### Massive MIMO with Low-Resolution Data Converters: Algorithm Design and Performance Evaluation

SVEN JACOBSSON



Department of Electrical Engineering Chalmers University of Technology Gothenburg, Sweden, 2017

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Department of Electrical Engineering Chalmers University of Technology SE-412 96 Gothenburg, Sweden Phone: +46 (0)31 772 1000 www.chalmers.se

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#### Abstract

Massive multiuser multiple-input multiple-output (MIMO) is foreseen to be a key technology in next-generation (5G) cellular communication systems, due to huge potential gains in spectral efficiency and energy efficiency. In this thesis, we investigate the performance of massive MIMO systems, which operate over a Rayleigh-fading channel, for the case when the base station (BS) is equipped with low-resolution data converters. More specifically, in the uplink the received signal at the BS is converted into the digital domain by a set of low-resolution analog-to-digital converters (ADCs). In the downlink, the transmit signal is generated by a set of low-resolution digital-to-analog converters (DACs).

First, we consider the narrowband massive MIMO uplink for the case when the BS is equipped with low-resolution ADCs. Our focus is on the case where neither the transmitter nor the receiver have any *a priori* channel state information (CSI), which implies that the fading realizations have to be learned through pilot transmission followed by channel estimation at the receiver, based on coarsely quantized observations. We derive a lowcomplexity channel estimator and present lower bounds and closed-form expressions for the achievable rates with the proposed channel estimator and linear detection algorithms.

Second, we consider the narrowband massive MIMO downlink for the case when the BS is equipped with low-resolution DACs. We derive lower bounds and closed-form expressions for the achievable rates with linear precoding under the assumption that the BS has access to perfect CSI. We also propose novel nonlinear precoding algorithms that are shown to significantly outperform linear precoders for the case of 1-bit DACs.

Finally, focusing on the case of 1-bit DACs and linear precoding, we extend our analysis to the case of frequency-selective channels and to oversampling DACs.

Our results suggest that the resolution of data converters in a massive MIMO system can be reduced significantly compared to what is used in today's state-of-the-art MIMO systems, without significant reductions in the overall system performance.

**Keywords:** massive MIMO, ADCs, DACs, quantization, hardware impairments, channel estimation, data detection, precoding, achievable rates, convex optimization

#### List of Publications

This thesis is based on the following publications:

- [A] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, "Throughput analysis of massive MIMO uplink with low-resolution ADCs", *IEEE Transactions* on Wireless Communications, vol. 16, no. 6, pp. 4038–4051, Jun. 2017..
- [B] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "Quantized precoding for massive MU-MIMO", to appear in *IEEE Transactions on Communications*, Jun. 2017,
- [C] S. Jacobsson, G. Durisi, M. Coldrey, and C. Studer, "Massive MU-MIMO-OFDM downlink with one-bit DACs and linear precoding", submitted to *IEEE Global Communications Conference (GLOBECOM)*, Apr. 2017.

Publications by the author not included in the thesis:

- [D] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, "One-bit massive MIMO: Channel estimation and high-order modulations", in *Proceedings of IEEE International Conference on Communications Workshop (ICCW)*, London, U.K., Jun. 2015, pp. 1304–1309.
- [E] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "Nonlinear 1-bit precoding for massive MU-MIMO with higher-order modulation", in *Proceedings* of Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA, Nov. 2016, pp. 763–767.
- [F] O. Castañeda, S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "1-bit massive MU-MIMO precoding in VLSI", Submitted to *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, Feb. 2017.

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> Sven Jacobsson Gothenburg, June 2017

### Acronyms

ADC:	Analog-to-digital converter	
AWGN:	Additive white Gaussian noise	
BER:	Bit error rate	
BS:	Base station	
CSI:	Channel state information	
DAC:	Digital-to-analog converter	
DPC:	Dirty-paper coding	
LTE:	Long-term evolution	
MRC:	Maximal-ratio combining	
MRT:	Maximal-ratio transmission	
MIMO:	Multiple-input multiple-output	
MMSE:	Minimum mean square error	
OFDM:	Orthogonal frequency-division multiplexing	
OOB:	Out-of-band	
QPSK:	Quadrature phase-shift keying	
RF:	Radio frequency	
SIC:	Successive interference cancellation	
SNR:	Signal-to-noise ratio	
TDD:	Time-division duplexing	
UE:	User equipment	
ZF:	Zero-forcing	
ZOH:	Zero-order hold	

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# Part I Overview

### CHAPTER 1

#### Background

#### 1.1 Background

We are on the brink of living in a networked society, where everyone and everything that can benefit from being connected will be connected. Due to the enormous increase in the number of mobile devices, mobile high-definition video streaming services, and the emergence of the internet of things, we are witnessing an explosion in global mobile data traffic. In 2016 alone, global mobile data traffic grew by over 60% to exceed 7 exabytes per month [1,2]. Furthermore, this growth is not likely to slow down anytime soon, as monthly global mobile data traffic is predicted to surpass 49 exabytes by 2021 [2].

