CHALMERS
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Using Digital Tools to Enhance Mathematical Modelling Processes

A Swedish Case Study on Effects of Upper Secondary Students' Usage of Digital Tools

Master's thesis in technology and learning

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SUMMARY

In the latest study plan, the Swedish National Agency for Education has emphasised students' ability to use digital tools. One subject area for which digital tools shall be used, and where they are known to be advantageous, is mathematical modelling. For example, digital tools are known to extend the range of problem situations that can be handled as well as enabling modellers to focus on modelling and strategic planning rather than calculations. How digital tools should be incorporated into the educational design to exploit these potential benefits is however not trivial.

By studying two small groups of upper secondary students, this study was designed to exemplify how the use of digital tools can affect students' processes and how the design of the learning environment can be adapted to improve the found effects. To gain an understanding in these topics, the students were recorded while engaging in their first modelling experience. Moreover, they were interviewed about their digital tools usage and previous experience of tools.

The students were found to benefit from the tools' collaborative support while remaining unaffected by speed enhancements and decreased cognitive load. In addition, they occasionally engaged in unnecessary tool usage and failed to use a tool's collaborative features in a new context where they would have benefited from using them.

The negative effects were found to be caused by students not having a habit of engaging in metacognitive activity and only consider tools which the teacher had demonstrated. To amend these issues, several instructional strategies were proposed. The ones associated with metacognition involve activities where students design their strategy, and are regularly prompted to present and evaluate it. To expand the number of tools known by students, teachers are recommended to engage students in activities where they acquaint themselves with a tool. In these activities, students practice various competencies associated with efficient tool usage, and their engagement therefore ranges from letting the teacher operate the tool or following a step-by-step instruction to exploring freely.

Keywords: digital tools, mathematical modelling, case study, upper secondary education, Sweden

Contents

1. Introduction

The incorporation of technology into mathematics education has been debated for several decades. In general, the opinions and beliefs held by teachers and researchers can be divided into two categories. On the one hand, some argue usage of digital tools can bridge knowledge and skill gaps by enabling students to engage in problems which would not have been accessible for them without the support of digital tools (Blum 1991; Geiger 2011). For instance, one teacher interviewed as part of Geiger's (2011) study exemplifies how some students who cannot integrate manually, but understands the concept of integration and when it is applicable, can distribute the calculation to digital tools. On the other hand, others reason digital tool usage before having gained the underlying conceptual and procedural knowledge will cause issues for students, for example impeding their knowledge acquisition or result in them accepting unreasonable output (Buchberger 1990; Geiger 2011; Kim 2007; Niss 1999). From this viewpoint, students should only use digital tools to automate calculations once they understand the involved procedures and have achieved the skill necessary to perform them routinely themselves (Buchberger 1990; Geiger 2011).

From 2017, the Swedish National Agency for Education can be argued to support the first category of beliefs as the agency modified the study plan for 10th to 12th grade mathematics education to emphasise the importance of utilising digital tools. In addition to extending the requirements on students' competencies from being able to perform procedures manually to also being able to perform these activities with the assistance of digital tools, the plan have in some cases replaced the previous requirement on manual procedural ability with being able to use digital tools for performing the same task. For example, the updated plan presented by the Swedish National Agency for Education (2017) requires students to use digital tools for regression analysis and statistical calculations, while the previous study plan required students to perform associated tasks manually. However, for most tasks, the previous requirements on procedural ability have been kept. For instance, this is the case for the ability to solve equations, handle algebraic expressions, calculate interest values, and determine number of possible permutations and combinations (Swedish National Agency for Education 2017). Furthermore, students are in this plan required to use programming for problem-solving and modelling activities, which was not required in the previous study plan.

The aim with these modifications is that students shall "develop their skills in using digital tools for solving problems, deepen their mathematical knowledge, and widen the areas in which their mathematical knowledge can be applied” (Swedish National Agency for Education 2017, p. 1, author’s translation). By emphasising the use of digital tools for solving problems and applying mathematics in non-mathematical contexts, the aim also stresses the importance of education in mathematical modelling and problem-solving, as well as how the use of digital tools are key to such activities. This belief is supported by previous research. For instance, Blum (1991) highlights that many realistic problem situations which are relevant for students on secondary and tertiary level can become accessible for students through the use of digital tools even in cases where students cannot understand the involved mathematical concepts. In addition, usage of digital tools can increase the rapidity of calculations and help students to retain focus on problem-solving activities (Blum 1991) such as strategic reflections.

Such focus on solution strategy design is important in problem-solving activities, as problem-solving occurs when faced with a problem for which no procedure for solving it is immediately known (Blum 1991; Wedelin 2015a). In turn, mathematical modelling can be regarded as a strategy for mathematical problem-solving. In short, a mathematical model is a mathematical representation which contains the main characteristics of a real-world situation or problem, and the process of mathematical modelling involves creating and solving this model to finally obtain a solution for the original problem (Galbraith 2006; Voskoglou 2007; Wedelin 2015a). Therefore, the incorporation of digital tools in mathematical modelling education can be regarded as highly beneficial. How the learning environment should be designed to take advantage of digital tools in a mathematical modelling context is however not obvious.

1.1. Thesis aim and delimitations

The aim of this thesis is to provide an example of how students use digital tools during the introduction of mathematical modelling and problem-solving, and how this usage can affect students’ classroom experience. Moreover, this study aims to propose how the learning environment can be adapted to enhance this experience. To reach these objectives, the following questions are investigated:

RQ1 How do the presence and use of digital tools directly affect mathematical modelling activities students engage in?

RQ2 How can the selection of problems, digital tools, and instructional design be adapted to increase found positive effects and decrease found negative effects?

Note that the word *directly* refers to short-term effects that can be observed during the studied activities, rather than development of behavioural patterns or knowledge and skill acquisition that can in turn affect future mathematical modelling activities.

Furthermore, the second research question only considers activities that can improve the effects found while investigating the first research question. It does thus not aim to propose suggestions for introducing effects that would potentially be beneficial, but was not found as part of the study.

Finally, as specified above, this study is a case study and findings can therefore not be generalised for other students. As only a few students are studied, the thesis does neither aim to discover all possible behavioural patterns, nor to draw conclusions for a general population.

2. Key factors for succeeding with mathematical modelling

As described above, a mathematical modelling process involves understanding a real-world problem situation, creating a mathematical model to represent it, solving the mathematical model, and interpreting the result in the real-world context (Galbraith 2006; Wedelin 2015a). This process is typically described with mathematical modelling cycles, of which one presented by Blum and Leiß has been created to model the actions of the problem-solving individual and is believed to be useful for cognitive analyses (Blum 2007; 2015).

Blum and Leiß' cycle consists of six states and seven transitions between them. Firstly, the modeller transition from a real-world situation and problem to a situation model (Blum 2007). For example, the modeller might construct this model by visualising the situation (Blum 2007). Secondly, the problem-solver simplifies and structures the situation model, thereby creating a real model (Blum 2007). Blum (2007) exemplifies that this step can include simplifying how a rope is spanned between two objects by assuming it is straightened rather than loosely spanned. Once a real model has been created, it is in the third transition mathematized to form a mathematical representation of the real model, for example a geometrical object (Blum 2007). However, the mathematical representation can consist of other mathematical elements such as mathematical symbols, relations, functions, equations, inequalities, and graphs (Meerschaert 2013; Voskoglou 2007). The mathematical model is then used to create a mathematical result, which is later interpreted to form a real result (Blum 2007), which constitutes transitions four and five. After a result has been found, the model and result are validated in relation to the situation model (Blum 2007). The seventh transition can then take two different directions, as the result can either be used to describe a solution to the real-world problem or undergo corrections, requiring the problem-solver to engage in the modelling cycle steps once more (Blum 2007).

Due to its orientation towards the cognition of the individual problem-solver, Blum and Leiß' cycle has also been used as a basis of several studies that focus on cognition. For example, there is research on what activities need to be completed to transition between different states in the modelling cycle, on what competencies are needed to successfully transition, and on what factors might cause barriers in the progress. All of these research areas are related, as they all focus on dividing the different states and transitions in the modelling cycle into smaller entities that can explain successfulness or failure in mathematical modelling. The sections below contains an introduction of these research areas.

2.1. Activities within the modelling cycle and causes of barriers

When transitioning from a real-world situation to a real model, students need to engage in several activities. To create an understanding, students need to begin by interpreting the context (Galbraith 2006). In this step, students are required to engage in several activities, including simplifying, idealising, and structuring the problem until only necessary characteristics remain (Blum 1991). In addition, modelling problems are often lacking information, and students are thus required to make assumptions (Blum 1991). Following this step, students can begin re-formalising the problem statement by identifying the strategic entities, for example what should be minimised or maximised in case of an optimisation problem (Galbraith 2007).

Here, Blum (1997) notes that sub-competencies necessary for succeeding in activities associated with constructing a real model include competency to make assumptions, simplify the situation, identify variables, identify relations between variables, find information, and distinguish relevant information from irrelevant information. Moreover, it can also be argued that students need competency in selecting representations used to structure data to support the competencies listed by Blum (1997). Different representations can reveal different aspects (Ainsworth 1999) and therefore be appropriate for different tasks. Simultaneously, including all information in one representation can cause it to become too complex to be useful (Ainsworth 1999). Instead, several representations can be used simultaneously to complement each other and thus provide a more complete set of information (Ainsworth 1999). For example, tables can be used to detect which results or data have been collected or remain to be explored, while graphs highlight trends clearly (Ainsworth 1999).

When developing a real model, barriers can be formed as students face difficulties in the above listed activities. When simplifying, students can misunderstand what simplification means, and thus create a low-quality model based on guesses (Maaß 2006). Moreover, they may believe that simplification is solely done for the purpose of simplifying calculations (Maaß 2006), causing the model to be inaccurate. Students may also fail to identify characteristics of the problem situation that motivate certain simplifications or approximations, resulting in a model with a high level of complexity. For instance, Blum (2007) exemplifies that a description of a mountain as very steep can indicate that an approximation with a right angle is suitable. As modelling problems often contain extensive information, one barrier students may face while structuring the situation is failing to identify the relevant information and disregarding the irrelevant one (Blum 1997; 2007). Additionally, inability to make assumptions has been classified as one of the most important barriers (Galbraith 2006). Finally, Blum (2007) notes an overreaching issue, namely that students may choose not to develop a situational model at all since they are trained in using information without having to understand the context.

When a real model has been constructed, it needs to be mathematized to form a mathematical model. In this transition, students need to represent the elements of the real model mathematically (Galbraith 2007). Success in this transition is therefore related to students' ability to translate data, concepts, relations, conditions, and assumptions into a mathematical language (Blum 1991). These abilities are in turn dependent on the students' capabilities in selecting mathematical notations and graphical representations (Blum 1997). Students might also need to reengage in activities associated with the previous transition, such as simplifying the model further by reducing the number and complexity of relations and variables (Blum 1997). A lack of mathematical competence is typically the reason for failure when mathematizing the real model. Maaß (2006) notes that mistakes that can occur include inadequate or incorrect use of mathematical symbols, formulas and algorithms.

Once a mathematical model of a problem has been created, it needs to be processed mathematically to find a solution to the problem (Galbraith 2006). Here, the complexity of the solution procedure depends on how the original problem has been modelled. If the students have designed a model which can be solved with known procedures, the activities and competencies presented by Galbraith (2007) and Blum (1997) are sufficient. Galbraith (2007) writes that the solution process can involve applying procedures, calculating, and creating graphical representations. In this step, mathematical knowledge related to conceptual understanding and procedural ability is regarded as a necessary competency (Blum 1997). Otherwise, the student is required to either devise an algorithm or realise the need of returning to previous stages to refine the model and adjusting the solution strategy, which is further expanded in section 2.2 below. Furthermore, even if the student can execute the selected strategy, barriers can form if the student is not able to realise that for example graphs or functions can be mathematical solutions (Maaß 2006), which will cause confusion in cases where a number is not retrieved at the end of the procedure.

The mathematical solution of the model cannot be used directly as a solution to the initial problem, and students are therefore required to interpret the former to acquire a real-world solution (Galbraith 2006). To complete the interpretation, students need to find the real-world counterpart of the mathematical solution to describe it in terms of the original situation (Galbraith 2007). In this step, students need competencies for relating mathematical results to the original context, generalising a solution that was originally found for a specific case, and communicating the solution (Blum 1997). Mistakes, which affect students' possibilities to present and validate the solution in a later stage, associated with this step include not interpreting the result at all (Maaß 2006) or not understanding the constraints set by reality. For instance, Schoenfeld (1987) exemplifies how students who are unable to realise the real-world implications of a result can make conclusions similar to that 31.333 buses need to be requested for a transport, despite that an order of 32 buses needs to be placed in a realistic situation.

