Analysis of Flexural Behaviour of Reinforced FRC Members

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ABSTRACT

In the present paper, the flexural behaviour and crack propagation in reinforced FRC members are analysed and discussed, this is also compared with conventional reinforced concrete members. Special attention is given to how the combined effect of fibre bridging and reinforcement bars influence the structural behaviour in the serviceability- and ultimate limit state. The effects of the fibre bridging are investigated by means of analytical models and finite element analyses, both based on non-linear fracture mechanics and uni-axial material properties.

Key words: Fibre reinforced concrete, flexural behaviour, analytical model, non-linear hinge, fracture mechanics.

1. INTRODUCTION

Industrialisation of the building industry is currently a very important topic and fibre reinforcement, as a replacement for ordinary reinforcement, could play an important role in this development. In some types of structures, like slabs on grade, foundations, and walls, fibres have the possibility to replace the ordinary reinforcement completely, while in other structures such as beams and slabs, fibres can be used in combination with pre-stressed or ordinary reinforcement. In both cases the potential benefits are due to economical factors, but also to rationalisation and improvement of the working environment at the construction sites. From a structural viewpoint, the main reason for incorporating fibres is to improve the cracking characteristics by the fibres ability to bridge across cracks. This mechanism influences both the serviceability- and the ultimate limit state. Service load behaviour, possibly the most difficult and least well understood aspect of the design of concrete structures, depends primarily on the properties of the concrete and the fibre bridging (the stress-crack opening relationship), which, at the design stage, are often not known reliably. However, with a rational design approach it would be possible to identify material properties to achieve optimal behaviour and cost effective performance.

Design and analysis models are available for performing cross-sectional analysis; these can be either numerical (e.g. FEM) or analytical. The proposed analytical models are based on different assumptions regarding kinematic (e.g. whether the crack surfaces remain plane or not) and constitutive conditions (e.g. the stress-crack opening relationship in tension and stress-strain relationship in compression); see RILEM TC 162-TDF ([1] and [2]). Depending on how the tensile response is represented two approaches exists, *viz.* the stress-strain (σ-ε) approach and the stress-crack opening (σ-w) approach. For the stress-strain approach two possibilities exists: to use the uni-axial behaviour or to represent the post-peak material behaviour with equivalent, or residual, flexural tensile strengths, which are determined from a three-point bending test (see e.g. [2]). Analysis can also be carried out by describing the cracked section as a non-linear hinge, as initially proposed by Ulfkjær *et al.* [3], and later by Pedersen [4], Cassanova & Rossi [5], and Olesen [6], based on a stress-crack opening relationship. The main drawback with the stress-strain approach is how the stress-strain relationship should be determined without violating the true fracture behaviour. Does the influence length (used to transform the stress-crack opening relationship into a stress-strain relationship) depend on the type of fibre, on loading conditions, on geometry, or, in the case where conventional reinforcement is combined with fibres, the average crack spacing? Furthermore, the equivalent, or residual, flexural strength is size dependent, as it is determined from a beam with a specific depth, and a size factor has to be introduced if the member depth is larger than the tested beam. There is also a disadvantage when designing structural members with a combination of bending moments, axial load, and restraint forces as the flexural strength is evaluated on a three-point bending specimen where no axial force is present. Also the stress-crack opening approach has its drawbacks: how to determine the length of the non-linear hinge; and are the kinematic assumptions correct?

Most of the proposed analytical models focuses on sections without conventional reinforcement; for example the fracture-mechanics based design approach for fibre-reinforced tunnel linings proposed by Nanakorn & Horii [7], or models for performing inverse analysis to determine material properties from a three-point bending test, see RILEM TC 162-TDF [1], Kitsutaka [8], and Stang & Olesen [9]. Moreover, most of the suggested models for reinforced FRC members are design models to predict the maximum moment resistance and do not provide much information of the crack propagation stage. Often the shape of the tensile softening relationship is ignored, to simplify analysis a plastic stress distribution is assumed, and the possible negative effect of a normal force acting on the cross section are also neglected in several models. Olesen [10] developed a non-linear hinge model to also consider de-bonding between reinforcement and concrete, the length of the hinge is the average crack spacing (which is a function of the load and changes during analysis), but the stress-crack opening relationship adopted is a drop-constant and in compression the concrete behaves elastic. Barros & Figueiras [11] proposed a layered approach for the analysis of SFRC cross sections under bending and axial forces. The model was based on a stress-strain concept, with a bi-linear tension softening relationship and a non-linear stress-strain relationship in compression. The fracture energy, G_F, together with the average crack spacing was used to determine the tension softening relationship. Furthermore, the tension-stiffening phenomenon was considered with a cracked reinforced concrete tie stiffening the reinforcement. The approach taken here is based on non-linear fracture mechanics, the fictitious crack (or cohesive crack) as suggested by Hillerborg *et al.* [12] and Hillerborg [13], and the concept of the non-linear hinge.

