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Path planning and decision making in a highway exit situation

Master's thesis in Systems, control and mechatronics

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CHALMERS UNIVERSITY OF TECHNOLOGY
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Abstract

Vehicle automation is an area where a lot of investments are currently made. Several driving assistance systems are already in place in many modern vehicles but more and more research is done to eventually achieve fully autonomous vehicles. Part of the automation is to calculate the optimal behaviour of the vehicle in every possible scenario. This thesis focuses on the scenario where a lane change must be made on a highway in order to take an exit. This requires decision making on a target lane and calculations of smooth paths. The proposed controller structure is divided into three parts: analysing the situation, planning the path for each possible action and comparing the calculated paths to choose the best one. This is done in a model predictive control framework where at each time two actions are evaluated: to stay in the current lane or to move into the adjacent lane. Simulations were done in Matlab and showed that the proposed controller generates smooth and safe paths when possible and is able to make decisions based on the current traffic situation. The implemented controller also shows good prospects to be fast enough for a real time implementation in an actual vehicle. Due to the fact that the method includes many tuning parameters it can be used for many purposes: for example to maximise energy efficiency in the control of a vehicle or to mimic the behaviour of different human drivers for prediction and simulation purposes.

Keywords: autonomous driving, path planning, decision making, optimisation, model predictive control, lane change, highway exit

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1
Introduction

During the past few years there has been a surge towards increased automated driving. Many driver assistance systems are already implemented in newer vehicles, such as adaptive cruise control [3, 4], collision avoidance systems [4, 5] and lane keeping assistance [6, 7], but much research is also done to eventually achieve fully autonomous vehicles [8–10].

A large number of traffic accidents or incidents today are due to human factors such as fatigue, inattention, risk taking and aggressive driving [11]. Increasing driving automation could thus lead to a great improvement in road safety [12]. Also energy efficiency could be much improved by introducing autonomous driving [13]. For example vehicle to vehicle or vehicle to infrastructure communication can help plan the speed to minimise braking. Autonomous vehicles also have the possibility to perform more complex calculations on energy optimisation than a human driver could.

To achieve full autonomy is very complex as it requires the driving to be energy efficient and guarantee safety in every seen and unforeseen situation. This puts high demands on both software and hardware. Sensors and signal processing must be in place to gather information about the surroundings and the best way to act on that information must then be determined for every possible scenario. Many aspects must be taken into consideration when planning the optimal path; safety, traffic regulations, energy efficiency and driver comfort are some of those aspects. A number of scenarios have already been researched, for example calculations of the optimal path in an overtaking situation [14,15], lane change manoeuvres [16–18] and intersections [19].

Highway driving is one area where automated driving is upcoming. A highway is a very controlled environment where the traffic is rather predictable since there are e.g. no pedestrians, traffic lights or intersections and since all traffic travels in the same direction. A highway is in general also relatively straight and well maintained with clear lane markings. It is also an environment where much can be gained by introducing automated driving: vehicles travel at high speeds making accidents severe, they also often travel long distances increasing the risk of driver inattention and fatigue. Therefore, a lot of research has been made specifically for highway scenarios.

Many methods have been proposed for automated driving. For example graph based search methods are common [20, 21]. The advantage of the graph search methods is that it is possible to evaluate a lot of different actions, the downside is that they require a significant amount of memory. Another common approach for path planning is to use a Model Predictive Control (MPC) framework [14, 16–19, 22–24]. In MPC a motion model is used to define a constrained optimal control problem that is solved over a receding horizon.

In an MPC approach, convex optimisation is often used because it is easier to solve than non-convex problems and guarantees that the optimal solution is found, assuming that the problem is feasible. One main difficulty is thus to define the constraints such that the problem becomes convex. In for example [14, 17, 22, 25] a few different ways of defining collision avoidance constraints are presented. In [17] they are defined by constraints on the time to collision, in [22] instead the distance to the surrounding vehicles are constrained using ramp barriers, in [14] a box constraint is used to constrain the distance to the leading vehicle and in [25] it is shown how safety critical zones can be defined with miscellaneous shapes.

1.1 Purpose

The purpose of this master's thesis is to develop a method for decision making and path planning in a highway exit situation. The aim is to develop a decision making and path planning algorithm where different actions are weighted to decide the optimal planned longitudinal and lateral motion. The decision should be based on a need to, if possible, perform the lane change before the exit, on reference tracking, energy efficiency and comfort. The resulting algorithm is meant to be used to control an ego vehicle and/or to be applied to all surrounding vehicles in order to do predictions and simulations.

The case considered is where a lane change must be made on a highway in order to take an exit, the exit could be either to the left or to the right. An exit to the left could be in a situation where the road splits in two and it is desired to take the road to the left. The problem can thus be formulated as switching to a reference lane before a certain point (the exit). The manoeuvre must be made to guarantee safety and that traffic rules are followed, the vehicle dynamics must also be taken into consideration.

1.2 Delimitations

The scope is limited to a straight, flat highway with two lanes of equal width. That the scope is limited to a highway means that following assumptions can be made: traffic flows in the same direction on both lanes, no pedestrians or bicycles are present and there are no intersections or roundabouts ahead. It is assumed that information about the position, velocity and acceleration of all surrounding vehicles as well as the road geometry are known by sensor data already filtered and fused.

The entire control problem can be divided into a route planner, a high level path planner and a low level controller as in [23]. The route planner decides on which route to take and the optimal speed over the entire trip, the high level path planner determines the short term optimal path in the given situation while the low level controller makes sure that the vehicle follows that path. Here it is assumed that the route planner and low level controller are available, this thesis instead focuses on the high level path planner.

In summary, the following assumptions are made

A1. Highway with two lanes of equal width.

A2. The road is straight and flat.

A3. The exact position, velocity and acceleration of the ego vehicle as well as the surrounding vehicles are known by perfect sensor data.

A4. A low level controller is available that can make sure that the vehicle follows the chosen path.

A5. A route planner is available that provides a reference speed and direction on which exit to take.

A6. None of the surrounding vehicles change lane, they can however change their position inside the lane.

1.3 Outline

The report is structured as follows. In Chapter 2 some terminology and the models used are presented, the problem formulation is stated as well. In Chapter 3 the overall solution design is described. The two major parts of the solution, path planning and decision making, are then presented in Chapters 4 and 5 respectively. Simulation results are presented in Chapter 6. A discussion about the performance and possible future work is provided in Chapter 7. Finally, a conclusion is presented in Chapter 8.

2

Preliminaries

This chapter first presents some definitions that are used in the report. Then the vehicle and environment model as well as the problem formulation is presented.

2.1 Terminology

Some definitions and notations will be used throughout the report, these are explained here.

2.1.1 Vehicle notations

Definition 1. The ego vehicle, short E, is the vehicle to be controlled. The index E always means a property or state connected to the ego vehicle.

Even though many surrounding vehicles are used in simulations and will be present in reality, only three are used to place constraints in the path planning. Hence, only these three will be named, they are denoted S_j with $j = 1, 2, 3$. The numbering of the vehicles is pictured in Figure 2.1. States and properties for surrounding vehicles are indexed with their number only, e.g. x_1 denotes the longitudinal position of vehicle S_1. If one of S_j is missing a virtual vehicle placed far away driving at the reference speed is used instead.

2.1.2 Coordinate system

A local coordinate system based on E's initial position at each iteration is used, see Figure 2.1. The coordinate system has the x-axis in the longitudinal direction, i.e. along the road. The y-axis is in the lateral direction, i.e. starting from the edge of the rightmost lane and going across the road. The origin is placed at the rightmost edge of the road at the longitudinal position of E. The reason the origin is placed at the edge of the road instead of in E is that this way the constraints on lateral position can be held constant with only the initial position of E changing between time instances. It would however not add much complexity to instead place the origin in the vehicle or to use a global coordinate system.

Definition 2. A vehicle's position in the coordinate system is measured to the middle of the vehicle's front, shown with a white cross in Figure 2.1.

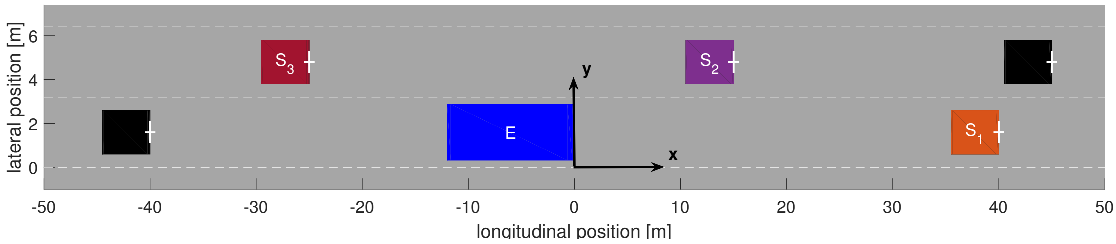


Figure 2.1: Vehicle notation and coordinate system. In blue, marked with E, is the ego vehicle, i.e. the vehicle to be controlled. The other three coloured vehicles are noted by S_j, $j = 1, 2, 3$ for surrounding. The leading vehicle in the same lane as E is S_1, the leading vehicle, with a positive x position, in the adjacent lane is S_2 and the vehicle behind E, negative x position, in the adjacent lane is S_3. The black vehicles are ones that will not be taken into account when planning the path. A vehicle's position is measured to the centre of the vehicle's front as is marked with a white cross.

2.1.3 Actions

As previously mentioned, the problem can be summarised as positioning the ego vehicle in a reference lane before reaching a certain point on the road (the exit). In this, two parts can be defined, decision making and path planning.

Definition 3. Decision making is to make the binary choice to position E either in the left lane or in the right lane.

Definition 4. Path planning is to, given a target lane, calculate the sequence of control signals that will generate the optimal path in terms of position, velocity and acceleration.

For the given scenario, the possible paths can be divided into two types, lane change and trailing. This results in two path planning problems.

Definition 5. A lane change is to switch from the lane E is currently positioned in into an adjacent lane.

Definition 6. Trailing is to maintain the current lane and trail the leading vehicle in that lane. If there is no leading vehicle a virtual leading vehicle is introduced.

2.2 Vehicle model

A point mass model is used to describe the vehicle dynamics of the ego vehicle. This is a very simplified model but it is often sufficient and used widely because of its simplicity and the lower computational complexity it gives compared to more advanced models. The point mass model is commonly used in literature, see e.g. [14, 17, 22, 23]. In [24] the point mass model was used for path planning for a car on a highway and it was shown that the more complex four wheel model was able to adequately follow the path generated with the point mass model, proving that the simple point mass model can in fact have real uses. It is in general good for simple vehicles, such as cars or rigid trucks. However, for longer vehicle combinations, more advanced models may be necessary, for example the single track model [26] or four wheel model.

In the point mass motion model, it is assumed that the vehicle behaves as a point mass, i.e. it has no orientation. States are the longitudinal and lateral position and speed, $\mathbf{x}_{\mathrm{E}}(t) = [x_{\mathrm{E}}(t), y_{\mathrm{E}}(t), v_{\mathrm{E},x}(t), v_{\mathrm{E},y}(t)]^T$ and the control inputs are the longitudinal and lateral acceleration $\mathbf{u}_{\mathrm{E}}(t) = [a_{\mathrm{E},x}(t), a_{\mathrm{E},y}(t)]^T$. The vehicle dynamics are thus given by

$$\begin{gathered}\dot{\mathbf{x}}_{\mathrm{E}}(t) = A\mathbf{x}_{\mathrm{E}}(t) + B\mathbf{u}_{\mathrm{E}}(t) \\ A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.\end{gathered} \tag{2.1}$$

The vehicle properties put the following constraints on the states and inputs

$$v_{\mathrm{E},x}(t) \in [0, v_{\mathrm{E},x}^{\max}] \tag{2.2a}$$
$$v_{\mathrm{E},y}(t) \in [v_{\mathrm{E},y}^{\min}, v_{\mathrm{E},y}^{\max}] \tag{2.2b}$$
$$a_{\mathrm{E},x}(t) \in [a_{\mathrm{E},x}^{\min}, a_{\mathrm{E},x}^{\max}] \tag{2.2c}$$
$$a_{\mathrm{E},y}(t) \in [a_{\mathrm{E},y}^{\min}, a_{\mathrm{E},y}^{\max}] \tag{2.2d}$$

The lateral speed is also constrained by the longitudinal speed

$$v_{\mathrm{E},y}(t) \in [-s, s]v_{\mathrm{E},x}(t) \tag{2.3}$$

where s comes from the maximum slip angle ensuring a path that E is able to follow, [14, 22].

