

Unified Model for Surface Fitting of Radio Telescope Reflectors

M. Lösler, C. Eschelbach, R. Haas

Abstract The main reflector of VLBI radio telescopes is affected by several disturbing forces. Temperature, wind, insolation or snow load deform the surface of the reflector and impair the receiving properties. Depending on the elevation orientation of the main reflector, the dead load of the dish w.r.t. the gravitation field of the Earth influence the surface negatively. In recent years, surface deformations and variations of the focal length have been analyzed by several groups. The common mathematical model to describe the main reflector is an ordinary rotational paraboloid. Due to the reflector design improvements, the surface of the main reflector of many of the upcoming VGOS radio telescopes cannot be parameterized by an ordinary rotational paraboloid. We present a unified mathematical model that overcomes this limitation and which is valid for the ordinary surface design as well as the new ring-focus reflector design of VGOS radio telescopes. The model is used for an independent confirmation of the specifications of the new Onsala twin telescopes at the Onsala Space Observatory.

Keywords Reverse Engineering, VGOS, Paraboloid, Ring-Focus, Focal-Length, Surface, Deformation, Close-Range Photogrammetry

1 Introduction

A VLBI radio telescope is a large geodetic space instrument that usually receives signals of quasi-stellar radio

sources in space. The receiving properties of such a telescope depend on the design of the radio telescope. The diameter of the main reflector of most of the existing radio telescopes lies within the range of 20 m up to 40 m, but there are also a few telescopes with larger diameter like the 100 m radio telescope Effelsberg. Due to the dimension, the main reflector is affected by several disturbance forces, e.g. the dead load of the dish w.r.t. the gravitation field of the Earth or weather conditions like insolation, wind or snow load. Some of the forces deform the main reflector as a function of the elevation orientation e.g. the path length of the signal. [Clark and Thomsen \(1988\)](#) parameterize the path length variations as a function of the change of the position of the vertex, the displacement of the receiver and the focal length variation w.r.t. the elevation orientation. The focal length is a design parameter of the main reflector and can be derived by e.g. high precision photogrammetric measurements of the reflector surface ([Fraser, 1986](#); [Luhmann, 2010](#)). These observations can also be used to validate the surface quality, i.e. the alignment of the panels of the reflector or to detect deformations ([Edmundson and Baker, 2001](#); [Shankar et al., 2009](#)). The root-mean-square (RMS) specifications of the surface quality of the reflectors and the path length variations are $<200 \mu\text{m}$ and $<300 \mu\text{m}$, respectively, for the upcoming VGOS – VLBI2010 Global Observing System – radio telescopes ([Petrachenko et al., 2009](#)). In the framework of reverse engineering, the parameter estimation of surfaces is a main part of industrial metrology.

2 Surface Model of Ordinary Paraboloid

The main reflector of most of the existing radio telescopes can be parameterized as a type of a quadric surface, i.e. a paraboloid. The canonical form of an ordinary paraboloid, i.e. the vertex is located at the origin and the principal axis is $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$, reads

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$$a^2 x_i^2 + b^2 y_i^2 = z_i, \quad (1)$$

where $\mathbf{p}_i = (x_i \ y_i \ z_i)^T$ represents the coordinates of the i -th object point and a and b are the datum-independent form-parameters of an ordinary elliptic paraboloid (OEP). By shifting and rotating, the object point \mathbf{p}_i can be transformed to a superordinate reference frame e.g. the measurement system, e.g. (Lösler, 2011):

$$\mathbf{p}_i = \mathbf{Q}(\mathbf{P}_i - \mathbf{P}_0). \quad (2)$$

Here point $\mathbf{P}_i = (X_i \ Y_i \ Z_i)^T$ corresponds to \mathbf{p}_i in the superordinate reference frame, $\mathbf{P}_0 = (X_0 \ Y_0 \ Z_0)^T$ is the translation vector and \mathbf{Q} represents the rotation sequence of the unit quaternion $\mathbf{q} = q_0 + \mathbf{q}$ with the scalar part q_0 and the imaginary part $\mathbf{q} = \varsigma q_1 + \xi q_2 + \zeta q_3$, with $\varsigma^2 = \xi^2 = \zeta^2 = \varsigma \xi \zeta = -1$, (Nitschke and Knickmeyer, 2000; Lösler and Nitschke, 2010), i.e.

$$\mathbf{Q} = (q_0^2 - \mathbf{q}^T \mathbf{q}) \mathbf{I} + 2(\mathbf{q} \mathbf{q}^T + q_0 [\mathbf{q} \times]), \quad (3)$$

with the skew-symmetric matrix

$$[\mathbf{q} \times] = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}, \quad (4)$$

where \mathbf{I} is the identity matrix.

