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Grid Frequency Estimation Using Multiple Model with Harmonic Regressor: Robustness Enhancement with Stepwise Splitting Method

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Abstract: Reduction of inertia in electricity networks due to high penetration level of renewable energy sources will require wind turbines to participate in frequency regulation via Active Power Control. The performance of frequency regulation and protection system depends strongly on the performance of network frequency estimation. Fast frequency variations and uncertainties associated with unknown harmonics and measurement noise in the network signals are the main obstacles to performance improvement of frequency estimation with classical zero crossing method, which is widely used in industry. The same uncertainties introduce challenges in model based frequency estimation. These challenges are addressed in this paper within the framework of multiple model with harmonic regressor.
Additional challenges associated with computational complexity of matrix inversion algorithms and accuracy of inversion of ill-conditioned matrices in the multiple model are also discussed in the paper. New high order algorithms with reduced computational complexity are presented. Instability mechanism is discovered in Newton-Schulz and Neumann matrix inversion techniques in finite precision implementation environment. A new stepwise splitting method is proposed for elimination of instability and for performance improvement of matrix inversion algorithms in the multiple model. All the results are confirmed by simulations.

Keywords: Accurate Frequency Tracking in Electricity Networks, Harmonic Regressor, Multiple Model, High Order Algorithms, Stepwise Splitting, Roundoff Errors, Numerical Instability, Newton-Schulz Algorithm, Neumann Series

1. INTRODUCTION

High penetration level of (1) renewable energy sources, (2) power electronics, (3) advanced transmission systems, and (4) higher nonlinear loads and new types of loads in future electricity networks will (a) essentially reduce grid inertia and (b) introduce significant distortions in voltage and current signals.
These distortions will result in fast deviations from fundamental frequency, appearance of additional harmonics and hence in reduction of efficiency of equipment, power losses, heating, increased noise levels and others.
Frequency regulation, enabled via active power control of wind turbines, load-side control and others will play an important role in future electricity networks for performance improvement. Notice that *reliable frequency measurement is necessary for high performance network control as well as for system protection.* Errors in frequency measurements will result in erroneous control action and even in frequency oscillations. This paper addresses a very important issue of accuracy improvement of frequency estimation algorithms in the presence of harmonics and noise (electrical noise, measurement noise and others) in future electricity networks. A brief overview of existing frequency estimation methods is given below.

1.1 Existing Frequency Estimation Methods and Further Developments

Zero crossing detection and calculation of the number of cycles that occur in a predetermined time interval is a direct, simple and widely used methodology for frequency detection, see Friedman (1994). However, the disturbances associated with harmonics and noise, which will appear around zero crossing points of the signals in future electricity networks deteriorate accuracy of the grid frequency estimation via classical zero crossing method. Modifications of zero crossing method (described for example in Stotsky (2016), see also references therein) aiming for improvement of estimation accuracy are all based on more accurate detection of zero crossing points. These methods require additional signal processing techniques, which introduce delays. The delays are significant for noise contaminated signals with a large number of harmonics and introduce significant limitations in the performance of modified zero crossing methods in the case of fast frequency tracking. Future frequency estimation algorithms should be model based, that allows complete reconstruction of the frequency contents of the signals for high performance frequency estimation.
A number of interesting surveys on model based frequency

estimation is available in the literature, see for example Quinn et al. (2013) and references therein. Promising multi-harmonic frequency estimators are based on optimization techniques, which maximize periodogram as a function of frequency, see Walker (1971) or minimize the error sum of squares with respect to unknown quantities, such as frequencies, phase shifts and coefficients. The best model matching provides the most accurate estimates. However, a number of extremum seeking algorithms, which are often realized as iterative search procedures can be ineffective due to local extrema and restricted region of attraction, which in turn are present due to a highly nonlinear nature of the problem. Computational complexity is an additional problem associated with the search procedures.

These difficulties can be avoided by applying multiple model approach (see for example Sakakura et al. (2016) for recent developments in general multiple model method), where the set of models is defined and each model is associated with different fundamental frequency. Residual error, which is associated with this set can be presented as a function of frequency and the frequency, which corresponds to the minimal value of residual error is the true frequency. Moreover, minimal residual error is also associated with the variance of the measurement noise.

All the residual errors can be calculated simultaneously, using parallel calculations, which essentially reduce execution time of the algorithm.

A simple and computationally efficient minimum seeking algorithm, realized as the interval reduction method is developed in Section 2 for fast and accurate calculation of the minimal value of the residual error and high performance estimation of the frequency and the variance of the measurement noise.

On the other hand, application of the multiple model (which consists of a large number of models) requires significant computational efforts, especially for a large number of harmonics. This introduces numerical challenges in finite precision implementation environment associated with inversion of information matrices of a large size and condition number (for fast varying frequency).

