756 \times 10^5 \text{ N}$

Section 7

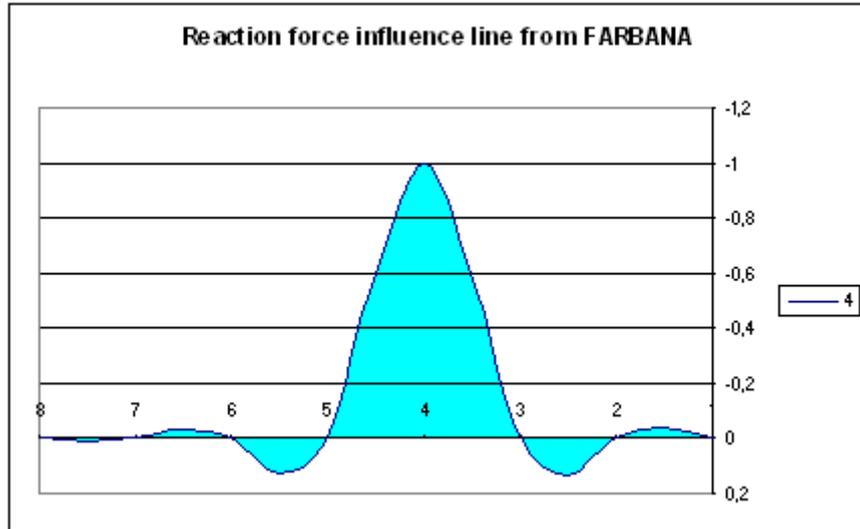


Figure 6. Influence line of section 4 of the bridge deck due to the load position.

max	$R_{z4} := A_{tot} \cdot (-0.98 - 1 - 0.96 - 0.03 - 0.04 - 0.05)$	$R_{z4} = -2.57 \times 10^6 \text{ N}$
till	$R_{z42} := A_{tot} \cdot (-0.01 + 0.05 - 0.98 - 0.96 - 0.83)$	$R_{z42} = -2.293 \times 10^6 \text{ N}$
till	$R_{z43} := A_{tot} \cdot (0.03 - 0.15 + 0.1 + 0.12 + 0.18)$	$R_{z43} = 2.352 \times 10^5 \text{ N}$
till	$R_{z45} := A_{tot} \cdot (-0.05 + 0.1 + 0.01 + 0.01 + 0.02)$	$R_{z45} = 7.56 \times 10^4 \text{ N}$
till	$R_{z46} := A_{tot} \cdot (0.01 - 0.02)$	$R_{z46} = -8.4 \times 10^3 \text{ N}$
till	$R_{z47} := A_{tot} \cdot (-0.01 + 0.01)$	$R_{z47} = 0 \text{ N}$

Section 9

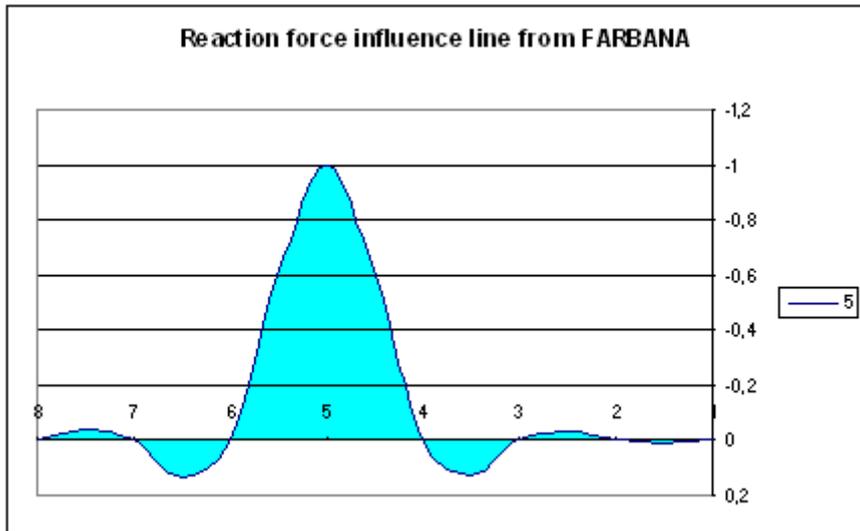


Figure 7. Influence line of section 5 of the bridge deck due to the load position.

max	$R_{z5} := A_{\text{tot}} \cdot (-0.978 - 1 - 0.956 - 0.022 - 0.278 - 0.033)$	$R_{z5} = -2.744 \times 10^6 \text{ N}$
till	$R_{z52} := A_{\text{tot}} \cdot (-0.011 - 0.178 - 0.233 - 0.44)$	$R_{z52} = -7.241 \times 10^5 \text{ N}$
till	$R_{z53} := A_{\text{tot}} \cdot (0.05 - 0.9 - 0.875 - 0.6875)$	$R_{z53} = -2.026 \times 10^6 \text{ N}$
till	$R_{z54} := A_{\text{tot}} \cdot (0.044 - 0.1556 + 0.077 + 0.0888 + 0.122)$	$R_{z54} = 1.48 \times 10^5 \text{ N}$
till	$R_{z56} := A_{\text{tot}} \cdot (-0.05 + 0.075)$	$R_{z56} = 2.1 \times 10^4 \text{ N}$
till	$R_{z57} := A_{\text{tot}} \cdot (0.0167 - 0.022)$	$R_{z57} = -4.452 \times 10^3 \text{ N}$

Section 11

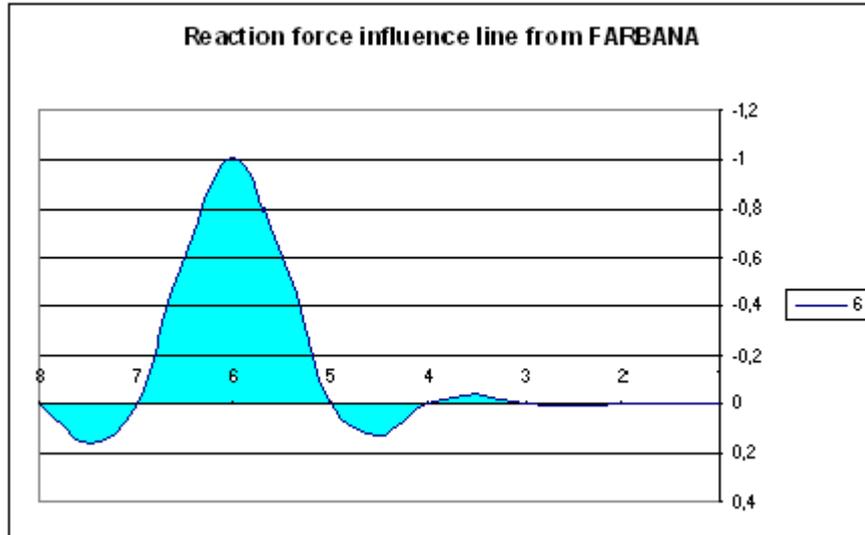


Figure 8. Influence line of section 6 of the bridge deck due to the load position.

max	$R_{z6} := A_{\text{tot}} \cdot (-0.98 - 1 - 0.92 - 3 \cdot 0.033)$	$R_{z6} = -2.519 \times 10^6 \text{ N}$
till	$R_{z62} := A_{\text{tot}} \cdot (0.04 + 0.055 + 0.0999)$	$R_{z62} = 1.637 \times 10^5 \text{ N}$
till	$R_{z63} := A_{\text{tot}} \cdot (-0.2 - 0.2375 - 0.4875)$	$R_{z63} = -7.77 \times 10^5 \text{ N}$
till	$R_{z64} := A_{\text{tot}} \cdot (0.045 - 0.944 - 0.911 - 0.711)$	$R_{z64} = -2.118 \times 10^6 \text{ N}$
till	$R_{z65} := A_{\text{tot}} \cdot (0.055 - 0.099 + 0.077 + 0.088 + 0.122)$	$R_{z65} = 2.041 \times 10^5 \text{ N}$
till	$R_{z67} := A_{\text{tot}} \cdot (-0.05 + 0.07)$	$R_{z67} = 1.68 \times 10^4 \text{ N}$

Section 13

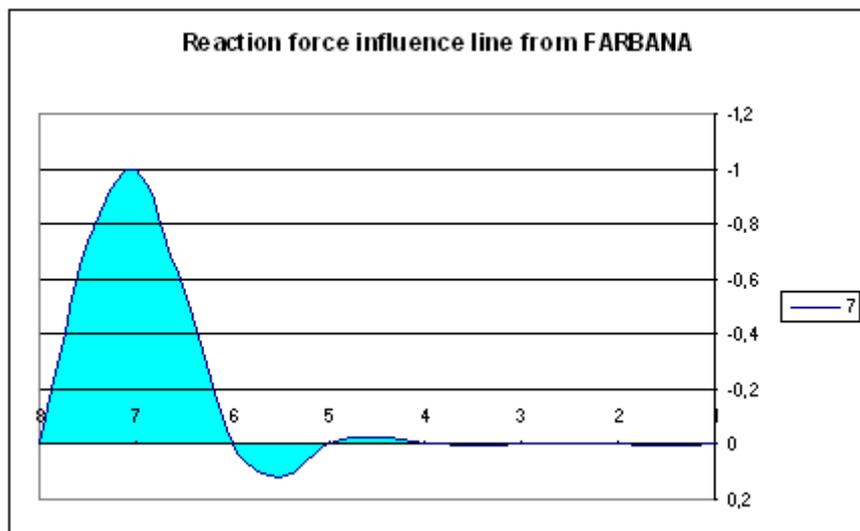


Figure 9. Influence line of section 7 of the bridge deck due to the load position.

max	$R_{z7} := A_{\text{tot}} \cdot (-1.05 - 1 - 0.889 - 0.028 - 0.02)$	$R_{z7} = -2.509 \times 10^6 \text{ N}$
till	$R_{z72} := A_{\text{tot}} \cdot (-0.0111 - 0.0167 - 0.0333)$	$R_{z72} = -5.132 \times 10^4 \text{ N}$
till	$R_{z73} := A_{\text{tot}} \cdot (0.05 + 0.0625 + 0.1125)$	$R_{z73} = 1.89 \times 10^5 \text{ N}$
till	$R_{z74} := A_{\text{tot}} \cdot (-0.011 - 0.128 - 0.244 - 0.489)$	$R_{z74} = -7.325 \times 10^5 \text{ N}$
till	$R_{z75} := A_{\text{tot}} \cdot (0.055 - 0.939 - 0.889 - 0.699)$	$R_{z75} = -2.076 \times 10^6 \text{ N}$
till	$R_{z76} := A_{\text{tot}} \cdot (0.03125 - 0.2125 + 0.08125 + 0.1 + 0.125)$	$R_{z76} = 1.05 \times 10^5 \text{ N}$

Evenly distributed load

	$p_1 := 4 \frac{\text{kN}}{\text{m}^2}$	$p_2 := 3 \frac{\text{kN}}{\text{m}^2}$	$p_3 := 2 \frac{\text{kN}}{\text{m}^2}$
Quantity	$n := 2$		
Lane width	$w := 3 \cdot \text{m}$		
Total	$p_{\text{tot}} := (p_1 + p_2 + p_3) \cdot n \cdot w$		$p_{\text{tot}} = 54 \frac{\text{kN}}{\text{m}}$

Formula: $Q_n = p_{\text{tot}} (\text{Influence line areas}) + R_{zn}$

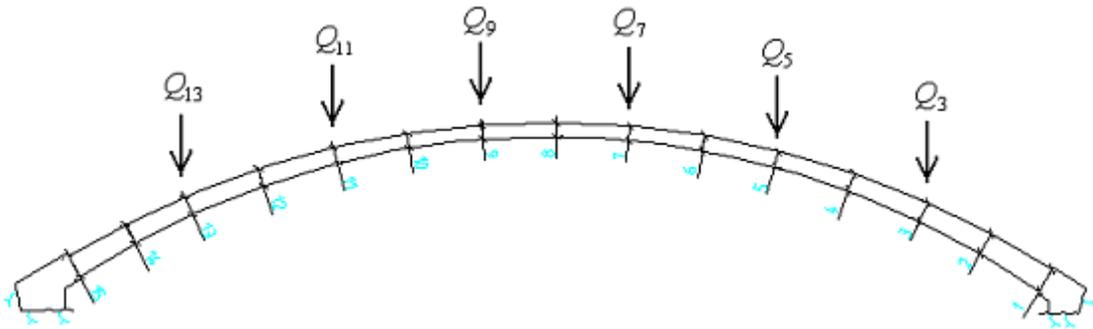


Figure 10. Influence line of section 2 of the bridge deck due to the load position.

Load case 3

max	$Q_3 := p_{\text{tot}} \cdot (-21.818 - 17.136 - 0.658 - 0.048) \cdot \text{m} + R_{z2}$	$Q_3 = -4.611 \times 10^6 \text{ N}$
till	$Q_5 := p_{\text{tot}} \cdot (3.769 - 18.252 + 2.669 + .196) \cdot \text{m} + R_{z23}$	$Q_5 = -5.854 \times 10^5 \text{ N}$
till	$Q_7 := p_{\text{tot}} \cdot (-1.01 + 2.74 - 17.998 - 0.737) \cdot \text{m} + R_{z24}$	$Q_7 = -9.351 \times 10^5 \text{ N}$
till	$Q_9 := p_{\text{tot}} \cdot (0.272 - 0.737 - 17.989 + 2.763) \cdot \text{m} + R_{z25}$	$Q_9 = -8.389 \times 10^5 \text{ N}$
till	$Q_{11} := p_{\text{tot}} \cdot (-0.072 + 0.197 + 2.679 - 18.252) \cdot \text{m} + R_{z26}$	$Q_{11} = -8.342 \times 10^5 \text{ N}$
till	$Q_{13} := p_{\text{tot}} \cdot (0.018 - 0.049 - 0.661 - 17.136) \cdot \text{m} + R_{z27}$	$Q_{13} = -9.627 \times 10^5 \text{ N}$

Load case 5

$$\begin{aligned}\text{max } Q_5 &:= p_{\text{tot}} \cdot (-18.252 - 17.935 - 0.719 - 0.073) \cdot m + R_{z3} & Q_5 &= -4.487 \times 10^6 \text{ N} \\ \text{till } Q_3 &:= p_{\text{tot}} \cdot (-17.136 + 2.455 + 0.177 + 0.018) \cdot m + R_{z32} & Q_3 &= -7.537 \times 10^5 \text{ N} \\ \text{till } Q_7 &:= p_{\text{tot}} \cdot (2.74 - 18.019 + 2.7 + 0.272) \cdot m + R_{z34} & Q_7 &= -6.132 \times 10^5 \text{ N} \\ \text{till } Q_9 &:= p_{\text{tot}} \cdot (-0.737 + 2.7 + -18.038 - 1.013) \cdot m + R_{z35} & Q_9 &= -2.868 \times 10^6 \text{ N} \\ \text{till } Q_{11} &:= p_{\text{tot}} \cdot (0.197 - 0.716 - 17.935 + 3.78) \cdot m + R_{z36} & Q_{11} &= -1.638 \times 10^6 \text{ N} \\ \text{till } Q_{13} &:= p_{\text{tot}} \cdot (-0.049 + 0.177 + 2.443 - 21.818) \cdot m + R_{z37} & Q_{13} &= -8.638 \times 10^5 \text{ N}\end{aligned}$$

Load case 7

$$\begin{aligned}\text{max } Q_7 &:= p_{\text{tot}} \cdot (-1.01 - 18.019 - 17.998 - 0.737) \cdot m + R_{z4} & Q_7 &= -4.61 \times 10^6 \text{ N} \\ \text{till } Q_3 &:= p_{\text{tot}} \cdot (-0.048 - 0.658 + 2.455 - 21.818) \cdot m + R_{z42} & Q_3 &= -3.377 \times 10^6 \text{ N} \\ \text{till } Q_5 &:= p_{\text{tot}} \cdot (0.196 + 2.669 - 17.935 + 3.769) \cdot m + R_{z43} & Q_5 &= -3.751 \times 10^5 \text{ N} \\ \text{till } Q_9 &:= p_{\text{tot}} \cdot (0.272 + 2.7 - 17.989 + 2.763) \cdot m + R_{z45} & Q_9 &= -5.861 \times 10^5 \text{ N} \\ \text{till } Q_{11} &:= p_{\text{tot}} \cdot (-18.252 + 2.679 - 0.716 - 0.072) \cdot m + R_{z46} & Q_{11} &= -8.919 \times 10^5 \text{ N} \\ \text{till } Q_{13} &:= p_{\text{tot}} \cdot (0.018 + 0.177 - 0.661 - 17.136) \cdot m + R_{z47} & Q_{13} &= -9.505 \times 10^5 \text{ N}\end{aligned}$$

Load case 9

$$\begin{aligned}\text{max } Q_9 &:= p_{\text{tot}} \cdot (-0.737 - 17.989 - 18.038 - 1.013) \cdot m + R_{z5} & Q_9 &= -4.784 \times 10^6 \text{ N} \\ \text{till } Q_3 &:= p_{\text{tot}} \cdot (0.018 + 0.177 - 0.658 - 17.136) \cdot m + R_{z52} & Q_3 &= -1.674 \times 10^6 \text{ N} \\ \text{till } Q_5 &:= p_{\text{tot}} \cdot (-0.073 - 0.719 + 2.669 - 18.252) \cdot m + R_{z53} & Q_5 &= -2.911 \times 10^6 \text{ N} \\ \text{till } Q_7 &:= p_{\text{tot}} \cdot (0.272 + 2.7 - 17.998 + 2.74) \cdot m + R_{z54} & Q_7 &= -5.154 \times 10^5 \text{ N} \\ \text{till } Q_{11} &:= p_{\text{tot}} \cdot (3.78 - 17.935 + 2.679 + 0.197) \cdot m + R_{z56} & Q_{11} &= -5.881 \times 10^5 \text{ N} \\ \text{till } Q_{13} &:= p_{\text{tot}} \cdot (-21.818 + 2.443 - 0.661 - 0.049) \cdot m + R_{z57} & Q_{13} &= -1.089 \times 10^6 \text{ N}\end{aligned}$$