In general, the main reflector is designed to be rotation-symmetrical. If $a \neq b$ the paraboloid is deformed w.r.t. the ideal design and the focal point degenerates to a focal line. By setting $a = b$, the ordinary elliptic paraboloid becomes an ordinary rotational paraboloid (ORP), i.e.

$$a^2 (x_i^2 + y_i^2) = z_i, \quad (5)$$

and the focal length f reads

$$f = \frac{1}{4a^2}. \quad (6)$$

The simplification of Eq. (1) is used by many groups as mathematical model, e.g. Sarti et al. (2009); Holst et al. (2012); Kallio et al. (2015), even though a difference between a and b impairs the receiving properties significantly.

3 Unified Model of Radio Telescope Main Reflector

Following the VLBI2010 agenda (Niell et al., 2006), a new generation of radio telescopes was designed. These so-called VGOS radio telescopes are much more com-

pact, i.e. the diameter of the main reflector is about 12 m, and faster, i.e. 12°/s in azimuth and 6°/s in elevation (Petrachenko et al., 2009). Moreover, the design of the main reflector is improved, in contrast to conventional radio telescopes. In conventional radio telescopes, the feed obstructs the path and results in fields of low intensity. As shown by Cutler (1947) this shading effect can be reduced by an improved reflector design. By stretching the paraboloid at the principal axis, the vertex as well as the focal point become circles (Prata et al., 2003). This design is known as ring-focus paraboloid.

In recent years, a lot of VGOS radio telescopes have been planned, were under construction or have already been installed. Most of these telescopes use the improved ring-focus design (Neidhardt et al., 2011; Gómez-González et al., 2014; Ipatov et al., 2015; Helldner et al., 2015). Due to the new main reflector design, a generalized mathematical model is needed to describe the surface because Eq. (1) as well as Eq. (5) are not suitable for ring-focus telescopes.

Therefore, we here propose an extended model of Eq. (1), which represents an elliptic ring-focus paraboloid (ERFP). The extended mathematical model is given by

$$a^2 (x_i - r n_{x,i})^2 + b^2 (y_i - r n_{y,i})^2 = z_i, \quad (7)$$

where r is the radius of the vertex circle and $\mathbf{n}_i = (n_x \ n_y)^T$ is the normalized vector that points in the direction of the elliptic paraboloid. The vector \mathbf{n}_i is not an unknown parameter because this vector can be expressed as a function of the point \mathbf{p}_i by substituting

$$n_{x,i} = \frac{x_i}{\sqrt{x_i^2 + y_i^2}} \quad (8)$$

and

$$n_{y,i} = \frac{y_i}{\sqrt{x_i^2 + y_i^2}}, \quad (9)$$

respectively.

In analogy to Eq. (5), Eq. (7) becomes a rotational ring-focus paraboloid (RRFP) by setting $a = b$, i.e.

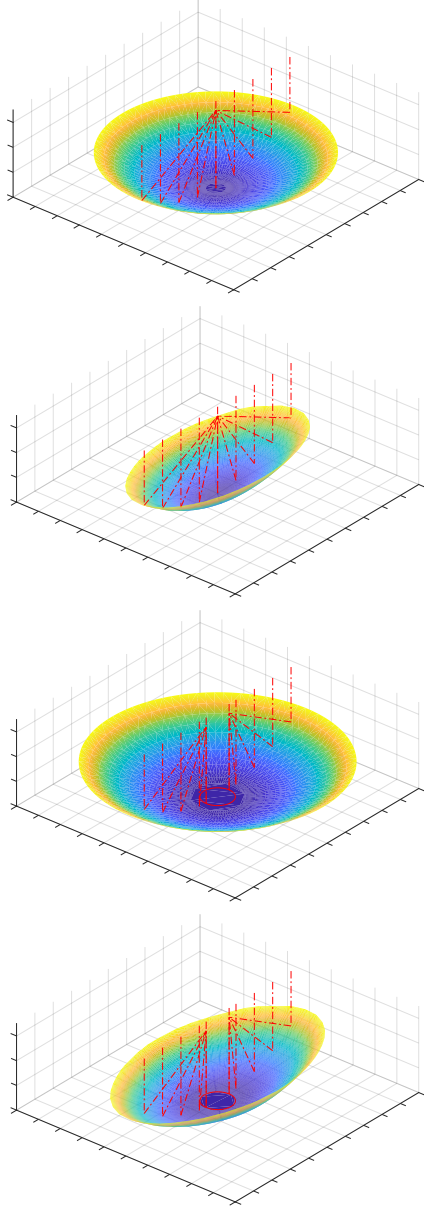
$$a^2 ((x_i - r n_{x,i})^2 + (y_i - r n_{y,i})^2) = z_i, \quad (10)$$