No air or rolling resistance is used in the optimisation. Including that would lead to additional, and more complicated, constraints on the velocity and acceleration.

2.3 Environment model

As assumptions A1. and A2. state, the environment is a straight and flat highway with two lanes; a road section is shown in Figure 2.1. The only objects on the road are vehicles driving in the same direction as E.

The road model contains lane width, distance to the exit, the reference lane from which the exit can be taken and the speed limit. The speed limit here is defined as the speed limit on the road in question for the ego vehicle specifically, i.e. it can depend on whether E is for example a car or a truck. The lane width in itself gives the total width of the road since it is assumed to have two lanes of equal width.

The road model places the following constraint on the lateral position, that E must always stay on the road

$$y_{\mathrm{E}}(t) \in [w_{\mathrm{E}}/2, 2w_l - w_{\mathrm{E}}/2] \tag{2.4}$$

where w_{E} is the width of the ego vehicle and w_l is the lane width. The road's speed limit puts an additional constraint on the longitudinal speed. The constraint on longitudinal speed can thus be defined as

$$v_{\mathrm{E},x}(t) \in [0, \min(v_{\mathrm{E},x}^{\max}, v_{\mathrm{speedlimit}})]. \tag{2.5}$$

2.4 Problem formulation

The problem is to find the control strategy that achieves the optimal lateral and longitudinal path in the scenario where a lane change is required to enable E to take a highway exit. The control problem is formulated as an MPC where the problem is solved over a receding horizon. The optimisation is done with the objective to position E in the reference lane and keep the reference speed while avoiding collision with surrounding vehicles, respecting the traffic rules and respecting E's physical limitations. This can be formulated into the following optimisation problem to be solved at every time instance

$$\begin{aligned} \min_{\text{path}} \quad & \text{Objective function} \\ \text{s.t.} \quad & \text{Vehicle dynamics, (2.1)} \\ & \text{Vehicle constraints, (2.2), (2.3)} \\ & \text{Collision avoidance constraints} \\ & \text{Traffic rules} \\ & \text{Initial state} \end{aligned} \tag{2.6}$$

where vehicle constraints are the physical constraints arising from the vehicle's capabilities and traffic rules include speed limit and staying on the road.

The objective function in this case will be taking comfort, tracking of references, energy efficiency and the destination into consideration. It can be defined as

$$J(.) = J_{\mathrm{exit}}(.) + \int_0^{t_f} V_{\mathrm{actuator}}(.) + V_{\mathrm{tracking}}(.)\,\mathrm{d}t \tag{2.7}$$

where $(.)$ denotes a function of the decision variables. Here J_{exit} is the cost that rises from the need to take the exit, V_{actuator} includes comfort and energy efficiency and V_{tracking} is the cost that rewards keeping to the reference speed and lateral position. Comfort includes minimising the acceleration in all directions as well as the lateral velocity. Energy efficiency is here only taken into consideration by minimising the longitudinal acceleration.

The main challenges are to define the path planning constraints such that the problem is convex while the constraints guarantee safety but are not too restrictive, to design the decision making in order to make as good decisions as possible and to design the complete controller structure. The method used can be seen as a development of the method presented in [14, 15] where path planning and decision making were done especially in an overtaking scenario but to some extent also in the highway exit scenario.

3

Solution design

This chapter presents the proposed solution in general terms. First the solution structure is presented and then each part is described.

3.1 Solution structure

The proposed solution is an MPC approach that includes three parts, situation analysis, calculation of optimal paths and decision making. The structure can be seen in Figure 3.1. The inputs to the controller is the information about the position of the exit and sensor data that gives information about the surrounding vehicles and road. The exit position includes the distance to the exit and from which lane it can be taken. The sensor data should be processed to give position, velocity and acceleration of all vehicles.

This information is sent to a situation analyst that evaluates the situation to set constraints and references for the path planning. Using this, the optimal path for two possible actions is calculated with model predictive control: making a lane change into the closest gap or to stay in the current lane and trail the leading vehicle. The paths are then compared in a decision manager that evaluates the paths and chooses the best one. The first control signal is then applied to the vehicle.

3.2 Situation analyst

The situation analyst is the part where the information from sensor readings and information about the exit is gathered and analysed. The sensor system should provide a list of the surrounding vehicles as well as information about the road. Also the current states of E is needed.

The situation analyst goes through the list of vehicles and identifies the ones of importance, i.e. S_1, S_2 and S_3, by checking the positions of the vehicles compared to the position of E. The surrounding vehicles could come with a predicted movement from another system, otherwise a prediction is done here, predicting the speed and position of the vehicles in the entire prediction horizon.

It also evaluates the lateral position by comparing it to the lane for the exit, the reference lane. This evaluation is used to determine appropriate references for lateral

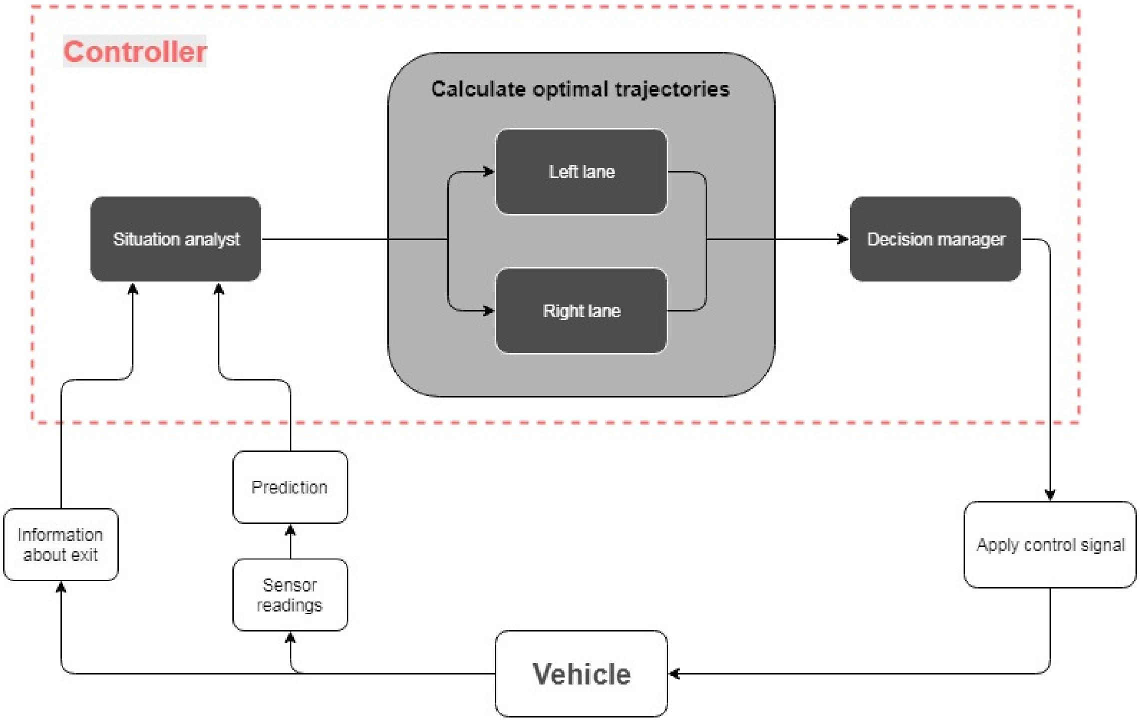


Figure 3.1: Structure of the algorithm. As input to the controller is sensor readings and information about the exit, i.e. from which lane and at what distance. A situation analyst then evaluates the situation and sets the constraints and references for path planning. Then the optimal trajectories are calculated for the two actions in a model predictive controller. The trajectories are then sent to the decision manager that chooses the best action. The first control signal for that path is then applied to the vehicle and the loop starts over.

position and longitudinal speed. For a lane change path the reference is always set to be the centre of the adjacent lane and the speed limit, respectively. For the trailing path, if the lane for trailing is the reference lane the lateral reference is the centre of the lane and the reference speed the speed limit. If the lane for trailing is not the reference lane then the same references are used when the exit is far away, when the exit is closer the lateral reference is moved closer to the reference lane and when the exit is closer still the reference speed will be to slow down.

3.3 Calculating paths

The optimal path for each lane is calculated using model predictive control. Input to the path planning is information about the surroundings and the exit as well as reference states and constraints. Using this, the optimal path both for positioning E in the right and in the left lane are calculated, an example of two generated paths is shown in Figure 3.2. These paths are then the output sent to the decision manager. The problem formulation for both types of path planning problems, trailing and lane change, is presented in continuous time in Chapter 4. The problems are then

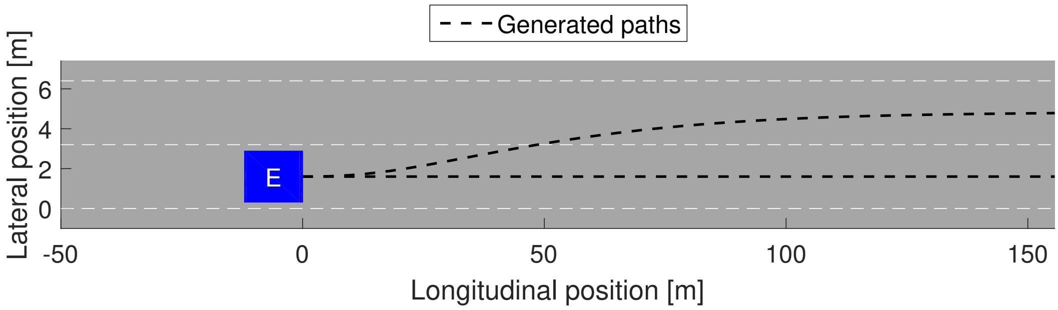


Figure 3.2: Example of the two paths generated in one iteration. One path that leads to the left lane and one path that stays in the right lane.

transformed into discrete time using Euler's first order discretisation.

3.4 Decision manager

The decision manager takes the two calculated paths and compares them to find the best one. It uses the information about the exit and the previous choices together with the state and input trajectories to compare and choose the optimal path. When the optimal path has been chosen the first control inputs belonging to that path are applied. The logic behind the decision making is presented in Chapter 5.

4

Path planning

This chapter describes the path planning optimisation problems.

4.1 Trailing

The trailing problem can at large be seen as an adaptive cruise control problem, i.e. it is mostly a longitudinal problem. However, lateral control is also done to keep a reference lateral position in the lane.

For this problem the only surrounding vehicle of any importance is the leading vehicle, S_1, assuming that no vehicles change lane into the gap between the leading vehicle and the ego vehicle. The collision avoidance constraint is formulated as a barrier where the longitudinal position of E is limited by the longitudinal position of the leading vehicle, the ego vehicle should always keep a safety margin to the leading vehicle. This is defined as

$$0 \leq x_{\mathrm{E}}(t) \leq x_1(t) - L_f(t) \tag{4.1}$$

where $L_f(t) = l_1 + \theta_f(v_{\mathrm{E},x}^{\mathrm{ref}} - v_{1,x}(t)) + \tau_f v_{\mathrm{E},x}^{\mathrm{ref}}$ is the safety margin to be kept, l_1 is the length of S_1, θ_f is the desired time gap to the vehicle in front and the term $\tau_f v_{\mathrm{E},x}^{\mathrm{ref}}$ makes sure that at higher velocities the gap is held larger. The collision avoidance constraint is pictured in Figure 4.1.