Load case 11

$$\begin{aligned}\text{max } Q_{11} &:= p_{\text{tot}} \cdot (-0.072 - 0.716 - 17.935 - 18.252) \cdot m + R_{z6} & Q_{11} &= -4.516 \times 10^6 \text{ N} \\ \text{till } Q_3 &:= p_{\text{tot}} \cdot (-0.048 + 0.177 + 2.455 - 21.818) \cdot m + R_{z62} & Q_3 &= -8.749 \times 10^5 \text{ N} \\ \text{till } Q_5 &:= p_{\text{tot}} \cdot (0.196 - 0.719 - 17.935 + 3.769) \cdot m + R_{z63} & Q_5 &= -1.57 \times 10^6 \text{ N} \\ \text{till } Q_7 &:= p_{\text{tot}} \cdot (-0.737 + 2.7 - 18.019 - 1.010) \cdot m + R_{z64} & Q_7 &= -3.039 \times 10^6 \text{ N} \\ \text{till } Q_9 &:= p_{\text{tot}} \cdot (2.763 - 18.038 + 2.7 + 0.272) \cdot m + R_{z65} & Q_9 &= -4.602 \times 10^5 \text{ N} \\ \text{till } Q_{13} &:= p_{\text{tot}} \cdot (-17.136 + 2.443 + 0.177 + 0.018) \cdot m + R_{z67} & Q_{13} &= -7.661 \times 10^5 \text{ N}\end{aligned}$$

Load case 13

$$\begin{aligned}\text{max } Q_{13} &:= p_{\text{tot}} \cdot (-0.049 - 0.661 - 17.136 - 21.818) \cdot m + R_{z7} & Q_{13} &= -4.651 \times 10^6 \text{ N} \\ \text{till } Q_3 &:= p_{\text{tot}} \cdot (0.018 - 0.048 - 0.658 - 17.136) \cdot m + R_{z72} & Q_3 &= -1.014 \times 10^6 \text{ N} \\ \text{till } Q_5 &:= p_{\text{tot}} \cdot (-0.073 + 0.196 + 2.669 - 18.252) \cdot m + R_{z73} & Q_5 &= -6.458 \times 10^5 \text{ N} \\ \text{till } Q_7 &:= p_{\text{tot}} \cdot (0.272 - 0.737 - 17.998 + 2.740) \cdot m + R_{z74} & Q_7 &= -1.582 \times 10^6 \text{ N} \\ \text{till } Q_9 &:= p_{\text{tot}} \cdot (-1.013 + 2.763 - 17.989 - 0.737) \cdot m + R_{z75} & Q_9 &= -2.993 \times 10^6 \text{ N} \\ \text{till } Q_{11} &:= p_{\text{tot}} \cdot (3.780 - 18.252 + 2.679 + 0.197) \cdot m + R_{z76} & Q_{11} &= -5.212 \times 10^5 \text{ N}\end{aligned}$$

APPENDIX C: LINEAR ANALYSIS

C1: Original box girder cross-section

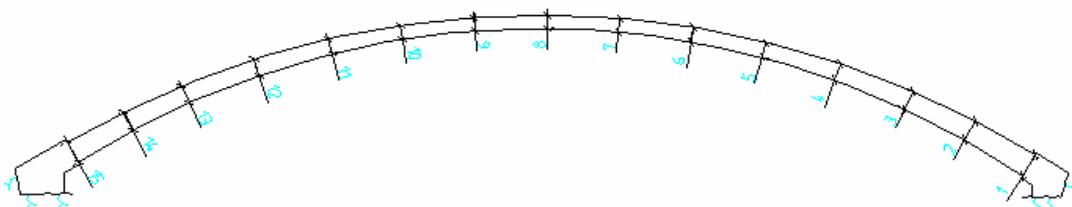
Input data for the structural analysis program

Table 1 Input data for the original arch cross-section.

Section	h [m]	A_c [m ²]	I_c [m ⁴]	Z_c [m]
1	5,340	19,674	84,988	2,714
2	4,782	18,211	63,155	2,374
3	4,272	16,825	46,415	2,064
4	3,800	15,5	33,524	1,779
5	3,434	14,914	26,251	1,609
6	3,184	14,514	21,858	1,494
7	3,036	14,278	19,47	1,425
8	2,989	14,202	18,744	1,404
9	3,042	14,287	19,563	1,428
10	3,195	14,532	22,041	1,499
11	3,449	14,938	26,53	1,616
12	3,818	15,529	33,908	1,788
13	4,292	16,857	46,939	2,074
14	4,763	18,181	62,555	2,365
15	5,305	19,618	83,659	2,696

The results obtained from the structural analysis program

Cross-section: Original



Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	42277	-147391	-67803	-158312
3	-2459	-143501	-50425	-134496
5	11816	-135917	-10187	-127434
7	54296	-132666	36279	-124291
9	63320	-133628	48055	-125231
11	41499	-129937	23192	-137981
13	17647	-148792	-10478	-139053
15	-180361	-154358	-217589	-144632

Load case 5

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	42277	-147391	-526	-166584
3	-26108	-135274	-79288	-153890
5	44732	-145297	-10187	-127434
7	50720	-124710	31835	-141795
9	80048	-143336	48055	-125231
11	41499	-129937	17933	-148066
13	14044	-139889	-17202	-157952
15	-152573	-164303	-217589	-144632

Load case 7

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	42277	-147391	-22594	-165463
3	-26108	-135274	-57216	-150981
5	7604	-127839	-21107	-143698
7	107698	-140175	36279	-124291
9	62499	-125566	36943	-141045
11	41499	-129937	-1870	-145771
13	20486	-156403	-10478	-139053
15	-123048	-161896	-217589	-144632

Load case 9

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	67951	-168200	6972	-146671
3	-26108	-135274	-61117	-154332
5	11841	-146510	-10187	-127434
7	50720	-124710	27180	-142863
9	113584	-144533	48055	-125231
11	41499	-129937	4226	-149061
13	14044	-139889	-23204	-159048
15	-156697	-165396	-217589	-144632

Load case 11

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	101718	-166367	6972	-146671
3	-26108	-135274	-62140	-153197
5	7604	-127839	-21398	-145487
7	68466	-142267	36279	-124291
9	62499	-125566	43959	-142861
11	84480	-149071	23403	-129623
13	14044	-139889	-33577	-158884
15	-182624	-145454	-253529	-164480

Load case 13

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	113426	-162957	6972	-146671
3	-25660	-150518	-50425	-134496
5	7604	-127839	-36085	-142493
7	53167	-139404	36279	-124291
9	95963	-140520	48055	-125231
11	41499	-129937	22493	-144866
13	30800	-157897	-10478	-139053
15	-182624	-145454	-304456	-162780

Maximum values

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	113426	-162957	-67803	-158312
3			-79288	-153890
5	44732	-145297	-36085	-142493
7	107698	-140175		
9	113584	-144533		
11	84480	-149071	-1870	-145771
13	30800	-157897	-33577	-158884
15			-304456	-162780

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
 MPa := 10^6 Pa kNm := 1000 N·m

Calculation for steel area (I cross-section)

Cross-section : Original

Material properties

Concrete C40/50

Partial safety factor

Safety class 3

$$\begin{aligned} \gamma_n &:= 1.2 & \eta\gamma_m &:= 1.5 \\ f_{cck} &:= 38 \text{ MPa} & \varepsilon_{cu} &:= 3.5 \cdot 10^{-3} \\ f_{cc} &:= \frac{f_{cck}}{(\eta\gamma_m \cdot \gamma_n)} & f_{cc} &= 2.111 \times 10^7 \text{ Pa} \end{aligned}$$

Stress block factors

$$\beta := 0.443 \qquad \alpha := 0.877$$

Steel

Partial safety factor

Safety class 3

$$\begin{aligned} \eta\gamma_m &:= 1.15 & \eta\gamma_{mes} &:= 1.05 & \gamma_n &:= 1.2 \\ f_{sk} &:= 500 \text{ MPa} & E_{sm} &:= 200 \text{ GPa} \\ f_{sd} &:= \frac{f_{sk}}{\eta\gamma_m \cdot \gamma_n} & f_{sd} &= 3.623 \times 10^8 \text{ Pa} \\ E_s &:= \frac{E_{sm}}{\eta\gamma_{mes} \cdot \gamma_n} & E_s &= 1.587 \times 10^{11} \text{ Pa} \\ \varepsilon_{sy} &:= \frac{f_{sd}}{E_s} & \varepsilon_{sy} &= 2.283 \times 10^{-3} \end{aligned}$$

For simplification of the calculations, we assumed that the cross-section is composed of 4 identical I-beams, see Figure 1.

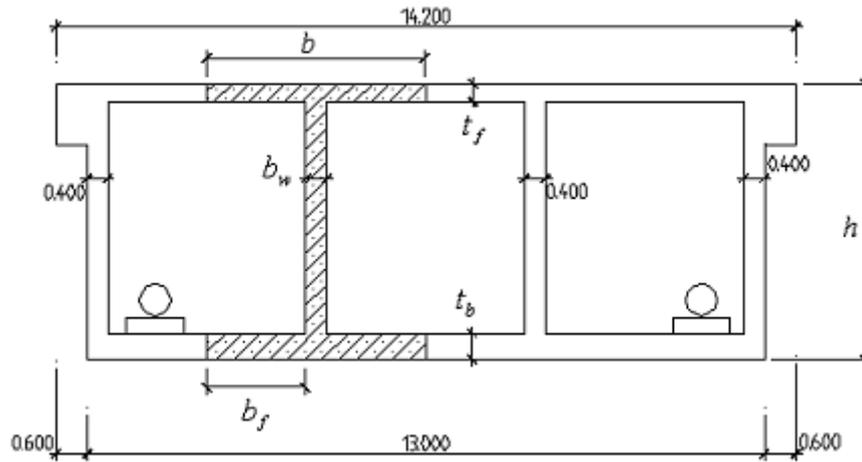


Figure 1. Simplified I-beam section

Section 1

Positive moment

Forces

$$M_{sd} := \frac{113426}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{162957}{4} \cdot \text{kN}$$

$$M_{sd} = 2.836 \times 10^4 \text{ kNm}$$

$$N_{sd} = 4.074 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 5.340 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 5.29 \text{ m}$$

$$e := d - \frac{h}{2}$$

$$e = 2.62 \text{ m}$$

$$b_f := 1.9 \text{ m}$$

$$b_w := 0.4 \text{ m}$$

$$t_f := 0.35 \text{ m}$$

$$x_1 := 0.0001 \text{ m}$$

$$b := 4.2 \text{ m}$$

$$t_b := 0.5 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 1.351 \times 10^5 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 x_1 \cdot (d - 0.4 x_1) - M_s, x_1\right]$$

$$x = 0.37\text{m}$$

$$0.8 \cdot x = 0.296\text{m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.046 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \quad F_c = 2.627 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \quad F_s = -1.447 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \quad A_s = -0.04\text{m}^2$$

Steel diameter

$$\phi := 20\text{-mm}$$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2}\right)^2 \quad A_{si} = 3.142 \times 10^{-4} \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -127.088$$

Section does not need reinforcement

Section 1

Negative moment

Forces

$$M_{sd} := \frac{67803}{4} \cdot \text{kNm} \quad N_{sd} := \frac{158312}{4} \cdot \text{kN}$$

$$M_{sd} = 1.695 \times 10^4 \text{ kNm} \quad N_{sd} = 3.958 \times 10^4 \text{ kN}$$

Cross-section constants

$$\begin{aligned}h &:= 5.340 \text{ m} & cc &:= 0.05 \text{ m} \\d &:= h - cc & d &= 5.29 \text{ m} \\b &:= 4.2 \text{ m} & t_b &:= 0.5 \text{ m} \\e &:= d - \frac{h}{2} & e &= 2.62 \text{ m} \\b_f &:= 1.9 \text{ m} & b_w &:= 0.4 \text{ m} \\t_f &:= 0.35 \text{ m} & x_1 &:= 0.0001 \text{ m} \\M_s &:= M_{sd} + N_{sd} \cdot e & M_s &= 1.206 \times 10^5 \text{ m kN}\end{aligned}$$

Assumption: Compressed area does fit in the flange

$$\begin{aligned}x &:= \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right] \\x &= 0.33 \text{ m} \\0.8 \cdot x &= 0.264 \text{ m} < t_b & \text{OK!} \\ \varepsilon_s &:= \frac{d - x}{x} \cdot \varepsilon_{cu} & \varepsilon_s = 0.053 > \varepsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\F_c &:= f_{cc} \cdot b \cdot 0.8 \cdot x & F_c = 2.339 \times 10^4 \text{ kN} \\F_s &:= F_c - N_{sd} & F_s = -1.619 \times 10^4 \text{ kN} \\A_s &:= \frac{F_s}{f_{sd}} & A_s = -0.045 \text{ m}^2\end{aligned}$$

Steel diameter $\phi := 20 \text{ mm}$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2}\right)^2 \quad A_{si} = 3.142 \times 10^{-4} \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -142.222$$

Section does not need reinforcement

Section 3

Negative moment

Forces

$$M_{sd} := \frac{79288}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{153890}{4} \cdot \text{kN}$$

$$M_{sd} = 1.982 \times 10^4 \text{ kNm}$$

$$N_{sd} = 3.847 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 4.272 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 4.222 \text{ m}$$

$$e := d - \frac{h}{2}$$

$$e = 2.086 \text{ m}$$

$$b_f := 1.9 \text{ m}$$

$$b_w := 0.4 \text{ m}$$

$$t_f := 0.35 \text{ m}$$

$$x_1 := 0.0001 \text{ m}$$

$$b := 4.2 \text{ m}$$

$$t_b := 0.4 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 1.001 \times 10^5 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.345 \text{ m}$$

$$0.8 \cdot x = 0.276 \text{ m} < t_b \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 0.039 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x$$

$$F_c = 2.451 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd}$$

$$F_s = -1.397 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}}$$

$$A_s = -0.039 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}}$$

$$n = -122.705$$

Section does not need reinforcement

Section 5

Positive moment

Forces

$$M_{sd} := \frac{44732}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{145297}{4} \cdot \text{kN}$$

$$M_{sd} = 1.118 \times 10^4 \text{ kNm}$$

$$N_{sd} = 3.632 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 3.434 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$b_f := 1.9 \text{ m}$$

$$b_w := 0.4 \text{ m}$$

$$d := h - cc$$

$$d = 3.384 \text{ m}$$

$$t_f := 0.35 \text{ m}$$

$$x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2}$$

$$e = 1.667 \text{ m}$$

$$b := 4.2 \text{ m}$$

$$t_b := 0.35 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 7.174 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root} \left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1 \right]$$

$$x = 0.31 \text{ m}$$

$$0.8 \cdot x = 0.248 \text{ m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 0.035 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x$$

$$F_c = 2.201 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd}$$

$$F_s = -1.432 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}}$$

$$A_s = -0.04 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}}$$

$$n = -125.796$$

Section does not need reinforcement

Section 5

Negative moment

Forces

$$M_{sd} := \frac{36085}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{142493}{4} \cdot \text{kN}$$

$$M_{sd} = 9.021 \times 10^3 \text{ kNm}$$

$$N_{sd} = 3.562 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 3.434 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$b_f := 1.9 \text{ m}$$

$$b_w := 0.4 \text{ m}$$

$$d := h - cc$$

$$d = 3.384 \text{ m}$$

$$t_f := 0.35 \text{ m}$$

$$x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2}$$

$$e = 1.667 \text{ m}$$

$$b := 4.2 \text{ m}$$

$$t_b := 0.35 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 6.841 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.295 \text{ m}$$

$$0.8 \cdot x = 0.236 \text{ m} < t_b \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 0.037 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x$$

$$F_c = 2.095 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd}$$

$$F_s = -1.468 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}}$$

$$A_s = -0.041 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}}$$

$$n = -128.95$$

Section does not need reinforcement

Section 7

Positive moment

Forces

$$M_{sd} := \frac{107698}{4} \cdot \text{kNm} \qquad N_{sd} := \frac{140175}{4} \cdot \text{kN}$$

$$M_{sd} = 2.692 \times 10^4 \text{ kNm} \qquad N_{sd} = 3.504 \times 10^4 \text{ kN}$$

Cross-section constants

$$\begin{aligned} h &:= 3.036 \text{ m} & cc &:= 0.05 \text{ m} & b_f &:= 1.9 \text{ m} & b_w &:= 0.4 \text{ m} \\ d &:= h - cc & d &= 2.986 \text{ m} & t_f &:= 0.35 \text{ m} & x_1 &:= 0.0001 \text{ m} \\ e &:= d - \frac{h}{2} & e &= 1.468 \text{ m} & b &:= 4.2 \text{ m} & t_b &:= 0.35 \text{ m} \end{aligned}$$

$$M_s := M_{sd} + N_{sd} \cdot e \qquad M_s = 7.837 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.39 \text{ m}$$

$$0.8 \cdot x = 0.312 \text{ m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \qquad \varepsilon_s = 0.023 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \qquad F_c = 2.769 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \qquad F_s = -7.35 \times 10^3 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \qquad A_s = -0.02 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \qquad n = -64.572$$

