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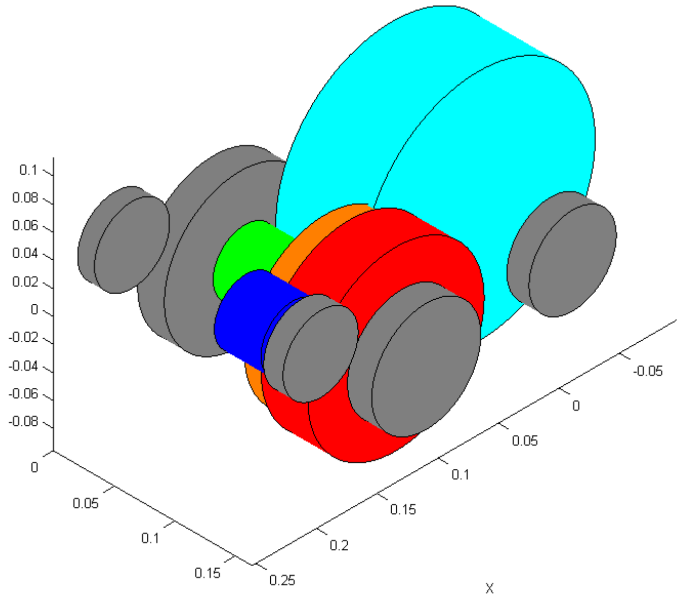

Optimization tool for EV-transmissions

Master’s thesis in Automotive Engineering

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Department of Mechanics and Maritime Sciences
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Cover:
Transmission layout representation used for quick analysis of result plausibility, see Section 3.2.9.

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Abstract

Due to the increasing concern for fossil fuel powered vehicles negative impact on the environment car manufacturers are looking more and more towards alternative drivetrains. One of the solutions is to use an electric motor also known as an e-machine. To be able to utilize an e-machine efficiently a transmission is needed between the e-machine and the driveshafts connecting to the wheels. The design process for this type of transmission is not as well mapped out as for a conventional transmission. The aim of the thesis work was to create a tool that aids the initial part of the design process. Matlab programing language was used to create a program that uses the basic parameters known in the start of a transmission project to provide a recommendation on where initial development should be started. The tool provides a variety of solutions ranked by different criteria selected by the user. The program is function based, starting by creating 2D layouts, calculating gear widths and progressing by calculating bearing forces and bearing sizes. Since the aim of the tool was to provide a quick estimation of the optimal layout, simplified calculations were used in the function files. The accuracy was verified using two methods in the Masta transmission program and the result showed that the transmissions recommended by the tool are optimized.

Key words: EV-transmission, Gearbox optimization, Electric vehicle transmission, transmission layout

Optimeringsverktyg for EV-transmissioner
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Sammanfattning

På grund av den ökande miljömedvetenheten i samhället börjar biltillverkare leta efter alternativ till fossila framdrivningssystem. En av lösningarna är att använda elmotordrivna bilar. För att använda en elmotor i dess effektivast körområde behövs en transmission mellan motorn och drivaxlarna som går till hjulen. Designprocessen för denna typ av transmission är inte lika väl utforskad som för konventionella transmissioner. Målet för examensarbetet var att skapa ett verktyg som hjälper till i början av designprocessen. Maltlab programmeringsspråk användes för att skapa ett program som använder de få parametrar som finns tillgängliga i början av designprocessen och ger en rekommendation av var fortsatt utveckling ska börja. Verktyget ger en mängd av möjliga lösningar rankade efter kriterier valda av användaren. Programmet är funktionsbaserat och börjar med att skapa 2D layouts, beräkna kuggbredd och fortsätter med att beräkna lagerlaster och lagerstorlekar. Eftersom målet med verktyget var att göra en snabb beräkning av den optimala lösningen har simplifierade ekvationer använts i funktionsfilerna. Noggrannheten utvärderades med två olika metoder i transmissionsprogrammet Masta vilket visade att de transmissioner som rekommenderades av verktyget var optimerade.

Nyckelord: EV-transmission, transmissionsoptimering, transmissionslayout, el-bilsväxellåda

Acknowledgements

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Contents

1 Introduction

Due to the drive towards more environmentally friendly transportation solutions electric and hybrid drivetrains for passenger vehicles are increasingly popular. To operate an electric motor in its efficient speed range a transmission is needed. The design process for constructing this type of transmission is not as well investigated as for conventional transmissions. To improve the range of the electric drivetrain a high efficiency is sought after in all parts of the drivetrain, including the transmission. When running a vehicle entirely on an electric drivetrain the Noise, Vibration and Harshness (NVH) from the transmission is easier to perceive since there is no combustion engine creating covering noise. This outlines efficiency and NVH as the two main performance parameters for an EV-transmission. Light weight is the third parameter considered in this thesis since a lighter vehicle requires less energy and has better driving performance. The transmission strength and durability are still required to fulfil the requirements for the application.

1.1 Objective

This thesis aims to improve the efficiency of the design process in start of the project and limit costly redesigns at the end of a transmission design project. To produce tools that will cost time in the start of the project but save time in the long run. The tool should rely on the basic input parameters that are available in the start of the project to produce close estimates of the optimum design or give multiple suggestions of layouts with high performance.

1.2 Limitations

The tool and guidelines will focus on EV-transmissions with a three-shaft design, input-, lay- and output shaft. It is intended to provide solutions for transmissions with one fixed gear ratio. Various simplifications have been made when calculating the internal forces so that they could be calculated quickly, one of the largest simplifications is concerning the direction of the park-lock force.

2 Background Study

In the beginning of the project a literature study was made using AVL internal educational material. It produced a background on how the three performance parameters described in section 1 should be evaluated. It also produced some information on how the Matlab tool should be focused concerning transmission layout. Lastly the design method for the chosen transmission will be described shortly.

2.1 Gear basics

Geared transmissions have been used for the last hundred years, methods for defining the macro- and micro geometry of the gears have been gradually developed. The basic gear theory has a vast amount of terminology that will not be explained in this thesis but can be found in the book by Mägi, Melkersson (2014). For example, helix angle is one of the parameters that has an impact on NVH, efficiency and strength/durability.

2.1.1 NVH

Noise, Vibration and Harshness characteristics of an automotive transmission become more important when there is not a combustion engine running that drowns any noise from the transmission. This means that when evaluating the performance of a transmission for an electric drivetrain gear whine and rattle are measured. The magnitude of gear whine and rattle are closely linked to Transmission error.

Transmission error is the difference in tooth speed between a perfectly machined and stiff gear transmitting a perfectly smooth speed profile compared to a gear with manufacturing variances and that deflects under load. The small variances in speed become vibrations and cause noise (Nilsson 2013).

Contact ratio is a dimensionless number that describes how many gear teeth have contact with each other on average. A small contact ratio of for example 1.2 means that the difference in stiffness if 1 or 2 gears are in contact is large. When 1 tooth is in contact it deflects more than if 2 teeth are in contact, this will cause transmission error. Compared to a contact ratio of for example 4.2 where either 4 or 5 teeth are in contact at any one time. Therefor NVH can be linked to contact ratio through transmission error.

The main gear parameters that influence contact ratio are helix angle, width of the gear and size of teeth relative to the gear size. This means that there are simple calculations that can be made to estimate the NVH performance of a transmission.

2.1.2 Efficiency

The losses in a transmission can be divided into three groups; mesh friction, splash losses and bearing losses. Mesh friction occurs in between two mating teeth when they contact and slide against each other. The amount of mesh loss is both speed and load dependant. Splash losses occur when the gear wheels pass through the oil sump in the transmission and churn the oil, these losses are speed dependant. Splash losses are hard to make a rough estimation of their magnitude since they depend on internal geometry of the transmission. To make a good estimation time demanding CFD simulations are needed.

The losses inside the bearing can be divided into three friction sources; rolling moment, sliding moment and drag moment, SKF (2018). Given the speed and load on the bearings SKF provides accurate ways of estimating the losses based on empirical studies. Although the amount of drag is dependent on the oil level in the bearing which can be hard to estimate inside the transmission.

2.1.3 Strength/durability

The basic parameters determining the strength of a gear and pinion are gear width and centre distance. A wider tooth will distribute the stress along a longer line along the tooth. A larger centre distance, the distance between the axles that the gears sit on, will create a longer moment arm for the gear teeth to work on reducing the force being transmitted through the teeth. Both width and centre distance decide the size and weight of the gears which usually are minimized for a transmission. Different widths and centre distances can be combined to satisfy the same strength requirement but give different characteristics when it comes to weight, NVH and efficiency.

2.2 Transmission concepts

An EV-transmission can have different configurations and functions depending on the application and price class of the vehicle it is used in. Some basic factors that determine what kind of configuration is required are speed range and if the electric powertrain in to be run together with a combustion powertrain. To give a background on which layout was chosen for this thesis some of the concepts commonly used are presented below.

2.2.1 Layshaft design

A layshaft design means that there are several different axis that normal spur gears sit on. There is an input and an output axel on different axis parallel to each other, the torque is transferred between these via a layshaft.

2.2.2 Planetary

Planetary gear sets can be used to provide the reduction for the whole transmission or combined with a layshaft design.

2.2.3 One or two gears

Even though an electric motor can provide torque through a large range of speed it may be more efficient at certain load points and provide more performance with a transmission using a 2-speed design. This adds complexity to the system and the shifting can be performed both with power and without depending on the components chosen.

2.2.4 Disconnect

For some vehicle solutions a motor and transmission disconnect function may be sought after to either limit the top speed of the e-machine or minimize the loses when the e-machine is not being used. It may be placed either directly after the input or just before the output.

2.2.5 Park lock

A Park lock for an Ev-transmission has a similar function to an automatic transmission's "P"-position and is often required as backup to stop the vehicle from rolling forwards or backwards when parked. It has specific requirements on which speed it should be able to be engaged. This puts load requirements on bearing and other components of the transmission.

The park lock should always engage when the vehicle moves below a certain speed for example 3kph, the park lock should not engage over a certain speed for example 5kph. This means that the system should handle torque and force pulses that are created from park lock engagement and a direct stop at slightly below 5kph.

2.3 Typical design process

To construct a useful tool for the design process, a picture of the typical design process was created in the start of the project. The process naturally differs from project to project but by studying the process used for specifically the type of transmission the tool is aimed for, some generalisations can be made.

- Firstly, a requirement summary is set up, this is usually done by or with the customer. It mainly specifies the lifespan, abuse load, weight and space.
- Layout concepts are created using tools like Paint or PowerPoint
- The most promising layouts are modelled in Masta to evaluate their load capability
- Simple CAD models are made to foresee clashes with auxiliary components
- Park lock load calculations are made varying the position of the park lock

2.4 Masta

Masta has been used in several parts of the thesis work and is mentioned many times throughout the report. Masta is a commercially used transmission modelling programme from SMT (Smart Manufacturing Technology). At AVL Vicura it is used as a part of the design process for a transmission. In this project it has been used to quickly set up a model with the basic layout of a transmission like in Figure 1 to extract bearing forces or other basic parameters. It has also been used to evaluate the whole Matlab tool.

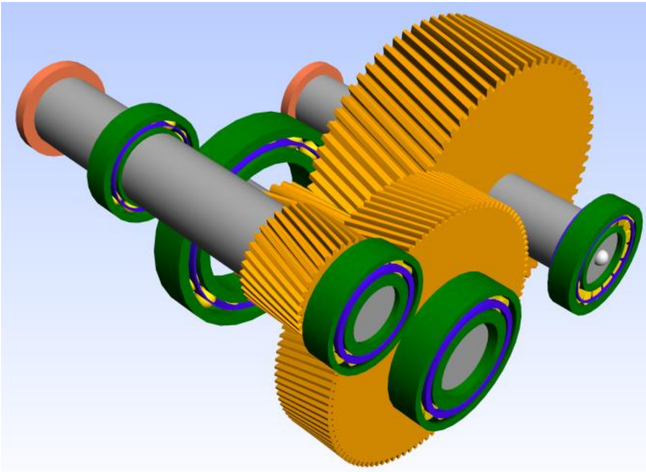


Figure 1 Masta model of EV-transmission

3 Method

The project started with a literature study to get a rough picture of the scope of the thesis and to create a base for the project. The major part of the project was dedicated to construct the Matlab tool, an extensive description of its functions is presented in the tool construction chapter, see 3.2. To complement the Matlab tool guidelines for efficiency evaluation were created.