In other words the development of new matrix inversion algorithms with reduced computational complexity and improved robustness is required. These algorithms are presented in Section 3 and Section 5.

2. MULTIPLE MODEL ESTIMATION

2.1 Description of the Minimal Residual Method

Suppose that a measured signal y_k can be presented in the following form

$$y_k = \varphi_k^T \theta_* + \xi_k \tag{1}$$

where θ_* is the vector of unknown constant parameters and φ_k is unknown harmonic regressor presented in the following form:

$$\varphi_k^T = [cos(q_0 k)\ sin(q_0 k)\ cos(2q_0 k)\\ sin(2q_0 k)\ ...\ cos(hq_0 k)\ sin(hq_0 k)] \tag{2}$$

where q_0 is unknown fundamental frequency of network (for example $q_0 = 50$ Hertz), h is unknown number of harmonics, and ξ_k is a zero mean white Gaussian noise, $k = 1, 2, ...$ is the step number. The system has four unknown quantities : 1) the fundamental frequency of network q_0, 2) the number of harmonics h, 3) the vector of parameters θ_*, and 4) the variance of the measurement noise. It is assumed that the upper bound $\overline{h}$ of the number of harmonics is known and $h \leq \overline{h}$. The algorithm for frequency and parameter estimation can be presented in the following steps, which are executed in each step k.

Step 1: Estimation for the Initial Set of Frequencies. Define the frequency interval as the following vector of size r:

$$f_1 = [\ \hat{q}_{11}\ \hat{q}_{12}\ \hat{q}_{13}\ ...\ \hat{q}_{1(r-1)}\ \hat{q}_{1r}] \tag{3}$$

where the frequencies $\hat{q}_{1i}$, $i = 1, ..., r$, $r \geq 3$ are presented in increasing order. The frequency interval should cover unknown fundamental frequency of the system q_0.

Substep 1: Estimation of the Variance. The regressor vector $\hat{\varphi}_i$ is introduced for each frequency $\hat{q}_{1i}$ as follows:

$$\hat{\varphi}_i^T = [cos(\hat{q}_{1i}k)\ sin(\hat{q}_{1i}k)\ cos(2\hat{q}_{1i}k)\\ sin(2\hat{q}_{1i}k)\ ...\ cos(\overline{h}\hat{q}_{1i}k)\ sin(\overline{h}\hat{q}_{1i}k)] \tag{4}$$

forming a multiple model of the regressor with the frequencies corresponding to the components of the vector (3). Notice that the size of the model of each regressor (4) is larger than or equal to the size of unknown regressor (2) since $h \leq \overline{h}$.

Multiple model of the signal (1) with adjustable parameters θ_i is presented in the following form:

$$\hat{y}_i = \hat{\varphi}_i^T \theta_i \tag{5}$$

The signal y_k is approximated by the multiple model $\hat{y}_i$ for each frequency corresponding to the components of the vector (3) in the least squares sense in each step k of a moving window of a size w.

The frequency estimation algorithm is based on minimization of the following error E_i with respect to argument i, which corresponds to the certain frequency in the multiple model (5):

$$E_i = \sum_{p=k-(w-1)}^{p=k} (\hat{y}_{pi} - y_p)^2 \tag{6}$$

for a fixed step k, where $k \geq w$.

The least squares solution for estimation of the parameter vector θ_i can be written as follows:

$$A_i \theta_i = b_i \tag{7}$$

$$A_i = \sum_{p=k-(w-1)}^{p=k} \hat{\varphi}_{pi}\ \hat{\varphi}_{pi}^T \tag{8}$$

$$b_i = \sum_{p=k-(w-1)}^{p=k} \hat{\varphi}_{pi}\ y_p \tag{9}$$

where the matrix A_i is an information matrix (see Stotsky (2010),(2015) where the properties of this matrix are discussed for systems with harmonic regressor), and the parameter vector θ_i satisfies (7). The parameter vector can be calculated with high accuracy in the finite precision implementation environment using high order algorithms, described in Section 3 and Section 5.

