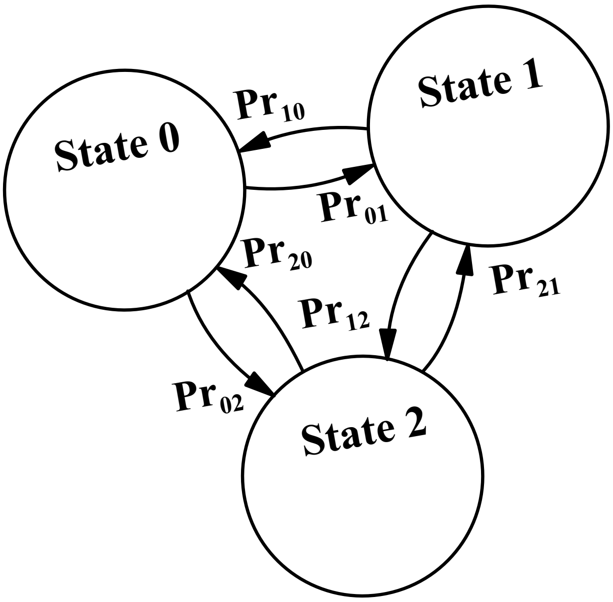

Assessment of Interruption Costs in Electric Power Systems using the Weibull-Markov Model

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Göteborg, Sweden, 2003

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by

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Dedicated to Wille

Preface

The liberalization of the electricity markets has increased the need for performance assessment. The ability to produce qualitative performance indicators for an existing or planned power system is essential in order to improve overall efficiency.

The reliability of supply is directly affected by a reduction in investment. To compare the gain in reliability against the required investments, a monetary evaluation of reliability performance is required. Such is possible by calculating expected interruption costs for groups of customers.

Currently used methods for assessing interruption costs are either slow and inexact, or bare the risk of producing unrealistic results. It was believed that it would be worth to try for an alternative calculation method that would be accurate, yet fast, and realistic enough to be practicable.

It was shown in the Licentiate thesis, which was published in February 2001, that an alternative stochastic model had been found in the Weibull-Markov model and it was shown that the methods based on that model had potential.

This thesis is based on and contains large parts of the licentiate thesis. The chapters in which the Weibull-Markov model was introduced are reproduced here without important changes. It is shown in this thesis that the Weibull-Markov methods and models are a good alternative for the commonly used homogenous Markov models.

This Doctor Thesis would not have been possible without the constructive cooperation between Chalmers University of Technology in Sweden and DIgSILENT GmbH in Germany; without the patience of Dr. Martin Schmieg (at DIgSILENT); without the suggestions of Dr. Markus Pöller (at DIgSILENT); without the discussions with Math Bollen (my examiner at Chalmers) or without the love of Wille Groenewolt (at home).

Gomaringen, Baden-Württenberg, May 12, 2003,
Jasper van Casteren

Abstract

Modern competitive electricity markets do not ask for power systems with the highest possible technical perfection, but for systems with the highest possible economic efficiency. Higher efficiency can only be reached when accurate and flexible analysis tools are used. In order to relate investment costs to the resulting levels of supply reliability, it is required to quantify supply reliability in a monetary way. This can be done by calculating the expected interruption costs.

Interruption costs evaluation, however, cannot be done correctly in all cases by methods which are based on the commonly used homogenous Markov model and is time consuming when using a Monte-Carlo simulation. It was the objective of this thesis to find a new way for calculating interruption costs which would combine the speed and precision of the analytic Markov method with the flexibility and correctness of the Monte Carlo simulation.

A new calculation method was found, based on a new stochastic model. This new model was called the “Weibull-Markov” model and is described in detail in this thesis. The new model and methods have been implemented in a computer program and the speed and accuracy of the calculation method was tested in various projects and by comparison with Monte-Carlo simulations.

It is shown in this thesis that disregarding the effects of the probability distribution of the interruption duration can lead to large errors, up to 40 % and more, in the calculated expected interruption costs. An estimation of the possible error has been made for a large number of published customer interruption cost functions. The actual error in specific reliability calculations is hard to estimate. It is however clear that this error cannot be simply ignored.

The use of the new Weibull-Markov model and the reliability assessment methods do not significantly slow down the calculation speed, offer more flexibility in reliability worth assessment and produce more accurate results. They can be used in all areas of power system reliability assessment which have always been the exclusive domain of homogenous Markov modeling.

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keywords: *power system reliability, reliability worth, semi-Markov model, interruption costs, customer damage function, duration distribution.*

Contents

Terms and Abbreviations

Terms

M_D^k	k^{th} central moment of D
V_D	variance of D
σ_D	standard variance of D
$G_D(t,\tau)$	remainder CDF for $D > t$
X_c	Stochastic component c
X_{c,n_c}	n_c'th successive state of X_c
T_{c,n_c}	n_c'th successive epoch at which X_c changes
$D_{c,i}$	duration of $X_c = i$
$\lambda_{c,ij}$	homogenous transition rate for $X_c = i$ to $X_c = j$
$\lambda_{c,i}$	homogenous state transition rate for $X_c = i$
S	Stochastic system
S_{n_s}	n_s'th successive state of S
T_{n_s}	n_s'th successive epoch at which S changes
X_{c,n_s}	X_c according to S_{n_s}
$X_{c,s}$	X_c according to $S = s$
$D_c(n_s)$	Remaining duration of X_{c,n_s} at T_{n_s}
$A_c(n_s)$	Passed duration of X_{c,n_s} at T_{n_s}
$\eta_{c,i}$	Weibull form factor for $X_c = i$
$\beta_{c,i}$	Weibull shape factor for $X_c = i$
$M_{c,i}$	Mean duration of $X_c = i$
$V_{c,i}$	Variance of the duration of $X_c = i$
$?_{c,n_s}$	= $?_{c,i}$ for $X_{c,n_s} = i$, i.e. η_{c,n_s}, β_{c,n_s}, etc.
$?_{c,s}$	= $?_{c,i}$ for $X_{c,s} = i$, i.e. $\eta_{c,s}$, $\beta_{c,s}$, etc.
$\mathrm{Fr}_{c,i}$	Frequency of occurrence of $X_c = i$
$\mathrm{Pr}_{c,i}$	Probability of occurrence of $X_c = i$
Fr_s	Frequency of occurrence of S=s
Pr_s	Probability of S=s

Abbreviations

TTF	Time To Failure
TTR	Time To Repair
MTBF	Mean Time Between Failures
TBM	Time Between Maintenance
CDF	Cumulative Density Function
PDF	Probability Density Function
SF	Survival Function
LPEIC	Load Point Expected Interruption Costs
ECOST	Expected Customer Outage Cost

Chapter 1

Introduction

In Western Europe, the availability of electric energy has been long regarded as a basic provision for economic development and prosperity. Centralized and monopolistic companies were therefore formed to design, construct and control the electrical power systems. These companies strove for technical perfection in a growing market where the risk of over-investment was considered low. Consequently, the electrical power systems in this part of the world are among the most reliable and well designed. Fig.1.1 shows some values for the unavailability of the electricity supply, expressed in average minutes lost per year ([45]). This figure shows large differences between various European countries. Where the average non-availability of electricity in Italy is around 200 minutes per year, it is about 25 minutes in the Netherlands. However, the probability of being not supplied in Italy is still very small and equals 200/(8760*60)=0.00038. This means that electric power is available in 99.962 % of time. In the Netherlands and Germany, where the average non-availability is less than 25 minutes per year, the availability is more than 99.995 % of time. This is astonishingly good, considering that we talk about a very large and complex technical system.

However, modern competitive electricity markets are believed to no longer demand for power systems with the highest technical perfection, but for the economically most efficient. The liberalization of the electricity markets that resulted from this believe has since forced electric companies to find a new balance between technical perfection and maximum profit.

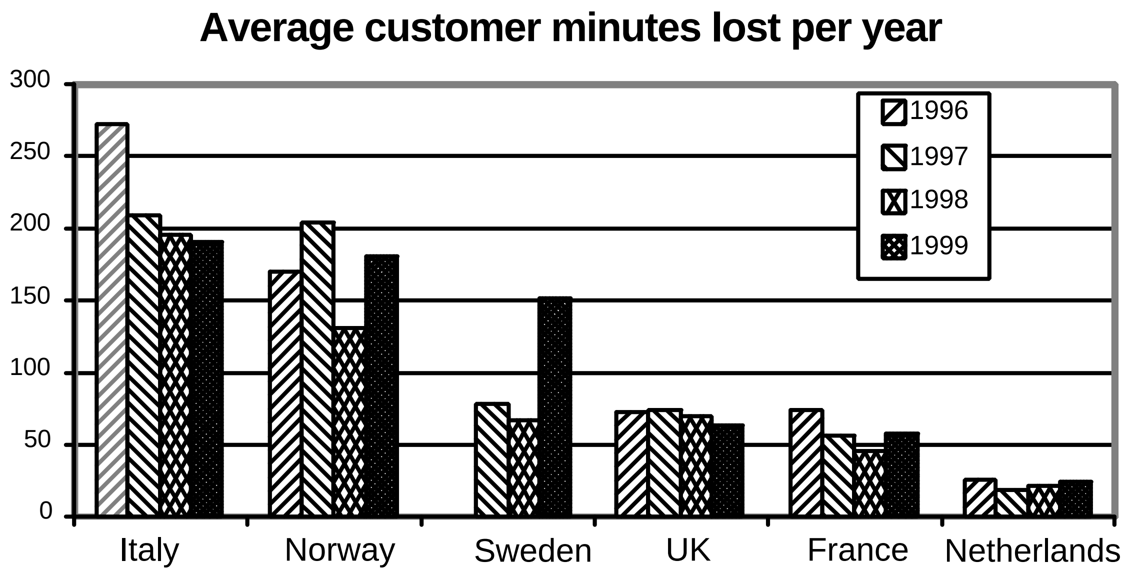


Figure 1.1: European Comparison of unavailability

A perfect power system is a power system where all loads are continuously supplied by electricity of constant nominal voltage without any waveform distortion. Of course, such systems do not exist. All loads will suffer supply interruptions sometimes, the voltage or current will always show deviations from nominal values and waveforms will always be distorted to some extent. The question is how bad this is. What levels of imperfection can be tolerated? At which levels will we have the highest achievable efficiency? High levels of perfection can only be reached by high investments and will thus cause high prices. Low levels, however, will lead to all kind of costs due to insufficient performance, such as ([45]) :

- Economic penalties payable to affected customers or into funds.
- Tariff reductions or other revenue affecting penalties by regulating.
- Customer loss due to negative publications.

The liberalization of the electricity markets forces the electricity companies to view their power plants, transmission networks and distribution systems in a new light. In order to survive in the open market, it is required to reduce costs and to maximize returns on investments.

It is the task of reliability engineers to support the decision processes in the planning phases by providing power system performance indicators. Investments can then be judged by the gain in performance.
The calculation of (expected) performance indicators in respect to supply reliability can be divided into the calculation of non-monetary interruption statistics and reliability worth assessment. Reliability worth assessment produces monetary indices, and is a relatively young discipline.

This thesis introduces a new stochastic model which can be used to produce monetary indices more correctly, in a fast analytic way, based on a flexible definition of damage caused by customer interruptions.

1.1 Quality and Reliability

Electric power is used by consumers to operate electrical machines and appliances. When that is possible without exceptions, then such consumers will be satisfied and we have sufficient **power quality**. Power quality is thus a measure for the ability of the system to let the customers use their electrical equipment. Any peculiarity or fault in the power system that (might) prevent the use of electrical devices or (might) interrupt their operation means a lack of power quality.

The ability of the system to let customers use their equipment is determined by the extent to which the voltage and/of the current of the consumer's power supply are ideal. All deviations from the ideal are called **power quality phenomena** or power quality disturbances. A power quality phenomenon can be a sudden change of values, a limited period of lesser quality, but also a continuous situation in which some (sensitive) equipment can not be used properly.

Continuous or slow changing offsets from a perfect power quality are called voltage variations or current variations. Phenomena which suddenly appear and which have a limited duration are called power quality events.

In [30], an exhaustive description of power quality variations and events is given. Possible power quality variations are:

- Voltage/current magnitude variations
- Voltage frequency variations
- Current phase variations
- Voltage and current unbalance
- Voltage fluctuations (“flicker”)
- Harmonic distortion
- Interharmonic components
- Periodic voltage notching
- Mains signaling voltages
- High frequency voltage noise

and some example of power quality events are:

- Interruptions
- Undervoltages
- Overvoltages
- Voltage magnitude steps
- Voltage sags
- Fast voltage events (“transients”)

Power quality variations, as listed above, are examples of the **voltage quality** or **current quality** of the electric supply. Power quality variations, however important and interesting, are not further considered in this thesis.

The power quality events can be divided into two groups:

- Interruptions
- Other voltage quality events

Depending on the duration of the event, momentary, temporary and sustained interruptions can be distinguished. Reliability analysis is the field of research which calculates the number and severity of power interruptions. It is divided into the field of security analysis and adequacy analysis. Security analysis calculates the number of interruptions due to the transition from one situation to the other. Adequacy analysis looks at interruptions which are due to the non-availability (“outage”) of one or more primary components in the system. The position of adequacy analysis in the whole area of power quality analysis is illustrated by Fig.1.2.

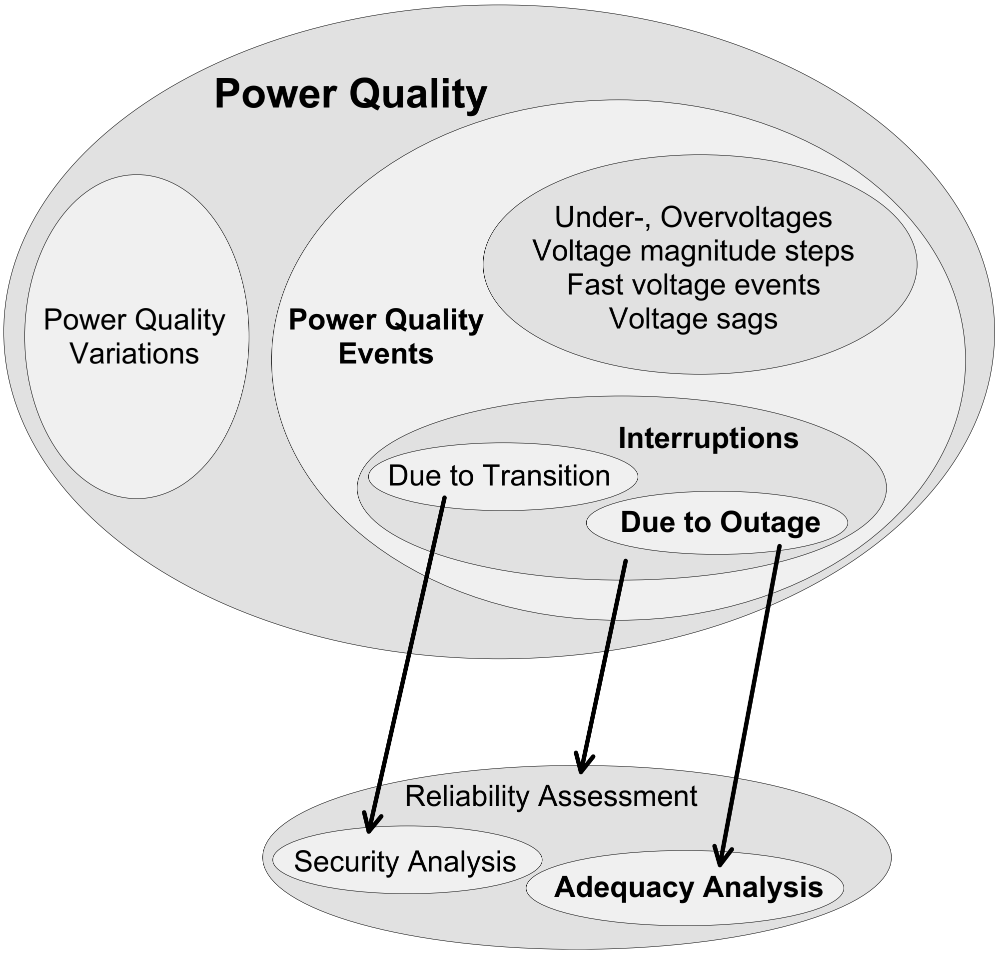


Figure 1.2: Power Quality and Adequacy Analysis

The subject of this thesis is power system adequacy analysis. Security analysis is not further addressed.

One method for determining the number of interruptions is by monitoring voltages and counting the number of interruptions.
For the use of deciding, from the monitored voltages, when an interruptions occurs, a criterion like the one in the IEEE std.1159-1995 can be used. This standard defines an interruption as an event during which the voltage is lower than 0.1 p.u. A sustained interruption is defined as an interruption with a duration longer than 1 minute.

When monitoring a real power system we normally have the situation that all kinds of events can cause interruptions, but that the exact causes for specific captured interruptions are often unknown. That is why some minimum voltage like 0.1 p.u. is required as criterion for an interruption. A criterion like “not longer connected to a source of electric power” cannot be used for measurements, as the network condition is normally unknown when classifying the voltage measurement.

A completely different situation occurs when reliability calculations are made with a computer program using digital network models. In that case, only a very specific set of events will happen, namely only those events which have been modeled. For each analyzed interruption, it is known in detail why the interruption occurred and the status of all parts of the power system is completely known. On the other hand, a voltage at the interrupted load points is normally not calculated. That is why we need a different definition for an interruption when we perform reliability analysis.

For the purpose of the assessment of reliability indices by system modeling and mathematical analysis, it can be assumed that as long as a customer is still physically connected to the power system, he will be supplied. Only when a customer is disconnected from the system, it is called an interruption.

The interruptions of customers are caused by failing equipment, or by maintenance which is performed to prevent equipment from failing. Failures are events where a device suddenly does not operate as intended, and failures occur in all parts of the power system. Cables are damaged, for example, by digging activities, lighting strikes, breakers suddenly open, transformers burn out, etc., etc. Every reliability analysis must start by recognizing and modeling all relevant failures which may affect the system's reliability.

A failure, or a possible future failure, may lead to the temporarily removal of one or more primary devices from the system. The situation where a primary device is removed is called an outage and the removed device is said to be 'outaged'. An outage will bring the system in a weakened condition.

Primary devices are removed in order to clear a fault, to prevent further damage or weakening of the system, to repair a faulted device or for performing maintenance. All equipment that is de-energized or isolated is said to be outaged. Not all outages result from failures, however, and not all failures will lead to outages. When devices are deliberately removed from the system for preventive maintenance, then that is called a "scheduled outage" or "planned outage". An outage which is an inevitable result of a failure is called a "forced outage" or "unplanned outage".

Outages may lead to situations in which customers are not longer supplied with electric power. As mentioned before, such an event is called an interruption. Not all outages will lead to interruptions. Many components, especially at higher voltage levels and in industrial systems, are operated in parallel. After the outage of one component, the parallel one will take over immediately and the load or customer will not experience an interruption. The reserve capacity of a power system which is used to compensate for an outage is called "redundancy". Redundancy improves the reliability.

Besides causing interruptions, outages may also affect the system's ability to perform scheduled transports of power without violating technical or operational constraints. Often, outages in meshed high-voltage transport systems will not lead to interruptions, but they generally affect the transport capacities. Outages in radially operated distribution systems on the other hand will normally not affect transport capacity, but will often lead to interruptions.

The relation between failures, outages, interruptions and reduced transport capacity is shown in Fig.1.3

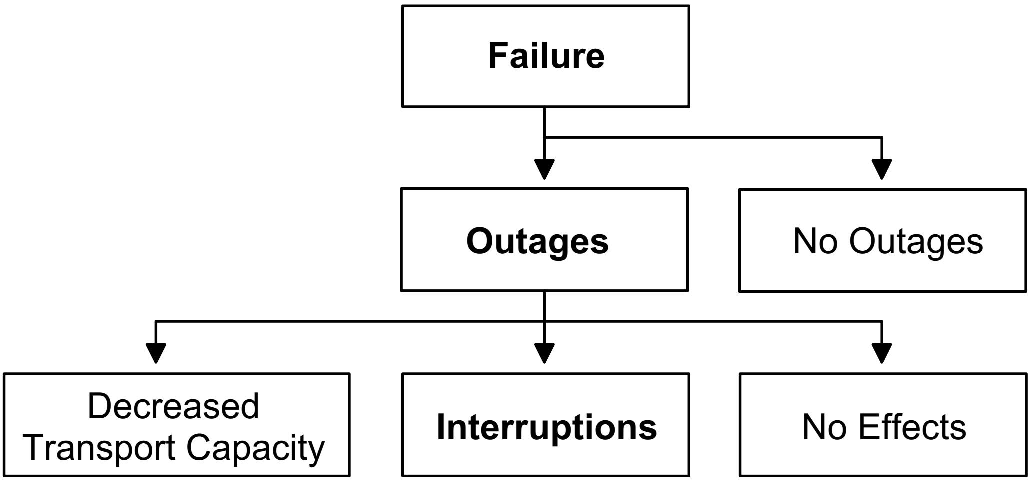


Figure 1.3: Failures and possible results

1.2 Reliability Assessment

Reliability assessment, although targeted at only one aspect of power quality, is nevertheless a wide area itself. Three basic types of reliability assessment can be recognized:

- Monitoring of system performance
- Deterministic reliability assessment
- Stochastic reliability assessment

The monitoring of system performance is performed by capturing the number, type and duration of interruptions. This is indispensable for gathering detailed information about the performance of power system components. It is also used for bench-marking purposes which are, for instance, used for market regulating.

System performance monitoring, however, can normally not be used to assess the reliability of individual parts of the networks. The size of such networks is often so small that years of monitoring would be required in order to reach some statistical significance. Of course, it can also not be used to compare design alternatives for system expansion planning.

Deterministic reliability assessment can be based on active failures (faults) or on outages. Deterministic outage-based reliability assessment will perform an outage effect analysis for each relevant outage in the system. Each outage may cause the interruption of one or more loads or may cause overloading of parallel parts of the system. The well-known “n-1”, or “n-k”, analysis is an example of such analysis. Deterministic outage-based reliability assessment produces qualitative results, telling if criteria are met or not, but may also produce quantitative results such as capacity margins, maximum loading, maximum interrupted power, etc.

Deterministic fault-based reliability assessment does not analyze the effect of outages, but the effects of faults (active failures). The difference is that the reactions of the power system are considered in a fault effect analysis whereas they are neglected in an outage effect analysis. Typical system reactions to a fault are (in chronological order)

- Fault clearance by protection
- Fault isolation
- Power restoration

A fault effect analysis may produce several sets of outage conditions, each with a different duration. Each of these conditions will be analyzed for interruptions and overloads. More detailed criteria can then be used to judge the adequacy of the system. It may, for instance, be acceptable to have some interruptions as long as power to the interrupted loads can be restored within 30 minutes, or short-term overloading can be tolerated. Such criteria cannot be tested with an outage effect analysis only.

Stochastic reliability assessment adds to a deterministic fault-based reliability assessment in two aspects:

- It may also consider passive failures
- It uses stochastic failure models

The use of stochastic models allows the calculation of averaged indices, by combining the results of the various failure effects analysis on the basis of the failure frequencies and/or probabilities.

Stochastic failure models define the expected number of times a failure will happen per unit of time, and how long it will take to repair the failed component. During the repair, the failed component will be out of service, and all customers which cannot be supplied without the component will be interrupted for the duration of the repair. Mathematical expressions are used to calculate the probability or the severity of the interruptions, using the stochastic failure data. The possibilities for calculating interruption severity depend on the used stochastic model. If the severity depends not only on the mean interruption duration, but also on the probability distribution of the duration, then the commonly used homogenous Markov models complicate such severity calculations. The new Weibull-Markov model, to be introduced in chapter 3 of this thesis, is more suited for calculating interruption severity indices.

In stochastic reliability assessment, each relevant failure is analyzed in a failure effect analysis (FEA). A typical FEA uses a multitude of power system analysis tools, such as

- topological analyses for finding isolated areas, power restoration switches, etc.
- loadflow calculations for overload detection
- linear optimization tools for overload alleviation
- short-circuit analysis for fault clearance
- interruption costs calculations

The combination of the results of the performed FEA's into averaged results, on the basis of the stochastic data, is currently done in roughly two ways

- By Monte Carlo simulation
- By analytic calculations, using Markov models

Monte Carlo simulation is the repeated chronological simulation of the power system. During each simulation, faults will occur randomly, as in real-life, and the reactions of the system to these faults is simulated chronologically. The performance of the system is then monitored during the simulations.

The advantage of Monte Carlo simulation is that all aspects of the power system can be addressed and there are no limits to the stochastic models or to the possibilities for measuring performance. The disadvantages are the often very high computational demands. Monte Carlo simulation produces stochastic results only, which are inexact, but for which it is possible to calculate variances and even probability distributions.

Analytic calculations range from matrix manipulations to state enumeration techniques. State enumeration means that all relevant failures will be analyzed one by one, as with the deterministic fault effect analysis. The results of each of these analysis are then added together by using the stochastic failure and repair data.
Analytic calculations on the basis of the Markov model are normally much faster than Monte Carlo simulations, and they produce exact results. Variances can be produced also, but probability distributions are normally not feasible. The disadvantage of the analytic Markov methods is that there are severe limitations to the calculation of interruption costs indices.

1.3 Objectives of this thesis

The main objective of this thesis is to present a new method with which a correct assessment of interruption costs indices by analytic calculations is possible. The presented method is based on a new stochastic model, which is called the “Weibull-Markov model”. The Weibull-Markov model is a semi-Markov model which uses Weibull distributions for the state durations.

The second objective of this thesis is to show that the Weibull-Markov model can replace the homogenous Markov model in reliability analyses, and that this replacement removes the limitations with regard to the calculation of interruption costs indices without introducing new restrictions.

1.4 Contents of this thesis

An introduction into power system reliability assessment is given in chapter 2. This introduction includes an overview over the various failure models that can be used for reliability assessment. The various steps and methods that are used for the failure effect analysis (FEA) are discussed in chapter 2.3. Stochastic models are introduced in chapter 3. This chapter covers the basics of homogenous Markov models and introduces the Weibull-Markov model. Interruption costs assessment is discussed in chapter 4. Chapter 5 discusses the results for an example distribution network.

Chapter 2

Power System Reliability Assessment

2.1 Basic Scheme

The assessment of the reliability of a power system means the calculation of a set of performance indicators. An example of such an indicator is the LPAIF, or the "Load Point Average Interruption Frequency", which equals the average number of times per year a load point suffers an interruption. Two different sets of indicators can be distinguished: local indicators and system indicators. Local indicators are calculated for a specific point in the system. Examples are

- The average time per year during which a specific generator is not able to feed into the network
- The average duration of the interruptions at a specific busbar.
- The interruption costs per year for a specific customer.

System indicators express the overall system performance. Examples are

- The total amount of energy per year that cannot be delivered to the loads.
- the average number of interruptions per year, per customer.
- The total annual interruption costs.

Power system reliability analysis can be split into three parts:

1. The definition of a model for the healthy, undisturbed, power system.
2. The definition of possible deviations from the healthy system state, such as possible failures, changes in demand, planned outages, etc.
3. The analysis of the effects of a large set of healthy and unhealthy system states.

The results of the analyses of the various system states is used to calculate the performance indicators. The basic diagram of the calculation procedure is depicted in Fig.2.1.
The definition of the initial healthy system state is the starting-point of the reliability assessment procedure. In this system state, all components are working as intended. The initial load profile, generator dispatch and switch settings allow for a normal, healthy, loadflow in which no overloading or voltage deviations should be present.

The set of definitions of possible deviations from the initial state is used to generate events. These events will bring the system in an disturbed state, such as may occur in the real system. This disturbed state is then processed by the failure effect analysis, which may react to present faults or other undesirable situations by opening switches, restoring power, performing repairs, or other changes to the system. This is repeated until enough unhealthy system states have been analyzed. Finally, the intermediate results are processed to produce the various performance indicators.

In order to perform a reliability assessment, we thus have to creates models for the various failures and other disturbances that that may occur in the system. Secondly, we have to be able to analyze the effects of these system disturbances. Thirdly, we have to select the relevant system conditions and failures that must be analyzed in order to calculate the reliability performance indicators.

The various failure models and the methods to select relevant system states are described in the following sections. The Failure Effect Analysis is the subject of the next chapter.

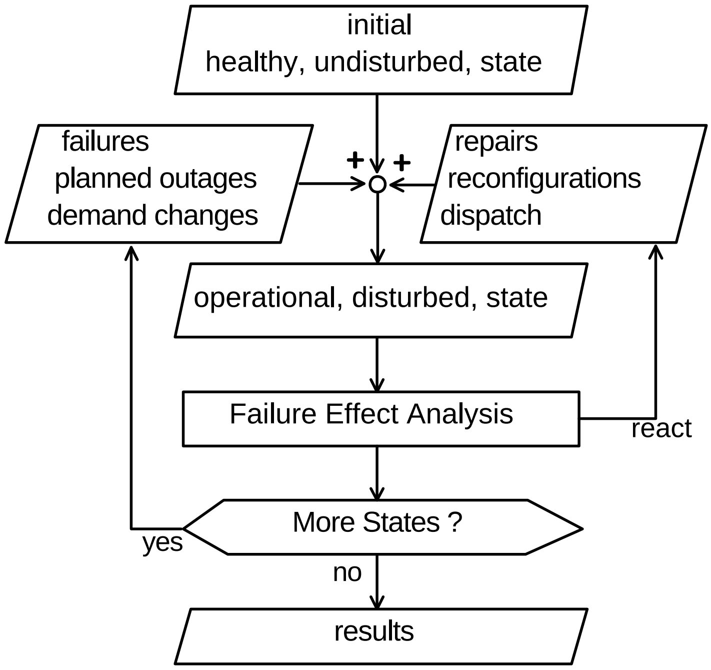


Figure 2.1: Basic reliability assessment scheme

2.2 Failure Models

Failures in power systems can be divided into active and passive failures. Active failures are failures that require an automatic intervention of protection devices. Passive failures will not directly provoke a reaction of the power system. Some passive failures will remain undetected until an inspection is carried out or until the device is called upon to perform its function.

2.2.1 Passive Failures

Passive failures may persist in the system for a long time without further affecting the system. Although they may cause the interruption of customers in some cases, they are seldom a direct threat to the system itself. The reaction time to passive failures is therefore in the range of minutes to hours or even days to months.

Passive failures can be divided into manifest failures and hidden failures. Manifest failures will show themselves directly in their effects, where hidden failures can exist undetected up to the next inspection or up to the moment the component is called upon.

Manifest Failures

The following manifest failures can be distinguished:

- **Inadvertent breaker opening (mal trip).** A mal-trip can occur in four ways:
  - Independent inadvertent breaker open. This is the sudden opening of a breaker either due to a human error, a breaker failure or an automation failure. The occurrence of these failure is independent of the existing state of the network.
  - Independent mal-trip. This is the incorrect opening of a breaker by a protection device without a short-circuit in the system.
  - Maintenance related inadvertent breaker open. This may happen due to incorrect switching in relation to maintenance where, for instance, backup-equipment is taken out inadvertently.
  - Fault-related mal-trip. This is the incorrect opening of a breaker by a protection device in relation to an existing short-circuit.
- **Independent Equipment Trip** Independent equipment trips are sudden outages primary components due to a fault in the component itself. Examples are generator trips due to turbine faults or the tripping of a Buchholz-relay at a transformer.

Hidden Failures

Hidden failures will cause the failed component to not perform the next time it will be called upon to function. Examples are undetected relay failures, undetected communication channel failures and stuck breakers.

2.2.2 Active Failures

Active failures are a direct threat to the power system. They may lead to severe damage and even total system collapse when they are not immediately cleared. It is the task of automatic protection equipment to react to active failures and to minimize direct damage to the system.

Four kinds of active failures can be distinguished:

- Single active short-circuits.
- Cross Country faults.
- Common mode faults.
- Overlapping faults.

Single Active Short-Circuits

A single active short-circuit is a fault at a single location which leads to the immediate isolation of the faulted area by automatic protection devices. The fault may be either a three-phase, a two phase, a two phase with earth or a single phase short-circuit.

In compensated networks, it is possible to operate the network with a single-phase short-circuit long enough to allow for load transfer or other preventive system reconfigurations. Such is done in order to isolate and repair the fault without or with a minimum of customer interruptions. The only contribution of such single phase failures to the unreliability would then be the reduced redundancy during the repair and the risk of inadvertent switching in relation to the network reconfigurations and repair.
Single phase short-circuits in compensated networks which do not lead to automatic protection intervention or load interruptions can be neglected in a reliability assessment.

Cross Country Faults

A cross-country failure starts with a single phase fault in a compensated network, which is not isolated by protection. Due to the presence of the single phase fault, the voltage at the other phases is raised by a factor of $\sqrt{3}$. This increased phase voltage may lead to a second short-circuit somewhere else in the network, after which one or both failures will be isolated by automatic protection intervention. The double short-circuit is called a cross country failure.

Common Mode Failures

A common mode failure is the simultaneous occurrence of two or more active failures due to a single common cause. A common mode failure will lead to the immediate isolation of all areas which contain the faults. Examples are

- lightning flash-over at multi-circuit towers
- damage to multi-circuit poles or towers due to car accidents
- multiple cable damage due to excavation works

Overlapping Faults

An overlapping fault is a fault which occurs during the outage of one or more relevant other components. These other components may be on forced outage, i.e. on repair, or on planned outage, i.e. during maintenance.

Important is that the outaged components are relevant to the faulted component. For the power system of Sweden, for example, it is likely that some repair or maintenance is going on somewhere at any time. That would mean that all faults in Sweden will happen during one or more forced or planned outages, and all faults in Sweden would therefore be “overlapping faults”. That would make no sense.
In defining overlapping faults, only those outages are regarded which cause a significant difference in the effects of the occurring fault. In the case of two parallel transformers, we would have an overlapping fault if the second transformer fails during the outage of the first. The outage of the first transformer is relevant, because this could mean that customers are interrupted where they would not have been when both transformers were available. In other cases, the occurrence of an overlapping fault could also mean that power is restored slower because relevant equipment is not available.

The problem with detecting relevant pre-fault outages is that we would have to make a failure effects analysis in order to find out that we do not need to analyze the combination of outage and fault. That would be to late, as we would already have analyzed it.

Another aspect of pre-fault outages is that they will change the operational state of the network. This normally leads to higher loading of backup equipment, which may lead to higher failure rates. Such accelerated faults due to pre-fault outages always lead to overlapping outages, as it is clear that the outaged and the faulted component are not independent.

This can also be generalized : we could detect at least a subset of all possible faults for which a certain outage is relevant by calculating the difference in pre-outage and during-outage loading of all components. Those components which show a significant difference in loading are affected by the outage and therefore good candidates for an overlapping fault analysis.

Overlapping faults differ from common mode failures because they do not occur at the same time. More important is that overlapping faults are often much more seldom than common mode failures, because their frequency of happening is the product of failure frequency and the probability of the pre-fault outage, which normally is very low.

This can be shown with the simple example of two parallel cables. Suppose the independent failure frequency for each cable is once every 5 years = 0.2/a, and that a repair takes 10 hours on average. The probability of being in repair for each cable is thus $10h \cdot 0.2/a = 2h/a = 2/8760$. A year of 8760 hours is used here, which is common practice in reliability assessment. This means that an overlapping failure will occur

$$2 \cdot \frac{2}{8760} \cdot 0.2/a = \frac{1}{10950}/a \tag{2.1}$$

i.e. once in 10950 years on average. The frequency for a common mode failure due to digging, for example, is often much higher than that.

2.3 Failure Effect Analysis (FEA)

The task of the Failure Effect Analysis (FEA) is to analyze the ability of the system to provide all loads with sufficient power after a failure, without violating the system constraints. If not all loads can be supplied with power, then the loads which suffer an interruption have to be identified, as well as the duration of these interruptions. The FEA is a quasi steady-state analysis.

Load interruptions due to the transient effects of the sudden introduction of short-circuits and the subsequent opening and closing of switches, are not considered here. Such transient effects are the subject of security analysis.

The FEA not only comprises the analyses of steady-state system states, but also includes the simulation of the reactions of the system and the system operators to faults and other emergency situations. The power system is actively changed during such a simulation, which may include

- switching actions in order to clear and isolate faults
- switching actions in order to restore power
- switching actions, re-dispatch and load shedding in order to alleviate overloading

The Failure Effect Analysis is a mixture of short-circuit analyses, load flow calculations, topological analysis and system simulation. The level of detail in the FEA is determined by the level of detail in these sub-analyses and simulations, and so is its complexity and calculating speed.