2. MODEL FOR FLEXURAL BEHAVIOUR

2.1 Analytical model for flexural behaviour for reinforced FRC members

The non-linear hinge approach, as described above, can also be used for beams with a combination of conventional reinforcement and fibres, see Figure 1. Based on the recommendations of RILEM TC 162-TDF [1], a simplified model for sectional analysis, based on the non-linear hinge concept, can be established. The following assumptions are introduced:

- the cross-section is subjected to a bending moment, M, and a normal force, N (no long-term effects are considered);
- the average strain in the reinforcement is related to the average elongation of the hinge (at the level of the reinforcement);
- tension stiffening and the distribution of stresses between the cracks is not considered at this stage;
- the length of the non-linear hinge is set to the average crack spacing;
- the crack surfaces remain plane and the crack opening angle equates the overall angular deformation of the non-linear hinge;
- a non-linear stress-strain relationship in compression (according to CEB-FIP MC90 [14]);
- a fictitious crack (or cohesive crack) is assumed with a bi-, poly- or non-linear tension softening relationship; and
- a bi-linear stress-strain relationship for the reinforcement.

The cross-sectional response can be determined through an iterative approach where the rotation for the considered cross section (see Figure 1) is increased and in each step the position of the neutral axis is determined by solving the equilibrium equation of sectional forces Equation (6) and the corresponding bending moment Equation (7) is calculated for each step.

The average curvature, κ_{m}, of the non-linear hinge is given by:

$$\kappa_m = \frac{\theta}{s} \tag{1}$$

The crack mouth opening displacement, w_{CMOD}, can be related to the crack opening angle, θ^*, and the length of the crack, a:

$$w_{CMOD} = \theta^* \cdot a \tag{2}$$

The average strain in the reinforcement is calculated as:

$$\varepsilon_s = \frac{\theta}{s} \cdot (d_1 - y_0) \tag{3}$$

The compressive strain in the concrete is calculated as:

$$\varepsilon_c = \frac{\theta}{s} \cdot (y - y_0) \tag{4}$$

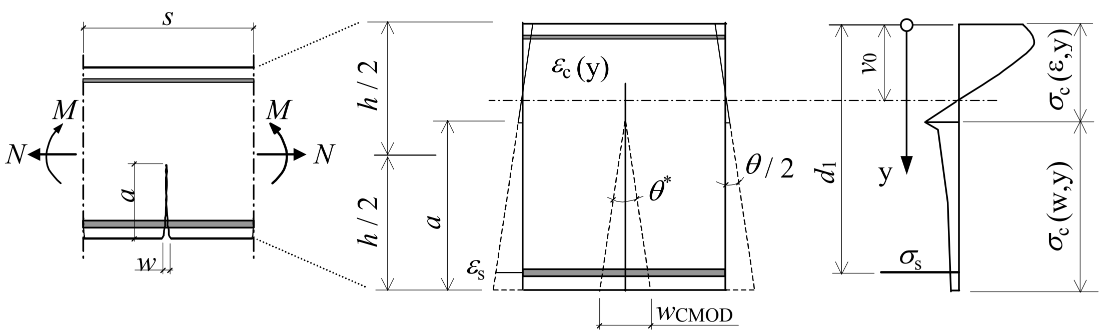

Figure 1. Regional analysis of beam/slab subjected to constant bending moment, the non-linear hinge and the stress distribution in a cracked section.

When the crack surfaces remain plane the overall angular deformation of the hinge, θ, is equal to the crack opening angle, θ^*. The crack mouth opening displacement, w_{CMOD}, can then be related to the depth of the neutral axis, y_0, the overall angular deformation of the hinge, θ, the tensile strength, f_t, the modulus of elasticity, E, the normal force, N, the cross-sectional area, A, and the length of the non-linear hinge, s, by:

$$w_{CMOD} = \theta(h - y_0) - \left(\frac{f_t}{E} - \frac{N}{A \cdot E}\right)s \tag{5}$$