3.1 Simple Guidelines

During the literature study at the start of the project, information about the design process was gathered. It was used to get an overlook on the project and condensed into some simple rules of thumb presented in the result chapter 4.1. These are meant to be used as a complement to the Matlab tool.

3.2 Tool construction

The tool is set up too solve a problem where a transmission should fit in a rectangular box, various input parameters are assigned that are needed to define the layout of the transmission. Firstly, the basic layout of the tool is described and then a more in-depth description of each function is given in chapter 3.2.2 and onwards. To aid the user in defining input parameters chapter 3.4 describes each of the input variables separately.

3.2.1 Layout

To get an initial overview of the layout of the tool a function diagram together with a solution diagram was created, see Figure 2. Matlab was chosen as the programming language since it is a common and easy to understand tool. A map was made to be able to present the basic layout of the tool, see Figure 3
.

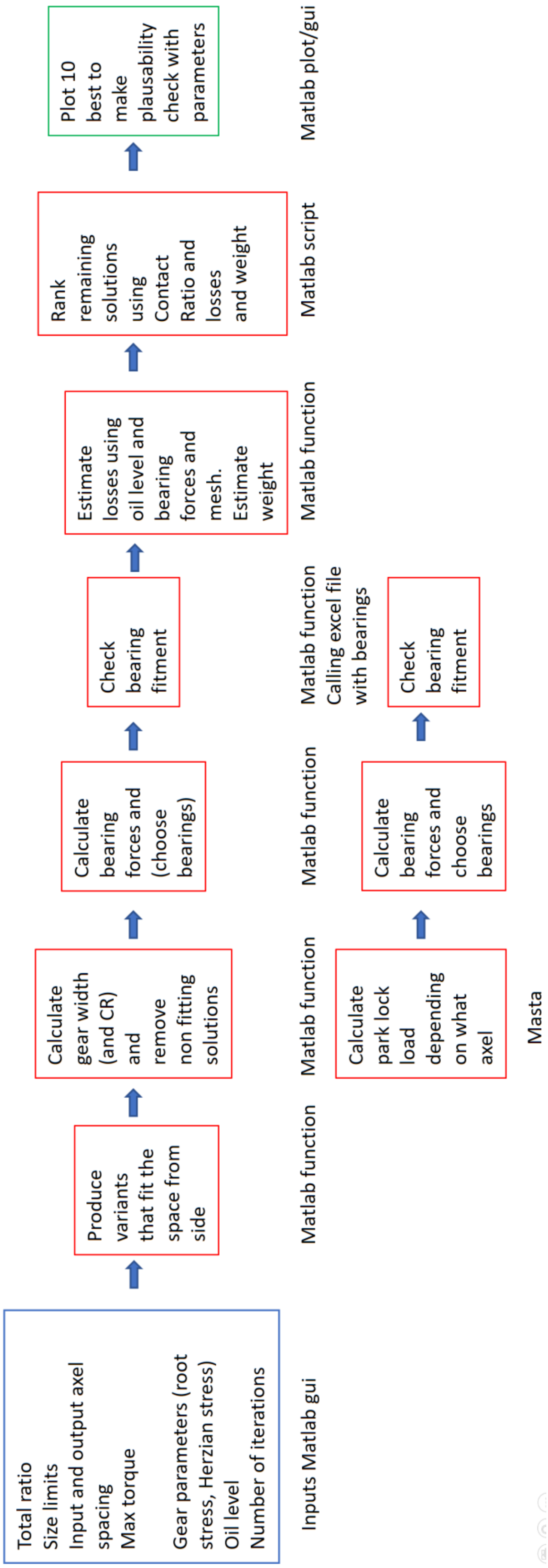


Figure 2 Solution diagram

Programming was started from the first box of the solution diagram. The function file *possibleCombinations2D* sets up a vast amount of gear dimensions and then sorts out the ones that are possible to combine and fit the basic criteria for space and gear ratio. It also checks so that the gear base circle does not collide with the minimum shaft diameter.

A function file, *gearWidthCalc*, was created to calculate the required width of each gear stage for each transmission combination. The Lewis formula was implemented together with its table of factors, see Equation 1 which has been taken from Boston gear (2018) where the tooth form factors can be found. The function file removes solutions where the combined gear width is greater than the transmission width. The function also calculated the contact ratio for later use to evaluate transmission error. The contact ratio equations were taken from MITCalc (2018).

$$W = \frac{SFY}{P}\left(\frac{600}{600+V}\right) \qquad \text{Equation 1}$$

Where:

$W = Tooth\ Load$
$S = Safe\ Material\ Stress$
$F = Face\ Width$
$Y = Tooth\ Form\ Factor\ (from\ table)$
$P = Diametral\ Pitch$
$D = Pitch\ Diameter$
$V = Pitch\ Line\ Velocity$

The next function in the function diagram calculates the forces that the bearings are subjected to and chooses bearings from a bearing catalogue. The forces were acquired using a free body diagram, see Appendix 1. The bearing forces were calculated in the main script of the program and the bearing choice calculations were done in a function file, *bearingSelector*. The SKF method listed in the SKF catalogue was used when choosing the right sized bearing (SKF 2018).

Weight is estimated by calculating the simple body volume of the gears as if they were cylinders with the diameter of the base circle and have the same width over the whole face. This was multiplied with the density of steel. The bearing weights are extracted from the SKF table. The sum of these are considered a reference weight for the transmission. This is very simplified since the transmission consists of many more parts such as axles and housings.

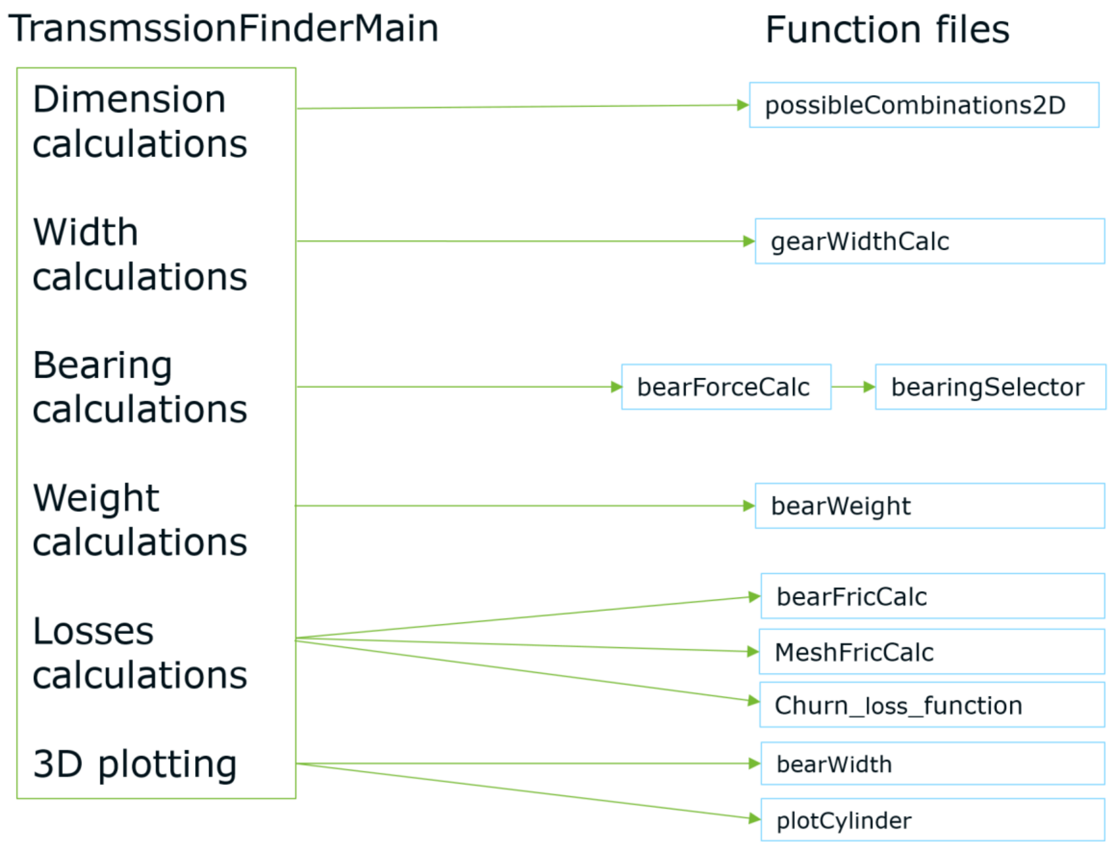


Figure 3 Tool layout map

The losses are calculated using three different categories; bearing friction, mesh friction and splash losses. The bearing friction is calculated in the *bearFricCalc* function using the SKF method from the "SKF friction calculation.pdf" (SKF 2018). The mesh friction power is calculated using ISO standard 14179 (Masta 2015) that the transmission development program Masta (Masta 2018) also uses. This is done in a function file called *meshFricCalc*. The splash loss prediction is based on an existing AVL-Vicura function file, *Churn_loss_function*, which is based on research paper 128_1.pdf (Changenet 2007).

Solutions where the bearings are bigger than the shaft distances so that the bearings clash or are closer to each other than the minimum distance are deleted. This is done using a simple "if" block in the main script with a specified minimum distance.

The transmission solutions are ranked using the three performance parameters, contact ratio NVH, efficiency and weight. The top 10 solutions are plotted to give a visual representation if they are plausible.

3.2.2 PossibleCombinations2D

The function "PossibleCombinations2D" uses the space requirements to set up a set of gear diameters and then sort them to see that they still fit inside the space requirements. Necessary input parameters are height and width of the rectangle, centre distance between input and output shafts. The span of ratios the transmission needs to be within and a minimum shaft diameter together with the resolution that the dimensions are divided into “dimensionSteps”. It also needs the angle between the line from input to output and the horizontal axis (Inclination).

The function file creates a vector of gear diameters for each gear ranging from the minimum shaft diameter to the whole width/height of the transmission. It then defines each combination in a for loop to check if it satisfies several different requirements. Total gear ratio and that the whole combination fits inside the defined rectangle, see Figure 4. It checks so that the gears do not clash with the minimum shaft diameter. If the combination satisfies the requirements it is saved in a matrix and output from the function file.

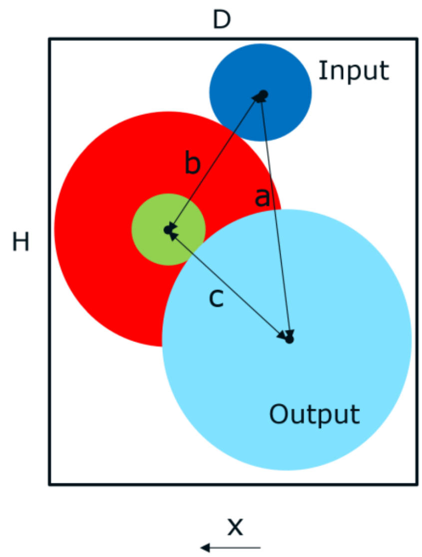


Figure 4 2D layout

3.2.3 GearWidthCalc

This function uses each layout from the previous function and computes required gear width for each of the stages. It uses the helix angles as input together with the abuse input torque and max stress, module and pressure angle are also inserted. “Abuse input torque” is a constant equivalent torque that matches the load situation that the varied torque input of real world usage will put on the transmission.

The gear width for each stage is calculated using the Lewis formula which gives a basic estimation of the required gear width. A quick and rough method was sought after because the focus of the tool was not to provide accurate gear width results. It uses empirical data in a formula to find the stress in a certain gear. It is then possible to calculate the contact ratio for each stage, this is done in line with the equations from MITCalc (2018). This will be used to estimate transmission error which is used to estimate NVH.