Since the emergence of the first generation cellular network in 1981, a new generation of cellular networks has appeared roughly every ten years, leading up to the first deployment of 4G in 2009. The fifth generation cellular network, 5G, has to meet the ever-increasing aggregate traffic demands from billions of connected devices, to enable fast and reliable access to information anywhere and at any time. In order to meet this demand, today's long term evolution (LTE) system embodying 4G has to be complemented with new, and possibly disruptive, technologies [3,4].

Multiple-input multiple-output (MIMO) technology, has been an active research area for over two decades, and is part of the current LTE standard [5]. Equipping cellular base stations (BSs) with a large number of active antenna elements compared to the number of active user equipments (UEs)—a system architecture solution often referred to as *massive* MIMO—is a potentially disruptive multiuser MIMO technology foreseen to be a key technology solution for 5G and beyond, which promises significant gains in terms of spectral efficiency, energy efficiency, reliability, and coverage compared to traditional small-scale MIMO systems [6-8].

Increasing, by orders of magnitude, the number of active antenna elements at the BS will, however, lead to significant increases in radio frequency (RF) circuitry power consumption and system costs. This calls for the use of low-cost and power-efficient hardware components, which will unavoidably reduce the signal quality due to an increased level of impairments. Therefore, practical deployment of massive MIMO systems will require novel design approaches that jointly reduce system costs and circuit power consumption, without degrading the spectral efficiency and reliability.

The specific focus of this thesis is on the impact of equipping the BS with low-resolution data converters. More specifically, it is assumed that in the *uplink* (UEs transmit to the BS), low-resolution analog-to-digital converters (ADCs) are used to convert the received analog baseband signal into digital domain. Conversely, in the *downlink* (BS transmits to UEs), low-resolution digital-to-analog converters (DACs) are used to generate the transmit signal.

High-speed high-resolution data converters are power-hungry devices [9, 10]. Hence, architectures involving low-resolution data converters are attractive for massive MIMO systems, where the total number of data converters could be in the order of hundreds or thousands. The question at the core of this thesis is whether the aforementioned massive MIMO gains, which were theoretically derived under the assumption of ideal hardware, survive in the presence of significant impairments due to low-cost data converter solutions.

#### 1.2 Scope of the Thesis

The scope of this thesis is to analyze the performance of massive MIMO systems for the case when the BS is equipped with low-resolution data converters. The specific objectives of this thesis are as follows:

- I To characterize the uplink and downlink throughput achievable in a massive MIMO system in the scenario when the BS employs low-resolution data converters.
- II To design low-complexity channel estimation and data detections algorithms that, together with modern coding techniques, are able to approach the uplink throughput unveiled in Objective I.
- III To develop low-complexity precoding algorithms that, together with modern coding techniques, are able to approach the downlink throughput unveiled in Objective I.

#### 1.3 Organization of the Thesis

Part I of the thesis is organized as follows. In Chapter 2, the multiuser MIMO system model is introduced both for the case of uplink and downlink transmission. In Chapter 3,

the basic operations of ADCs and DACs are presented. In Chapter 4, the contributions of the papers attached to the thesis are summarized.

### 1.4 Notation

This section describes the notation used in Part I of this thesis. Lowercase and uppercase boldface letters designate column vectors and matrices, respectively. For a matrix  $\mathbf{A}$ , its complex conjugate, transpose, and Hermitian transpose is denoted  $\mathbf{A}^*$ ,  $\mathbf{A}^T$ , and  $\mathbf{A}^H$ , respectively. The identity matrix of size  $N \times N$  is denoted by  $\mathbf{I}_N$  and the all-zero  $N \times 1$  vector is denoted by  $\mathbf{0}_N$ . The determinant of a matrix  $\mathbf{A}$  is denoted by  $\det(\mathbf{A})$ . The  $\ell_2$ -norm of a vector  $\mathbf{a}$  is  $\|\mathbf{a}\|$ . The operator  $\mathbb{E}_{\mathbf{A}}[\cdot]$  stands for the expectation over the random matrix  $\mathbf{A}$ . The complex-valued circularly-symmetric Gaussian probability density function with zero mean and covariance  $\mathbf{K} \in \mathbb{C}^{N \times N}$  is denoted by  $\mathcal{CN}(\mathbf{0}_N, \mathbf{K})$ .