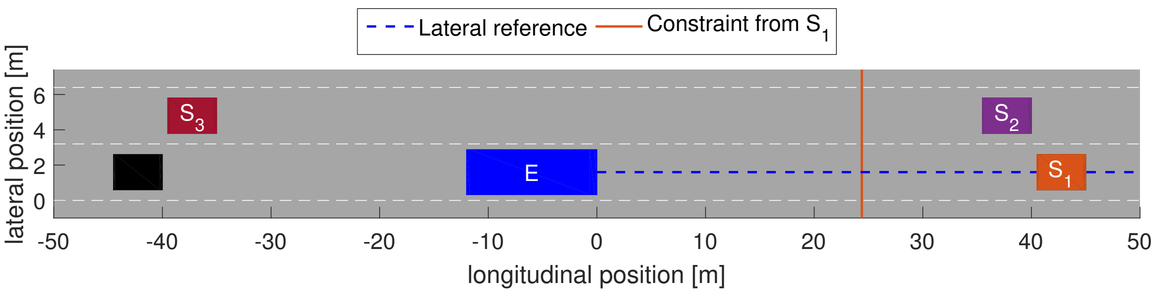


Figure 4.1: Lateral reference and collision avoidance constraint for the trailing path planning problem. Only S_1 places a safety constraint.

The cost function is defined as

$$J_{\text{trailing}}(\mathbf{x}_{\mathrm{E}}(t), \mathbf{u}_{\mathrm{E}}(t)) = \int_0^{t_f} ||\mathbf{x}_{\mathrm{E}}(t) - \mathbf{x}_{\mathrm{E}}^{\mathrm{ref}}(t)||_P^2 + ||\mathbf{u}_{\mathrm{E}}(t)||_Q^2 \, \mathrm{d}t \tag{4.2}$$

where P and Q are positive definite weight matrices and $||\mathbf{x}_\mathrm{E}(t)-\mathbf{x}_\mathrm{E}^\mathrm{ref}(t)||_P^2 = (\mathbf{x}_\mathrm{E}(t)-\mathbf{x}_\mathrm{E}^\mathrm{ref}(t))^T P(\mathbf{x}_\mathrm{E}(t) - \mathbf{x}_\mathrm{E}^\mathrm{ref}(t))$. The reference states are all zero except for the lateral position and longitudinal velocity, these are set by the situation analyst, see Section 3.2.

The full optimisation problem to be solved is

$$
\begin{aligned}
\min_{\mathbf{u}_\mathrm{E}} \quad & J_\mathrm{trailing}(\mathbf{x}_\mathrm{E}(t), \mathbf{u}_\mathrm{E}(t)) && \text{(4.3a)}\\
\text{s.t.} \quad & \dot{\mathbf{x}}_\mathrm{E}(t) = A\mathbf{x}_\mathrm{E}(t) + B\mathbf{u}_\mathrm{E}(t) && \text{(4.3b)}\\
& x_\mathrm{E}(t) \in [0, x_1(t) - L_f(t)] && \text{(4.3c)}\\
& y_\mathrm{E}(t) \in [w_\mathrm{E}/2, 2w_l - w_\mathrm{E}/2] && \text{(4.3d)}\\
& v_{\mathrm{E},x}(t) \in [0, v_{\mathrm{E},x}^\mathrm{max}] && \text{(4.3e)}\\
& v_{\mathrm{E},y}(t) \in [v_{\mathrm{E},y}^\mathrm{min}, v_{\mathrm{E},y}^\mathrm{max}] && \text{(4.3f)}\\
& a_{\mathrm{E},x}(t) \in [a_{\mathrm{E},x}^\mathrm{min}, a_{\mathrm{E},x}^\mathrm{max}] && \text{(4.3g)}\\
& a_{\mathrm{E},y}(t) \in [a_{\mathrm{E},y}^\mathrm{min}, a_{\mathrm{E},y}^\mathrm{max}] && \text{(4.3h)}\\
& v_{\mathrm{E},y}(t) \in [-s, s]v_{\mathrm{E},x}(t) && \text{(4.3i)}\\
& \mathbf{x}_\mathrm{E}(0) = \mathbf{x}_{E,0} && \text{(4.3j)}
\end{aligned}
$$

where constraints (4.3b)-(4.3i) are enforced for all $t \in [0, t_f]$.

4.2 Lane change

Two cases can be defined in the lane change situation that require different solutions to the path planning. One is that the leading vehicle, S_1, does not significantly limit E's movement, this is either when the leading vehicle is far away or when it drives with a speed that is close to or faster than the reference speed. The other case is when the leading vehicle is within the horizon and drives at a speed noticeably lower than the reference speed.

The reason this division is made is that a slow moving leading vehicle close complicates the problem significantly. For the adjacent vehicles, ramp barriers can easily be defined. However, for the leading vehicle a ramp barrier could prevent us from planning the path beyond the lane change. This is because a ramp barrier will block both lanes ahead of the leading vehicle. Other types of safety constraints affect the problem convexity and to attain convexity some reformulation must be made in this case.

4.2.1 Leading vehicle close to reference speed or outside horizon

The problem where the leading vehicle does not significantly constrain the path is solved in a similar way as the trailing problem. Here, however, the adjacent vehicles S_2 and S_3 must be taken into consideration.

In [14,22,23] ramp barriers were used as a way to implement convex safety constraints for the surrounding vehicles, this approach is adopted here as well. The ramp barriers can be divided into two types, one that keeps E from colliding with a vehicle ahead and one that keeps it from colliding with a vehicle behind. These are then rotated depending on whether the vehicle is positioned in the left or right lane. The ramp barriers used in this problem are pictured in Figure 4.2.

For a surrounding vehicle ahead of E, here S_1 and S_2, the ramp barrier can be defined as

$$\frac{x_j(t) - x_\mathrm{E}(t)}{L_f} \pm \frac{y_j(t) - y_\mathrm{E}(t)}{w_l} \geq 1 \tag{4.4}$$

with $L_f = l_j + \theta_f(v_{\mathrm{E},x}^{\mathrm{ref}} - v_{j,x}) + \tau_f v_{\mathrm{E},x}^{\mathrm{ref}}$, where l_j is the length of vehicle j, θ_f is the desired time gap to a vehicle in front and the term $\tau_f v_{\mathrm{E},x}^{\mathrm{ref}}$ makes sure that at higher velocities the gap is held larger. Which sign is to be used depends on which lane S_j is in, if it is in the left lane the positive sign is used, if it is in the right lane the negative sign is used.

Similarly, for a vehicle behind the ego vehicle, in this case S_3, the ramp is defined as

$$\frac{x_j(t) - x_\mathrm{E}(t)}{L_r} \mp \frac{y_j(t) - y_\mathrm{E}(t)}{w_l} \leq -1 \tag{4.5}$$

where $L_r = l_\mathrm{E} + \theta_r(v_{\mathrm{E},x}^{\mathrm{ref}} - v_{j,x}) + \tau_r v_{3,x}$, with θ_r the desired time gap to a vehicle behind. Here negative sign is used if the vehicle is in the left lane and positive if it is in the right lane.

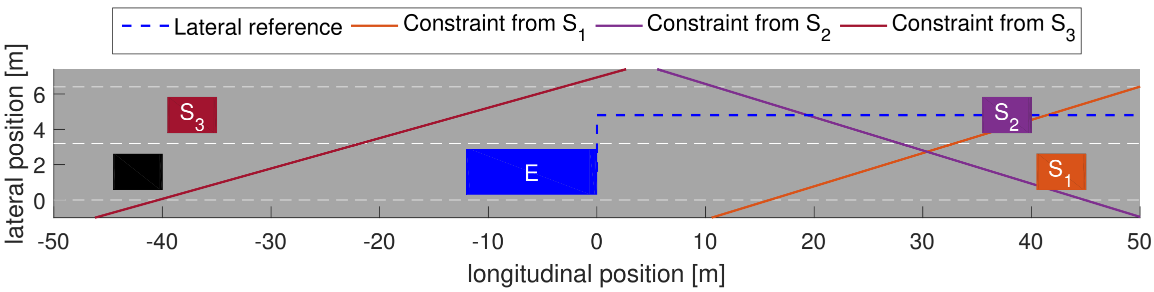


Figure 4.2: Lateral reference and collision avoidance constraints for the lane change path planning problem where the leading vehicle is driving at reference speed.

The cost function is the same as in the trailing problem

$$J_{\mathrm{lanechange}}^{1}(\mathbf{x}_\mathrm{E}(t), \mathbf{u}_\mathrm{E}(t)) = \int_0^{t_f} ||\mathbf{x}_\mathrm{E}(t) - \mathbf{x}_\mathrm{E}^{\mathrm{ref}}(t)||_P^2 + ||\mathbf{u}_\mathrm{E}(t)||_Q^2 \,\mathrm{d}t \tag{4.6}$$

where $J_{\mathrm{lanechange}}^{1}$ stands for the cost function in the first version of the lane change problem and P and Q are the same weight matrices as in the trailing problem.

The optimisation problem to be solved is thus

$$
\begin{aligned}
\min_{\mathbf{u}_\mathrm{E}} \quad & J^1_{\mathrm{lanechange}}(\mathbf{x}_\mathrm{E}(t), \mathbf{u}_\mathrm{E}(t)) && \text{(4.7a)}\\
\text{s.t.} \quad & \dot{\mathbf{x}}_\mathrm{E}(t) = A\mathbf{x}_\mathrm{E}(t) + B\mathbf{u}_\mathrm{E}(t) && \text{(4.7b)}\\
& x_\mathrm{E}(t) \in [0, \infty] && \text{(4.7c)}\\
& y_\mathrm{E}(t) \in [w_\mathrm{E}/2, 2w_l - w_\mathrm{E}/2] && \text{(4.7d)}\\
& v_{\mathrm{E},x}(t) \in [0, v^{\max}_{\mathrm{E},x}] && \text{(4.7e)}\\
& v_{\mathrm{E},y}(t) \in [v^{\min}_{\mathrm{E},y}, v^{\max}_{\mathrm{E},y}] && \text{(4.7f)}\\
& a_{\mathrm{E},x}(t) \in [a^{\min}_{\mathrm{E},x}, a^{\max}_{\mathrm{E},x}] && \text{(4.7g)}\\
& a_{\mathrm{E},y}(t) \in [a^{\min}_{\mathrm{E},y}, a^{\max}_{\mathrm{E},y}] && \text{(4.7h)}\\
& v_{\mathrm{E},y}(t) \in [-s, s]v_{\mathrm{E},x}(t) && \text{(4.7i)}\\
& \frac{x_j(t) - x_\mathrm{E}(t)}{L_f} \pm \frac{y_j(t) - y_\mathrm{E}(t)}{w_l} \geq 1, \quad j = 1, 2 && \text{(4.7j)}\\
& \frac{x_3(t) - x_\mathrm{E}(t)}{L_r} \mp \frac{y_3(t) - y_\mathrm{E}(t)}{w_l} \leq -1 && \text{(4.7k)}\\
& \mathbf{x}_\mathrm{E}(0) = \mathbf{x}_{E,0} && \text{(4.7l)}
\end{aligned}
$$

where constraints (4.7b)-(4.7k) are enforced for all $t \in [0, t_f]$.

4.2.2 Slow moving leading vehicle inside horizon

The approach presented above, in Section 4.2.1, is appropriate in the specified scenario, when the leading vehicle is far away or drives close to the reference speed. For a nearby leading vehicle that drives slowly there is however a problem with this approach. The problem is the ramp barrier constraint for the leading vehicle. If this is applied here it will lead to an odd behaviour of E or possibly that a lane change that should be possible becomes infeasible. The problem is that the ramp barrier for the leading vehicle will move at the same slow speed as S_1, thus when E has changed lane it will be blocked by the slow moving ramp and forced to slow down even if the lane is clear. If there is then a faster vehicle coming from behind there would be a collision if E is forced to move slowly, hence the lane change will become infeasible. Hence, another solution to the lane change problem is proposed, this is inspired by the solution to the similar overtaking scenario presented in [14].

On the other hand, the approach presented in this section cannot be used to handle the case where the leading vehicle drives close to the reference speed or faster. That is because the leading vehicle will here be used as a reference and the inverse of the relative speed will come into the equations, making it inappropriate to use when S_1 is driving close to the reference speed or is not inside the horizon. Because of this, the lower constraint on $v_{\mathrm{E},x}$ is here $v_{\mathrm{E},x}(t) \geq v_{1,x}(t) + \epsilon$ where $\epsilon > 0$.

For this path optimisation problem, two assumptions must be made in addition to those presented in Section 1.2. These should hold within a prediction horizon, however they do not have to hold between iterations.

A7. All surrounding vehicles have a constant longitudinal velocity

A8. All surrounding vehicles have a constant lateral velocity.