The gear widths, contact ratios and helix angles for each layout are put in a matrix together with the layout from the previous step. The matrix is the output from the function file.

3.2.4 Bearing force calculation and bearing size selection

The transmission parameters from the previous function are used together with the minimum distance between components in the y-direction as inputs. The bearing abuse Input torque is a different abuse torque compared to that for the gear width calculations since they may need to be varied compared to each other to get the most accurate estimation of real world usage. Bearing dimensioning also considers equivalent load, see section 3.4 for more details about the different load cases. Bearing data is read from excel sheets for each of the bearing types. The data in the excel sheets is gathered from the online SKF manual.

Bearing forces are calculated using classical mechanics using a free body diagram, see Appendix 1 starting with the input axle (axle 1). This is done inside a function file called *bearForceCalc*. The required bearing diameter and index are found using a function file called *bearingSelector*. The *bearingSelector* requires the inputs; bearing type, radial force, axial force, life, bearing data and static load. Depending on the bearing type it follows different procedures for calculating necessary bearing size, all according to the SKF catalogue (SKF 2018). As an input for the main program it can be selected which of the bearings on an axle shall handle the axial forces. This is so that for example cylindrical roller bearings can be placed in a position with high radial forces and combined with a ball bearing on the other side of the axle absorbing the axial loads. When tapered roller bearings are used for an axle a predefined side must be used since they only handle pressing forces and the direction of the helix defines the direction of the forces when applying a positive torque. Preload can be defined as an input since tapered bearings are often installed with preload. This has influence on both the max load the bearing can handle and the efficiency of the bearings.

Since the bearing choice method follows the SKF catalogue it will not be described in this thesis. The method needs some input variables that have been estimated using the recommended values from SKF and AVL Vicura. For example, the variable “static safety” (S0) factor has a heavy impact on the size selection for the static load case, it was chosen according to common practice at AVL Vicura, this method uses the value named “Normal” in the SKF catalogue.

The same procedure is used for axle 3, the calculated forces are then used as inputs for calculating the total forces on axle 2. Axle 2 is more complicated since two gears are acting on it. If the axles are not placed in a straight line the magnitude of the bearing forces will vary depending on the direction of the input torque. The equivalent negative, regen, torque is required as an input to be able to compare its magnitude with the positive torque. The *bearForceCalc* function checks which case will result in the highest bearing load and uses this load for the bearing selection based on the life requirement.

For static load dimensioning of the bearings six different load cases are calculated to see which is most critical. Firstly, the abuse torque transmitted through the whole transmission in both positive and negative direction. Secondly the park lock torque impulse upstream of the park lock wheel both negative and positive direction. Thirdly the torque pulse downstream of the park lock wheel, between the park lock and the wheels is considered.

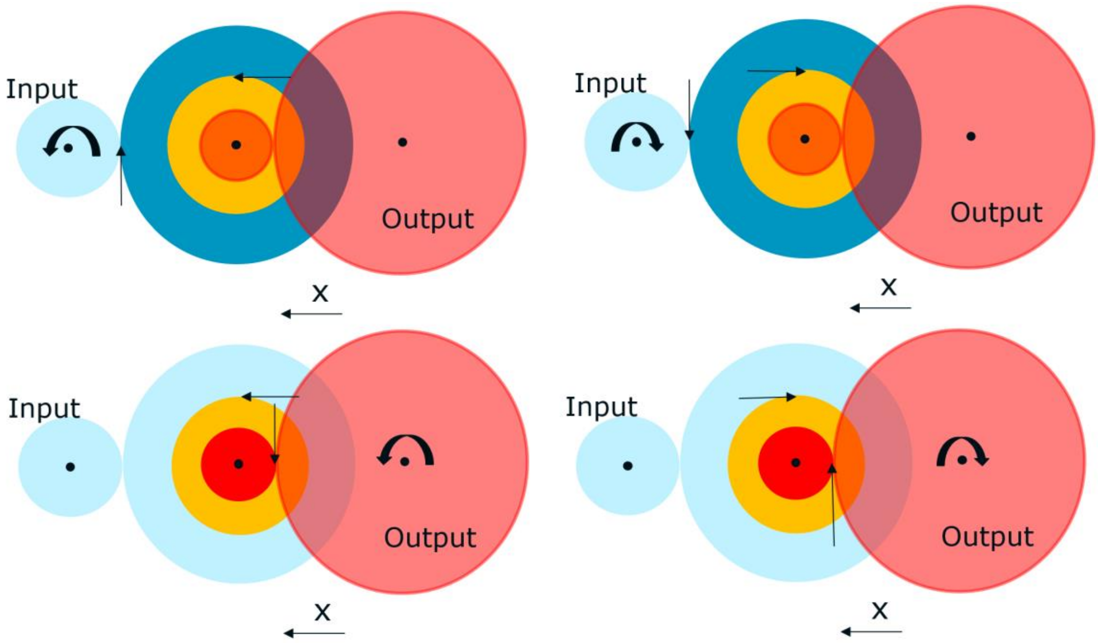


Figure 5 Park lock forces

When the park lock is engaged on the lay shaft the two largest masses that will generate forces in the transmission are the rotor inertia of the e-machine and the whole vehicle mass. The e-machine rotor has a certain inertia and often rotates at a high-speed meaning that it can create a large torque pulse when stopped rapidly. The stage 1 gears and the other components between the rotor and the park lock wheel are relatively stiff meaning that the torque pulse will be created and stopped relatively quickly upstream of the park lock. Downstream of the park lock a torque pulse is created due to the vehicle forward or rearward movement being abruptly stopped. The force is transmitted through the suspension and tires that have a relatively low stiffness, this means that the torque pulse will be at its peak slightly after the park lock is engaged.

The torque pulse upstream and downstream of the park lock are considered separately since they occur at different timesteps. The resulting force from the park lock wheel on the intermediate shaft is assumed to be distributed evenly between the two bearings, this would happen if the park lock is placed in the middle between the two bearings. The direction of the park lock force is dependent on the placement of the park pawl and the direction of rotation. To simplify things the park lock force is assumed to point perpendicular to the direction of the mesh forces. The mesh forces are assumed to act perpendicular to the line in between the two axles as if the pressure angle of the gear would be 0 degrees see Figure 5. It is assumed that input-, lay- and output shaft are positioned in a straight line, this means that the force upstream and downstream both act on the lay shaft perpendicular to the park lock force.

3.2.5 Rough weight estimation

The transmission weight is used to rank the relative weight of the transmissions against each other, therefore the exact result is not so important. The gear weight is represented by a cylinder with a diameter of the gear base circle and homogenous width, multiplied by the density of steel. The transmissions differential sits on the output shaft and makes up the center of the gear wheel on the second stage. Since the same differential will probably be used for all solutions this weight will not change, therefor only an outer ring of gear teeth are considered in the weight calculations for the gear wheel in the second stage.

The weight of the bearings is extracted from the bearing data using the function *bearWeight*. The inputs are the bearing data, bearing types and bearing indexes. The function extracts and sums up the weight of the bearings in the transmission. A for loop adds the weight value for each of the transmissions to the end of the list of parameters in the matrix.

3.2.6 Losses calculations

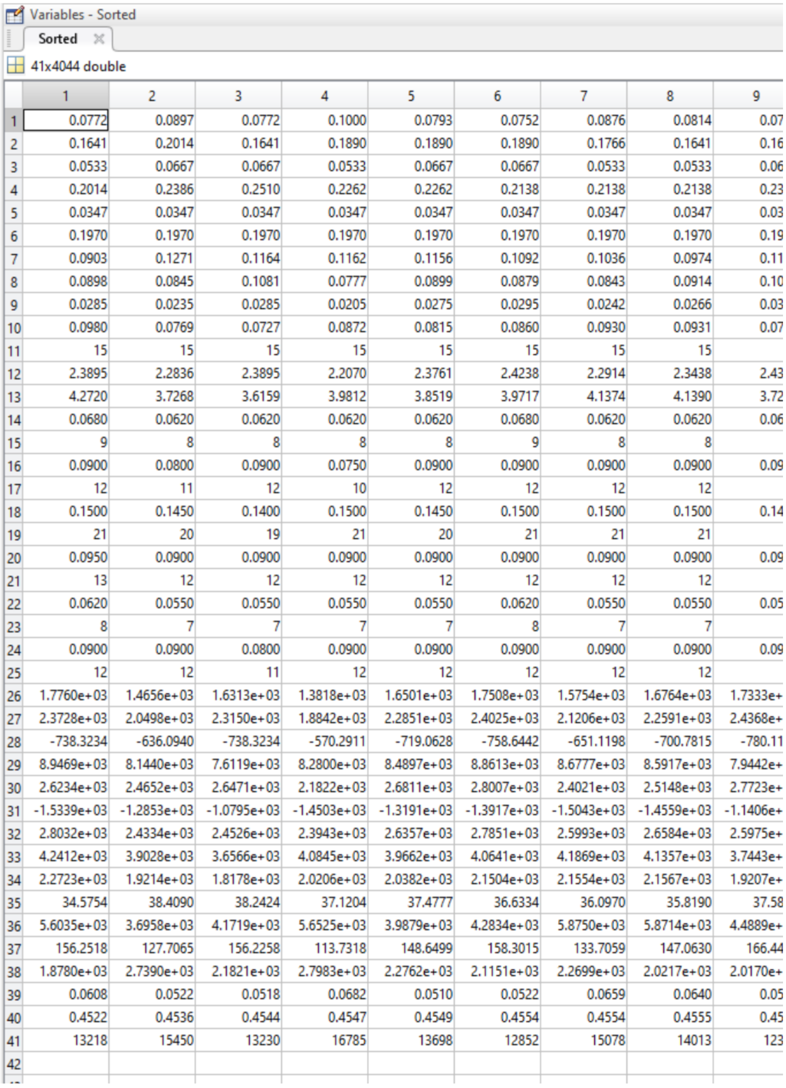
The losses are considered to come from three main sources; bearings, gear mesh and oil churning/splashing. The power of the losses for each bearing is calculated in the function file *bearFricCalc*, it follows the “SKF friction calculation.pdf” for calculating bearing losses (SKF 2018). The inputs are bearing type, radial load, axial load, bearing data, bearing index, rpm and oil viscosity. The axial and radial loads are taken from the earlier bearing force calculations. Rpm is equivalent input speed that represents the typical usage of the transmission and is a user input. Oil viscosity is estimated from the typical temperature and type of gearbox oil. Depending on the bearing type slightly different methods are used to calculate the losses. The losses are divided up into rolling-, sliding- and drag moment. The drag moment is highly dependent on the oil level the bearing is subjected to, the oil level needs to be estimated depending on the position of the bearing inside the transmission. The equations used can be found in the “SKF friction calculation.pdf” (SKF 2018).

The gear mesh losses are calculated using ISO 14179 (Masta 2015) standard which Masta also uses. The necessary inputs are RPM, torque, pressure angle, oil viscosity, module and various transmission parameters such as pitch diameter. The power loss from the mesh in stage one and two are summed and sent as an output from the function file.

The splash/churning losses are calculated using a function file that has been programmed by a Vicura employee, it calculates the churning losses from each gear separately. The inputs are gear diameter and width, number of teeth, tooth depth, pressure angle, oil density, oil viscosity, oil volume, oil immersion depth and RPM. Several outputs are available but only power loss is used for the Matlab tool. The equations in the function file are taken from (Changenet 2007) where the results have been experimentally validated.

3.2.7 Sorting to ranking criteria

A set of weighting factors are presented in the input section of the main script so that the user can select how the different criteria should be prioritised compared to each other on an infinite scale. Weight, efficiency and contact ratio ie NVH can then be evaluated together. The transmission solution that gets the lowest score is ranked first in the new matrix named “Sorted”. Therefore the “Sorted” matrix contains all the earlier transmission solutions but sorted from best to worst, see Figure 6. Each column represents a transmission solution where each of the rows define one of the solutions characteristics for example gear width and helix angle. This means that the best transmission solution is in column one and to the right in the matrix you can find worse and worse solutions. Since the transmission error does not decrease much more when the contact ratio exceeds 5, no performance is added when the contact ratio exceeds this value. In the ranking function this was considered by programming a max function so that contact ratio values exceeding the input variable “targetCR” were set to “targetCR”.