Section does not need reinforcement

Section 9

Positive moment

Forces

$$M_{sd} := \frac{113584}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{144533}{4} \cdot \text{kN}$$

$$M_{sd} = 2.84 \times 10^4 \text{ kNm}$$

$$N_{sd} = 3.613 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 3.042 \text{ m} \quad cc := 0.05 \text{ m} \quad b_f := 1.9 \text{ m} \quad b_w := 0.4 \text{ m}$$

$$d := h - cc \quad d = 2.992 \text{ m} \quad t_f := 0.35 \text{ m} \quad x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2} \quad e = 1.471 \text{ m} \quad b := 4.2 \text{ m} \quad t_b := 0.35 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 8.155 \times 10^4 \text{ mkN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.406 \text{ m}$$

$$0.8 \cdot x = 0.325 \text{ m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.022 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \quad F_c = 2.882 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \quad F_s = -7.312 \times 10^3 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \quad A_s = -0.02 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -64.242$$

Section does not need reinforcement

Section 11

Positive moment

Forces

$$M_{sd} := \frac{84480}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{149071}{4} \cdot \text{kN}$$

$$M_{sd} = 2.112 \times 10^4 \text{ kNm}$$

$$N_{sd} = 3.727 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 3.449 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$b_f := 1.9 \text{ m}$$

$$b_w := 0.4 \text{ m}$$

$$d := h - cc$$

$$d = 3.399 \text{ m}$$

$$t_f := 0.35 \text{ m}$$

$$x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2}$$

$$e = 1.675 \text{ m}$$

$$b := 4.2 \text{ m}$$

$$t_b := 0.35 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 8.352 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.362 \text{ m}$$

$$0.8 \cdot x = 0.289 \text{ m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 0.029 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x$$

$$F_c = 2.567 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd}$$

$$F_s = -1.16 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}}$$

$$A_s = -0.032 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}}$$

$$n = -101.923$$

Section does not need reinforcement

Section 11

Negative moment

Forces

$$M_{sd} := \frac{1870}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{145771}{4} \cdot \text{kN}$$

$$M_{sd} = 467.5 \text{ kNm}$$

$$N_{sd} = 3.644 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 3.449 \text{ m} \quad cc := 0.05 \text{ m} \quad b_f := 1.9 \text{ m} \quad b_w := 0.4 \text{ m}$$

$$d := h - cc \quad d = 3.399 \text{ m} \quad t_f := 0.35 \text{ m} \quad x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2} \quad e = 1.675 \text{ m} \quad b := 4.2 \text{ m} \quad t_b := 0.35 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 6.149 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.263 \text{ m}$$

$$0.8 \cdot x = 0.211 \text{ m} < t_b \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.042 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \quad F_c = 1.867 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \quad F_s = -1.777 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \quad A_s = -0.049 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -156.148$$

Section does not need reinforcement

Section 13

Positive moment

Forces

$$M_{sd} := \frac{30800}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{157897}{4} \cdot \text{kN}$$

$$M_{sd} = 7.7 \times 10^3 \text{ kNm}$$

$$N_{sd} = 3.947 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 4.292 \text{ m} \quad cc := 0.05 \text{ m}$$

$$b_f := 1.9 \text{ m} \quad b_w := 0.4 \text{ m}$$

$$d := h - cc \quad d = 4.242 \text{ m}$$

$$t_f := 0.35 \text{ m} \quad x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2} \quad e = 2.096 \text{ m}$$

$$b := 4.2 \text{ m} \quad t_b := 0.4 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 9.044 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.31 \text{ m}$$

$$0.8 \cdot x = 0.248 \text{ m} < t_f \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 0.044 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x$$

$$F_c = 2.196 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd}$$

$$F_s = -1.751 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}}$$

$$A_s = -0.048 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}}$$

$$n = -153.862$$

Section does not need reinforcement

Section 13

Negative moment

Forces

$$M_{sd} := \frac{33577}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{158884}{4} \cdot \text{kN}$$

$$M_{sd} = 8.394 \times 10^3 \text{ kNm}$$

$$N_{sd} = 3.972 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 4.292 \text{ m} \quad cc := 0.05 \text{ m} \quad b_f := 1.9 \text{ m} \quad b_w := 0.4 \text{ m}$$

$$d := h - cc \quad d = 4.242 \text{ m} \quad t_f := 0.35 \text{ m} \quad x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2} \quad e = 2.096 \text{ m} \quad b := 4.2 \text{ m} \quad t_b := 0.4 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 9.165 \times 10^4 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.314 \text{ m}$$

$$0.8x = 0.251 \text{ m} < t_b \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.044 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \quad F_c = 2.226 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \quad F_s = -1.746 \times 10^4 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \quad A_s = -0.048 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -153.364$$

Section does not need reinforcement

Section 15

Negative moment

Forces

$$M_{sd} := \frac{304456}{4} \cdot \text{kNm}$$

$$N_{sd} := \frac{162780}{4} \cdot \text{kN}$$

$$M_{sd} = 7.611 \times 10^4 \text{ kNm}$$

$$N_{sd} = 4.069 \times 10^4 \text{ kN}$$

Cross-section constants

$$h := 5.305 \text{ m} \quad cc := 0.05 \text{ m} \quad b_f := 1.9 \text{ m} \quad b_w := 0.4 \text{ m}$$

$$d := h - cc \quad d = 5.255 \text{ m} \quad t_f := 0.35 \text{ m} \quad x_1 := 0.0001 \text{ m}$$

$$e := d - \frac{h}{2} \quad e = 2.603 \text{ m} \quad b := 4.2 \text{ m} \quad t_b := 0.5 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 1.82 \times 10^5 \text{ m kN}$$

Assumption: Compressed area does fit in the flange

$$x := \text{root}\left[f_{cc} \cdot b \cdot 0.8 \cdot x_1 \cdot (d - 0.4 \cdot x_1) - M_s, x_1\right]$$

$$x = 0.508 \text{ m}$$

$$0.8 \cdot x = 0.406 \text{ m} < t_b \quad \text{OK!}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.033 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$F_c := f_{cc} \cdot b \cdot 0.8 \cdot x \quad F_c = 3.603 \times 10^4 \text{ kN}$$

$$F_s := F_c - N_{sd} \quad F_s = -4.664 \times 10^3 \text{ kN}$$

$$A_s := \frac{F_s}{f_{sd}} \quad A_s = -0.013 \text{ m}^2$$

Amount of steel needed

$$n := \frac{A_s}{A_{si}} \quad n = -40.974$$

Section does not need reinforcement

C2: Calculations for the solid beam section

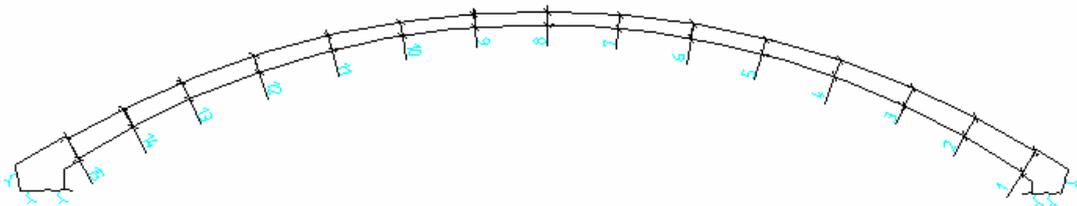
Input data for the structural analysis program

Table 1 Input data for the original arch cross-section.

Section	h [m]	A_c [m ²]	I_c [m ⁴]	Z_c [m]
1-15	1,5	19,5	3,66	0,75

The results obtained from the structural analysis program

Cross-section: Solid



Load case 3

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	132546	-173284	68317	-185166
3	-10694	-169719	-48412	-161209
5	-21544	-161393	-33201	-153000
7	26505	-157435	20128	-149142
9	46420	-149726	36532	-158615
11	31301	-154665	21251	-163839
13	39338	-174512	29168	-165375
15	-80538	-180622	-97164	-170942

Load case 5

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	141182	-193496	125405	-173089
3	-42993	-161005	-68215	-180944
5	13148	-170620	-33201	-153000
7	26177	-148938	14683	-167033
9	61136	-167890	40446	-150008
11	31301	-154665	16734	-173779
13	34581	-165099	22295	-184650
15	-56375	-190435	-97164	-170942

Load case 7

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	132546	-173284	107790	-191784
3	-40011	-177408	-48412	-161209
5	-26878	-152793	-37470	-169199
7	86438	-164354	20128	-149142
9	46420	-149726	29023	-165793
11	31301	-154665	-4590	-171084
13	34581	-165099	27413	-181898
15	-42076	-187671	-97164	-170942

Load case 9

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	158373	-194112	125405	-173089
3	-42993	-161005	-58780	-181063
5	-22002	-171480	-33201	-153000
7	26177	-148938	12058	-167728
9	98991	-168705	40446	-150008
11	31301	-154665	7647	-174394
13	34581	-165099	19051	-185375
15	-60812	-191162	-97164	-170942

Load case 11

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	190736	-192621	125405	-173089
3	-42993	-161005	-64054	-180284
5	-26878	-152793	-49072	-171428
7	40617	-166859	20128	-149142
9	46420	-149726	36305	-167983
11	79401	-174116	24813	-154947
13	34581	-165099	18419	-185488
15	-89895	-171215	-113285	-191062

Load case 13

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	194902	-189113	125405	-173089
3	-42993	-161005	-56432	-176710
5	-26878	-152793	-65496	-168304
7	26177	-148938	19521	-164063
9	81338	-164882	40446	-150008
11	37515	-170079	24813	-154947
13	65396	-183246	29168	-165375
15	-89895	-171215	-163314	-189220

Maximum values

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	194902	-189113		
3			-68215	-180944
5	13148	-170620	-65496	-168304
7	86438	-164354		
9	98991	-168705		
11	79401	-174116	-4590	-171084
13	65396	-183246		
15			-163314	-189220

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
 MPa := 10^6 Pa kNm := 1000 N·m

Calculation for steel area

Cross-section : Solid

Material properties

Concrete C40/50

Partial safety factor

Safety class 3

$$\gamma_n := 1.2$$

$$\eta\gamma_m := 1.5$$

$$f_{cck} := 38 \cdot \text{MPa}$$

$$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$$

$$f_{cc} := \frac{f_{cck}}{(\eta\gamma_m \cdot \gamma_n)}$$

$$f_{cc} = 2.111 \times 10^7 \text{ Pa}$$

Stress block factors

$$\beta := 0.443$$

$$\alpha := 0.877$$

Steel K500 (Kamstång B500B)

Partial safety factor

Safety class 3

$$\eta\gamma_m := 1.15$$

$$\eta\gamma_{mes} := 1.05$$

$$\gamma_n := 1.2$$

$$f_{sk} := 500 \cdot \text{MPa}$$

$$E_{sm} := 200 \cdot \text{GPa}$$

$$f_{sd} := \frac{f_{sk}}{\eta\gamma_m \cdot \gamma_n}$$

$$f_{sd} = 3.623 \times 10^8 \text{ Pa}$$

$$E_s := \frac{E_{sm}}{\eta\gamma_{mes} \cdot \gamma_n}$$

$$E_s = 1.587 \times 10^{11} \text{ Pa}$$

$$\varepsilon_{sy} := \frac{f_{sd}}{E_s}$$

$$\varepsilon_{sy} = 2.283 \times 10^{-3}$$

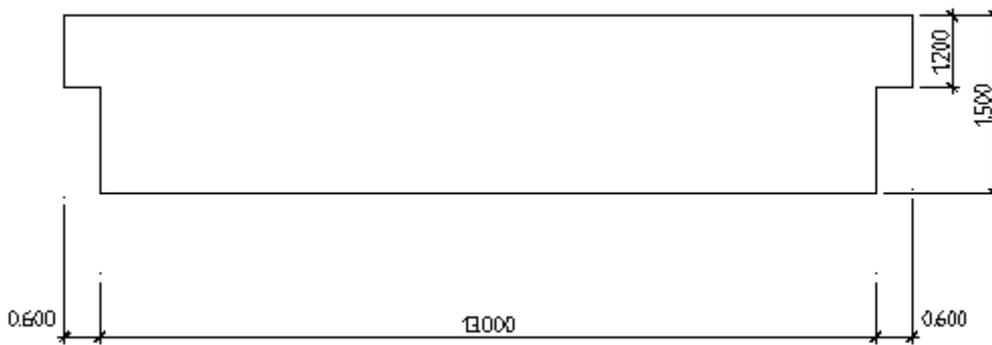


Figure 1. Solid beam cross-section

Section 1

Positive moment

Forces

$$M_{sd} := 194902 \text{ kNm}$$

$$N_{sd} := 189113 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 3.273 \times 10^5 \text{ kNm}$$

Steel diameter

$$\phi := 20 \text{ mm}$$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2} \right)^2$$

$$A_{si} = 3.142 \times 10^{-4} \text{ m}^2$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_T := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_T = 0.567 \quad m_T > m_{bal} \quad \text{NOT OK !}$$

The cross-section will be over reinforced

Try with putting compression reinforcement

$$d_t := 0.05 \text{ m}$$

Assume: $\sigma_{st} := f_{sd}$

$$A_{st} := \frac{M_s - m_{bal} \cdot b \cdot d^2 \cdot f_{cc}}{(d - d_t) \cdot \sigma_{st}} \quad A_{st} = 0.23 \text{ m}^2$$

$$n_t := \frac{A_{st}}{A_{si}} \quad n_t = 732.127$$

Choose: $n_t := 733$

$$M_{II} := A_{st} \cdot \sigma_{st} \cdot (d - d_t) \quad M_{II} = 1.167 \times 10^5 \text{ kNm}$$

$$M_I := M_s - M_{II} \quad M_I = 2.106 \times 10^5 \text{ kNm}$$

$$m_T := \frac{M_I}{b \cdot d^2 \cdot f_{cc}} \quad m_T = 0.365$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_T)} \quad \omega = 0.48$$

$$\varepsilon_{st} := \varepsilon_{cu} \cdot \left(1 - 0.8 \cdot \frac{d_t}{\omega \cdot d} \right)$$

$$\varepsilon_{st} = 3.299 \times 10^{-3} \quad \varepsilon_{st} > \varepsilon_{sy} \quad \text{OK!}$$

$$A_{s1} := \left[\frac{M_I}{d \cdot \left(1 - \frac{\omega}{2} \right)} + \frac{M_{II}}{d - d_t} \right] \cdot \frac{1}{f_{sd}} \quad A_{s1} = 0.758 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = 0.236 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}}$$

$$n = 750.168$$

Choose: $n := 750$

Calculation for x

Assume: $\varepsilon_s, \varepsilon_{s1}, \varepsilon_{s2}, \varepsilon_{s3} > \varepsilon_{sy}$

$\varepsilon_{st}, \varepsilon_{st1}, \varepsilon_{st2}, \varepsilon_{st3} > \varepsilon_{sy}$

Tension reinforcement:

$$d := 1.44 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \quad n_3 := 215 \quad A_{s3} := A_{si} \cdot n_3 \quad A_{s3} = 0.068 \text{ m}^2$$

$$d_3 := 1.29 \text{ m} \quad n_4 := 105 \quad A_{s4} := A_{si} \cdot n_4 \quad A_{s4} = 0.033 \text{ m}^2$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3} + A_{s4} \quad A_{stot} = 0.236 \text{ m}^2$$

Compression reinforcement:

$$d_t := 0.05 \text{ m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 215 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.068 \text{ m}^2$$

$$d_{t3} := 0.2 \text{ m} \quad n_{t4} := 88 \quad A_{st4} := A_{si} \cdot n_{t4} \quad A_{st4} = 0.028 \text{ m}^2$$

$$A_{sttot} := A_{st1} + A_{st2} + A_{st3} + A_{st4} \quad A_{sttot} = 0.23 \text{ m}^2$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{stot} + f_{sd} \cdot A_{sttot}, x_1)$$

$$x = 0.794 \text{ m}$$

Check assumption

$$\epsilon_s := \frac{d-x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 2.85 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s1} := \frac{d_1-x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s1} = 2.629 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s2} := \frac{d_2-x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s2} = 2.409 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s3} := \frac{d_3-x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s3} = 2.188 \times 10^{-3} < \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{NOT OK}$$

$$\epsilon_{st} := \frac{x-d_t}{x} \cdot \epsilon_{cu} \quad \epsilon_{st} = 3.28 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st1} := \frac{x-d_{t1}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st1} = 3.059 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st2} := \frac{x-d_{t2}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st2} = 2.839 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st3} := \frac{x-d_{t3}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st3} = 2.618 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Change assumption:

$$\epsilon_s, \epsilon_{s1}, \epsilon_{s2} > \epsilon_{sy}$$

$$\epsilon_{st}, \epsilon_{st1}, \epsilon_{st2}, \epsilon_{st3} > \epsilon_{sy}$$

$$\epsilon_{s3} < \epsilon_{sy}$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root} \left[\begin{array}{l} \alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot (A_{s1} + A_{s2} + A_{s3}) - E_s \cdot \epsilon_{cu} \cdot A_{s4} \cdot \left(\frac{d_3 - x_1}{x_1} \right) \dots, x_1 \\ + f_{sd} \cdot A_{sttot} \end{array} \right]$$

$$x = 0.792 \text{ m}$$

Check assumption

$$\epsilon_s := \frac{d-x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 2.864 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\begin{aligned} \epsilon_{s1} &:= \frac{d_1 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s1} = 2.643 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{s2} &:= \frac{d_2 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s2} = 2.422 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{s3} &:= \frac{d_3 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s3} = 2.201 \times 10^{-3} & < & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{st} &:= \frac{x - d_t}{x} \cdot \epsilon_{cu} & \epsilon_{st} = 3.279 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{st1} &:= \frac{x - d_{t1}}{x} \cdot \epsilon_{cu} & \epsilon_{st1} = 3.058 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{st2} &:= \frac{x - d_{t2}}{x} \cdot \epsilon_{cu} & \epsilon_{st2} = 2.837 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \\ \epsilon_{st3} &:= \frac{x - d_{t3}}{x} \cdot \epsilon_{cu} & \epsilon_{st3} = 2.616 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK!} \end{aligned}$$

Check moment capacity $M_d > M_s$

$$\begin{aligned} M_d &:= \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots \\ &\quad + f_{sd} \cdot A_{st4} \cdot (d - d_{t3}) - f_{sd} \cdot A_{s2} \cdot (d - d_1) - f_{sd} \cdot A_{s3} \cdot (d - d_2) \dots \\ &\quad + [-E_s \cdot \epsilon_{s3} \cdot A_{s4} \cdot (d - d_3)] \end{aligned}$$

$$M_d = 3.13 \times 10^5 \text{ kNm}$$

$M_d < M_s$ NOT OK!

$$M_s = 3.273 \times 10^5 \text{ kNm}$$

Increase compression steel area!