When a real-world meaning of the solution has been retrieved, the final step in the modelling cycle involves evaluating the solution and used model to decide whether the solution should be accepted or if another iteration of the cycle is necessary (Blum 1997). Competencies that are of importance include critiquing the solutions and model, and reflecting on alternative solution strategies (Blum 1997). In short, the validation regards investigating the sensitivity to variances in assumed values and relations, and if the results are valid when environmental factors are taken into account (Meerschaert 2013). In addition, any unexpected results need to be identified and recalculated. However, the validation step is complex and will vary depending on the model type that is selected, which is something students need to realise (Maaß 2006).

2.2. Importance of metacognition to plan and monitor the process

While the previous section especially highlights how cognitive competencies, such as being able to perform a calculation, is important for transitioning in the modelling cycle, this section focuses on the role of metacognition. As will be expanded below, metacognitive ability is often associated with planning of the solution strategy and in identifying and correcting issues within the process, and thus plays an important role when solving problems (Wilson 2004). In fact, it has been shown that in case of problem-solving, failure during the process is usually associated with lack of use of metacognition rather than low content knowledge (Schoenfeld 1987). Despite possessing high conceptual knowledge and procedural ability, novice modellers use these competencies inefficiently due to not engaging in metacognitive activities (Schoenfeld 1987).

The planning process is often argued to occur either during the modelling steps or during the solve phase. Regardless, it is an important step in the process, and insufficient planning is related to several blockages. For instance, Schoenfeld (1987) writes that students who lack metacognitive ability engage in meaningless calculations and switch between directions without reasoning about their sensibility, eventually causing failure to solve the problem. Moreover, Maaß (2006) describes an alternative result where students complete their process without actually having any result. This alternative ending suggests students not having engaged sufficiently in planning activities, as the planning should include the overreaching goal of the procedure.

The planning process includes key activities of applying heuristic strategies, such as dividing the problem into smaller ones; selecting suitable formulae; and choosing appropriate algorithms (Blum 1997; Galbraith 2006; Meerschaert 2013). The identification of available resources, such as content-specific knowledge, knowledge about problem-solving in general, and previous experience of problem-solving that can be of relevance for the current problem, is labelled as metacognitive awareness by (Wilson 2004). After awareness of resources has been successfully raised, students can engage in evaluation of their relevance, capacity, limitations, usefulness and efficiency in relation to what assignment is to be done as well as past experience in using them (Wilson 2004). Thereby, the evaluation also considers personal capability in using the resources. As several resources are compared, metacognitive regulation uses this information to optimise the use of resources (Wilson 2004).

In addition, Wilson (2004) states that metacognitive awareness includes identification of the current state, including what has been done, what needs to be done and what might be done as the work progresses. This is necessary as the regulation needs to take the current state and future steps into account (Wilson 2004). However, it also emphasises how metacognitive regulation involves revising a strategy that has already been selected when the evaluation labels the current solution strategy as unfeasible or ineffective.

The topic of strategy revising is further extended by Goos (2002), who emphasises both the ability to identify barriers that are forming and to react appropriately. To monitor the process, students can engage in routine monitoring and controlled monitoring. Routine monitoring is employed to confirm that no issues exists, for example by verifying that found solutions satisfy the conditions, while controlled monitoring is engaged when the presence of issues has been identified (Goos 2002). In order to have the possibility of solving a blockage, indications such as lack of progress or inconsistent results need to be identified (Goos 2002). If the student fails to notice these indications, metacognitive blindness occurs and the student will not be able to act on it, which eventually leads to a blockage (Goos 2002). Another metacognitive process that can cause blockages during the monitoring is labelled as metacognitive mirage (Goos 2002). It involves identifying false issues and engaging in regulation despite that no real issue existed, which may cause the student to abandon a strategy that would have been successful (Goos 2002).

Assuming that real issues are identified, students can either react by using an appropriate or inappropriate response, which affects their success. Goos (2002) exemplifies this by noting that lack of progress should trigger the student to reassess the plan and select to either replace it or continue with it depending on the results of the assessment. In terms of metacognitive categories, the student should re-evaluate the strategy, and then engage in metacognitive regulation by re-planning the solution strategy as needed. Here, the student also needs to apply metacognitive awareness to identify possible resources and strategies that can be used. Another example is that detected anomalous results should prompt the student to investigate the accuracy of the strategy execution, and correct any errors (Goos 2002). Here, the student is evaluating both the execution and the strategy, and then amends the strategy, corrects calculations or identifies and uses additional resources (Goos 2002). If the response is appropriate, the student achieves metacognitive success (Goos 2002). Otherwise, the student will engage in metacognitive vandalism (Goos 2002). Such actions include the re-modelling of a problem in order to be able to use a resource the student possess, where the resulting model enforces a concept that is not appropriate for the problem (Goos 2002).

3. Effects and requirements associated with usage of digital tools

The usage of digital tools can affect the successfulness of a student's transition in the modelling cycle. Firstly, it affects the ranking of importance of competencies and introduces additional requirements on ability. Secondly, the use of a tool can reach the user's cognition and alter it. While the effects on competency can be argued to be implied by the two extensions of Blum and Leiß' (2007) mathematical modelling cycle presented by Greefrath in 2011, the effects have also been studied in previous research.

In the first proposition of how the modelling cycle can be extended to include the notion of digital tools, Greefrath (2011) added a technological subsection between the mathematical model and solution states. Greefrath (2011) argued that the mathematical model first would have to be retranslated into the syntax and format used by the tool. Subsequently, the tool would be used to generate a result, which would in turn need to be retranslated into a mathematical result (Greefrath 2011). In other words, the digital tool would only be used in the solution phase, and put further requirements on the used model.

However, Greefrath (2011) noted that digital tools can be used for more purposes than generating a solution, which lead to the second proposition. For example, tools can assist the user while understanding the problem by providing a visualisation that is essential for clarifying the interpretation of the situation, assisting in information collection via the Internet while developing a model, and enabling experimenting of a complex situation by providing a simulation environment (Galbraith 2006; Greefrath 2011). During the solution phase, digital tools can support in solving the problem or testing conjectures by offering functionality for activities such as performing calculations and creating graphical representations (Galbraith 2007; Meerschaert 2013). In the verification phase, digital tools can provide means of validating the results numerically or algebraically (Greefrath 2011). Since digital tools also offer the possibility to automate calculations and the validation phase typically involves recalculating the solution multiple times with small variances in assumed values, they can increase the speed of which the validation phase is executed essentially (Blum 1991; Galbraith 2007; Meerschaert 2013).

3.1. Cognitive effects from using digital tools in the modelling process

Digital tools provide users with accessibility to high computation speed and dynamical representations. These characteristics in turn provide the possibility to handle more data, use strategies that would be unpractically time-consuming to use if the tool was not available, and engage in activities that revolve around dynamically exploring a model. For example, technology enables the creation of dynamical spreadsheet models and interactive simulations (Greefrath 2011; Salomon 2005). Simulations can be used both for exploring a complex situation and for creating a result when the situation includes features that results in the model not having an analytical solution (Greefrath 2011; Meerschaert 2013). While the results of these models probably could have been computed manually, the computation would have taken too much time to be practically usable in a classroom setting. Similar reasoning can also be used when highlighting that digital tools can be used to employ numerical solution algorithms (Meerschaert 2013). Furthermore, digital tools can be used to inspect the behaviour of functions in different intervals (Salomon 2005).

In addition, as highlighted in section 1, digital tools can potentially enable students to use procedures they would not have been possible to use if they had to execute them manually. This characteristic provides access to mathematical concepts and model types for which the student do not possess strategies to manage. As noted in section 1, teachers and researchers have however not agreed on whether this effect is positive or negative.

Another important aspect of the use of digital tool is their provision of services which can decrease the cognitive load of users. By using digital tools to perform cognitive tasks such as calculation, equation solving, and plotting, students are freed from the distractions of these tasks and can focus on the metacognitive activities that are vital for problem-solving performance (Blum 1991; Galbraith 2007; Kuzle 2017; Meerschaert 2013; Salomon 2005).

As a result, the digital tools can potentially enhance the cognitive performance of the user (Salomon 2005) and may thereby improve the student's successfulness. For instance, digital tools have been found to provide services, such as exploring and inspecting systems more closely, that activate awareness of mathematical knowledge the user would not have considered otherwise (Kuzle 2017). These effects can also be amplified by the speed at which the digital tools perform the delegated tasks. As the tools can provide accurate results rapidly (Blum 1991) when operated correctly, the user can observe the changes generated by small changes to the model or data without major interruption, and thus avoid losing the focus of the overall discussion when engaging in activities concerning the details.

However, digital tools can also discourage the user from engaging in metacognitive and reflective activities, thus generating an increased dependability on the tool. For example, Blum (1991) identifies a risk of that reflection and decision-making is distributed to tools, and notes this effect is not wanted as these activities should be left for the students in order for them to develop skills in for example comparing and selecting models. Ainsworth (1999) exemplifies how automation can be reasoned to hinder construction of thorough understanding by describing how a tool providing multiple representations with dynamic translation of modifications on one representation onto another can discourage reflection on the nature of the translation. This means that despite the tool's possibility of off-loading students' cognition and thereby enabling them to focus on learning about the relation between the used representations, the linking can also make the student unable to develop own knowledge. Additionally, Geiger (2010) found that students might face blockages due to not reflecting about the involved mathematics when believing digital tools can solve anything. Finally, cognitive resources will be allocated until the user is experienced in operating the tool, which hinders the user from focusing on any other aspect than the interface of the tool (Galbraith 2006; Kim 2007).

3.2. Dependencies toward tool mastery and metacognitive ability

As indicated above, the sought increased activation of metacognition requires competency in operating the used tools. Cognitive resources are not freed until the digital tool is operated well, partly because usage of tools will consume available cognitive resources of the user until the user understands its interface and has gained skill in using it (Artigue 2002; Galbraith 2006; Kim 2007; Kuzle 2017). In other words, students cannot take advantage of the potentials of the tool until they are able to efficiently operate it to exploit the features it provides (Artigue 2002; Kim 2007; Kuzle 2017). In addition, Galbraith (2007) stresses that students must be able to correctly use the syntax rules the digital tools require. Similarly, Greefrath (2011) presents a need of translating a model into the format used by the digital tool, which is to perform calculation on it, and then interpreting the results to mathematical results when using digital tools in the solution phase. However, this reasoning likely holds for all uses of the digital tool, meaning that the data need to be reformatted to adhere to the tool's constraints regardless of what step of the cycle the tool is used in.

For example, in case simulation models are implemented and run, students must understand the programming language to use before being able to focus on the construction of the model (Kim 2007). Furthermore, the argument about knowledge of programing languages also applies when computations are to be automated. In those cases, a programming language or algebra system scripting language need to be understood. Additionally, students need to be able to specify the steps involved in the solution clearly enough for the steps to constitute an algorithm that can be translated into the programming or scripting language.

Thus, students' awareness of the capabilities of provided digital tools as well as their own skill in using them become important. If calculations are involved in the solution strategy, Galbraith (2007) writes that students need to have competency in selecting technology to enable calculation, selecting technology to automate it for multiple cases, and selecting technology to verify that the retrieved output is a solution. In addition, when students have access to procedures supported by digital tools as well as procedures they can execute manually, they are required to recognise whether a digital tool needs to be used to enable calculation or if manual methods are feasible or even constitute a better solution strategy.

Furthermore, students' knowledge of how technology can process data might also influence the design of strategy and models (Galbraith 2006). Thus, the availability of digital tools might influence the design of the mathematical model. Students therefore need to be trained in identifying indications of subconsciously favouring a specific strategy and representation in addition to being able to identify separate options and evaluating them.

In fact, the requirements on students' metacognitive abilities related to identifying and evaluating representations become higher as the amount of resources that can be accessed by the user drastically increases when tools are available. As indicated above, digital tools provide a large amount of techniques, algorithms, representations and model types. Thus, there is an increased demand on the students' ability of recognising the possibility of using different resources as well as evaluating how appropriate they are to use for the problem at hand. This concerns both being able to select between mathematical representations, and knowing which ones are supported by the different tools (Galbraith 2006).