Variables - Sorted

Sorted

41x4044 double

	1	2	3	4	5	6	7	8	9
1	0.0772	0.0897	0.0772	0.1000	0.0793	0.0752	0.0876	0.0814	0.07
2	0.1641	0.2014	0.1641	0.1890	0.1890	0.1890	0.1766	0.1641	0.16
3	0.0533	0.0667	0.0667	0.0533	0.0667	0.0667	0.0533	0.0533	0.06
4	0.2014	0.2386	0.2510	0.2262	0.2262	0.2138	0.2138	0.2138	0.23
5	0.0347	0.0347	0.0347	0.0347	0.0347	0.0347	0.0347	0.0347	0.03
6	0.1970	0.1970	0.1970	0.1970	0.1970	0.1970	0.1970	0.1970	0.19
7	0.0903	0.1271	0.1164	0.1162	0.1156	0.1092	0.1036	0.0974	0.11
8	0.0898	0.0845	0.1081	0.0777	0.0899	0.0879	0.0843	0.0914	0.10
9	0.0285	0.0235	0.0285	0.0205	0.0275	0.0295	0.0242	0.0266	0.03
10	0.0980	0.0769	0.0727	0.0872	0.0815	0.0860	0.0930	0.0931	0.07
11	15	15	15	15	15	15	15	15	
12	2.3895	2.2836	2.3895	2.2070	2.3761	2.4238	2.2914	2.3438	2.43
13	4.2720	3.7268	3.6159	3.9812	3.8519	3.9717	4.1374	4.1390	3.72
14	0.0680	0.0620	0.0620	0.0620	0.0620	0.0680	0.0620	0.0620	0.06
15	9	8	8	8	8	9	8	8	
16	0.0900	0.0800	0.0900	0.0750	0.0900	0.0900	0.0900	0.0900	0.09
17	12	11	12	10	12	12	12	12	
18	0.1500	0.1450	0.1400	0.1500	0.1450	0.1500	0.1500	0.1500	0.14
19	21	20	19	21	20	21	21	21	
20	0.0950	0.0900	0.0900	0.0900	0.0900	0.0900	0.0900	0.0900	0.09
21	13	12	12	12	12	12	12	12	
22	0.0620	0.0550	0.0550	0.0550	0.0550	0.0620	0.0550	0.0550	0.05
23	8	7	7	7	7	8	7	7	
24	0.0900	0.0900	0.0800	0.0900	0.0900	0.0900	0.0900	0.0900	0.09
25	12	12	11	12	12	12	12	12	
26	1.7760e+03	1.4656e+03	1.6313e+03	1.3818e+03	1.6501e+03	1.7508e+03	1.5754e+03	1.6764e+03	1.7333e+
27	2.3728e+03	2.0498e+03	2.3150e+03	1.8842e+03	2.2851e+03	2.4025e+03	2.1206e+03	2.2591e+03	2.4368e+
28	-738.3234	-636.0940	-738.3234	-570.2911	-719.0628	-758.6442	-651.1198	-700.7815	-780.11
29	8.9469e+03	8.1440e+03	7.6119e+03	8.2800e+03	8.4897e+03	8.8613e+03	8.6777e+03	8.5917e+03	7.9442e+
30	2.6234e+03	2.4652e+03	2.6471e+03	2.1822e+03	2.6811e+03	2.8007e+03	2.4021e+03	2.5148e+03	2.7723e+
31	-1.5339e+03	-1.2853e+03	-1.0795e+03	-1.4503e+03	-1.3191e+03	-1.3917e+03	-1.5043e+03	-1.4559e+03	-1.1406e+
32	2.8032e+03	2.4334e+03	2.4526e+03	2.3943e+03	2.6357e+03	2.7851e+03	2.5993e+03	2.6584e+03	2.5975e+
33	4.2412e+03	3.9028e+03	3.6566e+03	4.0845e+03	3.9662e+03	4.0641e+03	4.1869e+03	4.1357e+03	3.7443e+
34	2.2723e+03	1.9214e+03	1.8178e+03	2.0206e+03	2.0382e+03	2.1504e+03	2.1554e+03	2.1567e+03	1.9207e+
35	34.5754	38.4090	38.2424	37.1204	37.4777	36.6334	36.0970	35.8190	37.58
36	5.6035e+03	3.6958e+03	4.1719e+03	5.6525e+03	3.9879e+03	4.2834e+03	5.8750e+03	5.8714e+03	4.4889e+
37	156.2518	127.7065	156.2258	113.7318	148.6499	158.3015	133.7059	147.0630	166.44
38	1.8780e+03	2.7390e+03	2.1821e+03	2.7983e+03	2.2762e+03	2.1151e+03	2.2699e+03	2.0217e+03	2.0170e+
39	0.0608	0.0522	0.0518	0.0682	0.0510	0.0522	0.0659	0.0640	0.05
40	0.4522	0.4536	0.4544	0.4547	0.4549	0.4554	0.4554	0.4555	0.45
41	13218	15450	13230	16785	13698	12852	15078	14013	123
42									

Figure 6 "Sorted" matrix

3.2.8 Delete clashing solutions

A small part in the main script is used to sort out and erase solutions that are not plausible considering bearing fitment. Solutions where the bearing size is relatively big compared to the distance between the axles can encounter problems where the bearings clash. A minimum distance of housing material is needed between the bearings to support them. This distance is defined as an input and a Matlab “if” statement checks if this distance is available in-between all the bearings.

3.2.9 Plot 10 highest ranking

This section of the script is dedicated to visualizing certain transmission solutions so that the user can make a quick judgement if the results are plausible. For a fast result the Matlab plot function is used, see Figure 7. It is possible to rotate the 3D-plot to get a feeling for the geometry. A function file called “plotCylinder” is used to plot both gear and bearings as solid cylinders. As a default the 10 first solutions in the “Sorted” matrix are plotted since they should theoretically be the best solutions.

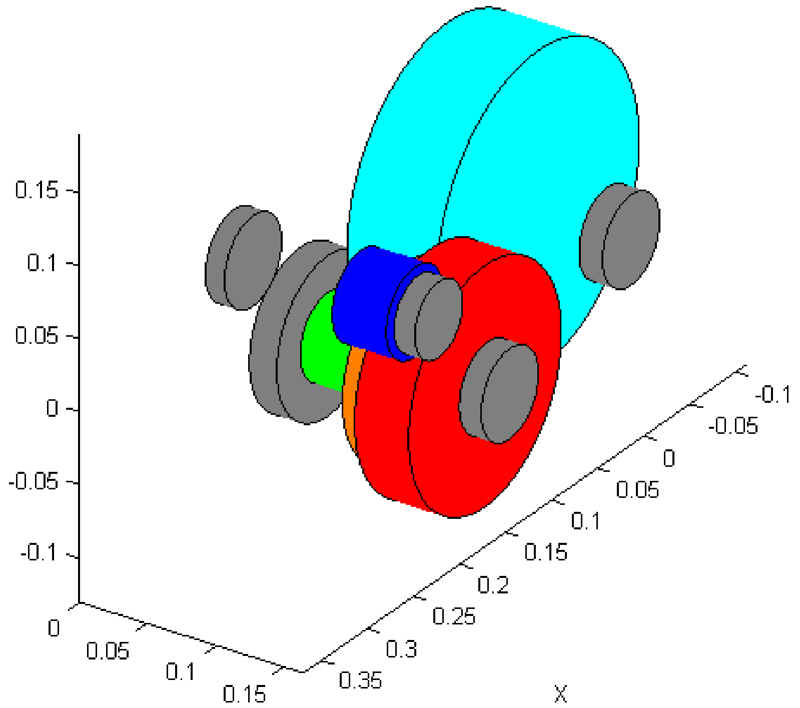


Figure 7 Presentation of a transmission solution

3.3 Tool testing, calibration and validation

The tool was tested and debugged during the construction process using common sense to figure out if the results were even plausible. When more accurate checks were needed a Masta model was set up to match one of the solutions created by the tool. It was mainly used to correct the force equations and verify that the forces were still correct when the helix angles were reversed.

When the dedicated tool testing period began, one of the final Masta models of a project at AVL Vicura was used as a reference to see if the bearing selection and the gear width calculation were correct. A simple load case was set up with a durability torque and an abuse torque to mimic the input parameters used by the Matlab tool. The magnitude was chosen so that the lowest of the safety factors for the bearings was 1 (at the bearing with designation B21). Since a quick and simple equation was used for the gear width calculation slight calibration was needed to match it to the reference project model which is considered very close to the optimum. This was done by adding a tuning parameter in the input section of the script to manipulate the maximum material stress input to the gear width equation, see Equation 2. It was also concluded that the bearings selected by the Matlab tool were slightly larger than the ones required by the Masta model. This was solved by adding a tuning parameter both for the lifetime durability torque and the static load torque for the bearing calculations, see Equation 3 and Equation 4. The parameters were lowered until the size of one of the bearings in the transmission matched the size of the bearing with a safety factor of 1 (B21) in the reference project file.

$$maxStress = MaterialLimit * tuningFactor \qquad \text{Equation 2}$$

$$bearingInputTorque = durabilityTorque * tuningFactor \qquad \text{Equation 3}$$

$$bearingAbuseInputTorque = staticTorque * tuningFactor \qquad \text{Equation 4}$$

To test the bearing static load calculations, one of the transmission solutions from the Matlab tool was chosen and was modelled in Masta. The same load case was used, and the bearing safety factors were checked according to the “ADV summary”.
Too get an as accurate test as possible of the complete Matlab tool, three transmission solutions from the “Sorted” matrix were chosen. The best ranked solution was chosen as a benchmark (#1), one solution with a significant reduction in efficiency (#46) and one solution with a significant reduction in Contact ratio (#75).

To check that the right three solutions were chosen four scatter plots were created in the last parts of the script, see Figure 11, Figure 12 and Figure 13. Pareto lines were created to see that the solutions were relatively close to the optimal design and that they were not outliers. It was made possible to select which points should be coloured red to see where the three chosen solutions were compared to most of the solutions.

3.4 Inputs description

Below follows a description of each of the input parameters in the input section of the main script to aid the user when running the tool and help to understand what the tool takes into consideration. The tool is meant to be used without a graphic user interface and the top of the main script be modified. This makes it easier to get an overview of what information is needed and making it easy to save different input combinations and modify them afterwards. A short description of each parameter can be found in the script as a reminder of what the variable represents. The description below goes into more detail and describes why each of the parameters were selected as input variables.

3.4.1 Tuning factors

tStress- adjusts the maximum tolerated material stress for the gears to improve the gear width estimation and calibrate it to the reference project.

tBearD- adjusts the torque that is used to calculate the size of the bearings for lifetime durability.

tBearS- adjusts the torque that is used to calculate the size of the bearings for static load capacity.

3.4.2 Dimensions

iSpan- defines a minimum and maximum gear ratio that is acceptable between the input and output shaft. This method was chosen since it is often possible to vary the ratio a little when designing a transmission.

dimensionSteps- sets the resolution of which transmission solutions will be calculated. A larger number here will exponentially increase the number of transmission solutions in the final matrix. The run time estimation displayed when the program is run gives an indication if dimensionSteps is larger than desired and needs to be reduced.

minShaft- The minimum shaft diameter represents the shaft diameter and minimum possible gear diameter used in the transmission. It restricts the gears of different stages clashing with axles and pinions being too small.

Width- is the amount of available space in the rectangular box described earlier, see Figure 4. It is defined in the x-direction of the vehicle coordinate system.

Depth- available space in the y-direction of the vehicle coordinate system

Height- available space in the z-direction of the vehicle coordinate system

a- distance between input and output axis of the transmission. This is often predefined since the output corresponds to the position of the axle for the vehicle. The input corresponds to the position of the e-machine which position and distance relative the output is defined by vehicle parameters and other factors such as the clearance between outer parts like the e-machine and the driveshafts.