Based on these assumptions and the stress distribution in Figure 1, the sectional forces can be written as:

$$\frac{N}{b} = \int_0^{h-a} \sigma_c(\varepsilon, y)dy + \int_{h-a}^{h} \sigma_f(w, y)dy + \sigma_s \cdot A_s \tag{6}$$

$$\frac{M}{b} = \int_0^{h-a} \sigma_c(\varepsilon, y)\left(y - \frac{h}{2}\right)dy + \int_{h-a}^{h} \sigma_f(w, y)\left(y - \frac{h}{2}\right)dy + \sigma_s \cdot A_s \cdot \left(d_1 - \frac{h}{2}\right) \tag{7}$$

2.2 Numerical model - Finite element analysis

A more general approach, than the analytical models, is the finite element method; were it is possible to take into account the effects of bond-slip, cracking, multi-axial stress states, the stress distribution between cracks, etc. The analysis, however, becomes, for larger structures, time consuming. On the other hand, an analysis can be used to investigate and check the assumptions of the non-linear hinge model and to compare the result.

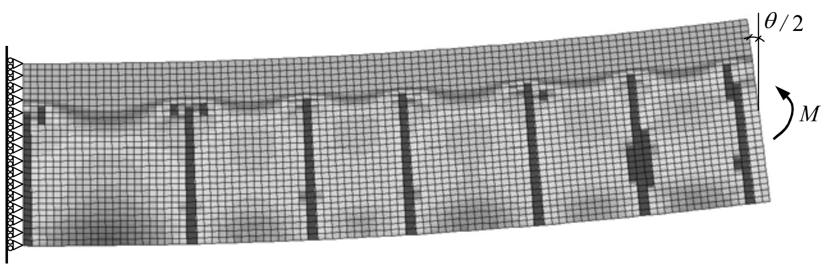

Figure 2. Detailed analysis of a beam segment for regional behaviour. The figure shows the longitudinal strain in the concrete, the dark regions indicate cracks.

3. ANALYSIS OF FLEXURAL BEHAVIOUR OF SLABS

3.1 Introduction

We consider a slab, 250 mm thick and 1 m wide, having 0.1% reinforcement placed 225 mm from the top of the slab. If otherwise not stated the following material properties have been used. For the concrete: tensile strength f_t = 2.5 MPa, compressive strength f_c = 38 MPa, and modulus of elasticity E_c = 30 GPa. For the reinforcement: a yield stress f_y = 500 MPa, a tensile strength f_u = 550 MPa (ε_u = 6%), and the elastic modulus E_s = 200 GPa. Furthermore, the FRCs were simulated by a bi-linear stress-crack opening relationship in tension (see Figure 3).

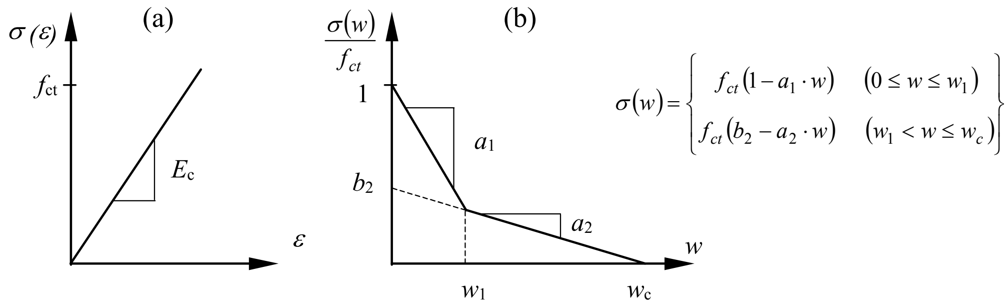

Figure 3. Definition of constitutive parameters used in the cohesive crack model. The material is assumed linear elastic in the pre-cracked region, (a), while a bi-linear relationship is used for the cracked region, (b).

3.2 Estimation of the length of the non-linear hinge

A reasonable assumption has to be made regarding the length of the non-linear hinge, *s*. As can be seen in Figure 4, the length, *s*, does not influences the maximum moment resistance to any large extent. However, the length has a large influence in the pre- and post-peak stages; a long hinge leads to lower moment for a given rotation and a short hinge length increases the ductility.