The output of the FEA is a list of busbars and terminals which are interrupted due to the failures in the analyzed operational state, each with a description of the duration of these interruptions. These descriptions can be a single value in case of a deterministic duration, or a probability distribution in case of a stochastic duration.

The input data required for performing the FEA are

- the basic system definition
- the operational system state
- the failures in the system

The FEA thus starts with a system in which one or more, possibly active, failures may be present.

The basic system definition includes all aspects of the power system which are assumed constant during the reliability assessment, which comprises:

- the topology of the system
- the electrical models for all primary equipment
- the models of all relevant protection devices
- all relevant loadflow controller models, such as
  - secondary controllers
  - voltage controllers, tap changer controllers
  - shunt and VAR compensator controllers
- emergency and maintenance switching protocols

The topology of the system is considered here independent of switch positions. The position of a switch, either open or closed, does therefore not change the basic topology. Also, any possible connection or generator which may be used in certain emergency conditions, i.e. for restoring power to important loads, must be defined as a part of the basic system topology.
The electrical models must be accurate enough to allow a valid loadflow calculation.

The operational system state comprises

- the set of forecasted peak load profiles
- the set of forecasted peak transport flows
- switch positions
- generator availabilities and capacities
- planned outages
- ongoing repairs

Each FEA starts by changing the system model to the current operational state, and then introduces one or more failures. The effects of these failures is then analyzed in three phases:

- Primary Failure Effect Analysis
- Secondary Failure Effect Analysis
- Tertiary Failure Effect Analysis

2.3.1 Primary Failure Effect Analysis

The primary failure effect analysis determines the effects of the automatic reactions of the power system to the active failures, i.e. the reactions of the system's automatic protection devices, such as over-current relays, fuses, distance relays, under-voltage or over-voltage relays, under-frequency relays, etc.
The results of the primary reactions are the opening of one or more breakers in the system. Typical reaction times are in the order of milliseconds to a few seconds. Due to these short reaction times, the primary failure effects cannot be prevented or modified after the occurrence of the failures. The objective of the primary reactions of the power system are to minimize the damage and to minimize the direct effects of the fault to non-faulted parts of the system.

All active failures must be cleared by the protection. To find out which relays will intervene, a short-circuit calculation is needed. The advantages of using short-circuit analysis is that fault currents can be established, and protection selectivity can thus be regarded. The disadvantage is that short-circuit analysis requires detailed overcurrent, differential and distance relay models. For many reliability assessment projects, such detailed information is not available, mostly because the reliability assessment is required in a stage of project development where the protection system has not been designed yet. Another argument against the use of full short-circuit calculation is the calculation speed. But even in cases where calculation speed is not a problem and detailed protection data is available, the use of short-circuit analysis could complicate the interpretation of the results, because both the primary and secondary system design would have to be regarded.

If perfect selectivity can be assumed, then a fast topological analysis can be used to simulate fault clearance. Instead of performing a short-circuit calculation, a topological search algorithm is started at the fault position. This algorithm will continue to search the network around the fault position up to the first relays. The smallest area around the fault which can be isolated by protection devices is thus found. These protection devices are then assumed to trip. The advantage of this method is, besides its speed, the minimum requirements on the protection modeling. The only required information is the position where protection devices are measuring fault currents and which breaker(s) they will open. In the case of a fuse, only the location is required, and this is also true for many overcurrent and distance relays. This limited demand for relay information makes it possible not only to start a reliability assessment in an early stage of system design, but also to quickly compare various basic protection system lay-outs.

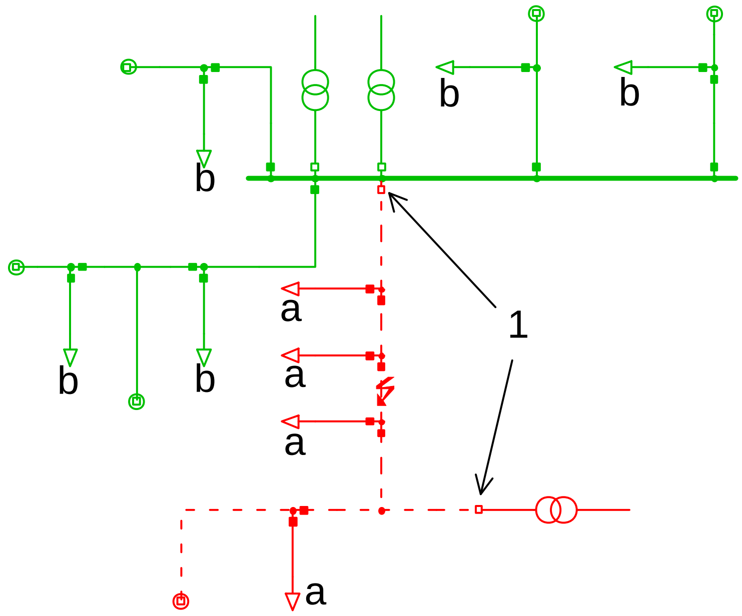


Figure 2.2: Cleared area and backup protection

In Fig.2.2, a part of a network is shown in which a short-circuit occurs. The topological search algorithm will find the dotted area as the smallest area that can be isolated in order to clear the fault. The relays at “1” are found as the relays that will trip. As the faulted area is fed from two sides, two relays have to be tripped. Breaker fail-to-trip, as would happen when a hidden failure is present in the relay or the breaker, can be accounted for by not opening the breaker and letting the topological algorithm search further. In Fig.2.2, it is assumed that the upper breaker at “1” fails to trip. The next smallest area to clear is by tripping the breakers at the parallel transformers. The tripping of these backup relays will mean a prolonged fault clearance time and the whole area in Fig.2.2 will be tripped.

For the purpose of reliability assessment, the fault clearance time is not important. Important is, however, the fact that a much wider area is tripped and thus more customers are interrupted than without the hidden failure which led to the fail-to-trip event. If we assume a failure frequency for the fault in Fig.2.2 of F $(1/a)$, and a fail-to-trip probability of $P = P_s + P_r$, with P_s for the breaker and P_r for the relay, then we will have a contribution to the interruption frequency for the loads "a" in Fig.2.2 of F $(1/a)$ and a contribution of $P \cdot F$ $(1/a)$ for the loads "b".

Important is that the short-circuit itself, i.e. the system state directly after the short-circuit appears and before the first protection devices intervene, is not regarded. The short-circuit itself and the detailed analysis of fault currents, remaining voltages and fault clearance times, including the effects of fail-to-trip events and backup protection, is the subject of voltage sag analysis.

2.3.2 Secondary Failure Effect Analysis

The secondary failure effect analysis considers the reaction of the power system and its operators to the condition which is reached after the primary reaction has finished. Typical durations for the secondary reactions are in the order of seconds to minutes for automatic reactions and in the order of minutes to hours for manual reactions. The objective of the secondary reactions are to minimize the duration of the interruptions and to alleviate any violated operational constraint.

Fault Isolation and Power Restoration

The task of the primary FEA is to clear active failures by isolating the smallest possible area containing the fault. The area which is isolated to clear the fault is called the "cleared area". All loads in the cleared area will suffer an interruption. It is the task of the fault isolation and power restoration to simulate the process of fault location, fault isolation, power restoration and repair. This is done by searching for the smallest possible area around the fault and subsequently restoring power to as many customers as possible and as quickly as possible ([57]). Fig.2.3 illustrates this process. The shown fault in this schematic example will lead to the isolation the whole bottom feeder by protection. This is the "cleared area". By manually opening two isolators, the fault can be isolated more selectively. Power to the remaining parts of the cleared area can then be restored and these areas are therefore called “restorable areas”. Areas that are actually restored by (re)closing switches are called “restored areas”.

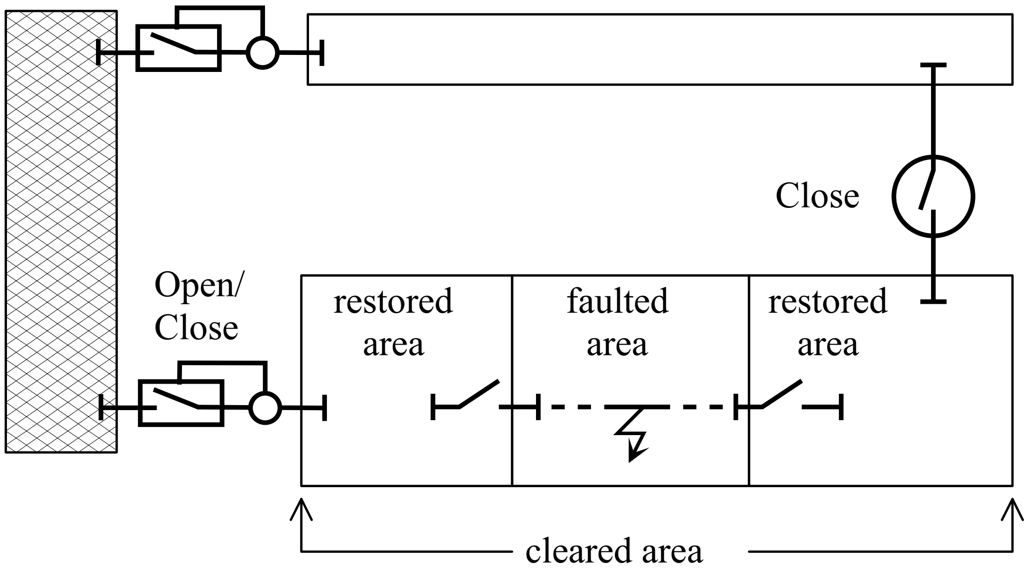


Figure 2.3: Cleared, faulted and restored areas

The fault isolation uses the same topological search for switches as has been used for the fault clearance. A topological search is started at the faulted components to find the smallest area that can be isolated in order to isolate the fault. However, in stead at protection devices, the search now stops at any switch that can be opened.

The faulted area is smaller than or equal to the cleared area. The areas between the faulted and the cleared area are the “restorable areas” because, principally, power may be restored to the customers in these areas. The actual power restoration is performed by finding all switches that can connect or reconnect a restorable area to a supplied area. One or more of these switches are then closed to restore power.

Important for the power restoration scheme is that the resulting interruption times are realistic. The used method must, for instance, regard the effects of remote sensing and switching. The interruption duration in the case of "switched" power restoration, i.e. in the case of power restoration by network reconfiguration, is determined by the time spend locating the fault, the timer needed to open the switches that isolate the fault and the time needed to determine and close the switches through which the power is restored. Each of these times depend on the amount of automation and remote control in the network, the presence of short-circuit detectors, the type of relays, the loading of the network, the accessibility of sub-stations, etc.

After the fault isolation and power restoration phase have finished, the network will be reconfigured and power flow will be different from the pre-fault situation. In some cases, for instance in many low voltage distribution networks, it can be assumed that overloading in the post-fault situation does not occur. Overload verification can then be omitted, thus increasing the calculation speed. A simple connection algorithm is used to check if a load is still physically connected to a generator or external network. When so, the load can be considered to be supplied.

In many cases, overloading can not be disregarded. This is especially the case in meshed transmission networks, or highly loaded distribution systems. Some kind of overload verification must then be used.

Overload Alleviation The observed overloads have to be alleviated. The way in which this is done depends on the system condition, the fault that caused the overloading, and on the type of overloading. Overload alleviation may include

- switching-in backup equipment and cold reserve,
- maintenance or repair cut-off, which means that components which are currently on outage are brought back into service as fast as possible,
- network reconfiguration,
- generator re-dispatch,
- load reduction, load transfer and load shedding.

These three methods are listed in order of applicability. Load shedding is a last-resort method when everything else fails. Load shedding will lead to customers interruptions. These interruptions, although caused by operator intervention, must be looked upon as forced interruptions, and not as planned interruptions. The frequency and duration of load shedding therefore must be added to the interruption statistics.

Linear programming techniques are often used to find the switches to open or close, the generators to change or the loads to reduce, transfer or shed. These optimization techniques try to minimize the effects of the overloads on the customers and are normally based on some linear network flow calculations.

In the case of (sub-)transmission reliability assessment, some load may be transferred from one feeder to the other by switching actions in the distribution networks. These networks, however, may only be included in the transmission model as single lumped loads. In that case, load transfer must be modeled by stating ‘transferable’ percentages and transfer targets for specific loads. The transfer targets are the loads to which the given percentage of power is transferred.

2.3.3 Manipulation of the Secondary FEA

The emergency reconfiguration schemes for fault isolation and power restoration normally have a significant effect on the calculated reliability indices. By correctly locating and isolating the fault and subsequent closing of normally open switches, the interruption duration can often be drastically reduced. Improving the supply reliability by improving the emergency reconfiguration schemes, however, is only feasible when it is possible to associate the impact of a fault with specific schemes. It must then be possible to analyze, adjust and check these schemes in detail.
It is therefore important to model the fault isolation and power restoration schemes in a way that they

- reflect reality
- allow fast modifications and experiments
- are transparent and can be checked easily

- allow for further investigations

The outcome of the secondary FEA is a list of interrupted loads with the duration of the interruptions. This duration will be deterministic in the case of a “switched” power restoration, i.e. in the case of power restoration by network reconfiguration. When switched power restoration is not possible, the faulted components have to be repaired in order to restore power. Such “repaired” power restoration will have a stochastic duration.

But even when we assume switched restoration to have a deterministic duration, then we often still have a different interruption duration for different loads in different restored areas. Another difference in interruption duration occurs when some loads have to be shed at once, while others will be shed not before some short-term overloading limit is violated.

It is obvious that the implementation of the secondary FEA will have a significant influence on the produced reliability indices. In [41], a method is described in which power restoration schemes for specific failures can be defined in an interactive way.

2.4 Tertiary Failure Effect Analysis

Typical reaction times for tertiary reactions are in the order of minutes to hours. The objective of the tertiary reactions are to optimize the operational state which is reached after the secondary reactions have finished. Tertiary reactions are not considered further in this thesis.

2.5 Stochastic Modeling and Interruption Costs Assessment

In order to produce qualitative reliability indices for a power system, we have to provide both electric system data and component failure data.
With the electric system data and the component failure data, contingencies can be created, i.e. system states in which one or more failures are present. A failure effect analysis, including a primary, secondary and tertiary FEA, is

then performed. The FEA produces a list of interrupted customers and will also calculate the duration of the interruption for each individual customer.

The duration is a very important aspect of the interruption, and its mean value can be used, for instance, to calculate the annual interruption probability, expressed in minutes per year of not being supplied. It depends on the type of the used failure models if it will also be possible to calculate the probability distribution of the interruption duration for a specific contingency. As the failure models use stochastic durations, i.e. stochastic life-times and repair durations, a repetition of the same contingency under the same conditions will lead to different interruption durations. The interruption duration is therefore a stochastic quantity.

This is important because the damage caused by an interruption will often depend largely on the duration of the interruption. If this dependency is not linear, then it will not be possible to calculate the expected interruption costs correctly when the probability distribution of the duration is not known.

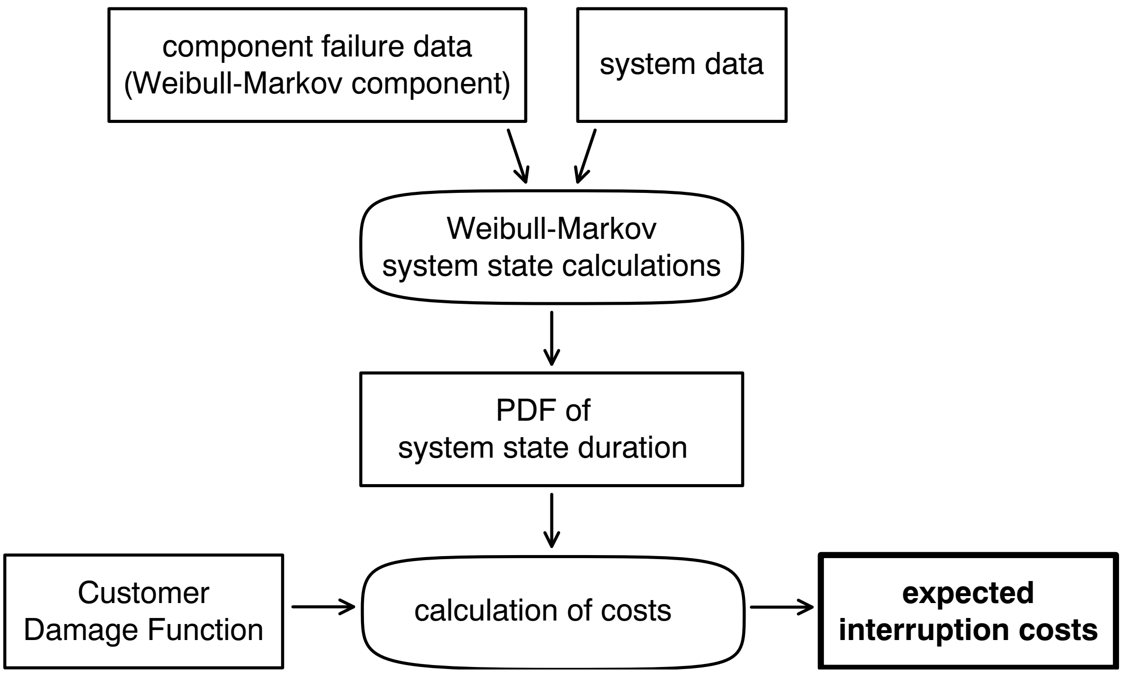


Figure 2.4: Utilization of the Weibull-Markov model

The commonly used failure model in reliability assessment is the so-called homogenous Markov model. It will be shown in the next chapter that a correct assessment of the probability distribution of the interruption durations is not possible with the homogenous Markov model.
Therefore, in order to assess expected interruption costs correctly, a different stochastic failure model is required that will enable the calculation of duration distributions. Such an alternative to the homogenous Markov model is the Weibull-Markov model.

The principle utilization of the Weibull-Markov model for the calculation of interruption costs is depicted in Fig.2.4. The failure data and the electric system data are used in a Weibull-Markov system state calculation. This calculation produces the means of the stochastic interruption durations as well as discrete probability density functions (PDF). Each interruption duration PDF is combined with an interruption damage function to produce the expected interruption costs for a customer, for the currently analyzed contingency. The expected annual customer interruption costs are then calculated by combining the results of all analyzed contingencies.

Chapter 3

Stochastic models

In order to perform a reliability assessment, we regard the electrical power system as a collection of components. Each component is a typical part of the electric power system which is treated as one single object in the reliability analysis. Examples are a specific load, a line, a generator, etc., but also a complete transformer bay with differential protection, breakers, separators and grounding switches may be treated as a single component in a reliability assessment.

A component may exhibit different ’component states’, such as ’being available’, ’being repaired’, etc. In the example of a transformer, the following states could be distinguished:

1. the transformer performs to its requirements
2. the transformer does not meet all its requirements
3. the transformer is available, but not used
4. the transformer is in maintenance
5. the transformer is in repair
6. the transformer is being replaced by another transformer
7. the transformer behaves in such a way as to trigger its differential relay

For some reliability calculations, all these possible states may have to be accounted for. Normally, a reduction is made to two or three states. A two state model would, for instance, only distinguish between

1. the transformer is available
2. the transformer is not available

A three state model could further distinguish between repairs (“forced outages”) and maintenance (“planned outages”), or between different levels of availability. Each of these states is described by

- an electrical model with electrical and operational constraints
- a duration distribution
- the possible transitions to the other states

The electric model for a transformer which is not available would be an infinite impedance, for instance. A model for a transformer which is only partly available would have a stuck tap changer, for instance, or would have a reduced capacity.

A stochastic component is a component with two or more states which have a random duration and for which the next state is selected randomly from the possible next states. A stochastic component changes abruptly from one state to another at unforeseen moments in time. If we would monitor such a stochastic component over a long period of time, while recognizing four distinct states – x_0, x_1, x_2 and x_3 –, a graph as depicted in Fig.3.1 could be the result.
Because the behavior is stochastic, another graph will appear even if we would monitor an absolute exact copy of the component under exactly the same conditions.

For all stochastic models, only the state duration and the next state are stochastic quantities. Each distinct functional state of a component is therefore regarded as being completely deterministic, apart from its duration. Phenomena like randomly fluctuating impedances or random harmonic distortions are therefore not part of the stochastic behaviour of a component.

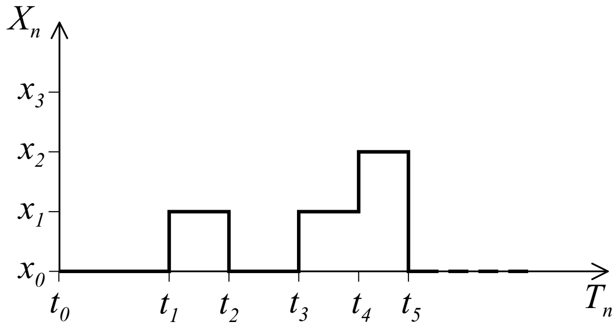


Figure 3.1: Example of monitored states of a component

If such phenomena are to be included in a reliability assessment, to assess the number of interruptions due to excessive harmonic distortion for example, the stochastic model must be extended by a number of states for which the fluctuating random quantity is considered constant.

This chapter introduces the stochastic models for electrical power system components. From these stochastic models, the model for a stochastic power system are then developed.

3.1 Stochastic Models

The basic quantity in reliability assessment is the duration D for which a component stays in the same state. This duration is a stochastic quantity, as its *precise* value is unknown. The word “precise” is emphasized here, as, although we don’t know the value of a stochastic quantity, we almost always know something about the possible values it could have. The time until the next unplanned trip of a generator, for example, is unknown, but nobody would expect a good generator to trip every day, as well as nobody would expect it to operate for 10 years continuously without tripping even once. This example range from 1 day to 10 years is too wide to be of practical use in a reliability assessment. However, for actual generators, a much smaller range of expected values can be obtained from measured data. The basic question about a stochastic quantity is thus about the range of its expected values, or ‘outcomes’: which outcomes can be expected and with what probability?

Both the outcome range and the outcome probabilities can be described by a single function; the "Cumulative Distribution Function" or CDF. This function is also called the "distribution function" and is written as $F_D(\tau)$ and defines the probability of D being smaller than τ, which is written as:

$$F_D(\tau) = \Pr(D \leq \tau) \tag{3.1}$$

For reliability purposes, the probability for a negative duration is zero and the probability that the duration will be smaller than infinity is one:

$$F_D(0) = 0 \tag{3.2}$$
$$F_D(\infty) = 1 \tag{3.3}$$

The "Probability Density Function" or PDF, for D, $f_D(\tau)$, is the derivative of the cumulative distribution function. The PDF, also known as "density function", gives a first insight into the possible values for D, and the likelihood of it taking a value in a certain range.

$$f_D(\tau) = \frac{d}{d\tau} F_D(\tau) \tag{3.4}$$
$$f_D(\tau) = \lim_{\Delta\tau \to 0} \frac{\Pr(\tau \leq D < \tau + \Delta\tau)}{\Delta\tau} \tag{3.5}$$
$$\int_0^\infty f_D(\tau) d\tau = 1.0 \tag{3.6}$$

The survival function (SF), $R_D(\tau)$, is defined as the probability that the duration D will be longer than τ.

$$R_D(\tau) = \Pr(D > \tau) = 1 - F_D(\tau) \tag{3.7}$$

For a specific component, i.e. a transformer, where D is the life time, which is also known as the time to failure (TTF), the survival function gives the probability for the component functioning for at least a certain amount of time without failures. For a group of identical components, the SF is the expected fraction of components that will 'survive' up to a certain time without failures.

The hazard rate function (HRF), $h_D(\tau)$, is defined as the probability density for a component to fail shortly after a certain time τ, given the fact that the component is still functioning at τ,

$$h_D(\tau) = \lim_{\Delta\tau \to 0} \frac{\Pr(\tau \leq D < \tau + \Delta\tau \mid D > \tau)}{\Delta\tau} = \frac{f_D(\tau)}{R_D(\tau)} \tag{3.8}$$

The HRF is an estimate of the unreliability of the components that are still functioning without failures after a certain amount of time. An increasing HRF signals a decreasing reliability. An increasing HRF means that the probability of failure in the next period of time will increase with age. A decreasing HRF could, for instance, occur when only the better components survive.

The expected value of a function g of a stochastic quantity D is defined as

$$E(g(D)) = \int_0^\infty g(\tau) f_D(\tau) d\tau \tag{3.9}$$

The expected value of D itself is its mean E_D, which is defined as

$$E_D = E(D) = \int_0^\infty \tau f_D(\tau) d\tau \tag{3.10}$$

The k'th central moment, M_D^k, is defined as

$$M_D^k = E([D - E(D)]^k) \tag{3.11}$$

The variance V_D is defined as the second central moment

$$V_D = M_D^2 = E([D - E(D)]^2) = E(D^2) - (E(D))^2 \tag{3.12}$$

The standard variance σ_D is defined as the square of the variance

$$\sigma_D = \sqrt{V_D} \tag{3.13}$$

The remainder CDF is defined as the CDF of the remaining duration after an inspection at time t has revealed that the component had not failed yet. Because the total duration D is a stochastic quantity, the remainder $D - t$ is also stochastic. The remainder CDF, $G_D(\tau, t)$, is defined as

$$\begin{aligned} G_D(\tau, t) &= \Pr(D \leq \tau \mid D > t) = \frac{\Pr(t < D \leq \tau)}{\Pr(t < D)} \\ &= \frac{F_D(\tau) - F_D(t)}{R_D(t)} \end{aligned} \tag{3.14}$$

3.1.1 The Exponential Distribution

The negative exponential distribution, or simply “exponential distribution”, is defined by

$$f_D(\tau) = \lambda e^{-\lambda\tau} \tag{3.15}$$

which makes that

$$\begin{aligned} F_D(\tau) &= 1 - e^{-\lambda\tau} && (3.16)\\ h_D(\tau) &= \lambda && (3.17)\\ E_D &= \frac{1}{\lambda} && (3.18)\\ V_D &= \frac{1}{\lambda^2} && (3.19) \end{aligned}$$

The HRF of the negative exponential distribution, $h_D(\tau)$, is not dependent of time which considerably simplifies many reliability calculations.

The remainder CDF, $G_T(\tau, t)$, for the negative exponential distribution equals

$$G_D(\tau, t) = 1 - e^{-\lambda(\tau - t)} = F_D(\tau - t) \tag{3.20}$$

The remainder of an exponential distributed duration thus has the same distribution as the total duration. The expected value, variance, etc. for the remainder are all the same as for the total duration, independent of the inspection time. This is a peculiarity unique to the exponential distribution. It also shows that the exponential distribution is very abstract. If, for instance, a repair duration would be negative exponentially distributed, then the expected time to finish the repair would be independent of the time already spend repairing.

The exponential distribution is a good approximation for events that are due to external circumstances. All failures which are not due to ageing could be modeled with an exponential distribution for the TTF. The exponential distribution, however, is not suitable for modeling repair and restoration processes.

3.1.2 The Weibull distribution

Where the exponential distribution uses only one parameter (λ), the Weibull PDF uses a shape factor β and a scale factor η. It is defined as

$$f_D(\tau) = \frac{\beta}{\eta^\beta}\tau^{\beta-1}\exp\left[-\left(\frac{\tau}{\eta}\right)^\beta\right] \tag{3.21}$$

which makes that

$$\begin{aligned}
F_D(\tau) &= 1-\exp\left[-\left(\frac{\tau}{\eta}\right)^\beta\right] && (3.22)\\
h_D(\tau) &= \frac{\beta}{\eta^\beta}\tau^{\beta-1} && (3.23)\\
E_D &= \eta\Gamma\left(1+\frac{1}{\beta}\right) && (3.24)\\
V_D &= \eta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\} && (3.25)
\end{aligned}$$

where

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$$

is the normal gamma function.

The Weibull PDF equals the negative exponential distribution when the shape parameter $\beta = 1.0$. Some examples of the Weibull PDF, for different means and variance are displayed in in Fig.3.2 and Fig.3.3.
The conditional CDF, $G_T(\tau,t)$, for the Weibull distribution equals

$$G_D(\tau,t) = \exp\left[\left(\frac{t}{\eta}\right)^\beta-\left(\frac{\tau}{\eta}\right)^\beta\right] \tag{3.26}$$

which is dependent on the inspection time t.

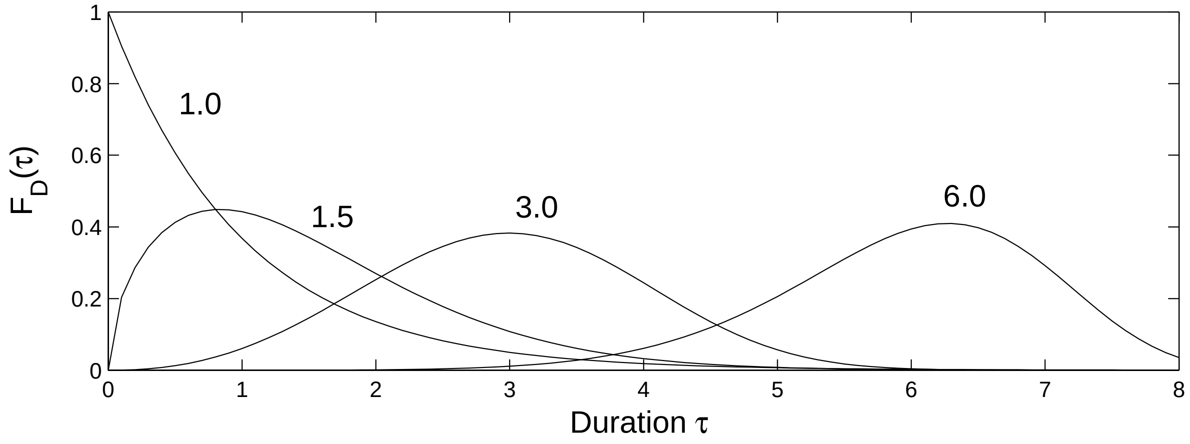


Figure 3.2: Weibull PDF for different means, variance = 1.0

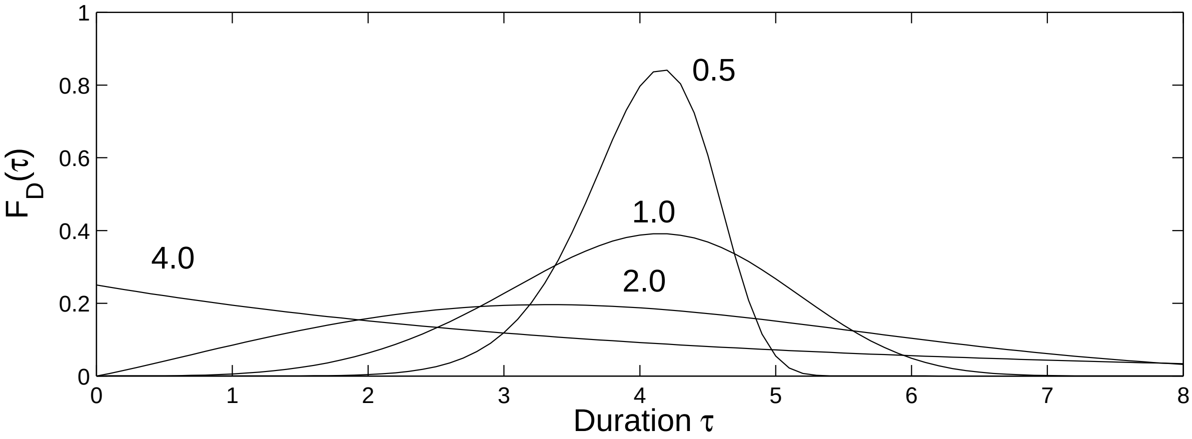


Figure 3.3: Weibull PDF for different variances, mean = 4.0

Weibull Probability Charts

The Weibull survival function can be transformed into a straight line by taking the double logarithm of the reciprocal of the survival function:

$$\ln(\frac{1}{R_D(\tau)}) = \left(\frac{\tau}{\eta}\right)^{\beta} \tag{3.27}$$

$$\ln(\ln(\frac{1}{R_D(\tau)})) = \beta \ln(\tau) - \beta \ln(\eta) \tag{3.28}$$

From (3.28), it is clear that a plot of $\ln(-\ln(R_D(\tau)))$ against $\ln(\tau)$ will produce straight lines. Because the transformation is independent of the shape and scale parameters, it is possible to draw plotting paper where the vertical axis is logarithmic and corresponds to the survival time τ, and the horizontal axis is transformed to correspond with $\ln(-\ln(1 - F_D(\tau)))$. Such “Weibull-probability charts” are a helpful tool in estimating shape and scale parameters without the risk of producing completely unrealistic parameters for measurements which do not fit a single Weibull distribution. Such unrealistic parameters would be produced without further warning by numerical parameter estimation algorithms.

3.1.3 Maintenance, Repair and Failure Density

In many reliability textbooks ([53], [119]), a distinction is made between repairable and non-repairable components. While in many systems the latter are of great importance, non-repairable components do not exist in electric power systems or are of no importance. Electric power systems are not build to perform once or twice, but to perform continuously, 24 hours a day, 365 days per year. Therefore, there is no such thing as a ‘mission time’ for electric power systems, nor for their components.

When speaking of the repair of a power system component, not the repair of the actually failed component is meant, but the restoration of the functionality of the component. Such may be achieved by repairing the component, for example by repairing a faulted cable, or by replacing it.

A repair can restore the component to a condition as if it was brand new again. Repair by replacement is the trivial example. Such a perfect repair is called a repair **“as-good-as-new”**. If the repair restores the component to about the same condition in which it was directly before the failure, it is called a repair **“as-bad-as-old”**.

If we would take a new component into service and just use it continuously, without preventive maintenance, it will fail after a certain time. This time is called the Time To First Failure or TTFF.

Many power system components, however, will undergo some scheme of preventive maintenance. Such maintenance is performed in order to prolong the lifetime of the component or, in better words, to significantly increase the mean time between failures. This means that the original stochastic TTFF of the component can therefore not be measured, as it is obscured by the maintenance. What is left to be measured is the time between the last maintenance and the failure, which is called the time to failure (TTF). For a component without maintenance the TTF equals the time between failures (TBF). For a component with maintenance, TBF is larger than or equal to TTF.

To investigate how the original component's TTFF, and the measurable TTF and TBF are related, we assume ideal maintenance and ideal repair. Ideal maintenance is performed at regular equidistant intervals, has no duration, and restores the component to a state as-good-as-new. Ideal repair also has no duration, and also makes the component as-good-as-new again.

Suppose that we have a component for which we know the PDF of the time to failure, i.e. without failure. If we then perform ideal preventive maintenance at fixed intervals T_M, we can calculate the new PDF for the time between failures for the maintained component.

The probability density of the TTFF for the period until the first maintenance, $f_1(t)$, has the same value as the original PDF because maintenance has not changed anything yet:

$$f_1(t) = \begin{cases} f_T(t) & \text{if } 0 < t \leq T_M \\ 0 & otherwise \end{cases} \tag{3.29}$$

Note that $f_1(t)$ is not a PDF, because it only describes the distribution in the first period and $\int_0^\infty f_1(t)dt$ will therefore be smaller than one.

The probability of surviving until $t = T_M$ is $R_T(T_M)$. This is also the probability of the component to fail at $t > T_M$ if the maintenance would not have been performed. Because the ideal maintenance has no duration, this remaining probability is distributed over all moments $t > T_M$. The shape of the distribution is again the same as the original because the maintenance is a repair-as-good-as-new. The probability distribution for the second period will therefore be

$$f_2(t) = \begin{cases} R_T(T_M) f_1(t - T_M) & \text{if } T_M < t \leq 2T_M \\ 0 & otherwise \end{cases} \tag{3.30}$$

The general solution is achieved by repeating this for $t = kT_M$ with $k \in \mathcal{N}^+$ (see [53, p.14])

$$f_T^*(t) = \sum_{k=0}^{\infty} R_T^k(T_M) f_1(t - kT_M) \tag{3.31}$$

Before calling $f_T^*(t)$ a probability distribution function, we have to show that $\int_0^{\infty} f_T^*(t) = 1$. Using (3.29), we can write

$$\int_0^{T_M} f_T^*(t)dt = \int_0^{T_M} f_T(t)dt = F_T(T_M)$$

and therefore

$$\begin{aligned} \int_0^{\infty} f_T^*(t)dt &= \sum_{k=0}^{\infty} R_T^k(T_M) F_T(T_M) \\ &= F_T(T_M) \cdot \sum_{k=0}^{\infty} (1 - F_T(T_M))^k \\ &= 1 \end{aligned}$$

The function $f_T^*(t)$ is thus a correct probability distribution function.