Inclination- is the angle from the output to the input axis relative to the xy-plane. This parameter also defines the position of the input relative the output and is set from the start due to the same reasons as for “a”.

distanceComponents- to simplify the design of the transmission, the distance between all components is set to be the same value. The distance between the center of the bearing to the gear sides in the y-direction or gear face to gear face between the stages is specified by this value. The magnitude of the value has greatest influence on the bearing forces.

helixAngles- the helix angles can be specified using a matrix, the first row specifies the helix angle for stage 1 and the second row for stage 2. Each column represents a transmission solution. If more than one column is added transmission solutions are created with the only thing varying being the helix angle. A positive value for stage 1 will result in a right-hand pinion, a positive value for stage 2 will result in a left-hand pinion.

module- the module for the gears in the two stages can be selected the same way as for the helix angles.

alpha- the pressure angle for the gears can be selected in the same way as for the helix angles.

3.4.3 Loads for gear calculation

gearInputTorque- is the input abuse torque used for the gear width calculations. Usually this will be specified as an abuse torque from an extreme load case, for example if the car goes over a bump and a wheel goes airborne and the driveline speeds up only to rapidly retard when the wheel hits the ground. The suspension is compressed and max friction force is achieved which results in the maximum torque transmitted through the transmission. The Lewis formula equation assumes that the gears will run at a constant torque, so the input torque can be slightly reduced since the gears will only see abuse torque for small parts of their lifetime.

RPM- the Lewis equation takes rpm into account since a higher rpm results in larger impacts when the teeth mesh. Therefore, higher rpm requires slightly wider teeth.

3.4.4 Bearings

bearingTypes- the matrix makes it possible to make different bearing combinations using three different type of bearings. The number 1 represents a single row ball bearing from the 6000 SKF series. Number 2 represents cylindrical roller bearings from the N 200 series and number 3 represents tapered roller bearings from 302 series. Each column represents a transmission solution in the same way as for the helix angles.

axLoadSide- This matrix specifies which bearing on an axle will absorb the axial load, if both bearings would absorb axial load the system would be undetermined. ‘R’ stands for the right side of the transmission (negative y-direction in the vehicle coordinate system) and ‘L’ stands for left. If there is a roller- and a ball bearing on an axle the ball bearing will absorb the axial load independent of the direction, the side that the ball bearing is placed should be selected in the matrix. If two tapered roller bearings are mounted on an axle they only handle force in one direction, the bearing which is exerted to pushing forces should be selected. If positive torque and positive helix angles are chosen, the configuration for tapered bearings on each of the axles will be: [‘R’;

‘R’; ‘R’]. Each row of the matrix is defined for each of the three axles. Each column of the matrix stands for each of the bearing combinations, so the number of columns in “bearingTypes” and “axLoadSide” should be equal.

life- the number of million revolutions on the input shaft the bearings should endure. This is used in the SKF formula and in this case uses the L10 prediction which means 90% probability that they will endure.

bearingInputTorque- this is an equivalent load if the bearings would be run at a constant torque throughout their whole life. This is simplified since the bearings will be run at different torque levels for different amounts of time, the equivalent torque should represent these different torque levels if they were simplified and combined to one torque figure.

bearingInputTorqueNegative- this is the equivalent torque if the transmission was run with negative torque. If the shafts of the transmission are not in a straight line the magnitude and direction of the bearing forces for the layshaft will vary depending if the torque is applied in the positive or negative direction. The bearingInputTorqueNegative is not relevant for bearing life calculations since the amount and size of regen torque is much “less” than the positive torque.

bearingAbuseInputTorque- this torque is closely related to the gearInputTorque but separated since the gear torque might need to be modified depending on the method of gear width calculation. The magnitude can be related to the maximum torque transmitted through the tire when maximum normal force is pressed upon it during a bump situation. The bearingAbuseInputTorque is used to created six different load cases. Firstly, the torque transmitted through the whole transmission in the positive direction and secondly in the negative direction.

This torque is also used for the park lock situation, if the park lock wheel is placed on the layshaft it is assumed that the system upstream is much stiffer than the system downstream of the layshaft. This means there will be two separate torque pulses on the layshaft. One from the inertia of the rotor and one from the wheels stopping the car. The third load case is if a positive torque is transmitted through the gears upstream of the park lock. The fourth is negative torque transmitted through the gears upstream of the park lock. The fifth and sixth load case is torque transmitted through the gears downstream of the park lock wheel in the positive and negative direction.

bearingClearance- is an estimated dimension value used to see which transmission solutions have bearings that clash or are too close together in the radial direction. If the bearings are too close together there will not be enough housing material in between them to hold them.

preload- the vector is used to define preload at the working condition of the transmission, if tapered bearings are used on an axle then the preload for this axle needs to be estimated since it effects both the forces the bearing needs to endure and how efficient the bearings are.

parkLockDia- specifies the diameter that the park lock force acts upon. The estimated diameter is crucial since it defines the magnitude of the force from the park lock that will be exerted on the bearings in a park lock load case.

parkLockWidth- is the width of the park lock wheel. It is used for the plot of the transmission

3.4.5 Losses

nRPM- is the input speed used for calculation of losses. It should be either considered as an equivalent speed to represent the whole speed range the transmission will be run in or a specific speed where the best efficiency is desired, for example at an RPM representing highway driving.

nyOil- is the viscosity of the oil in the transmission at an estimated running temperature.

lossesInputTorque- is the torque used for calculating the mesh losses. It should be considered in the same way as nRPM, either an equivalent torque or a specific load point.

rho- is the density of the oil and is used in the calculations of the churning losses.

Volume- is an estimated amount of oil inside the transmission used for churning loss calculations.

Immersion- short for Immersion factor is a percentage of the gear that is submerged in oil when the transmission is not rotating. It is defined as the height of oil on the gear divided by the pitch radius.

g- is the gravitational acceleration. The Churn loss function uses this as an input.

3.4.6 General

maxStress- the max stress the material in the gears can handle before yielding and is used for gear width calculations. It is currently defined in pounds per square inches since the Lewis formula handles this.

densitySteel- is a rough estimate of the density of the material to make the weight estimations.

sortingFactor- This value defines which row in the matrix should be used to sort the columns of the matrix. There are four alternatives, sort by contact ratio, weight, efficiency or the combined weight factor that combines the aforementioned three parameters.

weightFactorCR- defines how important contact ratio is compared to weight and efficiency. It is one of three weighting factors representing the performance parameters; contact ratio, weight and efficiency. Adding the three weighting factors should result in the value 1, since their actual value is not important rather their value compared to each other.

weightFactorWeight- the weight factor representing the importance of low weight for the transmission.

weightFactorEfficiency- the weight factor representing the importance of low losses for the transmission.

gearThickness- is the thickness of a gear ring in the radial direction, it is assumed that there is a gear ring surrounding a differential on the output shaft. This is taken into account since the differential for a certain application will be the same size for all variants and the weight stay constant. The only weight that should vary with gear size is the ring of gear teeth surrounding the differential.

targetCR- used to limit the values taken into account by the ranking function, see section 3.2.7.

4 Result

The result is divided into two parts, the simple guidelines that come from knowledge acquired throughout the project and the Matlab tool. These can be used in combination with each other at the start of a transmission design project.

4.1 Simple guidelines

The following points are short descriptions of some of the things that can be good to have in mind during the initial phase of the design process.

- Placing park-lock directly on input shaft may create high loads due to the inertia of the E-machine and the high stiffness in between them. Therefore it can be beneficial to place it on the lay shaft even though the torque from the wheels is higher there.
- More gears submerged in oil will give more splash loses, too little gear submerged in oil will give insufficient lubrication. This this is important to take into consideration when looking at the position of the intermediate shaft placement.
- Sometimes it is required to provide different gear ratio variants for a transmission. It can be beneficial to provide the different ratios on first step since it may simplify housing design and differential design.

4.2 Matlab tool

The main result of the thesis is a Matlab based tool that requires relatively few input parameters to compute a suggestion of a transmission layout for a two-stage constant ratio ev-transmission. The input method was chosen so that it would be as user friendly and intuitive as possible for someone with basic Matlab- or general programming knowledge. It is done by using an input section at the start of the main script where each parameter is briefly described. This makes it easy for a user who has run the program before to decide the required input. For a first-time user reading the thesis report and specifically the chapter called; Inputs description 3.4 is recommended for explanation on how to use the tool.

Depending on the parameter resolution requested, the program run-time is relatively short. This is one of the main benefits of the program, that is covers a vast space of layout solutions in a short amount of time. This gives a quick idea of where to start looking for the optimal solution. The output of the tool is a matrix and ten plots. The matrix provides all the necessary transmission parameters needed to continue with the design process. It is sorted so that the best solution is in column one and so forth. The Matlab plots visualize the ten best solutions to check if the results seem reasonable and that there are no erroneous inputs.

The tool has been validated using Masta to figure out if the result will be useful in a real case design scenario. When using the rough input parameters for the reference project and running the Matlab tool with efficiency as the only priority the layout of the transmission is close to the layout of the reference transmission, see Figure 8. Sorting solely on weight and contact ratio was also tested rendering solutions where the gear wheel in the second stage was larger than the reference project design.

Figure 8 Layout comparison

When inserting the bearings chosen by the Matlab tool into Masta it was found that the safety factors were slightly above the required value indicating that the correct bearings were selected by the Matlab tool.

When the three transmission solutions (#1, #46 and #75) were implemented as described in chapter 3.3 mixed results were found comparing the values from the Matlab tool and Masta. Focus is to be put on which order the values are ranked not on the specific value, see Figure 9 and Figure 10. Looking at the Masta results in the efficiency comparison solution #75 has slightly better efficiency compared to #1 which is not what the Matlab tool computed. When looking at the contact ratio the Matlab tool follows the same trend as the Masta result, the slight differences are due to the selection of micro geometry of the gear teeth.