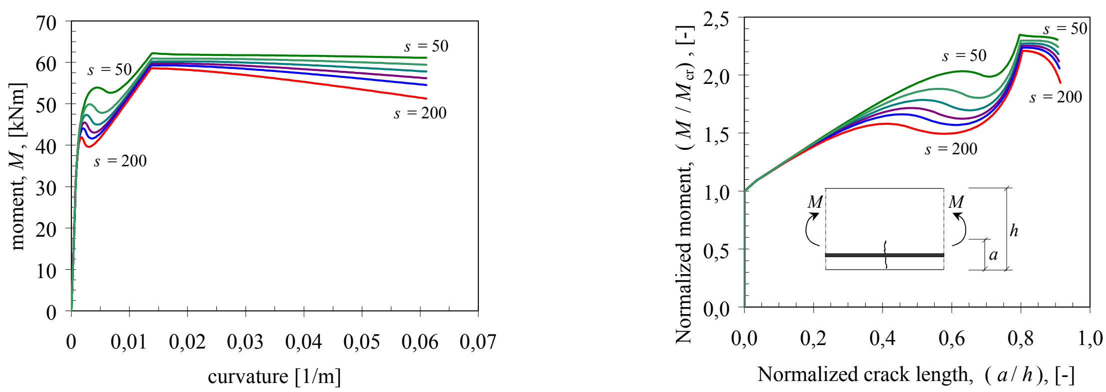

Figure 4. Effect of the length of the hinge on (a) the moment-curvature relationship and (b) the crack propagation. Length of the hinge, s: 50, 75, 100, 125, 150, and 200 [mm]. a_1=10 [mm^{-1}], a_2=0.025 [mm^{-1}], b_2=0.5.

To check the assumption of the hinge length a comparison was made between the non-linear hinge model and FE-analyses (see Figure 2). The FE-analyses were performed with the program DIANA (version 8.1 [15]). In the analyses, a smeared crack approach were used, the crack band with, *h*, were, for the FRC, set to 12.5 and 25 mm (two respectively four elements) as cracking did not localize in one element. The bi-linear stress-crack opening relationship was transformed into a stress-strain relationship by dividing the crack opening with the crack bandwidth. Furthermore, bond-slip behaviour was modelled with interface elements, which were given a bond-slip relationship according to the CEB-FIP MC90 [14], assuming confined concrete with good bond conditions. As can be seen in Figure 5, the overall behaviour is predicted fairly well and the peak-moment also corresponds. However, the FE-analyses show a somewhat stiffer behaviour during the cracking stage and yielding occurs at a smaller rotation. This is expected as tension stiffening was ignored in the simplified non-linear hinge model. Furthermore, the difference at the first crack development is due to convergence problems in the FE-analysis.

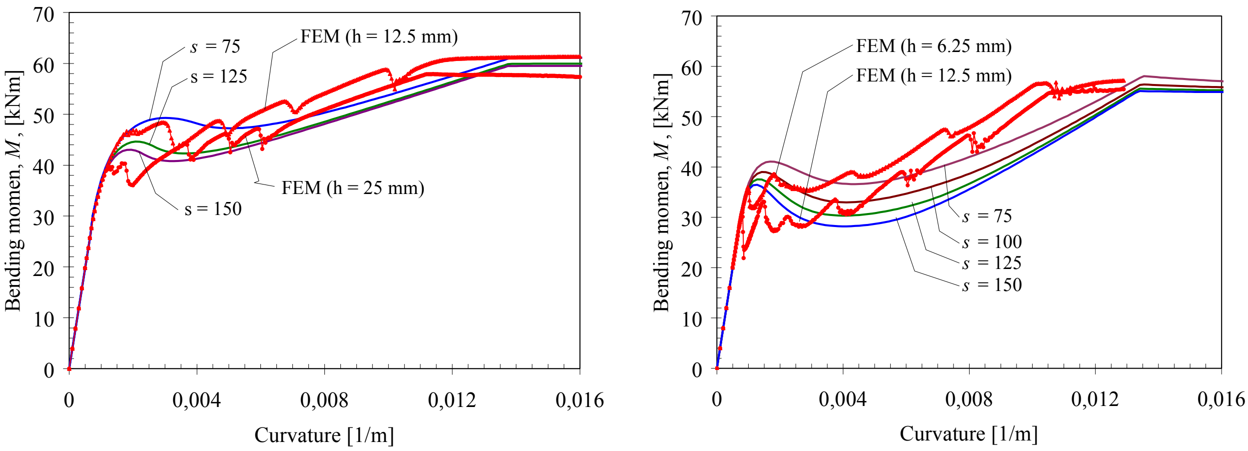

Figure 5. Comparison of the non-linear hinge model and FE-analysis, with different hinge lengths and crack bandwidths. (a) For fibre reinforced concrete with 0.1 % reinforcement and (b) for normal reinforced concrete with 0.2 % reinforcement, and a softening curve according to Cornelissen et. al. [16].