This is an important result, because the geometric expression $R_T^k(T_M)$ forces the PDF to fluctuate between two negative exponential curves. This is sometimes used as an argument in favor of using exponential distributions for the TTF, as TTF is forced to become 'more or less' exponential by the preventive maintenance.

The effect of ideal maintenance is illustrated here by a small example. A single component was taken with a Weibull distributed TTFF, with a scale factor of 1.0 and a shape factor of 4.0. Ideal maintenance was performed at $T = 1, 2, 3, \cdots$. The distribution of both the TTF and the TBF where simultaneously obtained from a chronological Monte Carlo simulation which ran up to 100.000 failures. The resulting distribution for the TTF is depicted

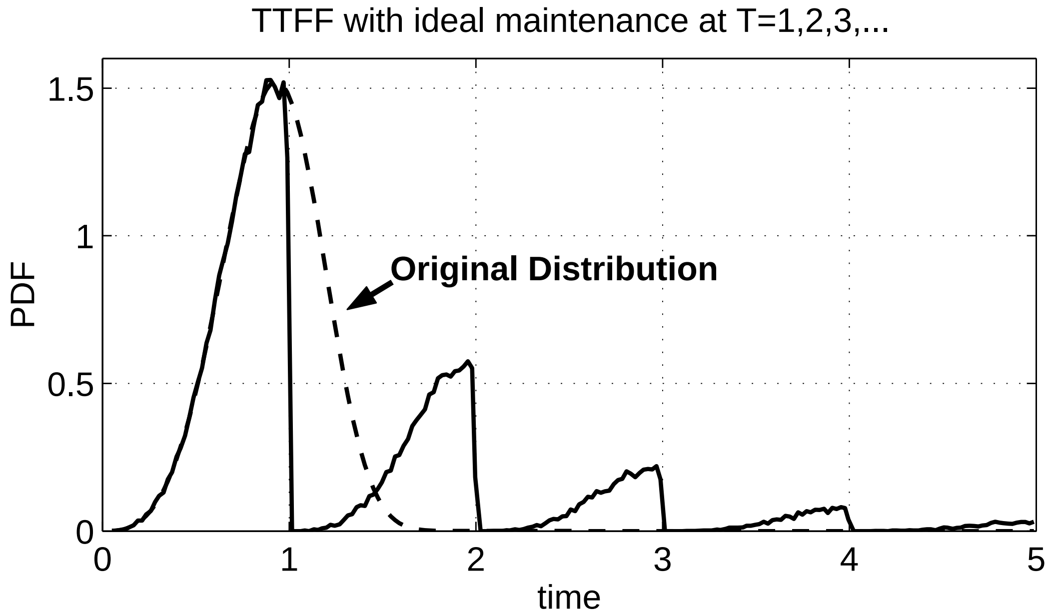


Figure 3.4: TTFF with ideal maintenance

in Fig.3.4. This figure also shows the original TTFF, which is bell-shaped with a mean of 0.91. The TTF has a mean of 1.32 and a standard deviation of 0.97. Maintenance for this component increases the TTF by about 45%.

The resulting distribution of the TBF is depicted in Fig.3.5 This figure shows high peaks of probability density around multiples of the time between maintenance. If we compare Fig.3.5 with Fig.3.4, then we see that where the distribution of the TTF is zero directly after each maintenance, the distribution of the TBF is almost symmetrical around the maintenance periods. The distribution of the TBF becomes more clear when we write the times at which failures occur as $t = nT - \delta$ where $0 < \delta < T$.
From Fig.3.4, we see that failures are more likely to occur for smaller δ. The TBF is the time between two subsequent failures, and can thus be written as $(n+k)T - \delta_1 - nT + \delta_2 = kT - \delta_1 + \delta_2$, $k = 0, 1, 2, \cdots$. As δ_1 and δ_2 are identically distributed, the distribution of the TBF is symmetrical around kT. The measured mean of the TBF (i.e. the MTBF) is 1.55.

The Monte-Carlo simulation was repeated for a exponential distributed TTFF by setting the form factor to 1.0. The resulting distribution of the TBF is

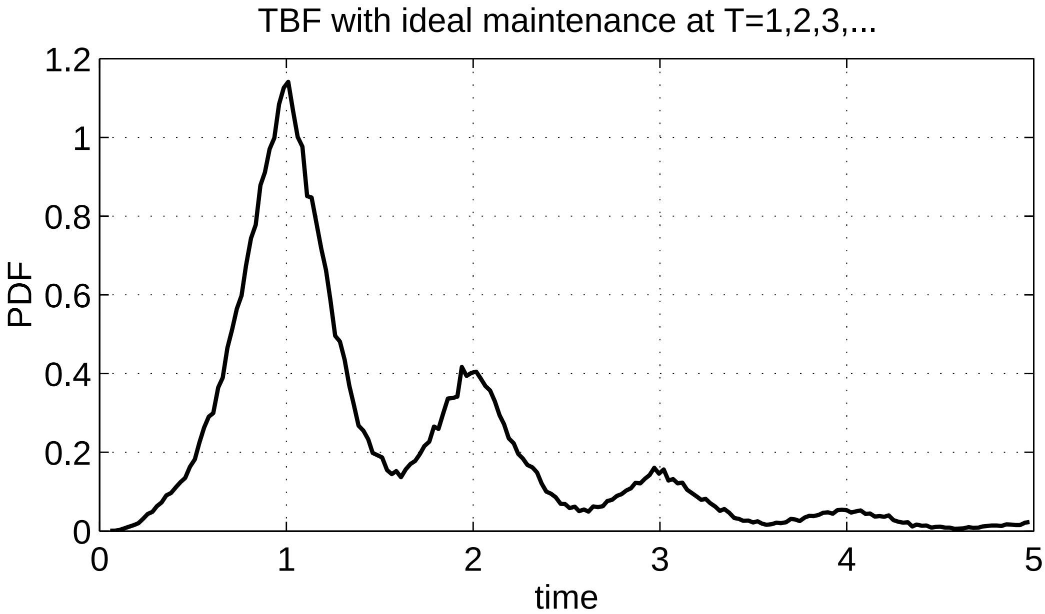


Figure 3.5: TBF with ideal maintenance

depicted in Fig.3.6. This TBF equals the original TTFF. Maintenance on a

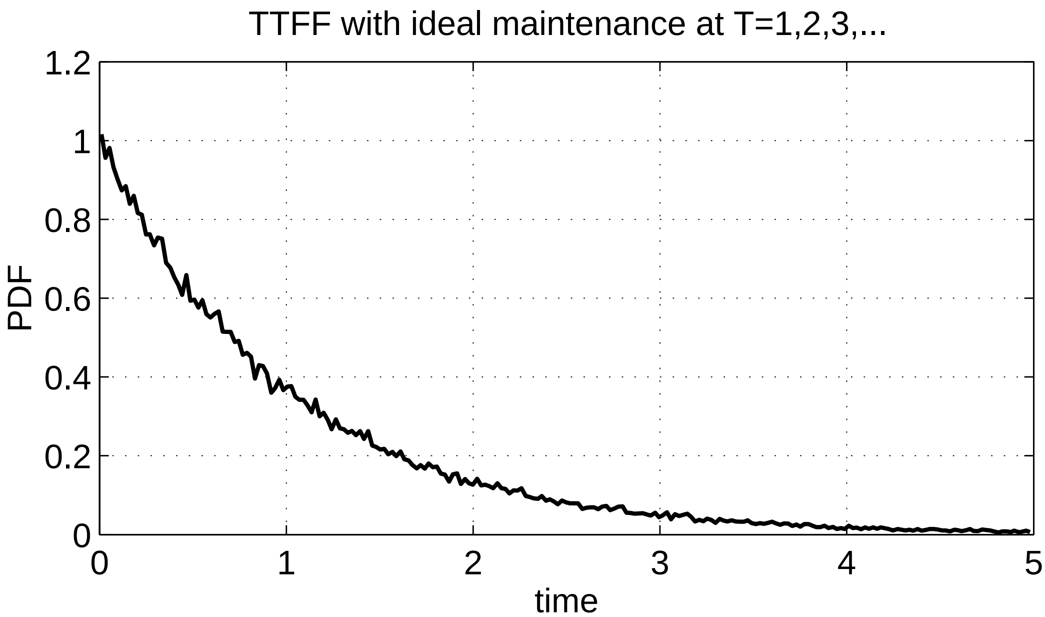


Figure 3.6: TBF with ideal maintenance, exponential TTFF

component with an exponentially distributed TTFF has no effect.

3.1.4 Measuring and Modeling TTF

In most cases, only the MTBF is known for a specific class of components, which is normally calculated as the number of failures divided by the number of monitored service-years. We cannot directly derive conclusions about the distribution of the TTF from that. Most utilities, however, keep records of failure time and performed maintenance. From these records, the distribution of the TTF could theoretically be derived. This would lead to a right-censored set of measured TTFF, with which Weibull form factors could be estimated.

Measuring TBF where regular maintenance is performed will lead to more or less negative exponentially probability density distributions. The conclusion that the component's TTFF is thus negative exponentially distributed is, however, wrong. The real distribution of the component's TTFF may be completely different. Nevertheless, homogeneous models can still be used in this case as they correctly model the physical TBF. It must however be kept in mind that the homogenous model models the combination of the component's failure process and the performed preventive maintenance.

Homogenous models can not be used for determining the effects of changed maintenance planning. This is caused by the fact that when using a homogenously distributed TBF for a component, additionally modeled maintenance will not affect the failure frequency. The conclusion would then be that additional maintenance will only increase the unavailability of the component (because additional planned unavailability is introduced), and less maintenance is therefore always better. Modeling maintenance in a reliability analysis where homogenous failure models are used exclusively is still sensible, as performed maintenance may decrease reliability performance not only by adding planned unavailability, but also because it introduces periods of reduced redundancy, and increased numbers of human failures. But again, all these effects will lower the reliability performance and only the maintenance that was performed during the measurement of the failure data may be modeled.

When performing a chronological Monte-Carlo simulation with homogenous failure models and imperfect maintenance in that sense that the maintenance duration is larger than zero, a small but systematic error is introduced. This error will result in small effects of changed maintenance intervals on the failure frequency despite the use of homogenous models. This is caused by the fact that, during the Monte-Carlo simulation, we have to draw a new TTF when maintenance is performed. If that new TTF is drawn at the end of the maintenance, then the failure process will be temporarily switched off, which will lead to slightly lower average failure frequencies. When the new TTF is drawn at the start of the maintenance, we would have to disregard all TTF that are smaller than the maintenance duration, which would also (normally) lead to slightly lower average failure frequencies. It would be wrong to interpret these changes in failure frequency as a realistic effect of changed maintenance intervals.

3.2 Homogenous Markov Models

One of the important qualities of the homogenous Markov model shows itself when we build a stochastic system from stochastic components. When all stochastic components are homogenous Markov models, then the system will be a homogenous Markov model also, only much larger.
This enables the calculation of state probability, frequency and duration by analytic matrix operations. Although this seems to be a great advantage, this is only partly so. The system state indices which can all be obtained together by matrix operations can much easier be calculated by summing the contributions of the system component.
The one exception is the calculation of system state duration distributions. These can not be calculated from the component models when the component life-times and repair durations have arbitrary probability distributions.
However, the system state duration distributions, although easily obtained for homogenous Markov model, will be unrealistic in that case when preventive maintenance is used.
This is so because the one important disadvantage of the homogenous Markov model is the exclusive use of the negative exponential distribution for all stochastic durations in the system. In the case of repair of maintenance durations, these distributions are already highly unlikely, but in the case of life time, they cannot be other than incorrect. Using a negative exponentially distributed lifetime will always cause the model to react to preventive maintenance by a lowered overall availability, which is surely not the case for normal power system equipment.

Nevertheless, the homogenous Markov model is very important due to its computational elegance. A good understanding of the basic properties of the homogenous Markov model is required for understanding other models and methods used in power system reliability assessment.

3.2.1 The Homogenous Markov Component

The monitored stochastic behaviour of a component can be described completely by a set of state and epoch combinations $(x_n, t_n)_{n=0}^{\infty}$. This is illustrated by Fig.3.7 which shows a possible graph of the monitored states of a component with four distinct states; x_0, x_1, x_2 and x_3.

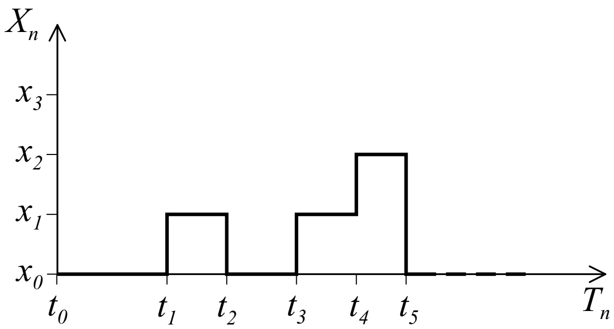


Figure 3.7: Example of monitored states of a component

If we monitor the same component under exactly the same conditions, another set $(x_n, t_n)_{n=0}^{\infty}$ will be the result, as each next state and each state duration $(t_{n+1} - t_n)$ are stochastic quantities. Each set $(x_n, t_n)_{n=0}^{\infty}$ is an outcome from a infinite number of possible outcomes, and is called a "component history". The stochastic history for the component with index "c", is written as "$(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$". Both X_{c,n_c} and T_{c,n_c} are stochastic quantities and we may talk, for example, about "the probability of $X_{c,23} = x_1$" or "the probability density function of $(T_{c,45} - T_{c,44})$".

The homogenous Markov model is now defined by:

- the set of possible states $\overline{x_c} = \{1, 2, \cdots, N_c\}$ where N_c is the number of possible states
- the stochastic history $(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$, where

- $\forall_{n_c}(X_{c,n_c} \in \overline{x_c}, X_{c,n_c} \neq X_{c,n_c+1})$
- $T_{c,0} = 0$ and $\forall_{n_c}(T_{c,n_c+1} > T_{c,n_c})$

• the set of continuous probability distribution functions $F_{c,ij}(t)$ for the conditional state durations $D_{c,ij}$

$$\begin{aligned} F_{c,ij}(t) &= \Pr(D_{c,ij} \leq t) \\ &= \Pr(X_{c,n_c} = i\ ,\ (T_{c,n_c+1} - T_{c,n_c}) \leq t \mid X_{c,n_c+1} = j) \\ &= 1 - \exp\left(-\frac{t}{\lambda_{c,ij}}\right) \end{aligned}$$

From the homogenous Markov model, the stochastic process $X_c(t)$ can be defined as

$$T_{c,n_c} \leq t < T_{c,n_c+1} \Rightarrow X_c(t) = X_{c,n_c} \tag{3.32}$$

The conditional state durations $D_{c,ij}$ are the stochastic durations for state i, given the fact that the next state will be j. An outcome of a history is obtained by drawing outcomes $d_{c,ik}$ for all conditional state durations $D_{c,ik}$ in each state, and selecting the smallest value. Then, with j such that $d_{c,ij} = \min(d_{c,ik})$, $X_{c,n_c+1} = j$ and $T_{c,n_c+1} = T_{c,n_c} + d_{c,ij}$.

The probability distribution for the duration $D_{c,i}$ of state $x_{c,i}$ is thus the distribution of the minimum of the conditional durations:

$$\begin{aligned} F_{c,i}(t) &= \Pr(D_{c,i} \leq t) = \Pr(\min_{j=1}^{N_c}(D_{c,ij}) \leq t) \\ &= 1 - \prod_{j=1}^{N_c} \Pr(D_{c,ij} > t) = 1 - \prod_{j=1}^{N_c} \exp\left(-\frac{t}{\lambda_{c,ij}}\right) \\ &= 1 - \exp\left(-\frac{t}{\lambda_{c,i}}\right) \qquad (3.33) \\ \frac{1}{\lambda_{c,i}} &= \sum_{j=1}^{N_c} \frac{1}{\lambda_{c,ij}} \qquad (3.34) \end{aligned}$$

The state duration is thus again exponentially distributed and is characterized by the single 'state transition rate' $\lambda_{c,i}$. The state transition rate is expressed in 'per time' units, and may thus be interpreted as a frequency. This frequency, however, expresses the number of transitions out of the state per time spend in the state, and not per total time. The state transition rate thus only equals the state frequency for components with just one state. For a component with two identical states, the state frequency will be half the state transition rate. The expected state duration can be directly calculated from the state transition rate as

$$E(D_{c,i}) = \frac{1}{\lambda_{c,i}} \tag{3.35}$$

For the transition probability $P_c(i,j) = \Pr(X_{c,n_c+1} = j \mid X_{c,n_c} = i)$ it follows that

$$\begin{aligned} P_c(i,j) &= \Pr(D_{c,ij} = \min_{k=1}^{N_c}(D_{c,ik})) \\ &= \int_0^\infty \Pr(\min_{k\neq j}(D_{c,ik}) \geq \upsilon)\ \frac{1}{\lambda_{c,ij}}\ e^{-\upsilon/\lambda_{c,ij}}\ d\upsilon \\ &= \int_0^\infty \frac{1}{\lambda_{c,ij}}\ e^{-\upsilon/\lambda_{c,i}}\ d\upsilon \\ &= \frac{\lambda_{c,i}}{\lambda_{c,ij}} \end{aligned} \tag{3.36}$$

Both the state duration distribution and the transition probabilities are thus independent of time and independent of the history of the system. By (3.36), a constant transition probability matrix $P_c = [P_c(i,j)]$ is defined for the Markov model. The fact that the transition probabilities are constant means that the sequence of states in a history of the component is independent of the time spend in those states. This sequence of states, $(X_{c,n_c})_{n_c=0}^{\infty}$, is called the “embedded Markov chain”. For Markov chains with stationary transition probabilities the so-called Markov-property holds, which can be written as

$$\begin{aligned} &\Pr(X_{c,n_c+m} = j \mid X_{c,0} = k, X_{c,1} = l, \cdots, X_{c,n_c} = i) \\ &\quad = \Pr(X_{c,n_c+m} = j \mid X_{c,n_c} = i) = P_c^{(m)}(i,j) \end{aligned} \tag{3.37}$$

where $P_c^{(m)}(i,j)$ is the value on the i,j position in the m^{th} power of P_c. For all $P_c^{(m)}$, $\forall_i \sum_{j=1}^{N_c} P^{(m)}(i,j) = 1.0$.

The Markov property tells us that the probability to find the component in a certain state after a certain number of transitions only depends on the number of transitions and on the starting state.

The homogenous Markov model may be graphically depicted as shown in Fig.3.8, which shows a Markov Model with three states. Because the state duration PDF and the transition probabilities can both be calculated from the transition rates, the only data needed to completely define a homogenous Markov model is the set of these transition rates λ_{ij}.

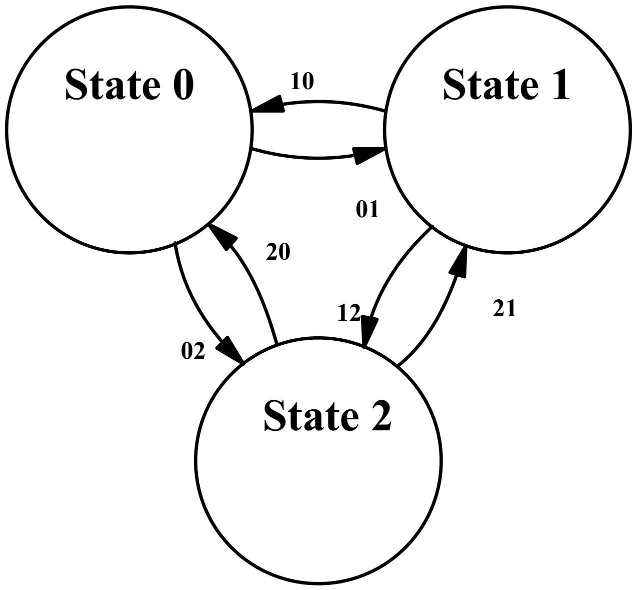


Figure 3.8: The homogenous Markov model

Both the transition probabilities and the conditional state durations in a homogenous Markov model are independent of the history of the system. This means that when the component is found to change to a certain state at a certain time, the probabilities for the next states and the distribution of the duration of the current state, are always known.

By calculating the state duration rates and the state transition probabilities from the transition rates, an alternative representation of the homogenous Markov model results. This representation is graphically depicted in Fig.3.9. Both representations are analogous. The transition rates can be calculated from the alternative model data as

$$\lambda_{ij} = \Pr_c(i,j) \cdot \lambda_i \tag{3.38}$$

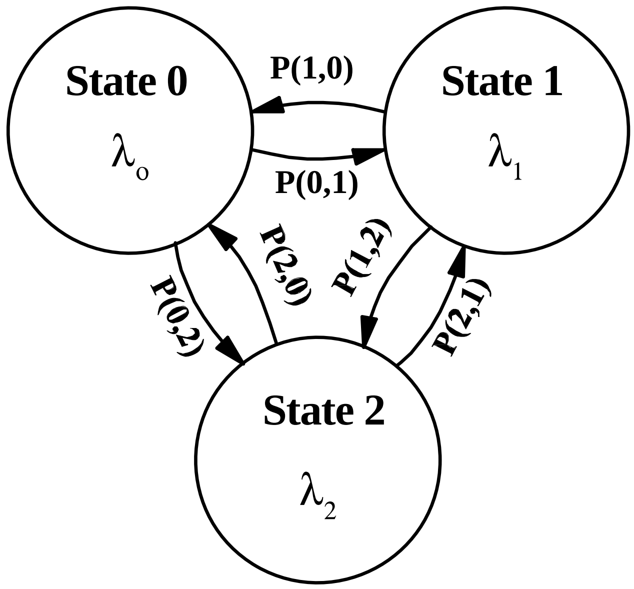


Figure 3.9: Alternative representation of the homogenous Markov Model

3.2.2 The Homogenous Markov System

A homogenous Markov system is a stochastic model of a power system for which all stochastic components are homogenous Markov components. The homogenous Markov system is nothing more than a combination of those components.

The homogenous Markov system is defined by

- the number of homogenous Markov components N
- the set of homogenous Markov components $\left((X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}\right)_{c=1}^{N}$
- the resulting stochastic system history $(S_{n_s}, T_{n_s})_{n_s=0}^{\infty}$, where
  - $S_{n_s} = (X_{1,n_s}, X_{2,n_s}, \cdots, X_{N,n_s})$ and $X_{c,n_s} = X_c(T_{n_s})$
  - $(T_{n_s})_{n_s=0}^{\infty} = \bigcup_{c=1}^{N}(T_{c,n_c})_{n=0}^{\infty}$

The system state $S(t)$ at any time t is thus the vector of component states $X_c(t)$ at that time. Because the number of possible states is limited for all components, the system state space is also limited. However, because the stochastic components are assumed to be stochastically independent, the number of possible system states is the product of the number of possible component states. For a moderate system of 100 components, each of which has two states, the size of the system state space is 2^{100} possible states.

The system changes state when at least one of its component changes its state. However, because all components are assumed to be stochastically independent, the probability of two of them changing state at the very same moment is zero:

$$\neg\exists_{a,b,n_a,n_b}(T_{a,n_a} = T_{b,n_b}) \tag{3.39}$$

For each system epoch T_{n_s}, there is thus exactly one component with the same epoch,

$$\forall_{n_s}\exists_c^1\exists_{n_c}(T_{n_s} = T_{c,n_c}) \tag{3.40}$$

The system thus changes state because one component changes state, and that component is therefore called the “causing component”. Each system state S_{n_s} has a single causing component. Two succeeding system states may have the same causing component.

For a system which changes to state S_{n_s} at time T_{n_s}, the remaining state duration for component c is defined as

$$D_c(n_s) = T_{c,n_{sc}+1} - T_{n_s} \tag{3.41}$$
$$n_{sc} = \sup\{n_c \in \mathbb{N}^+ \mid T_{c,n_c} \leq T_{n_s}\} \tag{3.42}$$

and the age of the component state as

$$A_c(n_s) = T_{n_s} - T_{c,n_{sc}} \tag{3.43}$$

These basic relations are illustrated by Fig.3.10.
The age of a component state is the time it has already spend in its current state at the start of a new system state, and the remaining duration is the

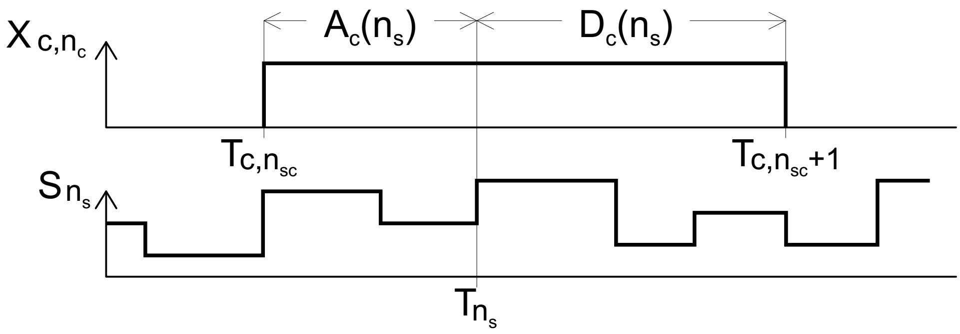


Figure 3.10: Component Age and Remaining Duration

time for which it will continue to stay in that state. The duration of the system state is the minimum of all remaining component state durations. According to (3.34), the state durations for the homogenous component are negative exponentially distributed. Because the conditional density function of a exponentially distributed duration for all inspection times equals the original distribution, the distribution of the remaining state duration is independent of the age of the state and equals the total component state duration distribution. Therefore,

$$F_{D_c(n_s)}(t) = 1 - \exp\left(-\frac{t}{\lambda_{n_{sc}}}\right) \tag{3.44}$$

where $\lambda_{n_{sc}}$ is the state transition rate for the state $X_{c,n_{sc}}$.

Because the system state duration is the minimum of the remaining component durations, the distribution of the system state duration can be calculated as:

$$\begin{aligned} F_{D_{n_s}}(t) &= \Pr(D_{n_s} < t) = \Pr(\min(D_c(n_s)) \leq t) \\ &= 1 - \exp\left(-\frac{t}{\lambda_{n_s}}\right) \end{aligned} \tag{3.45}$$

$$\frac{1}{\lambda_{n_s}} = \sum_{c=1}^{N} \frac{1}{\lambda_{n_{sc}}} \tag{3.46}$$

where $D_{n_s} = T_{n_s+1} - T_{n_s}$ is a stochastic system state duration.

From (3.45), it is clear that all system state durations are negative exponentially distributed. Because the minimum outcome for the remaining state durations for all components also determines the next system state, the same expressions as used in (3.36) can be used to show that the system state transition probabilities are independent of time. The conclusion is that the homogenous Markov system is itself again a homogenous Markov model. This is the most important quality of the homogenous Markov model.

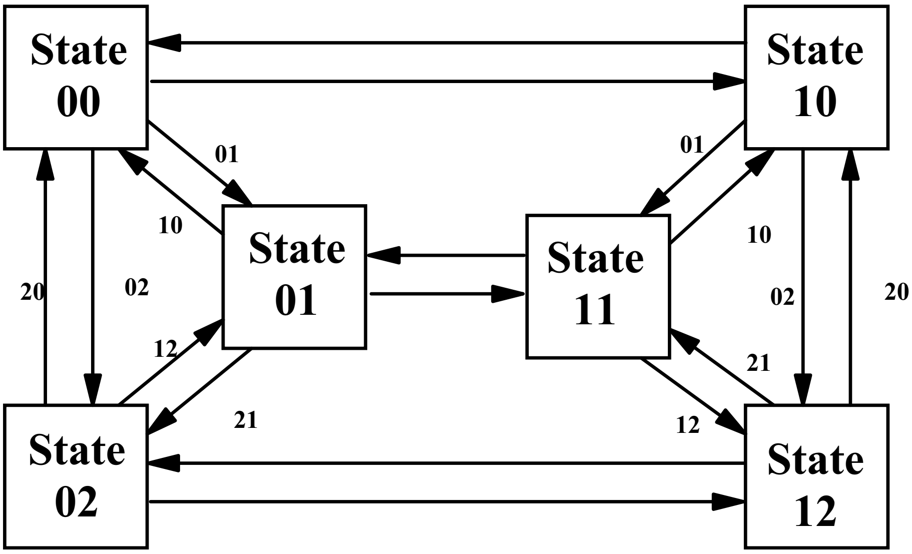


Figure 3.11: A homogenous Markov system

An example of a homogenous system is graphically depicted in Fig.3.11. This example system consists of two components, one with three and one with two states. The horizontal transitions in this picture are caused by a change of the two-state component, the other transitions are caused by the three-state component. From (3.44), it follows that the transition rates of the Markov system equal the corresponding component transition rates. In Fig.3.11, the transition rates of the three state component are shown. If the three state component would be the one shown in Fig.3.8, then the transition rates shown in Fig.3.11 and Fig.3.8 would be the same.

3.2.3 Device of Stages

As we will see later, we will need the distribution of system state durations in order to calculate some reliability worth indices correctly. The homogenous Markov model makes it possible to quickly calculate this for any system state, because all components use negative exponential distributions exclusively. The system state duration is therefore also distributed according to a negative exponential distributions, according to (3.45) and (3.46).

The exponential distribution, however, is not used because it is a good model for the component state durations, but only because it simplifies the calculations. The distribution of the Markov system therefore has no relation to the actual state duration distribution. It cannot be used for that reason to calculate reliability worth indices.

The alternative to using homogenous Markov models is to use non-homogenous models. The problem with these models, however, is that it is normally very hard, if not impossible, to calculate a system state duration distribution. The Weibull-Markov model, which will be introduced in the next chapters, forms an exception to this rule.

For non-homogenous models, it is commonly tried to convert the model into a Markovian model, preferably a homogenous one. One of the possible ways for such conversion is the method of the device of stages. Because this method is considered a general solution for the case of having non-homogenous component models, it is introduced here. It will be shown, however, that this method is not a solution to the problem of assessing interruption costs.

The method of the device of stages represents each state which has a non-exponential duration distribution by a combination of ‘virtual’ states that are exponentially distributed. The series and/or parallel combinations and the transitions rates of those virtual states are chosen so as to make the duration distribution of the transitions through the group of virtual states as good an approximation as possible of the original non-exponential distribution. The representation of a single non-exponential state by a combination of exponential states is illustrated by Fig.3.12, for a two-state component. The state “1” of this component is non-exponential and is therefore converted to a series-parallel combination of 5 exponentially distributed states. The transition rates for these 5 virtual states should now be chosen so that the distribution of the duration between entering state “a” or “c” and leaving state “e” should equal, or approximate, the duration distribution of state “1”.

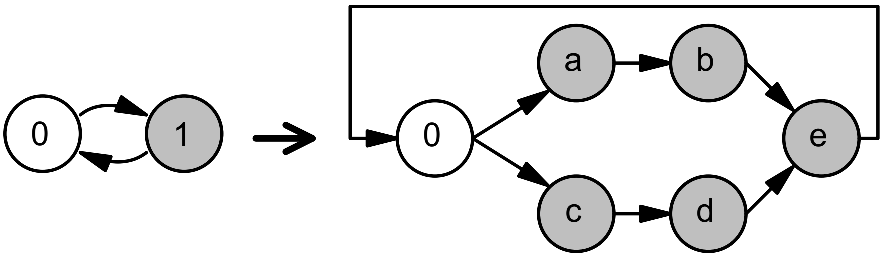


Figure 3.12: Example of a device of stages

The method of the device of stages can be used to calculate time dependent state probabilities, and may thus be used for addressing ageing effects, effects of preventive maintenance, or other time-dependent behaviour of the component. However, it is not a solution to the problem of calculating system state duration distributions. This is best illustrated by an example. In Fig.3.13, a system with two components is depicted, each of which has two states. Assumed is that the UP state of these components (“0” and “a”) are negative-exponentially distributed, but the DOWN states (“1” and “b”) are Weibull distributed. The system is supposed to function when at least one component is in the UP state. The question therefore is to find an expression of the distribution of the system down time, which is the distribution of the duration of system state “1b”, which is shown in grey in Fig.3.13.

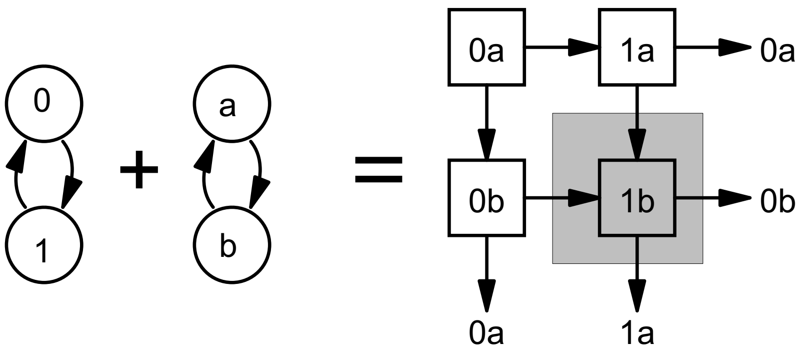


Figure 3.13: System with two two-state components

Even for this very simple system, the expression for the system down time distribution is not simple because it is the distribution of the minimum of a Weibull distribution with the remaining distribution of another Weibull distribution, for which the age is unknown.

In Fig.3.14, the two down states of the two components have been converted to a series of two and three virtual states: ‘1’, ‘2’ and ‘b’, ‘c’, ‘d’. The resulting system now has 12 possible states, of which 6 are down states. The system has now become homogenous Markovian, and for each of the states in Fig.3.14, the duration is negative exponentially distributed. However, in order to develop an expression of the duration distribution of the combination of the six system down states, we would have to account not only for the six states, but also for every way in which the system could change from an up state to a down state and back again, and for every possible route between entering and leaving the part of the state space with down states. For a combination of a number of virtual states in series, or a number of virtual states in parallel, such overall distributions can be found. For more complex virtual state spaces, such as the one in Fig.3.14, no general way of calculating the overall duration distribution was found in literature.

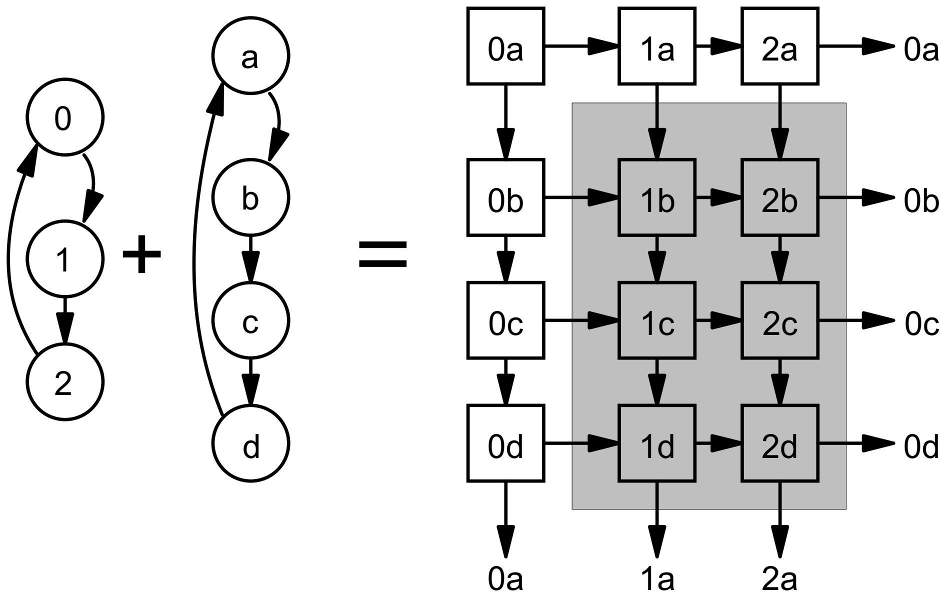


Figure 3.14: Converted components and resulting system

The “device of stage” method therefore offers no solution to the problem of finding the system state duration distribution.

3.3 Weibull-Markov Models

The Weibull-Markov model is a non-homogenous Markov model. Although the mathematics involved in defining and using the Weibull-Markov model are somewhat more complex than those used for the homogenous Markov model, it will be shown that this model lends itself for all types of reliability calculations possible with the homogenous Markov model, and yet enables a correct analytical calculation of interruption costs.