These are made since a translation from time to relative distance, as will be used here, cannot be made before the optimal velocity has been calculated. Hence, a prediction of the surrounding vehicle's velocity made in time cannot be taken into consideration unless it is assumed to be constant.

4.2.2.1 Collision avoidance constraints

To solve this problem some reformulation is needed, this is done with inspiration from [14]. The main issue was the ramp barrier for S_1, this is thus replaced by a box constraint around the leading vehicle saying that close to S_1 the lateral position of E must be in the adjacent lane. The collision avoidance constraints for S_2 and S_3 are still ramp barriers as in the previous lane change problem.

If the lane change is to be done into the left lane the constraint for S_1 is defined as

$$y_{\mathrm{E}}(t) \in [y_{\mathrm{E}}^{\min}(t), y_{\mathrm{E}}^{\max}(t)] \tag{4.8}$$

$$y_{\mathrm{E}}^{\min}(t) = \begin{cases} w_l + w_{\mathrm{E}}/2, & \text{for } x_{\mathrm{E}}(t) \in x_1(t) + [-L_r, L_f] \\ w_{\mathrm{E}}/2, & \text{otherwise} \end{cases} \tag{4.9}$$

$$y_{\mathrm{E}}^{\max}(t) = 2w_l - w_{\mathrm{E}}/2. \tag{4.10}$$

If the lane change is instead to be done to the right the constraint is

$$y_{\mathrm{E}}(t) \in [y_{\mathrm{E}}^{\min}(t), y_{\mathrm{E}}^{\max}(t)] \tag{4.11}$$

$$y_{\mathrm{E}}^{\min}(t) = w_{\mathrm{E}}/2 \tag{4.12}$$

$$y_{\mathrm{E}}^{\max}(t) = \begin{cases} w_l - w_{\mathrm{E}}/2, & \text{for } x_{\mathrm{E}}(t) \in x_1(t) + [-L_r, L_f] \\ 2w_l - w_{\mathrm{E}}/2, & \text{otherwise} \end{cases}. \tag{4.13}$$

Using this constraint the safety critical zone around the leading vehicle is a rectangle that does not stretch into the adjacent lane, see Figure 4.3. Hence, when E has made a lane change its continued movement will not be constrained by the leading vehicle thus preventing the problem with misplaced deceleration and infeasibility. However, introducing this constraint makes the problem non-convex since the constraint depends on the time E will be close to S_1, something that is not known beforehand. In Sections 4.2.2.2 to 4.2.2.5 a number of steps will be taken to reformulate the problem to restore convexity.

4.2.2.2 Change of reference frame

The first step is to reformulate the problem to instead of absolute longitudinal position work with position relative to S_1. This means that S_1 appears stationary and the road moves with the speed $-v_{1,x}$.

With this change of reference frame, the new states and control inputs are $\tilde{\mathbf{x}}_{\mathrm{E}}(t) = [\tilde{x}_{\mathrm{E}}(t), y_{\mathrm{E}}(t), \tilde{v}_{\mathrm{E},x}(t), v_{\mathrm{E},y}(t)]^T$ and $\tilde{\mathbf{u}}_{\mathrm{E}}(t) = [a_{\mathrm{E},x}(t), a_{\mathrm{E},y}(t)]^T$, where $\tilde{x}_{\mathrm{E}}(t) = x_{\mathrm{E}}(t) -$

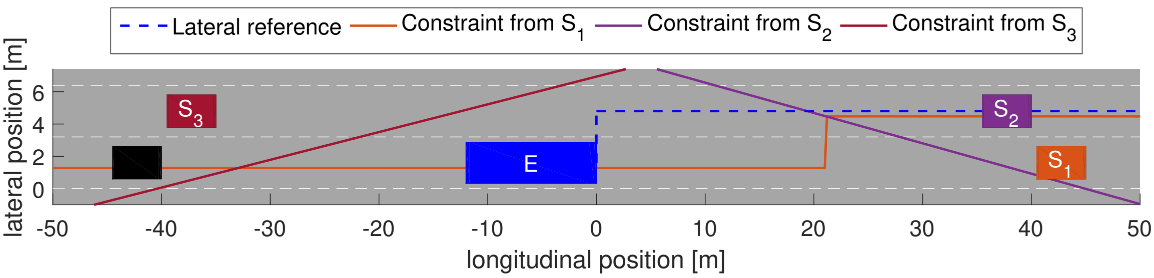


Figure 4.3: Lateral reference and safety constraint for the lane change path planning problem where the leading vehicle is driving slower than the reference speed and inside the horizon.

$x_1(t)$ and $\tilde{v}_{\mathrm{E},x}(t) = v_{\mathrm{E},x}(t) - v_{1,x}$. Note that the states concerning lateral motion are unchanged, it is only for the longitudinal motion that S_1 is used as a reference. Because of the assumption of constant velocity for S_1 the inputs are also unchanged.

The motion model is now

$$\begin{gathered}\dot{\tilde{\mathbf{x}}}_{\mathrm{E}}(t) = \tilde{A}\tilde{\mathbf{x}}_{\mathrm{E}}(t) + \tilde{B}\tilde{\mathbf{u}}_{\mathrm{E}}(t) \\ \tilde{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\end{gathered} \tag{4.14}$$

and constraints have become

$$\tilde{v}_{\mathrm{E},x}(t) \in [\epsilon, v_{\mathrm{E},x}^{\max} - v_{1,x}] \tag{4.15}$$
$$v_{\mathrm{E},y}(t) \in [v_{\mathrm{E},y}^{\min}, v_{\mathrm{E},y}^{\max}] \tag{4.16}$$
$$v_{\mathrm{E},y}(t) \in [-s, s](\tilde{v}_{\mathrm{E},x}(t) + v_{1,x}) \tag{4.17}$$
$$a_{\mathrm{E},x}(t) \in [a_{\mathrm{E},x}^{\min}, a_{\mathrm{E},x}^{\max}] \tag{4.18}$$
$$a_{\mathrm{E},y}(t) \in [a_{\mathrm{E},y}^{\min}, a_{\mathrm{E},y}^{\max}] \tag{4.19}$$

where $\epsilon > 0$ is introduced to guarantee that E will always drive faster than S_1.

The ramp barriers for S_2 and S_3 become

$$\frac{x_2(t) - \tilde{x}_{\mathrm{E}}(t) - (v_{1,x} - v_{2,x})t}{L_f} \pm \frac{y_2(t) - y_{\mathrm{E}}(t)}{w_l} \geq 1 \tag{4.20}$$

and

$$\frac{x_3(t) - \tilde{x}_{\mathrm{E}}(t) - (v_{1,x} - v_{3,x})t}{L_r} \mp \frac{y_3(t) - y_{\mathrm{E}}(t)}{w_l} \leq -1 \tag{4.21}$$

with signs as before.

The critical zone around S_1 now appears stationary in the moving frame. With $x_{1,0}$

being the initial longitudinal position of S_1 it is

$$y_E(t) \in [y_E^{\min}(t), y_E^{\max}(t)] \tag{4.22}$$

$$y_E^{\min}(t) = \begin{cases} w_l + w_E/2, & \text{for } x_E(t) \in x_{1,0} + [-L_r, L_f] \\ w_E/2, & \text{otherwise} \end{cases} \tag{4.23}$$

$$y_E^{\max}(t) = 2w_l - w_E/2 \tag{4.24}$$

or

$$y_E(t) \in [y_E^{\min}(t), y_E^{\max}(t)] \tag{4.25}$$

$$y_E^{\min}(t) = w_E/2 \tag{4.26}$$

$$y_E^{\max}(t) = \begin{cases} w_l - w_E/2, & \text{for } x_E(t) \in x_{1,0} + [-L_r, L_f] \\ 2w_l - w_E/2, & \text{otherwise} \end{cases}. \tag{4.27}$$

The problem with convexity remains however, the time for which E is in the zone depends on the optimal speed that is in turn determined by solving the optimisation problem.

4.2.2.3 Sampling in relative distance

In order to make the constraint on y_E convex the system is reformulated to be sampled in space instead of time, more specifically in the relative distance $\tilde{x}_E$. For simplicity the notation $\tilde{x} = \tilde{x}_E$ will be used hereafter.

This means that $\tilde{x}_E$ can be removed from the state vector, instead the state $\tilde{t}(\tilde{x})$, representing the time, is introduced, giving the state vector $\hat{\mathbf{x}}_E = [\tilde{t}(\tilde{x}), y_E(\tilde{x}), \tilde{v}_{E,x}(\tilde{x}), v_{E,y}(\tilde{x})]^T$.

The time derivatives describing the vehicle dynamics must now be replaced by space derivatives. Here the notation $(.)' = \mathrm{d}/\mathrm{d}\tilde{x}$ is used. The space derivatives of the states are

$$\tilde{t}'(\tilde{x}) = \frac{1}{\tilde{v}_{E,x}(\tilde{x})} \tag{4.28}$$

$$y_E'(\tilde{x}) = \frac{\mathrm{d}y_E(\tilde{x})}{\mathrm{d}t(\tilde{x})} \cdot \frac{\mathrm{d}t(\tilde{x})}{\mathrm{d}\tilde{x}} = v_{E,y}(\tilde{x}) \frac{1}{\tilde{v}_{E,x}(\tilde{x})} \tag{4.29}$$

$$\tilde{v}_{E,x}'(\tilde{x}) = \frac{\mathrm{d}\tilde{v}_{E,x}(\tilde{x})}{\mathrm{d}t(\tilde{x})} \cdot \frac{\mathrm{d}t(\tilde{x})}{\mathrm{d}\tilde{x}} = a_{E,x}(\tilde{x}) \frac{1}{\tilde{v}_{E,x}(\tilde{x})} \tag{4.30}$$

$$v_{E,y}'(\tilde{x}) = \frac{\mathrm{d}v_{E,y}(\tilde{x})}{\mathrm{d}t(\tilde{x})} \cdot \frac{\mathrm{d}t(\tilde{x})}{\mathrm{d}\tilde{x}} = a_{E,y}(\tilde{x}) \frac{1}{\tilde{v}_{E,x}(\tilde{x})} \tag{4.31}$$

The constraint on lateral position is now finally convex

$$y_E(\tilde{x}) \in [y_E^{\min}(\tilde{x}), y_E^{\max}(\tilde{x})] \tag{4.32}$$

$$y_E^{\min}(\tilde{x}) = \begin{cases} w_l + w_E/2, & \text{for } \tilde{x} \in x_{1,0} + [-L_r, L_f] \\ w_E/2, & \text{otherwise} \end{cases} \tag{4.33}$$

$$y_E^{\max}(\tilde{x}) = 2w_l - w_E/2 \tag{4.34}$$

or

$$y_{\mathrm{E}}(\tilde{x}) \in [y_{\mathrm{E}}^{\min}(\tilde{x}), y_{\mathrm{E}}^{\max}(\tilde{x})] \tag{4.35}$$

$$y_{\mathrm{E}}^{\min}(\tilde{x}) = w_{\mathrm{E}}/2 \tag{4.36}$$

$$y_{\mathrm{E}}^{\max}(\tilde{x}) = \begin{cases} w_l - w_{\mathrm{E}}/2, & \text{for } \tilde{x} \in x_{1,0} + [-L_r, L_f] \\ 2w_l - w_{\mathrm{E}}/2, & \text{otherwise} \end{cases}. \tag{4.37}$$

4.2.2.4 Change of variables

In all state derivatives in (4.28)-(4.31) the inverse relative longitudinal speed, $1/\tilde{v}_{\mathrm{E},x}(\tilde{x})$, comes in, since this is not a linear relation a variable change is made, the new state $z(\tilde{x}) = 1/\tilde{v}_{\mathrm{E},x}(\tilde{x})$ is introduced instead of $\tilde{v}_{\mathrm{E},x}(\tilde{x})$.