### CHAPTER 2

#### Multiuser MIMO Systems

Throughout the thesis, a single-cell multiuser MIMO system, as illustrated in Fig. 2.1, is considered. Here, U single-antenna UEs communicate, in the same time-frequency resource, with a BS, which is equipped with  $B \ge U$  antenna elements. The wireless channel connecting the UEs to the BS is modeled as a memoryless block-fading channel, i.e., a channel that remains constant during a coherence interval of T consecutive symbol transmissions, before changing into a new independent realization. Let  $\mathbf{H} \in \mathbb{C}^{B \times U}$  denote the channel matrix connecting the U UEs to the B BS antennas. Throughout this thesis, the entries of  $\mathbf{H}$  are modeled as independent and  $\mathcal{CN}(0, 1)$ -distributed (Rayleigh fading).

The system operates in time-division duplexing (TDD) mode, which means that uplink and downlink transmissions take place in the same frequency spectrum but in different time slots. The TDD frame structure is illustrated in Fig. 2.2. During each coherence interval, in the uplink phase, the UEs transmit pilot symbols and data symbols. The pilots allow the BS to acquire channel state information (CSI), which is later used to detect the data symbols sent from the UEs, and to precode the data symbols to the UEs during the downlink phase.

In the remainder of this chapter, the BS is assumed to have access to perfect CSI, i.e., the BS knows perfectly the realizations of the channel matrix  $\mathbf{H}^{1}$ . For now, it is also assumed that the ADCs and DACs at the BS are ideal.

<sup>&</sup>lt;sup>1</sup>This assumption is relaxed in Paper A.



Figure 2.1: A single-cell multiuser MIMO system where U single-antenna UEs are served by a B-antenna BS in the same time-frequency resource.

#### Coherence interval

Uplink pilot	Uplink data	Downlink data
$\operatorname{transmission}$	$\operatorname{transmission}$	transmission

Figure 2.2: TDD frame structure. The UEs transmit pilots and data symbols in the uplink. The BS performs channel estimation and data detection in the uplink. The obtained channel estimates are then used to precode data symbols to the UEs in the downlink.

#### 2.1 Uplink Transmssion

In the uplink, the U single-antenna UEs transmit simultaneously to the BS. The discretetime complex baseband received signal at the BS,  $\mathbf{y}_{ul} \in \mathbb{C}^B$ , can be written as

$$\mathbf{y}_{\rm ul} = \sqrt{\rho_{\rm ul}} \mathbf{H} \mathbf{s}_{\rm ul} + \mathbf{n}_{\rm ul} \tag{2.1}$$

Here,  $\rho_{ul}$  is the uplink signal-to-noise ratio (SNR). The vector  $\mathbf{s}_{ul} \in \mathbb{C}^U$ , which has independent and identically distributed entries, denotes the transmitted symbols from the UEs. This vector must satisfy the average power constraint  $\mathbb{E}[\|\mathbf{s}_{ul}\|^2] \leq U$ . The vector  $\mathbf{n}_{ul} \sim \mathcal{CN}(\mathbf{0}_B, \mathbf{I}_B)$  is the additive white Gaussian noise (AWGN) at the BS.

With perfect CSI at the BS, the ergodic sum-rate capacity of the channel input-output model (2.1) is (see, e.g., [11])

$$C_{\rm ul} = \mathbb{E}_{\mathbf{H}} \left[ \log_2 \left( \det \left( \mathbf{I}_B + \rho_{\rm ul} \mathbf{H} \mathbf{H}^H \right) \right) \right].$$
(2.2)

The sum-rate capacity (2.2) can be achieved by performing successive interference cancellation (SIC) at the BS [11]. Unfortunately, the computational complexity associated with implementing SIC is prohibitively high, especially for massive MIMO systems that are equipped with a large number of BS antennas.