The definition of the Weibull-Markov component starts by altering the homogenous component by using not a negative exponential distribution, but a Weibull distribution for the conditional state durations. The result is a stochastic component, defined by

- the set of possible states $\overline{x_c} = \{1, 2, \cdots, N_c\}$ where N_c is the number of possible states
- the stochastic history $(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$, where
  - $\forall_{n_c}(X_{c,n_c} \in \overline{x_c}, X_{c,n_c} \neq X_{c,n_c+1})$
  - $T_{c,0} = 0$ and $\forall_{n_c}(T_{c,n_c+1} > T_{c,n_c})$
- the stochastic process $X_c(t) = X_{c,n_c}$ for $T_{c,n_c} \leq t < T_{c,n_c+1}$
- the set of continuous probability distribution functions $F_c(t)$ for the conditional state durations $D_{c,ij}$

$$
\begin{aligned}
F_{c,ij}(t) &= \Pr(D_{c,ij} \leq t) \\
&= \Pr(X_{c,n_c} = i\ ,\ (T_{c,n+1} - T_{c,n_c}) \leq t \mid X_{c,n+1} = j) \\
&= 1 - \exp\left(-\left(\frac{t}{\eta_{c,ij}}\right)^{\beta_{c,ij}}\right)
\end{aligned}
$$

This model with Weibull distributions, which has independent duration distributions defined for each transition separately, equals the homogenous Markov model when all shape factors $\beta_{c,ij}$ equal one.

The above model is mathematically problematic in the sense that it it is even hard to derive a useful expression for the component state duration distribution. As with the homogenous model, the next state and the current state duration are determined by drawing outcomes for all conditional state durations and selecting the lowest one. The probability distribution for the duration $D_{c,i}$ of state $x_{c,i}$ is therefore the distribution of the minimum of the conditional durations:

$$\begin{aligned}
F_{c,i}(t) &= \Pr(D_{c,i} \leq t) = \Pr(\min_{j=1}^{N_c}(D_{c,ij}) \leq t) \\
&= 1 - \prod_{j=1}^{N_c} \Pr(D_{c,ij} > t) = 1 - \prod_{j=1}^{N_c} \exp\left(- \left(\frac{t}{\eta_{c,ij}} \right)^{\beta_{c,ij}} \right) \\
&= 1 - \exp\left(\sum_{j=1}^{N_c} - \left(\frac{t}{\eta_{c,ij}} \right)^{\beta_{c,ij}} \right) \qquad (3.47)
\end{aligned}$$

Expression (3.47) can be simplified drastically by taking a same shape factor for all conditional state durations in a same state:

$$\beta_{c,ij} = \beta_{c,i} \qquad (3.48)$$

For such “same shape” models,

$$F_{c,i}(t) = 1 - \exp\left(\left(- \frac{t}{\eta_{c,i}} \right)^{\beta_{c,i}} \right) \qquad (3.49)$$

$$\left(\frac{1}{\eta_{c,i}} \right)^{\beta_{c,i}} = \sum_{i=1}^{N_c} \left(\frac{1}{\eta_{c,ij}} \right)^{\beta_{c,i}} \qquad (3.50)$$

The state duration for a “same-shape” Weibull distributed component is thus again Weibull distributed with the scale factor given by (3.50). The stochastic component as defined above, together with (3.48) is called a **“Weibull-Markov”** component.

With the expression for the state duration distribution, the transition probability matrix for the Weibull-Markov component can be derived as

$$
\begin{aligned}
P_c(i,j) &= \Pr(D_{c,ij} = \min_{k=1}^{N_c}(D_{c,ik})) \\
&= \int_0^\infty \Pr(\min_{k\neq j}(D_{c,ik}) \geq \upsilon)\ \frac{\beta_{c,i}\upsilon^{\beta_{c,i}-1}}{\eta_{c,ij}^{\beta_{c,i}}}\ e^{-(\upsilon/\eta_{c,ij})^{\beta_{c,i}}}\ d\upsilon \\
&= \int_0^\infty \frac{\beta_{c,i}\upsilon^{\beta_{c,i}-1}}{\eta_{c,ij}^{\beta_{c,i}}}\ e^{-(\upsilon/\eta_{c,i})^{\beta_{c,i}}}\ d\upsilon \\
&= \frac{\eta_{c,i}}{\eta_{c,ij}} \qquad (3.51)
\end{aligned}
$$

The transition probabilities for the Weibull-Markov component are thus independent of time and independent of the history of the component. The Weibull-Markov model is thus a semi-Markov model. With the stationary transition probabilities, it is clear that X_{c,n_c} is again an embedded Markov-chain for which

$$
\begin{aligned}
&\Pr(X_{c,n_c+m} = j \mid X_{c,0} = k, X_{c,1} = l, \cdots, X_{c,n_c} = i) \\
&\qquad = \Pr(X_{c,n_c+m} = j \mid X_{c,n_c} = i) = P_c^{(m)}(i,j)
\end{aligned}
$$

where $P_c^{(m)}(i,j)$ is the value on the i,j position in the m^{th} power of P_c. For all $P_c^{(m)}$, $\forall_i \sum_{j=1}^{N_c} P^{(m)}(i,j) = 1.0$.

Expressions (3.50) and (3.51) are equivalent to their homogenous counterparts (3.36) and (3.34). A graphical representation of the Weibull-Markov component is shown in Fig.3.15.

The definition and basic equations of the Weibull-Markov model look very much alike those of the homogenous Markov model, except for the additional distribution shape parameter β. The requirement that the new model should be compatible with the homogenous Markov model is met: all homogenous Markov models can be transformed into Weibull-Markov models without a need for new reliability data, by using $\beta = 1$ in all states. Existing measurements of component reliability can be used at any convenient time to calculate more realistic shape factors. It is possible to gradually substitute homogenous data by more realistic Weibull-Markov data.

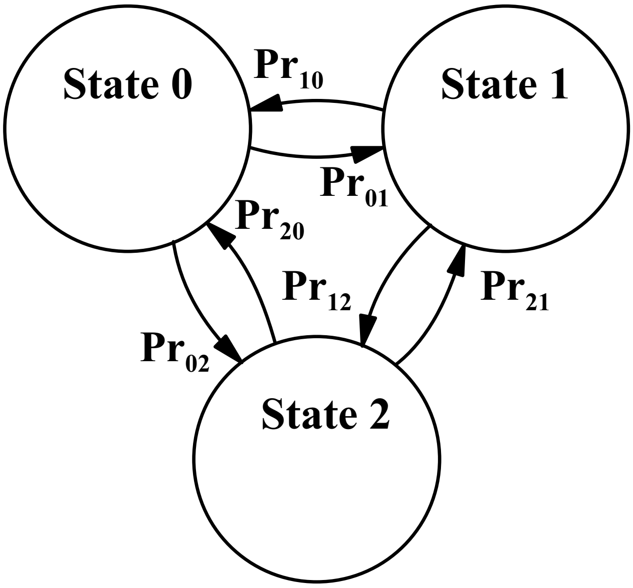


Figure 3.15: The Weibull-Markov Model

3.3.1 Component State Probability and Frequency

The fact that the Weibull-Markov model is fully compatible with the homogenous Markov model is very important. Even more important is the requirement that it should be possible to analyze Weibull-Markov components by fast and exact methods. These component analyses are needed to calculate component state probabilities and frequencies, which are needed to speed up a system analysis. The assessment of system reliability indices does not ask for 'solving' a large Weibull-Markov model. As will be shown, a system build from Weibull-Markov models is very hard to analyze as a whole with analytic methods. However, by 'pre-processing' the components, the system properties can be assessed by adding the contributions of the components.

The state probabilities and frequencies of a Weibull-Markov component can be calculated by first regarding the embedded Markov chain. For any Weibull-Markov model, leaving out the component index, the chain state probability vector is defined as

$$Q_n = [\Pr(X_n = 1), \Pr(X_n = 2), \cdots, \Pr(X_n = N_c)] \tag{3.52}$$

The chain state probability vector can be calculated from the initial state probability after n state transitions as

$$Q_n = P^{(n)} \cdot Q_0 \tag{3.53}$$

where

$$P = [P(i,j)] = \left[\begin{array}{ccccc} P(1,1) & P(1,2) & P(1,3) & \cdots & P(1,N) \\ P(2,1) & P(2,2) & P(2,3) & \cdots & \\ \vdots & & & & \vdots \\ P(N,1) & \cdots & & & P(N,N) \end{array} \right]$$

and $P(i,j) = \Pr(X_{n+1} = j \mid X_n = i)$

In [119] it is shown that for a Markov chain with stationary transition probabilities, the long term state probabilities can be found by solving

$$Q = P \cdot Q \tag{3.54}$$

where

$$Q = [Q(1), Q(2), \cdots, Q(N)] = \lim_{n \to \infty} Q_n \tag{3.55}$$

With the Weibull-Markov component, there are no self-transitions, and therefore $\forall_i (P(i,i) = 0)$. The long term Markov chain state probabilities can be solved by using

$$\forall_i \left(\sum_{j=1}^{N_c} P(i,j) = 1.0 \right) \tag{3.56}$$

$$\sum_{i=1}^{N_c} Q(i) = 1.0 \tag{3.57}$$

and by taking

$$\begin{aligned} A_{ij} &= \begin{cases} P(i,N) + 1 & \text{if } i = j \\ P(i,n) - P(i,j) & \text{if } i \neq j \end{cases} \\ b_i &= P(i,N) \\ Q' &= [Q(1), Q(2), \cdots, Q(N-1)] \end{aligned}$$

A solution for Q' can then be found by solving

$$A \cdot Q' = b \tag{3.58}$$

and $Q(N)$ is found by using (3.57) again.

The state probabilities for the Weibull-Markov component can be calculated from the embedded state probabilities as

$$\mathrm{Pr}(i) = \frac{Q(i) * E(D_i)}{\sum_{i=1}^{N} Q(i) * E(D_i)} \tag{3.59}$$

where $E(D_i) = \eta_i \Gamma\left(1 + \frac{1}{\beta_i}\right)$ is the expected duration of state i.

With the known Weibull-Markov state probabilities $\mathrm{Pr}(i)$, the state frequencies $\mathrm{Fr}(i)$ can be calculated as

$$\mathrm{Fr}(i) = \frac{\mathrm{Pr}(i)}{E(D_i)}$$

3.4 The Weibull-Markov System

The Weibull-Markov system is defined in the same way as the homogenous Markov system; as a stochastic model of a power system for which all stochastic components are Weibull-Markov components.

The Weibull-Markov system is thus defined by

- the number of Weibull-Markov components N
- the set of Weibull-Markov components $\left((X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}\right)_{c=1}^{N}$
- the resulting stochastic system history $(S_{n_s}, T_{n_s})_{n_s=0}^{\infty}$, where
  - $S_{n_s} = (X_{1,n_s}, X_{2,n_s}, \cdots, X_{N,n_s})$ and $X_{c,n_s} = X_c(T_{n_s})$
  - $(T_{n_s})_{n_s=0}^{\infty} = \bigcup_{c=1}^{N} (T_{c,n_c})_{n=0}^{\infty}$

Except for the different component state duration distributions, the Weibull-Markov system behaves in the same way as the homo/-genous Markov system. Again, the probability of two components changing state at the very same moment is zero:

$$\neg\exists_{a,b,n_a,n_b}(T_{a,n_a} = T_{b,n_b}) \tag{3.60}$$

and thus

$$\forall_{n_s}\exists^1_c\exists_{n_c}(T_{n_s} = T_{c,n_c}) \tag{3.61}$$

Identical definitions for the remaining state duration and the age of the components c for all system states are used.

$$D_c(n_s) = T_{c,n_{sc}+1} - T_{n_s} \tag{3.62}$$

$$n_{sc} = \sup\{n_c \in \mathbb{N}^+ \mid T_{c,n_c} \leq T_{n_s}\} \tag{3.63}$$

$$A_c(n_s) = T_{n_s} - T_{c,n_{sc}} \tag{3.64}$$

However, where the distribution of the remaining duration of the state of a homogenous component equals the distribution of the complete duration, this is generally not the case for a Weibull-Markov component. For a Weibull-Markov component, the distribution of the remaining duration normally depends on the history of the system. As the system state duration is the minimum of the remaining component state durations, this dependency makes it very hard, if not impossible, to derive exact expressions for the system state duration distribution as a whole. Even more important is that also the probabilities for the following system state become history dependent, as they are too determined by the smallest outcome of the remaining component state durations.

From this it is clear that the Weibull-Markov system is not only a non-homo/-genous model, but even not a Markov model. It will however be shown that the Weibull-Markov system will becomes a semi-Markov system again when it is assumed to be stationary.

3.4.1 Weibull-Markov System State Probability and Frequency

The Weibull-Markov model can only be an alternative to the homogenous Markov model if it is possible to calculate state probabilities and frequencies analytically with comparable computational efforts.

The probability for a system state s is defined as the probability to find the system in that state at time t and is written as $\mathrm{Pr}(s,t) = \mathrm{Pr}(S(t) = s)$. For any moment in time:

$$\sum_{s=1}^{N_s} \mathrm{Pr}(S(t) = s) = 1 \tag{3.65}$$

where N_s is the number of possible system states.
As all component are assumed to be statistically independent, the system state probability is the product of the component state probabilities:

$$\mathrm{Pr}(s,t) = \prod_{c=1}^{N} \mathrm{Pr}(X_c(t) = x_c) \tag{3.66}$$

The frequency of a system state s is defined as the density of the number of transitions into the system state per unit of time, for a certain moment in time.

$$\mathrm{Fr}(s,t) = lim_{\Delta t \to 0} \frac{E(\text{number of transitions to s in}[t, t+\Delta t])}{\Delta t} \tag{3.67}$$

If we write a system with only two components as S^2, the upper index being the number of components, then the frequency of the system state $s^2 = (x_1, x_2)$ at t is the frequency of $X_1(t) = x_1$ times the probability of $X_2(t) = x_2$, plus the frequency of $X_2(t) = x_2$ times the probability of $X_1(t) = x_1$.

$$\mathrm{Pr}(s^2,t) = \mathrm{Pr}(x_1,t) \cdot \mathrm{Pr}(x_2,t) \tag{3.68}$$
$$\mathrm{Fr}(s^2,t) = \mathrm{Fr}(x_1,t) \cdot \mathrm{Pr}(x_2,t) + \mathrm{Fr}(x_2,t) \cdot \mathrm{Pr}(x_1,t) \tag{3.69}$$

This can be repeated for a third component and the two-component system:

$$\mathrm{Pr}(s^3,t) = \mathrm{Pr}(s^2,t) \cdot \mathrm{Pr}(x_3,t) \tag{3.70}$$
$$\mathrm{Fr}(s^3,t) = \mathrm{Fr}(s^2,t) \cdot \mathrm{Pr}(x_3,t) + \mathrm{Fr}(x_3,t) \cdot \mathrm{Pr}(s^2,t) \tag{3.71}$$

and, by induction, for the whole system:

$$\mathrm{Pr}(s,t) = \mathrm{Pr}(s^{N-1},t) \cdot \mathrm{Pr}(x_N,t) \tag{3.72}$$
$$\mathrm{Fr}(s,t) = \mathrm{Fr}(s^{N-1},t) \cdot \mathrm{Pr}(x_N,t) + \mathrm{Fr}(x_N,t) \cdot \mathrm{Pr}(s^{N-1},t) \tag{3.73}$$

These recursive equations for probability and frequency are independent of the state duration distributions. The recursive equation for the system state frequency can be rewritten into

$$\mathrm{Fr}(s,t) = \sum_{c=1}^{N} \left(\mathrm{Fr}(x_c,t) \cdot \prod_{d=1,d\neq c}^{N} \mathrm{Pr}(x_d,t) \right) \tag{3.74}$$

or into

$$\mathrm{Fr}(s,t) = \mathrm{Pr}(s,t) \cdot \sum_{c=1}^{N} \frac{\mathrm{Fr}(x_c,t)}{\mathrm{Pr}(x_c,t)} \tag{3.75}$$

The equality (3.74) is known as the state frequency balance.

For the stationary system, for $t \to \infty$, the component state probabilities and frequencies become time independent and are written as $\mathrm{Pr}(x_c)$ and $\mathrm{Fr}(x_c)$. The system state probability and frequency will then also become time independent as

$$\mathrm{Pr}(s) = \lim_{t\to\infty} \mathrm{Pr}(s,t) = \prod_{c=1}^{N} \mathrm{Pr}(x_c) \tag{3.76}$$

$$\mathrm{Fr}(s) = \lim_{t\to\infty} \mathrm{Fr}(s(t)) = \mathrm{Pr}(s) \cdot \sum_{c=1}^{N} \frac{\mathrm{Fr}(x_c)}{\mathrm{Pr}(x_c)} \tag{3.77}$$

For the stationary system, the expected state duration, or ‘state expectancy’, in units per time per unit of time, equals the state probability. The expected system state duration is then calculated by dividing the system state expectancy by the system state frequency:

$$E(D_s) = \mathrm{Pr}(s)/\,\mathrm{Fr}(s) \tag{3.78}$$

In most reliability assessment calculations, the unit of frequency is taken as $1/a$, where a = annum = 8760 hours, and the expectancy is expressed in hours. In that case,

$$E(D_s) = \mathrm{Pr}(s) \cdot 8760/\,\mathrm{Fr}(s) \text{ hours} \tag{3.79}$$

3.4.2 Weibull-Markov System State Duration Distribution

For homogenous systems, the system state duration distribution is found without problems. Because the system too is a homogenous Markov model, the system state duration will be distributed according to a negative exponential distribution, with a duration rate which is the reciprocal sum of all corresponding component transition rates.

To find an expression for the state duration distribution of a Weibull-Markov system, it is important that we consider stationary systems only. This means that all component models in the system are stationary, and the history for each component beyond the last state change is irrelevant.

Suppose a stationary system with two components. If we would monitor such a system for a short period, a graph as depicted in Fig.3.16 could be the result. It is now possible to regard the epochs of the one component as

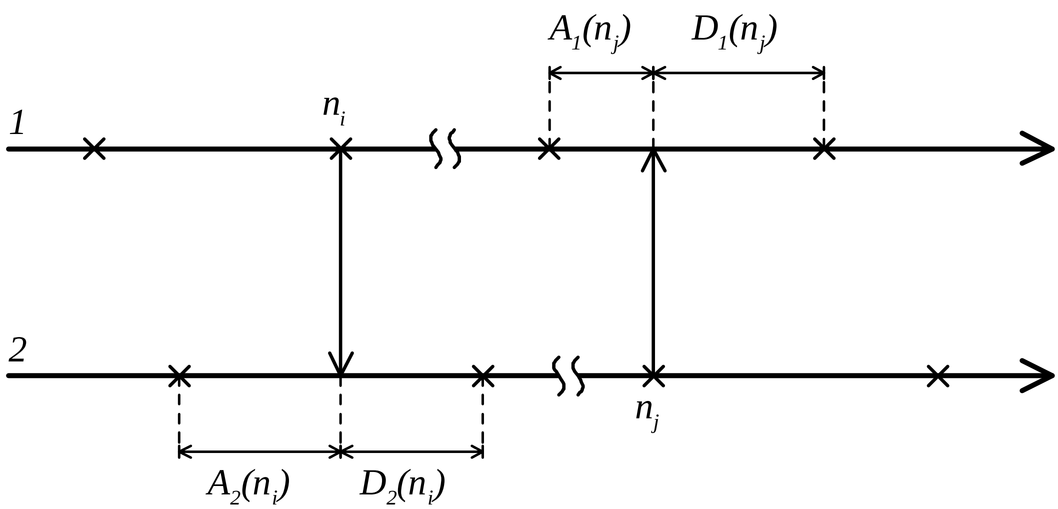


Figure 3.16: Epochs and inspection times for a two-component system

inspection times for the other, and vice versa, as is depicted in Fig.3.16. As long as each system state is treated separately, and because both components are stationary and stochastically independent, each separate epoch of the one is a random inspection time for the other.

The derivation of the expression for the system state duration distribution starts with a well known result from renewal theory, according to which a component X_c, when found in state i at a random inspection time τ, has a remaining state duration distribution according to

$$\Pr(D_{c,i} - \tau < D \mid D_{c,i} > \tau) = \frac{1}{M_{c,i}} \int_0^D [1 - F_{c,i}(t)]dt \tag{3.80}$$

where $M_{c,i}$ is the mean duration of $X_c = i$ and $F_{c,i}(t) = \Pr(D_{c,i} < t)$.

By using the random epochs of the one component as an inspection time for the other, the remaining state duration $D_1(n_j)$ and $D_2(n_i)$ in Fig.3.16 will thus respect (3.80).

According to (3.80), the distribution of the remaining state duration is independent of the passed state duration, $D_{c,i} - \tau$, at the moment of inspection τ, for all types of distributions for the total state duration $F_{c,i}(t)$. From this, it follows that the probability of X_c, when found in state i at time τ, to not change state in the interval $\tau \leq t \leq \tau + D$ is

$$\Pr(X_c(\tau + D) = i \mid X_c(\tau) = i) = \frac{1}{M_{c,i}} \int_D^\infty [1 - F_{c,i}(t)]dt \tag{3.81}$$

Because a system will change as soon as one of its components changes, the probability of a system, consisting of N components, which is found in state s at time τ, with $T_{n_s} < \tau < T_{n_s+1}$, to not change state in the interval $\tau \leq t \leq \tau + D$ is

$$\Pr(S(\tau + D) = s \mid S(\tau) = s) = \prod_{c=1}^{N} \frac{1}{M_{c,n_s}} \int_D^\infty [1 - F_{c,n_s}(t)]dt \tag{3.82}$$

where M_{c,n_s} is the mean duration of X_{c,n_s} and $F_{c,n_s}(t) = \Pr(D_{c(n_s)} < t)$.

Expression (3.82) is an important first result, because it shows that it is possible to express the remaining system state duration distribution in terms of component state properties. However, we do not want to calculate the distribution of the remaining system state duration for arbitrary inspection times, but the distribution for the whole state duration. The inspection time in (3.82) then equals the moment at which the system state starts, which is the moment at which the causing component changes its state. The distribution of the ‘remaining’ state duration for that one component will thus equal the distribution of the total state duration. For all other components, we have to use (3.81), as they have already spend some time in their state at the moment the new system state starts.

If we suppose that component X_c is the causing component for system state S_{n_s}, then it follows that he probability of that system state, when its starts at epoch T_n because component X_c changed state at T_n, to last longer than D, is

$$\begin{aligned}&\Pr(S(T_{n_s}+D)=S_{n_s} \mid X_{c,n_s}\neq X_{c,n_s-1})=\\&\quad=[1-F_{c,n_s}(D)]\prod_{k\neq c}^{N}\left(\frac{1}{M_{k,n_s}}\int_D^\infty[1-F_{k,n_s}(t)]dt\right)\end{aligned} \tag{3.83}$$

All that is left now to do is to find an expression for the probability for each component to be the causing component for system state S_{n_s}. Equation (3.83) can then be weighted by that probability and the distribution of the system state duration can then be found by summing the weighted expressions for each component.

The probability that X_c is the causing component of system state S is the fraction of occurrences of that system state which happen due to a change of X_c. That fraction of occurrences is the relative system state frequency due to X_c, which is written as $\mathrm{Fr}_s(X_c)$. If $s=(x_{1,s},\cdots,x_{c,i},\cdots,x_{N,s})$, then s can be reached by any transition of component X_c from $x_{c,j\neq i}$ to $x_{c,i}$. Possible previous states of s are therefore all $(x_{1,s},\cdots,x_{c,j\neq i},\cdots,x_{N,s})$. This is depicted in Fig.3.17.

For each of the transitions shown in Fig.3.17, the absolute transition frequency equals

$$\mathrm{Fr}(X_{c,n_s-1}=j \mid X_{c,n_s}=i)=\mathrm{Fr}_c(j,i)\cdot\prod_{k\neq c}^{N}\Pr(x_{k,s}) \tag{3.84}$$

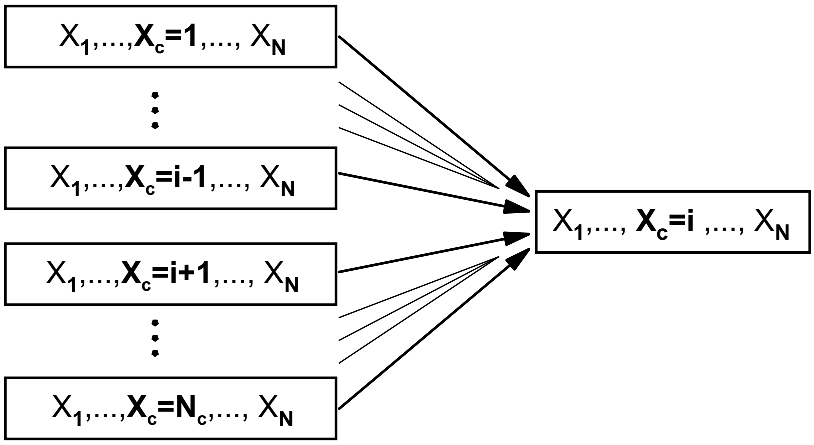


Figure 3.17: Transitions due to changing X_c

and the total transition frequency for any of these states to S thus equals

$$\begin{aligned}\mathrm{Fr}(X_{c,n_s-1} \neq i \mid X_{c,n_s} = i) &= \sum_{j \neq i}^{N_c} \mathrm{Fr}(X_{c,n_s-1} = j \mid X_{c,n_s} = i) \\ &= \mathrm{Fr}_{c,i} \prod_{k \neq c}^{N} \Pr(x_{k,s}) \qquad (3.85)\end{aligned}$$

For (3.85), it was used that $\mathrm{Fr}_{c,i} = \sum_{j \neq i}^{N_c} \mathrm{Fr}_c(j,i)$, which expresses the fact that a component state frequency is the sum of the absolute transition frequencies into that state.

With (3.85), the probability that X_c causes S is now expressed as

$$\begin{aligned}\Pr(X_{c,n_s-1} \neq X_{c,n_s}) &= \frac{\mathrm{Fr}(x_{c,s}) \prod_{k \neq c}^{N} \Pr(x_{k,s})}{\sum_{c=1}^{N} \left(\mathrm{Fr}(x_{c,s}) \prod_{k \neq c}^{N} \Pr(x_{k,s}) \right)} \qquad (3.86) \\ &= \frac{\mathrm{Fr}(x_{c,s}) / \Pr(x_{c,s}))}{\sum_{c=1}^{N} \mathrm{Fr}(x_{c,s}) / \Pr(x_{c,s})} \qquad (3.87)\end{aligned}$$

which results in

$$\Pr(X_{c,n_s-1} \neq X_{c,n_s}) = \frac{1/M_{c,s}}{\sum_{c=1}^{N} 1/M_{c,s}} \qquad (3.88)$$

For a Weibull-Markov system, (3.83) takes the form of

$$\begin{aligned}
&\Pr(S(T_{n_s}+D) = S_{n_s} \mid X_{c,n_s} \neq X_{c,n_s-1}) \\
&= e^{-\left(\frac{D}{\eta_{c,s}}\right)^{\beta_{c,s}}} \prod_{k\neq c}^{N} \frac{1}{M_{k,s}} \int_k^{\infty} e^{-(\frac{t}{\eta_{k,s}})^{\beta_{k,s}}} dt && (3.89) \\
&= e^{-\left(\frac{D}{\eta_{c,s}}\right)^{\beta_{c,s}}} \prod_{k\neq c}^{C} \frac{\eta_{k,s}}{M_{k,s}\beta_{k,s}} \left[\Gamma\left(\frac{1}{\beta_{k,s}}\right) - \Gamma\left(\frac{1}{\beta_{k,s}}, (\frac{D}{\eta_{k,s}})^{\beta_{k,s}}\right)\right] && (3.90)
\end{aligned}$$

where $\Gamma(x, y)$ denotes the incomplete gamma function for x from 0 to y.

The combination of (3.88) and (3.90) leads to the following expression for the probability for the duration of system state S to last longer than D:

$$\begin{aligned}
&\Pr(T_{n_s+1} - T_{n_s} > D) = \\
&\sum_{c=1}^{N} (\Pr(X_{c,n_s-1} \neq X_{c,n_s}) \cdot \Pr(S(T_{n_s}+D) = S_{n_s} \mid X_{c,n_s} \neq X_{c,n_s-1})) \\
&\qquad = \frac{\prod_{c=1}^{N} \gamma_{c,s}(D)/M_{c,s}}{\sum_{c=1}^{N} 1/M_{c,s}} \sum_{c=1}^{N} \frac{\exp\left(-(D/\eta_{c,s})^{\beta_{c,s}}\right)}{\gamma_{c,s}(D)} && (3.91)
\end{aligned}$$

where

$$\gamma_{c,i}(D) = \frac{\eta_{c,i}}{\beta_{c,i}} \left[\Gamma\left(\frac{1}{\beta_{c,i}}\right) - \Gamma\left(\frac{1}{\beta_{c,i}}, \left(\frac{D}{\eta_{c,i}}\right)^{\beta_{c,i}}\right)\right] \qquad (3.92)$$

From equation 3.91 and 3.92, it is clear that the system state duration distribution is independent of the previous system states in the steady state case. The $\gamma_{c,i}(D)$ function only depends on the component state duration distribution parameters β and η. The $\gamma_{c,i}(D)$ values for each component can thus be calculated prior to the actual reliability assessment.

3.5 Basic Power System Components

This chapter shows a possible implementation of a Weibull-Markov model for modeling stochastic power system components. The proposed methods are introduced on the basis of a model for the synchronous generator.

3.5.1 Defining a Weibull-Markov Model

The Weibull-Markov model is determined by the following set of parameters.

- N, the number of states
- $\{\beta_i\}$, the set of form-factors, one for each state
- $\{\eta_i\}$, the set of characteristic times, one for each state
- P, the transition probability matrix
- Electrical parameters, which define the electrical model for the component for each state

It is however possible to enter the state duration parameters other than by β and η, as in many cases, these are unknown. Alternatively,

- $\{\mu_i\}$, the mean state durations
- $\{\sigma_i\}$, the state duration variances

should be possible too.

Any two of the resulting possible state duration parameters, β_i, η_i, μ_i, σ_i, will determine the other two. The following conversion formulas can be used:

$$
\begin{aligned}
\mu(\beta,\eta) &= \eta\Gamma_1(\beta) \\
\mu(\beta,\sigma) &= \Gamma_1(\beta)\sqrt{\frac{\sigma^2}{\Gamma_2(\beta)}} \\
\sigma(\beta,\eta) &= \sqrt{\eta^2\Gamma_2(\beta)} \\
\sigma(\beta,\mu) &= \frac{\mu\sqrt{\Gamma_2(\beta)}}{\Gamma_1(\beta)} \\
\beta(\eta,\mu) &= \Gamma_1^{\text{inv}}\left(\frac{\mu}{\eta}\right) \\
\beta(\eta,\sigma) &= \Gamma_2^{\text{inv}}\left(\frac{\mu}{\eta}\right) \\
\beta(\mu,\sigma) &= \Gamma_3^{\text{inv}}\left(\frac{\mu^2}{\eta^2}\right) \\
\eta(\beta,\mu) &= \frac{\mu}{\Gamma_1(\beta)} \\
\eta(\beta,\sigma) &= \sqrt{\frac{\sigma^2}{\Gamma_2(\beta)}}
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma_1(\beta) &= \lambda\Gamma(1+\frac{1}{\beta}) \\
\Gamma_2(\beta) &= \Gamma(1+\frac{2}{\beta}) - \Gamma(1+\frac{1}{\beta})^2 \\
\Gamma_3(\beta) &= \frac{\Gamma_1(\beta)^2}{\Gamma_2(\beta)}
\end{aligned}
$$

The inversion of $\Gamma_1(\beta)$, $\Gamma_2(\beta)$ and $\Gamma_3(\beta)$ can be performed by newton methods. De values for $\mu(\eta,\sigma)$, $\sigma(\eta,\mu)$ and $\eta(\mu,\sigma)$ can be calculated by first calculating the corresponding β.

The above set of conversion formulas enables an easy definition of a Weibull-Markov model. However, it is also possible to leave all form-factors $\{\beta_i\}$ to their default values of one, and enter the Weibull-Markov model as a homogenous Markov model. Such would be needed if no other than homogenous model data is available. In stead of $\{\beta_i\}$, $\{\eta_i\}$ and P, the homogenous Markov model requires the input of:

- $\{\lambda_{ij}\}$, the matrix of state transition rates

This asks for the conversion of the state transition matrix to the transition probability matrix P. Such conversion can be done by using (3.36).

The back transformation from the state probability matrix to a state transition matrix can be performed by using the calculated state duration means and (3.38). This back transformation makes it possible to switch between 'homogenous input mode' and 'Weibull-Markov input mode', which may be used to check the validity of the model or to check the correct transformation of a homogenous model into a more realistic Weibull-Markov model.

3.5.2 A Generator Model

The basic stochastic generator model is fairly primitive, as it only defines states for the generator being available or not available. In many applications, such a two-state model is sufficient. However, there are several reasons to include more states to account for partial outages of the generator. Such a partial outage is a condition in which the generator is still connected to the net and is still producing power, but in which the maximum output is limited. Such a situation may occur when

- A run-of-the-river hydro turbine is reduced in capacity due to a low river level.
- A large, multi-machine thermal power plant is reduced in capacity due to the outage of one or more generator units.
- The available power is reduced due to the outage of a sub-component, such as a pulverizer, a fan, a feed water or cooling water pump, etc.

A state that models a partial outage is called a "derated state". In order to account for derated states, which number may differ from generator to generator, it must be possible to freely define new states.

The implementation of the dialog which is used to define a stochastic generator is shown in Fig.3.18. This example shows a generator with two derated states which has been defined as a Weibull-Markov model after which the dialog was changed to the homogenous Markov mode. It therefore shows the transition rate matrix and it would be possible to edit this matrix. This might be useful when stochastic data is available in several different formats, which are all to be translated into state transition rate models. The entry for the “dependent state” in the figure is needed to calculate the remaining transition probability, in order to make sure that the transition probabilities add up to one for each state.

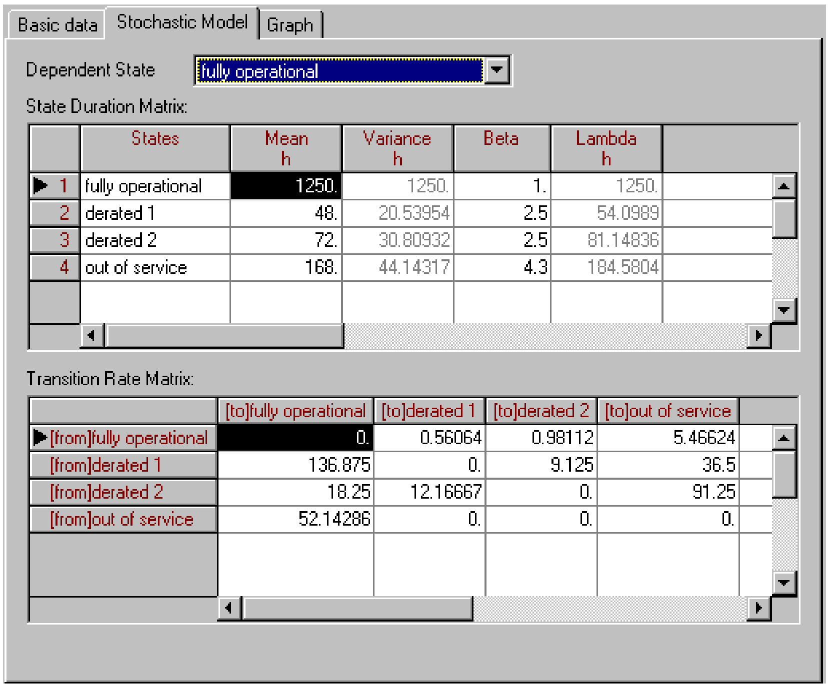


Figure 3.18: The stochastic model for a synchronous generator

The electrical models for the different states of the synchronous generator only differ in the maximum available active power and in the number of available machines. The latter is only of importance when the stochastic model is used for a multi-machine power plant. The dialog for defining the electrical model is shown in Fig.3.19.