3.3 Influence of the stress-crack opening relationship

Figure 6 and Figure 7 shows how the parameters of the bi-linear stress-crack opening influence the moment-curvature relationship. As can be seen in Figure 6, a_1 mainly have an influence on the pre-peak stage until a critical value for a_1 is reached (corresponds to low values of a_1) after which it also influences the maximum moment resistance of the cross-section. Further, a_2 mainly influences the shape of the moment-curvature relationship after that the maximum moment of the cross-section is reached. For low values, the moment do not decrease with increasing rotation, but as a_2 is increased the maximum moment will be decreased and the moment decreases with increased rotation. The parameter b_2 has a large influence on the maximum moment and the pre-peak part; as b_2 increases the moment resistance is increased, as can be seen in Figure 7(a). The concrete quality (compressive and tensile strength and modulus of elasticity) mostly influences the maximum moment (which increases for concrete with a higher tensile and compressive strength), see Figure 7(b). But also the pre- and post-peak part of the moment-curvature relationship is affected; the drop at the first peak becomes steeper and larger for concrete with a higher strength, this is also valid for the post-peak stage.

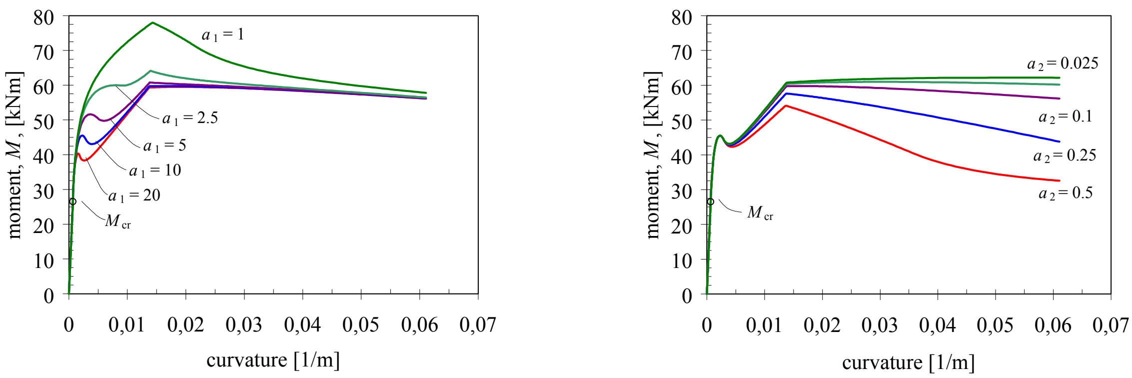

Figure 6. Effect of a_1 and a_2 on the moment-curvature relationship. (a) a_1: 20, 10, 5, 2.5, and 1, a_2=0.025 and b_2=0.5. (b) a_2: 0.5, 0.25, 0.1, 0.05, and 0.025, a_1=10 and b_2 =0.5.

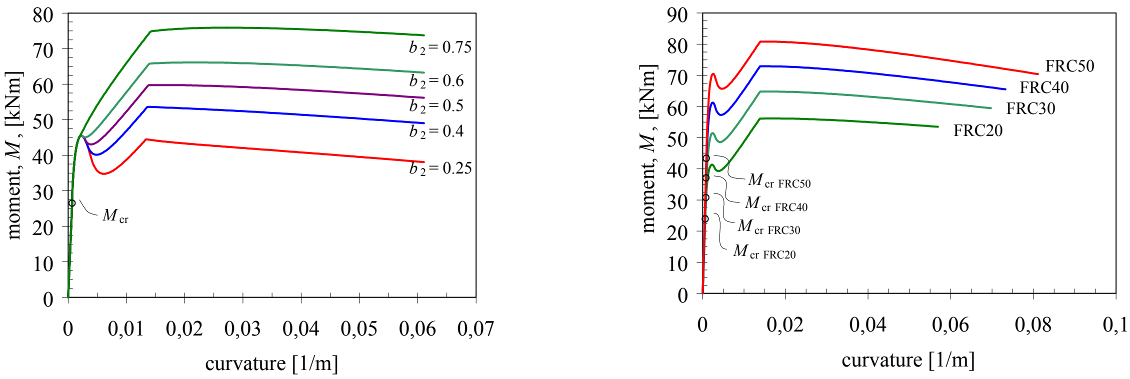

Figure 7. *Effect of* b_2 *and concrete quality on the moment-curvature relationship. (a)* b_2*: 0.25, 0.4, 0.5, 0.6, and 0.75,* a_1*=10 and* a_2*=0.025. (b) Concrete: C20 (*f_c*=28 MPa,* f_t*=2.25 MPa,* E_c*=29 GPa), C30 (*f_c*=38 MPa,* f_t*=2.9 MPa,* E_c*=30.5 GPa), C40 (*f_c*=48 MPa,* f_t*=3.5 MPa,* E_c*=35 GPa), and C50 (*f_c*=58 MPa,* f_t*=4.1 MPa,* E_c*=37 GPa).*

Figure 8 and Figure 9 shows how the parameters of the bi-linear stress-crack opening relationship influence the crack propagation, which is visualised as the normalized length of the crack. From the figures it can be seen that the peak-moment occurs at a crack length around 0.8 of the depth.