This gives the new state vector $\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) = [\tilde{t}(\tilde{x}), y_{\mathrm{E}}(\tilde{x}), z(\tilde{x}), v_{\mathrm{E},y}(\tilde{x})]^T$ with the derivatives

$$\tilde{t}'(\tilde{x}) = z(\tilde{x}) \tag{4.38}$$

$$y_{\mathrm{E}}'(\tilde{x}) = v_{\mathrm{E},y}(\tilde{x})z(\tilde{x}) \tag{4.39}$$

$$z'(\tilde{x}) = -\frac{1}{\tilde{v}_{\mathrm{E},x}(\tilde{x})^2}\tilde{v}_{\mathrm{E},x}'(\tilde{x}) = -\frac{1}{\tilde{v}_{\mathrm{E},x}(\tilde{x})^3}\tilde{a}_{\mathrm{E},x}(\tilde{x}) = -z^3(\tilde{x})\tilde{a}_{\mathrm{E},x}(\tilde{x}) \tag{4.40}$$

$$v_{\mathrm{E},y}'(\tilde{x}) = a_{\mathrm{E},y}(\tilde{x})z(\tilde{x}) \tag{4.41}$$

By defining the input $u_x(\tilde{x}) = -z'(\tilde{x})$ the nonlinearity in (4.40) is removed, the state derivatives then become $\hat{\mathbf{x}}_{\mathrm{E}}'(\tilde{x}) = [z(\tilde{x}), v_{\mathrm{E},y}(\tilde{x})z(\tilde{x}), -u_x(\tilde{x}), a_{\mathrm{E},y}(\tilde{x})z(\tilde{x})]^T$. However, there is still nonlinearity in the state derivatives, for the states concerning the lateral motion, $y_{\mathrm{E}}'(\tilde{x}) = v_{\mathrm{E},y}(\tilde{x})z(\tilde{x})$ and $v_{\mathrm{E},y}'(\tilde{x}) = a_{\mathrm{E},y}(\tilde{x})z(\tilde{x})$. A new change of state is thus introduced, instead of using $v_{\mathrm{E},y}(\tilde{x})$ as state the space derivative of $y_{\mathrm{E}}(\tilde{x})$ is used, i.e. $\nu_y(\tilde{x}) = y_{\mathrm{E}}'(\tilde{x}) = v_{\mathrm{E},y}(\tilde{x})z(\tilde{x})$. The derivative of this state is

$$\begin{aligned}\nu_y'(\tilde{x}) &= v_{\mathrm{E},y}'(\tilde{x})z(\tilde{x}) + v_{\mathrm{E},y}(\tilde{x})z'(\tilde{x}) \\ &= a_{\mathrm{E},y}(\tilde{x})z^2(\tilde{x}) + v_{\mathrm{E},y}(\tilde{x})\left(-\frac{1}{\tilde{v}_{\mathrm{E},x}(\tilde{x})^2}\tilde{v}_{\mathrm{E},x}'(\tilde{x})\right) \\ &= a_{\mathrm{E},y}(\tilde{x})z^2(\tilde{x}) - v_{\mathrm{E},y}(\tilde{x})z^3(\tilde{x})\tilde{a}_{\mathrm{E},x}(\tilde{x}).\end{aligned} \tag{4.42}$$

Using this as an input, $u_y(\tilde{x}) = \nu_y'(\tilde{x})$, gives the states $\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) = [\tilde{t}(\tilde{x}), y_{\mathrm{E},y}(\tilde{x}), z(\tilde{x}), \nu_y(\tilde{x})]^T$ and state derivatives $\hat{\mathbf{x}}_{\mathrm{E}}'(\tilde{x}) = [z(\tilde{x}), \nu_y(\tilde{x}), -u_x(\tilde{x}), u_y(\tilde{x})]^T$. Now a linear motion model is achieved, with the input vector $\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}) = [u_x(\tilde{x}), u_y(\tilde{x})]^T$ it is

$$\begin{gathered}\hat{\mathbf{x}}_{\mathrm{E}}'(\tilde{x}) = \hat{A}\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) + \hat{B}\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}) \\ \hat{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \hat{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}.\end{gathered} \tag{4.43}$$

The constraints on $v_{\mathrm{E},x}(\tilde{x})$ and $v_{\mathrm{E},y}(\tilde{x})$ are translated into constraints on $z(\tilde{x})$ and

$\nu_y(\tilde{x})$ as

$$z(\tilde{x}) \in \left[\frac{1}{v_{\mathrm{E},x}^{\max} - v_{1,x}}, \frac{1}{\epsilon}\right] \tag{4.44}$$

$$\nu_y(\tilde{x}) \in [-s, s]\left(\frac{1}{z(\tilde{x})} + v_{1,x}\right) z(\tilde{x}) = [-s, s](1 + v_{1,x} z(\tilde{x})) \tag{4.45}$$

which are convex.

Constraints on the inputs are however not convex, as a final step for making the optimisation convex these are linearized around $z_{\mathrm{ref}} = 1/(v_{\mathrm{E},x}^{\mathrm{ref}} - v_{1,x})$ and $a_{\mathrm{E},x}^{\mathrm{ref}} = 0$. This gives

$$u_x(\tilde{x}) \in [a_{\mathrm{E},x}^{\min}, a_{\mathrm{E},x}^{\max}](-2z_{\mathrm{ref}}^3 + 3z_{\mathrm{ref}}^2 z(\tilde{x})) \tag{4.46}$$

$$u_y(\tilde{x}) \in [a_{\mathrm{E},y}^{\min}, a_{\mathrm{E},y}^{\max}](-z_{\mathrm{ref}}^2 + 2z_{\mathrm{ref}} z(\tilde{x})) \tag{4.47}$$

4.2.2.5 Final optimisation problem

Similarly as in the previous optimisation problems, the cost function is defined as

$$J_{\mathrm{lanechange}}^2(\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x})) = \int_0^{\tilde{x}_f} ||\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) - \hat{\mathbf{x}}_{\mathrm{E}}^{\mathrm{ref}}(\tilde{x})||_R^2 + ||\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x})||_S^2 \,\mathrm{d}\tilde{x} \tag{4.48}$$

where R and S are positive definite weight matrices, $\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) = [\tilde{t}(\tilde{x}), y_{\mathrm{E}}(\tilde{x}), z(\tilde{x}), \nu_{\mathrm{E},y}(\tilde{x})]^T$ and $\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}) = [z^3(\tilde{x})\tilde{a}_{\mathrm{E},x}(\tilde{x}), a_{\mathrm{E},y}(\tilde{x})z^2(\tilde{x}) - v_{\mathrm{E},y}(\tilde{x})z^3(\tilde{x})\tilde{a}_{\mathrm{E},x}(\tilde{x})]^T$.

The final convex optimisation problem is thus

$$\begin{aligned}
\min_{\hat{\mathbf{u}}_{\mathrm{E}}} \quad & J_{\mathrm{lanechange}}^2(\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x})) && \text{(4.49a)}\\
\text{s.t.} \quad & \hat{\mathbf{x}}_{\mathrm{E}}'(t) = \hat{A}\hat{\mathbf{x}}_{\mathrm{E}}(t) + \hat{B}\hat{\mathbf{u}}_{\mathrm{E}}(t) && \text{(4.49b)}\\
& \tilde{t}(\tilde{x}) \in [0, \infty) && \text{(4.49c)}\\
& y_{\mathrm{E}}(\tilde{x}) \in [y_{\mathrm{E}}^{\min}(\tilde{x}), y_{\mathrm{E}}^{\max}(\tilde{x})] && \text{(4.49d)}\\
& z(\tilde{x}) \in \left[(v_{\mathrm{E},x}^{\max} - v_{1,x}(\tilde{x}))^{-1}, \epsilon^{-1}\right] && \text{(4.49e)}\\
& \nu_y(\tilde{x}) \in [-s, s](1 + v_{1,x}(\tilde{x})z(\tilde{x})) && \text{(4.49f)}\\
& u_x(\tilde{x}) \in [a_{\mathrm{E},x}^{\min}, a_{\mathrm{E},x}^{\max}](-2z_{\mathrm{ref}}^3 + 3z_{\mathrm{ref}}^2 z(\tilde{x})) && \text{(4.49g)}\\
& u_y(\tilde{x}) \in [a_{\mathrm{E},y}^{\min}, a_{\mathrm{E},y}^{\max}](-z_{\mathrm{ref}}^2 + 2z_{\mathrm{ref}} z(\tilde{x})) && \text{(4.49h)}\\
& \frac{x_2(\tilde{x}) - \tilde{x} - (v_{1,x} - v_{2,x})\tilde{t}(\tilde{x})}{L_f} \pm \frac{y_2(\tilde{x}) - y_{\mathrm{E}}(\tilde{x})}{w_l} \geq 1 && \text{(4.49i)}\\
& \frac{x_3(\tilde{x}) - \tilde{x} - (v_{1,x} - v_{3,x})\tilde{t}(\tilde{x})}{L_r} \mp \frac{y_3(\tilde{x}) - y_{\mathrm{E}}(\tilde{x})}{w_l} \leq -1 && \text{(4.49j)}\\
& \mathbf{x}_{\mathrm{E}}(0) = \mathbf{x}_{E,0} && \text{(4.49k)}
\end{aligned}$$

where constraints (4.49b)-(4.49j) are enforced for all $\tilde{x} \in [0, \tilde{x}_f]$.

The optimal path will have states and inputs as a function of relative distance, this is translated to depend on time by using the calculated velocity.

5

Decision making

This chapter describes the logic behind the decision manager.

The goal of the decision manager is to choose the best path, the one to the left lane or the one to the right lane. This is structurally very simple, the cost, according to a specified cost function, is calculated for both paths and the one with a lower cost is chosen. If one action is infeasible the other will always be chosen. If both actions are infeasible then the path calculated at the previous time step will be used.

The real work on this part is defining the cost function. To really choose the best path several aspects need to be weighed together. The cost includes

- State and input costs. Similar to the cost functions in the path planning algorithms.
- The distance to the exit. The closer the exit is, the more costly it is not to be in the reference lane.
- A cost for switching between lane choices. This is to prevent situations where the controller switches back and forth between choosing the left or right lane.

The complete cost function is then the weighted sum of the three parts presented above

$$J = q_{\text{states}} J_{\text{states}} + q_{\text{exit}} J_{\text{exit}} + q_{\text{switch}} J_{\text{switch}} \tag{5.1}$$

where q_{states}, q_{exit} and q_{switch} are positive weights used to balance the different parts. The weights are chosen to achieve the desired behaviour in some chosen scenarios.

5.1 States and inputs

The cost for states and inputs is included to reward a path that is comfortable and energy efficient. It is defined in the same way as the objective functions in the path planning algorithms

$$J_{\text{states}}(.) = \sum_{t=0}^{t_f} ||\mathbf{x}_{\text{E}}(t) - \mathbf{x}_{\text{E}}^{\text{ref}}(t)||_P^2 + ||\mathbf{u}_{\text{E}}(t)||_Q^2 \tag{5.2}$$

where P and Q are the same weight matrices as was used in Sections 4.1 and 4.2.1. The comparison is made in time domain, if one path is defined in space domain the states and inputs are translated into time domain for the comparison. In this part neither the longitudinal or lateral position has any importance so these weights are set to zero. The lateral position instead comes into the final cost by the exit part of the cost.

5.2 Exit

The cost on choosing the path that does not end up in the reference lane is dependent on the distance to the exit. If the exit is far away there is time to wait for a better opportunity to move into the reference lane while if the exit is very close it is urgent to position E in the reference lane in order to not miss the exit.

The cost is defined as

$$J_{\text{exit}}(.) = \begin{cases} (1 - (d_{\text{exit}}/d_{\max})^{0.4})|\delta_{\text{path}} - \delta_{\text{ref}}|, & \text{for } d_{\text{exit}} \in [0, d_{\max}] \\ 0, & \text{otherwise} \end{cases} \tag{5.3}$$

where d_{exit} is the distance to the exit, $d_{\max} = 2000\,\text{m}$ is the maximum distance for which the exit gives rise to a cost, $\delta_{\text{path}}^{\textbf{candidate}}$ stands for the lane in which the path in question ends up in and δ_{ref} stands for the reference lane. The power of the function, 0.4, is chosen as a tuning parameter to achieve an appropriate distance dependency on the cost. The function can be seen in Figure 5.1.