Because the Weibull-Markov model allows for defining the shape of the state duration distribution, the dialog has ‘graph’ page, which is shown in Fig.3.20. This page shows the duration distribution and a set of state parameters,

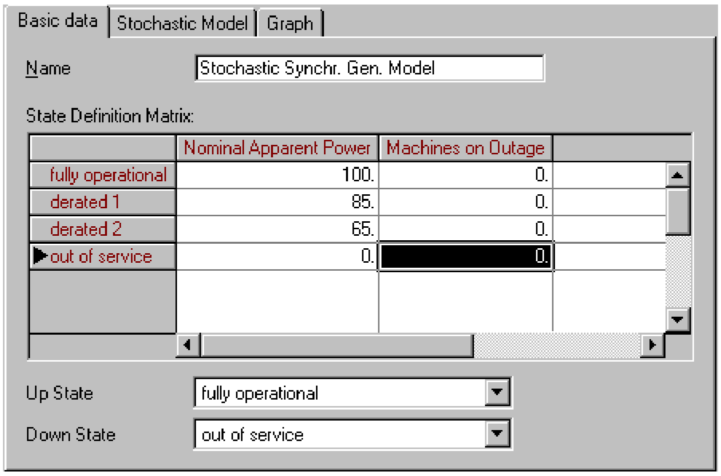


Figure 3.19: Electrical parameters for the stochastic generator

such as the frequency and the MTBS (mean time between states). The state parameters are calculated from the entered model and are shown for reference and checking purposes. The shown graph page shows the distribution and state parameters for the ‘out of service’ state. The state probability informs the user that this generator is out of service for more than 10% of time (942.1851 h/a), from which he may decide to check the stochastic data used.

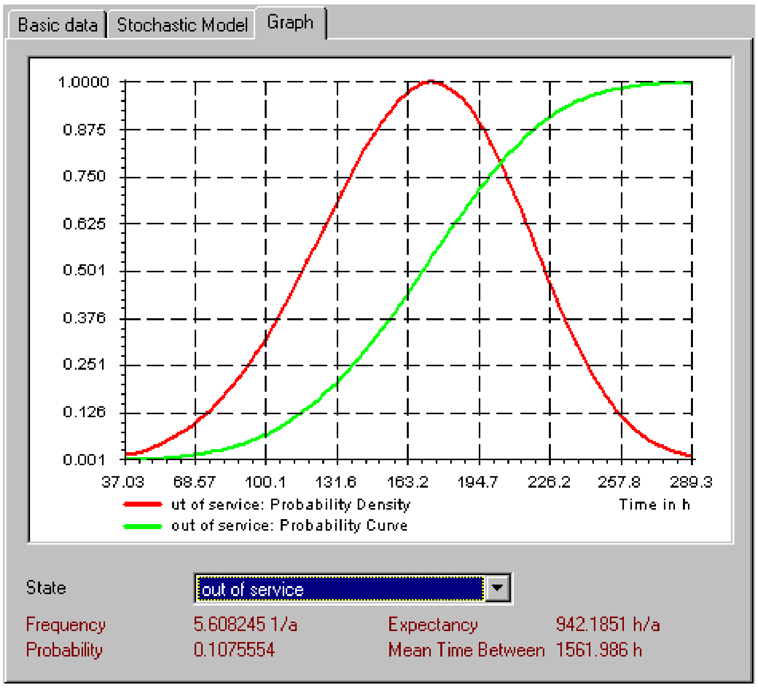


Figure 3.20: State duration graph and state parameters

Chapter 4

Interruption Costs Assessment

4.1 Reliability Worth

Power system utilities try to maximize efficiency. They have to find the best balance between performance and cost. Regarding the reliability of supply, they have to avoid too low a level of investments, because that will result in excessive damage caused by supply interruptions, as well as over-investments.

The highest level of efficiency can only be reached by comparing the increase of performance with the required investment costs. The calculation of (expected) performance indicators in respect to supply reliability is the task of the reliability assessment. This task can be divided into the calculation of non-monetary interruption statistics and the calculation of reliability worth indices. Reliability worth assessment produces monetary indices, and is a relatively young discipline. The calculation of non-monetary interruption statistics is more established ([2], [3], [4], [12], [13], [16], [53]). This includes the calculation of

- Interruption frequency.
- Mean interruption duration.
- Probability of not being supplied.

- Energy not supplied and mean interrupted power.

Two major problems arise if we use interruption statistics for investment justification or for estimating the (relative) importance of the various parts of the system.

The first problem is the fact that we cannot directly compare the costs of increased reliability, which is in monetary units, with a non-monetary reliability performance index. Non-monetary reliability indices can be used to establish minimum system requirements or to rank different design alternatives. But they cannot be related to the investment costs.

The second problem is that the interruption statistics express the supply reliability from the system's point of view, i.e. they express the system performance within the context of the system alone. The supplied customers are only regarded as far as their electrical effects on the system. Customers with identical electrical behaviour are treated equally. It is therefore difficult, if not impossible, to account for the effects of specific customer importance or interruption damage functions in the calculated interruption statistics. In other words, if the power consumption and the frequency and mean duration of the power interruptions are identical for a hospital, for instance, and a shopping mall, then their calculated interruption statistics will also be identical. It will not be possible to bias the non-monetary reliability performance indices for the difference in importance of these two loads, even when that importance is known to be completely different.

One possible way to accommodate for customer importance is to use the costs for the energy not supplied (money/kWh) and/or a cost per interrupted power (money/kW) as an adjustable measure for interruption severity. This can produce useful indices, but it is often insufficient for more detailed planning or selection of alternatives. This linearization of the costs with the duration of the interruption does not consider the fast increase with duration that occurs for individuals as well as for aggregated loads ([124]).

For investment justification or for comparing different design alternatives on a monetary basis or on the basis of interruption severity, we will have to consider a more detailed interruption damage model.

4.2 Interruption Damage Functions

The assessed severity of the impact of a supply interruption, either planned or unplanned, depends on many aspects. The major factors are the definition of the severity that is used, the type of interrupted customers, the duration of the interruption and the extent of the interrupted area.

The consequences of an interruption are quantified by a so-called “interruption damage function”, which gives the consequences in monetary units as a function of the characteristics of the interruption.

The interruption damage function for a single point interruption and for multi-point interruptions, for example for large area interruptions, can be modeled in the same way. Single point damage models describe the interruption damage for a single load in the analyzed system. This load may model a single household or a single asynchronous motor in an industrial process, but may also model a whole distribution network or a large factory. The single point load will be interrupted as soon as the supply to the busbar to which it is connected is interrupted. A multi-point damage model describes the interruption damage for an area or a large industrial proces which are supplied by more than one bus. The multi-point model has an additional trigger condition which defines the combinations of busbars that must be interrupted in order for the damage function to apply.

In [46], the impacts of interruptions are classified as direct vs. indirect and economic vs. otherwise (social). Short interruptions in a small residential area will normally only cause direct damage, such as food spoilage or inconvenient building temperatures. More wide-spread interruptions with a longer duration will also cause indirect damage such as civil disobedience, breakdown of logistics chains, etc.
The classification into direct and indirect costs, and into economic and non-economic costs, is not further discussed here. It is assumed that the impacts of interruptions can be quantified into cost functions, regardless of the nature of the impacts.

Interruption damage functions are often abstract estimations of the actual interruption damage. Actual interruption damage may depend on

- The type of interrupted customers
- The duration of the interruption
- The situation in which the interruption occurs: day of week, time of day, customer's activities at the moment of interruption, etc.

As a different damage function can be assigned to each specific customer or customer mix, the dependency of the damage function itself can be further confined to

- The duration of the interruption
- The moment of occurrence

When we consider the moment of occurrence, then we would have to define damage functions for all recurrent situations for which the interruption damage is know to differ significantly. That could mean, for instance, that we have to define different damage functions for different times of the day, for different days of week, etc. Although it would be possible to create and use arrays of damage functions for each customer, we would like to avoid it and use only one single damage function for each customer.

Specifying damage functions for specific intervals in time will only make a difference when there is also a significant difference in the probability of occurring for specific types of interruptions in these intervals. If interruptions are equally likely to occur during periods of high interruption costs as during low cost periods, then we can average the damage costs by using the relative length of each period.

If we do not consider a time-dependent probability for the occurrence of interruptions, then we can thus use interruption damage models which are only dependent on the duration of the interruption. This approximation is used in most studies. Considering the time-dependency of the interruption damage function only affects that results when also the time dependency of the failure rates are included in the calculations.

4.2.1 Sector Customer Damage Function

Data collected from customer surveys is used to create damage functions for certain classes or "sectors" of customers. These surveys give information about the perceived interruption costs for each specific customer separately, and may contain information about the effect of the duration of the interruption, the time of occurrence, the amount of interrupted power or energy, etc.

The “raw” data from the customer surveys has to be processed and transformed in order to create customer damage functions (CDF) which can also be projected upon customers which have not been surveyed.

The first step in the data transformation is performed by grouping all raw damage functions according to some customer classification, for instance the SIC (Standard Industrial Classification). Typical customer classes, or sectors, are “residential”, “industrial”, “commercial” , “government”, etc.

The raw damage functions are then normalized per group by using a specific customer parameter. This normalization parameter may be the measured yearly peak demand, the yearly energy consumption, a combination of peak demand and energy consumption, or other values. In some cases, normalization for shorter durations is done to annual peak demand, and to annual energy consumption for longer durations ([19]).

When the normalization parameter is known for each raw damage function, then averaged damage functions can be calculated as

$$C(s,d) = \sum_{i=1}^{N(s)} \frac{C(s,i,d)}{F(s,i)} \tag{4.1}$$

where, for sector s and interruption duration d,

- $C(s,d)$ is the averaged damage function,
- $N(s)$ is the number of raw damage functions,
- $C(s,i,d)$ is the raw damage function for customer i, and
- $F(s,i)$ is the normalization factor for customer i

When the normalization factor is only known for the whole group of raw damage functions, then an aggregating process can be used, for which

$$C'(s,D) = \frac{\sum_{i=1}^{N(s)} C(s,i,d)}{\sum_{i=1}^{N(s)} F(s,i)} \tag{4.2}$$

duration	SCDF $/kWh	5000 kWh $	8000 kWh $
20 min.	0.0000235	0.12	0.19
1 hr	0.0000972	0.49	0.78
2 hrs	0.000273	1.37	2.18
4 hrs	0.000520	2.60	4.16
8 hrs	0.00129	6.45	10.32
1 day	0.00597	29.85	47.76
2 days	0.0138	69.00	110.40

Table 4.1: SCDF and scaled CDF for two customers

and $C'(s,D) \leq C(s,D)$. Mixtures of averaging and aggregating can be used when some, but not all, normalization factors are known.

The resulting averaged or aggregated damage function for a specific sector of customers is called a Sector Customer Damage Function or SCDF.

4.2.2 Composite Customer Damage Function

The SCDF is not used in the actual reliability assessment itself. It is only used to create damage functions for single customers or for mixes of customers. The normalization process leads to SCDFs which have units like $/kWh or $/kW. In some cases these units have been misinterpreted as costs per energy not supplied or per interrupted power.

If we want to create a customer damage function (CDF) for a single customer, then we need the SCDF for the sector and the normalization factor for the customer. If yearly energy consumption has been used, then the SCDF will be in $/kWh. The CDF is then obtained by multiplying the SCDF with the customer's yearly consumption. This is illustrated in table 4.1, where a SCDF is scaled to a customer of 5000 kWh and to a customer with 8000 kWh.

If we need to create damage functions for a mix of customers, then we have to create a composite customer damage function (CCDF). The CCDF is basically the sum of the individual customer damage functions in the customer mix. It can be constructed by taking the relative contribution of the various customer sectors to the overall normalization factor. The weighted sum of the different SCDFs according to these relative contributions is then multiplied by the overall factor.

4.3 Use of Damage Functions

The CDF and CCDF that result from the scaling of one or more SCDF's define the damage for each interruption as a function of the interruption duration. This information is combined with the results of the FEA to calculate annual interruption costs for each load point and for the whole system.

4.3.1 Interruption Costs Calculations

The stochastic failure models are used to create specific system states which are then analyzed. Such analysis may include load flow calculations and topological analysis, but may also include power system protection algorithms, power restoration procedures and optimization methods, during which the network is reconfigured and generation may be rescheduled.

The principle objective of the system state analysis is to express the ability of the system to meet the load demands in preliminary performance indicators. These preliminary indicators are then used, at the end of the system state creation and analysis phase, to calculate overall performance indicators. The interruption costs indicators express the expected costs per year due to load interruptions. For each load point, the LPEIC (load point expected interruption costs, $/a) indicator expresses the total expected costs per year due to the interruptions of the load point.

By drawing outcomes for the stochastic conditional state durations for each component, the duration of the current state and the number of the next state are determined. When the current state duration has passed, a transition to the next state is made and new outcomes can be drawn to determine the new duration and the following state. In this way, a possible history for the component can be simulated in time. By performing a parallel simulation of all components in the system, the whole system can be simulated in time. This way of generating system states in a chronological order is called "Monte-Carlo simulation".

Each time a new system state is reached during the Monte-Carlo simulation, it is analyzed for performance. Because the simulation of one single period of, for instance, 5 years, would not make it possible to derive accurate overall performance indicators, it is necessary to simulate and analyze the same period a large number of times. The LPEIC is then calculated as

$$LPEIC = \frac{\sum_{i=1}^{I} C(d_i)}{K \cdot \text{analyzed period}} \tag{4.3}$$

where $C(d_i)$ is a single outcome of the interruption cost function C, for the simulated duration d_i, I is the total number of interruptions of the considered load point during the whole Monte-Carlo analysis, and K is the number of times the analyzed period (the length of which is expressed in years) was simulated.

The Monte-Carlo simulation technique is very powerful because it allows for the simulation of about any possible event in the system and it does not put any restriction on the stochastic component models. Its big disadvantage, however, is its high computational demand. The technique of system state enumeration is therefore often used instead.

In a system state enumeration, all relevant system states are created and analyzed one by one. In this case, no outcomes are drawn for the stochastic durations, but the probabilities and frequencies of the system states are calculated directly. An analytical reliability assessment by state enumeration produces exact results for the performance indicators.

For the reliability costs indicators, such as the LPEIC, the state enumeration methods, however, are problematic. In many textbooks and articles (e.g. [16],[66]), the following equation is used for calculating the LPEIC.

$$LPEIC = \sum_{i=1}^{M} f_i \cdot C(d_i) \tag{4.4}$$

where f_i is the frequency of the i^{th} system state that causes an interruption of the load point and M is the number of different system states which lead to such interruption. This equation, however, is principally wrong, as it assumes that all interruptions caused by the i^{th} system state are of duration d_i. This is an assumption that can not be justified, as the durations of repairs or maintenance are stochastic.

The use of (4.4) is often defended by the assumption that power to an interrupted load will be restored by network reconfiguration, and not by repair. Network reconfiguration is often modeled by deterministic switching durations. Three arguments against the use of (4.4) in a state enumerated reliability assessment can be formulated:

1. It is often unknown if power can be restored to all interrupted loads by network reconfiguration alone in all cases. One objective of a reliability assessment may be to find cases where switched power restoration is not possible.

2. The financial risk related to interruption costs cannot be assumed to be determined by the majority of cases in which the restoration procedures work as planned. Load interruption during unusual system conditions may lead to unexpected high restoration durations. Such may happen in the case of failing protection devices, stuck breakers, unavailable (backup) transmission lines, peak load situations, etc.

3. Wide area power systems, or systems in rural areas, may lack the required network reconfiguration options or may require the modeling of switching times as stochastic quantities.

4. Statistical data shows a considerable spread in interruption duration ([30], [97], [43])

For a correct calculation of interruption costs or other reliability cost indicators, it is therefore necessary to assess the duration distribution of all interruptions during a state enumerated reliability assessment. This is possibly by using the Weibull-Markov model, as introduces in chapter 3. The calculation proceeds as follows:

- Expression (3.92) is evaluated for each stochastic model prior to the actual state enumeration

- Expression (3.91) is evaluated for each system state for which the probability distribution of the duration is required.

This means that, if

- a CDF or CCDF has been defined as a vector $\{C(T_j)\}$ for the interruption durations $\{T_1, \cdots, T_{N_c}\}$
- the probability $\Pr(T_{j-1} < D_i < T_j)$ for the state duration D_i can be calculated using (3.92).

then the expected interruption costs for the for the i^{th} system state can be calculated as

$$E(C_i(d)) = \sum_{j=1}^{N_c} \Pr(T_{j-1} < D_i < T_j) \cdot C(T_j) \tag{4.5}$$

where D_i is the stochastic system state duration.

With (4.5), the annual LPEIC can then be calculated as

$$LPEIC = \sum_{i=1}^{M} f_i \cdot E(C_i(d)) \tag{4.6}$$

where f_i is the system state frequency.

The above method for calculating the LPEIC, i.e. using (3.92), (3.91) and (4.5) to evaluate (4.6) seems very complicated and time consuming. Indeed, when compared with (4.4), a slight increase in efforts cannot be denied. However, it has been found in practice that the evaluation of the LPEIC in the described way increases the total computation time only very slightly. When the proposed method is analyzed for required computational demands, it is clear why the computational demand is very moderate:

- Expression (3.92) is evaluated for each stochastic component separately, and only has to be analyzed once, prior to the reliability analysis. The results can be stored with the stochastic model model. The computational demand is thus very low or zero.

- Expression (3.91) is evaluated for each analyzed system state. The computation demand is very small, as it only requires the multiplication or addition of a few known factors.
- The evaluation of (4.5) and (4.6) is very fast, and linear with the number of load points.

The low computational demands and the very simple expressions that are to be evaluated, of which the most complex can be evaluated a-priori, make the proposed method of calculating the LPEIC elegant, transparent and practical.

4.4 Effects of the Interruption Duration Distribution

The ability to calculate reliability worth on the basis of the probability distribution of the interruption duration is only of practical use when the shape of this distribution is relevant. It is easy to define customer damage functions for which the LPEIC would become highly dependent on the interruption duration distributions. It is also clear that the use of linear damage functions, results in a LPEIC that is completely independent of the interruption duration distribution. It is therefore required to evaluated the importance of the interruption duration distributions on the basis of realistic damage functions.

A number of customer surveys have been undertaken in various countries over the whole world in the last decades. The Cigré Task Force 38.06.01 discusses the use of interruption costs functions in power system analysis, and has published an overview of surveyed data in ([46]).

One of the conclusions of the task force is that "The actual event based approach can provide ECOST values which are significantly different from those obtained by the average event (analytical) method if the applied CDF are highly non-linear." ([46]). Although the report continues by saying that "In most cases, however, either technique will provide acceptable results.", it does not indicate in which cases average event methods give acceptable results and in which cases they produce unacceptable errors.

In order to quantify the errors in calculated LPEIC values, the published customer damage functions in [46] where analyzed for their sensitivity to the probability distribution of the interruption duration. This analysis was performed in the following way:

- A Weibull distribution of the interruption duration was presumed.
- The mean costs per interruption was evaluated for various values of the β-shape factor of the duration distribution, and for four different mean durations. A linear interpolation of the customer damage was used for interruption durations in between the discrete values of the damage function.
- The resulting four functions, each of which gives the mean costs as function of the β factor for a certain mean duration, are divided by the mean costs for a unity shape factor, i.e. for $\beta = 1$.
- The value of the relative mean costs was interpreted as an "error function" which indicates the possible error that could result when the damage function is used in a reliability calculation in which the duration distribution is either not considered or modeled incorrectly.

The Weibull distribution is chosen as a model for the interruption duration distribution because measurements indicate that interruption durations are often more or less Weibull distributed ([128]). Another reason is that the Weibull distribution also covers the exponential distribution, and a comparison with results from a homogenous Markov model is therefore possible.

The SCDF for commercial customers in Canada, according to [46], is used here as example, to describe the above analysis method in more detail. The SCDF is shown in table 4.2, in 1999 US$. The mean interruption costs for a Weibull distributed interruption duration, was calculated as

$$E_C(\beta, \mu) = E_C(\beta, \eta(\mu)) = \int_0^\infty C(\tau) \cdot \frac{\beta}{\eta^\beta} \tau^{\beta-1} \exp\left[-\left(\frac{\tau}{\eta}\right)^\beta \right] d\tau \quad (4.7)$$

where $\eta = \eta(\mu)$ is the scale factor of the Weibull distribution which corresponds to a mean of μ and where $C(\tau)$ is the SCDF, in this example according to table 4.2, which was linearly interpolated.

The function $E_C(\beta, \mu = M)$ describes the dependency of the interruption costs to the β form factor, for a fixed mean duration M. Each of these

	\$/kW
2 sec.	0.232
1 min.	1.63
20 min.	4.83
1 hr	13.0
2 hrs	27.3
4 hrs	65.7
8 hrs	106
1 day	127

Table 4.2: SCDF for “Canada, Commercial, \$/kW”

functions is subsequently divided by $E_C(1.0, M)$, which results in an “error function” which describes the deviations for various β-factors independently of the used SCDF. These error functions have been calculated for a mean duration (μ) of 10 minutes, 30 minutes, 1 hour and 4 hours. The results for the example SCDF are depicted in Fig.4.1.
The error function for a mean duration of 4 hours in Fig.4.1 shows values in excess of 20% for $\beta > 2.5$. This means, for the example SCDF, that not regarding the effects of the interruption duration distribution correctly could lead to errors of 20% or more in the calculated LPEIC index.

The analysis described above was carried out on all damage functions published in ([46]). The largest deviations which were found are listed in table 4.3. The results of all analyzed damage functions are shown in appendix A.

Note that these results are based on “average customers” and interruption durations with a Weibull probability distribution. For individual customers and other distributions, the results may even be larger. However, no data was available to substantiate this.

From the analysis of the damage functions, it is clear that the dependency of the interruption costs to the interruption duration and the probability distri-

SCDF	largest deviation
Saudi Arabia Residential	66 %
Saudi Arabia Industrial	45 %
Saudi Arabia Commercial	39 %
Norway Agriculture	33 %
Greece Companies+organisations	27 %
Australia Commercial	25 %
Canada Commercial	25 %
Nepal Commercial	25 %
Greece Commercial total	23 %
Denmark Household	22 %
Sweden Commercial	20 %
Australia Major User	19 %
Greece Business	19 %
Sweden Agriculture	19 %
Australia Residential	18 %
Sweden Residential	18 %
Great Britain Industrial	16 %
Greece Industrial	16 %

Table 4.3: Largest deviations for analyzed international CDF.

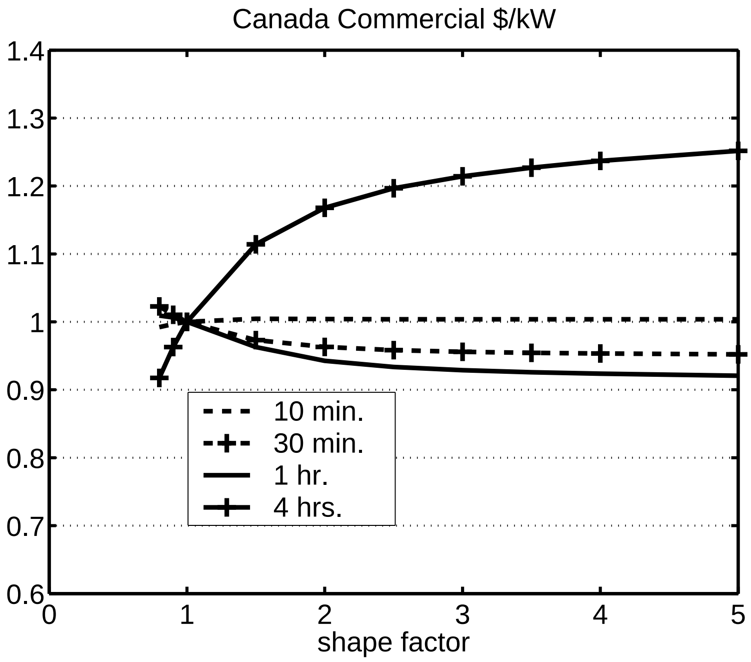


Figure 4.1: PDF dependency for “Canada, Commercial, $/kW”

bution of that duration cannot always be ignored. It would be allowed to ignore these factors when damage function are used which are (almost) linear functions of the interruption durations. The use of homogenous calculation methods will produce correct results for such functions. In all other cases, the use of homogenous methods bears the risk of producing reliability worth indices with large errors.

Chapter 5

Example

An example is shown here for an 11kV distribution system. This system is a modification of the test system described in [6], which was based on a real UK system. This system is chosen here because of its size and complexity.

5.1 Description of the Test System

A single line diagram of the test system is displayed in Fig.5.1. Some parameters describing the size of the test system are given in table 5.1.
The test system is a radially operated distribution network, in which all loads can be fed from two sides.

Test System Parameters	
Number of Nodes	85
Number of Lines	30
Number of transformers	13
Number of Loads	56
Total Demand	52.08 MW

Table 5.1: Size of the test system.

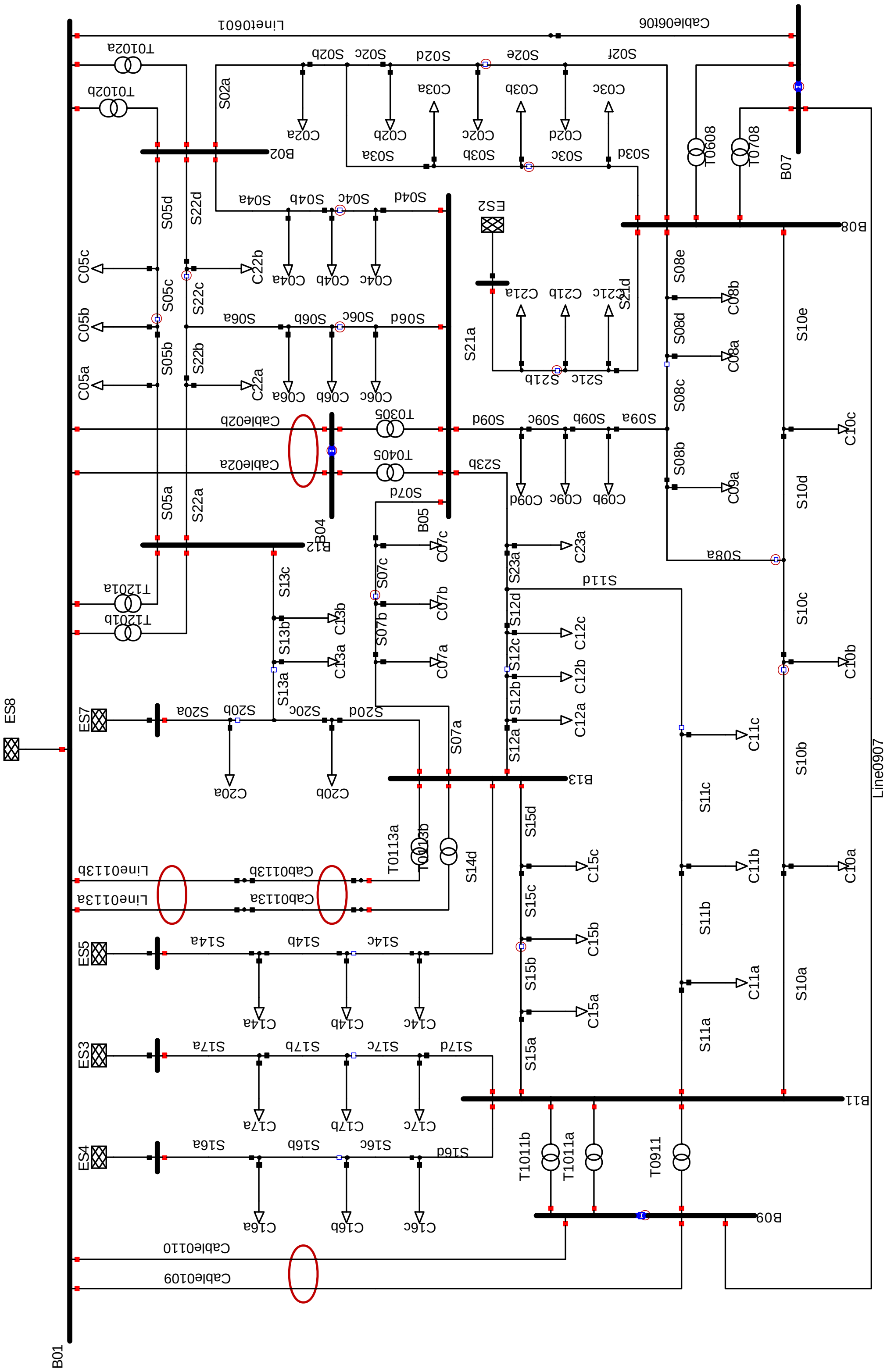


Figure 5.1: The distribution test system

	$	$/kW	$/kWh
2 sec.	122	0.232	0.00105
1 min.	148	1.63	0.000986
20 min.	347	4.83	0.00235
1 hr	1023	13.0	0.00700
2 hrs	1806	27.3	0.0123
4 hrs	3767	65.7	0.0278
8 hrs	6756	106	0.0477
1 day	14831	127	0.0987

Table 5.2: Interruption Cost Function

The peak load demands for the load points are listed in table 5.3. Each load point was given a fixed number of customers,which is also listed in table 5.3. All loads have the same interruption cost function, according to table 5.2. This cost function is the one for the commercial customers in Canada, according to [46]. The SCDF per kW peak demand was used by all loads and was automatically scaled to the kW peak demand (according to table 5.3) during the reliability assessment.

The failure data used for lines, cables, transformers and busbars is listed in tables 5.4 and 5.5. Common mode failure models were defined for the cables which are encircled in the single line diagram. The data for these common failures is shown in table 5.4.

Name	MW	MVAr	Cust.	Name	MW	MVAr	Cust.
C02a	0.50	0.020	100	C11a	0.40	0.081	80
C02b	0.52	0.020	104	C11b	0.30	0.061	60
C02c	0.40	0.020	80	C11c	0.40	0.081	80
C02d	0.90	0.183	180	C12a	1.50	0.305	300
C03a	0.54	0.020	108	C12b	1.20	0.244	240
C03b	0.62	0.020	124	C12c	1.20	0.244	240
C03c	1.10	0.223	220	C13a	2.20	0.447	440
C04a	0.90	0.183	180	C13b	2.20	0.447	440
C04b	0.40	0.081	80	C14a	0.10	0.020	20
C04c	1.60	0.325	320	C14b	0.10	0.020	20
C05a	1.00	0.203	200	C14c	1.70	0.345	340
C05b	1.40	0.284	280	C15a	1.80	0.366	360
C05c	2.30	0.020	460	C15b	0.70	0.142	140
C06a	1.00	0.203	200	C15c	1.20	0.244	240
C06b	1.40	0.284	280	C16a	0.10	0.020	20
C06c	1.90	0.386	380	C16b	0.10	0.020	20
C07a	1.20	0.244	240	C16c	0.80	0.162	160
C07b	0.90	0.183	180	C17a	0.10	0.020	20
C07c	2.10	0.426	420	C17b	0.10	0.020	20
C08a	0.74	0.150	148	C17c	1.40	0.284	280
C08b	0.90	0.183	180	C20a	0.10	0.020	20
C09a	0.74	0.150	148	C20b	1.40	0.284	280
C09b	0.74	0.150	148	C21a	0.10	0.020	20
C09c	0.74	0.150	148	C21b	0.30	0.061	60
C09d	0.74	0.150	148	C21c	0.50	0.102	100
C10a	0.90	0.183	180	C22a	1.40	0.284	280
C10b	0.90	0.183	180	C22b	1.70	0.345	340
C10c	0.40	0.081	80	C23a	1.50	0.305	300

Table 5.3: Load Data

Name	Failure Frequency 1/a	Repair Mean h	Repair β
Cables 0113a + 0113b	0.01	300	2.0
Cables 02a + 02b	0.03	300	2.0
Lines 0109 + 0110	0.05	300	2.0
Lines 0113a + 0113b	0.05	300	2.0
11kV Cables	3.2/100km	33.5	3.5
33kV Cables	3.2/100km	33.5	3.5
33kV Overhead Lines	2.5/100km	212	3.5
Transformers	0.02	343	1.5

Table 5.4: Common mode and branch failure data

Name	BusBar Failure Freq. 1/a	Field Failure Freq. 1/a	Repair Mean h	Repair β
11kV Bar	0.002	0.005	14	2.5
33kV Bar	0.0025	0.015	24	2.5

Table 5.5: Busbar failure data

System Indices		
Customer Avg. Interruption Frequency Index	CAIFI	0.128 1/Ca
Customer Avg. Interruption Duration Index	CAIDI	5.92 h
Avg. Service Availability Index	ASAI	99.9913 %
Energy Not Supplied	ENS	39.6 MWh/a
Avg. Customer Curtailment Index	ACCI	3.80 kWh/Ca
Expected Interruption Cost	EIC	259 k$/a
Interrupted Energy Assessment Rate	IEAR	6.55 $/kWh

Table 5.6: Overall system reliability indices

A reliability assessment was made for the whole system in the single line diagram. Overlapping failures were not considered. The reliability assessment produced overall system indices as well as indices for all individual load points in the system. Interruption costs were also calculated for every individual load point and for the overall system.

The first calculation was made using the β-factors for the Weibull distributions of the repair durations, as shown in the tables 5.4 and 5.5. The results for some system indices are shown in table 5.6. In table 5.7, the results for the 20 load points with the highest interrupted customer-frequency are shown.

The 10 highest and 10 lowest values for the ACIF and the ACIT are shown in the bar-graphs of Fig.5.2 and Fig.5.3.

Load Point Indices							
Name	**LPIF C/a**	**LPIT Ch/a**	**AID h**	**LPENS MWh/a**	**LPEIC k$/a**	**ACIF 1/a**	**ACIT h/a**
C14c	70.3	239.0	3.40	1.19	9.49	0.207	0.703
C20b	67.6	166.6	2.46	0.83	8.20	0.242	0.595
C12a	64.1	293.2	4.58	1.47	11.69	0.214	0.977
C07a	52.2	184.7	3.54	0.92	5.96	0.218	0.770
C15c	52.2	251.8	4.82	1.26	12.76	0.218	1.049
C12b	51.2	234.5	4.58	1.17	9.35	0.214	0.977
C23a	47.4	272.0	5.74	1.36	7.63	0.158	0.907
C07c	42.0	247.2	5.89	1.24	7.13	0.100	0.589
C07b	39.2	143.1	3.65	0.72	4.65	0.218	0.795
C06c	38.0	306.9	8.08	1.53	7.55	0.100	0.808
C12c	37.9	139.6	3.68	0.70	6.06	0.158	0.582
C06b	37.8	221.3	5.86	1.11	6.72	0.135	0.791
C22a	37.8	157.1	4.16	0.79	4.48	0.135	0.561
C13a	33.9	471.4	13.91	2.36	13.36	0.077	1.071
C13b	33.9	471.4	13.91	2.36	13.36	0.077	1.071
C04c	32.0	176.8	5.53	0.88	4.85	0.100	0.553
C09a	30.5	149.5	4.90	0.75	5.48	0.206	1.010
C09b	30.5	108.9	3.57	0.54	4.07	0.206	0.736
C09c	30.5	103.0	3.37	0.51	3.65	0.206	0.696
C09d	30.5	97.1	3.18	0.49	3.24	0.206	0.656

Table 5.7: Load point reliability indices

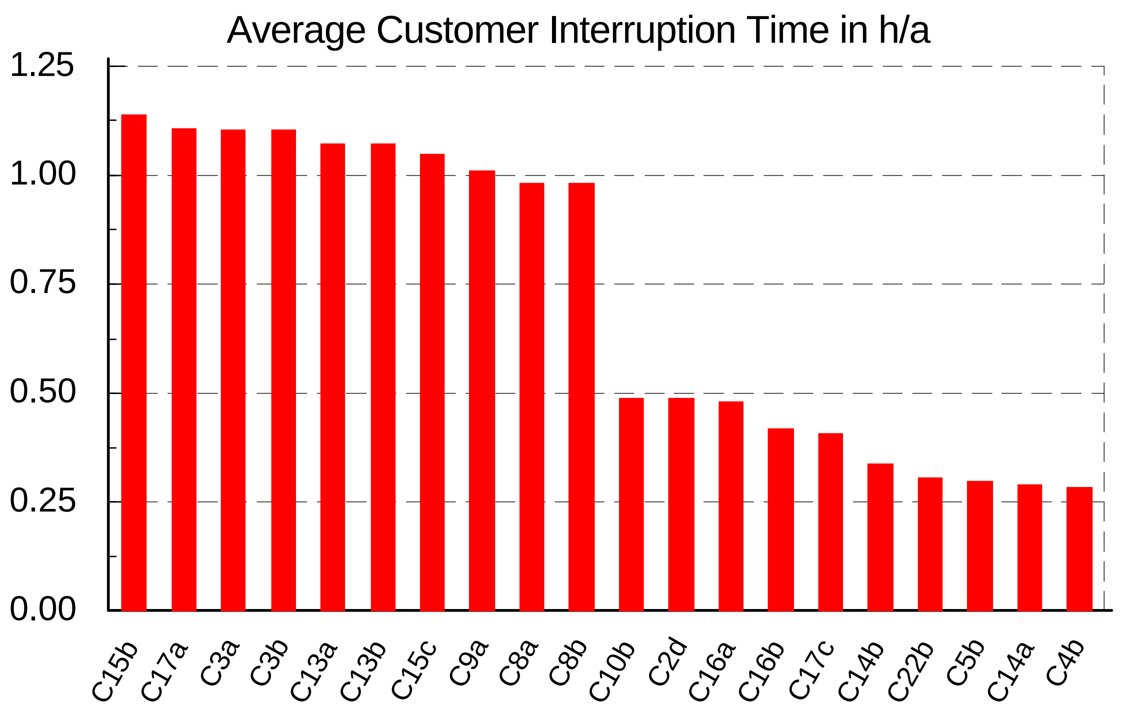


Figure 5.2: Average Customer Interruption Time

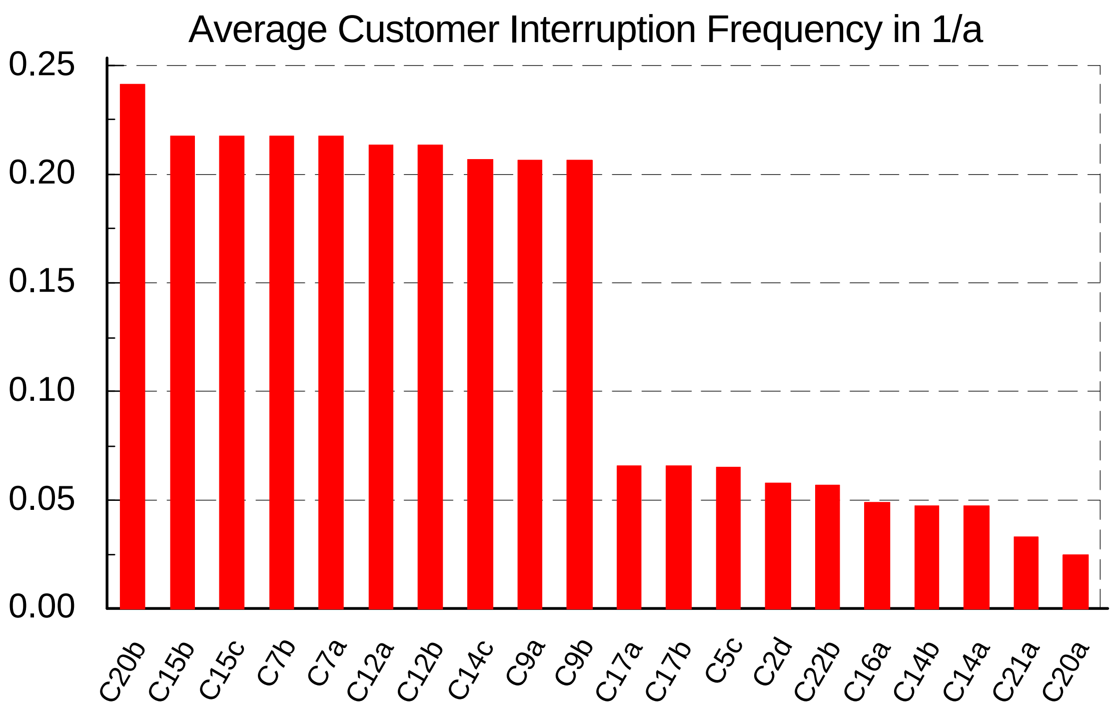


Figure 5.3: Average Customer Interruption Frequency

5.2 Markov vs. Weibull-Markov Models

After the first "Weibull-Markov" calculation all β-factors in the failure models were set to unity. This results in a homogenous Markov system. A reliability calculation for the homogenous system was then started without further changes. The results for all non-monetary indices were the same, as expected. Only the values for the interruption costs where different. Fig.5.4 shows the bar graphs for the LPEIC index, for both the Weibull-Markov calculation and the homogenous calculation.