Linear detection algorithms—although inferior to nonlinear processing algorithms such as SIC—are less computationally demanding and, as will be demonstrated later, yield near-optimal performance when the number of BS antennas exceed by far the number of UEs. With linear detection at the BS, an estimate  $\hat{s}_{ul}$  of the transmitted symbols  $s_{ul}$  is obtained as follows:

$$\hat{\mathbf{s}}_{\mathrm{ul}} = \mathbf{A}^H \mathbf{y}_{\mathrm{ul}}.\tag{2.3}$$

Here,  $\mathbf{A}$  is the detection matrix. It can be shown (see, e.g., [11, 12]) that the sum-rate achievable with Gaussian signaling and linear detection is

$$R_{\rm ul}^{\rm sum} = \mathbb{E}_{\mathbf{H}} \left[ \sum_{u=1}^{U} \log_2 \left( 1 + \frac{\rho_{\rm ul} \left| \mathbf{a}_u^H \mathbf{h}_u \right|^2}{\rho_{\rm ul} \sum_{v \neq u} \left| \mathbf{a}_u^H \mathbf{h}_v \right|^2 + \left\| \mathbf{a}_u \right\|^2} \right) \right]$$
(2.4)

where  $\mathbf{a}_u$  is the *u*th column of the matrix  $\mathbf{A}$ .

Three conventional linear detection schemes are maximal-ratio combining (MRC), zeroforcing (ZF), and minimum mean-square error (MMSE) detection [12]. The detection matrices associated with these detectors are

$$\mathbf{A} = \begin{cases} \mathbf{H} & \text{for MRC,} \\ \mathbf{H} (\mathbf{H}^{H} \mathbf{H})^{-1} & \text{for ZF,} \\ \mathbf{H} (\rho_{ul} \mathbf{H}^{H} \mathbf{H} + \mathbf{I}_{U})^{-1} & \text{for MMSE.} \end{cases}$$
(2.5)

In Fig. 2.3a, the sum-rate achievable with linear detection (2.4) is shown, as a function of the number of BS antennas B, for the case U = 10 and  $\rho_{ul} = -10$  dB. For comparison, the rate achievable with SIC (2.2) has also been plotted. Note that as B grow large, the rate achievable with ZF and MMSE detection approaches the rate achievable with the optimal receiver. This demonstrates that linear detection is near-optimal as the number of BS antennas grow large.

#### 2.2 Downlink Transmission

In the downlink, the discrete-time complex baseband channel input-output relation can be written as

$$\mathbf{y}_{\rm dl} = \sqrt{\rho_{\rm dl}} \mathbf{H}^T \mathbf{x}_{\rm dl} + \mathbf{n}_{\rm dl} \tag{2.6}$$



Figure 2.3: Uplink and downlink throughput. As the number of BS antennas grow large, linear detection and precoding offers near-optimal performance.

Here,  $\mathbf{y}_{dl} \in \mathbb{C}^U$  denotes the received signal at the U UEs. Furthermore,  $\mathbf{n}_{dl} \sim \mathcal{CN}(\mathbf{0}_U, \mathbf{I}_U)$  is the AWGN at the UEs and  $\rho_{dl}$  is the downlink SNR. The transmitted signal over the B BS antennas,  $\mathbf{x}_{dl} \in \mathbb{C}^B$ , must satisfy the average power constraint

$$\mathbb{E}\left[\|\mathbf{x}_{\mathrm{dl}}\|^2\right] \le 1. \tag{2.7}$$

The capacity of the multiuser downlink channel has been characterized in [13–16]. When perfect CSI is available at the BS, dirty-paper coding (DPC) [17] is known to achieve the sum-rate capacity of the channel (2.6). Practical implementations of DPC are, however, computationally demanding, with a complexity that scales unfavorably with the number of BS antennas. Linear precoding [7,18], on the other hand, is an attractive low-complexity approach to massive MIMO precoding, which offers competitive performance to DPC for large antenna arrays [7]. With linear precoding, the transmitted signal  $\mathbf{x}_{dl}$  can be written as

$$\mathbf{x}_{\rm dl} = \frac{1}{\beta} \mathbf{P} \mathbf{s}_{\rm dl} \tag{2.8}$$

where  $\mathbf{P} \in \mathbb{C}^{B \times U}$  is the precoding matrix and  $\mathbf{s}_{dl} \in \mathbb{C}^U$  is a vector whose *u*th entry is