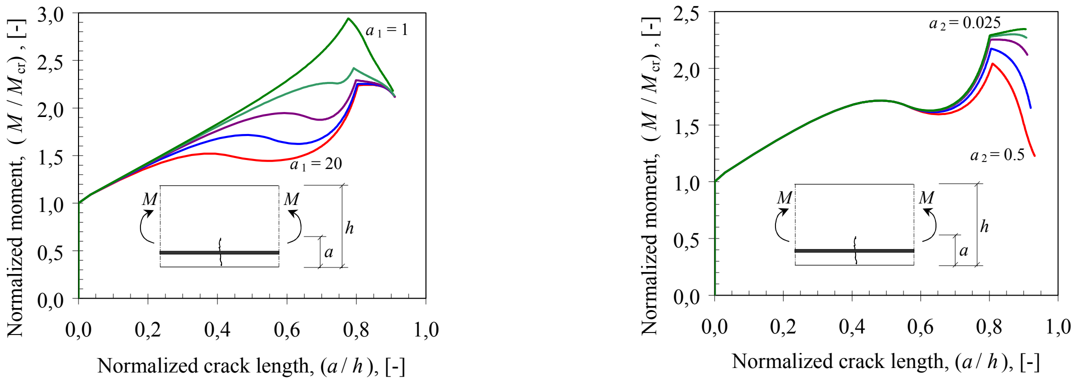

Figure 8. *Effect of* a_1 *and* a_2 *on the crack propagation. (a)* a_1*: 20, 10, 5, 2.5, and 1,* a_2*=0.025 and* b_2*=0.5. (b)* a_2*: 0.5, 0.25, 0.1, 0.05, and 0.025,* a_1*=10,* a_2*=0.025,* b_2*=0.5.*

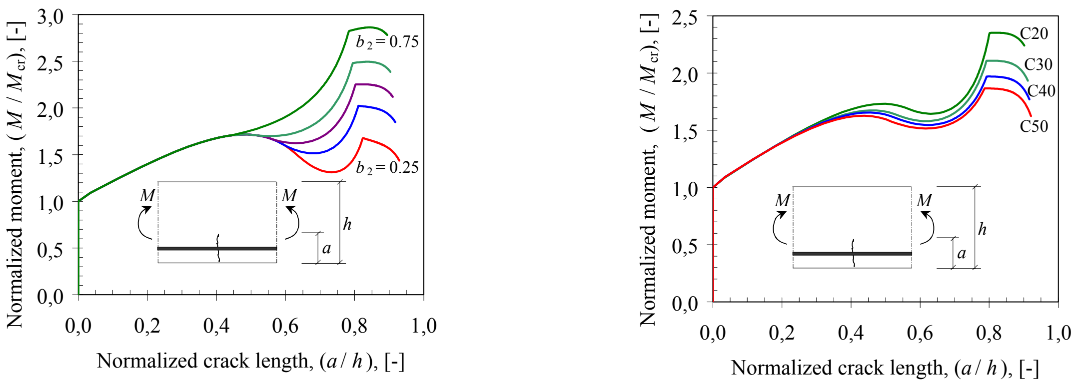

Figure 9. *Effect of* b_2 *and concrete quality on the crack propagation. (a)* b_2*: 0.2, 0.4, 0.5, 0.6, and 0.8,* a_1*=10 and* a_2*=0.025. (b) Concrete: C20, C30, C40, and C50.*

3.4 Comparison of conventional RC-members and FRC-members

In Figure 10 a comparison is made between a slab with conventional concrete and a fibre-reinforced slab. Different cases, providing the same maximum moment, have been investigated. The slab with conventional concrete was analysed considering the fracture energy, G_f, and with a shape of the softening curve according to Cornelissen *et. al.* [16]. Different reinforcement ratios, ρ, were investigated as well as reinforcement with higher yield strength, f_y. For the FRC, four cases were investigated, for concretes with two different strength classes, with: $b_2 = 0.5$ and reinforcement with yield strength of either 500 or 600 MPa; $b_2 = 0.4$; and $b_2 = 0.25$. For the conventional concrete two cases were investigated for each type of concrete: reinforcement with yield strength of either 500 or 600 MPa. The reinforcement ratios, ρ, were chosen so that the moment resistance were the same for all the investigated cases.