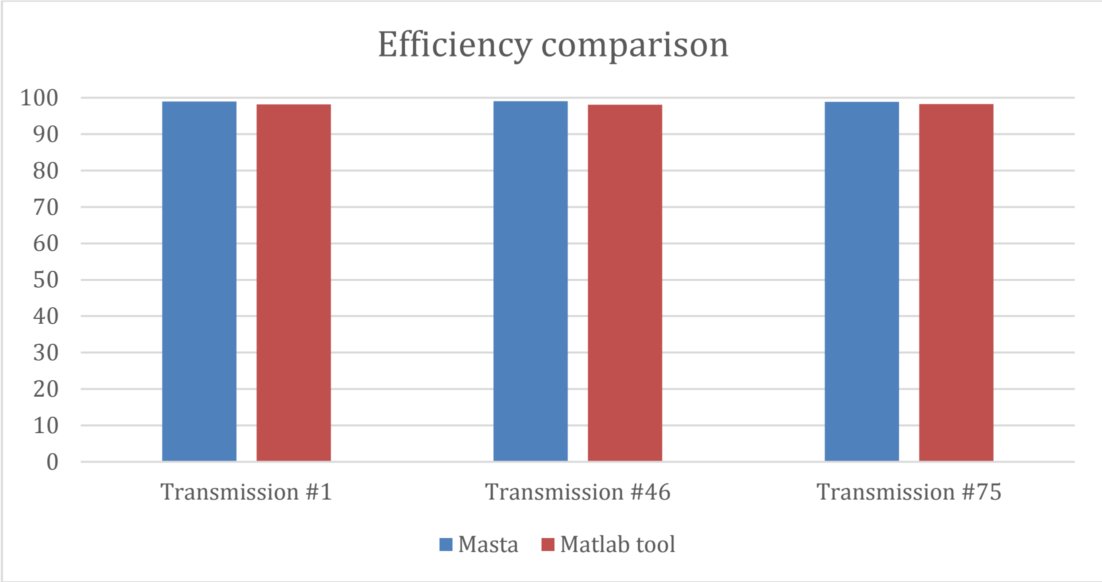


Figure 9 Efficiency comparson

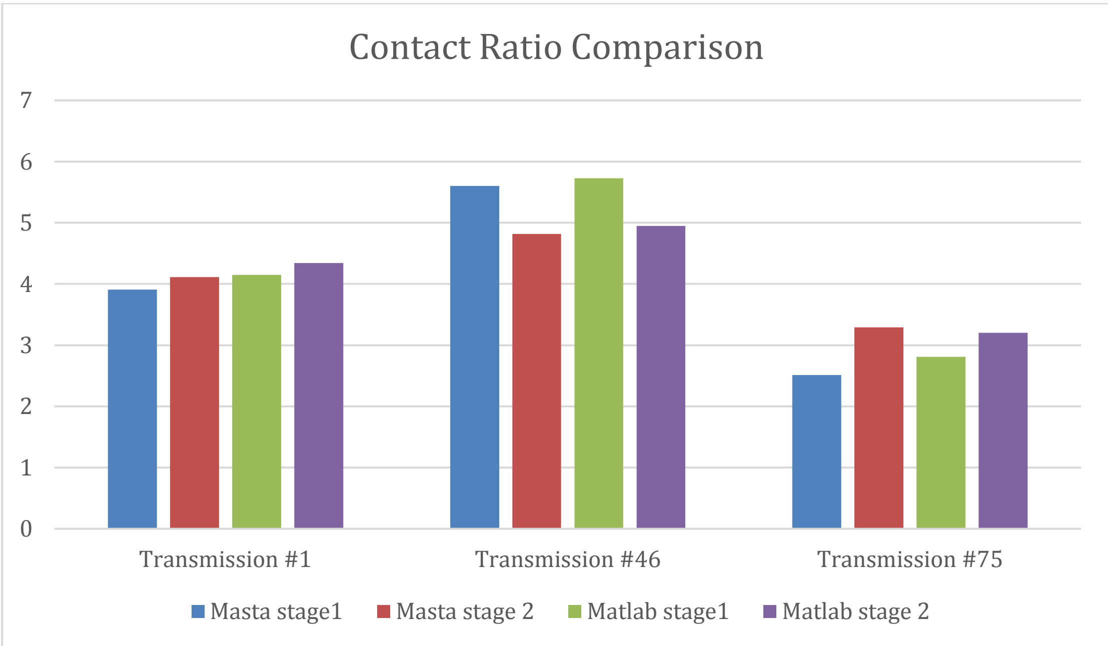


Figure 10 Contact Ratio Comparison

To see that the three selected transmissions were close to the optimal solution in the different optimization parameters three scatter plots were used together with Pareto lines. Since a contact ratio higher than 5 does not add performance the Pareto line is not drawn for lines over 5 where contact ratio is on one of the axis, see Figure 11, Figure 12 and Figure 13. The red dots mark the transmissions that are ranked as the ten best solutions by the Matlab tool. Green arrow indicates in which direction it is desirable for the transmissions to be. It can be seen in the figures that the red dots are not always on the pareto line where the best solution for that plot would be. This is because the best solution is a compromise between all three performance parameters.

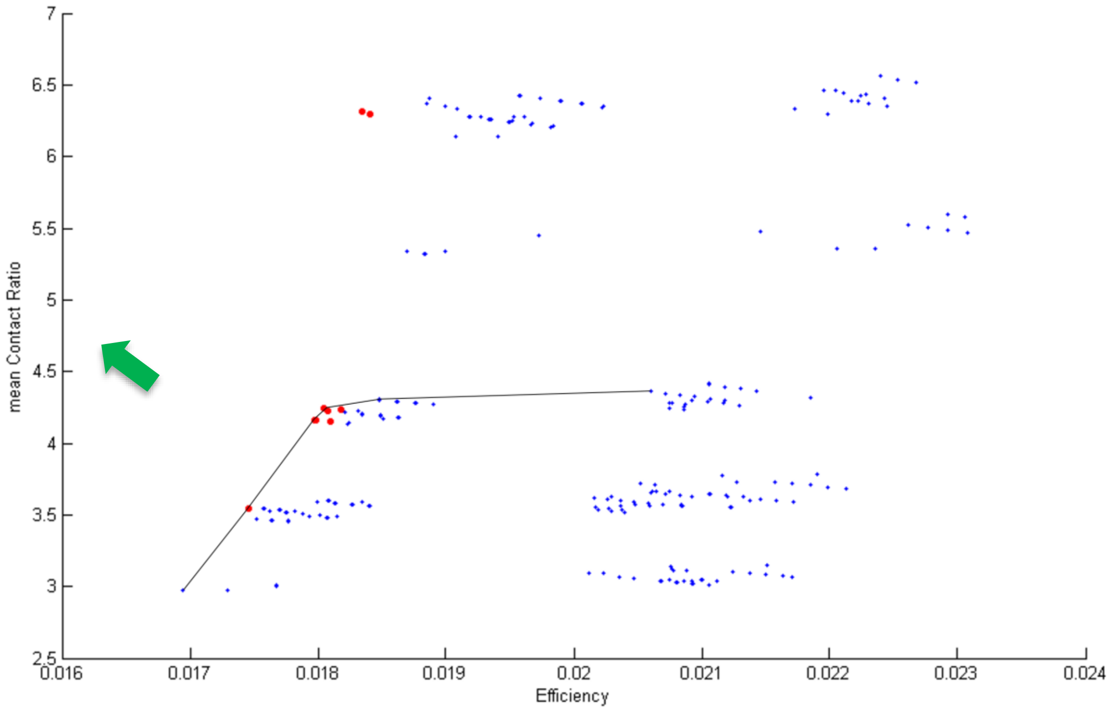


Figure 11 Efficiency vs contact ratio with pareto line

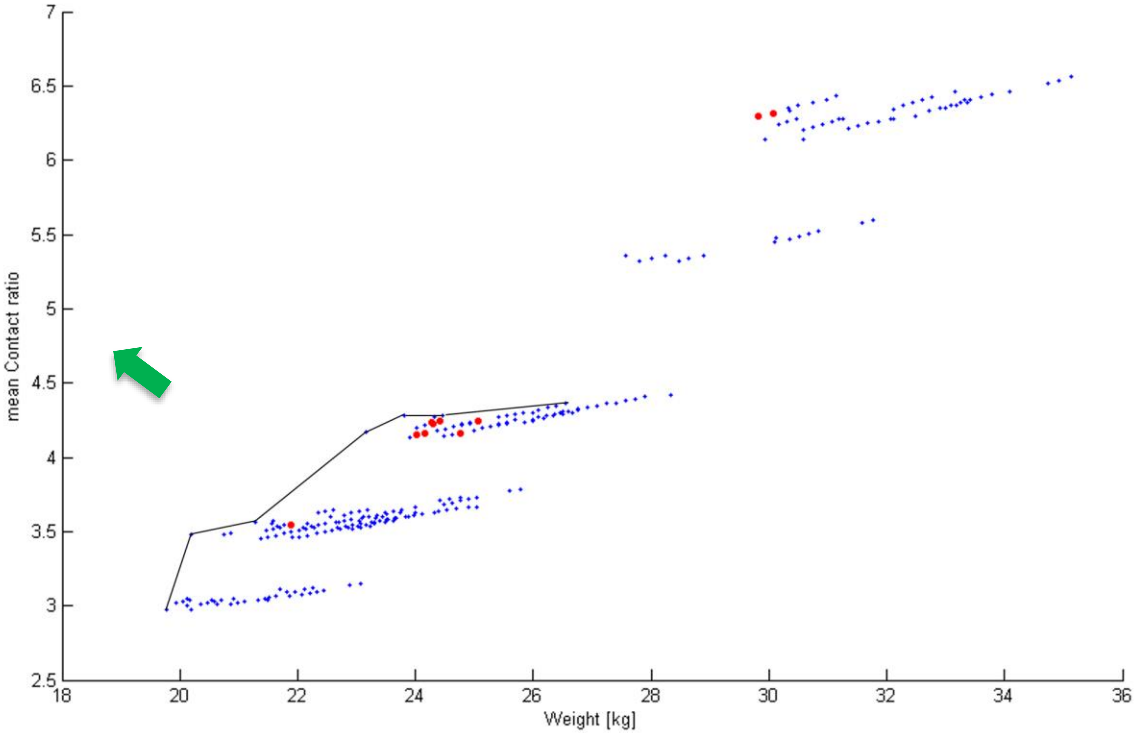


Figure 12 Weight vs Contact Ratio with pareto line

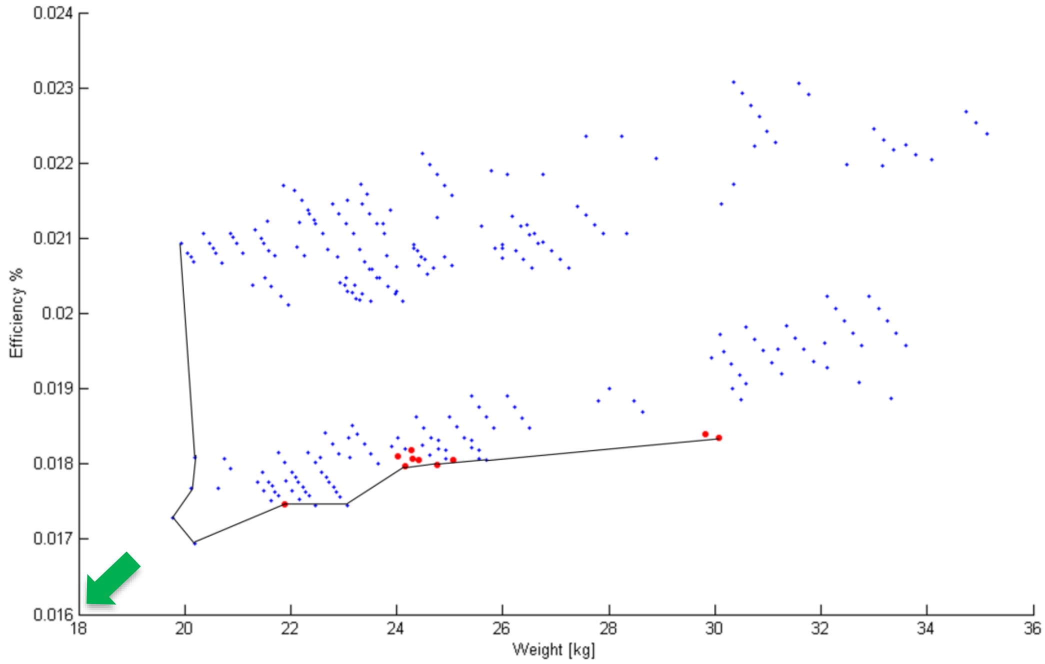


Figure 13 Weight vs Efficiency with pareto line

5 Discussion/Conclusion

The aim of the thesis was to create a tool that saves time in the total design process, this is hard to prove until it has been tested in an actual design process but the forecast looks promising. The input section of the program is highly versatile and provides a span of functions. The bearing size selection is proven to be accurate with confirmation through Masta.

Reading the Pareto lines, it is possible to see how the programs basic optimization function works. It shows that the tool priorities the solutions that have good general characteristics and not solutions that are extremely good in one direction and bad in others.

The program layout makes it customizable, since there are function files for each type of calculation these can be changed if for example more detailed efficiency calculations are required. This also opens the possibility to use existing tools like Masta for accurate calculations in parts of the Matlab tool. The tool is setup up so that optimization software like AVL Cameo can be used together with the tool to create faster and more accurate optimizations.

6 References

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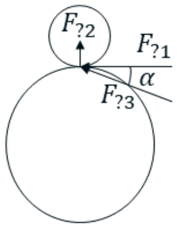
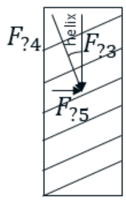
Appendix 1

The free body diagrams and the force equations used by the Matlab tool to calculate the bearing forces are presented below.