The necessity of increased conceptual knowledge and metacognitive activation also becomes visible in research regarding barriers related to the validation step, in which error-checking and evaluation of alternatives is characterised by being a complex process that cannot be standardised. In this step, students need knowledge about how to validate the specific type of model that has been chosen, since different model types require different strategies for validation. For example, the validation of probabilistic simulations requires rerunning the simulation multiple times for each set of configuration settings and noting the variance in output, while deterministic models only require assumed values and relationships to be varied once when they are evaluated (Meerschaert 2013). This also requires understanding of the underlying concepts used by the tool, as the students need to realise that, for example, usage of an inappropriate step size in simulation of discrete systems can cause chaotic behaviour where the system should be stable (Meerschaert 2013).

In addition, monitoring processes require the ability of validating the output of the digital tools. As stated by Kim (2007), students need to estimate the reasonability of the tool's output. Furthermore, it is necessary that they understand the interaction provided by the tool. For example, students need to be able to note and understand the implications of warning messages and indications of errors. Lingefjärd's (2010) study exemplifies a case where students are unable to understand an indicator of error size, which results in them losing motivation and hiding the issue rather than correcting it.

4. Methodology

From the literature study, it has become apparent that several factors affect the result of modelling sessions. Regardless of if digital tools are involved in the process or not, these factors mainly concerns students' cognitive capacity, abilities related to strategic planning, procedural competency, concept knowledge, and monitoring skills.

As such, the methodological questions were designed to investigate how digital tools reach the cognition of students and how important their use are for students' progress in the modelling cycle. Firstly, studying students' interactions with each tool can indicate their proficiency in using it, amount of barriers formed by its usage, what output it provides to the students, and if it enables students to use concepts and procedures they would not have been able to use if it was not present. Secondly, students' metacognitive activities in general and in association with their tool usage can be studied to understand if the digital tools encourage or discourage metacognitive discussions, if they generate awareness of resources students had not considered otherwise, to what degree students plan their tool usage, what expectations students have on their tool usage, if students can detect issues with their tool usage, how students react to barriers, and how dependent students are on the presence of tools.

While some of these methodological considerations target the direct effects by the tools, others can be used to explain those effects and thus provide input to the second research question. This question aims to discuss how the educational design can be altered to increase the observed positive effects and decrease the observed negative effects, and thus requires an understanding of the causes of the observed effects to both discuss design elements from the case study and general design recommendations.

Due to the focus on understanding the reasoning of the participating students and the causes of observed positive and negative effects, a qualitative methodology was selected for both the data collection, data processing, analysis, and interpretation. This is reasonable since the study focuses on identifying and explaining behaviour of specific individuals, and does not attempt to generalise findings (Trost 2010). In short, the cognitive and metacognitive actions of two groups of 12^{th} grade students were observed by recording their modelling process and complementing the data with individual interviews. The material was later labelled and organised into lists and tables to provide answers to the methodological questions. Subsequently, the findings were analysed and strategies for improving the effects were developed from the experience of the study as well as material from the literature study. Further details relating to the design of the study and implications of its execution can be found below.

4.1. Elicitation of thoughts and reasoning

One method for capturing the thought processes of participants involves instructing them to articulate what they are thinking while they are engaging in modelling activities. However, studies have found that it is necessary to continuously prompt participants to continue speaking, since they easily forget to and become silent (Wilson 2004). Since the teachers that were contacted for participation in this study requested that the thesis student participated as an extra instructor in the classroom, such monitoring and prompting was not be possible, and it was therefore likely that this methodology would generate gaps in the data.

An alternative to using this strategy is to attempt to generate natural conversation. This strategy involves grouping students into small groups, for example pairs, and record their discussion. Since the students need to interact with each other while solving problems, their reasoning and explanations can then be recorded with a sound recorder, and there is a lower risk that they will stop communicating their thoughts. As the strategy also relies on students' thoughts being verbalised as part of a discussion or explanation, minimising the group size is preferable as it decreases the risk of students considering ideas without verbalising them while waiting for an opportunity to speak. Moreover, a larger group size increases the risk of students forming sub-groups which interact concurrently. In turn, this may create sound records where it is difficult to distinguish the different discussions that are ongoing simultaneously.

Another option for exploring students' reasoning is interviews. Interviews are suitable for exploring how the interviewee thinks (Charmaz 2006) and can be used to get an understanding of how students make strategic decisions. For example, Kuzle (2017) used interviews to ask questions about the typical behaviour of different steps in the problem-solving process and to determine why participants decided to use a particular tool feature. In Kuzle's (2017) study, this generated an understanding of students' reasoning or lack of reasoning behind choices as well as how the use of a feature influenced further actions (Kuzle 2017). Interviews also allows for stopping on a question to gain more details or explore how the interviewee thinks (Charmaz 2006), which gives the opportunity to thoroughly explore thought processes.

For an interview to be successful in eliciting feelings and thoughts, it is important that the interviewer focuses the questions on events, uses additional questions to elicit more detail when needed, and avoids statements and leading questions (Ejvegård 2009; Trost 2010). Thus, successfully conducting interviews requires skill. For example, choice of words, pace between questions, how words are emphasized, the amount of eye contact, and nervousness of the interviewer are factors that can affect the outcome (Charmaz 2006; Trost 2010; Ejvegård 2009). On the other hand, ambiguous questions or other issues that may cause confusion or uncertainty can be directly noted and clarified by the interviewer, and are thus less likely to generate issues compared to when including them in for example questionnaires (Ejvegård 2009). In order to ask follow-up questions, the interviewer must however be able to focus on the interviewee. Charmaz (2006) and Trost (2010) recommends using an audio recorder to generate this focus.

One strategy that can be combined with the interview format is self-reporting, where participants can report which activities they have engaged in and in which order they engaged in them (Wilson 2004). This data collection method can either be done without tools, or with the help of for example action cards that are sorted or video playback, as was done in the study by Wilson (2004).

However, retrospective accounts, both during regular interviews or with self-reporting strategies, have been found to be inaccurate. When using self-report strategies, Wilson (2004) found that the use of video playback can cause participants to reorganise and correct action cards. Due to this reason, Wilson (2004) concluded that the inclusion of video playback was vital for the validity and reliability, which indicates that studies only based on participants' own reporting should be questioned as participants might not report cognitive and metacognitive actives accurately. Trost (2010) discusses the general use of retrospective accounts during interviews, and also warns about inaccurate response. He notes that retrospective questions will give input on how the interviewee currently perceive what happened then, rather than what actually happened (Trost 2010). Furthermore, it has been shown that the connection between what someone did at a specific point in time and how the same person remembers the same event is weak due to memory loss, simplifications, and reinterpretations (Trost 2010). To lower the risk of inaccurate accounts, it is important to ask about different parts of the event rather than the event in whole (Trost 2010), and to hold the interview or self-report session close in time to the events that are studied. However, in Wilson's (2004) study, these unwanted effects appeared even though students were engaged in the self-reporting session shortly after the problem-solving episode was completed. While the negative experiences from only one study cannot be used to predict effect on future studies, it is apparent that there is a risk involved in relying on retrospective accounts for the analysis, and that this risk can be mediated by using video playback of the studied events.

Since using video playback for the whole problem-solving and modelling process would require a long time to be reserved for interviews, alternative thinking cues was considered for this study. For example, if students are asked to submit a report following a scheme in which they are required to motivate and document their activities within modelling cycle, the report can be used as a guide during the interview. Similarly to how video records help participants recall memories of the event, seeing figures and texts might help students remember how they were reasoning.

Based on the above discussion, a combination of data collection activities was considered when designing the study. Records of group activities were used instead of think-aloud strategies, since the risk of students not sharing own reasoning with team members was considered low as all students need to have an understanding of the strategy to contribute to its execution. It was therefore assumed students explain their reasoning to the other members. Interviews could therefore be used to complement the recorded data by providing the opportunity of thoroughly exploring processes that could not be fully understood from the records. Additionally, one risk that was identified when using interviews is that the sequence of and contents in events and thoughts may be inaccurately reported, which indicates that data collection of events should mainly be based on a more reliable source, such as the records. Since the number of classroom visits was limited, and that the problem-solving process was anticipated to allocate much time, the interviews were planned to be short and focused to complement recorded data. To increase efficiency of interviews, it was decided to use working documents or screen records to create thinking hooks. However, such tools were never needed during the interviews, as they occurred during the modelling sessions and students' memories could be activated by describing a specific scenario.

While it was concluded that the risk of inaudibility of sound records and students not verbalising their thoughts would be minimal if students worked it pairs, the participating teacher decided to divide the students into groups of four. To compensate for the increased risk of inaudible records, the groups were placed with a relatively large distance to other groups and the number of sound recorders was increased. The latter was later found to be necessary, as records occasionally was inaudible and then could be replaced with records from other recorders. The main reason for the inaudibility was however that students' computers were used as sound recorders and that there were a large amount of background noise in the records when students were typing on the corresponding computer. It would therefore have been necessary to use several sound recorders even in the case of pairs.

Another deviation from the initial plan was the placement of the interviews. Due to there being few lessons allocated for education about and oral exams of mathematical modelling and a three-week break in the middle of the sequence, interviews had to be held either during the second and third modelling sessions or after the break. The first option was selected, as risk of effects on students' memory and thesis time plan were estimated as high. However, it resulted in the interviews being short and only based on data from the first two sessions.

The length of the interviews in combination with other factors caused the opportunity of arranging additional interviews after the break as well as collecting data from students' oral exams to be considered as beneficial. When analysing the third modelling session, it was found that video records of students' tool usage and working documents were missing. Furthermore, the session records and interviews indicated inconsistent results regarding the metacognitive activities of students. There is a possibility that these data collection methods are complementary. However, this behaviour can also be explained by students not using metacognition during the session although having the competency necessary to use it, as suggested in section 5.2. As further discussed in section 6.1, this can in turn be explained by students lacking the habit of engaging in metacognitive activities, or regarding the engagement as too expensive in regards to cognitive demand. In addition, it is possible students might have processed and thought about the problem outside of the modelling sessions, despite being asked to avoid doing so. However, the lessons that were allocated for mathematical modelling education after the break were cancelled due to unrelated reasons, and further data collection was therefore not possible. Moreover, access was not given to the students' documents. Consequentially, the conclusions that could be drawn were limited.

4.2. Recording of cognitive processes and actions

In addition to the reasoning of students, the actions they engage in as well as how and when they use digital tools were decided to be studied. This data can be recorded by having an observer take notes, by using a video recorder, or by self-reporting. However, as described in section 4.1, observations was not be possible. Furthermore, due to the issues associated with self-reporting which were discussed in the same section, it was also regarded as inappropriate.

Therefore, use of video recorders was selected to collect data of students' actions. The study was designed to use screen recorders since the studied tools are digital and interactions with them therefore involves the use of computers. In addition, this alternative enabled simultaneous interactions with multiple computers to be recorded, which was regarded as necessary in case students decide to work on separate computers. However, this alternative also generates data loss in cases where students use smartphones. This effect was not considered during the design of the study due to it being planned to only study a few selected tools that were introduced to students during the sessions and which could only be used in a computer environment. However, the results identified Google Search as one of the used digital tools. This tool could have been accessed via smartphones in addition to the recorded computer interactions, and there might therefore be gaps in the data concerning the use of this tool.

On the other hand, not all actions that are relevant to the study will require the use of a computer. For example, design of a mathematical model can be created with pen and paper even if digital tools are selected to assist in the solution phase. In addition, actions such as solving simpler equation systems might not require support from digital tools.

An external video recorder would be necessary to capture actions which do not involve the use of computers. The external recorder might however require continuous adjustments due to students concealing the papers with their bodies. Thus, it is likely that this data cannot be recorded without constantly adjusting the recorder. An alternative to using this method involves instructing students to include all models and calculations in their working document. The document can then be used to better understand the context when analysing the discussions, as the recorded sound should also include associated discussions. Since access to working documents was not provided, the study relied on the content from sound records and screen records, which in turn occasionally recorded the contents of the working document. There were few scenarios where this type of actions occurred, out of which the most notable loss was that of visual cues when students pointed at the screen with their fingers while discussing graphs and figures. However, access to information regarding exactly what students pointed at was not vital to analyse the behaviour and discussion.

However, this complementary information was missing from the third modelling session, where students were not able to initiate the screen recorder. While collected sound records indicated cognitive activity and captured discussions, the students' tool interaction from the session could not be studied. This includes both groups' use of GeoGebra and one group's usage of python scripts. The sound records did however include students noting if they had issues with the tools and what tasks tools were used for. Another effect was that the development environment used for editing the python script could not be identified.