Calculation for x

Assume: $\epsilon_s, \epsilon_{s1}, \epsilon_{s2}, \epsilon_{s3} > \epsilon_{sy}$ $\epsilon_{st}, \epsilon_{st1}, \epsilon_{st2}, \epsilon_{st3} > \epsilon_{sy}$ $\epsilon_{st4} < \epsilon_{sy}$

$$d := 1.44 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$\begin{aligned}
d_2 &:= 1.34 \text{ m} & n_3 &:= 215 & A_{s3} &:= A_{si} \cdot n_3 & A_{s3} &= 0.068 \text{ m}^2 \\
d_3 &:= 1.29 \text{ m} & n_4 &:= 105 & A_{s4} &:= A_{si} \cdot n_4 & A_{s4} &= 0.033 \text{ m}^2 \\
A_{stot} &:= A_{s1} + A_{s2} + A_{s3} + A_{s4} & & & & & A_{stot} &= 0.236 \text{ m}^2 \\
d_t &:= 0.05 \text{ m} & n_{t1} &:= 215 & A_{st1} &:= A_{si} \cdot n_{t1} & A_{st1} &= 0.068 \text{ m}^2 \\
d_{t1} &:= 0.1 \text{ m} & n_{t2} &:= 215 & A_{st2} &:= A_{si} \cdot n_{t2} & A_{st2} &= 0.068 \text{ m}^2 \\
d_{t2} &:= 0.15 \text{ m} & n_{t3} &:= 215 & A_{st3} &:= A_{si} \cdot n_{t3} & A_{st3} &= 0.068 \text{ m}^2 \\
d_{t3} &:= 0.2 \text{ m} & n_{t4} &:= 215 & A_{st4} &:= A_{si} \cdot n_{t4} & A_{st4} &= 0.068 \text{ m}^2 \\
d_{t4} &:= 0.25 \text{ m} & n_{t5} &:= 215 & A_{st5} &:= A_{si} \cdot n_{t5} & A_{st5} &= 0.068 \text{ m}^2 \\
A_{sttot} &:= A_{st1} + A_{st2} + A_{st3} + A_{st4} + A_{st5} & & & & & A_{sttot} &= 0.338 \text{ m}^2
\end{aligned}$$

Horizontal equilibrium gives:

$$\begin{aligned}
x_1 &:= 0.0001 \text{ m} \\
x &:= \text{root} \left[\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{stot} + f_{sd} \cdot (A_{sttot} - A_{st5}) \dots, x_1 \right. \\
&\quad \left. + E_s \cdot \epsilon_{cu} \cdot A_{st5} \cdot \left(\frac{x_1 - d_{t4}}{x_1} \right) \right] \\
x &= 0.639 \text{ m}
\end{aligned}$$

Check assumptions:

$$\begin{aligned}
\epsilon_s &:= \frac{d - x}{x} \cdot \epsilon_{cu} & \epsilon_s &= 4.39 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK !} \\
\epsilon_{s1} &:= \frac{d_1 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s1} &= 4.116 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK !} \\
\epsilon_{s2} &:= \frac{d_2 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s2} &= 3.842 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK !} \\
\epsilon_{s3} &:= \frac{d_3 - x}{x} \cdot \epsilon_{cu} & \epsilon_{s3} &= 3.568 \times 10^{-3} & > & \epsilon_{sy} = 2.283 \times 10^{-3} & \text{OK !}
\end{aligned}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.226 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st1} := \frac{x - d_{t1}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st1} = 2.952 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st2} := \frac{x - d_{t2}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st2} = 2.678 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st3} := \frac{x - d_{t3}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st3} = 2.404 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st4} := \frac{x - d_{t4}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st4} = 2.13 \times 10^{-3} < \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots \\ + f_{sd} \cdot A_{st4} \cdot (d - d_{t3}) + E_s \cdot \varepsilon_{st4} \cdot A_{st5} \cdot (d - d_{t4}) - f_{sd} \cdot A_{s2} \cdot (d - d_1) - f_{sd} \cdot A_{s3} \cdot (d - d_2) \dots \\ + [-f_{sd} \cdot A_{s4} \cdot (d - d_3)]$$

$$M_d = 3.283 \times 10^5 \text{ kNm}$$

$M_d > M_s$ OK!

$$M_s = 3.273 \times 10^5 \text{ kNm}$$

Section 3

Negative moment

Forces

$$M_{sd} := 68215 \text{ kNm}$$

$$N_{sd} := 180944 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 1.949 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_r = 0.338 \quad m_r < m_{bal} \quad \text{OK !}$$

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)} \quad \omega = 0.43 \quad \omega < \omega_{bal} \quad \text{OK !}$$

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}} \quad A_{s1} = 0.473 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = -0.027 \text{ m}^2$$

Section 3 does not need reinforcement.

Minimum reinforcement

$$d := 1.44 \text{ m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.672 \text{ m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 4 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.24 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 5

Negative moment

Forces

$$M_{sd} := 65496 \text{ kNm}$$

$$N_{sd} := 168304 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_S := M_{sd} + N_{sd} \cdot e$$

$$M_S = 1.833 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_T := \frac{M_S}{b \cdot d^2 \cdot f_{cc}}$$

$$m_T = 0.318$$

$$m_T < m_{bal}$$

OK !

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{1 - 2 \cdot m_T}$$

$$\omega = 0.396$$

$$\omega < \omega_{bal}$$

OK !

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}}$$

$$A_{s1} = 0.435 \text{ m}^2$$

$$A_S := A_{s1} - \frac{N_{sd}}{f_{sd}}$$

$$A_S = -0.029 \text{ m}^2$$

Section 5 does not need reinforcement.

Minimum reinforcement

$$d := 1.44 \text{ m}$$

$$n := 64$$

$$A_S := n \cdot A_{si}$$

$$A_S = 0.02 \text{ m}^2$$

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.617\text{m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 4.673 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.216 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 5

Positive moment

Forces

$$M_{sd} := 13148\text{kNm}$$

$$N_{sd} := 170620\text{kN}$$

Cross-section constants

$$h := 1.5\text{m}$$

$$cc := 0.05\text{m}$$

$$d := h - cc$$

$$d = 1.45\text{m}$$

$$tp := 0.75\text{m}$$

$$b := 13\text{m}$$

$$e := d - tp$$

$$e = 0.7\text{m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 1.326 \times 10^5 \text{kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_r = 0.23 \quad m_r < m_{bal} \quad \text{OK !}$$

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)} \quad \omega = 0.265 \quad \omega < \omega_{bal} \quad \text{OK !}$$

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}} \quad A_{s1} = 0.291\text{m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = -0.18\text{m}^2$$

Section 5 does not need reinforcement.

Minimum reinforcement

$$d := 1.44\text{ m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02\text{m}^2$$

$$d_t := 0.05\text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.403\text{m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 8.995 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.066 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 7

Positive moment

Forces

$$M_{sd} := 86438\text{kNm} \quad N_{sd} := 164354\text{kN}$$

Cross-section constants

$$h := 1.5\text{ m} \quad cc := 0.05\text{ m}$$

$$d := h - cc \quad d = 1.45\text{ m}$$

$$tp := 0.75\text{ m} \quad b := 13\text{ m}$$

$$e := d - tp \quad e = 0.7\text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 2.015 \times 10^5\text{ kNm}$$

Values for balanced reinforcement K500

$$m_{\text{bal}} := 0.365$$

$$m_T := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_T = 0.349 \quad m_T < m_{\text{bal}} \quad \text{OK !}$$

$$\omega_{\text{bal}} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_T)} \quad \omega = 0.451 \quad \omega < \omega_{\text{bal}} \quad \text{OK !}$$

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}} \quad A_{s1} = 0.495 \text{m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = 0.041 \text{m}^2$$

$$n := \frac{A_s}{A_{si}} \quad n = 132.088$$

Choose: $n := 133$

Calculation for x

Assume: $\varepsilon_s > \varepsilon_{sy}$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_s, x_1)$$

$$x = 0.745 \text{m}$$

Check assumption

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.309 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK !}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x)$$

$$M_d = 2.009 \times 10^5 \text{ kNm}$$

$$M_d < M_s$$

NOT OK!

$$M_s = 2.015 \times 10^5 \text{ kNm}$$

Increase the amount of steel!

$$n_{ny} := 150$$

$$A_{sny} := n_{ny} \cdot A_{si}$$

$$A_{sny} = 0.047 \text{ m}^2$$

Assume: $\epsilon_s > \epsilon_{sy}$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{sny} \cdot x_1)$$

$$x = 0.754 \text{ m}$$

Check assumption

$$\epsilon_s := \frac{d-x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 3.233 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x)$$

$$M_d = 2.025 \times 10^5 \text{ kNm}$$

$$M_d > M_s$$

OK!

$$M_s = 2.015 \times 10^5 \text{ kNm}$$

Minimum compression reinforcement:

$$d := 1.44 \text{ m} \quad n := 150 \quad A_s := n \cdot A_{si} \quad A_s = 0.047 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root}[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1]$$

$$x = 0.705 \text{ m}$$

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.648 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x-d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.252 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 9

Positive moment

Forces

$$M_{sd} := 98991 \text{ kNm}$$

$$N_{sd} := 168705 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.171 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}}$$

$$m_r = 0.376$$

$$m_r > m_{bal}$$

NOT OK !

The cross-section will be over reinforced

Try with putting compression reinforcement

$$d_t := 0.05 \text{ m}$$

Assumption: $\sigma_{st} := f_{sd}$

$$A_{st} := \frac{M_s - m_{bal} \cdot b \cdot d^2 \cdot f_{cc}}{(d - d_t) \cdot \sigma_{st}}$$

$$A_{st} = 0.013 \text{ m}^2$$

$$n_t := \frac{A_{st}}{A_{si}} \quad n_t = 40.616$$

Choose: $n_t := 41$

$$M_{II} := A_{st} \cdot \sigma_{st} \cdot (d - d_t) \quad M_{II} = 6.472 \times 10^3 \text{ kNm}$$

$$M_I := M_S - M_{II} \quad M_I = 2.106 \times 10^5 \text{ kNm}$$

$$m_T := \frac{M_I}{b \cdot d^2 \cdot f_{cc}} \quad m_T = 0.365$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_T)} \quad \omega = 0.48$$

$$\varepsilon_{st} := \varepsilon_{cu} \cdot \left(1 - 0.8 \cdot \frac{d_t}{\omega \cdot d} \right)$$

$$\varepsilon_{st} = 3.299 \times 10^{-3} \quad \varepsilon_{st} > \varepsilon_{sy} \quad \text{OK!}$$

$$A_{s1} := \left[\frac{M_I}{d \cdot \left(1 - \frac{\omega}{2} \right)} + \frac{M_{II}}{d - d_t} \right] \cdot \frac{1}{f_{sd}} \quad A_{s1} = 0.54 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = 0.075 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}} \quad n = 237.948$$

Choose: $n := 238$

Calculation for x

Assume: $\varepsilon_s, \varepsilon_{s1} > \varepsilon_{sy} \quad \varepsilon_{st} > \varepsilon_{sy}$

Tension reinforcement

$$d := 1.44 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 23 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 7.226 \times 10^{-3} \text{ m}^2$$

$$n_{\text{tot}} := n_1 + n_2 \quad n_{\text{tot}} = 238$$

$$A_{\text{stot}} := A_{s1} + A_{s2} \quad A_{\text{stot}} = 0.075\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_t := 41 \quad A_{\text{st}} := A_{\text{si}} \cdot n_t \quad A_{\text{st}} = 0.013\text{m}^2$$

Horizontal equilibrium gives:

$$x_1 := 0.0001\text{m}$$

$$x := \text{root}(\alpha \cdot f_{\text{cc}} \cdot b \cdot x_1 - N_{\text{sd}} - f_{\text{sd}} \cdot A_{\text{stot}} + f_{\text{sd}} \cdot A_{\text{st}}, x_1)$$

$$x = 0.794\text{m}$$

Check assumption

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{\text{cu}} \quad \varepsilon_s = 2.847 \times 10^{-3} > \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{s1} := \frac{d_1-x}{x} \cdot \varepsilon_{\text{cu}} \quad \varepsilon_{s1} = 2.626 \times 10^{-3} > \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{\text{st}} := \frac{x-d_t}{x} \cdot \varepsilon_{\text{cu}} \quad \varepsilon_{\text{st}} = 3.28 \times 10^{-3} > \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{\text{cc}} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{\text{sd}} \cdot A_{\text{st}} \cdot (d - d_t) - f_{\text{sd}} \cdot A_{s2} \cdot (d - d_1)$$

$$M_d = 2.143 \times 10^5 \text{ kNm}$$

$$M_d < M_{\text{sd}} \quad \text{NOT OK!}$$

$$M_s = 2.171 \times 10^5 \text{ kNm}$$

Increase the amount of compression reinforcement.

$$d_t := 0.05\text{m} \quad n_t := 100 \quad A_{\text{st}} := A_{\text{si}} \cdot n_t \quad A_{\text{st}} = 0.031\text{m}^2$$

$$\text{Assume: } \varepsilon_s, \varepsilon_{s1} > \varepsilon_{\text{sy}} \quad \varepsilon_{\text{st}} > \varepsilon_{\text{sy}}$$

Tension reinforcement

$$\begin{aligned}
 d &:= 1.44 \text{ m} & n_1 &:= 215 & A_{s1} &:= A_{s1} \cdot n_1 & A_{s1} &= 0.068 \text{ m}^2 \\
 d_1 &:= 1.39 \text{ m} & n_2 &:= 23 & A_{s2} &:= A_{s1} \cdot n_2 & A_{s2} &= 7.226 \times 10^{-3} \text{ m}^2 \\
 n_{\text{tot}} &:= n_1 + n_2 & n_{\text{tot}} &= 238 & & & & \\
 A_{\text{stot}} &:= A_{s1} + A_{s2} & & & A_{\text{stot}} &= 0.075 \text{ m}^2 & &
 \end{aligned}$$

Horizontal equilibrium gives:

$$\begin{aligned}
 x_1 &:= 0.0001 \text{ m} \\
 x &:= \text{root}\left[\left(\alpha \cdot f_{\text{cc}} \cdot b \cdot x_1 - N_{\text{sd}} - f_{\text{sd}} \cdot A_{\text{stot}}\right) + f_{\text{sd}} \cdot A_{\text{st}} \cdot x_1\right] \\
 x &= 0.766 \text{ m}
 \end{aligned}$$

Check assumption

$$\begin{aligned}
 \varepsilon_s &:= \frac{d-x}{x} \cdot \varepsilon_{\text{cu}} & \varepsilon_s &= 3.078 \times 10^{-3} & > & \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} & \text{OK!} \\
 \varepsilon_{s1} &:= \frac{d_1-x}{x} \cdot \varepsilon_{\text{cu}} & \varepsilon_{s1} &= 2.85 \times 10^{-3} & > & \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} & \text{OK!} \\
 \varepsilon_{\text{st}} &:= \frac{x-d_t}{x} \cdot \varepsilon_{\text{cu}} & \varepsilon_{\text{st}} &= 3.272 \times 10^{-3} & > & \varepsilon_{\text{sy}} = 2.283 \times 10^{-3} & \text{OK!}
 \end{aligned}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{\text{cc}} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{\text{sd}} \cdot A_{\text{st}} \cdot (d - d_t) - f_{\text{sd}} \cdot A_{s2} \cdot (d - d_1)$$

$$M_d = 2.187 \times 10^5 \text{ kNm}$$

$$M_d > M_s \quad \text{OK!}$$

$$M_s = 2.171 \times 10^5 \text{ kNm}$$

Section 11

Positive moment

Forces

$$M_{sd} := 79401 \text{ kNm}$$

$$N_{sd} := 174116 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.013 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}}$$

$$m_r = 0.349$$

$$m_r < m_{bal}$$

OK !

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)}$$

$$\omega = 0.45$$

$$\omega < \omega_{bal}$$

OK !

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}}$$

$$A_{s1} = 0.494 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}}$$

$$A_s = 0.014 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}}$$

$$n = 44.081$$

Choose: $n := 45$

Calculation for x

$$\text{Assume: } \varepsilon_s > \varepsilon_{sy}$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_s, x_1)$$

$$x = 0.744 \text{ m}$$

Check assumption

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.319 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x)$$

$$M_d = 2.007 \times 10^5 \text{ kNm}$$

$$M_d < M_s \quad \text{NOT OK!}$$

$$M_s = 2.013 \times 10^5 \text{ kNm}$$

Increase the steel area.

$$n_{ny} := 64 \quad A_{sny} := n_{ny} \cdot A_{si} \quad A_{sny} = 0.02 \text{ m}^2$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{sny}, x_1)$$

$$x = 0.754 \text{ m}$$

Check assumption

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.234 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x)$$

$$M_d = 2.025 \times 10^5 \text{ kNm}$$

$$M_s = 2.013 \times 10^5 \text{ kNm}$$

$$M_d > M_s \quad \text{OK!}$$

Minimum compression reinforcement:

$$d := 1.44 \text{ m} \quad n_1 := 64 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.02 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.704 \text{ m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.659 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.251 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 11

Negative moment

Forces

$$M_{sd} := 4590 \text{ kNm}$$

$$N_{sd} := 171084 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 1.243 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_r = 0.216 \quad m_r < m_{bal} \quad \text{OK !}$$

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)} \quad \omega = 0.246 \quad \omega < \omega_{bal} \quad \text{OK !}$$

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}} \quad A_{s1} = 0.27 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = -0.202 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}} \quad n = -644.11$$

Section 11 does not need reinforcement.