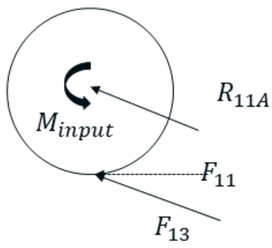
$$F_{?1} = \frac{2 * M_{in}}{d_1}$$

$$F_{?3} = \frac{F_{?1}}{\cos(\alpha)}$$

$$F_{?5} = F_{?3} * \tan(helix)$$

$\rightarrow: F_{13} = F_{?3}$

$"M_{tot}": M_{input} = F_{11} * \frac{d_1}{2}$



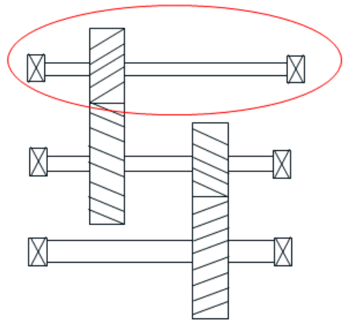
Axle 1

A

$$\rightarrow: R_{12y} = 0$$

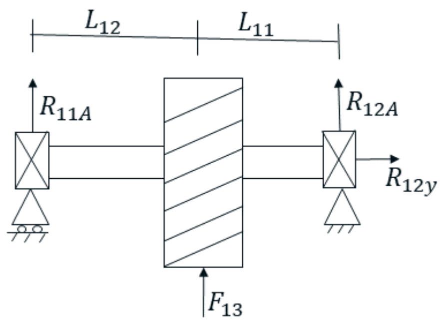
$$\uparrow: R_{11A} + R_{12A} + F_{13} = 0$$

$$"M_1": F_{13} * L_{12} + R_{12A} * (L_{12} + L_{11}) = 0$$



$$R_{12A} = \frac{-F_{13} * L_{12}}{L_{12} + L_{11}}$$

$$R_{11A} = -R_{12A} - F_{13}$$



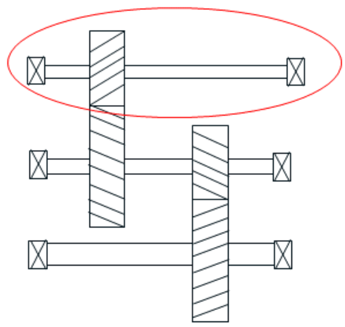
Axle 1

B

$$\rightarrow: R_{12y} = -F_{15}$$

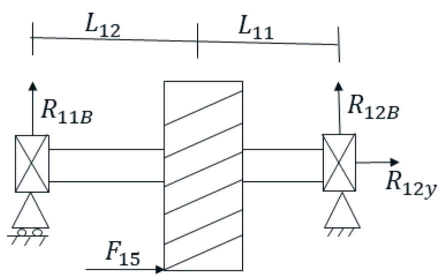
$$\uparrow: R_{11B} + R_{12B} = 0$$

$$"M_1": F_{15} * \frac{d_1}{2} + R_{12B} * (L_{12} + L_{11}) = 0$$

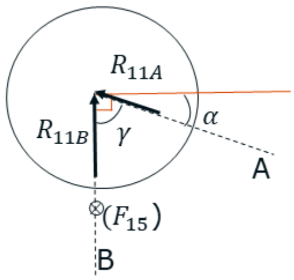


$$R_{12B} = \frac{-F_{15} * \frac{d_1}{2}}{L_{12} + L_{11}}$$

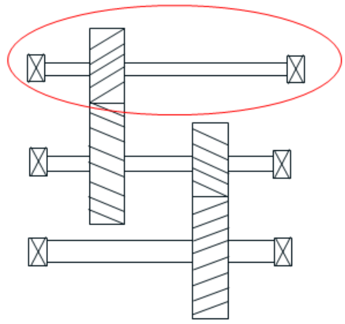
$$R_{11B} = -R_{12B}$$

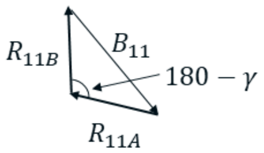


Axle 1



$$\gamma = 90 - \alpha$$





$$B_{11} = \sqrt{R_{11A}^2 + R_{11B}^2 - 2 * R_{11A} * R_{11B} * \cos(180 - \gamma)}$$

$$B_{12} = \sqrt{R_{12A}^2 + R_{12B}^2 - 2 * R_{12A} * R_{12B} * \cos(180 - \gamma)}$$

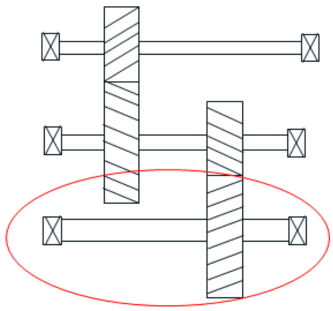
Axle 3

A

$$\rightarrow: R_{42y} = 0$$

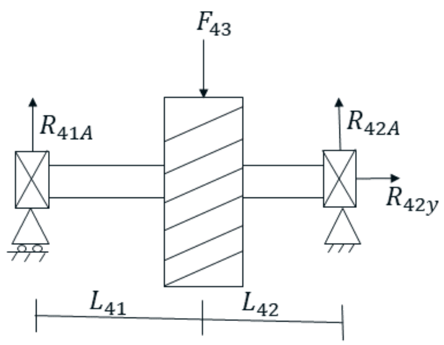
$$\uparrow: R_{41A} + R_{42A} - F_{43} = 0$$

$$\mathrm{M}_1: F_{43} * L_{42} - R_{42A} * (L_{42} + L_{41}) = 0$$



$$R_{42A} = \frac{F_{43} * L_{42}}{L_{42} + L_{41}}$$

$$R_{41A} = -R_{42A} + F_{43}$$



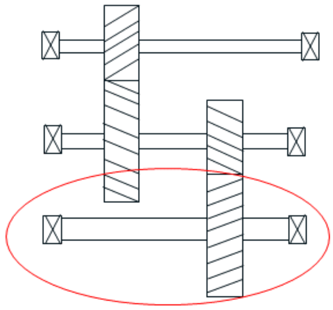
Axle 3

B

$\rightarrow: R_{42y} = F_{45}$

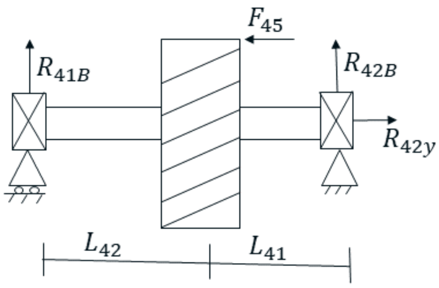
$\uparrow: R_{41B} + R_{42B} = 0$

$"M_1": -F_{45} * \frac{d_4}{2} - R_{42B} * (L_{42} + L_{41}) = 0$

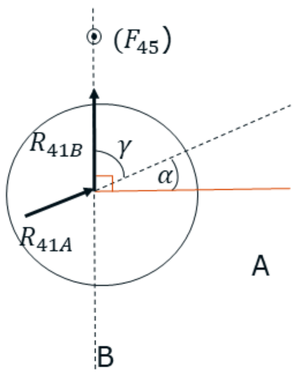


$$R_{42B} = \frac{-F_{45} * \frac{d_4}{2}}{L_{42} + L_{11}}$$

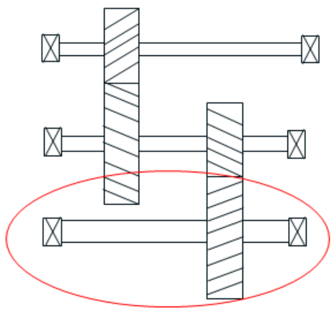
$$R_{41B} = -R_{42B}$$

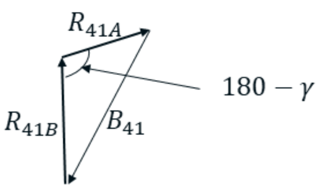


Axle 3



$\gamma = 90 - \alpha$



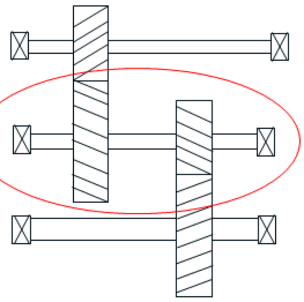


$$B_{41} = \sqrt{R_{41A}^2 + R_{41B}^2 - 2 * R_{41A} * R_{41B} * \cos(180 - \gamma)}$$

$$B_{42} = \sqrt{R_{42A}^2 + R_{42B}^2 - 2 * R_{42A} * R_{42B} * \cos(180 - \gamma)}$$