with focal length $f = \frac{1}{4a^2}$, cf. Eq. (6).

The generalized mathematical model (7) becomes universal because four types of paraboloids can be described, cf. Table 1, and the parameters of conventional radio telescopes as well as VGOS related radio telescopes with ring-focus can be fitted. Figure 1 depicts the four estimable paraboloid types.

Table 1: Paraboloid types that can be expressed by the universal model (7) depending on the form-parameter a , b and r .

	$a = b$	$a \neq b$
$r = 0$	ordinary rotational	ordinary elliptic
$r \neq 0$	rotational ring-focus	elliptic ring-focus

**Fig. 1:** Paraboloid types (from top to bottom): ordinary rotational paraboloid (ORP), ordinary elliptic paraboloid (OEP), rotational ring-focus paraboloid (RRFP) and elliptic ring-focus paraboloid (ERFP), and ray path lines (red dashed lines).

4 Parameter Estimation

As well-known from linear algebra, Eq. (1) is a special form of a quadric surface specified by

$$\mathbf{P}_i^T \mathbf{U} \mathbf{P}_i + \mathbf{P}_i^T \mathbf{u} + u_0 = 0, \quad (11)$$

where \mathbf{U} is a symmetric matrix that contains the elements of the so-called quadratic function

$$\mathbf{U} = \begin{pmatrix} u_1 & \frac{u_4}{\sqrt{2}} & \frac{u_5}{\sqrt{2}} \\ \frac{u_4}{\sqrt{2}} & u_2 & \frac{u_6}{\sqrt{2}} \\ \frac{u_5}{\sqrt{2}} & \frac{u_6}{\sqrt{2}} & u_3 \end{pmatrix} \quad (12)$$

and the vector \mathbf{u} contains the coefficients

$$\mathbf{u}^T = (u_7 \ u_8 \ u_9) \quad (13)$$

of the quadric (Drixler, 1993). To eliminate the mixed terms in Eq. (11), each quadric can be transferred into a certain normal form by a principal axis transformation. As a result of the principal axis transformation the translation vector \mathbf{P}_0 as well as the quaternion q , which defines the rotation sequence, are determined, cf. Eq. (2). Moreover, the type of the quadric can be classified and specific form-parameters are estimable.

The big advantage of using Eq. (11) instead of Eq. (1) is the bilinear normal equation systems. Thus, rough approximation values are sufficient to solve the unknown parameters u_i , with $i = 0 \dots 9$. On the other hand, the drawback of Eq. (11) is the universal scope, i.e. the lack of specifying the desired form-type. Beside the paraboloid, Eq. (11) describes sixteen further forms like plane, ellipsoid, cylinder or cone (Khan, 2010). If the observed point cloud is close to the vertex, the quadric surface may describe an ellipsoid instead of a paraboloid (Drixler, 1993; Lösler and Nitschke, 2010). Thus, explicit formulated models like Eq. (1) are needed to force the desired form-type. Furthermore, Eq. (7) is out of scope of Eq. (11). For this reason, the use of Eq. (11) is only recommended for deriving appropriate approximation values for the least-squares adjustment. Of course, for some radio telescopes an approximation of the focal length f can be taken from the main reflector design and initial spatial transformation parameters \mathbf{P}_0 and q can be predicted by the measurement setup (Sarti et al., 2009; Holst et al., 2012). But in the framework of reverse engineering, advanced information is seldomly available (Dutescu et al., 2009) and, thus, appropriate approximation values are important to ensure convergence.

To solve Eq. (7) within a least-squares adjustment, an error in variables model (EIV) is needed. Such a EIV model can be expressed as mixed model, also known as Gauß-Helmert model (Koch, 2014; Lösler et al., 2016). Due to the different paraboloid types, cf. Table 1, a Gauß-Helmert model with restrictions is recommended (Casparly and Wichmann, 2007; Lösler and Nitschke, 2010). Table 2 summarizes the required restrictions to switch over the four models.