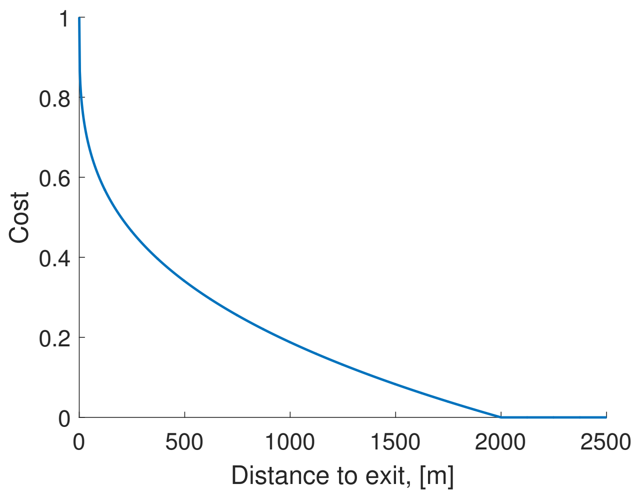


Figure 5.1: J_{exit} as a function of distance to the exit in the case where the final position of the path is not in the reference lane. When the exit is further away than 2 kilometres the cost is zero, otherwise it increases the closer the exit is.

5.3 Switching between lane choices

The last part of the cost is to avoid unnecessary switches in the choice of lane. Switching back and forth is something that could for example happen when there is a lot of sensor noise or when the surrounding vehicles behave in an unpredictable manner. In order to not cause confusion and discomfort it is desirable that once a manoeuvre has been initialised it is completed. The cost for switching is defined as

$$J_{\text{switch}}(.) = \sum_{k=1}^{10} \rho^k |\delta_{\text{path}}^{\text{candidate}} - \delta_{\text{path}}^{\text{previous}}(k)| \tag{5.4}$$

where $\rho \in (0,1)$ is a forgetting factor, $\delta_{\text{path}}^{\text{candidate}}$ is the final lane the candidate path would be positioned in and $\delta_{\text{path}}^{\text{previous}}(k)$ is the final lane the k'th previously chosen path was positioned in, $k = 1$ means one time step ago, $k = 2$ means two time steps ago and so on.

6

Results

The chapter describes how simulations were performed and presents the results of simulating a few different scenarios.

6.1 Simulations

Simulations were performed in Matlab. As a solver HPIPM, which stands for High Performance Interior Point Method, was used. This is very appropriate for this kind of problems since it is a multistage solver. Multistage means that it takes into consideration that the same states and inputs are calculated for several time steps, where at one time the states and inputs only depend on the previous states and inputs. This can be compared with a single stage solver, which puts all states and inputs for all times in one state vector. This means that transition and weight matrices will be very sparse and will hence loose speed compared to a multistage solver. Presented in Figure 6.1 is the computation time as a function of the number of samples used, for a PC with a 2.69 GHz processor and 16 GB RAM. The computation time is the time for one entire iteration of the controller pictured in Figure 3.1 with a prediction horizon of 10 seconds. In the simulations the number of samples used is 100, giving a computation time of approximately 8 ms.

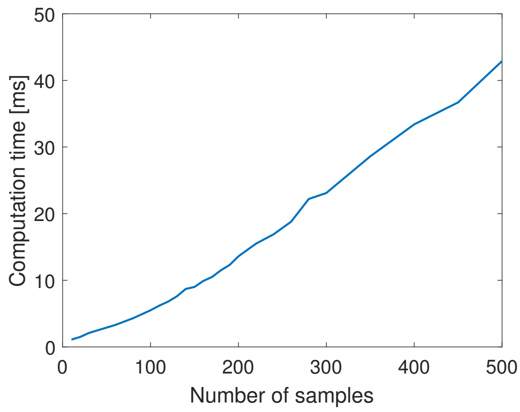


Figure 6.1: Computation time as a function of the number of samples for one iteration of the controller pictured in Figure 3.1. The prediction horizon is here 10 seconds. The computation was performed on a PC with a 2.69 GHz processor and 16 GB RAM.

6.2 Scenarios

Below, a few simulated scenarios are presented. For simplicity, in all scenarios the exit to be taken is from the left lane, meaning that the left lane is the reference lane. However, the same results would be achieved if the exit was instead from the right lane. Parameters common for all scenarios are presented in Table 6.1. The time horizon used is $t_f = 10\,\mathrm{s}$ and the horizon in relative distance is $\tilde{x}_f = 50\,\mathrm{m}$. Unless otherwise stated, no vehicles have an initial acceleration or lateral speed. The weights used for the path planning and decision making, common for all simulations, are presented in 6.2.

$l_j = 4.5\,\mathrm{m}$	$w_j = 2\,\mathrm{m}$	$l_\mathrm{E} = 12\,\mathrm{m}$	$w_\mathrm{E} = 2.55\,\mathrm{m}$
$w_l = 3.2\,\mathrm{m}$	$v_\mathrm{ref} = 80\,\mathrm{km/h}$	$t_f = 10\,\mathrm{s}$	$\tilde{x}_f = 50\,\mathrm{m}$
$v_{\mathrm{E},x}^{\min} = 0\,\mathrm{km/h}$	$v_{\mathrm{E},x}^{\max} = 90\,\mathrm{km/h}$	$v_{\mathrm{E},y}^{\min} = -4\,\mathrm{m/s}$	$v_{\mathrm{E},y}^{\max} = 4\,\mathrm{m/s}$
$a_{\mathrm{E},x}^{\min} = -4\,\mathrm{m/s^2}$	$a_{\mathrm{E},x}^{\max} = 1\,\mathrm{m/s^2}$	$a_{\mathrm{E},y}^{\min} = -1\,\mathrm{m/s^2}$	$a_{\mathrm{E},x}^{\max} = 1\,\mathrm{m/s^2}$
$\theta_f = 1\,\mathrm{s}$	$\tau_f = 0.5\,\mathrm{s}$	$\theta_r = 0.5\,\mathrm{s}$	$\tau_r = 0.25\,\mathrm{s}$
$s = 0.18$			

Table 6.1: Parameters common for all simulations.

$P = \mathrm{diag}(0,2,1,4)$	$Q = \mathrm{diag}(4,4)$	$R = \mathrm{diag}(0,2,4,4)$	$S = \mathrm{diag}(3,4)$
$q_\mathrm{states} = 1$	$q_\mathrm{exit} = 600$	$q_\mathrm{switch} = 30$	

Table 6.2: Weights for path planning and decision making, common for all simulations.

6.2.1 Scenario 1 – Typical case

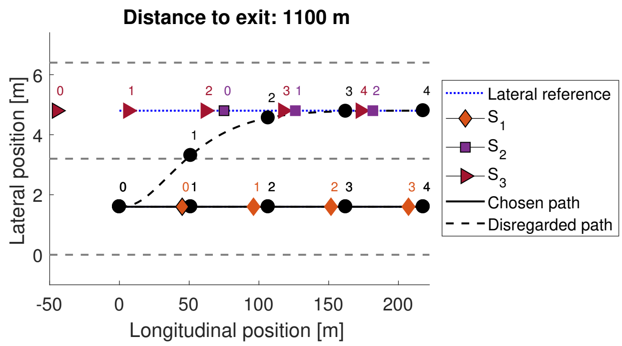
The first scenario is a simple scenario where all surrounding vehicles keep a constant speed equal to the reference speed and are spaced such that a lane change is feasible. See Table 6.3 for the initial values. The generated paths are shown for three time instances in Figure 6.2, the velocity and acceleration profiles are shown in Figure 6.3.

$x_1(0) = 45\,\mathrm{m}$	$x_2(0) = 75\,\mathrm{m}$	$x_3(0) = -45\,\mathrm{m}$
$v_{1,x}(0) = 80\,\mathrm{km/h}$	$v_{2,x}(0) = 80\,\mathrm{km/h}$	$v_{3,x}(0) = 80\,\mathrm{km/h}$
$v_{\mathrm{E},x}(0) = 80\,\mathrm{km/h}$	$d_\mathrm{exit}(0) = 1100\,\mathrm{m}$	

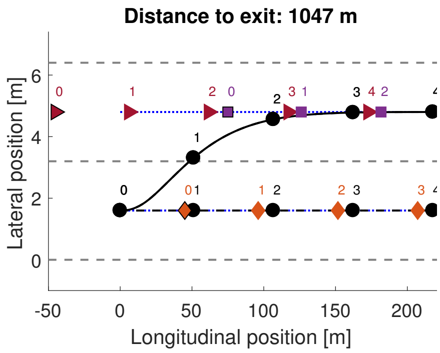
Table 6.3: Initial values for scenario 1.

At first, Figure 6.2a, the exit is so far away that the cost from states and inputs for performing the lane change is higher than the cost for staying in the right lane, hence the lane change is not initialised. Since all vehicles drive at the same speed the scenario has not changed for the second time instance, Figure 6.2b, except for the distance to the exit, hence the cost from states and inputs for performing the lane change stays the same. However, when the exit comes closer this cost eventually becomes higher than the cost for the lane change and the lane change into the reference lane is initialised.

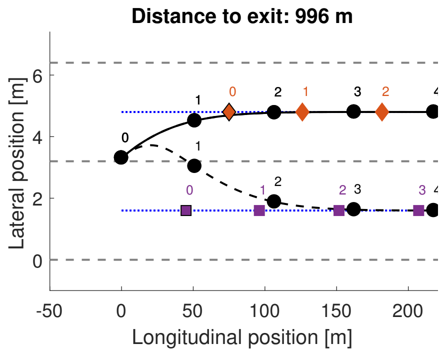
At the final time instance, presented in Figure 6.2c, E has moved into the other lane. The path for going back into the right lane is still calculated as an abortion option. At this time it can be seen how, when E moves into another lane, the situation analyst changes the labelling of the surrounding vehicles, S_j. The path planning for the left lane is now a trailing problem while the path planning for the right lane is a lane change problem.



(a) Time 0.



(b) After 2.5 seconds.



(c) After 4.8 seconds.

Figure 6.2: Generated paths for scenario 1 at three different times. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. At the initial time, presented in (a), both paths are feasible but the exit is still so far away that a the lane change is more costly than staying in the right lane. At the second time, (b), the exit is closer which adds more to the cost of staying in the right lane and the lane change into the reference lane is initialised. Lastly, in (c) E has passed into the reference lane, however, the path for going back to the right lane is still calculated as an abortion option. Here it can be seen how the labels, S_j, of the surrounding vehicles change as E passes into the other lane.

6.2.2 Scenario 2 – Accelerating surrounding vehicle

This scenario shows how the controller can take the predicted behaviour of the surrounding vehicles into consideration. Here a comparison is made between a case

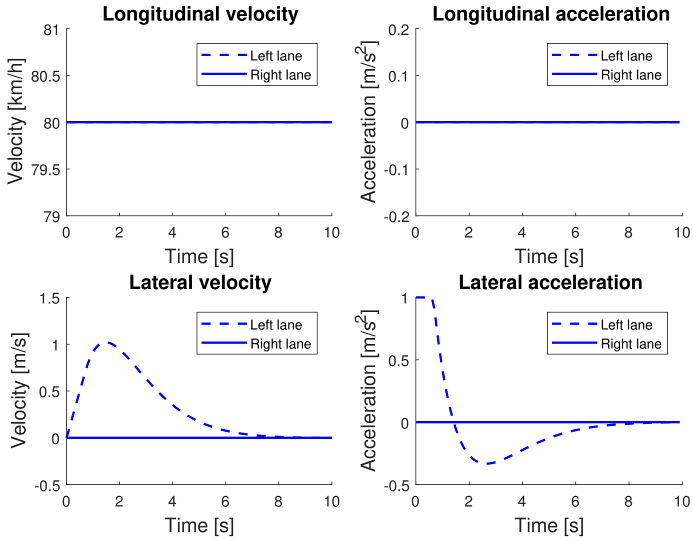


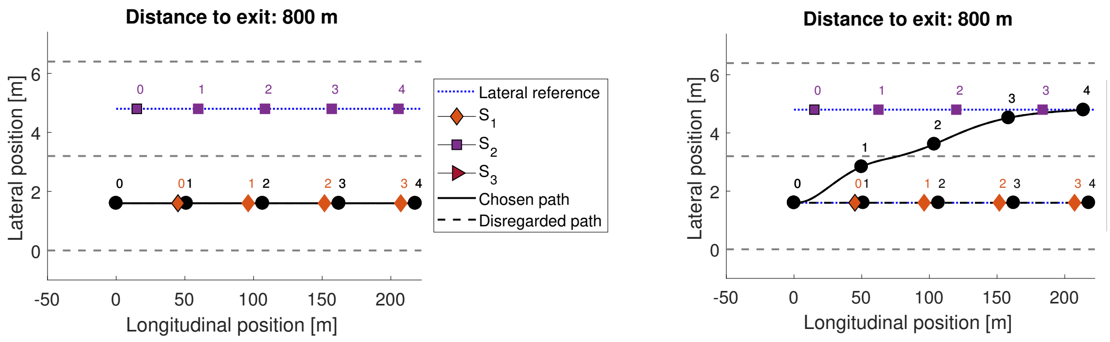
Figure 6.3: Longitudinal and lateral velocity and acceleration over the prediction horizon at the initial time in scenario 1. The solid line indicates the path chosen by the decision manager, compare to Figure 6.2a. In the top two plots it is clear that there is no longitudinal acceleration for any of the two paths. There is however a change in lateral velocity and acceleration for the lane change into the left lane. Both the velocity and acceleration varies smoothly despite the fact that there is no constraint or cost on the jerk.

where all surrounding vehicles drive with constant velocity and the same situation but where one of the vehicles have a constant acceleration. The initial positions and velocities for all vehicles are equal in the two cases, the only thing that differs is the acceleration for S_2. The initial values is presented in Table 6.4.