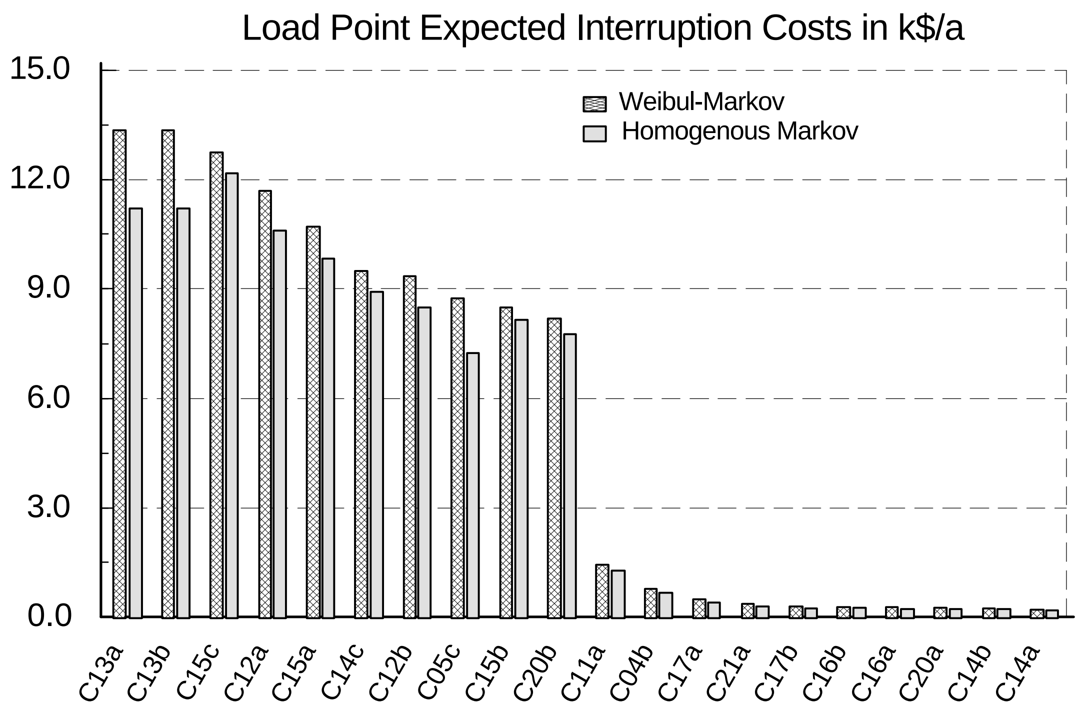


Figure 5.4: Load Point Expected Interruption Costs

The difference between the Markov and the Weibull-Markov calculation is well over 16 % for the higher values of the LPEIC, and up to 18 % for all loads. These differences are calculated as relative to the Weibull-Markov results and can thus be interpreted as the relative error of the homogenous results.

The differences of 16 % and more are remarkable because power to interrupted loads in the test system can almost always be restored by switching

actions. Interruptions which are restored by switching have short duration and therefore cause low costs. Repaired interruptions, on the other hand, have long duration and therefore cause higher costs. Switching actions were modeled with deterministic durations. This means that the interruption duration could be different for different failures, but had always the same duration for the same failure. The duration of switched restorations is therefore not affected by a change in a β parameter of a stochastic mode, as no stochastic mode is used for determining the duration of switched restorations.
However, from the results it is clear that, although the repaired restorations are less frequent than the switched restorations, they nevertheless have a significant effect on the overall interruption costs due to the higher cost per interruption. Otherwise there would not be a difference between the homogenous and the Weibull-Markov results.

A second comparison was made in which a 'single penalty' interruption costs function was used. This costs function was zero for all interruptions shorter than 8 hours and equaled 50$ for interruptions with a duration longer than 8 hours. Again, a Weibull-Markov calculation was made, with the original β-factors, and a homogenous Markov calculation, with all β-factors set to unity again. The resulting LPEIC values for the Weibull-Markov and the Markov calculation are shown in Fig.5.5. The difference between the Markov and the Weibull-Markov results are now exceeding 30 %.

5.3 Monte Carlo Simulation vs. State Enumeration

To investigate the correctness of the interruption costs calculations by analytic state enumeration methods, using Weibull-Markov models, comparisons with Monte Carlo simulations were made.

The Monte-Carlo simulation produces averaged values, but also confidence limits. A 95% confidence level was used. The results of the comparison are shown in Fig.5.6.

The Fig.5.6 shows that the results of the state enumeration method are within the 95 % confidence intervals as calculated by the Monte Carlo simulation.

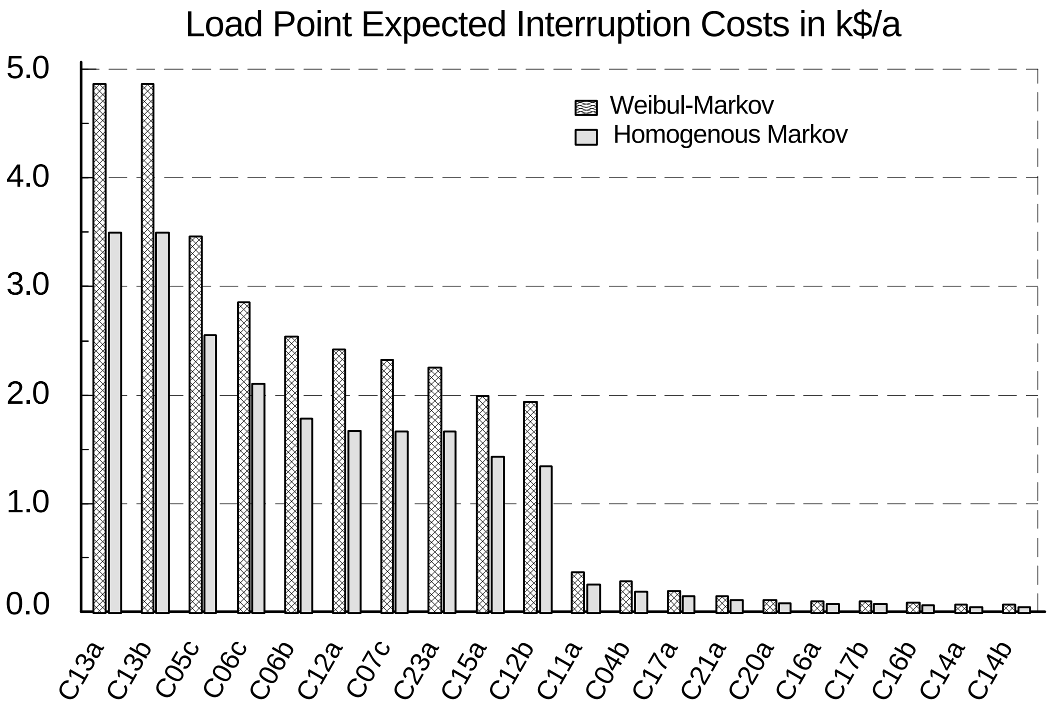


Figure 5.5: LPEIC for 'single penalty' interruption costs

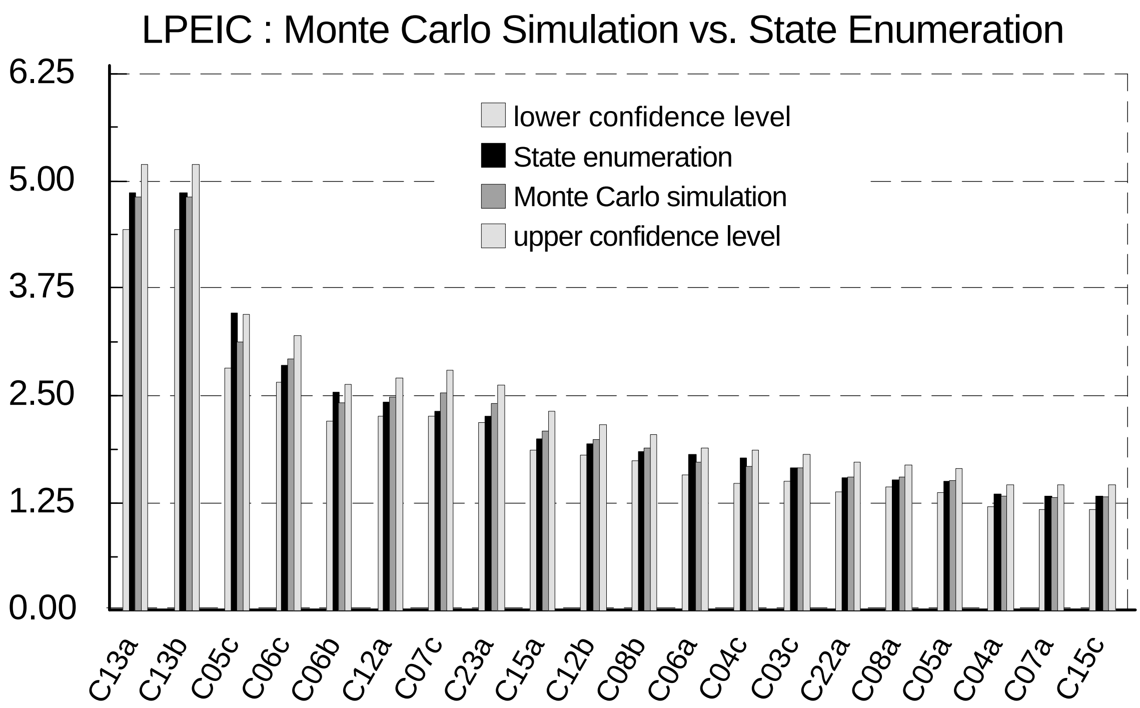


Figure 5.6: LPEIC, comparison

Chapter 6

Conclusions

The commonly used homogenous Markov model can be extended to include Weibull-distributed state durations. The so-called ťťWeibull-Markov” component is a semi-Markov model of a power-system component. Instead of an exponential distribution, a Weibull distribution is used for the state duration.

A system consisting of Weibull-Markov components is referred to as a Weibull-Markov system. A Weibull-Markov system does not longer obey the Markov property. It is therefore not a semi-Markov model. It is shown in this thesis that it is possible to obtain an analytical expression for the state-duration distribution for each of the states of the Weibull-Markov system.

The Weibull-Markov stochastic model as introduced in this thesis is “upward compatible” with the homogenous Markov model. All existing failure data can also be used for the Weibull-Markov model. The Weibull-Markov model expands the homogenous Markov model by enabling the assessment of the probability distribution for the system state durations.

The Weibull-Markov model can be used to obtain more correct results for the probability distribution of interruptions of the power supply. This distribution is especially important when estimating the worth of reliability.

The expected annual interruption costs are normally calculated on the basis of customer damage functions. These damage functions define the costs of an interruption as a function of the interruption duration.

The mathematically correct way of calculating the expected interruption cost is by multiplying the damage function by the probability density function of the interruption duration. Calculation methods, however, which exclusively use the homogenous Markov model, are not able to assess this density function. It is shows in this thesis that, when homogenous Markov models are used despite this inability, errors of 40 % and more in the calculated expected interruption costs may result.

It has been shown that the results of an analytical state enumeration method which uses the Weibull-Markov model, are comparable with the results of a Monte-Carlo simulation. The computational demands for the proposed calculation methods are comparable with the demands for the homogenous Markov methods.

It should also be noted that an analytical method allows more conclusions to be drawn from the results than a Monte-Carlo simulation.

Summarizing, the proposed methods for calculating expected yearly interruption costs, based on the Weibull-Markov model,

- give more accurate results for the expected yearly interruption costs
- offer more flexibility in the use of customer damage functions
- do not effectively increase the computational demand
- are simple to implement, and easy to use.

Future work on power-system reliability should include a thorough investigation of the various duration distributions needed for the component models. Interesting examples are the modeling of life-time distributions in combination with maintenance for the time-to-failure, and the modeling of component repair-time distributions in combination with switching actions for the time-to-restoration. plement, and easy to use.

Appendix A

Analyzed International CDF

This appendix gives customer damage functions for a range of customer sectors in various countries. All damage functions are based on the data given in [46].

All interruption costs functions shown in this sections are given in 1999 US$.

A.1 Australia

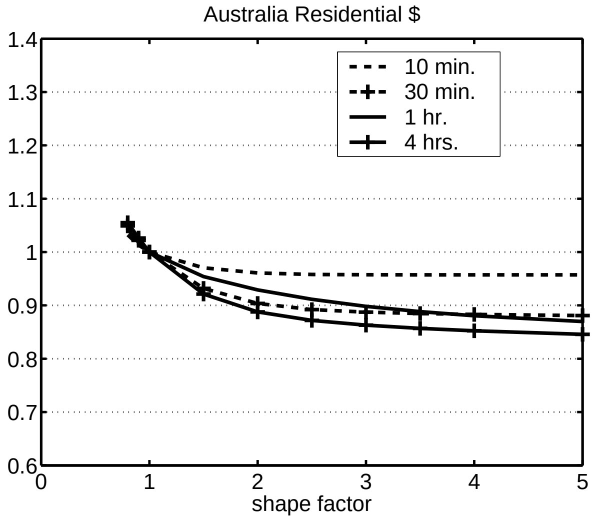
The costs data for Australia was obtained in studies conducted in 1996 and 1997 by the Reliability Research Group at the CEPE, Monash University.

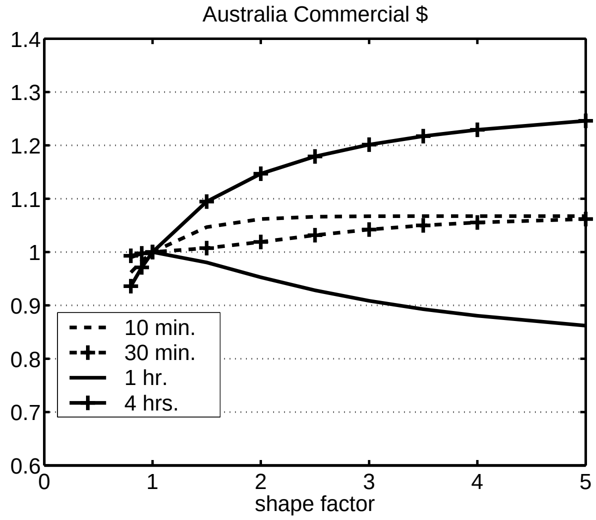
	Residential		Commercial		Agricultural	
	$	$/kWh	$	$/kWh	$	$/kWh
2 sec.			281	0.00188		
1 min.			511	0.00163	17.3	0.00456
20 min.	0.162	0.0000235	1156	0.00376	61.0	0.00658
1 hr	0.591	0.0000972	1277	0.00654	126	0.00915
2 hrs	1.60	0.000273	2908	0.00527	184	0.0111
4 hrs	3.30	0.000520	5178	0.00939	433	0.0336
8 hrs	8.04	0.00129	6789	0.0302	885	0.0549
1 day	35.2	0.00597	10468	0.0410	1926	0.151
2 days	81.5	0.0138				

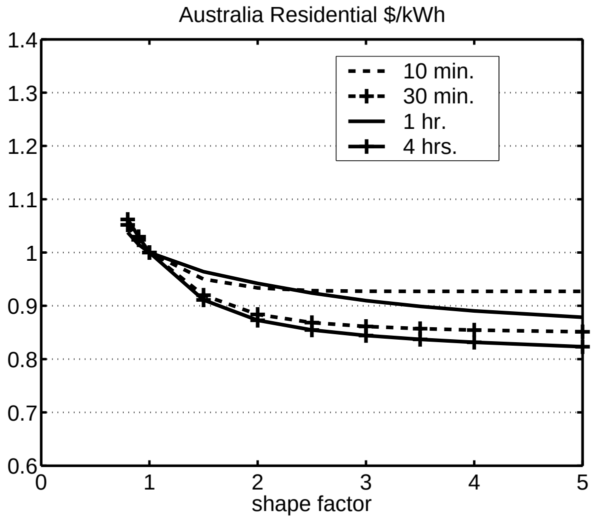
Table A.1: Australia, SCDF for “Residential”, “Commercial” and “Agricultural”

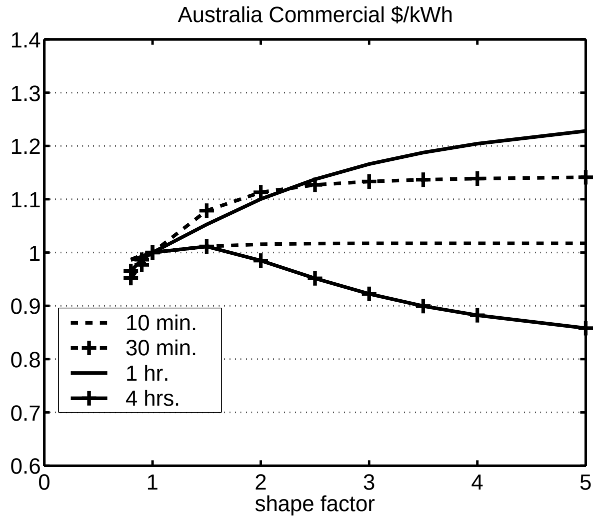
	Industrial		Major User		Ind. + Major	
	$	$/kWh	$	$/kWh	$	$/kWh
2 sec.	1187	0.000331	4902	0.000175	2383	0.000202
1 min.	1827	0.00115	6114	0.000175	3135	0.000341
20 min.	2176	0.00150	7517	0.000373	3425	0.000593
1 hr	3579	0.00207	15169	0.00121	6944	0.00133
2 hrs	4092	0.00328	14580	0.000709	7089	0.00114
4 hrs	8019	0.00745	22344	0.00133	12269	0.00236
8 hrs	15223	0.0144	57464	0.00439	26286	0.00610
1 day	19974	0.0155	94709	0.00822	40437	0.00933

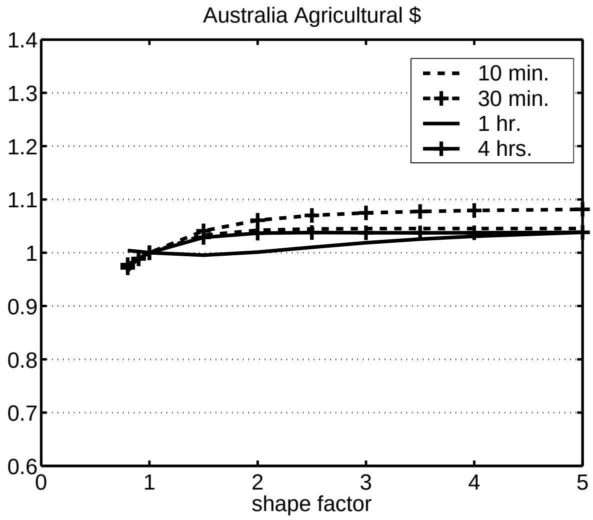
Table A.2: Australia, SCDF for “Industrial”, “Major User” and “Industrial + Major User”

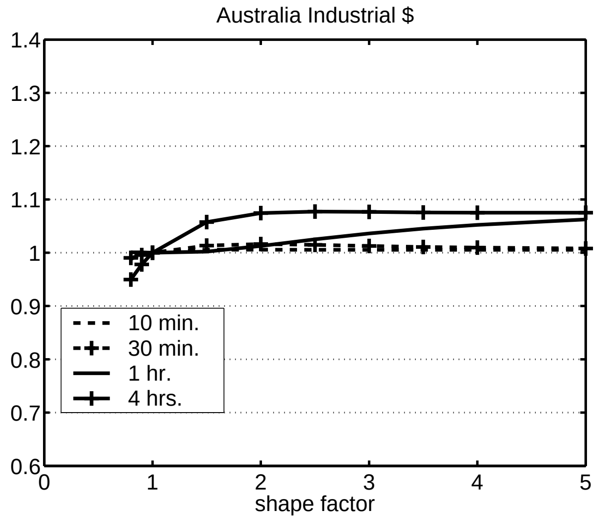

Australia Residential $
10 min.
30 min.
1 hr.
4 hrs.
shape factor

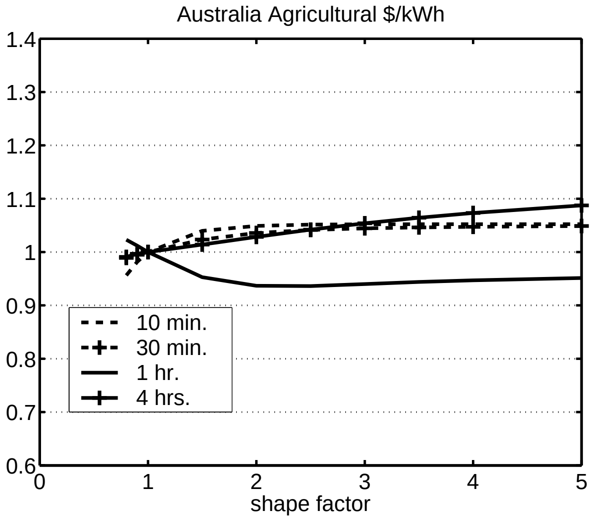

Australia Commercial $
10 min.
30 min.
1 hr.
4 hrs.
shape factor

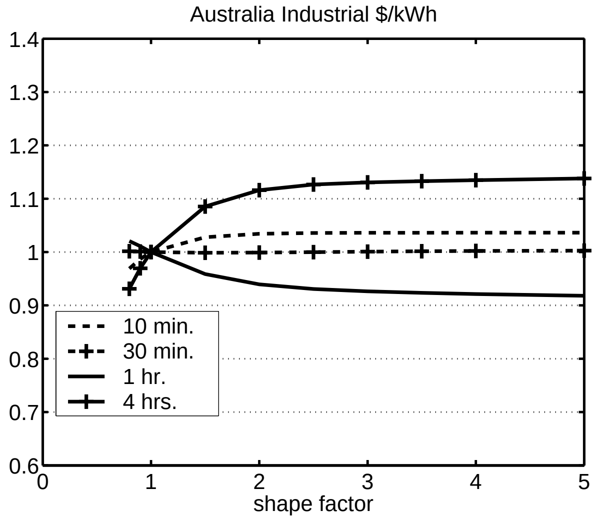

Australia Residential $/kWh
10 min.
30 min.
1 hr.
4 hrs.
shape factor

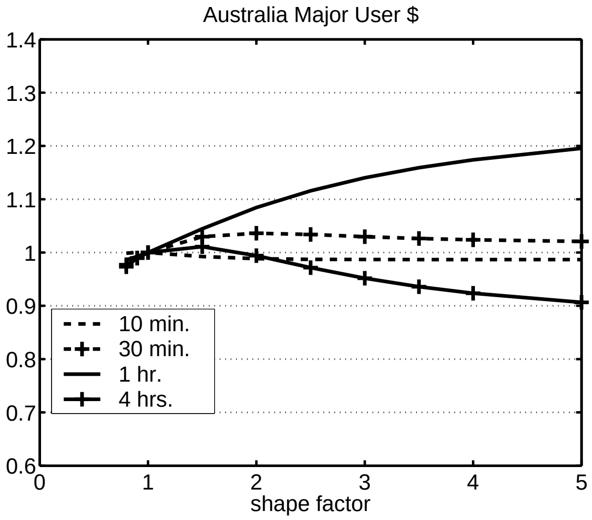

Australia Commercial $/kWh
10 min.
30 min.
1 hr.
4 hrs.
shape factor

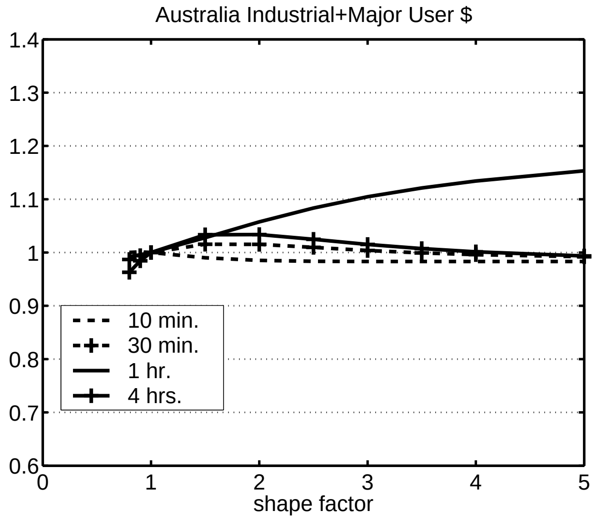

Australia Agricultural $
10 min.
30 min.
1 hr.
4 hrs.
shape factor

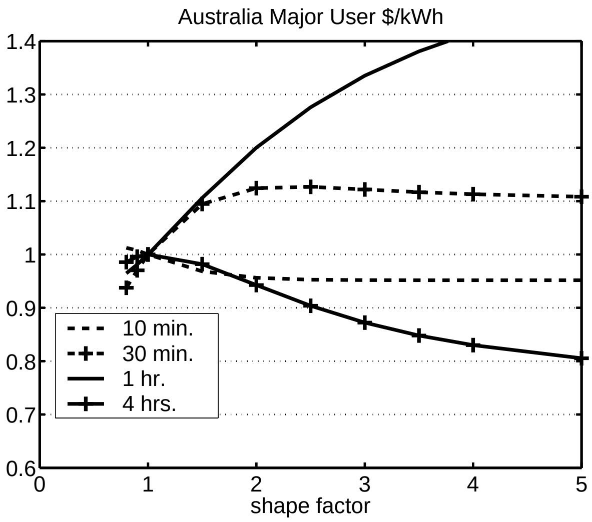

Australia Industrial $
10 min.
30 min.
1 hr.
4 hrs.
shape factor

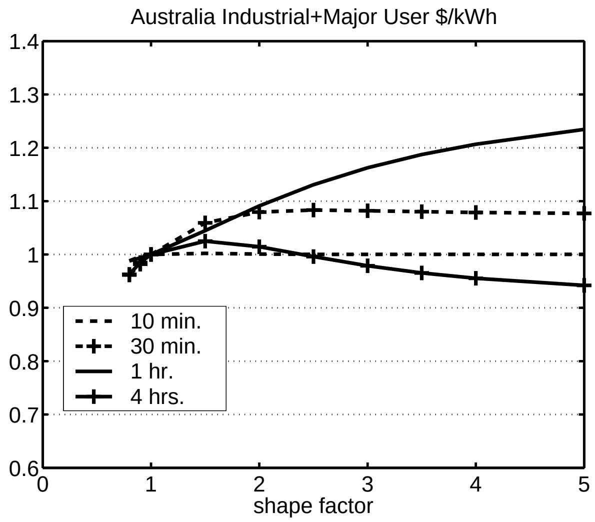

Australia Agricultural $/kWh
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.


Australia Industrial $/kWh
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.


Australia Major User $
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.


Australia Industrial+Major User $
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.


Australia Major User $/kWh
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.

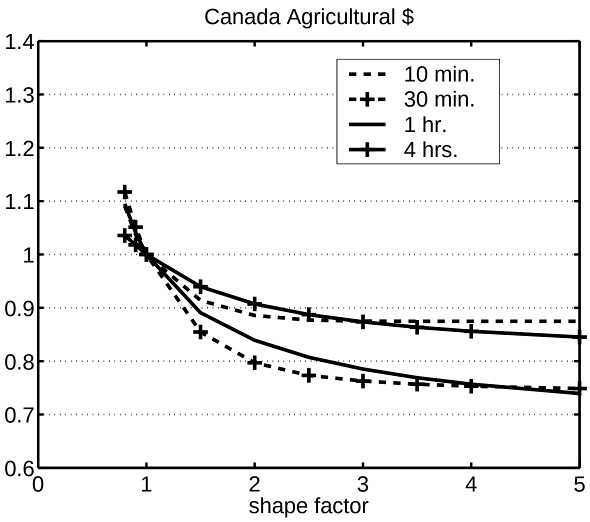

Australia Industrial+Major User $/kWh
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.

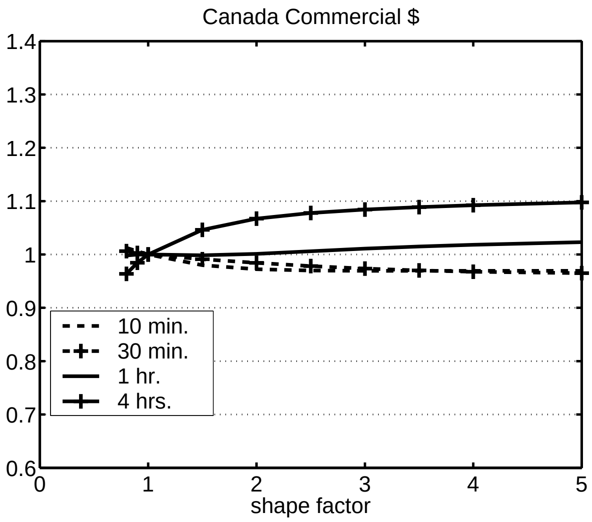
A.2 Canada

The university of Saskatchewan conducted various customer damage survey in Canada, with some regularity from the 1970's onwards. Here published data is taken from the 1987 survey into agricultural interruption costs, the 1991 surveys into residential, commercial and industrial customers, and the 1995 study of interruption costs for government, institute and office buildings (GIO).