Minimum reinforcement

$$d := 1.44 \text{ m} \quad n_1 := 64 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.02 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.372 \text{ m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.01 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.03 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 13

Positive moment

Forces

$$M_{sd} := 65396 \text{ kNm}$$

$$N_{sd} := 183246 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 1.937 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}}$$

$$m_r = 0.336$$

$$m_r < m_{bal}$$

OK !

$$\omega_{bal} := 0.480$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)}$$

$$\omega = 0.427$$

$$\omega < \omega_{bal}$$

OK !

$$A_{s1} := \frac{\omega \cdot d \cdot b \cdot f_{cc}}{f_{sd}}$$

$$A_{s1} = 0.469 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}}$$

$$A_s = -0.037 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}}$$

$$n = -118.274$$

Section 13 does not need reinforcement.

Minimum reinforcement

$$d := 1.44 \text{ m} \quad n_1 := 64 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.02 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.666 \text{ m}$$

$$\epsilon_s := \frac{d - x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 4.067 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st} := \frac{x - d_t}{x} \cdot \epsilon_{cu} \quad \epsilon_{st} = 3.237 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Section 15

Negative moment

Forces

$$M_{sd} := 163314 \text{ kNm} \quad N_{sd} := 189220 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m} \quad cc := 0.05 \text{ m}$$

$$d := h - cc \quad d = 1.45 \text{ m}$$

$$tp := 0.75 \text{ m} \quad b := 13 \text{ m}$$

$$e := tp - cc \quad e = 0.7 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 2.958 \times 10^5 \text{ kNm}$$

Values for balanced reinforcement K500

$$m_{bal} := 0.365$$

$$m_r := \frac{M_s}{b \cdot d^2 \cdot f_{cc}} \quad m_r = 0.513 \quad m_r > m_{bal} \quad \text{NOT OK !}$$

The cross-section will be over reinforced

Try with putting compression reinforcement $d_t := 0.05 \text{ m}$

Assumption: $\sigma_{st} := f_{sd}$

$$A_{st} := \frac{M_s - m_{bal} \cdot b \cdot d^2 \cdot f_{cc}}{(d - d_t) \cdot \sigma_{st}} \quad A_{st} = 0.168 \text{ m}^2$$

$$n_t := \frac{A_{st}}{A_{si}} \quad n_t = 534.375$$

Choose: $n_t := 535$

$$M_{II} := A_{st} \cdot \sigma_{st} \cdot (d - d_t) \quad M_{II} = 8.516 \times 10^4 \text{ kNm}$$

$$M_I := M_s - M_{II} \quad M_I = 2.106 \times 10^5 \text{ kNm}$$

$$m_r := \frac{M_I}{b \cdot d^2 \cdot f_{cc}} \quad m_r = 0.365$$

$$\omega := 1 - \sqrt{(1 - 2 \cdot m_r)} \quad \omega = 0.48$$

$$\varepsilon_{st} := \varepsilon_{cu} \cdot \left(1 - 0.8 \cdot \frac{d_t}{\omega \cdot d} \right)$$

$$\varepsilon_{st} = 3.299 \times 10^{-3} \quad \varepsilon_{st} > \varepsilon_{sy} \quad \text{OK!}$$

$$A_{s1} := \left[\frac{M_I}{d \cdot \left(1 - \frac{\omega}{2} \right)} + \frac{M_{II}}{d - d_t} \right] \cdot \frac{1}{f_{sd}} \quad A_{s1} = 0.695 \text{ m}^2$$

$$A_s := A_{s1} - \frac{N_{sd}}{f_{sd}} \quad A_s = 0.173 \text{ m}^2$$

$$n := \frac{A_s}{A_{si}} \quad n = 551.475$$

Choose: $n := 552$

Calculation for x

$$\text{Assume: } \varepsilon_s, \varepsilon_{s1}, \varepsilon_{s2} > \varepsilon_{sy} \qquad \varepsilon_{st}, \varepsilon_{st1}, \varepsilon_{st2} > \varepsilon_{sy}$$

$$d := 1.44 \text{ m} \qquad n_1 := 215 \qquad A_{s1} := A_{si} \cdot n_1 \qquad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \qquad n_2 := 215 \qquad A_{s2} := A_{si} \cdot n_2 \qquad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \qquad n_3 := 122 \qquad A_{s3} := A_{si} \cdot n_3 \qquad A_{s3} = 0.038 \text{ m}^2$$

$$n_{\text{tot}} := n_1 + n_2 + n_3 \qquad n_{\text{tot}} = 552$$

$$A_{\text{stot}} := A_{s1} + A_{s2} + A_{s3} \qquad A_{\text{stot}} = 0.173 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \qquad n_{t1} := 215 \qquad A_{st1} := A_{si} \cdot n_{t1} \qquad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \qquad n_{t2} := 215 \qquad A_{st2} := A_{si} \cdot n_{t2} \qquad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \qquad n_{t3} := 105 \qquad A_{st3} := A_{si} \cdot n_{t3} \qquad A_{st3} = 0.033 \text{ m}^2$$

$$n_{\text{ttot}} := n_{t1} + n_{t2} + n_{t3} \qquad n_{\text{ttot}} = 535$$

$$A_{\text{sttot}} := n_{\text{ttot}} \cdot A_{si} \qquad A_{\text{sttot}} = 0.168 \text{ m}^2$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{\text{stot}} + f_{sd} \cdot A_{\text{sttot}}, x_1)$$

$$x = 0.794 \text{ m}$$

Check assumption

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \qquad \varepsilon_s = 2.846 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \qquad \text{OK!}$$

$$\varepsilon_{s1} := \frac{d_1-x}{x} \cdot \varepsilon_{cu} \qquad \varepsilon_{s1} = 2.626 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \qquad \text{OK!}$$

$$\varepsilon_{s2} := \frac{d_2-x}{x} \cdot \varepsilon_{cu} \qquad \varepsilon_{s2} = 2.405 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \qquad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.28 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st1} := \frac{x - d_{t1}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st1} = 3.059 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st2} := \frac{x - d_{t2}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st2} = 2.839 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots \\ + [-f_{sd} \cdot A_{s2} \cdot (d - d_1)] - f_{sd} \cdot A_{s3} \cdot (d - d_2)$$

$$M_d = 2.876 \times 10^5 \text{ kNm}$$

$M_d < M_s$ NOT OK!

$$M_s = 2.958 \times 10^5 \text{ kNm}$$

Increase compression steel area!

Assume: $\varepsilon_s, \varepsilon_{s1}, \varepsilon_{s2} > \varepsilon_{sy}$ $\varepsilon_{st}, \varepsilon_{st1}, \varepsilon_{st2} > \varepsilon_{sy}$

$$d := 1.44 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \quad n_3 := 122 \quad A_{s3} := A_{si} \cdot n_3 \quad A_{s3} = 0.038 \text{ m}^2$$

$$n_{tot} := n_1 + n_2 + n_3 \quad n_{tot} = 552$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3} \quad A_{stot} = 0.173 \text{ m}^2$$

$$d_t := 0.05 \text{ m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 215 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.068 \text{ m}^2$$

$$d_{t3} := 0.2 \text{ m} \quad n_{t4} := 100 \quad A_{st4} := A_{si} \cdot n_{t4} \quad A_{st4} = 0.031 \text{ m}^2$$

$$n_{ttot} := n_{t1} + n_{t2} + n_{t3} + n_{t4} \quad n_{ttot} = 745$$

$$A_{sttot} := n_{ttot} \cdot A_{si} \quad A_{sttot} = 0.234 \text{ m}^2$$

Horizontal equilibrium gives:

$$x_1 := 0.0001 \text{ m}$$

$$x := \text{root}(\alpha \cdot f_{cc} \cdot b \cdot x_1 - N_{sd} - f_{sd} \cdot A_{stot} + f_{sd} \cdot A_{sttot}, x_1)$$

$$x = 0.695 \text{ m}$$

Check assumption

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.753 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{s1} := \frac{d_1 - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{s1} = 3.501 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{s2} := \frac{d_2 - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{s2} = 3.249 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.248 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st1} := \frac{x - d_{t1}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st1} = 2.996 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st2} := \frac{x - d_{t2}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st2} = 2.744 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st3} := \frac{x - d_{t3}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st3} = 2.493 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

Check moment capacity: $M_d > M_s$

$$M_d := \alpha \cdot f_{cc} \cdot b \cdot x \cdot (d - \beta \cdot x) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots$$

$$+ f_{sd} \cdot A_{st4} \cdot (d - d_{t3}) - f_{sd} \cdot A_{s2} \cdot (d - d_1) - f_{sd} \cdot A_{s3} \cdot (d - d_2)$$

$$M_d = 2.992 \times 10^5 \text{ kNm} \quad M_d > M_s \quad \text{OK!}$$

$$M_s = 2.958 \times 10^5 \text{ kNm}$$

C3: Compilation of the results for the different cross-sections

Reduced height of the box girder cross-section, variable height

Table 3 *Dimensions of the reduced height arch cross-section.*

Section	h [m]	Bottom slab [m]
1	4	0,40
2	3,58	0,35
3	3,216	0,30
4	3,000	0,25
5	2,872	0,25
6	2,663	0,25
7	2,539	0,25
8	2,500	0,25
9	2,544	0,25
10	2,672	0,25
11	2,885	0,25
12	3,193	0,25
13	3,590	0,30
14	3,984	0,35
15	4,473	0,40

Table 4 *Obtained bending moments, normal forces and amount of the reinforcement bars.*

Section	1	3	5	7	9	11	13	15
M_{sd}	127482	12356	42737	94074	98372	77739	38484	
$-N_{sd}$	146971	129033	131997	127330	131614	135625	143462	
n	-80	-131	-107	-40	-36	-79	-130	

$-M_{sd}$	34398	51923	32669			2843	16912	260578
$-N_{sd}$	142675	139773	129167			132748	144764	147511
n	-134	-112	-112			-141	-145	-20

Table 5 The traffic load coefficient increased to 1.7.

Section	1	3	5	7	9	11	13	15
M_{sd}	136813	15741	47788	101543	105051	83457	40761	
$-N_{sd}$	149047	130149	134361	129426	134166	138187	145862	
n	-74	-129	-105	-30	-28	-76	-131	
$-M_{sd}$	43829	55209	36203			6245	19878	272099
$-N_{sd}$	144245	142392	131174			134934	147418	149930
n	-130	-112	-111			-140	-146	-16

Table 6 The traffic load coefficient increased to 1.9.

Section	1	3	5	7	9	11	13	15
M_{sd}	146144	19127	52839	109011	111731	89176	43037	
$-N_{sd}$	151122	131265	136724	131522	136717	140749	148262	
n	-68	-128	-103	-21	-24	-71	-132	
$-M_{sd}$	53261	58495	39737			9646	22843	283621
$-N_{sd}$	145814	145012	133182			137120	150072	152349
n	-126	-113	-110			-140	-147	-12

Reduced height of the box girder cross-section, constant height

Table 7 Obtained bending moments, normal forces and amount of the reinforcement bars.

Section	1	3	5	7	9	11	13	15
M_{sd}	132370	11558	38125	99224	105740	87871	67392	
$-N_{sd}$	144694	127669	130762	125715	130114	134481	141557	
n	2	-126	-104	-29	-20	-52	-81	
$-M_{sd}$		49885	44074					164450
$-N_{sd}$		138824	128470					145099
n		-100	-95					65

APPENDIX D: NON-LINEAR ANALYSIS

D1: Iteration one, calculations and results from the Strip Step 2

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
 MPa := 10^6 Pa kNm := 1000 N·m

Calculation for the compressive zone

Cross-section : Solid

Iteration number: 1 Reduced reinforcement amount!

Material properties

Concrete C40/50

Partial safety factor

Safety class 3

$$\gamma_n := 1.2$$

$$\eta\gamma_m := 1.5$$

$$f_{cck} := 38 \cdot \text{MPa}$$

$$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$$

$$f_{cc} := \frac{f_{cck}}{(\eta\gamma_m \cdot \gamma_n)}$$

$$f_{cc} = 2.111 \times 10^7 \text{ Pa}$$

Stress block factors

$$\beta := 0.443$$

$$\alpha := 0.877$$

Steel K500 (Kamstång B500B)

Partial safety factor

Safety class 3

$$\eta\gamma_m := 1.15$$

$$\eta\gamma_{mes} := 1.05$$

$$\gamma_n := 1.2$$

$$f_{sk} := 500 \cdot \text{MPa}$$

$$E_{sm} := 200 \cdot \text{GPa}$$

$$f_{sd} := \frac{f_{sk}}{\eta\gamma_m \cdot \gamma_n}$$

$$f_{sd} = 3.623 \times 10^8 \text{ Pa}$$

$$E_s := \frac{E_{sm}}{\eta\gamma_{mes} \cdot \gamma_n}$$

$$E_s = 1.587 \times 10^{11} \text{ Pa}$$

$$\varepsilon_{sy} := \frac{f_{sd}}{E_s}$$

$$\varepsilon_{sy} = 2.283 \times 10^{-3}$$

Steel diameter

$$\phi := 20 \cdot \text{mm}$$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2}\right)^2$$

$$A_{si} = 3.142 \times 10^{-4} \text{ m}^2$$

Section 1

Positive moment

Forces

$$M_{sd} := 125661 \text{ kNm}$$

$$N_{sd} := 188632 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.517 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.933 \text{ m}$$

$$x_1 := 0.000001 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 3.017 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.45 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \quad n_3 := 170 \quad A_{s3} := A_{si} \cdot n_3 \quad A_{s3} = 0.053 \text{ m}^2$$

$$n_{tot} := n_1 + n_2 + n_3 \quad n_{tot} = 600$$

$$A_s := n_{tot} \cdot A_{si} \quad A_s = 0.188 \text{ m}^2$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3} \quad A_{stot} = 0.188 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 215 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.068 \text{ m}^2$$

$$d_{t3} := 0.2 \text{ m} \quad n_{t4} := 215 \quad A_{st4} := A_{si} \cdot n_{t4} \quad A_{st4} = 0.068 \text{ m}^2$$

$$n_{ttot} := n_{t1} + n_{t2} + n_{t3} + n_{t4}$$

$$n_{ttot} = 860$$

$$A_{st} := n_{ttot} \cdot A_{si}$$

$$A_{st} = 0.27m^2$$

$$A_{sttot} := A_{st1} + A_{st2} + A_{st3} + A_{st4}$$

$$A_{sttot} = 0.27m^2$$

Assume: $\varepsilon_s, \varepsilon_{s1}, \varepsilon_{s2}, \varepsilon_{st}, \varepsilon_{st1}, \varepsilon_{st2}, \varepsilon_{st3} > \varepsilon_{sy}$

$$x := \text{root} \left[\begin{array}{l} \alpha \cdot f_{cc} \cdot b \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots, x_1 \\ + f_{sd} \cdot A_{st4} \cdot (d - d_{t3}) - M_s - f_{sd} \cdot A_{s2} \cdot (d - d_1) \dots \\ + [-f_{sd} \cdot A_{s3} \cdot (d - d_2)] \end{array} \right]$$

$$x = 0.621m$$

Check assumptions:

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 4.676 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{s1} := \frac{d_1-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{s1} = 4.338 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{s2} := \frac{d_2-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{s2} = 4.056 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x-d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.218 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st1} := \frac{x-d_{t1}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st1} = 2.936 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st2} := \frac{x-d_{t2}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st2} = 2.654 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st3} := \frac{x-d_{t3}}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st3} = 2.372 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

x = 0.621m

Section 3

Negative moment

Forces

$$M_{sd} := 20204 \text{ kNm}$$

$$N_{sd} := 180533 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.371 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 1.079 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.15 \times 10^5 \text{ kNm}$$

Minimum reinforcement

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n := 64$$

$$A_s := n \cdot A_{si}$$

$$A_s = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02 \text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.777 \text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 2.989 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.275 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.777 \text{ m}}$$

Section 5

Positive moment

Forces

$$M_{sd} := 72044 \text{ kNm}$$

$$N_{sd} := 170373 \text{ kN}$$

Cross-section constants

$$\begin{aligned}h &:= 1.5 \text{ m} & cc &:= 0.05 \text{ m} \\d &:= h - cc & d &= 1.45 \text{ m} \\tp &:= 0.348 \text{ m} & b &:= 13 \text{ m} \\e &:= tp - cc & e &= 0.298 \text{ m} \\M_s &:= M_{sd} + N_{sd} \cdot e & M_s &= 1.228 \times 10^5 \text{ kNm}\end{aligned}$$

Minimum reinforcement

Tension reinforcement

$$d := 1.44 \text{ m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.366 \text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 0.01 > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.022 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.366 \text{ m}}$$