As can be seen in Figure 10, the main differences between the conventional slab and the FRC slab are the increased moment resistance and stiffness. After crack initiation the crack propagates fast in the conventional concrete, to a height of 0.7 compared to 0.5 for the FRC (see Figure 11). Moreover, the flexural stiffness is larger for the FRC member, which would lead to less deflection for a fibre-reinforced member, see Figure 12.

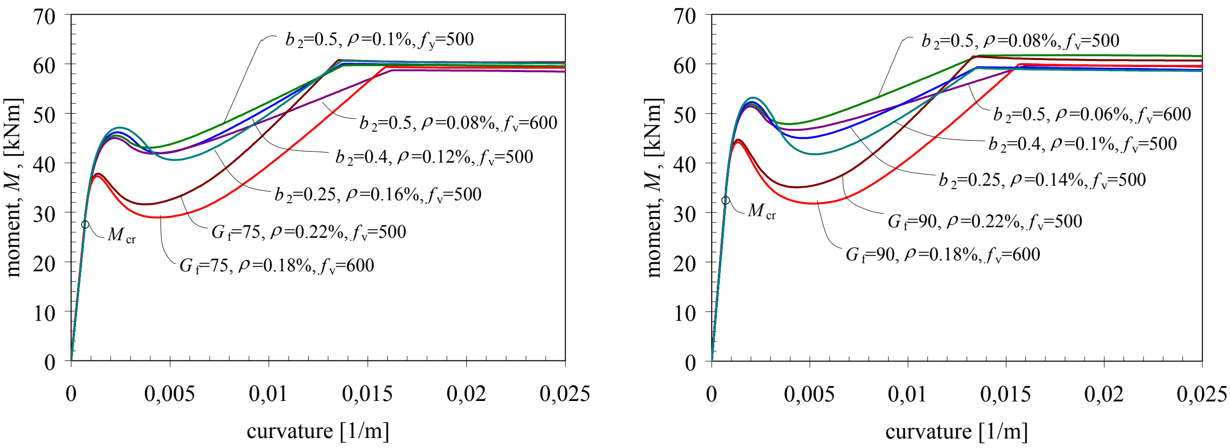

Figure 10. Comparison between FRC and conventional concrete. (a) f_c=38 MPa, f_t=2.5 MPa, E_c=30 GPa. (b) f_c=48 MPa, f_t=3.0 MPa, E_c=35 GPa.

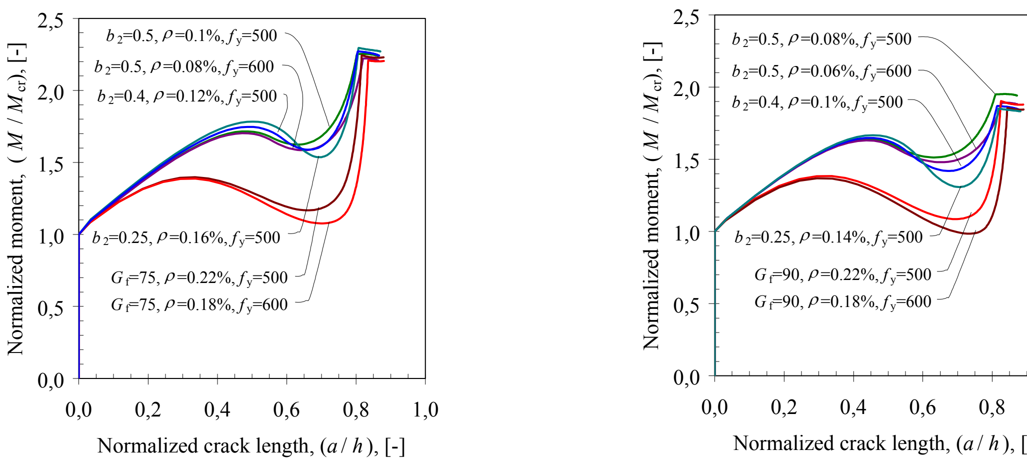

Figure 11. Comparison between crack propagation in FRC and conventional concrete. (a) f_c=38 MPa, f_t=2.5 MPa, E_c=30 GPa. (b) f_c=48 MPa, f_t=3.0 MPa, E_c=35 GPa.