$x_1(0) = 45\,\mathrm{m}$	$x_2(0) = 15\,\mathrm{m}$	$x_3(0) = -300\,\mathrm{m}$
$v_{1,x}(0) = 80\,\mathrm{km/h}$	$v_{2,x}(0) = 70\,\mathrm{km/h}$	$v_{3,x}(0) = 70\,\mathrm{km/h}$
$a_{2,x}(0) = 0\,\mathrm{m/s^2}$ or $0.5\,\mathrm{m/s^2}$	$v_{\mathrm{E},x}(0) = 80\,\mathrm{km/h}$	$d_{\mathrm{exit}}(0) = 800\,\mathrm{m}$

Table 6.4: Initial values for scenario 2.

The generated paths for the two cases is presented in Figure 6.4 and the velocity and acceleration for the case when S_2 accelerates is shown in Figure 6.5. In Figure 6.4a it is shown that when S_2 does not accelerate, making a lane change is infeasible and E is forced to stay in its lane. Comparing this to Figure 6.4b, where the initial positions and velocities are the same, it can be seen how the controller takes the acceleration of S_2 into account and is able to generate a feasible lane change.



(a) No surrounding vehicle accelerates.

(b) S_2 accelerates.

Figure 6.4: Generated paths for scenario 2 where in (a) no surrounding vehicles accelerate while in (b) S_2 accelerates with $a_{2,x} = 0.5\,\mathrm{m/s^2}$. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. In (a) S_2 is positioned such that a lane change into the left lane is infeasible. Since the velocity is constant and equal to the velocity of E the relative position will remain the same during the prediction horizon. In (b) the initial velocity and position is the same as in (a) but here S_2 accelerates. The controller takes the acceleration into regard and predicts that S_2 will speed up and increase its distance to E. This makes it possible to generate a feasible path that takes E into the left lane.

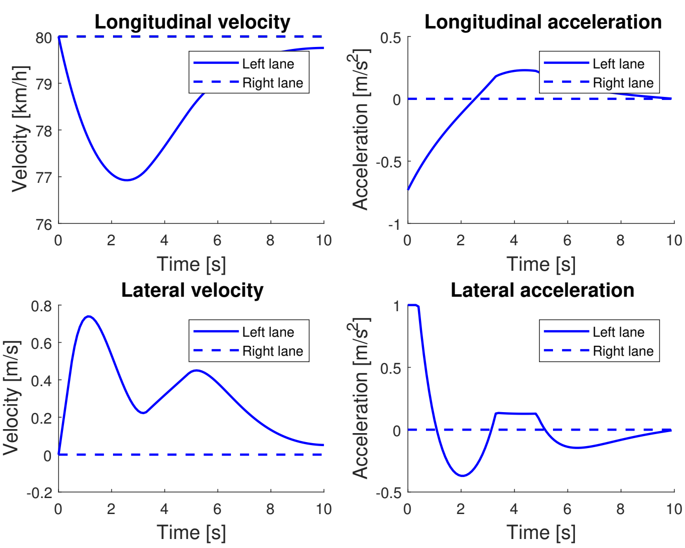


Figure 6.5: Longitudinal and lateral velocity and acceleration over the prediction horizon at the initial time for scenario 2 in the case where S_2 accelerates. Compare to Figure 6.4b. The solid line indicates the path chosen by the decision manager.

6.2.3 Scenario 3 – Abortion of lane change

The benefit of continuing to generate two paths even when a lane change into the reference lane has been initialised is shown in this scenario. Here going back to the original lane is used for aborting the lane change when that becomes infeasible. In this scenario S_3 suddenly starts to accelerate when a lane change has been initialised, making it impossible to safely go through with the lane change. The initial values of the scenario is presented in Table 6.5.

$x_1(0) = 45\,\mathrm{m}$	$x_2(0) = 150\,\mathrm{m}$	$x_3(0) = -45\,\mathrm{m}$
$v_{1,x}(0) = 80\,\mathrm{km/h}$	$v_{2,x}(0) = 80\,\mathrm{km/h}$	$v_{3,x}(0) = 80\,\mathrm{km/h}$
$v_{\mathrm{E},x}(0) = 80\,\mathrm{km/h}$	$d_{\mathrm{exit}}(0) = 800\,\mathrm{m}$	

Table 6.5: Initial values for scenario 3.

The generated paths for three time instances are shown in Figure 6.6. The velocity and acceleration for the final time is presented in Figure 6.7. At the initial time, Figure 6.6a, both paths are feasible and making a lane change into the reference lane is chosen. Shortly thereafter, at the time instance presented in 6.6b, S_3 has just started to accelerate which makes it infeasible to go through with the lane change, hence E will fall back into the right lane. As moving into the left lane is now infeasible, E has no choice but to wait in the right lane. As the exit moves closer the lateral reference is moved closer to the reference lane, see Figure 6.6b and 6.6c. Also, the longitudinal reference speed is set lower for staying in the right lane, see Figure 6.7. The combination of S_3's acceleration and E's deceleration eventually leads to S_3 passing by E and E can then make the lane change behind it, see Figure 6.6c.

6.2.4 Scenario 4 – When to initialise a lane change

In the final scenario the balance between the different parts in the decision making cost function is shown. This is done by comparing the distance to the exit where a lane change is initialised in three different cases. In Table 6.6 the initial values that are common for all three cases are presented. The difference between the cases lies in $v_{1,x}(0)$ and $v_{3,x}(0)$ as well as the distance to the exit. The values of these variables for the three cases are presented in Table 6.7.

$x_1(0) = 45\,\mathrm{m}$	$x_2(0) = 500\,\mathrm{m}$	$x_3(0) = -30\,\mathrm{m}$
$v_{2,x}(0) = 80\,\mathrm{km/h}$	$v_{\mathrm{E},x}(0) = 80\,\mathrm{km/h}$	

Table 6.6: Initial values common for all three cases in scenario 4.

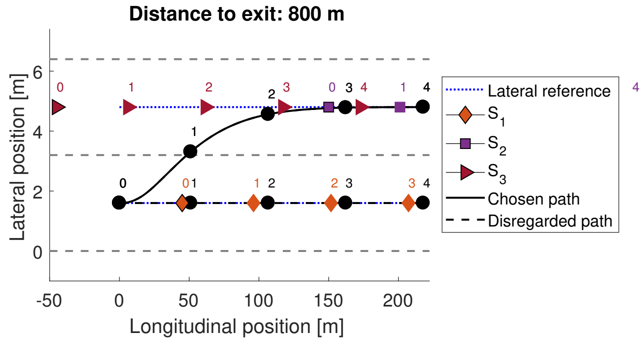
	$v_{1,x}(0)$	$v_{3,x}(0)$
Case 1	80 km/h	80 km/h
Case 2	72 km/h	80 km/h
Case 3	80 km/h	84 km/h

Table 6.7: Initial values that differ between the three cases in scenario 4.

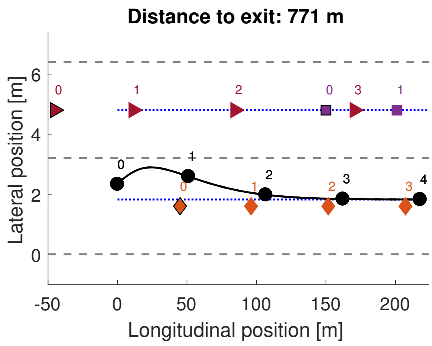
The generated paths as well as velocity and acceleration profiles at the initial time for Case 1 are shown in Figure 6.8. In this case the surrounding vehicles drive at the reference speed and are spaced such that they do not restrict E's behaviour significantly. No longitudinal acceleration is needed for either of the paths. Under these circumstances the lane change into the reference lane is initialised when there is just over 1 km to the exit, $d_{\text{exit}} = 1053\,\text{m}$.

In the second case, the leading vehicle S_1 drives slower than the reference speed. The resulting paths, velocity and acceleration are shown in Figure 6.9. In this case the E would have to slow down in order to stay in the right lane. This makes the cost for trailing higher than in the previous case, hence the lane change is initialised earlier, at $d_{\text{exit}} = 1515\,\text{m}$.

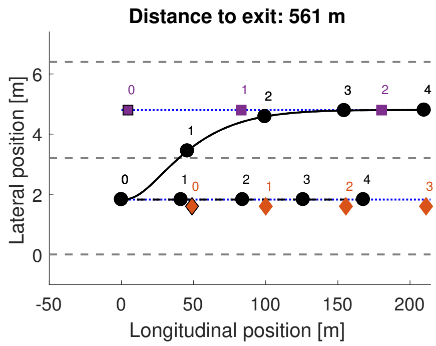
In the third case, S_3 drives faster than the reference speed. The resulting paths, velocity and acceleration are shown in Figure 6.9. In this case the E would have to increase its speed in order to be able to perform the lane change in front of S_3. This makes the cost for lane change higher than in the first case, hence the exit is closer, $d_{\text{exit}} = 922\,\text{m}$, before the cost for lane change is lower than the cost for trailing and the lane change is initialised.



(a) Time 0.



(b) After 1.4 seconds. S_3 has now started to accelerate with $a_{3,x} = 1\,\mathrm{m/s^2}$.



(c) After 11.0 seconds.

Figure 6.6: Generated paths for scenario 3 at three different times. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. At the initial time, presented in (a), both paths are feasible and a lane change into the reference lane is initialised. At the second time, (b), S_3 suddenly starts to accelerate with $a_{3,x} = 1\,\mathrm{m/s^2}$, making the lane change infeasible. As an abortion manoeuvre the path for staying in the right lane is instead chosen. E is then forced to stay in the right lane as S_3 slowly passes it. In (c) S_3 has finally passed E, and hence become S_2, the lane change is again feasible and thus initialised. Comparing the lateral reference for the right lane in (a) to the one in (b) and (c) it can be seen how the reference is moved towards the reference lane as the exit comes closer than 800 meters.