During the interviews, the students claimed to not be affected by the screen records. In addition, they stated it did not affect their participation or amount of computers used. The records showed that all students actively participated. However, the records indicated students were aware of the recording, as they opened discussions related to whether they should start a recording on a specific computer several times during each session. However, it also revealed that students did not refrain from using computers when being unable to initiate the recording. Therefore, it is unlikely that the awareness affected their tool usage.

4.3. Analysis of participants' thoughts and actions

From the information in section 2 and 3, it can be concluded that thoughts and actions are expected to be interconnected, and that they must therefore be related during the analysis. Moreover, there exist several strategies for managing, storing, and interpreting the data retrieved from the sessions and interviews.

Firstly, the sound, video and interview records can either be transcribed before being analysed or being analysed directly from the records. The former would allow for storing the data in textual format and therefore make it searchable. However, transcribing data takes long time (Trost 2010), and when transcription of videos were done to estimate the time for transcribing, it was found that one hour of material would take seven hours to transcribe. Considering that the study includes material from several classroom visits and groups, the time estimate for transcribing the material was determined to be impractically high in relation to the advantages it would provide. An alternative methodology involves analysing the visual and audio material while it is played, for example by classifying or summarising important content. Since this option requires the records to be replayed for every analysed category, it also allocates extensive time resources. As the number of categories to analyse is low, the repetitive playback was evaluated to have a minimal risk of exceeding the time needed for transcription, and the study was therefore planned not to use transcription. However, once the analysis was conducted, it was found necessary to have a textual representation in order to find patterns and reanalyse specific actions. A table including actions, statements, posed questions, arguments etcetera was constructed. In contrast to a transcription, the table entries included the essence of each element, but did not include exact wording. Consequentially, it did not require as long time to create.

Another design choice relating to the analysis of gathered data is that it either can be coded to form new categories or directly categorised into pre-determined categories. The codes can be used to summarise what happened, what a participant emphasises, and involved emotions (Charmaz 2006). In grounded theory practices, new categories will be generated from the codes and the coding will ensure no data is ignored (Charmaz 2006). Categorising data into pre-determined categories is likely to allocate less time, but will only allow for analysis that the categories and associated frameworks have been designed for (Charmaz 2006). There is therefore a risk that any data not fitting any framework will be lost, even though it could be interesting for the conclusions. However, grounded theory practices also requires extensive data to be collected in order for categories to become saturated (Charmaz 2006). Thus, it would require a larger sample as well as a narrower limitation of the extent of the study in order to be able to process the data in a way that both captures the variation and enables deep understanding of thoughts and behaviours. As information from data categorised into pre-existing categories was not regarded as uninteresting, it was concluded the use of grounded theory practices were neither practical in terms of the time frame of the study, nor required to draw interesting conclusions.

During the study, the analysis was implemented in three steps. The first step involved creating tables of all events from the sessions without analysing them. The table was then reviewed to generate a first impression of the results, which was in turn used to create interview questions. In the second step, the table was extended with columns dedicated to notes on tool usage, strategy considerations, blockages, identification and reactions to blockages, and key insights and actions. It was read through once for each of these categories. Interview data were processed in a similar manner and related to the data from the modelling sessions. After initial results had been collected, the sound and video records from modelling sessions and interviews were revisited to verify that the tables contained correct and complete information. Moreover, the data was reorganised into separate tables for each analysis orientation to verify that the initial results were consistent with the data. In the third step, the results were related to methodological decisions and previous research to answer the second research question.

4.4. Collection of previous research

The study of previous research was organised into two phases: the orientation phase, and the refinement phase. During the orientation phase, the objective was to get an overview and find inspiration, and literature was found via the Chalmers library by searching for keywords related to mathematical modelling, digital tools, problem design, and study methodologies. As content was read, interesting authors, keywords and conferences were noted. For example, keywords related to cognitive tools, metacognitive theories, and cognitive barriers were introduced. These entities were then used to extend the covered material until a thorough understanding of the research area and well-defined analysis categories had been formed. The refinement phase was used to revisit previous notes as importance of content had shifted, fill gaps in the data, replace any references to accounts of material by reading the original source, and investigating if the authors have written any newer pieces with new results. For instance, what material was important to answer the second research question was not known before the first had been answered, and thus required notes from the orientation study to be revisited.

The used material related to previous research was also evaluated to ensure it is scientific, used good methodologies, and was up-to-date, as recommended by Ejvegård (2009) and Trost (2013). Moreover, academic papers were preferred as they had already undergone peer review, and thus evaluated to meet those requirements. Conference articles, theses, and textbooks were also categorised to have a high academic trustworthiness.

4.5. Design of modelling problem

During the orientation study, it was found that modelling problem should be designed to be challenging, ill-structured, meaningful, and have multiple solution paths and solutions. In order for students to engage in problem-solving and modelling activities, both understanding and finding a strategy for solving the problem should be challenging, but not impossible (Wedelin 2015a; Blomhøj 2006). The modelling cycle typically includes the actions of making assumptions and simplifications as well as predicting sensitivity to such assumptions (Blum 1991; Meerschaert 2013). Therefore, it is reasonable to design problems which enable students to engage in such activities. These characteristics also implies that multiple models may exist, and that the design of these models depends on the chosen representation, solution strategy, simplifications and assumptions. Students therefore need to engage in discussions about the possible models and solutions. Additionally, Wedelin (2015a) notes that students should not be formed to learn extensive new theory to solve the problem, and Blomhøj (2006) writes that problems should be realistic enough for students to be able to use their own experience. Thus, students need to be able to engage in these reflective activities using their previous knowledge, experience and intuition.

Several sets of problems were created to fulfil the criteria outlined above as well as requirements set by teachers who were considering participating in the study. Existing problems were collected by investigating which problems had been used in previous research and which problems were available in textbooks about mathematical modelling and databases of problems. They were then rated in regards to how well they satisfied the design criteria and modified as necessary. Thereafter, solution examples were produced for the problems which had the highest conformance to the criteria. Finally, the problems and solution examples were discussed with the teachers. The last step was important since the involved teachers could not offer classroom visits prior to the study, and an estimation of whether the difficulty level of the problems were reasonable in relation to students' knowledge level and the allocated time had to be made by each respective teacher. Simultaneously, the changes requested by teachers were evaluated to ensure they did not involve simplifying the problems to a degree where the need of interpretation and simplification, which are activities that are characteristic for modelling tasks, would be removed. In situations where the needs of both the teacher and study could not be ensured, the participation of that teacher was declined.

Finally, the modelling problem which was selected for the participating students was the road network problem developed by Wedelin (2015a; 2015b), who also presents details related to the design of the modelling problem and on how to obtain it. The problem description presents a situation where two cities are connected by two separate paths, which are divided by a river. The posed problem concerns to what degree a bridge built to connect the two paths would off-load the network. Two supportive questions, both targeting the situation before and after the bridge is built, are stated in the problem: if an equilibrium state is likely to appear, and what the travel time will be. The participating teacher had two requirements in addition to the problem being solvable within the allocated time, namely that the session series would focus on algorithms and programming, and that it should not require the use of digital tools. By modifying this problem to request an answer on how each driver would reason rather than on if an equilibrium state would appear, a greater attention was put on algorithmically describing the behaviour of drivers. In addition, the problem was found to fulfil the teacher’s second criterion, as it could be solved by very simple calculation (Wedelin 2015b). Furthermore, the same result was found to also be obtainable by implementing a relatively simple simulation. The remaining students of the class, who did not participate in the study, were instead assigned linear optimisation problems by their teacher.

4.6. Selection of digital tools

Initially, the study was designed to introduce digital tools the students had no prior experience in using, and study the effects of those tools being present. The selection of digital tools was closely related to the selection of problem, since one requirement of the problem design was that students should be able to solve the problem with several solution strategies. Therefore, it is preferable that either several tools or numerous features of the digital tools can be used to solve the problem, since the number of possible solution strategies would otherwise be limited by how the tools can be used. However, the background study found that unfamiliarity with a tool can block the modeller from progressing, as described in section 3. Thus, there is a need of a compromise between the number of selected tools and their complexity on the one hand, and the need of learning how to operate the tool for it to become efficient on the other hand. In addition, the participating teacher requested a focus on algorithms and programming.

Another aspect that is important for the tool selection is the availability of the tools. It was decided that the students must be able to install the tools or access them online. Moreover, the digital tools need to be free of use for schools or small groups, or have a trial period long enough to be used during the period students will be engaging in the activities.

As a result of these considerations, two digital tools were selected. The first tool was Wolfram Cloud, an online computer algebra system which includes a large number of features, for example plotting, solutions algorithms, and simulation possibilities. The second tool was the integrated development environment Repl.it, which the teacher recommended since the students who had participated in a selectable programming course had experience in using it for Java programming. Since the teacher had the prerequisite that students should not be required to use a digital tool, the students were not forced to use these tools or any other tools.

In order to receive a variance in the tool usage and solution strategy, the teacher designed two groups of students with varying educational background. The first group consisted of four students their teacher believed to be likely to choose an analytical approach when solving mathematical problems. The second group, also consisting of four students, were instead believed to be more likely to use knowledge from an introductory programming course they had selected. Here, it should be noted that the students of the first group had not participated in the programming course.

However, the results of the thesis study revealed that the students did not use any of the two tools. This result is however not unexpected since there was little time allocated for modelling sessions and that students were required to spend cognitive resources on both understanding the mathematical modelling cycle and on solving the presented problem. As presented in section 3, usage of digital tools does not become efficient until students have learnt how to operate them, which in turn requires both cognitive resources and time. Furthermore, selecting a problem which does not require the use of digital tools is likely to have had an impact, as is further discussed in section 6. It would therefore have been interesting to analyse students' reasoning and behaviour in a case where tool usage would have either been more beneficial or enforced. Nevertheless, a consequence of this result was that the study was expanded to reveal why students decided not to use the introduced tools and to explore how other digital tools were used. Thus, the analysis involved all digital tools used by the students and interviews questions focusing on how students select between digital tools were designed.

4.7. Instructional design

The instructional design associated with the introductory lesson and three modelling sessions was aimed to provide students with the support necessary to understand the modelling process and break barriers related to concept understanding and tool operation. Simultaneously, the support was designed not to be too extensive, since students need to be challenged by the modelling problem.

The introductory lesson was planned to introduce the mathematical modelling cycle and the digital tools. The lesson was initiated with a presentation which explained the concepts of problem-solving, mathematical modelling, and programming. The remaining part of the lesson was then allocated for students to study an example problem. The example was designed to show a relatively realistic scenario with two different solutions. The reason for including solutions was that worked examples are known to be effective for introducing novice problem-solvers to problem-solving since studying worked example requires lower cognitive capacity than what would be allocated to find a solution (Renkl 2010). The plan was therefore that students could direct their cognitive capacity to relate the example to the modelling cycle, and thus get an understanding of the same, and to familiarise themselves with the tools by operating them as instructed in the examples.

The example included one solution strategy for each tool. The solution example using Wolfram Cloud demonstrated an analytical approach where the involved equation system was relatively simple to understand, yet required assistance from the tool to be solved. The second solution example used a Java based simulation framework developed in Repl.it, which provided an example of how behaviour could be described and simulated to provide an answer. Moreover, both solutions included intermediate models and strategies, including motivations of how they were developed or why they were abandoned, to emphasise the importance of strategy revision when engaging in modelling activities. Additionally, the examples also presented the tool implementation to simplify students' acquaintance with the tools and their interfaces. In addition to introducing the students to the tools' capabilities and the modelling cycle, including several solution examples was also hoped to raise students' awareness of the existence of multiple solution strategies and thereby increase their motivation, as described by Leikin (2007).

However, the teacher's introduction to the subject area, video and sound recording as well as the introductory presentation to modelling and programming caused students not to have time to study the example. Instead, a few minutes of the first modelling session was allocated for this activity. However, since the students could only study the solved example for a short time, they only had time to focus on recognising that there exists several solution strategies for a modelling problem and understanding how the problem required assumptions to be made. They did therefore not gain any experience with the tools.