	Agricultural			Commercial		
	$	**$/kW**	**$/kWh**	**$**	**$/kW**	**$/kWh**
2 sec.				122	0.232	0.00105
1 min.				148	1.63	0.000986
20 min.	1.13	0.0487	0.0000229	347	4.83	0.00235
1 hr	5.71	0.243	0.000115	1023	13.0	0.00700
2 hrs				1806	27.3	0.0123
4 hrs	47.2	2.02	0.000966	3767	65.7	0.0278
8 hrs	133	5.64	0.00270	6756	106	0.0477
1 day				14831	127	0.0987

Table A.3: Canada, SCDF for "Agricultural" and "Commercial"





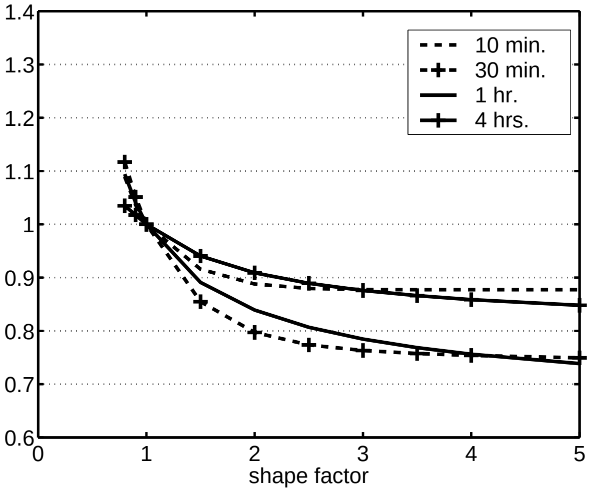
	Gov., Inst., Off.			Industrial		
	$	**$/kW**	**$/kWh**	**$**	**$/kW**	**$/kWh**
2 sec.	108	1.21	0.000258	907	0.782	0.00790
1 min.				1032	1.87	0.00821
20 min.	344	2.39	0.000803	1489	2.67	0.00868
1 hr	782	5.30	0.00197	2876	5.65	0.0109
2 hrs				4161	10.0	0.0128
4 hrs	2757	15.6	0.00686	7352	20.6	0.0195
8 hrs				12825	38.1	0.0364
1 day				21381	60.7	0.0485

Table A.4: Canada, SCDF for “Government, Institutes, Office Buildings” and “Industrial”

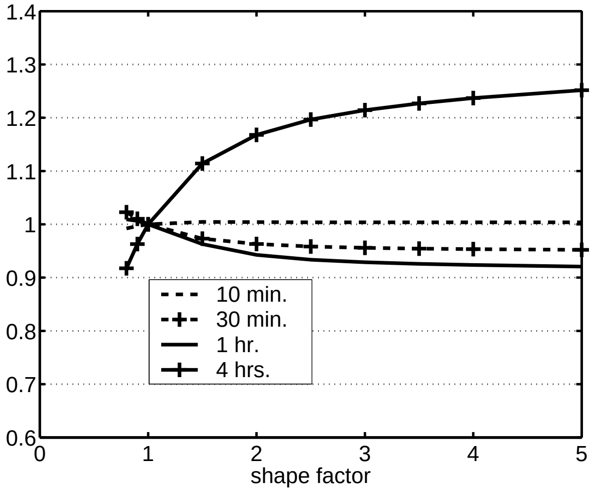
	Residential		
	$	**$/kW**	**$/kWh**
20 min.	0.190	0.0241	0.0000121
1 hr	1.10	0.141	0.0000701
4 hrs	12.1	1.57	0.000779
8 hrs	25.8	3.46	0.00172
1 day	117	15.8	0.00784

Table A.5: Canada, SCDF for “Residential”

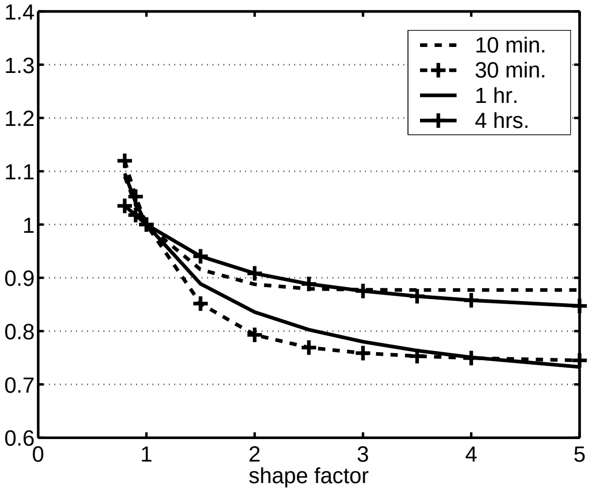
Canada Agricultural $/kW



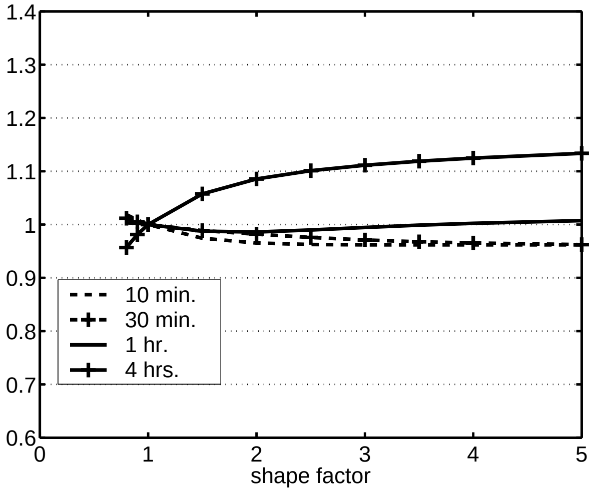
Canada Commercial $/kW



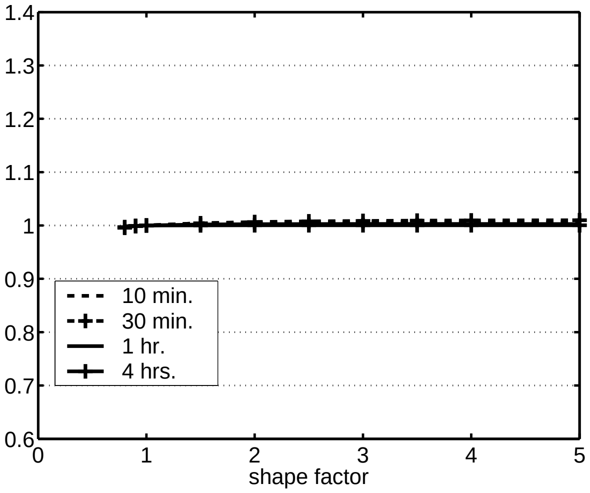
Canada Agricultural $/kWh



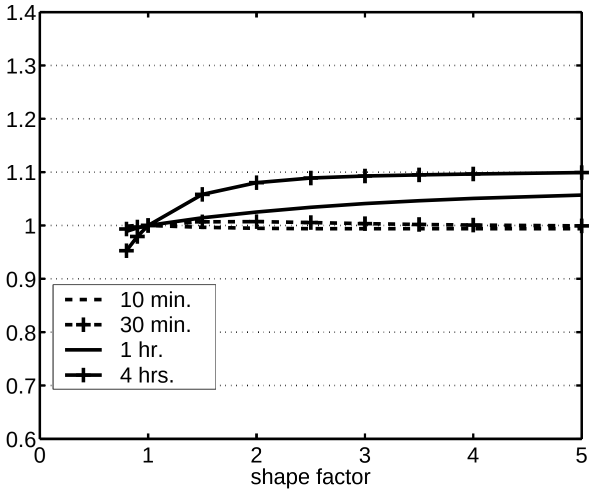
Canada Commercial $/kWh

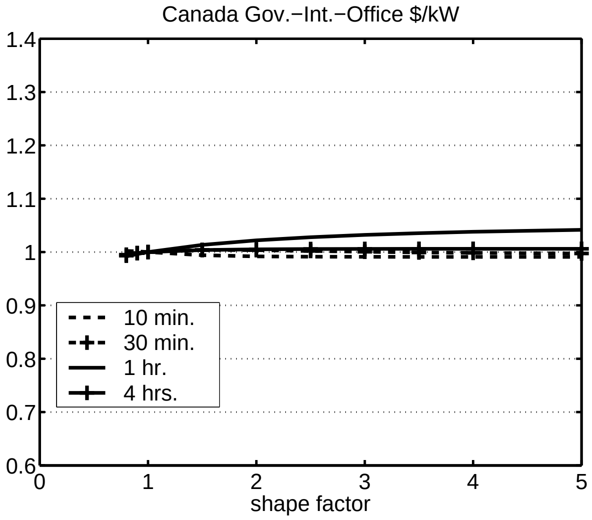


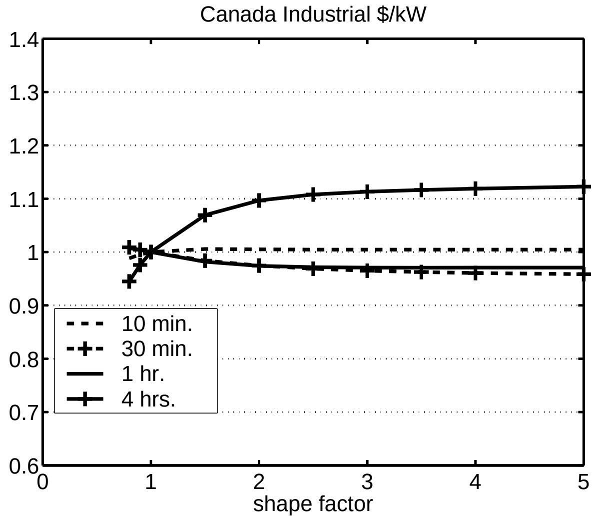
Canada Gov.−Int.−Office $

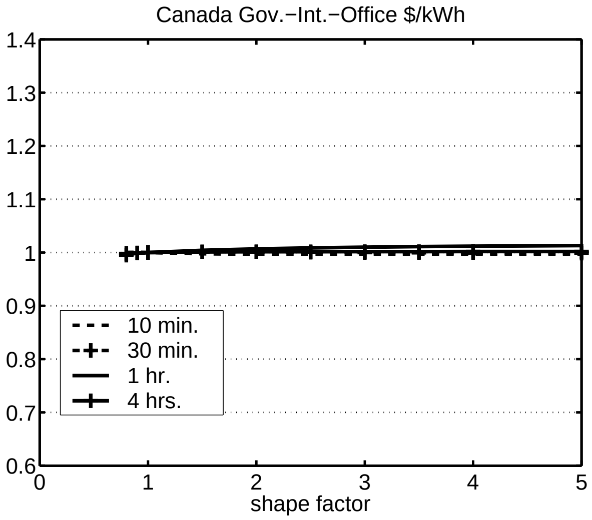


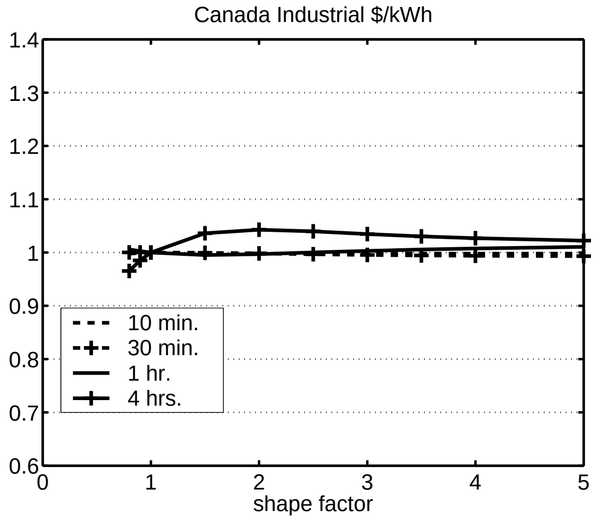
Canada Industrial $

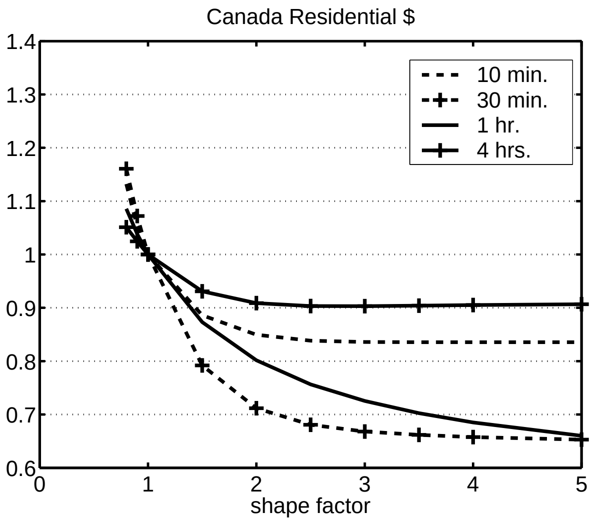


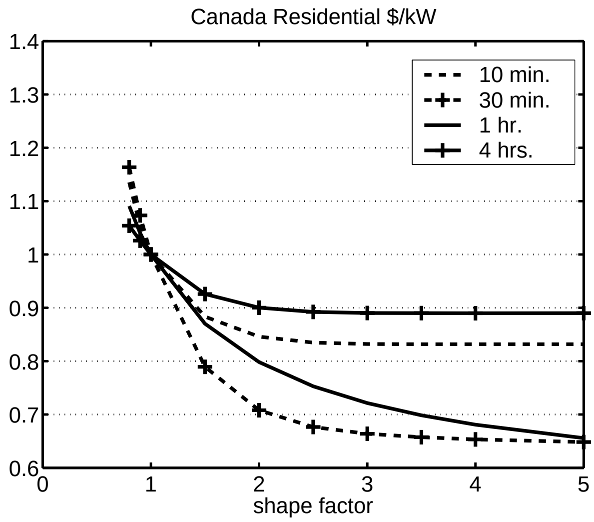

Canada Gov.−Int.−Office $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor

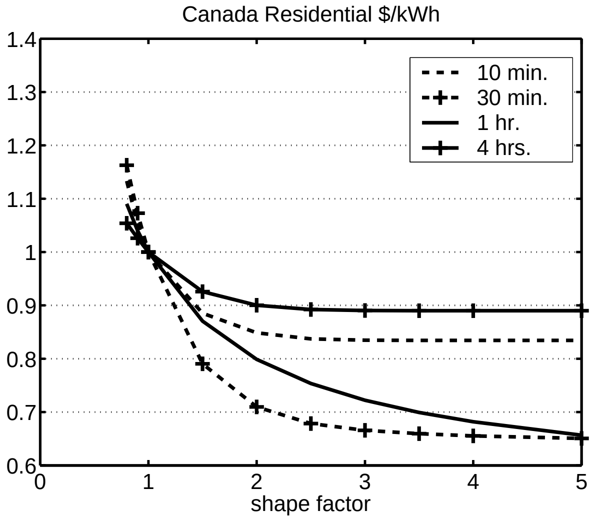

Canada Industrial $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Canada Gov.−Int.−Office $/kWh
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Canada Industrial $/kWh
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Canada Residential $
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Canada Residential $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor

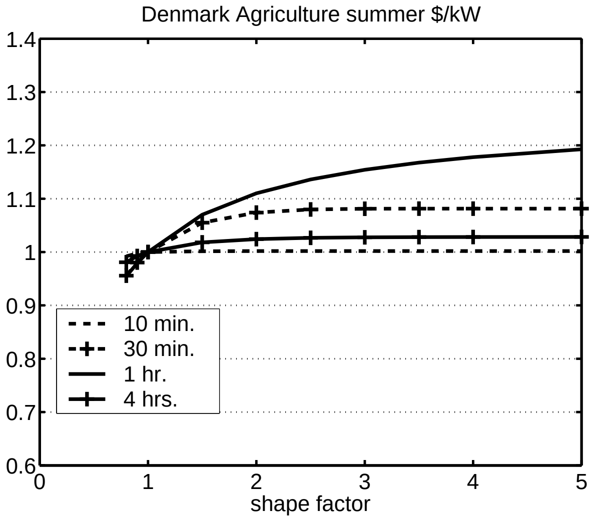


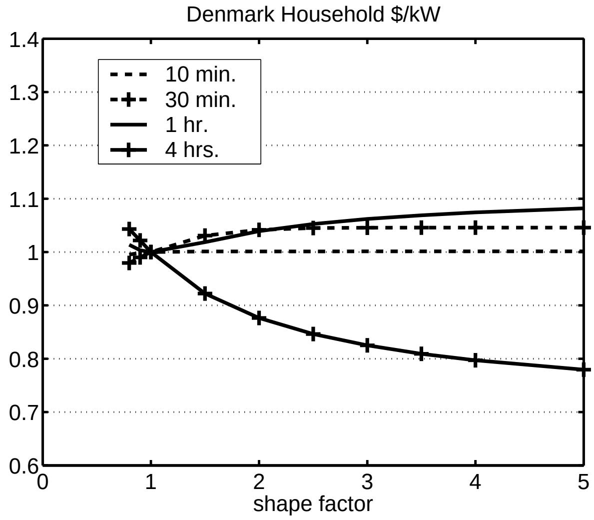
A.3 Denmark

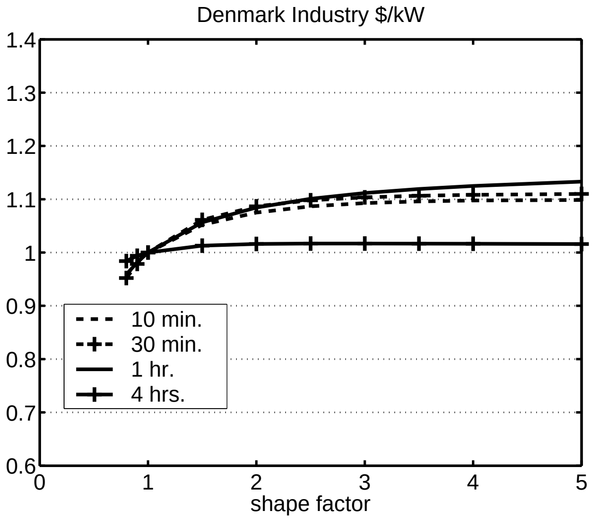
The damage functions which have been analyzed are from the 1994 investigation, which was initiated by the Nordic Council of Ministers. The shown costs are the interruptions losses as estimated by the customers.

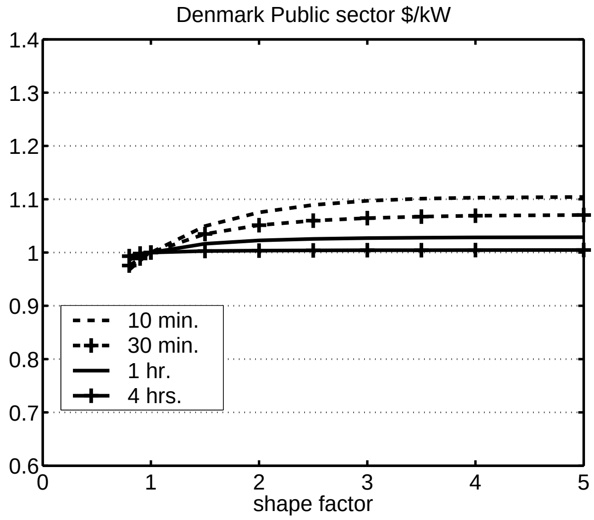
	Agri.	Household	Industry	Public	Ret. + Serv.
	$/kW	$/kW	$/kW	$/kW	$/kW
1 sec.			1.17	0.339	0.508
1 min.			3.28	0.339	0.555
15 min			9.57	2.99	1.40
1 hr	55.0	5.42	18.2	7.44	7.04
4 hrs		16.4	35.3	24.7	22.6
8 hrs	226	50.4	58.9	47.5	46.6

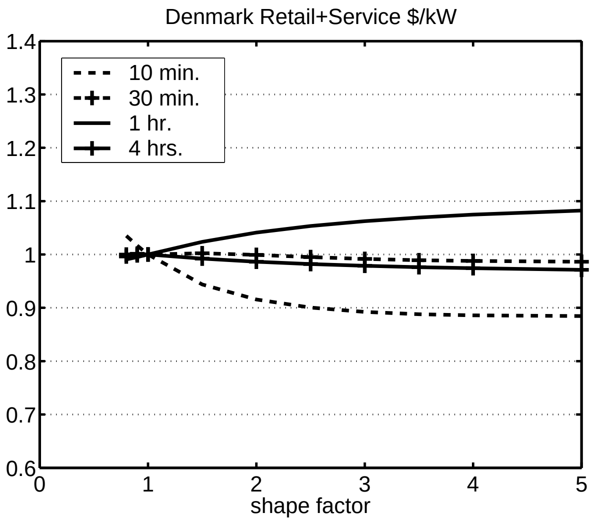
Table A.6: Denmark, SCDF for “Agriculture-summer”, “Household”, “Industry”, “Public sector” and “Retail+Service”


Denmark Agriculture summer $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Denmark Household $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Denmark Industry $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Denmark Public sector $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor

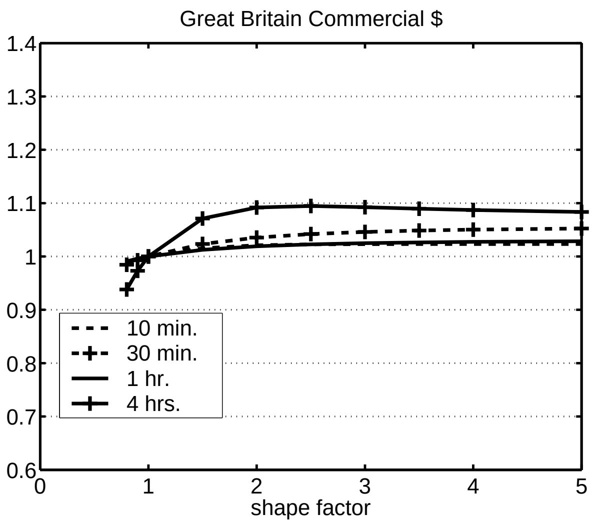

Denmark Retail+Service $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor

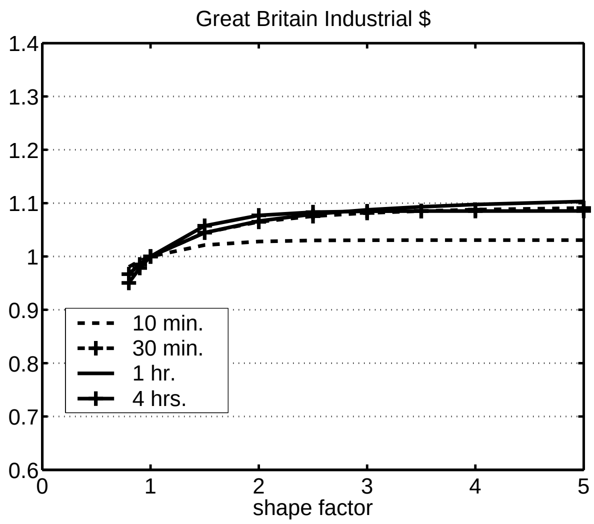
A.4 Great Britain

The analyzed costs are from the 1993 survey, conducted by the Manchester Centre for Electrical Energy at UMIST (University of Manchester Institute of Science and Technology). The shown costs are the interruption costs as perceived by the customers.

	Commercial			Industrial		
	$	**$/kW**	**$/kWh**	**$**	**$/kW**	**$/kWh**
1 sec.	18.2	1.57	730	1905	9.76	4794
1 min.	18.6	1.62	762	2381	10.3	4968
20 min.	78.0	6.17	2603	4603	22.7	10032
1 hr	168	16.9	7794	6825	40.1	18952
4 hrs	548	62.0	28778	12063	115	51730
8 hrs	1141	125	58825	19048	191	84698
1 day	1587	159	75524	25873	239	106508

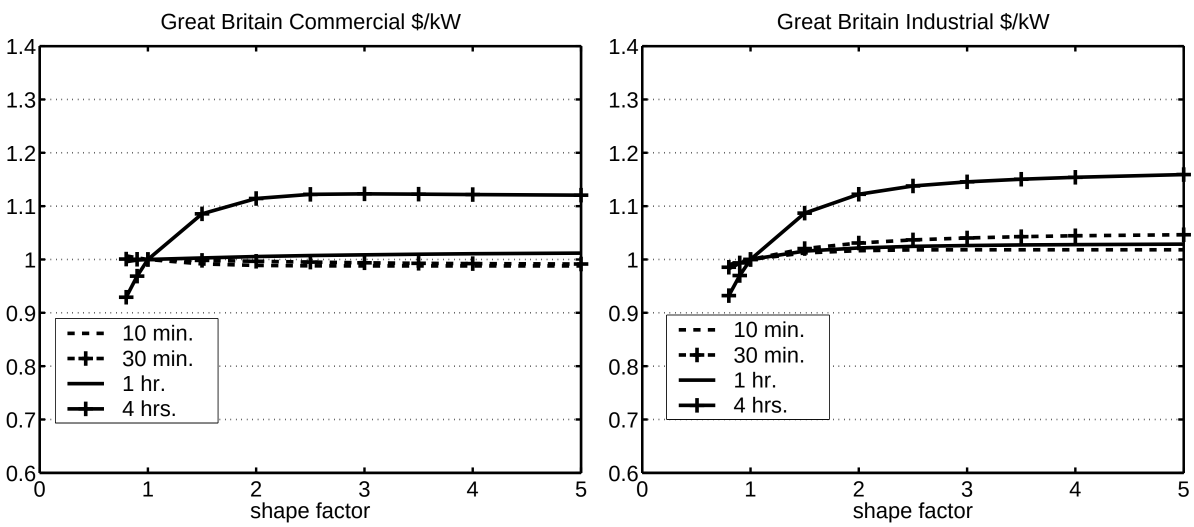
Table A.7: Great Britain, SCDF for “Commercial” and “Industrial”

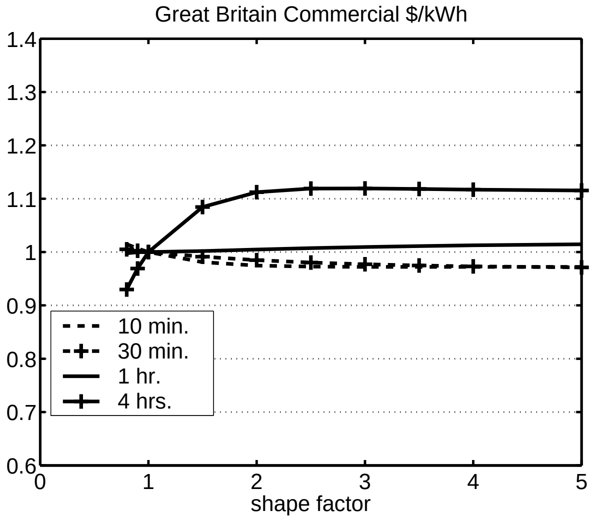


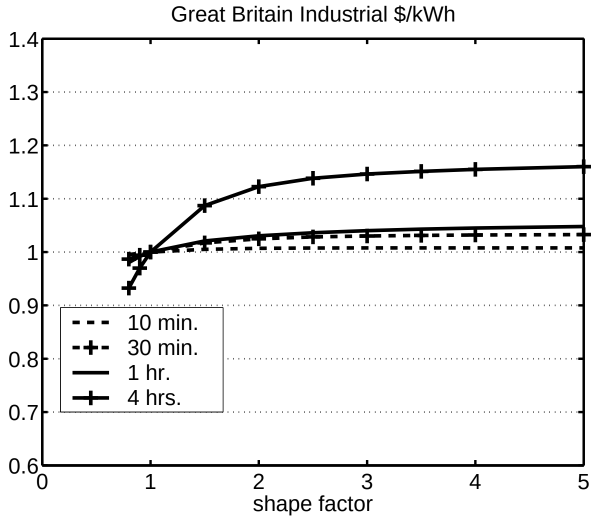


	Large user			Residential		
	$	$/kW	$/kWh	$	$/kW	$/kWh
1 sec.	342857	10.7	1698			
1 min.	342857	10.7	1698			
20 min.	347619	10.9	1730	0.302	0.238	95.2
1 hr	369841	11.4	2159	1.11	0.857	333
4 hrs	522222	14.1	2413	7.59	5.92	2286
8 hrs	655556	15.4	2714			
1 day	922222	21.2	3794			

Table A.8: Great Britain, SCDF for “Large user” and “Residential”




Great Britain Commercial $/kWh
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Great Britain Industrial $/kWh
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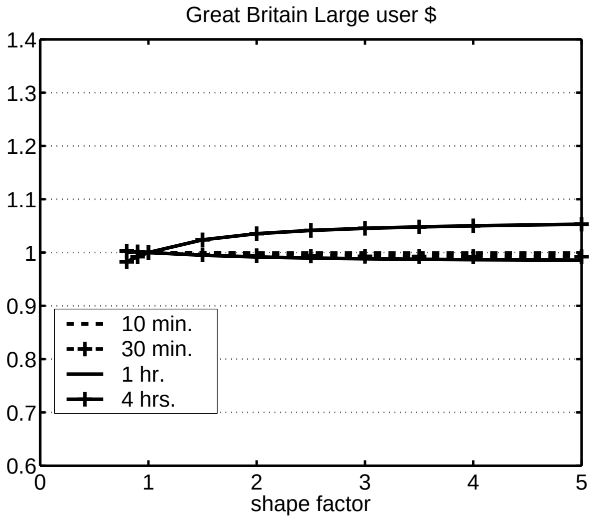
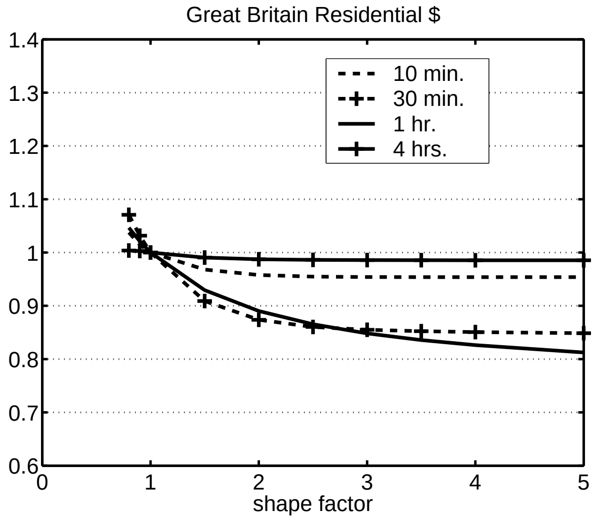
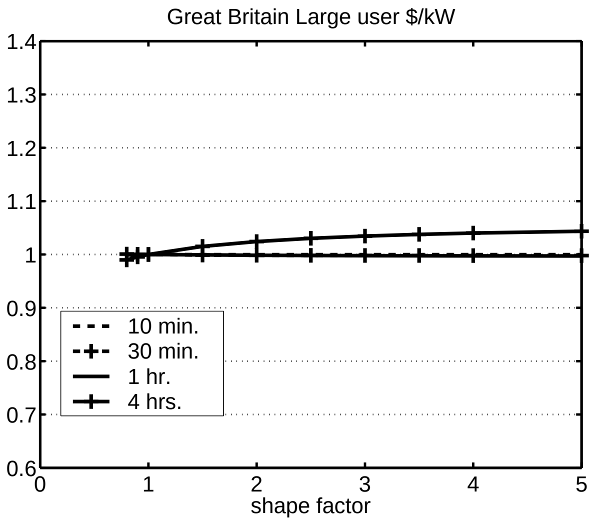
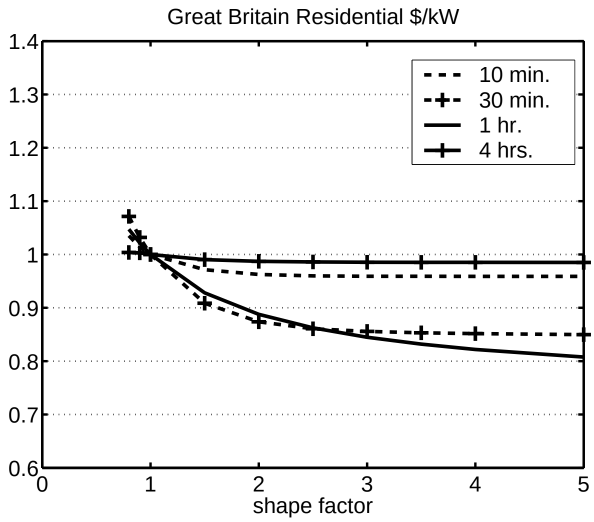
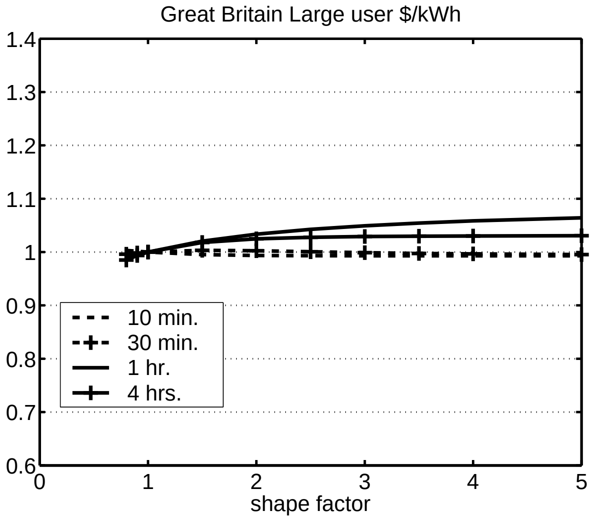
30 min.
1 hr.
4 hrs.
shape factor

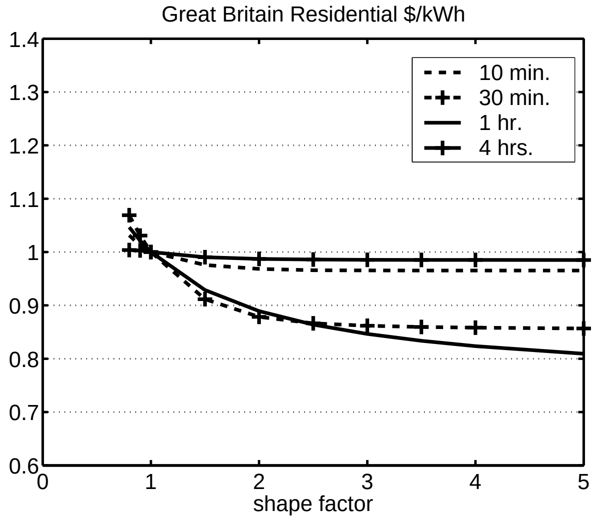

Great Britain Large user $
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Great Britain Residential $
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Great Britain Large user $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor


Great Britain Residential $/kW
10 min.
30 min.
1 hr.
4 hrs.
shape factor



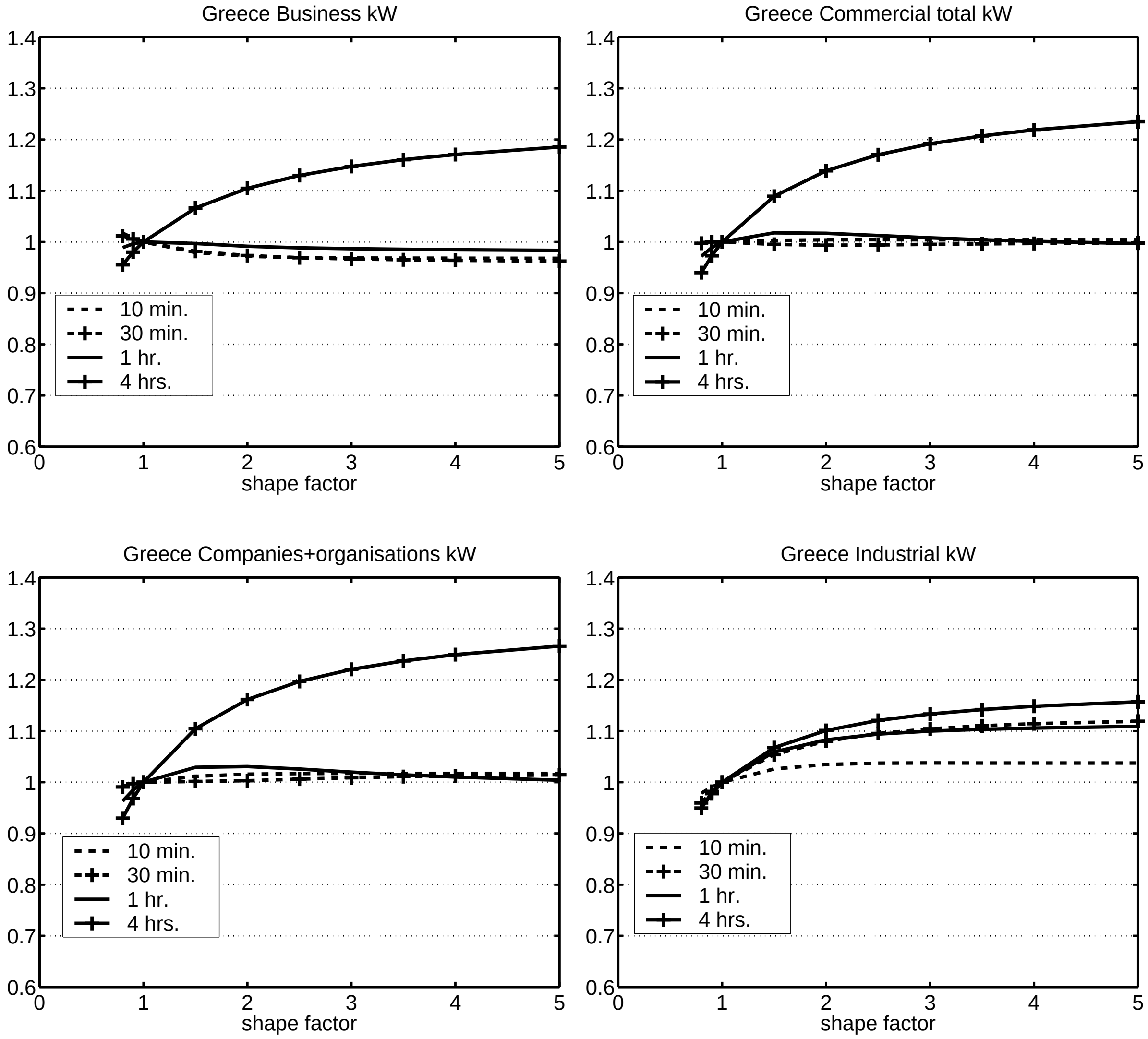


A.5 Greece

The analyzed costs functions are from the 1997 and 1998 surveys by the Power Systems Laboratory of the National Technical University of Athens (NTUA).