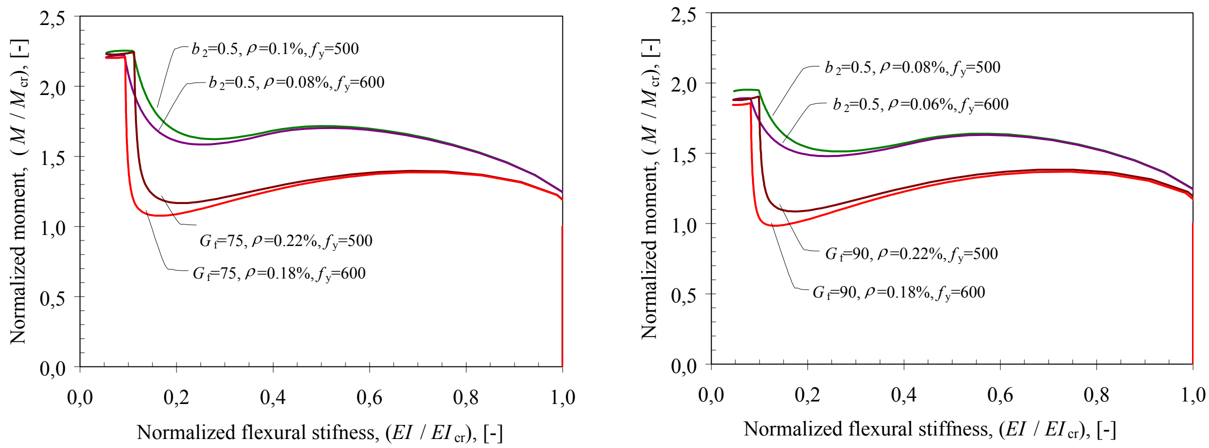

Figure 12. Comparison between flexural stiffness of FRC and conventional concrete. (a) f_c=38 MPa, f_t=2.5 MPa, E_c=30 GPa. (b) f_c=48 MPa, f_t=3.0 MPa, E_c=35 GPa.

4. CONCLUSIONS

With a simplified analytical model it was possible to analyse the flexural behaviour and to investigate how the shape of the stress-crack opening relationship, the material properties of the concrete, the reinforcement ratio and yield strength of the reinforcement influences the flexural behaviour. The model assumptions, however, need to be verified with full-scale experiments and a more comprehensive numerical study. Furthermore, long-term effects, like creep and shrinkage, as well as the effects of tension stiffening should be included in the model.

It was shown that the shape of the tensile softening curve not only influences the maximum moment but also the crack propagation stage is highly influenced. In this study a bi-linear stress-crack opening relationship was used and from this it can be concluded that:

- The slope of the first part of the stress-crack opening relationship, a_1, mainly influences the crack propagation stage. This corresponds to the serviceability limit state.
- The slope of the second part of the stress-crack opening relationship, a_2, mainly influences the shape of the moment-curvature relationship after the peak-moment is reached. For higher values of a_2 (corresponding to short fibres, fibres breaking, or fibres with a poor pull-out behaviour) the maximum moment is reached early and decreases with increased curvature. This is not a preferred behaviour for continuous members where moment redistribution takes place.
- The parameter b_2 (related to the volume fraction and efficiency of the fibres) influences the moment level, i.e. a higher value leads to a higher moment resistance. However, for high values it leads to an almost elastic plastic behaviour.

The main difference between conventional RC-members and FRC-members is the increased stiffness during the initial crack propagation. As can be seen in Figure 10 & Figure 11; for the RC-member, after crack initiation the moment continue to increase until the flexural moment is about 1.4 times the cracking moment (at a crack height of about 0.3 to 0.4). After this point, the moment decreases until the crack has propagated to a height of 0.8 of the section depth and the moment starts to increase (i.e. the reinforcement starts to work). For the FRC-member, the moment continues to increase until a crack height of about 0.5, at which the flexural moment is about 1.6 to 1.7 times the cracking moment. After this point is reached the moment slightly decreases until the reinforcement starts to work and the moment increases again. Another difference is that the maximum moment is reached early in the moment-curvature relationship and after that the moment decreases with increased rotation.

The concluding remark is that models able to predict the service load behaviour as well as the post-peak stage are required because:

- the design requirements for the serviceability limit state are often governing; and
- development of mixes for a certain structural application requires knowledge of the structural behaviour both during the cracking stage and the post-peak stage.

The view held by the author is that this is best achieved through relating the uni-axial material properties to the structural behaviour by adopting a fracture mechanical model, based on for example the fictitious crack model. Moreover, the model could then be used as a rational methodology to identify the material properties to achieve optimal behaviour and effective performance to cost ratio of RFC-members, particularly if used in conjunction with micro-mechanical models for materials development, see Stang & Li [17].

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