Otherwise, the strategy for instruction during the modelling sessions was designed to support the creativity of students. To achieve this, students must be able to design their own solution strategy, which in turn requires involved teachers to listen to students' ideas and not direct them to a great extent (Lingefjärd 2010). Simultaneously, the teachers need to offer assistance that is necessary to break blockages. If students are not given content-related help when needed, students may be lost, lose motivation or try to hide the issues when presenting their solution (Lingefjärd 2010). Lingefjärd's (2010) study concluded that achievement of these factors requires teachers to be able to understand the students' representations and strategies, ask questions that encourages students to reflect in a way that is necessary to continue with their solution path, and clear any conceptual misunderstandings without offering too much support. Using the same reasoning, teachers should also help students with the use of the tool, for example by correcting syntax, while letting the students decide how the tool should be used and what feature they want to use. To support the participating teacher in providing this support, detailed solution examples were created for the modelling problem. Thereby, involved teachers could be aware of some different solution strategies and tool features students might use.

Moreover, additional information was provided to students to assist them in their process. During the beginning of the second modelling session, the participating students were given an explanation of the concept of traffic intensity, which was a key concept of the problem description and which both groups of students struggled to understand. In addition, a presentation material about model validation was designed to assist students with questions and examples that could guide them when engaging in the activities themselves.

However, the teacher participating in the study did not want to offer students help, especially during the first session. This request covered both the case of students asking for assistance as well as teachers actively interrupting students to discuss their process. Although it cannot be determined whether this had any effect on students, it was found that the students did not engage in conversations with teachers to a great extent. This finding also supported in deciding if interviews should be held before or after the three-week break. Since the students did not seem to be relying on the support of teachers, holding the interviews during the modelling sessions was estimated to not have an impact on students' progress.

5. Findings from the case study

As described in section 4.6, two groups consisting of four students each were studied. The students of one of the groups, below denoted as group A, have no formal programming experience while the ones of the second group, below denoted as group B, have participated in an introductory programming course.

The students of group A spend the first session and parts of the second session on understanding the problem, remaining parts of the second session forming conjectures, and the third session on further conjectures and finding a solution. In the understanding phase, the students search for information, discuss various interpretation possibilities, formalise a goal, discuss the dynamics of the situation, and create a simplified Paint figure of the image presented in the problem description after noticing a connection to previous knowledge. While conjecturing and attempting to find solutions, they initially use their simplified image to discuss and GeoGebra to plot graphs. However, they realise the relations between the traffic intensity of each path require multiple dependent variables, and finally develop a set of dependent functions that describe the travelling time and a script to find a solution based on those functions.

In contrast, the students of group B quickly reach a common understanding and a solution to both partial problems during the first session. In this process, they do not use any digital tools for calculations or plotting. However, they believe they reached a solution too quickly, and spend the remaining time creating a more complex model of how the traffic intensity varies. They use GeoGebra for plotting any graphs involved in this process. This step is furthermore characterised by the students revisiting the discussion relating to the dynamics of the problem situation and searching for information. In comparison to group A, the students of group B are less prone to consider different interpretation possibilities and verify that they are answering the posed questions. While group A focuses on finding a distribution of traffic intensity and on the situation of many drivers, the students of group B focus on providing a recommendation for one specific driver.

In other words, the two groups used two distinctly different approaches resulting from differing interpretations of the problem situation. While this variance is important to consider when reading the results, the exact strategies and answers will not be expanded further. Instead, the below section includes a description of the observed behaviour of the participating students, with a focus on how they used digital tools and how the presence of the tools affected their processes.

5.1. Overview of usage and operation of digital tools

During the sessions, the students select to use tools they have prior experience in using. Neither of the groups select nor mention the two tools used in the example problem, that is the Wolfram Cloud tool and the simulation framework created in the Repl.it environment. Instead, they use Google Drive, Google Search, Microsoft Paint, GeoGebra, and a script development environment. The sections below describes the purpose of the students' usage of each digital tool and how well they could operate each tool.

5.1.1. Google Drive (file storage and collaborative editor)

Both groups of students use Google Drive documents to document their process and thoughts. For example, they document assumptions, models, motivations and screen shots from their tool use. They also use the tool in order to be able to store a photograph of the problem description.

During the interviews, the students speak about extensive experience with Google Drive documents and the records show they are easily handling the tool. There is no discussion on creating the document, and it is rapidly prepared by both groups. Although the students mention that documents get a bad format when pictures are pasted, they quickly correct the format. Several students manage to write concurrently, and they also cooperate in correcting each other's writing and formulation. However, they only use the functionality they have experience in using. As will be discussed below, Google Drive also supports a tool for creating and editing drawings which is likely to be more useful and efficient to create figures than Microsoft Paint.

The main issue the students experience with Google Drive is its dependability on Internet connectivity. During the third session, bad connectivity repeatedly hinders the students from making notes and summaries. Moreover, it also occasionally consumes their focus. The sound records indicate that the students are occasionally successful in writing text and adding pictures to the document, but most of the time struggle with these operations due to connectivity issues. As the students did not perform video records during the third session and their notes were not shared, it is unknown how much information was lost due to the connectivity issues. However, at the end of the third session, group B uses the sound recorder to summarise their progress since Google Drive was not functional. Group A does not seem to attempt to replace functionality normally supported by Google Drive.

5.1.2. Google Search (search engine)

Google Search is used to retrieve explanations of concepts unknown for the students, confirm suspicions or understanding the students already possess, or to retrieve facts. Google Search is used by group A during the first session to get an understanding of the concept of traffic intensity, verify if the presented problem is related to the concept of game theory, and to confirm that previous knowledge about graph theory and Braess' paradox is related to the problem. The latter two attempts are successful for retrieving information, while the students do not find any informative results when searching for a definition of traffic intensity. The description of game theory results in the students concluding drivers need to think rationally, but they later occasionally engage in discussions more related to perception and feeling, which demonstrate they have not fully understood the concept. The search about the paradox is done by one student and not shared with the other students of the group directly or indirectly. Group B uses Google Search during the second modelling session when failing to agree on which times of the day are rush hours, and succeed in retrieving this information and using it.

The interface of Google Search is not complex and handled well by the students, but it is also clear from the records that the students do not attempt to refine their search when the search results are not informative. In the scenario related to gaining an understanding of traffic intensity, the students quickly watch the list of results and decide they are irrelevant. They do not attempt strategies such as translating their expression from Swedish to English or add additional words to their query.

5.1.3. Microsoft Paint (raster graphics editor)

Several versions of a figure created with Microsoft Paint are used by group A as an aid in discussions that occur during the second session. The figure is a weighted graph which constitutes a simplification of the illustration provided in the problem description, but also includes expressions of travel time for each road as weights. The figure is used to clarify what road, path, or point students are referring to, and for providing less detail while still including extra information. Despite being a graph, the initial version is also used as a geometrical representation where the travel time of a path is represented by the length of a set of sides or edges. The figures seem to be vital for making important realisations and decisions. However, both the interviews and session records indicate that the figure rather than the tool was important for their progress. During the interviews, the students state the reason for selecting Microsoft Paint was that they needed to easily add the figure to the document for grading purposes, thus indicating they might not have used a computer to create it unless the teachers had requested all models to be presented. From the records, it is also questionable whether the tool aided their progress, or if their progress can be attributed to the created figure.

As the students use the figure in their discussions and make progress, they continuously improve the figure. One of the students are responsible for managing the figure, and he or she does not have any major struggles with achieving the sought result. For example, when the student cannot find a way to rotate a rectangle or to create a dotted line, others immediately come with suggestions which solve the blockage.

5.1.4. GeoGebra (computer algebra system and interactive geometry environment)

Both group A and B start using GeoGebra during the second session to plot graphs describing the travel time of some different paths from the second partial problem in order to compare them. While interview data show that students of group B selected this approach to find which path that would give the most optimal travel time depending on traffic intensity, the students of group A used it to observe a relationship they had conjectured during a previous session. Another notable difference is that groups A and B interpreted the intensity presented in the problem description differently, and that of group A conflicts with their use of GeoGebra, which they also recognise while inspecting the graph they had created. When concluding they need to introduce several variables to represent the intensity of different paths, they decide GeoGebra cannot be used for their multivariable analysis and seek other options.

During the third session, the students of group B use GeoGebra to find a function of how the traffic intensity varies during the rush hours. Two options they explore is a linear function and a periodic function. The former is plotted to decide at what time drivers should select different roads, as the first use of GeoGebra mentioned above had given students input on at what intensities drivers should select specific roads. The students later use GeoGebra to fit the sinus function to match the shape they had in mind by testing different values of various constants, and then use it to measure at what time drivers should select different roads for that function.

Both groups of students seem to operate the tool efficiently for their purpose of adding single variable functions, modifying functions, and reading values at specific points. The students of group B is temporarily confused by that a variable denoted as x in the problem presentation becomes y in GeoGebra, which is a limitation of the software if inequalities are used. They occasionally face issues with the tool creating items, such as gliders, when they did not intend to, but quickly correct these unintended changes. At one point, the students suggest solving the issues they have with curve fitting by changing between using degrees and radians, but cannot find how to change between the two. Despite this seems to generate frustration among some students, it does not block their progress.

It is however apparent that the students of both groups are not using features they are not experienced in using. For example, group A could have used GeoGebra to investigate how the related variables affect the total travel time of the paths by using constants that can be varied with gliders and substituting one of the variables with a function of the total intensity and the other variables. Instead, they reason GeoGebra cannot be used when multiple variables are involved, and are thus forced to adapt their strategy and use other resources to continue in their problem-solving process. As such, their knowledge of GeoGebra blocks their current strategy, but they are able to find an alternative strategy.

5.1.5. Programming and scripting

Although group A lacks students with formal programming experience, it is the group that uses programming. They reuse and modify an implementation one of the students have created for a separate modelling problem that was assigned to another group. In short, the script is implemented to retrieve all allowed combinations of traffic intensity of the different paths, and determine which combination returns the lowest summarised travel time of all paths.

The student implementing the script seem to understand the syntax and logic well, but have some issues with careless mistakes. When they get unexpected results, one of the group members without any programming experience asks control questions that help the student who implemented the script identify the source of the error and correct it within a short amount of time.

5.2. Nature of students' strategic planning and goal-setting

The records display a low level of metacognitive engagement for both groups of students. These observations are consistent with findings from the literature study in section 2.2, which state that it is expected for novice modellers not to engage in metacognitive activities. The records show that selection and implementation of a representation or procedure normally are made directly after an idea is presented, and it is unusual for the students to raise awareness of and compare different procedures, strategies or solution paths. The students of group B reach the first solution without any metacognitive activity, they formulate the aim of providing a suggestion of what path a driver should select without discussing other possible interpretations of the questions, engage in creating a function to describe how the traffic intensity varies over time without discussing why, and decide they cannot vary the numbers given in the problem description without motivating why. Sometimes, the students also ignore alternative suggestions without motivating why. In one situation, a student of group A suggests they could write an equation to solve the problem while other students improve their Microsoft Paint figure, but immediately withdraws the suggestion and states that it does not matter. The other students continue with their activity without considering the proposal.

There are occasions where students discuss different alternatives or return to evaluating an alternative that was previously suggested but never explored, but it is unusual. The students of group A initially select a point at which they believe the traffic intensity specified in the problem description has been measured and later discuss other possibilities, while the students of group B directly discuss different alternatives. The students of group B also discuss the suitability and difficulty of using various function types for describing the traffic intensity over time. They begin by using an alternative that is simpler to implement, and later increase the difficulty. However, as is further exemplified in section 5.3, the students multiple times return to attempting to use a function type they have evaluated as inapplicable, indicating they do not fully use their metacognitive evaluation. Moreover, their final selection of function type is not motivated, as they select a periodic function without thoroughly discussing its suitability.

This behaviour does not change considerably when the students make decisions related to tool usage. They do not discuss different tool options and seldom discuss alternatives to using the tools. Both groups use Google Search to find information without discussing other alternatives for retrieving the same information, use GeoGebra for comparing driving time or traffic intensity of different paths without considering other alternatives, and intuitively use a Google Drive document for documenting their process. The students' decision to use Microsoft Paint for creating the figure is however not recorded, as the record starts when the figure has been partly created. However, the students of group A discuss alternatives to writing a script. Substituting variables is not chosen due to the functions containing a higher number of variables than what can be substituted, and unnamed alternatives are not selected due to there being too many possible combinations. They also note that a disadvantage with using the script is that only integer solutions can be found. Nevertheless, the students do not thoroughly explore the alternative options. For example, they do not consider adding equations describing equilibrium criteria, described as a key step by Wedelin (2015b), which could have enabled the substitution strategy to be useful.