	Business	**Comm.**	**Comp. & Org.**	**Industrial**
	$/kW	**$/kW**	**$/kW**	**$/kW**
1 sec.	0.217	0.592	1.03	1.97
3 min.	0.250	0.723	1.28	2.37
20 min.	0.706	1.71	2.85	6.02
1 hr	2.02	3.73	5.48	9.52
2 hrs	4.09	7.55	11.1	13.7
4 hrs	8.34	12.9	16.9	18.7
1 day	27.7	31.5	31.4	41.0

Table A.9: Greece, SCDF for “Business”, “Commercial total”, “Companies+organisations” and “Industrial”

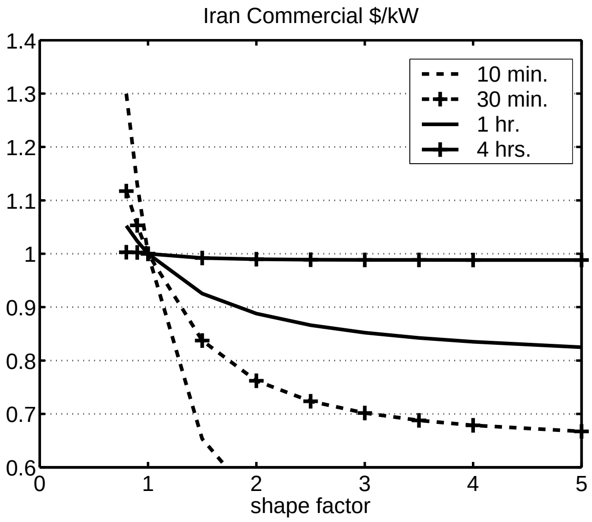


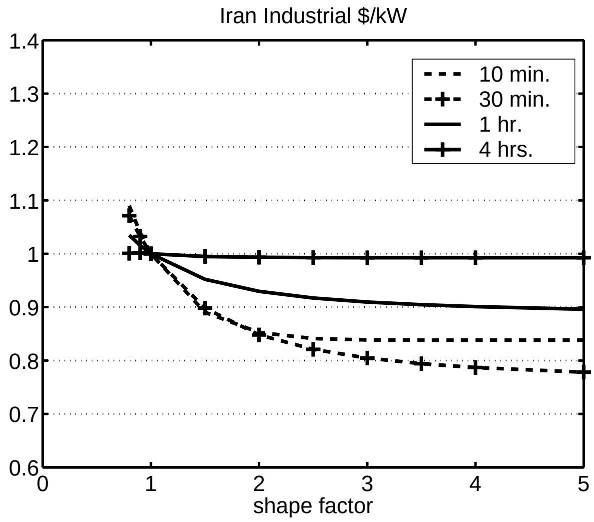
A.6 Iran

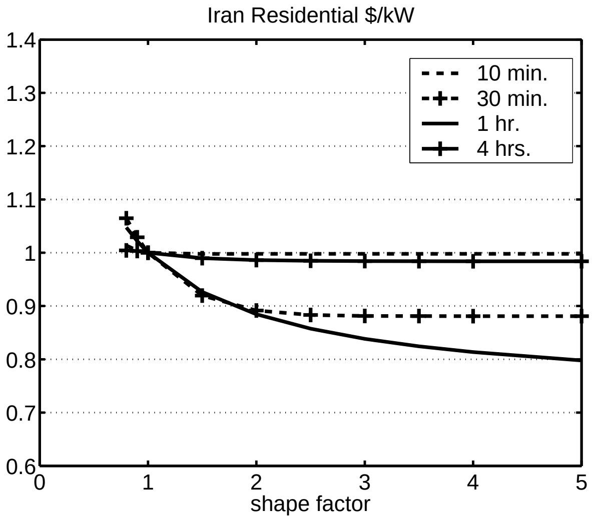
The analyzed interruption cost functions originate from the 1995 survey by the TAVANIR company in Iran.

	Commercial	Industrial	Residential
	$/kW	**$/kW**	**$/kW**
2 sec.	0.00566	0.180	
1 min.	0.00566	0.180	
20 min.	0.0646	0.304	
1 hr	1.21	1.92	0.793
2 hrs	3.64	4.80	2.38

Table A.10: Iran, SCDF for “Commercial”, “Industrial” and “Residential”







A.7 Nepal

The shown interruption costs functions are from the 1996 surveys conducted in Nepal by the power Systems Research Group at the University of Saskatchewan.

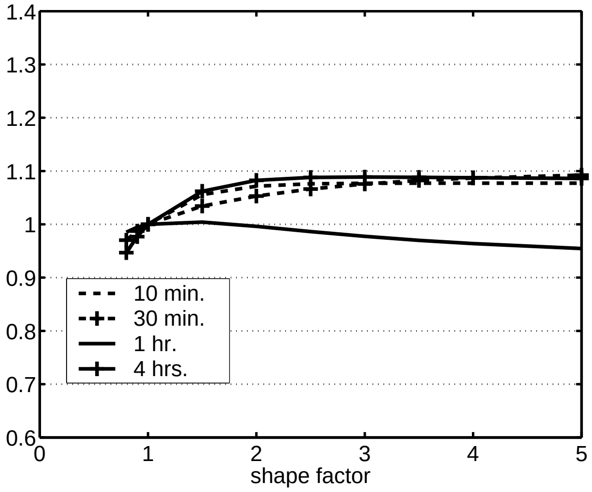
	Commercial			Industrial		
	$	**$/kW**	**$/kWh**	**$**	**$/kW**	**$/kWh**
1 min.	4.74	0.678	0.000195	32.9	0.0917	0.0000233
20 min.	8.49	2.32	0.000664	66.3	0.201	0.0000510
1 hr	9.04	5.00	0.00143	196	0.364	0.0000924
2 hrs	14.4	12.0	0.00344	263	0.502	0.000127
4 hrs	21.8	19.7	0.00565	362	1.29	0.000327
8 hrs	38.6	27.9	0.00799	532	2.58	0.000655
1 day	48.0	36.1	0.0104	852	9.45	0.00240

Table A.11: Nepal, SCDF for “Commercial” and “Industrial”

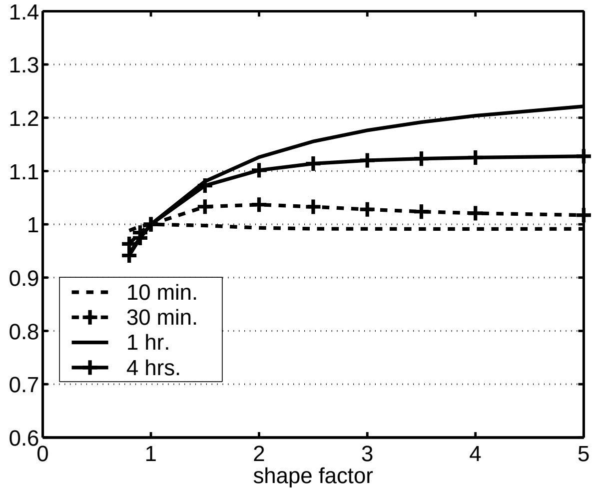
	Residential		
	$	**$/kW**	**$/kWh**
20 min.	0.0320	0.0270	0.0000155
1 hr	0.169	0.143	0.0000815
4 hrs	0.873	0.747	0.000427
8 hrs	1.78	1.54	0.000876
1 day	6.40	5.51	0.00314
2 days	13.8	11.8	0.00675

Table A.12: Nepal, SCDF for “Residential”

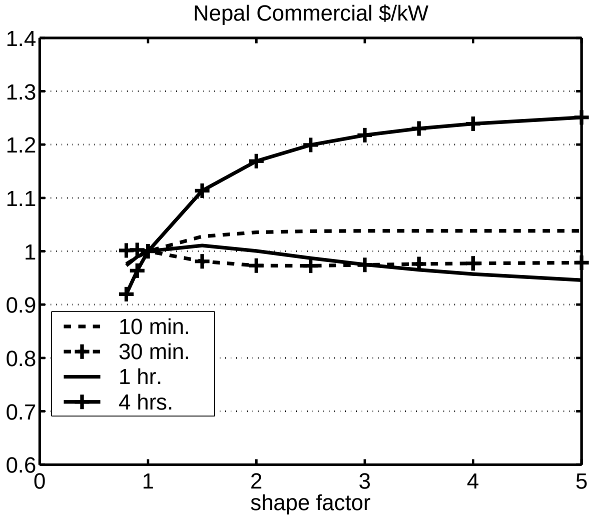
Nepal Commercial $



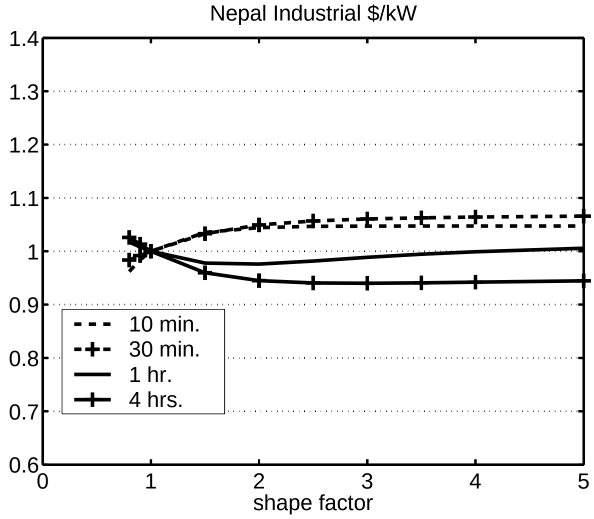
Nepal Industrial $



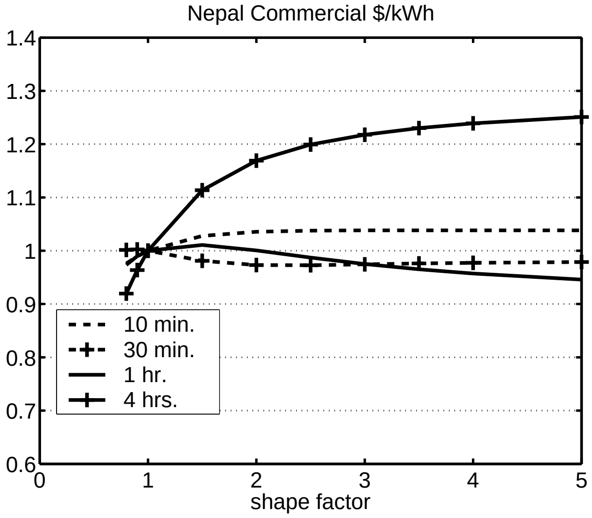
Nepal Commercial $/kW



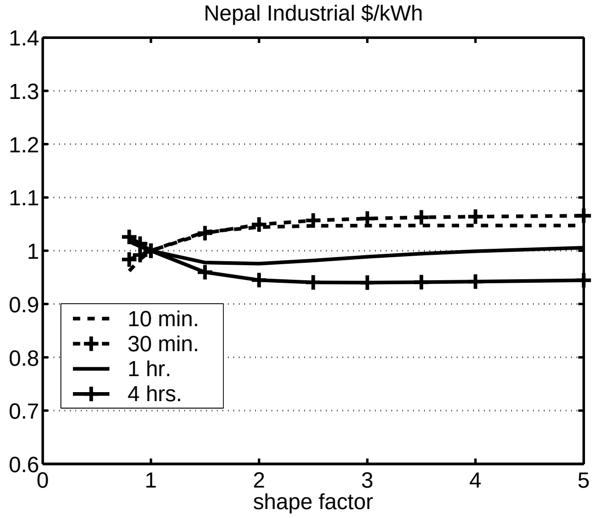
Nepal Industrial $/kW

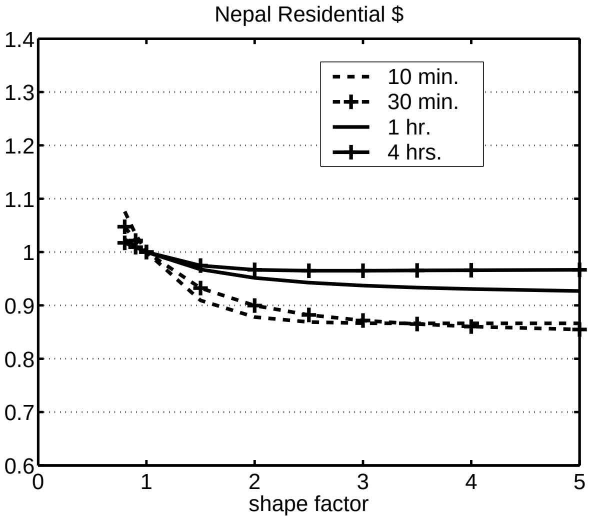


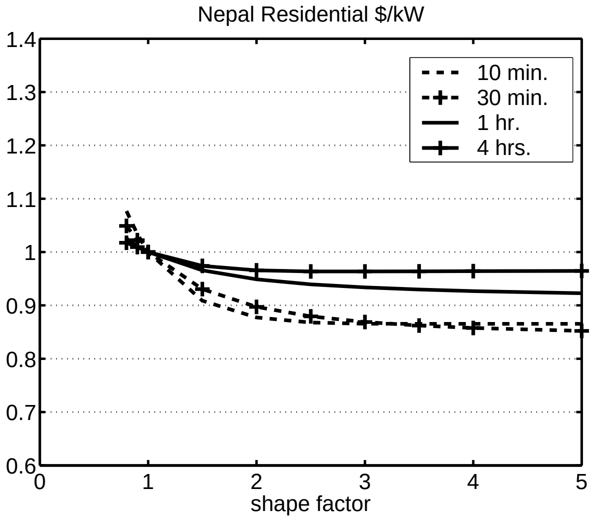
Nepal Commercial $/kWh

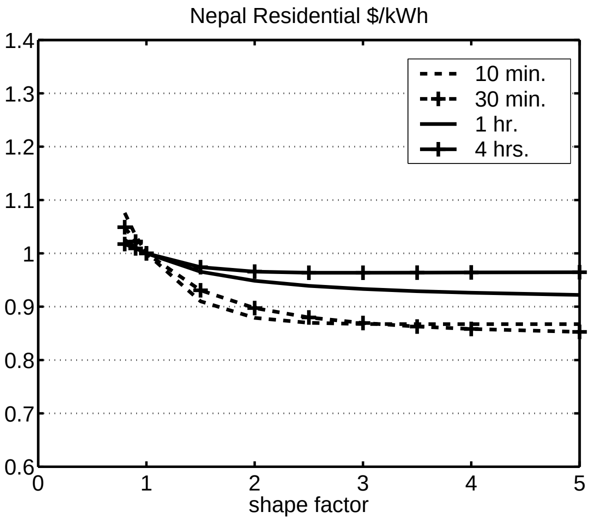


Nepal Industrial $/kWh







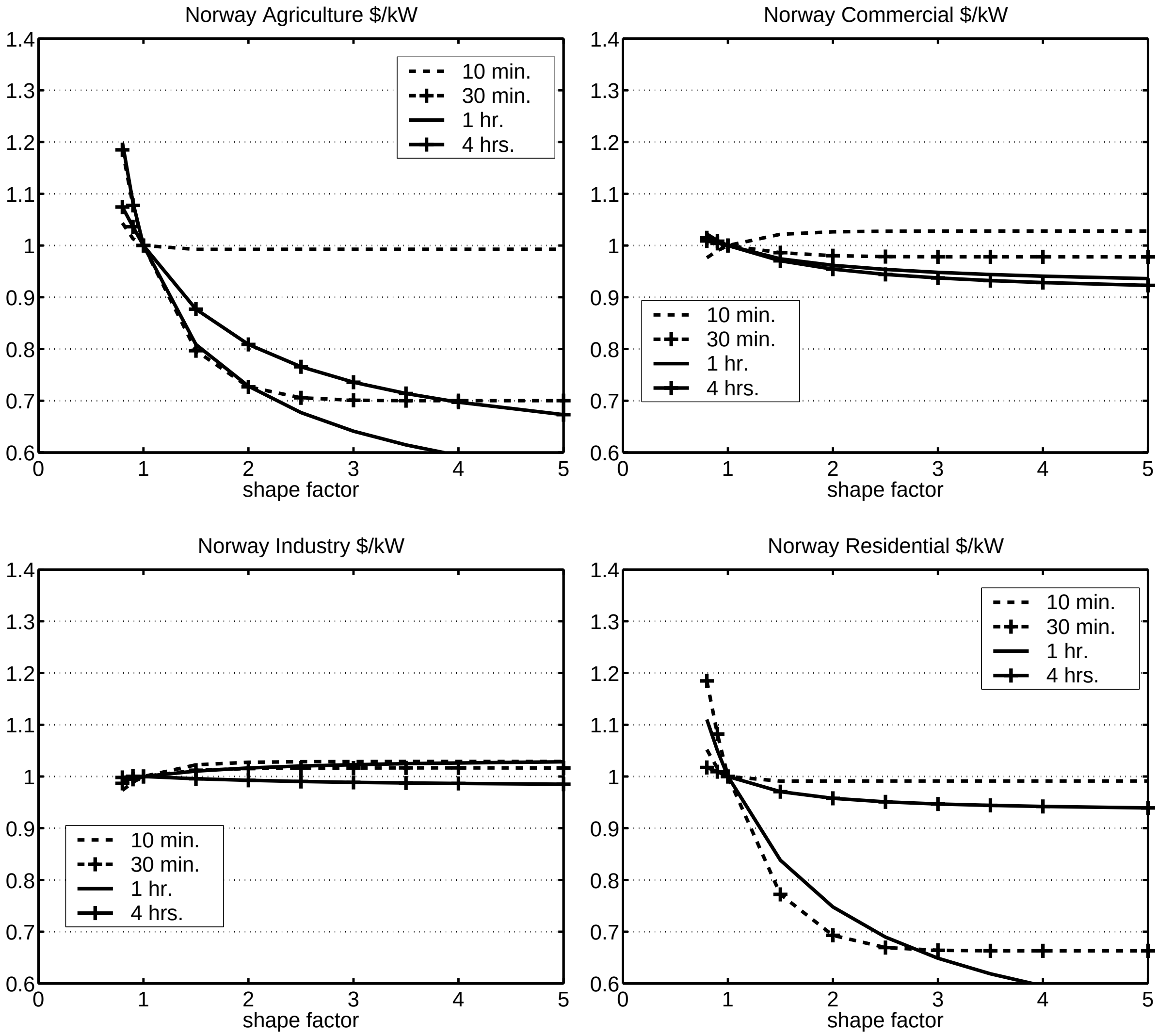


A.8 Norway

The analyzed interruption costs functions are from the 1989-1991 survey which was initiated by the Norwegian Water Resources and Energy Directorate (NVE) and performed by the Centre for Research in Economics and Business Administration (SNF).

	Agriculture	Commercial	Industry	Residential
	$/kW	$/kW	$/kW	$/kW
1 min.		0.907	1.33	
1 hr	0.147	4.73	6.79	0.293
4 hrs	1.97	19.8	21.4	4.48
8 hrs	8.32	45.5	42.5	10.7

Table A.13: Norway, SCDF for “Agriculture”, “Commercial”, “Industry” and “Residential”

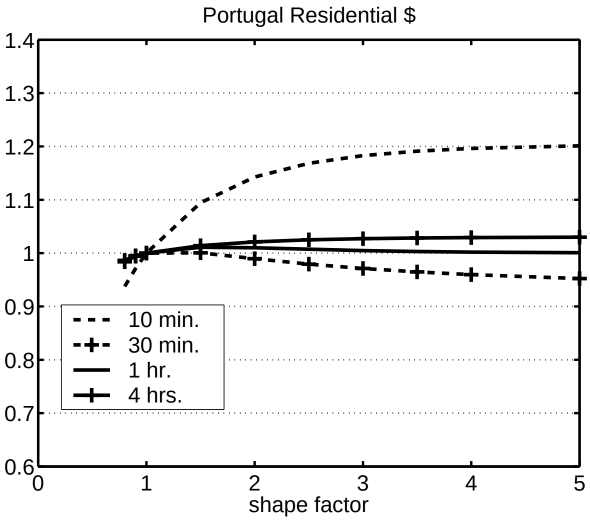


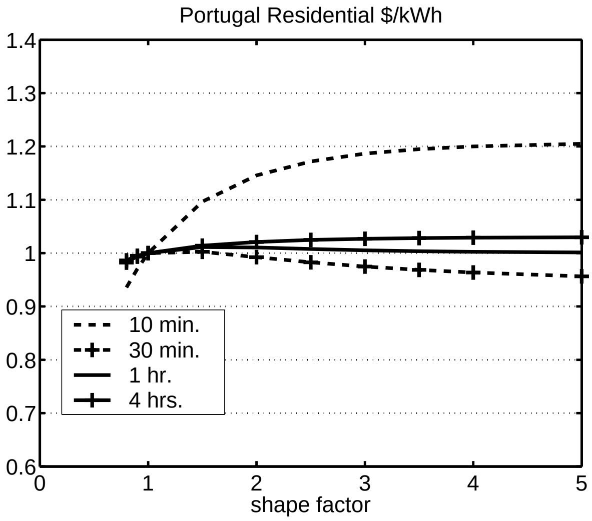
A.9 Portugal

The analyzed interruption cost functions are from the 1997-1998 surveys.

	Residential	
	$	**$/kWh**
5 min.	5.36	0.00168
30 min.	7.78	0.00238
2 hrs	25.4	0.00753
6 hrs.	59.4	0.0175

Table A.14: Portugal, SCDF for “Residential”



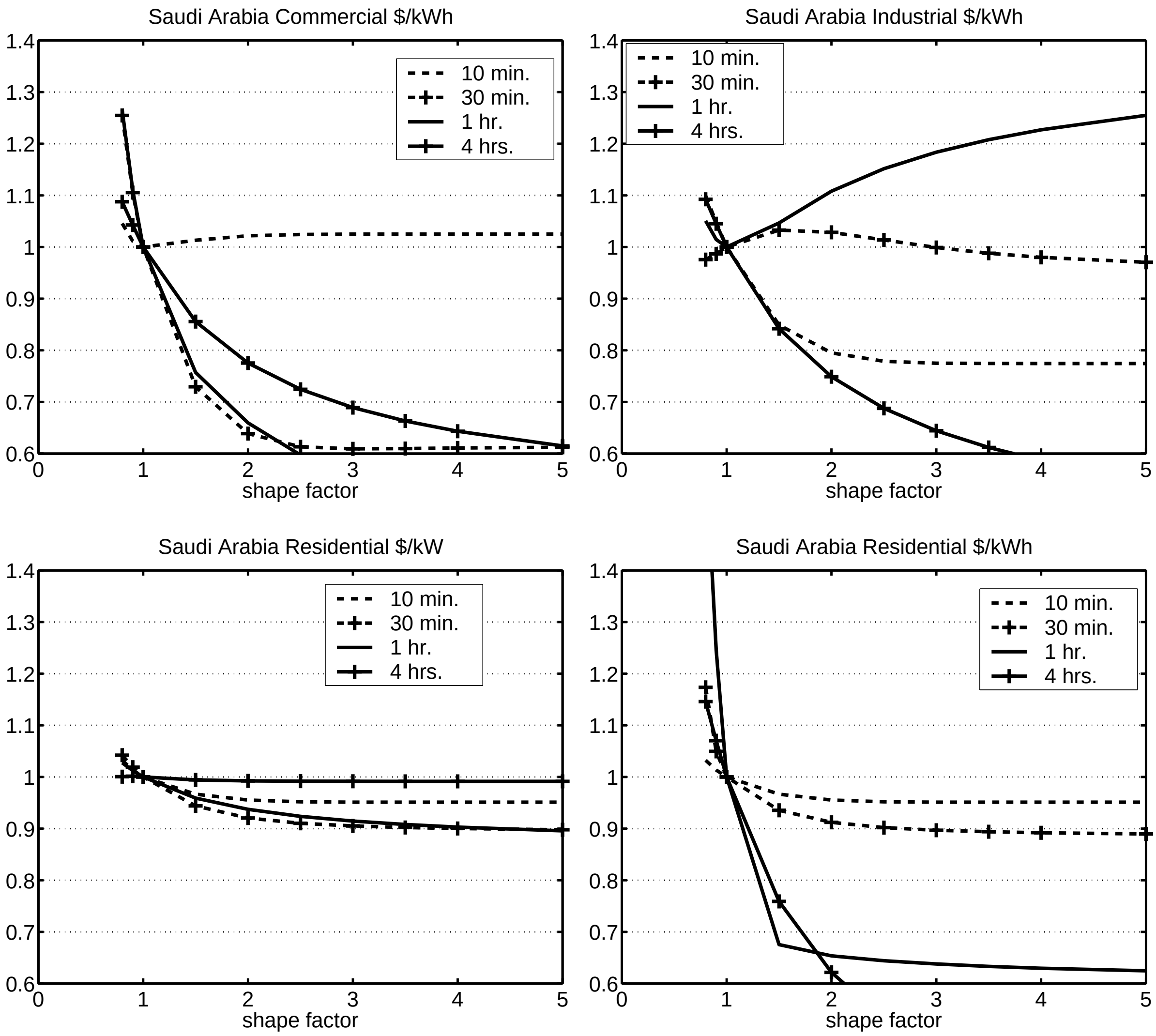


A.10 Saudi Arabia

The analyzed interruption costs functions are from the surveys which started in 1998 and from the surveys on 1992. The residential costs was based on willingness to pay.

	Commercial	**Industrial**	**Residential**	
	$/kWh	**$/kWh**	**$/kW**	**$/kWh**
20 min.	1.05	17.9	0.120	0.0572
1 hr	2.60	132	0.451	0.215
4 hrs	52.7	261	2.45	1.17
8 hrs	263	1558		26.0

Table A.15: Saudi Arabia, SCDF for “Commercial”, “Industrial” and “Residential”



A.11 Sweden

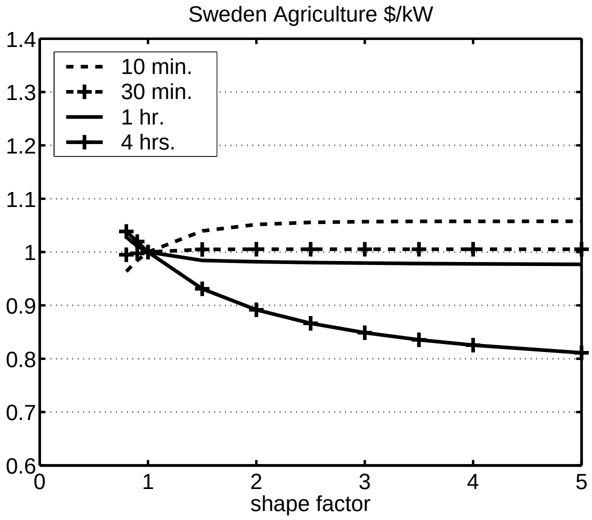
The analyzed interruption cost functions are from the surveys which have been published in 1994, and which were funded by the Swedish Electricity Distributors.

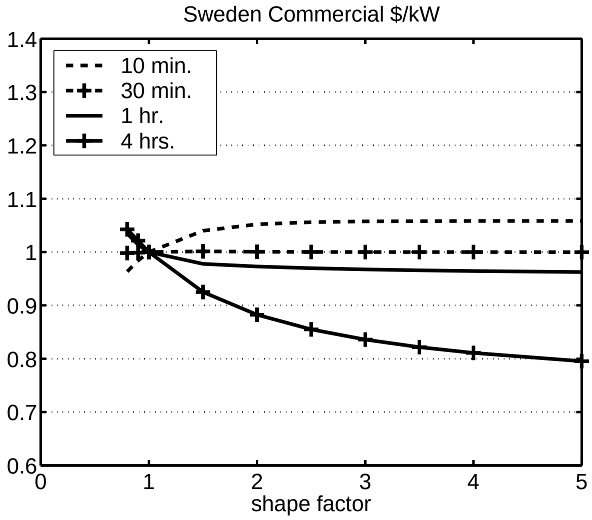
	Agriculture	Commercial	Large Ind.	Mining Ind.
	$/kW	**$/kW**	**$/kW**	**$/kW**
2 min.	0.581	1.74	2.35	0.375
1 hr	2.71	8.00	5.46	0.775
4 hrs	9.68	29.6	12.1	2.79
8 hrs	27.8	88.2	22.2	4.44

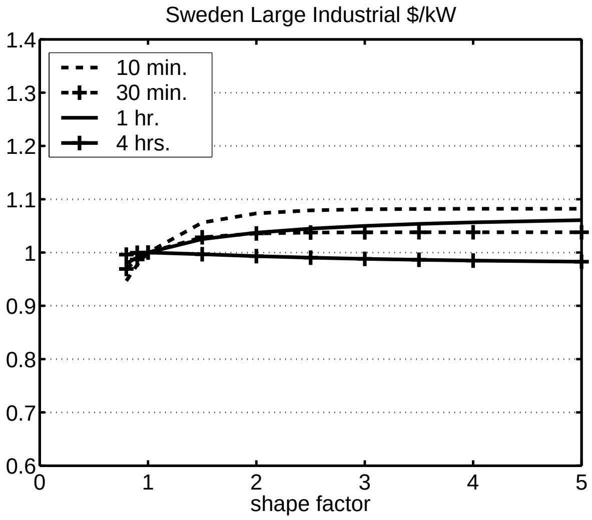
Table A.16: Sweden, SCDF for “Agriculture”, “Commercial”, “Large Industrial” and “Mining Industry”

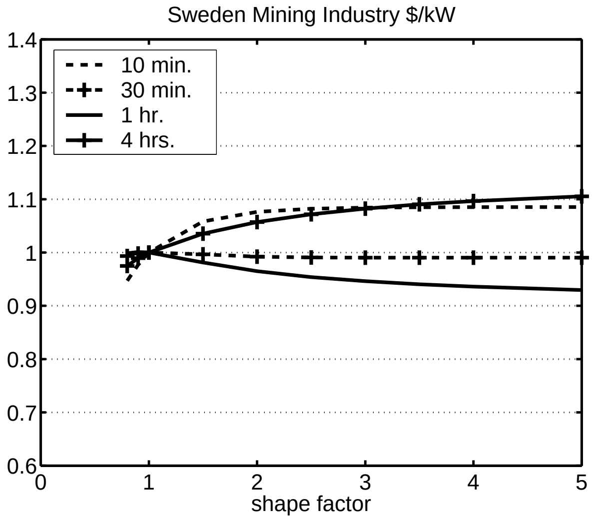
	Residential	Small Industrial	Textile Industry
	$/kW	**$/kW**	**$/kW**
2 min.	0.103	1.20	11.7
1 hr	0.310	4.70	15.8
4 hrs	1.18	19.2	31.1
8 hrs	3.31	41.3	53.0

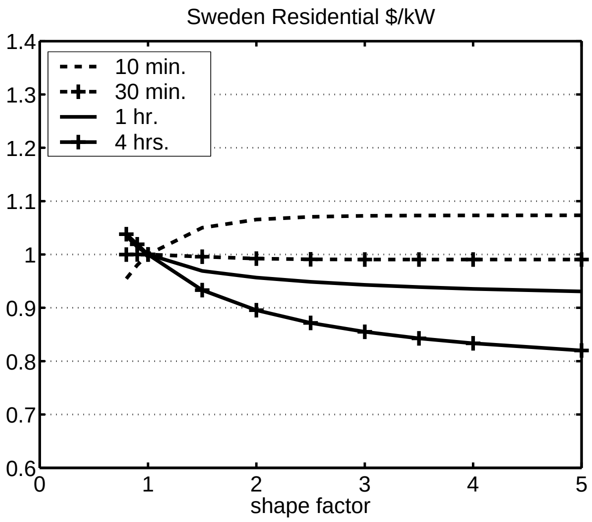
Table A.17: Sweden, SCDF for “Residential”, “Small Industrial” and “Textile Industry”

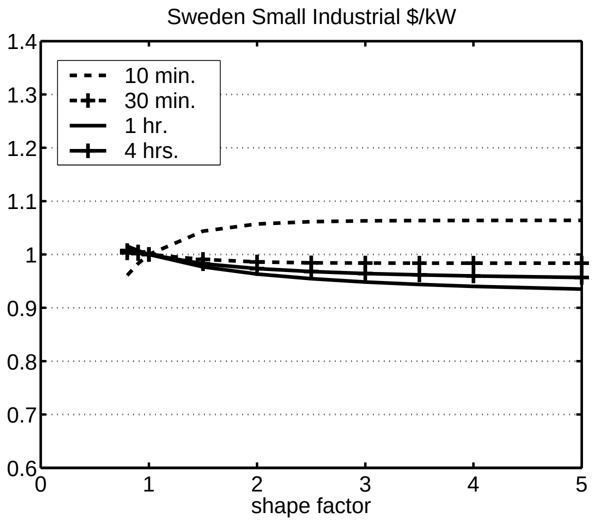

Sweden Agriculture $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor


Sweden Commercial $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor


Sweden Large Industrial $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor


Sweden Mining Industry $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor


Sweden Residential $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
1
2
3
4
5
shape factor


Sweden Small Industrial $/kW
10 min.
30 min.
1 hr.
4 hrs.
1.4
1.3
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0.6
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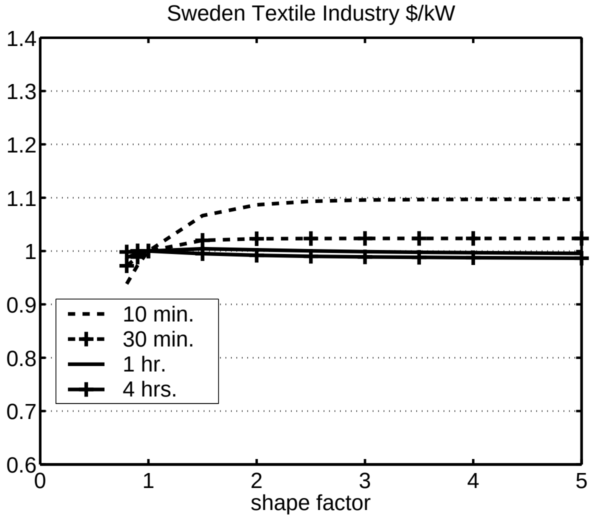
4
5
shape factor


Sweden Textile Industry $/kW
1.4
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1.2
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1
2
3
4
5
shape factor
10 min.
30 min.
1 hr.
4 hrs.

Appendix B

Glossary of Terms

Stochastic power system component. A power system component is said to be stochastic when it has more than one functional states in which it may reside, and when the time spend in each state, and the number of the next state are both stochastic quantities.

Stochastic component behavior. The stochastic behavior of a component is the description of the way in which the component changes from one state to another. This includes

- all the possible transitions from one state to another
- the distribution function for the stochastic durations for each possible state
- the probabilities for the transitions to other states, given the current state.

Component A component is a typical part of the electric power system which is treated as one single object in the reliability analysis. Examples are a single breaker, a transformer, a line, a generator, etc. Components may reside in different states, such as 'being available', 'being repaired', etc.

System A 'system' is short for an electrical power system. A system is build from components and changes state when one of its components changes state. The resulting 'system state' is the combination of all component states.

Event An event is a transition of a component between its states.

Outage An component outage is defined as the situation in which the component cannot be used, either because it has been taken out of service deliberately ('planned' outage), either because it has failed ('unplanned', or 'forced', outage).

Healthy State A healthy system state is a situation in which no unplanned outages are present.

Contingency A contingency is a system state in which one or more unplanned outages are present.

Coherent System Also called "consistent system". A system in which additional outages will never improve the system's performance.

Active Failure A failure of a component which activates the automatic protection system. Active failures are normally associated with short-circuits.

Adequacy The ability of the electrical power system to meet the load demands under various steady state system conditions.

Availability The fraction of time a component is able to operate as intended, either expressed as real fraction, or as hours per year.

Base State A system state in which no failures or outages are present.

Distribution Function The distribution function for the stochastic quantity X equals the cumulative density function CDF(x).

CDF(x) = the probability of X to take a value smaller than x.

Failure A failure is an undesirable event of a component.

Failure Effect Analysis (FEA) The electrical steady state and/or dynamic analysis of the system, possibly combined with network reconfiguration, generator rescheduling, or other alleviation techniques, in order to assess the loads which are lost or which have to be curtailed and the duration for which these loads are not supplied.

Hazard Rate Function The function HRF(x), describing the probability of a stochastic quantity to be larger than x+dx, given the fact that it is larger than x, divided by dx. The hazard rate may thus describe the probability of a element to fail in the next period of time, given the fact that it is still functioning properly. The hazard rate is often used to describe ageing and wear out. A famous example is the so-called “bath-tub” function which describes the probability of a device to fail in the next period of time during wear-in, normal service time and wear-out.

Hidden Failure An event of a component which will prevent it from operating as intended the next time will be called upon.

Interruption An unplanned zero-voltage situation at one or more load points due to outages in the system.

Maintenance The planned removal of one or more primary components from the system.

(n-1) system A system for which all relevant components are redundant units.

(n-k) system A system for which the outage of any k components will never lead to an interruption in the base state.

Outage The removal of a primary component from the system.

Passive Failure A failure of a component which does not activate the automatic protection system.

Probability Density Function The function PDF(x), describing the probability of the stochastic quantity to take a value from an interval around x, divided by the length of that interval. The PDF(x) is the derivative of the CDF(x).

Redundant Unit A component which outage will never lead to an interruption in the base state, but for which at least one contingency state exists for which its additional outage will lead to an interruption.

Repair The restoration of the functionality of a component, either by replacing the component or by repairing it.

Scheduled Outage The planned removal of a primary component from the system, i.e. for preventive maintenance.

Security The ability of the system to meet the loads demands during and after a transient or dynamic disturbance of the system.

Spare Unit A reserve component, not connected to the system, which may be used as a replacement for a component on outage by switching or replacing.

Statistic Statistic calculation methods are used to analyze stochastic quantities. A simple example is the method for calculating a mean repair duration by dividing the total time spend repairing by the number of repairs performed.

Information obtained by using statistic tools on measured data can be used to build stochastic models of the observed equipment.

Stochastic A quantity is said to be stochastic when its value is random and thus unknown. The range of possible values, however, may be known as well as the likelihood of these possible values. The number of eyes thrown with a dice is random, the possible outcomes are {1,2,3,4,5,6} and the likelihood is $\frac{1}{6}$ for each outcome. For a continuous range of possible outcomes, the likelihood is a continuous function, known as the Probability Density Function or “PDF”.

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