Axle 2

A1

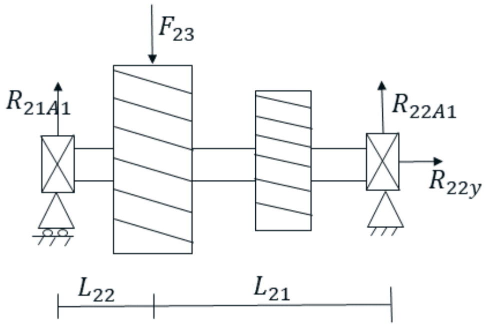


$$F_{23} = F_{13}$$

$$\rightarrow: R_{22yA1} = 0$$

$$\uparrow: R_{21A1} + R_{22A1} - F_{23} = 0$$

$$\mathrm{M}_1: F_{23} * L_{22} - R_{22A1} * (L_{22} + L_{21}) = 0$$

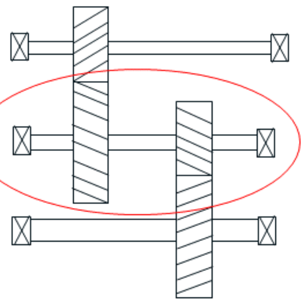


$$R_{22A1} = \frac{F_{23} * L_{22}}{L_{22} + L_{21}}$$

$$R_{21A1} = -R_{22A1} + F_{23}$$

Axle 2

B1

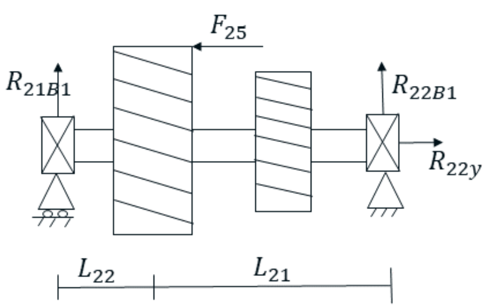


$$F_{25} = F_{15}$$

$$\rightarrow: R_{22y} = F_{25}$$

$$\uparrow: R_{21B1} + R_{22B1} = 0$$

$$\mathrm{M}_1: -F_{25} * \frac{d_2}{2} - R_{22B1} * (L_{22} + L_{21}) = 0$$

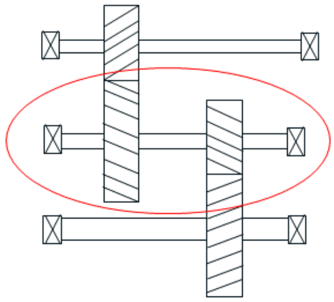


$$R_{22B1} = \frac{-F_{25} * \frac{d_2}{2}}{L_{22} + L_{21}}$$

$$R_{21B1} = -R_{22B1}$$

Axle 2

A2

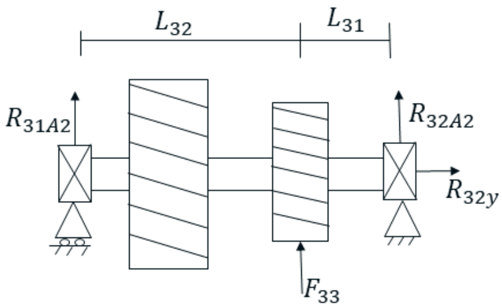


$$F_{33} = F_{43}$$

$$\rightarrow: R_{32y} = 0$$

$$\uparrow: R_{31A2} + R_{32A2} + F_{33} = 0$$

$$\mathrm{M}_1: -F_{33} * L_{32} - R_{32A2} * (L_{32} + L_{31}) = 0$$

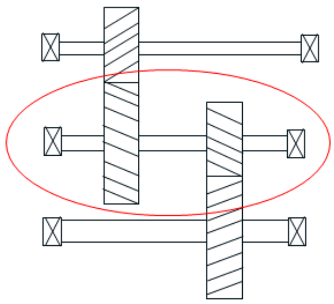


$$R_{32A2} = \frac{-F_{33} * L_{32}}{L_{32} + L_{31}}$$

$$R_{31A2} = -R_{32A2} - F_{33}$$

Axle 2

B2

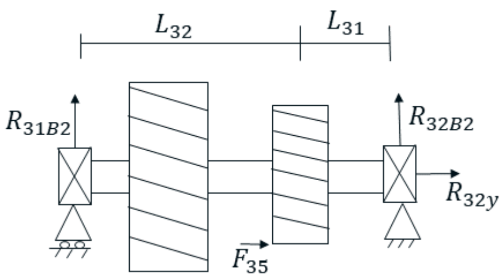


$$F_{35} = F_{45}$$

$$\rightarrow: R_{32y} = -F_{35}$$

$$\uparrow: R_{31B2} + R_{32B2} = 0$$

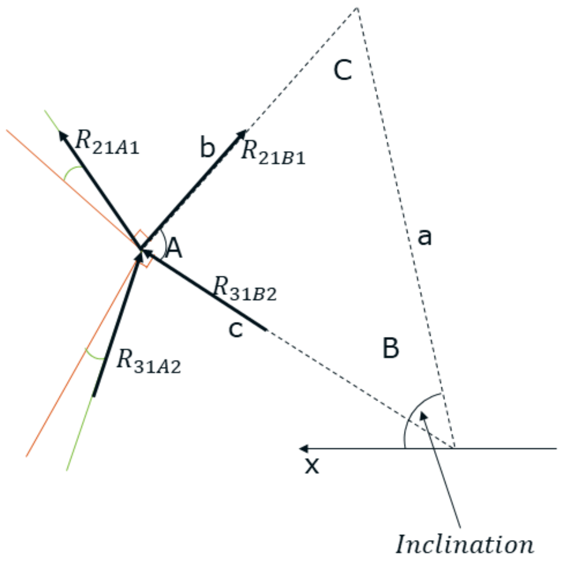
$$\mathrm{M}_1: -F_{35} * \frac{d_3}{2} - R_{32B2} * (L_{32} + L_{31}) = 0$$

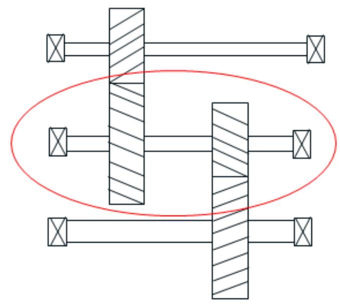


$$R_{32B2} = \frac{-F_{35} * \frac{d_3}{2}}{L_{32} + L_{31}}$$

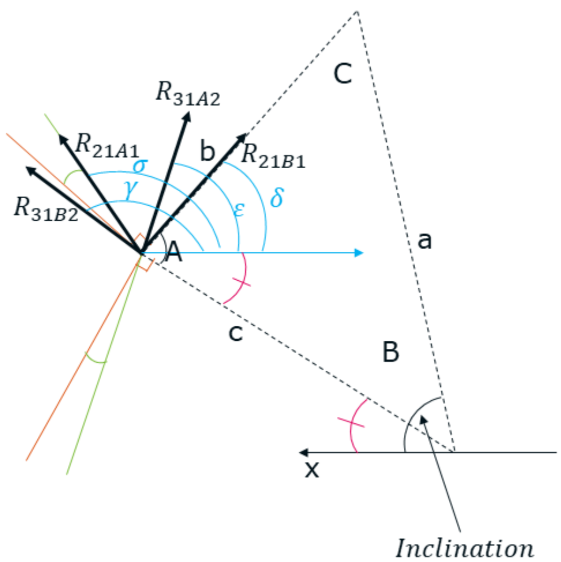
$$R_{31B2} = -R_{32B2}$$

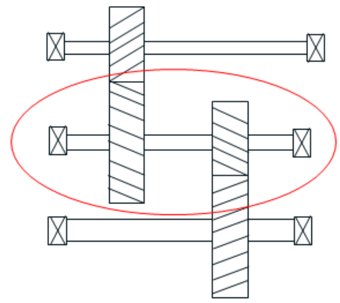
Axle 2 B21





Axle 2 B21





$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2 * b * c}\right)$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2 * a * c}\right)$$

$$\delta = A - (Inclination - B)$$

$$\sigma = \delta + 90 - \alpha$$

$$\gamma = -(Inclination - B) + 18$$

$$\varepsilon = -(Inclination - B) - 90$$

$$x = \text{Force} * \cos(angle)$$
$$y = \text{Force} * \sin(angle)$$

$$B_{21z} = \sqrt{sum(x)^2 + sum(y)^2}$$
$$B_{22z} = \sqrt{sum(x)^2 + sum(y)^2}$$

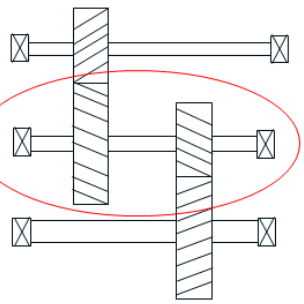
where

$$sum(x) = x_{R21A1} + x_{R21B1} + x_{R31A2} + x_{R32B2}$$

$$B_{22y} = abs(R_{32y} + R_{22y})$$

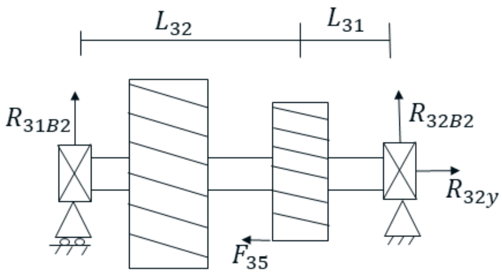
Axle 2
Negative torque test pretty much the same if you change the direction of the resultant forces

B2



$$F_{35} = F_{45}$$

$$\rightarrow: R_{32y} = -F_{35}$$
$$\uparrow: R_{31B2} + R_{32B2} = 0$$
$$\text{M}_1: F_{35} * \frac{d_3}{2} + R_{32B2} * (L_{32} + L_{31}) = 0$$

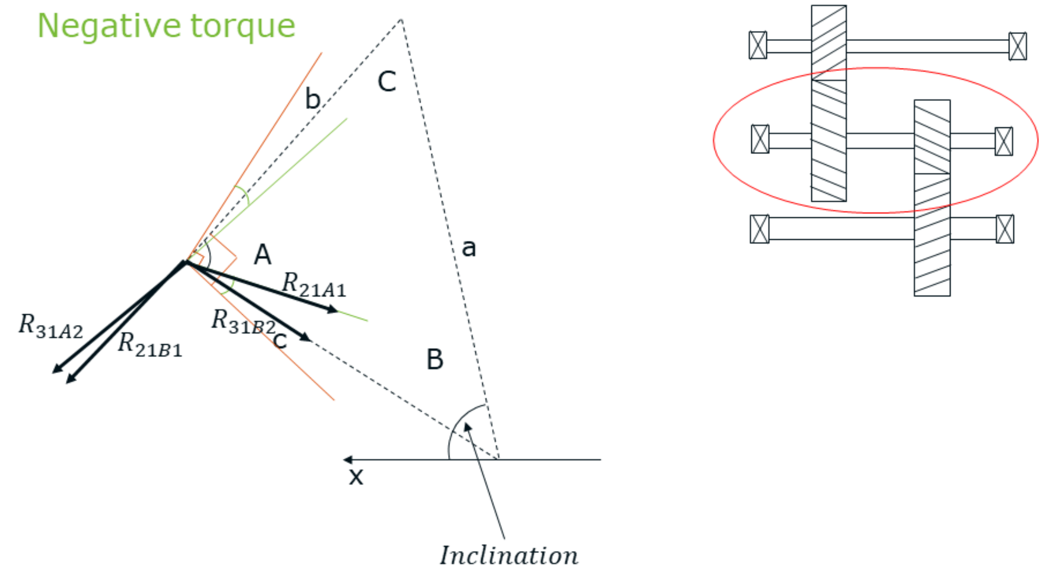


$$R_{32B2} = \frac{-F_{35} * \frac{d_3}{2}}{L_{32} + L_{31}}$$

$$R_{31B2} = -R_{32B2}$$

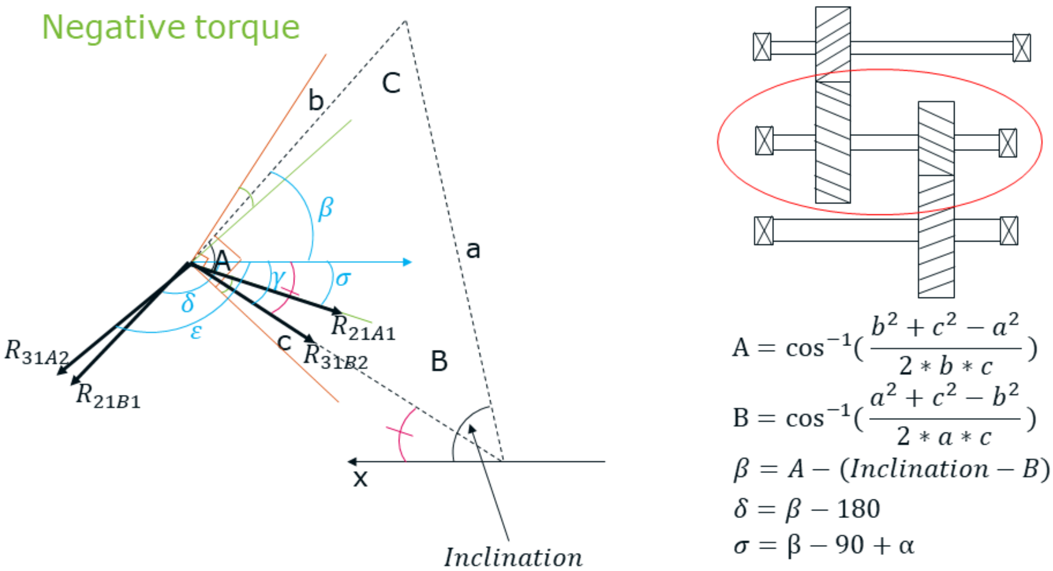
Axle 2 B21

Negative torque



Axle 2 B21

Negative torque



$$\mathrm{A} = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2 * b * c}\right)$$

$$\mathrm{B} = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2 * a * c}\right)$$

$$\beta = A - (Inclination - B)$$

$$\delta = \beta - 180$$

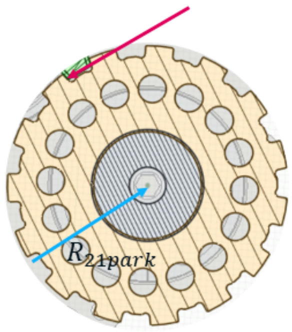
$$\sigma = \beta - 90 + \alpha$$

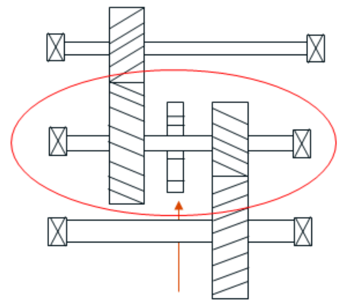
$$\gamma = -(Inclination - B)$$

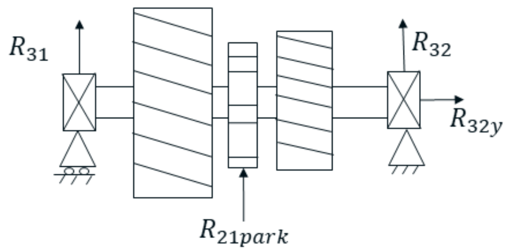
$$\varepsilon = -(Inclination - B) - 90$$

Axle 2

Park lock torque with park lock on layshaft







Four park lock cases