Simultaneously, the interviews suggest the students have competency necessary to engage in metacognitive considerations. During the interviews, the students display an awareness of generic and situational advantages and disadvantages with different tools. When prompted, they can explain why another tool would not be useful, mention some limitations with some of the tools, describe when a tool is useful, verbalise how they select between tools, and note that the use of tools was not always necessary. The students mention that the benefits with using Google Drive include not being dependent on a specific computer being present, saving time as students can write concurrently, and gaining the possibility of complementing with information or correcting others' writing immediately. GeoGebra is presented as an alternative to graphing calculators, which can be used to calculate areas of geometrical figures and plot graphs. However, the students state it cannot be used to solve differential equations, and that an alternative tool named Wolfram Alpha needs to be utilised in such cases. Moreover, some students describe having experience with a digital tool named Vernier Logger Pro, but claim it would not be useful in this problem situation since it is only beneficial when measurement data exist. Some students are asked how they would be affected if they did not have access to computers, and those students claim other strategies, such as using graphing calculators or plotting graphs and figures by hand, could be equally useful, and describe cases where a tool selection was made solely to meet the teacher's request of recording the process with screen recorders. Some students also make comments on how the tool usage resulted in realisations about their procedure, for example noting that using the tool made them realise their selection of representation and tool was incorrect and that this realisation was key for considering better models and strategies. Other students instead believed the digital tool did not provide any benefits in those cases.

Therefore, the interview results and session records can be argued to be contradictory. There is a possibility that selection of tools is not visible in the records since the considerations might have become automated due to students' extensive experience with the used tools, and related reflections therefore are no longer conscious. It is also possible that the lack of recorded metacognitive activity can be due to the methodological choices, related to session design and strategy for eliciting thoughts combined with the group size, resulting in students not verbalising their thoughts. This alternative is further discussed in section 4.1. However, all these explanations do, in turn, imply that the students did not engage in metacognitive activity. Firstly, metacognitive activity is not subconscious, but require explicit awareness of resources, as described in section 2.2. Secondly, lack of comments and questioning from group members regarding suggestions, even if the student who suggested an approach did evaluate the option, indicates that the group does not actively co-engage in regulation activities. Thus, it is likely that the students did not engage in metacognitive activity although they possess the competency.

Another finding is that most students seem to be dependent on their teacher's instruction when selecting between tools. For example, all students generally describe the same functionality, advantages, and disadvantages when interviewed about the tools they have experience with. In some cases, students describe limitations with tools for which inspection following the interviews reveals that these limitations do not always exist. In turn, this indicates the students have not explored the tools extensively, but only used a predefined set of functionality. For example, one student stated GeoGebra cannot be used to solve second order differential equations, which GeoGebra does support. Furthermore, the interviews also show that most students only know one tool that can offer a specific functionality or operation, and they thus cannot select between multiple tools when a specific model or strategy has been selected. Moreover, most of the students only mention having experience with tools teachers have introduced, instructed with and recommended during education in different subjects. One student state that since the class has not been required to buy graphing calculators and the student therefore does not own one, he or she is forced to use GeoGebra. In contrast, a few students explain they have experience with digital tools that is not related to their teacher's instructions. Regardless of the amount of tools the students have experience with, they describe tending to select the ones they have most experience with and know best. One student exemplifies that their mathematics teachers have used GeoGebra when instructing about examples, and therefore prefers using GeoGebra. During the sessions, both groups of students also demonstrate a will to incorporate the tools or strategies used in the presentation material. They occasionally try to find suggestions of how programming can be included in their strategy while referring to that it has been highlighted during presentations, although the records show they do not need to involve programming in those scenarios.

As described above, the students who have an experience with a larger number of tools also tend to select the tools with which they have most experience. One student describes having trained himself or herself with using graphing calculators and uses it as an alternative to GeoGebra when there is no access to computers. Another student mentions having used a tool named Symbolab to receive instruction on how to solve specific tasks if needed, but would not use it otherwise. One student also have engaged in relevant tool usage outside of the educational context, for example learning programming during his or her free-time and watching content about technical systems and graph theory on YouTube. The reason for engaging in such an activity is described as either the subject being interesting or the tool mastery possibly having future benefits, such as being a competence employers seek. The student explains tending to select the tools he or she has most experience with when having several options, but mainly considers which option would take the least time to use to produce sought results. These two factors are not contradicting, as efficient operation of tools take time to develop (Kim 2007), and when achieved results in a more rapid arrival at results. This evaluation of time requirements is thus likely to involve both evaluation of the tool functionality and limitations, as well as the own ability of operating it.

Although an increased metacognitive activity is displayed during interviews in terms of awareness and evaluation of digital tools, both the session records and interviews indicate a low degree of strategic planning and goal-setting for the modelling activities. The students seldom express why they are selecting a specific procedure or tool, what result they which to produce, or what question they aim to investigate. In cases where they do express such goals, either during interviews or sessions, there are only a few occurrences where they mention what they aim to do with the result or information once it have been retrieved. For example, there are occurrences, both during interviews and session records, where students of group B either state they have not thought about what they will do with the results or continue with a procedure despite recognising that they will not gain anything from executing it. The students of group A show a slightly higher awareness of an overall aim than group B. For example, both groups express that they want to identify when different paths have a shorter or equal travel time compared to others. However, while group B does not mention what conclusions can be drawn from detecting these points, the students of group A state that the aim with the investigation is to identify when drivers gain from selecting different paths. Nevertheless, group A only demonstrates such awareness during the third session when they are engaging in the solve phase, which involves using GeoGebra for the reason described above or implementing a script for finding an optimal traffic intensity distribution over the paths.

5.3. Monitoring and correction of process and results

Another category of metacognitive activities that can affect those related to awareness, evaluation and regulation of strategies is the awareness of state. As described in section 2.2, this can include awareness and evaluation of both state and results in relation to the posed problem statement, progress, agreement with the real-world situation, and noted constraints. From the records, it becomes apparent that both groups of students engage in routine monitoring to some degree, where the students of group A successfully engage in these activities to a larger degree than students of group B. Furthermore, the students of group A engage in controlled monitoring upon noticing issues with their strategy, while the students of group B are less reactive after identifying issues.

During the first session, both groups of students identify a feeling of insecurity when there is no progress, they are unsure of what activities to engage in, or when they have reached a solution unreasonably quickly. The students of group B respond to these feelings by rereading the problem description to verify they have not disregarded any information. As they cannot find any apparent issues, they instead ask the teacher for validation and support. The students of group A also become insecure multiple times during their discussions. In some of these situations, they review the presentation material. While the intention with using it is not verbalised, the students state they would like to view the description of each step of the modelling cycle while discussing, and they seem to rely on the material for guidance. In some cases, they also ask teachers for help to understand the dynamics involved in the problem, for example if all drivers are included in the measured traffic intensity. At one time, they decide to use Google Search to find an explanation of traffic intensity, but fail to find any results.

As mentioned in section 5.2, the students of group B ignore one of the students when he or she notes that even if they succeed in finding a more complex model for describing how traffic intensity varies and can find the time at which the intensity is a certain number, they will not be able to draw any conclusions from it. This is an example of how the students of group B engage in metacognitive vandalism as they are not adjusting their process according to identified issues. Thus, they allocate the cognitive and time related resources they have available to actions they know are meaningless. Another related situation group B experience is when they deviate from a strategy which could lead to them succeeding in creating a more complex model when they discover that GeoGebra can create graphs of normal distributions. Prior to this situation, the group had, at multiple times, abandoned the strategy of using the normal distribution after realising that it is not applicable. In short, the students are persistent in attempting to use a strategy which they have already evaluated as irrelevant. As they fail to provide GeoGebra with the input it needed for generating the distribution graph, they did however return to the original strategy. In comparison, the students of group A did never experience any similar situation.

Another difference between the two groups is that the students of group A repeatedly remind themselves of the questions posed in the problem. In some situations, they also return to the problem description to reread the questions. By using this strategy, the students can monitor whether their discussions and answers are relating to the posed questions. This leads them to consider how individual drivers reason when they are egoistic, and how other drivers are affected by such behaviour. In turn, these considerations result in the students concluding that all drivers will realise that they, in order to optimise the travel time of themselves in a long-term scenario, should be distributed to minimise the travel time of all drivers rather than a few drivers. In comparison, the students of group B create a description of how one driver may reason and how that driver's travel time will be affected by his or her own decision if the traffic intensity is constant. Thus, they fail to realise that they are to answer how all drivers reason and how long the travel time of all drivers will be.

During the third session, both groups express expectations on the results that will be produced with the digital tools they have selected. Group B is using GeoGebra to create a graph which fits a set of constraints by using a trial and error strategy to assign values to constants. Group A is instead using programming to find a distribution of traffic intensity that optimises the travel time for all drivers. The students of group A decide to use an expression that describes the summarised travel time of all roads, and specify a conjecture for when that expression will be the lowest. As they obtain a result which differs from their expected result, they correct their implementation. As such, they achieve metacognitive success. However, the final result does not adhere to the students' conclusion about how the traffic intensities will be balanced, and they correct the result manually. Thus, they do not correct the script to return a balanced result. They do succeed in obtaining an answer, but not correcting the logic in the script can also cause delays when they are to engage in the validation step.

One monitoring process which evaluates the used model and solution strategy is also triggered by the usage of GeoGebra. The students of group A use GeoGebra to plot all graphs describing the travel time of the different paths in order to investigate at what intensity the different paths will have a lower or equal travel time compared to the others, and draw conclusions from these graphs. However, the graphs also cause the students to realise their conclusions are based on the summarised traffic intensity being higher than the one described in problem presentation. Consequentially, the students perform an evaluation of their current solution strategy and model, and refines them to solve the issue.

However, the prediction generated by the students of group A to validate the script also exemplifies how the students fail to recognise when they have achieved a result. Similarly to how students may fail to understand that a result does not need to be numerical, which was described as a potential barrier in section 2.1, the students seem to believe the result needs to be obtained by using a tool, despite that they are able to find it analytically. For example, the students of group A could analytically minimise the expression by reasoning about when it would be minimal. In addition, it would also have been possible to first simplifying the expression by substituting variables, which was a suggestion that was ignored without really being considered by the students, or to use GeoGebra to view how the value of the expression changes when varying variables, as suggested in section 5.1.4. Instead, this caused the students to use cognitive and time related resources for implementing a script that was unnecessary. Furthermore, they also had used another reasoning, which was also based on the drivers reasoning rationally, during the second session and found a different result. While it was clear that the students came to a conclusion from that discussion, it is not apparent that they recognised the conclusion as a result. However, as the students' notes were not shared, it is not possible to know if they described it as a result or not. While the students succeeded in constructing a script, this failure in monitoring their state could have resulted in metacognitive mirage if their attempt at implementing a script would have failed and they would not have been able to recognise that they had found a solution. Nevertheless, failure to identify the state as that a result had been achieved negatively impacted their resource allocation.

5.4. Generation of conceptual awareness

The category of metacognition for which the tool usage had the greatest impact was generation of awareness of mathematical concepts the students would have been unlikely to consider if the tools had not been used. The use of Microsoft Paint and GeoGebra generated this type metacognitive activity for the students of group A. In addition, they also accessed content knowledge about graph theory, which one of the students had acquired outside of the school context. Since the awareness of graph theory was generated from previous use of YouTube rather than use of digital tools during the session, it is not covered in the analysis. However, the records did not indicate that the tool usage group B engaged in resulted in any additional conceptual awareness.

The students of group A use Microsoft Paint to create a graph that represents the problem situation. In the interviews, the students state the figure supported them in their discussions, which is also visible from the audio and video records from the session. These depict how the figure was vital for making important realisations and decisions. For example, the figure caused them to access their knowledge about geometrical shapes to calculate the travel time by assigning the expressions for travel time as the length of different sides. In turn, this caused the students to realise there will be an even distribution of vehicles on the two paths.

Moreover, the students' usage of GeoGebra causes them to realise their modelling of the travel time for different paths is insufficient. They had modelled the travel time with one function for each path, where all paths used the same variable for intensity. By inspecting the graphs plotted by GeoGebra, they realise they need to create an equation system consisting of several functions, where a separate variable denotes the intensity of each path, and an equality describing how they are related to the summarised intensity of all paths. In this situation, the students recognise that they have to include content knowledge about equation systems and multi-variable functions.

However, it is questionable whether the digital tools can be perceived as the main contributors to the generation of the awareness described above, or if it rather should be attributed to the created figures and graphs. From the records of the interviews and sessions, it is apparent that the students use the produced material while becoming aware of the mathematical concepts. However, the material is static and not used as dynamic representations, thus implying there were no additional benefit provided by them being digital. Moreover, the students could have produced the material manually without major effort, since the depicted functions were linear and the graph was simplistic. Thus, it is likely that the students would have received the same effect even if computers would not have been used.