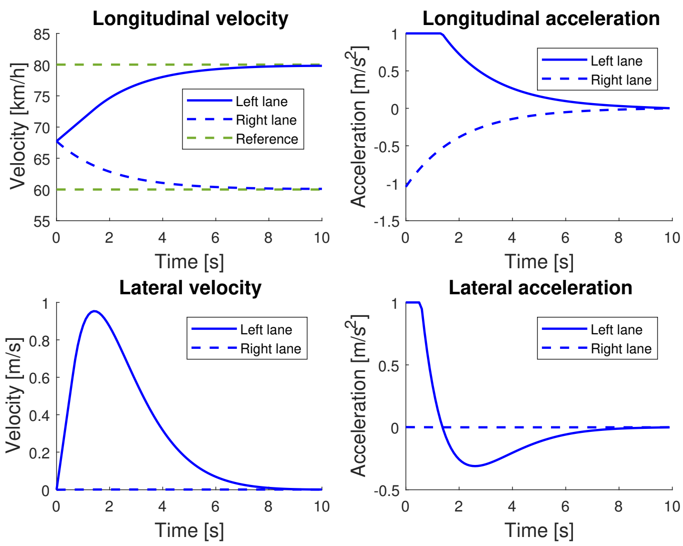
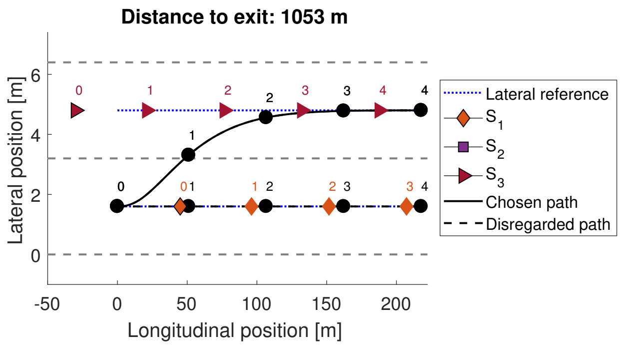
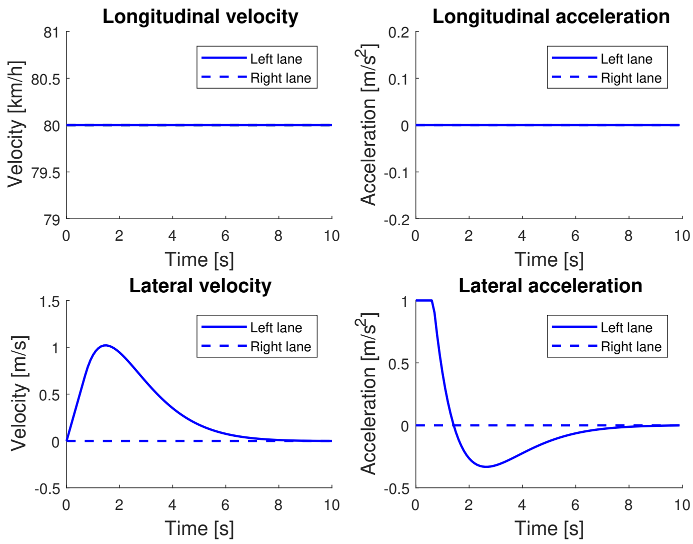


Figure 6.7: Longitudinal and lateral velocity and acceleration over the prediction horizon after 11.0 seconds in scenario 3. The solid line indicates the path chosen by the decision manager, compare to Figure 6.6c. Here it is seen how the reference for the longitudinal velocity for staying in the right lane has been increased because of the proximity of the exit. The two paths hence have different references for the longitudinal velocity and both paths require nonzero acceleration to reach the reference. Comparing the lateral velocity and acceleration profiles to the first scenario, Figure 6.3, it is clear that the lateral motion for the lane change is very similar despite the fact that in the abortion scenario longitudinal acceleration is involved.

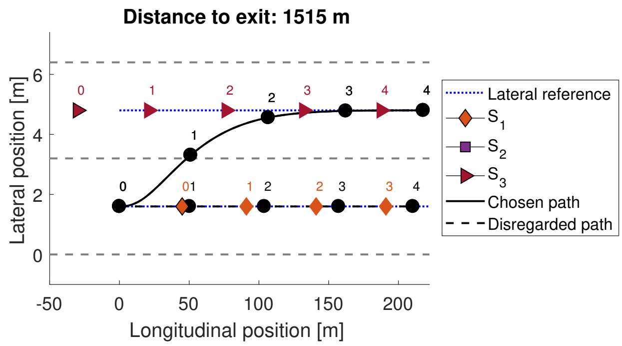


(a) Generated paths.

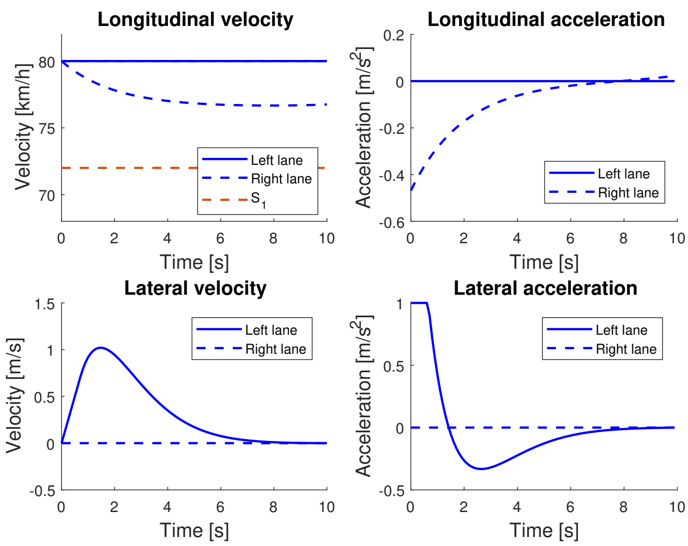


(b) Velocity and acceleration profiles.

Figure 6.8: Case 1 of scenario 4, both S_1 and S_3 drive at 80 km/h. In (a), the generated paths at the initial time. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. In (b), the corresponding velocity and acceleration profiles are shown. In this case no surrounding vehicle restricts E's motion which means that both paths are feasible and the longitudinal acceleration can be kept at zero. For this case the decision manager initialises the lane change into the reference lane when the exit is just over 1 kilometre away, 1053 m to be exact.

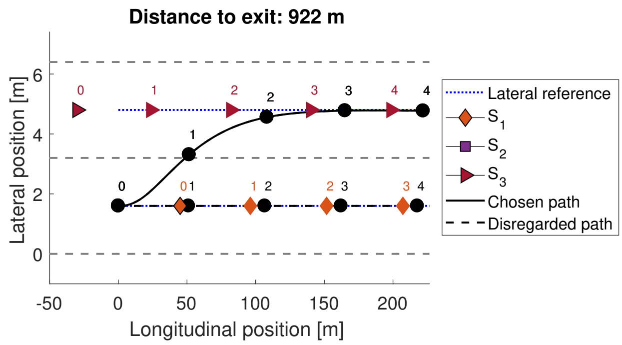


(a) Generated paths.

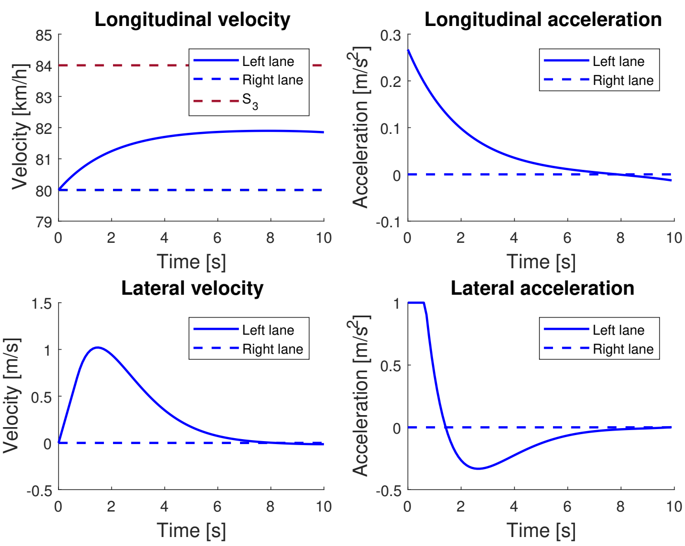


(b) Velocity and acceleration profiles.

Figure 6.9: Case 2 of scenario 4, $v_{1,x}(0) = 72\,\text{km/h}$ and $v_{3,x}(0) = 80\,\text{km/h}$. In (a), the generated paths at the initial time. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. In (b), the corresponding velocity and acceleration profiles are shown. In this case S_1 drives at a speed slower than the reference speed which makes it more expensive to stay in the right lane. That in combination with the upcoming exit makes it advantageous to initialise the lane change earlier than in case 1. Here the lane change is initialised already when the exit is 1515 metres away.



(a) Generated paths.



(b) Velocity and acceleration profiles.

Figure 6.10: Case 3 of scenario 4, $v_{1,x}(0) = 80\,\text{km/h}$ and $v_{3,x}(0) = 84\,\text{km/h}$. In (a), the generated paths at the initial time. The predicted position of the vehicles are shown for 5 time instances, 0 indicates the initial positions. In (b), the corresponding velocity and acceleration profiles are shown. In this case S_3 drives at a speed higher than the reference speed which means that E must also speed up in order to be able to change lane in front of it. This makes it more expensive to change lane, hence the lane change isn't initialised until the exit is 922 metres away.

7
Discussion

In this chapter first the results and general performance of the controller are discussed. Then some possibilities for future work are presented.

7.1 Performance

The evaluation of computation time indicates that the method will be fast enough to allow for a real time implementation in an actual vehicle. Also, the calculations might be possible to make even faster with an implementation of the entire controller in for example C.

The simulations also show that the implemented controller is very good at generating smooth and safe paths when possible and to make decisions based on the current traffic situation.

Comparing the lateral velocity and acceleration profiles for the lane change in the different scenarios it can be seen that they are very alike, except for scenario 2. This shows that the longitudinal and lateral motion are rather independent. In general it can be interpreted such that the longitudinal motion places the ego vehicle in a position and with a speed that will allow the lateral motion to perform the lane change in an optimal way. The connection between the longitudinal and lateral motion lies in the constraint the longitudinal velocity places on the lateral velocity and in the collision avoidance constraints where both longitudinal and lateral position are involved. The simulation of scenario 2 shows that the longitudinal and lateral motion are indeed coupled.

A downside of this controller structure is that at each time only two options are evaluated, this could for example mean that E chooses to perform a costly lane change into the closest gap when it perhaps would have been better to wait for the next gap and make the lane change then. For example a graph search method could evaluate more possible actions and choose the one that is best. The advantage of the method presented here compared to the graph search method is that this controller requires less memory and is very fast. The method presented here is able to indirectly evaluate more options in the way that it weighs the cost of states and inputs against the urgency to change lane due to the exit, hence it can wait for a better opportunity to change lane.

A part of the controller implementation that can be seen as both a great advantage and as a disadvantage is that it requires a lot of tuning of weights in both the path planning and the decision making. The strength is in the fact that the behaviour can be tuned in many different ways to achieve for example the best energy efficiency, to maximise the comfort of the passenger or to mimic the behaviour of different types of human drivers. The fact that there are so many weights involved however can make it very hard to define the optimal behaviour and there is no definite answer to the optimal tuning.

7.2 Future work

In future work focus could be on taking more complex behaviour of the surrounding vehicles into consideration, to evaluate more possible actions and/or to implement more advanced vehicle and environment models.

Taking more complex behaviour of the surrounding vehicles into consideration could mean to allow for the surrounding vehicles to change lane. It could also include taking into consideration that the surrounding vehicles will adapt to the motion of the ego vehicle. For instance, this could mean to allow a lane change that positions E in front of a faster moving vehicle and trust that it will slow its pace to adapt to E. Doing this would lead to a less conservative and more human-like behaviour.

Evaluating more possible actions could be to calculate the optimal path for positioning E in another gap than the closest one and including that in the decision making. It could also mean evaluating the cost of turning on the turning lights, moving toward the reference lane and slowing down, the cost for these actions would come from predicting how the surrounding vehicles would be affected.

The models for both vehicles and the environment are very simplified. As discussed above, a point mass model for planning the motion of simple vehicle types is commonly used and performs well. However, for more advanced vehicle, including for example trailers, a more advanced vehicle model might be necessary. Also, the air and rolling resistance could be included in the vehicle model to allow for a better optimisation in terms of energy efficiency as well as more accurate constraints on acceleration. As it is now, an assumption of a straight and flat road was made but this should be possible to extend to curved and inclining roads without too much effort.

8 Conclusion

The purpose of this master's thesis was to develop a method for automated decision making and path planning in a highway exit situation. The proposed solution is a model predictive controller in three parts where at each time two paths are generated, one for staying in the current lane and one for moving into the adjacent lane.

Using simulations in Matlab the method was evaluated in a few scenarios. The simulations show that the method is able to generate smooth paths that guarantee safety when so is possible. They also show how the decision manager takes the current traffic situation into consideration to decide on when to change lane and when to stay in the current lane. The MPC approach makes it possible to continuously adapt to changes and uncertainties in the surroundings. The method also shows good prospects for a real time implementation in regards to computation time, on a PC the average computation time for one iteration of the controller was 8 ms.

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