Section 7

Positive moment

Forces

$$M_{sd} := 38888 \text{ kNm} \quad N_{sd} := 163884 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m} \quad cc := 0.05 \text{ m}$$

$$d := h - cc \quad d = 1.45\text{m}$$

$$tp := 0.438\text{m} \quad b := 13\text{m}$$

$$e := d - tp \quad e = 1.012\text{m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 2.047 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.44\text{m} \quad n := 120 \quad A_s := n \cdot A_{si} \quad A_s = 0.038\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.722\text{m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.483 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.258 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

x = 0.722m

Section 9

Positive moment

Forces

$$M_{sd} := 69112\text{kNm}$$

$$N_{sd} := 168287\text{kN}$$

Cross-section constants

$$h := 1.5\text{m} \quad cc := 0.05\text{m}$$

$$d := h - cc \quad d = 1.45\text{m}$$

$$tp := 0.493\text{m} \quad b := 13\text{m}$$

$$d := 1.44\text{ m} \quad n_1 := 190 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.06\text{ m}^2$$

$$n_{\text{tot}} := n_1 \quad n_{\text{tot}} = 190$$

$$A_{\text{stot}} := A_{s1} \quad A_{\text{stot}} = 0.06\text{ m}^2$$

Compression reinforcement

$$d_t := 0.05\text{ m} \quad n_t := 80 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.025\text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.85\text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 2.432 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.294 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

x = 0.85m

Section 11

Positive moment

Forces

$$M_{sd} := 29105\text{ kNm} \quad N_{sd} := 173883\text{ kN}$$

Cross-section constants

$$h := 1.5\text{ m} \quad cc := 0.05\text{ m}$$

$$d := h - cc \quad d = 1.45\text{ m}$$

$$tp := 0.384\text{ m} \quad b := 13\text{ m}$$

$$e := d - tp \quad e = 1.066\text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 2.145 \times 10^5\text{ kNm}$$

$$d_t := 0.05\text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.774\text{m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.014 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.274 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

x = 0.774m

Section 13

Positive moment

Forces

$$M_{sd} := 6045\text{kNm}$$

$$N_{sd} := 182866\text{kN}$$

Cross-section constants

$$h := 1.5\text{m}$$

$$cc := 0.05\text{m}$$

$$d := h - cc$$

$$d = 1.45\text{m}$$

$$tp := 0.368\text{m}$$

$$b := 13\text{m}$$

$$e := d - tp$$

$$e = 1.082\text{m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.039 \times 10^5\text{kNm}$$

Tension reinforcement

$$d := 1.44\text{m}$$

$$n_1 := 64$$

$$A_{s1} := A_{si} \cdot n_1$$

$$A_{s1} = 0.02\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02\text{m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x = 0.717\text{m}$$

Check assumptions:

$$\varepsilon_s := \frac{d-x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.525 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x-d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.256 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.717\text{m}}$$

Section 15

Negative moment

Forces

$$M_{sd} := 108697\text{kNm}$$

$$N_{sd} := 188549\text{kN}$$

Cross-section constants

$$h := 1.5\text{m}$$

$$cc := 0.05\text{m}$$

$$d := h - cc$$

$$d = 1.45\text{m}$$

$$tp := 0.497\text{m}$$

$$b := 13\text{m}$$

$$e := d - tp$$

$$e = 0.953\text{m}$$

$$x_1 := 0.000001\text{m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.884 \times 10^5 \text{kNm}$$

Tension reinforcement

$$d := 1.44\text{m}$$

$$n_1 := 215$$

$$A_{s1} := A_{si} \cdot n_1$$

$$A_{s1} = 0.068\text{m}^2$$

$$d_1 := 1.39\text{m}$$

$$n_2 := 215$$

$$A_{s2} := A_{si} \cdot n_2$$

$$A_{s2} = 0.068\text{m}^2$$

$$d_2 := 1.34\text{m}$$

$$n_3 := 12$$

$$A_{s3} := A_{si} \cdot n_3$$

$$A_{s3} = 3.77 \times 10^{-3} \text{m}^2$$

$$n_{tot} := n_1 + n_2 + n_3$$

$$n_{tot} = 442$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3}$$

$$A_{stot} = 0.139\text{m}^2$$

Compression reinforcement

$$\begin{aligned}
 d_t &:= 0.05\text{-m} & n_{t1} &:= 215 & A_{st1} &:= A_{si} \cdot n_{t1} & A_{st1} &= 0.068\text{m}^2 \\
 d_{t1} &:= 0.1\text{-m} & n_{t2} &:= 215 & A_{st2} &:= A_{si} \cdot n_{t2} & A_{st2} &= 0.068\text{m}^2 \\
 d_{t2} &:= 0.15\text{-m} & n_{t3} &:= 166 & A_{st3} &:= A_{si} \cdot n_{t3} & A_{st3} &= 0.052\text{m}^2 \\
 n_{ttot} &:= n_{t1} + n_{t2} + n_{t3} & & & n_{ttot} &= 596 & & \\
 A_{sttot} &:= n_{ttot} \cdot A_{si} & & & A_{sttot} &= 0.187\text{m}^2 & &
 \end{aligned}$$

Assume: $\epsilon_s, \epsilon_{s1}, \epsilon_{s2}, \epsilon_{st}, \epsilon_{st1}, \epsilon_{st2} > \epsilon_{sy}$

$$x := \text{root} \left[\begin{aligned} & \alpha \cdot f_{cc} \cdot b \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots, x_1 \\ & + \left[-M_s - f_{sd} \cdot A_{s2} \cdot (d - d_1) - f_{sd} \cdot A_{s3} \cdot (d - d_2) \right] \end{aligned} \right]$$

$$x = 0.742\text{m}$$

Check assumptions:

$$\epsilon_s := \frac{d - x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 3.288 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s1} := \frac{d_1 - x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s1} = 3.052 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s2} := \frac{d_2 - x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s2} = 2.817 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st} := \frac{x - d_t}{x} \cdot \epsilon_{cu} \quad \epsilon_{st} = 3.264 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st1} := \frac{x - d_{t1}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st1} = 3.029 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st2} := \frac{x - d_{t2}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st2} = 2.793 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.742\text{m}}$$

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
MPa := 10^6Pa kNm := 1000 N·m

Calculation for cross-sectional constants

Cross-section : Solid

Iteration number: 1

Material properties

Concrete C40/50

Partial safety factor

e-module $\eta\gamma_{\text{mec}} := 1.2$ $\gamma_{\text{n}} := 1.2$

$E_{\text{ck}} := 35 \cdot \text{GPa}$

$$E_{\text{c}} := \frac{E_{\text{ck}}}{\eta\gamma_{\text{mec}} \cdot \gamma_{\text{n}}} \quad E_{\text{c}} = 2.431 \times 10^{10} \text{ Pa}$$

Steel K500 (Kamstång B500B)

Partial safety factor

e-module $\eta\gamma_{\text{mes}} := 1.05$ $\gamma_{\text{n}} := 1.2$

$E_{\text{sk}} := 200 \cdot \text{GPa}$

$$E_{\text{s}} := \frac{E_{\text{sk}}}{\eta\gamma_{\text{mes}} \cdot \gamma_{\text{n}}} \quad E_{\text{s}} = 158.73 \text{ GPa}$$

Condition

Concrete cover : Very aggressive environment, life span L2
cc > 40 mm and we choose 50 mm

Reinforcement spacing:

Parallel: 2ϕ

Vertical : 1.5ϕ



Cross-section constants:

$$h := 1.5\text{ m}$$

$$b := 13\text{ m}$$

$$t_f := 0.12\text{ m}$$

$$\alpha := \frac{E_s}{E_c}$$

$$\alpha = 6.531$$

Steel area

Steel diameter

$$\phi := 20\text{ mm}$$

$$A_{s1} := \pi \cdot \left(\frac{\phi}{2}\right)^2$$

$$A_{s1} = 3.142 \times 10^{-4}\text{ m}^2$$

Long-term effects

creep

Outside structure without heating: Humidity = 75%

$$\psi := 2$$

creep

$$\alpha_{ef} := \alpha \cdot (1 + \psi)$$

$$\alpha_{ef} = 19.592$$

Section 1

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{ m}$$

$$n_1 := 215$$

$$A_{s1} := A_{s1} \cdot n_1$$

$$A_{s1} = 0.068\text{ m}^2$$

$$\begin{aligned}
d_1 &:= 1.39\text{m} & n_2 &:= 215 & A_{s2} &:= A_{si} \cdot n_2 & A_{s2} &= 0.068\text{m}^2 \\
d_2 &:= 1.34\text{m} & n_3 &:= 170 & A_{s3} &:= A_{si} \cdot n_3 & A_{s3} &= 0.053\text{m}^2 \\
n_{\text{tot}} &:= n_1 + n_2 + n_3 & & & & & n_{\text{tot}} &= 600 \\
A_{\text{stot}} &:= A_{s1} + A_{s2} + A_{s3} & & & & & A_{\text{stot}} &= 0.188\text{m}^2
\end{aligned}$$

Compression reinforcement

$$\begin{aligned}
d_t &:= 0.05\text{m} & n_{t1} &:= 215 & A_{st1} &:= A_{si} \cdot n_{t1} & A_{st1} &= 0.068\text{m}^2 \\
d_{t1} &:= 0.1\text{m} & n_{t2} &:= 215 & A_{st2} &:= A_{si} \cdot n_{t2} & A_{st2} &= 0.068\text{m}^2 \\
d_{t2} &:= 0.15\text{m} & n_{t3} &:= 215 & A_{st3} &:= A_{si} \cdot n_{t3} & A_{st3} &= 0.068\text{m}^2 \\
d_{t3} &:= 0.2\text{m} & n_{t4} &:= 215 & A_{st4} &:= A_{si} \cdot n_{t4} & A_{st4} &= 0.068\text{m}^2 \\
n_{\text{ttot}} &:= n_{t1} + n_{t2} + n_{t3} + n_{t4} & & & & & n_{\text{ttot}} &= 860 \\
A_{\text{sttot}} &:= A_{st1} + A_{st2} + A_{st3} + A_{st4} & & & & & A_{\text{sttot}} &= 0.27\text{m}^2
\end{aligned}$$

x-value: $x := 0.621\text{m}$

$$A_{\text{ekv}} := b \cdot x + \alpha_{\text{ef}} \cdot A_{\text{stot}} + (\alpha_{\text{ef}} - 1) \cdot A_{\text{sttot}} \quad A_{\text{ekv}} = 16.789\text{m}^2$$

$$x_s := \alpha_{\text{ef}} \cdot (A_{s1} \cdot d + A_{s2} \cdot d_1 + A_{s3} \cdot d_2)$$

$$x_{\text{st}} := (\alpha_{\text{ef}} - 1) \cdot (A_{st1} \cdot d_t + A_{st2} \cdot d_{t1} + A_{st3} \cdot d_{t2} + A_{st4} \cdot d_{t3})$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{\text{tp}} := \frac{x_c + x_s + x_{\text{st}}}{A_{\text{ekv}}} \quad x_{\text{tp}} = 0.493\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{\text{tp}} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_{s1} \cdot (d - x_{tp})^2 + A_{s2} \cdot (d_1 - x_{tp})^2 \dots \right. \\ \left. + A_{s3} \cdot (d_2 - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st1} \cdot (x_{tp} - d_t)^2 + A_{st2} \cdot (x_{tp} - d_{t1})^2 \dots \right. \\ \left. + A_{st3} \cdot (x_{tp} - d_{t2})^2 + A_{st4} \cdot (x_{tp} - d_{t3})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 4.226 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 3.424 \times 10^{10} \text{m}^2 \text{N}$$

Section 3

Negative moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.777 \text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 10.869 \text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.415 \text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 0.979 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 7.931 \times 10^9 \text{ m}^2 \text{ N}$$

Section 5

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{ m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

x-value: $x := 0.366 \text{ m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 5.526 \text{ m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.264 \text{ m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 0.646 \text{ m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 5.236 \times 10^9 \text{ m}^2 \text{ N}$$

Section 7

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{ m} \quad n := 120 \quad A_s := A_{si} \cdot n \quad A_s = 0.038 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.722 \text{ m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 10.498 \text{ m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.426 \text{ m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 1.26 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 1.021 \times 10^{10} \text{m}^2$$

Section 9

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{m} \quad n := 190 \quad A_s := A_{si} \cdot n \quad A_s = 0.06 \text{m}^2$$

Compression reinforcement:

$$d_t := 0.05 \text{m} \quad n_t := 80 \quad A_{st} := A_{si} \cdot n_t \quad A_{st} = 0.025 \text{m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.85 \text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 12.687 \text{m}^2$$

$$x_s := \alpha_{ef} \cdot (A_s \cdot d)$$

$$x_{st} := (\alpha_{ef} - 1) \cdot (A_{st} \cdot d_t)$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.505 \text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \left[A_{st} \cdot (x_{tp} - d_t)^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 1.855 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 1.503 \times 10^{10} \text{m}^2 \text{N}$$

Section 11

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{m} \quad n := 64 \quad A_s := A_{si} \cdot n \quad A_s = 0.02 \text{m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{m}^2$$

x-value: $x := 0.774 \text{m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 10.83 \text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.414\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 0.974\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 7.89 \times 10^9 \text{ m}^2 \text{ N}$$

Section 13

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m} \quad n := 64 \quad A_s := A_{si} \cdot n \quad A_s = 0.02\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

x-value: $x := 0.717\text{m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 10.089\text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.389\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 0.886\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 7.179 \times 10^9 \text{ m}^2 \text{ N}$$

Section 15

Negative moment

Cross-section constants

Tension reinforcement

$d := 1.44\text{m}$	$n_1 := 215$	$A_{s1} := A_{si} \cdot n_1$	$A_{s1} = 0.068\text{m}^2$
$d_1 := 1.39\text{m}$	$n_2 := 215$	$A_{s2} := A_{si} \cdot n_2$	$A_{s2} = 0.068\text{m}^2$
$d_2 := 1.34\text{m}$	$n_3 := 12$	$A_{s3} := A_{si} \cdot n_3$	$A_{s3} = 3.77 \times 10^{-3} \text{ m}^2$
$n_{tot} := n_1 + n_2 + n_3$			$n_{tot} = 442$
$A_{stot} := A_{s1} + A_{s2} + A_{s3}$			$A_{stot} = 0.139\text{m}^2$

Compression reinforcement

$d_t := 0.05\text{m}$	$n_{t1} := 215$	$A_{st1} := A_{si} \cdot n_{t1}$	$A_{st1} = 0.068\text{m}^2$
$d_{t1} := 0.1\text{m}$	$n_{t2} := 215$	$A_{st2} := A_{si} \cdot n_{t2}$	$A_{st2} = 0.068\text{m}^2$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 166 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.052 \text{ m}^2$$

$$n_{ttot} := n_{t1} + n_{t2} + n_{t3} \quad n_{ttot} = 596$$

$$A_{sttot} := A_{st1} + A_{st2} + A_{st3} \quad A_{sttot} = 0.187 \text{ m}^2$$

x-value: $x := 0.742 \text{ m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_{stot} + (\alpha_{ef} - 1) \cdot A_{sttot} \quad A_{ekv} = 15.848 \text{ m}^2$$

$$x_s := \alpha_{ef} \cdot (A_{s1} \cdot d + A_{s2} \cdot d_1 + A_{s3} \cdot d_2)$$

$$x_{st} := (\alpha_{ef} - 1) \cdot (A_{st1} \cdot d_t + A_{st2} \cdot d_{t1} + A_{st3} \cdot d_{t2})$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.489 \text{ m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_{s1} \cdot (d - x_{tp})^2 + A_{s2} \cdot (d_1 - x_{tp})^2 \dots \right. \\ \left. + A_{s3} \cdot (d_2 - x_{tp})^2 \right]$$

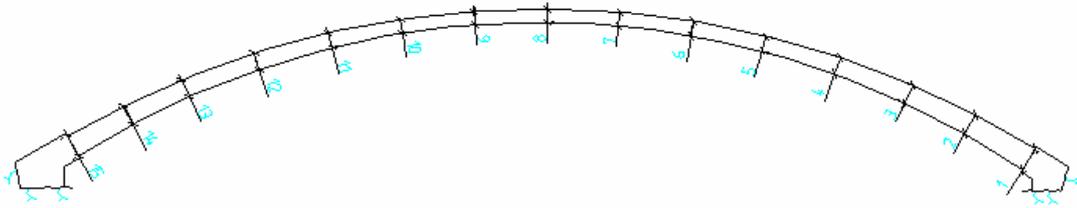
$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st1} \cdot (x_{tp} - d_t)^2 + A_{st2} \cdot (x_{tp} - d_{t1})^2 \dots \right. \\ \left. + A_{st3} \cdot (x_{tp} - d_{t2})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 3.445 \text{ m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 2.791 \times 10^{10} \text{ m}^2 \text{ N}$$

Results from Strip Step

Iteration: 2



Load case 3

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	82334	-172206	701	-183760
3	48753	-168407	18449	-159804
5	-76023	-151563	-80895	-159961
7	8775	-147686	4553	-155961
9	39792	-148484	33704	-156945
11	2054	-153466	-5398	-162217
13	-14696	-173286	-20441	-163945
15	-46004	-179148	-61078	-169812

Load case 5

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	84138	-192530	76401	-171986
3	22115	-160036	3509	-179650
5	-45979	-169498	-79957	-151734
7	8775	-147686	-6224	-165847
9	51496	-166741	36173	-148735
11	2054	-153466	-13001	-172633
13	-16888	-164252	-27944	-183289
15	-7152	-189401	-61078	-169812

Load case 7

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	82334	-172206	49524	-190672
3	28152	-176418	18449	-159804
5	-76023	-151563	-92252	-167900
7	65890	-163060	5218	-147854
9	39792	-148484	24493	-164468
11	2054	-153466	-31864	-169798
13	-16888	-164252	-21732	-180405
15	4657	-186507	-61078	-169812

Load case 9

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	103855	-193042	76401	-171986
3	22115	-160036	16578	-179661
5	-76023	-151563	-81348	-170419
7	8775	-147686	-5135	-166433
9	92336	-167451	36173	-148735
11	2054	-153466	-20417	-173150
13	-16888	-164252	-31512	-183927
15	-13564	-190043	-61078	-169812

Load case 11

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	146121	-191627	76401	-171986
3	22115	-160036	16779	-178981
5	-76023	-151563	-100223	-170278
7	19811	-165734	5218	-147854
9	39792	-148484	28549	-166843
11	43714	-173053	-2183	-153718
13	-16888	-164252	-38436	-184193
15	-54744	-170115	-71402	-190066

Load case 13

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	147185	-187765	76401	-171986
3	27242	-175261	18449	-159804
5	-76023	-151563	-111469	-166763
7	10041	-162341	5218	-147854
9	74828	-163391	36173	-148735
11	2705	-168654	-2183	-153718
13	1246	-182199	-20441	-163945
15	-54744	-170115	-134550	-187898