5.5. Achievement of speed improvements and associated enhancements

As highlighted above, tool output with large effects on metacognition could have been produced without using the digital tools and within a relatively short time. The functions graphed by the students were linear, and the students mention that they could effortlessly have produced the same visualisation due to this characteristic. The graph created with Microsoft Paint has a simplistic form and is painted in black and white. Therefore, similar arguments can be used for motivating that the digital tool itself was not important. In short, the tool usage did not have a considerable effect on time allocation. Consequentially, the speed improvement is unlikely to explain the metacognitive effects.

Another occurrence of tool usage with questionable results in terms of speed improvement is the adaption and use of a script for optimising the distribution of traffic intensity over different paths. In the session, the students motivate their choice of scripting by noting there are many possible combinations and that it would have taken long time to manually calculate the results for all combinations, which is a valid observation. However, as described in section 5.3, they fail to realise they could have found the result analytically by using the same reasoning as they used when finding the expectation used to validate the retrieved result. Therefore, the use of the script is unnecessary, and its development used time resources the group could have used for a more meaningful activity. In fact, the script usage exemplifies how students' use of digital tools can have a negative impact on time allocation. One of the students in the group suggested substituting variables, a suggestion that was not considered by the other students and which could have resulted in an expression with only one variable. Even though one of the students manages to create an expectation of the combination that would have generated the lowest summarised travel time from the expression with three variables, the alternative expression with one variable would have been simpler to optimise. This situation can therefore constitute an example of the importance of metacognitive activity, discussed in section 3.2, to achieve the potential benefits, such as speed improvement, described in sections 3.1. In other words, being able to operate the tool well is not the only prerequisite for achieving sought improvements.

The only occurrence where a digital tool is likely to have benefited the students in lowering cognitive load and increasing execution speed is when the students of group B try to fit a sinus curve to match a set of constraints. In this activity, students first use GeoGebra to fit the curve. When later manually solving a set of equations to obtain the same values for involved constants, they use GeoGebra to plot the resulting function to validate it. Firstly, finding the values required about seven minutes while using the digital tool, while it took ten minutes with the analytical approach. Secondly, the time and cognitive resources required for the analytical approach was lowered by having engaged in the trial-and-error approach first, as the students could effortlessly test conjectures about where alterations were to be made to the expression to gain different visual effects, such as increasing the amplitude or horizontally offset the graph, and watch the results until being able to create the wanted modification. Thus, the students knew where to modify the expression to create specific modifications to the graph when starting the analytical approach. Thirdly, being able to use GeoGebra to validate the function saved time during the analytical approach, as the alternative of calculating values of the expression for specific argument values would have taken time if done manually.

However, the speed improvement generated by the students' use of GeoGebra did not result in them allocating the freed cognitive resources efficiently and had no impact on what results they were able to achieve within the time assigned for the modelling activities. Since the students decided to use the freed time resources to recalculate the same results manually in order to be able to motivate why those values should be used, they did not use the cognitive resources for discussing another topic and did not progress further in their modelling process. While this behaviour can be regarded as inefficient use of resources, it can also be considered to demonstrate that the students are not dependent on the tools and that they have the competences necessary to execute the procedure by themselves.

5.6. Impact on collaboration and participation

One effect on the students' processes that is apparent from session and interview records, but which was not highlighted in the literature study, is the support of students' collaboration. The main effects are found during both groups' usage of Google Drive and from group A's usage of Microsoft Paint. However, another finding is that the students do not consider to achieve these positive effects when using other tools although technology can support it.

One tool that was selected by students due to its collaborative support is Google Drive. During the interviews, the students explain that Google Drive enables all students to see the updates on their own screen, which makes the discussion and what is written easier to follow. Furthermore, it allows all students to write simultaneously, which enables them to add information as soon as they get an idea. Thus, there is no need to transfer the text to different computers, working sequentially, or relying on the presence of a specific computer. The session records support that the students are exploiting these benefits, as they are recorded discussing what is written, correcting each other's writing and working simultaneously. Furthermore, the records show that the students intuitively create a Google Drive document, and the students also describe that they do not consider not using Google Drive when interviewed as they have become used to using it regardless of what subject they are studying.

Another tool which improved the understanding and clarity of discussions was Microsoft Paint. During the second modelling session, the students of group A used Microsoft Paint to create a graph that simplified the figure provided in the problem description. While being interviewed, the students describe how the figure has supported them in their discussions, especially when clarifying which road or path they are referring to and for identifying which paths they need to continue investigating. They describe having used it when investigating how a road is affected by changes in traffic intensity, and to follow different paths to reason about how they and the intensity split and later are combined. The students highlight how the partial problems have become considerably simpler to solve when having a simplified visualisation, and mention having based most of their ideas on the figure. The audio and video records from the sessions are consistent with these results, and depict how the figure was vital for making important realisations and decisions. However, these benefits are more associated with the created figure than the use of the digital tool, and, as discussed above, the figure could have been created and used equally well without using Microsoft Paint.

Although using the functionalities provided by Google Drive and highlighting the importance of being able to simultaneously display the document on all students' screens and not requiring to send the document between students, the students do not attempt to use similar functionality when watching figures created in GeoGebra or Microsoft Paint. For example, one interviewed student describes tilting his or her screen while using GeoGebra to enable other students to watch the results. Similarly, sound records from group A's modelling process reveal them moving their chairs to more easily view the created Paint figure.

6. Strategies for improving the effects of digital tools

The main effects portrayed in section 5 that can be attributed to the usage of digital tools include enhancement of students' collaboration and both positive and negative effect on their speed. One of the groups of students also gained a conceptual awareness from using representations created with tools, but this impact seems more related to the representation itself than the tool usage. In short, the students' sparse engagement in metacognitive activities in combination with their dependency to tools they have been trained in using can be used to explain the observed effects. This section elaborates this as well as suggests how the learning environment should be adapted in regards to these identified factors.

6.1. Fostering a habit of engaging in metacognitive activity

Students' lack of metacognitive activity can be used to explain why many effects occur or do not occur. While digital tools could enhance the pace with which students were executing a task, students did not exploit the freed resources to progress further. This behaviour indicates low metacognitive awareness of the state and what remains to be done. Moreover, the students did not transfer their needs from one context into another, despite it resulting in them experiencing the issues they had planned to avoid in one situation when engaging in activities in another. In turn, this is likely associated with students' inability to realise how the situations are similar, lack of habit in comparing different resources before execution, and narrow knowledge of tool features.

As emphasised in sections 2.2 and 3.2, engagement in metacognitive activity is an important factor for successfully transitioning in the mathematical modelling cycle. Thus, although this section only aims to suggest strategies for improving the technological effects found in the case study, their application likely affects the effectiveness in students' allocation of freed cognitive resources in general. Regardless of the purpose of the education targeting metacognitive competency, students need to repeatedly and actively engage in metacognitive activities in order for the associated competencies and habit in using them shall be developed (Maaß 2006; Schoenfeld 1987). Schoenfeld (1987) therefore proposed two suggestions for how to incorporate such reflection in classroom education.

The first strategy involves the class cooperating to solve a problem, and revolves around the teacher posing metacognitive questions. While the students focus on strategic decisions, calculation is distributed to the teacher to direct students' focus to metacognitive considerations (Schoenfeld 1987). The teacher presents a problem situation, and continues by asking all students if they are certain to have understood the problem before any solution strategy is employed (Schoenfeld 1987). Once all students claim they have a clear understanding, they are allowed to propose and discuss strategies (Schoenfeld 1987). While the teacher moderates the discussion, the responsibility of the students is emphasised by the teacher asking students for confirmation on what strategy they should employ (Schoenfeld 1987). An important factor highlighted by Schoenfeld (1987) is that the students select the strategy, and that the teacher accepts their choice regardless of its appropriateness. The teacher also regularly, independent of how the solution process is proceeding, notifies students on that a few minutes have passed, questions if they are progressing successfully, and reminds students that they should continue in case of success or otherwise adjust their strategy (Schoenfeld 1987). Depending on the students' reply, the class either continues or revises the strategy (Schoenfeld 1987). In the latter case, the teacher should prompt students on if they should reuse any part of the current strategy, and if there are any other suggestions they should revisit (Schoenfeld 1987). Only when the class have completed the problem-solving process, the teacher comments on the efficiency of the strategy, how it could have been improved, and alternative approaches (Schoenfeld 1987).

The second strategy builds upon a concept similar to the first one, but involves students working independently in small groups. In this suggestion, the teacher coaches the students in using the instructed techniques efficiently by regularly asking them what they are currently doing, why they are doing it, and how they plan on using the results once they have been received (Schoenfeld 1987). The aim with this strategy is for students to internalise these questions, and thereby foster students to reflect sufficiently for their problem-solving process to become structured and efficient (Schoenfeld 1987). As such, the teacher should initially interrupt the students frequently, and gradually reduce the support (Schoenfeld 1987). Moreover, Schoenfeld (1987) mentions that the support the teacher provides should include answering questions and offering advice, in addition to initiating metacognitive discussions.

While the extent of this consulting is not specified by Schoenfeld (1987), different implementations were observed and analysed by Lingefjärd (2010). The two teachers who participated in Lingefjärd's (2010) study agreed not to direct students, and instead support them in developing their own strategy. The results revealed that not offering content-related support or hints to clear uncertainties and blockages can cause students to fail to complete their process (Lingefjärd 2010). Thus, teachers need to provide support which does not only consist of diagnostic questions or encouraging statements, although Lingefjärd (2010) also found that type of support to be important as well.

6.2. Highlighting a variance of tools and enforcing practice in using them

As suggested above, not all observed effects can be attributed exclusively to the use of metacognition. The students' narrow range of tool competency obtained from classroom experience as well as tendency not to explore tools or features by themselves can explain why they, for example, did not use an alternative tool to Microsoft Paint to enhance their collaboration or concluded GeoGebra could not be used for studying an equation system with multiple variables. Moreover, it is known that the students' competency in using a tool is important to be able to exploit its functionalities, and that gaining such competency requires the student to have used it many times during a long time period (Kim 2007). Thus, the learning environment must be designed to repeatedly expose the students to tools. Moreover, the design must also consider the limited cognitive capacity of learners, which was indicated in section 3, and therefore not require students to focus on too many aspects simultaneously.

Since students displayed little interest to familiarise with new tools and were likely to select tools that had been included in teachers' instructions even in cases where they had experience of other tools, it becomes increasingly important to include a larger number of tools and tool features in the instruction. By demonstrating how various tools function, students gain an awareness of the existence of several alternatives and build familiarity with them without having to explore themselves. Thus, students can evaluate an increased amount of options when engaging in modelling activities, and consider functionality, benefits, and potential issues experienced in the example. Furthermore, instruction of several applications of tools to reach a solution or alternative tools to perform the procedure can cause students to retain motivation while solving problems. Although not discussing technology, Leikin (2007) in general emphasises the importance of students being aware that multiple solution strategies exist in order not to give up when they experience difficulties in solving a problem.

However, Renkl (2003) notes that skills such as programming and problem-solving need to be practiced by the student in order to be developed, and that studying of examples is not sufficient when accuracy and rapidity are sought. By using similar reasoning, students need to experience tools themselves in order to gain proficiency in using them. As highlighted in section 3.1, the students should not be left unassisted in this exploration, as they then risk facing a cognitive overload. Instead, the teacher can direct the students by assigning them specific tools for practicing, acting as a guide in using the tool, acting as an interface towards the tool, or suggesting tools based on the students' needs while they are engaging in modelling activities.

The first suggestion involves students practicing to use a tool to gain knowledge of what features it provides as well as how to operate it. As argued in section 4.7, students' cognitive resources should be allocated for understanding the tool when using this option. As a consequence, students should not be required to engage in metacognitive activities while focusing on learning the tool. One option for implementing this strategy involves using worked examples, as argued in section 4.7, since they require lower cognitive capacity than used when engaging in problem-solving. These worked examples can either be produced by an external source, or be based on previous problems the students have solved. Alternatively, the students can be assigned problems that require limited tool usage, for example small linear optimisation problems, and be given a task of automating their solution during the validation phase. As explained in section 3.1, this phase typically involves employing the same solution strategy several times with small variances in assumed values, and will therefore be time-consuming unless tools are used to automate the operations.