the symbol intended for the *u*th UE. Furthermore,  $\beta \in \mathbb{R}$  is chosen to satisfy the power constraint (2.7). It can be shown (see, e.g., [18]) that the ergodic sum-rate achievable with Gaussian signaling and linear precoding is

$$R_{\rm dl}^{\rm sum} = \mathbb{E}_{\mathbf{H}} \left[ \sum_{u=1}^{U} \log_2 \left( 1 + \frac{\rho_{\rm dl} \left| \mathbf{h}_u^T \mathbf{p}_u \right|^2}{\rho_{\rm dl} \sum_{v \neq u} \left| \mathbf{h}_u^T \mathbf{p}_v \right|^2 + \beta^2} \right) \right]$$
(2.9)

where  $\mathbf{p}_u$  is the *u*th column of the precoding matrix  $\mathbf{P}$ .

Three conventional linear precoders are maximal-ratio transmission (MRT), ZF, and MMSE precoding The precoding matrices associated with these precoders are

$$\mathbf{P} = \begin{cases} \mathbf{H}^* & \text{for MRT,} \\ \mathbf{H}^* (\mathbf{H}^T \mathbf{H}^*)^{-1} & \text{for ZF,} \\ \mathbf{H}^* (\mathbf{H}^T \mathbf{H}^* + \frac{U}{\rho_{d1}} \mathbf{I}_U)^{-1} & \text{for MMSE.} \end{cases}$$
(2.10)

In Fig. 2.3b, the sum-rate achievable with linear precoding (2.9) is shown, as a function of the number of BS antennas B, for the case U = 10 and  $\rho_{dl} = -5$  dB. For comparison, the rate achievable with DPC (see, e.g., [14]) is also illustrated (assuming equal power allocation to all UEs). Again, as B grow large, the rate achievable with ZF and MMSE precoding approaches the rate achievable with the optimal precoder, which demonstrates that linear precoding is near-optimal for large antenna arrays.

## CHAPTER 3

#### Data Converters

Digital signal processing is an integral part of all modern cellular systems [19]. In the uplink, in order to process data digitally, the received signal at the BS has to be converted into the digital domain, which requires conversion in time and amplitude. The device that performs these operations is called an ADC. Conversely, in the downlink, the digital transmit signal at the BS is converted into analog domain by a DAC, before being transmitted over the wireless channel. In this chapter, the basic building blocks of ADCs and DACs are introduced.

#### 3.1 Analog-to-Digital Converters

A block diagram of the basic functions of an ADC, drawn according to [20, Fig. 1.1a], is shown in Fig. 3.1a. An ADC with sampling rate  $f_s$  Hz and a resolution of b bits maps each sample of a continuous-time, continuous-amplitude signal to one out of  $2^b$  possible quantization labels, by operating  $f_s \cdot 2^b$  conversion steps per second.

The process of converting a continuous-time signal into a discrete-time signal is known as *sampling*. To ensure that the input to the sampling circuit adheres (at least approximately) to the sampling theorem, the analog input signal is passed through an anti-aliasing filter (a low-pass filter) prior to the sampling circuit. In this thesis, it is assumed that the sampling circuit is ideal, i.e., that the amplitude of the output of the sampling circuit at any sampling instant is exactly the amplitude of the input signal. It is further assumed that the anti-aliasing filter is an ideal low-pass filter with a cutoff frequency that equals the symbol rate, such that any out-of-band (OOB) disturbance



Figure 3.1: Block diagram of the basic functions of an ADCs and a DAC.

present in the analog input does not enter into the sampling circuit.

The process of converting a continuous-amplitude signal into a discrete-amplitude signal is known as *quantization*, which is a process that occurs whenever physical quantities are represented numerically. Fig. 3.2 illustrates the input-output relation for a 1-bit *zero-threshold* (the quantizer is symmetric around zero) quantizer and for a 4-bit uniform quantizer. While the sampling operation incurs no loss of information for band-limited signals, the mapping of a continuous-amplitude signal into a finite set of possible outcomes will cause an error between the input and output of the quantizer, which can be made smaller by increasing the resolution of the quantizer. Unfortunately, increasing the resolution of an ADC also increases the consumed power [9, 10].

#### 3.2 Digital-to-Analog Converters

A block diagram of the basic functions of a DAC, drawn according to [20, Fig. 1.1b], is shown in Fig. 3.1b. As shown in Fig. 3.1b, a DAC consists of a transcoder followed by a reconstruction stage. The transcoder produces an analog sequence whose amplitude is the analog representation of a digital code. The number of discrete amplitude levels in the transcoder output is determined by the number of DAC bits. The reconstruction block transforms this sequence into an analog signal. Typically, this block consists of a cascade of a sample-and-hold circuit and a reconstruction filter [20]. Often, the sample-and-hold circuit and the transcoder are implemented in the same circuit.