Table 2: Restrictions to transform the universal model of an elliptic ring-focus paraboloid (ERFP) to its simplifications, i.e. an ordinary rotational paraboloid (ORP), an ordinary elliptic paraboloid (OEP) and a rotational ring-focus paraboloid (RRFP).

Restriction / Type	ORP	OEP	RRFP	ERFP
$a - b = 0$	×	–	×	–
$q_1 q_2 + q_0 q_3 = 0$	×	–	×	–
$r = 0$	×	×	–	–
$\mathbf{q}^T \mathbf{q} = 1$	×	×	×	×

5 Onsala twin telescopes

The Onsala Space Observatory is located 45 km south of Göteborg at the Swedish west coast. Since the end of the 1960s, the observatory has been participating in numerous geodetic and astrometric VLBI campaigns.

The observatory is part of the International VLBI Service (IVS). In 2016, the building phase of the Onsala twin telescopes (OTT) project started. This project includes two VGOS specified radio telescopes, named OTT-N and OTT-S. The inauguration and start of operation of the two new VGOS systems is planned for 2017 (Haas, 2013; Elgered et al., 2017).

Figure 5 depicts the new VGOS-type radio telescopes at Onsala Space Observatory during the construction phase 2016.



Fig. 2: The Onsala twin telescopes – two identical VGOS-type radio telescopes (Photo: Roger Hammargren).

Continuous quality inspections, which prove the compliance with the specification e.g. the focal length or the alignment of the panels, were carried out by MT Mechatronics GmbH. The main reflector was observed using high precision close-range photogrammetric methods, cf. Fig. 3. The measurement accuracy is specified as $5 \mu\text{m} + 5 \mu\text{m/m}$ by the manufacturer (personal communication MT Mechatronics GmbH). In total, 224 representative adjustment points were measured. These points are distributed on six rings on the main reflector,

cf. Fig. 4. For an independent validation of the data, the 224 discrete points were introduced to the proposed mathematical model (7) and the form-parameters were derived. It should be noted that the panel adjustment is not finalized yet. Thus, the derived results must be interpreted as preliminary results but they prove the high-grade execution of construction work of the mechanical engineers.



Fig. 3: Observing representative points on the main reflector of OTT-S using high precision close-range photogrammetric methods (Photo: Rüdiger Haas).

The estimated results are presented in Table 3 and Table 4 for OTT-N and OTT-S, respectively. The approximation values were derived by Eq. (11) using a principal axis transformation. For both radio telescopes, the form-parameters a and b of the ERFP differ in a range of about $2.2e - 5$, which corresponds to a focal length variation of about 0.6 mm, cf. Eq. (6).

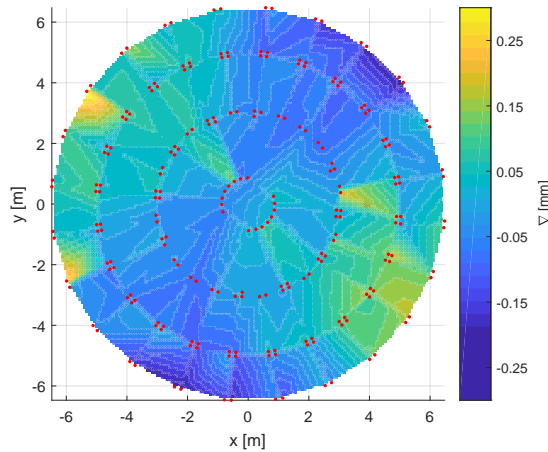
A further analysis was carried out by applying the restriction for a RRFP type radio telescope, cf. Table 2, and the parameter a was transformed to the focal length f by Eq. (6). The overall RMS of the RRFP adjustment was $82 \mu\text{m}$ and $102 \mu\text{m}$ for OTT-N and OTT-S, respectively. As an example, Figure 4 depicts the estimated deviations for OTT-N, i.e. the signed orthogonal distance ∇ , between the observed points and the estimated RRFP surface.

Table 3: Approximation values and adjustment results of OTT-N for ERFP and RRFP paraboloid types, respectively.