Maximum values

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	147185	-187765		
3	48753	-168407		
5			-111469	-166763
7	65890	-163060	-6224	-165847
9	92336	-167451		
11	43714	-173053	-31864	-169798
13	1246	-182199	-38436	-184193
15	4657	-186507	-134550	-187898

D2: Iteration five, calculations and results from the Strip Step 2

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
 MPa := 10^6 Pa kNm := 1000 N·m

Calculation for the compressive zone

Cross-section : Solid

Iteration number: 5

Material properties

Concrete C40/50

Partial safety factor

Safety class 3

$$\gamma_n := 1.2$$

$$\eta\gamma_m := 1.5$$

$$f_{cck} := 38 \text{ MPa}$$

$$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$$

$$f_{cc} := \frac{f_{cck}}{(\eta\gamma_m \cdot \gamma_n)}$$

$$f_{cc} = 2.111 \times 10^7 \text{ Pa}$$

Stress block factors

$$\beta := 0.443$$

$$\alpha := 0.877$$

Steel K500 (Kamstång B500B)

Partial safety factor

Safety class 3

$$\eta\gamma_m := 1.15$$

$$\eta\gamma_{mes} := 1.05$$

$$\gamma_n := 1.2$$

$$f_{sk} := 500 \text{ MPa}$$

$$E_{sm} := 200 \text{ GPa}$$

$$f_{sd} := \frac{f_{sk}}{\eta\gamma_m \cdot \gamma_n}$$

$$f_{sd} = 3.623 \times 10^8 \text{ Pa}$$

$$E_s := \frac{E_{sm}}{\eta\gamma_{mes} \cdot \gamma_n}$$

$$E_s = 1.587 \times 10^{11} \text{ Pa}$$

$$\varepsilon_{sy} := \frac{f_{sd}}{E_s}$$

$$\varepsilon_{sy} = 2.283 \times 10^{-3}$$

Steel diameter

$$\phi := 20 \text{ mm}$$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2}\right)^2$$

$$A_{si} = 3.142 \times 10^{-4} \text{ m}^2$$

Section 1

Positive moment

Forces

$$M_{sd} := 215925 \text{ kNm}$$

$$N_{sd} := 194056 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.499 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.951 \text{ m}$$

$$x_1 := 0.000001 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 4.005 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.45 \text{ m} \quad n_1 := 215 \quad A_{s1} := A_{si} \cdot n_1 \quad A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \quad n_3 := 170 \quad A_{s3} := A_{si} \cdot n_3 \quad A_{s3} = 0.053 \text{ m}^2$$

$$n_{tot} := n_1 + n_2 + n_3 \quad n_{tot} = 600$$

$$A_s := n_{tot} \cdot A_{si} \quad A_s = 0.188 \text{ m}^2$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3} \quad A_{stot} = 0.188 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 215 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.068 \text{ m}^2$$

$$d_{t3} := 0.2 \text{ m} \quad n_{t4} := 215 \quad A_{st4} := A_{si} \cdot n_{t4} \quad A_{st4} = 0.068 \text{ m}^2$$

$$n_{\text{ttot}} := n_{t1} + n_{t2} + n_{t3} + n_{t4}$$

$$n_{\text{ttot}} = 860$$

$$A_{\text{st}} := n_{\text{ttot}} \cdot A_{\text{si}}$$

$$A_{\text{st}} = 0.27\text{m}^2$$

$$A_{\text{sttot}} := A_{\text{st1}} + A_{\text{st2}} + A_{\text{st3}} + A_{\text{st4}}$$

$$A_{\text{sttot}} = 0.27\text{m}^2$$

Assume: $\epsilon_s, \epsilon_{s1}, \epsilon_{s2} < \epsilon_{\text{sy}}$ $\epsilon_{\text{st}}, \epsilon_{\text{st1}}, \epsilon_{\text{st2}}, \epsilon_{\text{st3}} > \epsilon_{\text{sy}}$

$$x := \text{root} \left[\begin{array}{l} \alpha \cdot f_{\text{cc}} \cdot b \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{\text{sd}} \cdot A_{\text{st1}} \cdot (d - d_t) + f_{\text{sd}} \cdot A_{\text{st2}} \cdot (d - d_{t1}) + f_{\text{sd}} \cdot A_{\text{st3}} \cdot (d - d_{t2}) \dots, x_1 \\ + f_{\text{sd}} \cdot A_{\text{st4}} \cdot (d - d_{t3}) - M_s - E_s \cdot \epsilon_{\text{cu}} \cdot \left(\frac{d_1 - x_1}{x_1} \right) \cdot A_{\text{s2}} \cdot (d - d_1) \dots \\ + \left[-E_s \cdot \epsilon_{\text{cu}} \cdot \left(\frac{d_2 - x_1}{x_1} \right) \cdot A_{\text{s3}} \cdot (d - d_2) \right] \end{array} \right]$$

$$x = 1.269\text{m}$$

Check assumptions:

$$\epsilon_s := \frac{d - x}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_s = 4.995 \times 10^{-4} < \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s1} := \frac{d_1 - x}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{s1} = 3.34 \times 10^{-4} < \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s2} := \frac{d_2 - x}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{s2} = 1.961 \times 10^{-4} < \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{\text{st}} := \frac{x - d_t}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{\text{st}} = 3.362 \times 10^{-3} > \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{\text{st1}} := \frac{x - d_{t1}}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{\text{st1}} = 3.224 \times 10^{-3} > \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{\text{st2}} := \frac{x - d_{t2}}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{\text{st2}} = 3.086 \times 10^{-3} > \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{\text{st3}} := \frac{x - d_{t3}}{x} \cdot \epsilon_{\text{cu}} \quad \epsilon_{\text{st3}} = 2.948 \times 10^{-3} > \epsilon_{\text{sy}} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 1.269\text{m}}$$

$$M_{sd} := 110025 \text{ kNm}$$

$$N_{sd} := 182336 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.248 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := tp - cc$$

$$e = 0.198 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 1.461 \times 10^5 \text{ kNm}$$

Minimum reinforcement

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n := 64$$

$$A_s := n \cdot A_{si} \quad A_s = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.457 \text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 7.54 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.117 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.457 \text{ m}}$$

Section 5

Positive moment

Forces

$$M_{sd} := 104537 \text{ kNm}$$

$$N_{sd} := 172324 \text{ kN}$$

Cross-section constants

$$\begin{aligned}h &:= 1.5\text{ m} & cc &:= 0.05\text{ m} \\d &:= h - cc & d &= 1.45\text{ m} \\tp &:= 0.297\text{ m} & b &:= 13\text{ m} \\e &:= tp - cc & e &= 0.247\text{ m} \\M_s &:= M_{sd} + N_{sd} \cdot e & M_s &= 1.471 \times 10^5 \text{ kNm}\end{aligned}$$

Minimum reinforcement

Tension reinforcement

$$d := 1.44\text{ m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02\text{ m}^2$$

Compression reinforcement

$$d_t := 0.05\text{ m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root}\left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1\right]$$

$$x = 0.46\text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 7.446 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.12 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.46\text{ m}}$$

Section 7

Positive moment

Forces

$$M_{sd} := 46474\text{ kNm} \quad N_{sd} := 165890\text{ kN}$$

$$h := 1.5 \cdot \text{m} \qquad \text{cc} := 0.05 \cdot \text{m}$$

$$d := h - \text{cc} \qquad d = 1.45 \text{m}$$

$$\text{tp} := 0.486 \text{m} \qquad b := 13 \cdot \text{m}$$

$$e := d - \text{tp} \qquad e = 0.964 \text{m}$$

$$M_s := M_{sd} + N_{sd} \cdot e \qquad M_s = 2.064 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.44 \text{m} \qquad n := 120 \qquad A_s := n \cdot A_{si} \qquad A_s = 0.038 \text{m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{m} \qquad n_t := 64 \qquad A_{st} := n_t \cdot A_{si} \qquad A_{st} = 0.02 \text{m}^2$$

Assume: $\epsilon_{st} > \epsilon_{sy}$ $\epsilon_s < \epsilon_{sy}$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.73 \text{m}$$

Check assumptions:

$$\epsilon_s := \frac{d - x}{x} \cdot \epsilon_{cu} \qquad \epsilon_s = 3.4 \times 10^{-3} < \epsilon_{sy} = 2.283 \times 10^{-3} \qquad \text{OK!}$$

$$\epsilon_{st} := \frac{x - d_t}{x} \cdot \epsilon_{cu} \qquad \epsilon_{st} = 3.26 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \qquad \text{OK!}$$

x = 0.73m

Section 9

Positive moment

Forces

$$M_{sd} := 68323 \text{ kNm}$$

$$N_{sd} := 170349 \text{ kN}$$

Cross-section constants

$$\begin{aligned} h &:= 1.5\text{-m} & cc &:= 0.05\text{ m} \\ d &:= h - cc & d &= 1.45\text{m} \\ tp &:= 0.557\text{-m} & b &:= 13\text{-m} \\ e &:= d - tp & e &= 0.893\text{m} \end{aligned}$$

$$M_s := M_{sd} + N_{sd} \cdot e \quad M_s = 2.204 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$\begin{aligned} d &:= 1.44\text{ m} & n_1 &:= 190 & A_{s1} &:= A_{si} \cdot n_1 & A_{s1} &= 0.06\text{m}^2 \\ n_{\text{tot}} &:= n_1 & n_{\text{tot}} &= 190 \\ A_{\text{stot}} &:= A_{s1} & A_{\text{stot}} &= 0.06\text{m}^2 \end{aligned}$$

Compression reinforcement

$$d_t := 0.05\text{ m} \quad n_t := 80 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.025\text{m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.793\text{m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 2.856 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.279 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.793\text{m}}$$

Section 11

Positive moment

Forces

$$M_{sd} := 26814 \text{ kNm}$$

$$N_{sd} := 175882 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.436 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 1.014 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.052 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n_1 := 64$$

$$A_{s1} := A_{si} \cdot n_1$$

$$A_{s1} = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02 \text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.724 \text{ m}$$

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_s = 3.462 \times 10^{-3}$$

>

$$\varepsilon_{sy} = 2.283 \times 10^{-3}$$

OK!

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu}$$

$$\varepsilon_{st} = 3.258 \times 10^{-3}$$

>

$$\varepsilon_{sy} = 2.283 \times 10^{-3}$$

OK!

$$\mathbf{x = 0.724 \text{ m}}$$

Section 13

Positive moment

Forces

$$M_{sd} := 5464 \text{ kNm}$$

$$N_{sd} := 184788 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.371 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 1.079 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.049 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n_1 := 64$$

$$A_{s1} := A_{si} \cdot n_1$$

$$A_{s1} = 0.02 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02 \text{ m}^2$$

Assume: $\varepsilon_s, \varepsilon_{st} > \varepsilon_{sy}$

$$x := \text{root} \left[f_{cc} \cdot b \cdot \alpha \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st} \cdot (d - d_t) - M_s, x_1 \right]$$

$$x = 0.722 \text{ m}$$

Check assumptions:

$$\varepsilon_s := \frac{d - x}{x} \cdot \varepsilon_{cu} \quad \varepsilon_s = 3.477 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\varepsilon_{st} := \frac{x - d_t}{x} \cdot \varepsilon_{cu} \quad \varepsilon_{st} = 3.258 \times 10^{-3} > \varepsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\mathbf{x = 0.722 \text{ m}}$$

Section 15

Negative moment

Forces

$$M_{sd} := 97735 \text{ kNm}$$

$$N_{sd} := 190459 \text{ kN}$$

Cross-section constants

$$h := 1.5 \text{ m}$$

$$cc := 0.05 \text{ m}$$

$$d := h - cc$$

$$d = 1.45 \text{ m}$$

$$tp := 0.528 \text{ m}$$

$$b := 13 \text{ m}$$

$$e := d - tp$$

$$e = 0.922 \text{ m}$$

$$M_s := M_{sd} + N_{sd} \cdot e$$

$$M_s = 2.733 \times 10^5 \text{ kNm}$$

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n_1 := 215$$

$$A_{s1} := A_{si} \cdot n_1$$

$$A_{s1} = 0.068 \text{ m}^2$$

$$d_1 := 1.39 \text{ m}$$

$$n_2 := 215$$

$$A_{s2} := A_{si} \cdot n_2$$

$$A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m}$$

$$n_3 := 12$$

$$A_{s3} := A_{si} \cdot n_3$$

$$A_{s3} = 3.77 \times 10^{-3} \text{ m}^2$$

$$n_{tot} := n_1 + n_2 + n_3$$

$$n_{tot} = 442$$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3}$$

$$A_{stot} = 0.139 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_{t1} := 215$$

$$A_{st1} := A_{si} \cdot n_{t1}$$

$$A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m}$$

$$n_{t2} := 215$$

$$A_{st2} := A_{si} \cdot n_{t2}$$

$$A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m}$$

$$n_{t3} := 166$$

$$A_{st3} := A_{si} \cdot n_{t3}$$

$$A_{st3} = 0.052 \text{ m}^2$$

$$n_{ttot} := n_{t1} + n_{t2} + n_{t3}$$

$$n_{ttot} = 596$$

$$A_{sttot} := n_{ttot} \cdot A_{si}$$

$$A_{sttot} = 0.187 \text{ m}^2$$

Assume: $\epsilon_s, \epsilon_{s1}, \epsilon_{s2}, \epsilon_{st}, \epsilon_{st1}, \epsilon_{st2} > \epsilon_{sy}$

$$x := \text{root} \left[\begin{array}{l} \alpha \cdot f_{cc} \cdot b \cdot x_1 \cdot (d - \beta \cdot x_1) + f_{sd} \cdot A_{st1} \cdot (d - d_t) + f_{sd} \cdot A_{st2} \cdot (d - d_{t1}) + f_{sd} \cdot A_{st3} \cdot (d - d_{t2}) \dots, x_1 \\ + \left[-M_s - f_{sd} \cdot A_{s2} \cdot (d - d_1) - f_{sd} \cdot A_{s3} \cdot (d - d_2) \right] \end{array} \right]$$

$$x = 0.666\text{m}$$

Check assumptions:

$$\epsilon_s := \frac{d - x}{x} \cdot \epsilon_{cu} \quad \epsilon_s = 4.069 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s1} := \frac{d_1 - x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s1} = 3.806 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{s2} := \frac{d_2 - x}{x} \cdot \epsilon_{cu} \quad \epsilon_{s2} = 3.543 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st} := \frac{x - d_t}{x} \cdot \epsilon_{cu} \quad \epsilon_{st} = 3.237 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st1} := \frac{x - d_{t1}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st1} = 2.974 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

$$\epsilon_{st2} := \frac{x - d_{t2}}{x} \cdot \epsilon_{cu} \quad \epsilon_{st2} = 2.712 \times 10^{-3} > \epsilon_{sy} = 2.283 \times 10^{-3} \quad \text{OK!}$$

x = 0.666m

N := newton kN := 1000 N GPa := $10^9 \cdot \text{Pa}$
MPa := 10^6Pa kNm := 1000 N·m

Calculation for cross-sectional constants

Cross-section : Solid

Iteration number: 2

Material properties

Concrete C40/50

Partial safety factor

e-module $\eta\gamma_{\text{mec}} := 1.2$ $\gamma_{\text{n}} := 1.2$

$E_{\text{ck}} := 35 \cdot \text{GPa}$

$$E_{\text{c}} := \frac{E_{\text{ck}}}{\eta\gamma_{\text{mec}} \cdot \gamma_{\text{n}}} \quad E_{\text{c}} = 2.431 \times 10^{10} \text{ Pa}$$

Steel K500 (Kamstång B500B)

Partial safety factor

e-module $\eta\gamma_{\text{mes}} := 1.05$ $\gamma_{\text{n}} := 1.2$

$E_{\text{sk}} := 200 \cdot \text{GPa}$

$$E_{\text{s}} := \frac{E_{\text{sk}}}{\eta\gamma_{\text{mes}} \cdot \gamma_{\text{n}}} \quad E_{\text{s}} = 158.73 \text{ GPa}$$

Condition

Concrete cover : Very aggressive environment, life span L2

cc > 40 mm and we choose 50 mm

Reinforcement spacing:

Parallel: 2ϕ

Vertical : 1.5ϕ



Cross-section constants:

$$h := 1.5\text{-m} \qquad b := 13\text{-m}$$

$$t_p := 0.75\text{m}$$

$$\alpha := \frac{E_s}{E_c} \qquad \alpha = 6.531$$

Steel area

Steel diameter $\phi := 20\text{-mm}$

$$A_{si} := \pi \cdot \left(\frac{\phi}{2}\right)^2 \qquad A_{si} = 3.142 \times 10^{-4} \text{m}^2$$

Long-term effects

creep

Outside structure without heating: Humidity = 75%

$$\psi := 2$$

$$\text{creep} \qquad \alpha_{ef} := \alpha \cdot (1 + \psi) \qquad \alpha_{ef} = 19.592$$