A common sample-and-hold circuit is a zero-order hold (ZOH) filter, which holds the amplitude of the each sample for a prescribed time duration. Unfortunately, a ZOH filter has a frequency response with infinite support, which causes undesired OOB emissions. To reduce these OOB emissions, a reconstruction filter (a low-pass filter) is installed after



Figure 3.2: Input-output relation for a (a) 1-bit zero-threshold quantizer and a (b) 4-bit uniform quantizer. The quantizer maps the continous-amplitude input into a discrete set of outcomes.

the sample-and-hold circuit. In this thesis, the OOB emissions caused by the DACs are ignored and it is assumed that no reconstruction filter is present in the DACs. In other words, the reconstruction stage consists of a ZOH filter only.

### CHAPTER 4

#### Contributions

The list of publications that are appended to this thesis is provided in Section 4.1. The list of additional publications by the author that are not included in this thesis is provided in Section 4.2

#### 4.1 Included Publications

#### [A] "Throughput analysis of massive MIMO uplink with low-resolution ADCs"

In this paper, we consider the massive MIMO uplink for the case when the BS is equipped with low-resolution ADCs and has no *a priori* CSI. Using Bussgang's decomposition, we propose a novel low-complexity channel estimator, which takes the distortion caused by the finite-resolution ADCs into account. We develop closed-form approximations for the rate achievable with Gaussian signaling, linear detectors, and low-resolution ADCs, which turn out accurate for a large range of system parameters. We also present easy-to-evaluate approximations on the rate achievable with finite-cardinality constellations, e.g., quadrature phase-shift keying (QPSK). By comparing this approximation with a numerically computed lower bound on said rates, we show that it is accurate for a large range of SNR values. Finally, through a numerical study, we show that only 3–4 bits are required to achieve a performance close to the infinite-resolution (no quantization) case for a large range of system parameters. This holds also when the UEs are received at vastly different power levels due to imperfect power control.

#### [B] "Quantized Precoding for Massive MU-MIMO"

In this paper, we investigate the performance achievable in the massive MIMO downlink for the case when the BS is equipped with low-resolution DACs and has perfect CSI. For the case of linear precoding and 1-bit DACs, we develop a lower bound on the rate achievable with Gaussian inputs. We further develop closed-form rate approximations, which are accurate over a large range of system parameters for DACs of arbitrary resolution. Our results suggest that high information rates are achievable despite the adverse impact of the low-resolution DACs. Indeed, 3–4 bit DACs are shown to be sufficient to close the gap to infinite-resolution performance. For 1-bit DACs, linear precoding is, however, far from optimal. We develop nonlinear precoders by formulating the MSE-optimal precoding problem and by relaxing it to a convex problem that can be solved in a computationally efficient manner. Through numerical simulations, we demonstrate the superiority of the proposed nonlinear precoders over linear precoders.

# [C] "Massive MU-MIMO-OFDM downlink with one-bit DACs and linear precoding"

In this paper, we investigate the downlink performance achievable with linear precoders over frequency-selective fading channels for the case of 1-bit DACs. In contrast to previous works, we here consider DACs that operate at a sampling rate that exceeds the symbol rate. Furthermore, to mitigate the frequency-selectivity of the fading channel, we assume that orthogonal frequency-division multiplexing (OFDM) is used. Focusing on the case of 1-bit DACs and linear precoding, we derive an approximation of the bit error rate (BER) achievable with QPSK signaling, as well as a lower bound on the rate achievable with Gaussian signaling.

#### 4.2 Publications Not Included

Publications by the author that are not included in the thesis are listed below.

- [D] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, "One-bit massive MIMO: Channel estimation and high-order modulations", in *Proceedings of IEEE International Conference on Communications Workshop (ICCW)*, London, U.K., Jun. 2015, pp. 1304–1309.
- [E] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "Nonlinear 1-bit precoding for massive MU-MIMO with higher-order modulation", in *Proceedings* of Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA, Nov. 2016, pp. 763–767.
- [F] O. Castañeda, S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "1-bit massive MU-MIMO precoding in VLSI", Submitted to *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, Feb. 2017.

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