Regardless of how worked examples are created and introduced, they can be used to teach students about the functionality, benefits, disadvantages, syntax and interface of the involved tools. If the worked examples include instructions on which tool commands were used, the student does not need to allocate cognitive resources for translating the used model into the format required by the tool, nor focus on exploring what operations the tool provides. Instead, the student can focus on familiarising with the tool's interface and reasoning about how the exemplified model was translated. If the example contains a solution strategy, but not any instruction on how the strategy can be executed with a tool, the students need to focus on all steps presented in section 3. These steps include finding suitable tool operations, making adaptions necessary to use the tool, operating the tool, and interpreting the result into the context of the original model.

Another strategy involves the teacher guiding students in their tool usage during modelling sessions by acting as tool expert. In sessions where students are engaging in problem-solving activities, the literature study in section 3 revealed that they have limited resources to allocate for exploring the tool without it negatively impacting other cognitive functions which are crucial for designing and revising the strategy. In these cases, students can practice their metacognitive competency as well as gaining awareness in regard to the features, benefits and disadvantages associated with the tool. One role the teacher can employ is to guide students in their tool usage. The interaction can then be designed to allow students to either ask questions about different tools to decide which strategy to use, or to describe their needs and receive suggestions. Moreover, the risk of cognitive overload while familiarising with the interface can be decreased by the teacher directing students toward specific operations and documentation, or assisting students in troubleshooting activities when they do not manage to produce the expected results. An alternative also involves the teacher acting as an interface to the tool, thus removing the cognitive load associated with adapting the input and operating the tool. Instead, students select the tool as described above, but the teacher operates it. The students thereby become acquaint with the functionality of the tool without investing resources in understanding its interface.

Finally, another suggestion for the interactivity with the teacher involves him or her analysing the students' processes and suggesting how tools can be used to improve it. This strategy can be used when students fail to transfer experiences from one context to another. The usage of Microsoft Paint that was observed during the case study can be used as an example of this strategy. While the students efficiently used Google Drive documents for removing some issues that hindered their collaboration, they did not realise they experienced corresponding issues while using Microsoft Paint, as further described in section 5.6. Instead of using Microsoft Paint, the students could have utilised the drawing functionality incorporated into Google Drive to share the figure on all screens while modifying it. Furthermore, the same tool can replace the need of pointing with fingers, as it enables students to mark individual graphical elements and share their marker with each other.

Regardless of which strategy is employed, it is important that students are aware of the aim with the activity and that they are given appropriate time to explore. If students have become adjusted to engage in metacognitive activities for evaluating their progress and note that their tool usage is not efficient, they may adjust their strategy and refrain from tool usage. As a consequence, they would lose their opportunity to familiarise with the tool.

6.3. Incorporating both competence areas and adjusting to students' development

As described by Renkl (2003) and implied by the literature study in section 3, students have a limited cognitive capacity available. If students use their cognitive resources to focus on one activity or aspect which allocate large amounts of cognitive capacity, their learning and engagement of other aspects risk to be impeded (Renkl 2003). The educational design needs to reflect this limitation, and it can therefore not be recommended to engage in introduction to both metacognitive reflection and a digital tool in the same activity. However, the same limitation does not require the two areas to be processed sequentially.

Instead, Wedelin (2015a; 2015b) suggests creating sets of small problems where each set covers three dimensions: realism, involved conceptual and procedural knowledge, and possible modelling approaches and solution processes. While skill in operating tools can be regarded as part of the second dimension, metacognitive activity itself is more related to the third. Each set of problems should according to Wedelin (2015a) display variance in each dimension, as students are required to experience differences and similarities in order to discern patterns. As each problem also can be designed to be challenging by varying the difficulty in understanding and solving it in relation to students' prior competencies (Wedelin 2015a), it is not necessary that all problems are challenging in all dimensions simultaneously. Instead, the combination of problems used in one set can be designed to develop students in all three dimensions.

Moreover, Wedelin (2015a) writes that procedural skills, such as competency in operating a digital tool, does not necessarily need to be developed prior to engaging in mathematical modelling activities. This suggestion can seem contradictory to the initial recommendation of separating the development of the two competence areas. In fact, Wedelin (2015a) discourages usage of problems that require acquisition of new theory and methods if the aim with the session is for students to develop a more overreaching modelling competency. However, this implies that this suggestion is contradictory only when students have little familiarity with the tool and when metacognitive competency is targeted. Furthermore, as students are developing their knowledge associated with the digital tool, its usage will allocate less cognitive resources and an increased amount of resources will remain available for acquiring other knowledge during the activity. Eventually, the allocation of resources will be lower than what is freed by procedural execution being distributed to the tool, and it becomes useful.

Therefore, it is important to note that the design of problems and problem sets affects what knowledge acquisition is possible and what effects can occur. This was also observed in this study. As the used problem was originally designed to emphasise how simplistic models and basic mathematical procedures can be used for realistic problem situations (Wedelin 2015b), it is not surprising students did not find it necessary to use digital tools. Moreover, the design of the problem provided challenges in understanding how flow affects what simplifications can be made, making assumptions, and precisely formulating conditions and assumptions mathematically (Wedelin 2015b). Thus, the challenge is related to the first phases in the mathematical modelling cycle rather than the execution of the procedure for obtaining a solution, and it is therefore likely students need assistance in the actions related to the former phases. This was observed during the study, as the main findings included the collaborative support that was vital for reaching a common understanding of the problem situation.

In addition to planning and problem design, practicing the strategies described above requires a large amount of expertise. As also described in section 4.7, Lingefjärd (2010) concluded teachers need to be able to quickly understand students' ideas and strategies as well as possessing ability to identify what issues they are facing and directly clarify misconceptions, extend their knowledge, or present hints as needed for the students to progress. In addition, teachers must be able to balance students' need of support with the limitation of guidance needed in order for students to develop autonomy and metacognitive skill. In other words, teachers must become skilled in providing minimal guidance to students. Lingefjärd (2010) suggests arranging classroom visits to improve these skills. For example, the observing teacher could identify cases when the practicing teacher does not take appropriate time to listen to students' explanations or refrains from answering a question, which are two factors Lingefjärd (2010) found to be significant for clearing blockages. Additionally, correct and rapid understanding of students' ideas requires extensive mathematical knowledge, as exemplified in Lingefjärd's (2010) study. As reasoned in section 4.7, lack of experience with students' problem-solving processes can likely be amended by selecting problems which have been practiced by other teachers or researchers previously, and for which there thus exists lists covering several interpretation possibilities, solution strategies, and examples of blockages.

Moreover, teachers must also be able to handle students' development. For example, Schoenfeld (1987) notes that students initially are uncomfortable by the teacher's involvement, but eventually become accustomed to engaging in the metacognitive discussions. Teachers therefore need social competence necessary for handling the frustration which can be caused by the initial feeling of discomfort. While Schoenfeld (1987) suggests mediating it by prior to the sessions declaring this specific questioning will occur, he provides no suggestion on how to handle such feelings when they appear. Instead, Schoenfeld (1987) notes that students eventually begin formulating answers before the teacher arrives to feel prepared and less uncomfortable. As this habit develops, the teacher's interruption to activate metacognitive thinking become superfluous, and the teacher should therefore withdraw the support (Schoenfeld 1987). Similarly, in case of the strategies suggested for extending students' tool expertise, the teacher needs to identify when students possess an understanding of a tool's capabilities or ability to operate it, and accordingly shift between the strategies or introduce other functionalities.

7. Conclusions

Digital tools are known to be beneficial when used in mathematical modelling. As explored in section 3.1, the support they provide in tasks such as calculating, graphing functions and simulating can in turn generate a better understanding, generate awareness of relevant mathematical concepts, increase speed, minimise interruptions in the overall focus, increase metacognitive activity, provide access to additional model types and procedures, and provide access to realistic problems with more complex data.

However, the introduction of digital tools into mathematics education does not automatically generate these positive effects. The research presented in section 3.2 describes how inefficiency in the operation of tools can hinder students' modelling processes. In fact, one conclusion from the research is that not knowing how to use a digital tool for a task will cause a blockage regardless of if the student knows it can be used for the task or not. One example further expanded in section 3.2 involves that students' cognitive resources will be fully allocated while they are trying to understand either the interface or syntax rules associated with the tool. In addition, the increased amount of concepts and procedures offered by the tools increase the demand on students' metacognitive competencies, as also explored further in section 3.2. As presented in sections 2.2 and 3.2, students need to be able to discern suitable concepts and procedures from inapplicable ones, as well as knowing how to verify the ones that are selected. However, when appropriate tool features are selected and the tools are efficiently operated, the research seems to be positive in regards to that the tools will not cause any barriers and that increased metacognitive activation will be generated as cognition is off-loaded.

Despite that the students who participated in the case study operated the selected tools well, no significant effects on metacognitive activity were found. One effect that did occur involved students' usage of tools to create graphical representations, which in turn resulted in them gaining awareness of constraints and mathematical concepts they had not considered otherwise. However, it was concluded that these effects could be attributed to the representations themselves rather than the tools, especially as usage of the tools was not necessary to create the representations. The lack of effects can partly be explained by the students mostly using the digital tools for tasks they could have performed manually without any additional effort. However, the same observation was also made in cases where students' allocation of resources was lowered by the tool usage. For instance, some students used GeoGebra for fitting a curve, which enabled them to test conjectures within a shorter time, but did not use the freed time for any activity that would have led to further progress. Thus, it was concluded that students need to actively allocate freed cognitive and time-based resources to metacognitive activity in order for positive effects on their metacognition to occur.

Students were also found to have a low engagement in metacognitive activity in general, although being able to reflect on tools' advantages, disadvantages, situational usefulness, and their strategy for selecting between tools when interviewed. They did not plan their strategy to a large degree regardless of if tools were used or not. In some cases, they could identify issues in their process without engaging in further metacognitive activities to adjust it. In other cases, they failed to gain awareness of their state, for example when not realising they had arrived at a solution, and thus engaged in unnecessary activities. Moreover, the students failed to translate their needs from one context to another, and did not realise they experienced the same difficulties in one context as they had successfully avoided in another. These observations in combination with that students did not seem to allocate available cognitive resources for metacognitive activities, led to the conclusion that students lacked a habit of using metacognition.

Consequentially, two strategies for training students' metacognitive competency and develop a habit of employing it were suggested in section 6.1. The first strategy involves the class discussing and selecting solution strategies, while the teacher executes the selected strategies and regularly prompts students to evaluate the current state. The second strategy instead involves the students engaging in modelling activities in small groups while the teacher regularly initiates metacognitive reflection by asking the students about their state and planning. If blockages appear, the teacher also offers the support necessary for students to independently continue with their process. As students eventually start internalising these questions, the teacher gradually decreases his or her involvement.

However, increasing the students' usage of metacognition was not found to be the only approach necessary to improve the observed effects. An issue that was observed was students' tendency to favour tools the teacher had included in his or her instruction, and reluctance to explore new tools and features. Even if their metacognitive activity would increase, students would only consider the, relatively small, range of tool features they had been instructed in using. For example, one prominent effect of the usage of one digital tool was its support of collaboration. However, when engaging in another collaborative activity, the same students faced the same issues as they had avoided in the first situation. Therefore, students need to become aware of a larger variance of tools and tool features.

Consequently, a set of strategies for repeatedly engaging students in activities where they gain awareness of tool features and train their proficiency in using them was suggested in section 6.2. The first suggestion involves the teacher using several tools and tool features while presenting examples. The remaining strategies instead involve students participating in modelling activities individually or in small groups. In these suggestions, students can follow an instruction involving tool usage step by step, implement an algorithm for an exemplified solution strategy, or use the teacher as a tool consultant or tool interface. Here, the teacher can observe the students tool usage and needs, and in turn suggest or enforce usage of specific tools. In the example specified above, the teacher could have intervened and suggested a tool which could functionally replace the tool they had selected and which contains collaborative functionalities.

Finally, the instruction needs to be designed to develop both areas of competency while not overloading students' cognitive capacity. Thus, section 6.3 suggested not to initially aiming at developing both competencies simultaneously. Instead, designing sets of problems where each problem focus on one aspect while the complete set covers all was suggested. Moreover, it was concluded teachers need to be aware of how the effects that can be generated by the tool usage are dependent on how the problem is designed. For example, problems aimed to train students in using a specific digital tool should be designed to exploit the capabilities of the tool. Furthermore, students claimed to select procedures and tools which saves time, and problems involving time-consuming, tedious or challenging procedures are therefore more likely to cause students to delegate execution of those procedures to digital tools.

8. References

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