Section 1

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m} \qquad n_1 := 215 \qquad A_{s1} := A_{si} \cdot n_1 \qquad A_{s1} = 0.068\text{m}^2$$

$$d_1 := 1.39 \text{ m} \quad n_2 := 215 \quad A_{s2} := A_{si} \cdot n_2 \quad A_{s2} = 0.068 \text{ m}^2$$

$$d_2 := 1.34 \text{ m} \quad n_3 := 170 \quad A_{s3} := A_{si} \cdot n_3 \quad A_{s3} = 0.053 \text{ m}^2$$

$$n_{\text{tot}} := n_1 + n_2 + n_3 \quad n_{\text{tot}} = 600$$

$$A_{\text{stot}} := A_{s1} + A_{s2} + A_{s3} \quad A_{\text{stot}} = 0.188 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068 \text{ m}^2$$

$$d_{t1} := 0.1 \text{ m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068 \text{ m}^2$$

$$d_{t2} := 0.15 \text{ m} \quad n_{t3} := 215 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.068 \text{ m}^2$$

$$d_{t3} := 0.2 \text{ m} \quad n_{t4} := 215 \quad A_{st4} := A_{si} \cdot n_{t4} \quad A_{st4} = 0.068 \text{ m}^2$$

$$n_{\text{ttot}} := n_{t1} + n_{t2} + n_{t3} + n_{t4} \quad n_{\text{ttot}} = 860$$

$$A_{\text{sttot}} := A_{st1} + A_{st2} + A_{st3} + A_{st4} \quad A_{\text{sttot}} = 0.27 \text{ m}^2$$

x-value: $x := 0.745 \text{ m}$

$$A_{\text{ekv}} := b \cdot x + \alpha_{\text{ef}} \cdot A_{\text{stot}} + (\alpha_{\text{ef}} - 1) \cdot A_{\text{sttot}} \quad A_{\text{ekv}} = 18.401 \text{ m}^2$$

$$x_s := \alpha_{\text{ef}} \cdot (A_{s1} \cdot d + A_{s2} \cdot d_1 + A_{s3} \cdot d_2)$$

$$x_{\text{st}} := (\alpha_{\text{ef}} - 1) \cdot (A_{st1} \cdot d_t + A_{st2} \cdot d_{t1} + A_{st3} \cdot d_{t2} + A_{st4} \cdot d_{t3})$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{\text{tp}} := \frac{x_c + x_s + x_{\text{st}}}{A_{\text{ekv}}} \quad x_{\text{tp}} = 0.51 \text{ m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{\text{tp}} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_{s1} \cdot (d - x_{tp})^2 + A_{s2} \cdot (d_1 - x_{tp})^2 \dots \right. \\ \left. + A_{s3} \cdot (d_2 - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st1} \cdot (x_{tp} - d_t)^2 + A_{st2} \cdot (x_{tp} - d_{t1})^2 \dots \right. \\ \left. + A_{st3} \cdot (x_{tp} - d_{t2})^2 + A_{st4} \cdot (x_{tp} - d_{t3})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 4.281 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 3.469 \times 10^{10} \text{m}^2 \text{N}$$

Section 3

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.32 \text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 4.928 \text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.254 \text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 0.642 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 5.201 \times 10^9 \text{m}^2 \text{N}$$

Section 5

Negative moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{m} \quad n := 64 \quad A_s := n \cdot A_{si} \quad A_s = 0.02 \text{m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02 \text{m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.461 \text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 6.761 \text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.291 \text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 0.67 \text{ m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 5.427 \times 10^9 \text{ m}^2 \text{ N}$$

Section 7

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44 \text{ m}$$

$$n := 120$$

$$A_s := A_{si} \cdot n$$

$$A_s = 0.038 \text{ m}^2$$

Compression reinforcement

$$d_t := 0.05 \text{ m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02 \text{ m}^2$$

x-value:

$$x := 0.882 \text{ m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st}$$

$$A_{ekv} = 12.578 \text{ m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.488\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 1.51\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 1.223 \times 10^{10} \text{m}^2 \text{N}$$

Section 9

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m} \quad n := 190 \quad A_s := A_{si} \cdot n \quad A_s = 0.06\text{m}^2$$

Compression reinforcement:

$$d_t := 0.05\text{m} \quad n_t := 80 \quad A_{st} := A_{si} \cdot n_t \quad A_{st} = 0.025\text{m}^2$$

$$\mathbf{x\text{-value:}} \quad x := 0.985\text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 14.442\text{m}^2$$

$$x_s := \alpha_{ef} \cdot (A_s \cdot d)$$

$$x_{st} := (\alpha_{ef} - 1) \cdot (A_{st} \cdot d_t)$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}}$$

$$x_{tp} = 0.555\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (x_{tp} - d_t)^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

$$I_{ekv} = 2.12\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 1.718 \times 10^{10} \text{ m}^2 \cdot \text{N}$$

Section 11

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m}$$

$$n := 64$$

$$A_s := A_{si} \cdot n$$

$$A_s = 0.02\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m}$$

$$n_t := 64$$

$$A_{st} := n_t \cdot A_{si}$$

$$A_{st} = 0.02\text{m}^2$$

x-value:

$$x := 0.822\text{m}$$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st}$$

$$A_{ekv} = 11.454\text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot (A_{st} \cdot d_t)$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.555\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (x_{tp} - d_t)^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 2.12\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 1.718 \times 10^{10} \text{m}^2 \text{N}$$

Section 11

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m} \quad n := 64 \quad A_s := A_{si} \cdot n \quad A_s = 0.02\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

x-value: $x := 0.822\text{m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 11.454\text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.435\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 1.061\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 8.597 \times 10^9 \text{ m}^2 \text{ N}$$

Section 13

Positive moment

Cross-section constants

Tension reinforcement

$$d := 1.44\text{m} \quad n := 64 \quad A_s := A_{si} \cdot n \quad A_s = 0.02\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_t := 64 \quad A_{st} := n_t \cdot A_{si} \quad A_{st} = 0.02\text{m}^2$$

x-value: $x := 0.67\text{m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_s + (\alpha_{ef} - 1) \cdot A_{st} \quad A_{ekv} = 9.478\text{m}^2$$

$$x_s := \alpha_{ef} \cdot A_s \cdot d$$

$$x_{st} := (\alpha_{ef} - 1) \cdot A_{st} \cdot d_t$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.37\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_s \cdot (d - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st} \cdot (d_t - x_{tp})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st} \quad I_{ekv} = 0.826\text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv} \quad EI_{ekv} = 6.69 \times 10^9 \text{ m}^2 \text{ N}$$

Section 15

Negative moment

Cross-section constants

Tension reinforcement

$d := 1.44\text{m}$	$n_1 := 215$	$A_{s1} := A_{si} \cdot n_1$	$A_{s1} = 0.068\text{m}^2$
$d_1 := 1.39\text{m}$	$n_2 := 215$	$A_{s2} := A_{si} \cdot n_2$	$A_{s2} = 0.068\text{m}^2$
$d_2 := 1.34\text{m}$	$n_3 := 12$	$A_{s3} := A_{si} \cdot n_3$	$A_{s3} = 3.77 \times 10^{-3} \text{ m}^2$
$n_{tot} := n_1 + n_2 + n_3$			$n_{tot} = 442$

$$A_{stot} := A_{s1} + A_{s2} + A_{s3} \quad A_{stot} = 0.139\text{m}^2$$

Compression reinforcement

$$d_t := 0.05\text{m} \quad n_{t1} := 215 \quad A_{st1} := A_{si} \cdot n_{t1} \quad A_{st1} = 0.068\text{m}^2$$

$$d_{t1} := 0.1\text{m} \quad n_{t2} := 215 \quad A_{st2} := A_{si} \cdot n_{t2} \quad A_{st2} = 0.068\text{m}^2$$

$$d_{t2} := 0.15\text{m} \quad n_{t3} := 166 \quad A_{st3} := A_{si} \cdot n_{t3} \quad A_{st3} = 0.052\text{m}^2$$

$$n_{ttot} := n_{t1} + n_{t2} + n_{t3} \quad n_{ttot} = 596$$

$$A_{sttot} := A_{st1} + A_{st2} + A_{st3} \quad A_{sttot} = 0.187\text{m}^2$$

x-value: $x := 0.897\text{m}$

$$A_{ekv} := b \cdot x + \alpha_{ef} \cdot A_{stot} + (\alpha_{ef} - 1) \cdot A_{sttot} \quad A_{ekv} = 17.863\text{m}^2$$

$$x_s := \alpha_{ef} \cdot (A_{s1} \cdot d + A_{s2} \cdot d_1 + A_{s3} \cdot d_2)$$

$$x_{st} := (\alpha_{ef} - 1) \cdot (A_{st1} \cdot d_t + A_{st2} \cdot d_{t1} + A_{st3} \cdot d_{t2})$$

$$x_c := \frac{b \cdot x^2}{2}$$

$$x_{tp} := \frac{x_c + x_s + x_{st}}{A_{ekv}} \quad x_{tp} = 0.527\text{m}$$

$$I_c := \frac{b \cdot x^3}{12} + b \cdot x \cdot \left(\frac{x}{2} - x_{tp} \right)^2$$

$$I_s := \alpha_{ef} \cdot \left[A_{s1} \cdot (d - x_{tp})^2 + A_{s2} \cdot (d_1 - x_{tp})^2 \dots \right. \\ \left. + A_{s3} \cdot (d_2 - x_{tp})^2 \right]$$

$$I_{st} := (\alpha_{ef} - 1) \cdot \left[A_{st1} \cdot (x_{tp} - d_t)^2 + A_{st2} \cdot (x_{tp} - d_{t1})^2 \dots \right. \\ \left. + A_{st3} \cdot (x_{tp} - d_{t2})^2 \right]$$

$$I_{ekv} := I_c + I_s + I_{st}$$

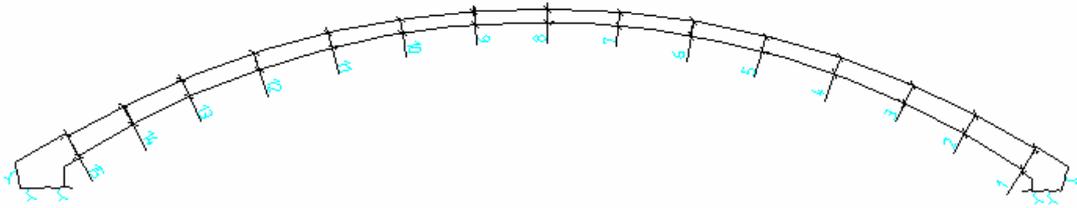
$$I_{ekv} = 3.644 \text{m}^4$$

$$EI_{ekv} := \frac{E_c}{1 + \psi} \cdot I_{ekv}$$

$$EI_{ekv} = 2.952 \times 10^{10} \text{m}^2 \text{N}$$

Results from Strip Step

Iteration: 6



Load case 3

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	90860	-171989	4078	-183415
3	54396	-168008	27292	-159551
5	-77099	-151280	-83015	-159525
7	8113	-147383	4230	-155497
9	38151	-148174	33004	-156467
11	915	-153155	-5739	-161739
13	-16550	-172817	-21976	-163642
15	-46058	-178684	-59391	-169512

Load case 5

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	95139	-192299	85067	-171770
3	30839	-159783	14571	-179381
5	-46519	-169198	-80941	-151456
7	8113	-147383	-6563	-165530
9	49805	-166411	34594	-148425
11	915	-153155	-14221	-172303
13	-18448	-163945	-29672	-182967
15	-5311	-189077	-59391	-169512

Load case 7

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	90860	-171989	56985	-190381
3	36660	-176079	27292	-159551
5	-77099	-151280	-93394	-167529
7	65679	-162660	4590	-147556
9	38151	-148174	23487	-164059
11	915	-153155	-32459	-169388
13	-18448	-163945	-23321	-180006
15	5772	-186107	-59391	-169512

Load case 9

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	114322	-192818	85067	-171770
3	30839	-159783	27058	-179398
5	-77099	-151280	-82293	-170129
7	8113	-147383	-5887	-166122
9	90351	-167127	34594	-148425
11	915	-153155	-21762	-172824
13	-18448	-163945	-33160	-183609
15	-11424	-189723	-59391	-169512

Load case 11

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	158292	-191445	85067	-171770
3	31707	-178997	27292	-159551
5	-77099	-151280	-101066	-170040
7	18612	-165472	4590	-147556
9	38151	-148174	25999	-166574
11	41726	-172782	-3283	-153408
13	-18448	-163945	-40317	-183928
15	-53114	-169811	-68856	-189803

Load case 13

Section	COMBI MAX-M		COMBI MIN-M	
	M	N	M	N
1	158447	-187587	85067	-171770
3	37748	-175049	27292	-159551
5	-77099	-151280	-112574	-166528
7	8778	-162081	4590	-147556
9	72350	-163123	34594	-148425
11	1021	-168384	-3283	-153408
13	-257	-181930	-21976	-163642
15	-53114	-169811	-131532	-187636

Maximum values

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	158447	-187587		
3	54396	-168008		
5			-112574	-166528
7	65679	-162660	-6563	-165530
9	90351	-167127		
11	41726	-172782	-32459	-169388
13			-40317	-183928
15	5772	-186107	-131532	-187636

D3: Compilation of the results from the iterations

Compilation of the iteration results

The amount of reinforcement obtained by linear analysis

Section	Tension	Compression
1	750	1075
3	64	64
5	64	64
7	150	64
9	238	100
11	64	64
13	64	64
15	552	745

Strip Step results

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	194902	-189113		
3			-68215	-180944
5	13148	-170620	-65496	-168304
7	86438	-164354		
9	98991	-168705		
11	79401	-174116	-4590	-171084
13	65396	-183246		
15			-163314	-189220

Cross-sectional constants

Section	x_{tp}	l_{ekv}	A_{ekv}	x_{tp} for Strip Step 2
1	0,517	4,9	19,033	0,233
3	0,371	0,828	9,504	0,379 negativ
5	0,348	0,77	8,789	0,402 negativ
7	0,438	1,43	10,462	0,312
9	0,493	2,015	11,903	0,257
11	0,384	0,868	9,92	0,366
13	0,368	0,821	9,426	0,382
15	0,497	3,975	16,563	0,253 negativ

Reduced amount of reinforcement for iterations

Section	Tension	Compression
1	600	860
3	64	64
5	64	64
7	120	64
9	190	80
11	64	64
13	64	64
15	442	596

Strip Step results for iteration one

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	125661	-188632		
3	24062	-169191	-20204	-180533
5	72044	-170373		
7	38888	-163884	-33530	-166704
9	69112	-168287		
11	29105	-173883	-44391	-170507
13	6045	-182866	-32930	-184936
15	31621	-187100	-108697	-188549

Cross-sectional constants

Section	x_{tp}	I_{ekv}	A_{ekv}	x_{tp} for Strip Step 2
1	0,493	4,226	16,789	0,257
3	0,415	0,979	10,869	0,335 negativ
5	0,264	0,646	5,526	0,486
7	0,426	1,26	10,498	0,324
9	0,505	1,855	12,687	0,245
11	0,414	0,974	10,83	0,336
13	0,389	0,886	10,089	0,361
15	0,489	3,445	15,848	0,261 negative

Strip Step results for iteration two

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	147185	-187765		
3	48753	-168407		
5			-111469	-166763
7	65890	-163060	-6224	-165847
9	92336	-167451		
11	43714	-173053	-31864	-169798
13	1246	-182199	-38436	-184193
15	4657	-186507	-134550	-187898

Cross-sectional constants

Section	x_{tp}	l_{ekv}	A_{ekv}	x_{tp} for Strip Step 2
1	0,51	4,281	18,401	0,24
3	0,254	0,642	4,928	0,496
5	0,291	0,67	6,761	0,459 negativ
7	0,488	1,51	12,578	0,262
9	0,555	2,12	12,687	0,195
11	0,435	1,061	11,454	0,315
13	0,37	0,826	9,478	0,38
15	0,527	3,644	17,863	0,223 negativ

Strip Step results for iteration three

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	217161	-194055		
3			-121900	-182323
5	104981	-172320		
7	46590	-165878		
9	67819	-170344		
11	26636	-175878	-48835	-172550
13	5430	-184783	-33416	-186881
15	41745	-189048	-97279	-190454

Cross-sectional constants

Section	x_{tp}	l_{ekv}	A_{ekv}	x_{tp} for Strip Step 2
1	0,641	5,611	25,07	0,109
3	0,308	0,692	7,398	0,442 negative
5	0,29	0,669	6,722	0,46
7	0,428	1,268	10,589	0,322
9	0,485	1,775	11,933	0,265
11	0,392	0,896	10,18	0,358
13	0,392	0,895	10,167	0,358
15	0,475	3,395	14,847	0,275 negative

Strip Step results for iteration four

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	159995	-187589		
3	53042	-168021		
5			-111931	-166535
7	65730	-162673	-6510	-165538
9	90028	-167140		
11	40921	-172795	-33273	-169405
13	2341	-181945	-37710	-183942
15	4290	-186127	-133087	-187651

Cross-sectional constants

Section	xtp	$iekv$	$Aekv$	xtp for Strip Step 2
1	0,499	4,24	17,4	0,251
3	0,248	0,641	4,304	0,502
5	0,297	0,678	7,008	0,453 negative
7	0,486	1,498	12,5	0,264
9	0,557	2,133	14,507	0,193
11	0,436	1,069	11,506	0,314
13	0,371	0,828	9,504	0,379
15	0,528	3,655	17,941	0,222 negative

Strip Step results for iteration four

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	215925	-194056		
3			-110025	-182336
5	104537	-172324		
7	46474	-165890	-25941	-168769
9	68323	-170349		
11	26814	-175882	-48765	-172564
13	5464	-184788	-33371	-186887
15	41423	-189061	-97735	-190459

Cross-sectional constants

Section	xtp	$iekv$	$Aekv$	xtp for Strip Step 2
1	0,644	5,666	25,213	0,106
3	0,29	0,668	6,709	0,46 negative
5	0,291	0,669	6,748	0,459
7	0,429	1,269	10,602	0,321
9	0,485	1,776	11,946	0,265
11	0,392	0,896	10,18	0,358
13	0,371	0,893	10,154	0,379
15	0,475	3,396	14,86	0,275 negativ

Strip Step results for iteration five

Section	M_{sd}	N_{sd}	M_{sd}	N_{sd}
1	158447	-187587		
3	54396	-168008		
5			-112574	-166528
7	65679	-162660	-6563	-165530
9	90351	-167127		
11	41726	-172782	-32459	169388
13			-40317	-183928
15	5772	-186107	-131532	-187636