Parameter	Approx	ERFP	RRFP
X_0	+4.507 m	+0.2187 m	+0.2187 m
Y_0	-0.007 m	+0.0296 m	+0.0296 m
Z_0	-1.746 m	-1.7203 m	-1.7203 m
q_0	+0.30447	+0.458686	+0.313149
q_1	-0.64258	-0.542601	-0.638285
q_2	-0.30107	-0.457090	-0.309742
q_3	-0.63540	-0.535032	-0.631340
a	+0.21611	+0.259967	+0.259955
b	+0.21615	+0.259945	+0.259955
r	+0 m	+0.7402 m	+0.7402 m
f	–	–	+3.6995 m

Table 4: Approximation values and adjustment results of OTT-S for ERFP and RRFP paraboloid types, respectively.

Parameter	Approx	ERFP	RRFP
X_0	+4.403 m	+0.7902 m	+0.7902 m
Y_0	-0.224 m	-0.4263 m	-0.4263 m
Z_0	-1.938 m	-3.3518 m	-3.3518 m
q_0	+0.82484	+0.825731	+0.824836
q_1	+0.05458	+0.027741	+0.056183
q_2	-0.56150	-0.563408	-0.561274
q_3	+0.03716	-0.003509	+0.038230
a	+0.22583	+0.259930	+0.259933
b	+0.22574	+0.259937	+0.259933
r	+0 m	+0.7396 m	+0.7396 m
f	–	–	+3.7001 m

**Fig. 4:** Deviations, i.e. the signed orthogonal distance ∇ , between the observed points (red dots) and the estimated rotational ring focus paraboloid surface of the OTT-N radio telescope.

To evaluate the influence of the radius r of the ring-focus design on the focal length f , the correlation coefficient can be used (Casparly and Wichmann, 2007):

$$\rho_{r,f} = \frac{\text{cov}(r,f)}{\sigma_r \sigma_f}. \quad (14)$$

The correlation coefficient results in values between -1 and $+1$ and describes the linear dependence between two parameters. Whereas $\rho = 0$ implies no linear correlations, a coefficient of $\rho = \pm 1$ represents a total dependence. The estimated $\rho_{r,f} \approx 0$ and, therefore, the dependence of both parameters is negligible.

Up to now, no advanced information was used during the analysis process, which is usual in reverse engineering. To evaluate the influence of a known radius \tilde{r} of the ring-focus design on the focal length f , the form-parameter can be restricted. The specified radius of the Onsala twin telescopes is $\tilde{r} = 0.74$ m and the further restriction reads

$$\tilde{r} = r. \quad (15)$$

Applying Eq. (15), the focal length becomes $f = 3.6997$ m for both radio telescopes. Compared to the intended focal length of $\tilde{f} = 3.7$ m, this value demonstrates the high quality of the pre-adjustment of the main reflectors.

6 Conclusion

In recent years, many research groups investigated on the force-deformation behavior of the main reflector of radio telescopes. Most of these investigations focused on the surface deformation or on the variations of the focal length. Technical innovations were introduced to the measurement process to achieve reliable results, and improvement of computer technology was used to process large amounts of data, but the mathematical model was kept unchanged.

Due to the reflector design improvements, the surface of the main reflector of the upcoming VGOS radio telescopes cannot be parameterized by an ordinary rotational paraboloid. The goal of our investigation was to formulate a unified mathematical model that describes the previous surface design as well as the ring-focus reflector design. The proposed data analysis concept is a two-stage process. Firstly, appropriate approximation values are derived by a quadric surface using the principal axis transformation. Thus, advanced information about the paraboloid type becomes unnecessary. Especially in the framework of reverse engineering it is important to have a self-provided and independent algorithm because prior information is maybe incomplete or rare. The second step is the adjustment process using the unified model, Eq. (7). Additional restriction can be introduced to switch over four surface types including the simplest case, i.e. an ordinary rotational paraboloid, and the most complex case, i.e. an elliptic ring-focus paraboloid.

At Onsala Space Observatory two VGOS-type radio telescopes are under construction, which are identical in design. For construction supervision several quality inspections were carried out to prove the compliance with the specifications. Two photogrammetric data sets were provided by MT Mechatronics GmbH. These data sets were used to verify the proposed algorithm and to derive independent results. Since the panel adjustment is not finished yet, the derived results must be interpreted as preliminary results. However, the results proved the high-grade execution of construction work of the mechanical engineers up to now.

Acknowledgements We gratefully acknowledge MT Mechatronics GmbH for supporting this research by providing two photogrammetric data sets of the Onsala twin telescopes.

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