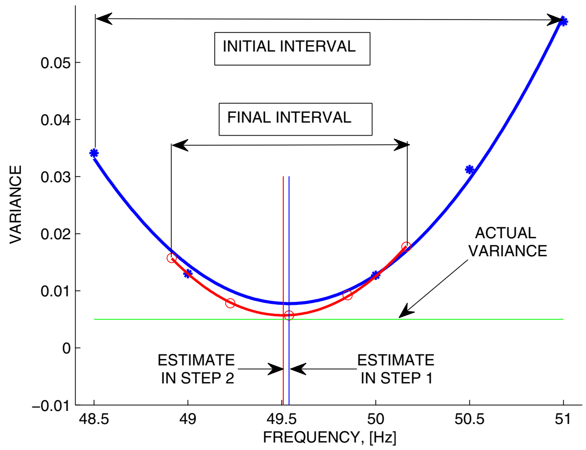


FIG 1. Estimation of the frequency in two steps. The following set of frequencies is selected in the first step of estimation [48.5 49 50 50.5 51] Hertz. This set covers actual frequency of 49.5 Hertz. Each point in this set is plotted with a star sign of a blue color. The frequency of 49.5382 Hertz was determined after the first step of estimation as an argument corresponding to minimal value of parabola plotted with a blue line. The frequency of 49.5382 Hertz is used as a central point a new set of frequencies with the range, which is reduced compared to the range of initial set. The following set of frequencies was determined in the second step of estimation [48.9132 49 49.2257 49.5382 49.8507 50 50.1632] Hertz, where the frequencies 49 and 50 Hertz were transferred from the initial interval. Each point in this set (excepting the frequencies of 49 and 50 Hertz) is plotted with a round sign of a red color. The frequency of 49.5079 Hertz was determined in the second step of estimation in the same way. Minimal value of parabola plotted with a red line corresponds to the actual value of the variance of measurement noise, which is equal to 0.005.

Finally, the variance V_i of the measurement noise ξ_k, associated with the multiple model is defined as follows :

$$V_i = \frac{E_i}{w - 2\overline{h} - 1} \tag{10}$$

Substep 2: Calculation of the Frequency via Minimization of the Variance. The model for V_i is defined as follows:

$$\hat{V} = aq^2 + bq + c \tag{11}$$

where q is the frequency and a, b and c are the coefficients calculated using least squares method to provide the best fit of V_i over the frequency interval f_1. Estimated frequency is defined as

$$\overline{q}_1 = -\frac{b}{2a} \tag{12}$$

which corresponds to the minimal value of $\hat{V}$, which is also an estimate of the variance of measurement noise ξ_k.
The frequency $\overline{q}_1$ is used as the central point for interval, which should be chosen in the next step.
Notice that the initial frequency interval (3) should be sufficiently large in order to cover unknown frequency. However, inaccuracies in calculations of the variances V_i for each model, especially in the points located close to the boundaries result in a biased estimate of the frequency. Therefore the frequency interval should be reduced in the next step of estimation for the sake of accuracy improvement.
Step 2: Estimation for Updated Set of Frequencies. Define updated frequency interval as the following vector of size r:

$$f_2 = [\ \hat{q}_{21}\ \hat{q}_{22}\ ...\ \overline{q}_1\ ...\ \hat{q}_{2(r-1)}\ \hat{q}_{2r}] \tag{13}$$

where the range of the frequency interval defined by f_2 in (13) is reduced with respect to the range of the frequency interval defined by f_1 in (3) , i.e. $(\hat{q}_{2r} - \hat{q}_{21}) <<< (\hat{q}_{1r} - \hat{q}_{11})$.
Notice that the frequencies from the interval f_1 can be included in the interval f_2 (if they fit to this interval), which improves curve fitting accuracy without additional computational effort. The substeps 1 and 2 are repeated for this new set of frequencies (13) resulting in updated estimate of the frequency $\overline{q}_2$, which is used as central point for interval defined in the next step.
Reduction of the range of the interval in each step ensures the convergence of estimated frequency $\overline{q}_z$, where $z = 1, 2, ...$ to its true value q_0 as the step number increases and the range of the interval reaches its minimal value.
This algorithm is the fast convergent algorithm due to the model based minimization of the variance in each step and few steps are required only for estimation of the fundamental frequency with a very high accuracy. Notice that two steps is usually sufficient for accurate estimation of the frequency.
The algorithm of minimization of the variance and estimation of the frequency in two steps is illustrated in Figure 1. Notice that any overtones, including half harmonics can be included in the model (4) provided that these overtones are present in the signal (1). Inclusion of additional number of overtones increases the size of the regressor vector. This size together with the window size w can be reduced using the stepwise regression method of subsequent inclusion of the harmonics/overtones in the regressor, see for example Stotsky (2009).
Algorithm with multiple model provides significant improvement of frequency estimation performance compared to classical zero crossing algorithm, see Figure 2.
However, multiple model algorithm requires significant computational efforts in each step associated with inversion of a number of information matrices. Moreover, reduction of the window size is required for accurate tracking of fast varying frequency, that introduces ill-conditioning in information matrices.
The next Section describes new high order matrix inversion algorithms with reduced computational complexity, and Section 5 includes new robust inversion algorithms for ill-conditioned matrices.

3. HIGH ORDER ALGORITHMS

Positive definite and symmetric matrix A can be split as follows (see for example Chen (2005) and references therein):

$$A = S - D \tag{14}$$

where S is a positive definite and symmetric matrix, and D is a symmetric matrix. This splitting facilitates calculation of the inverse of matrix A, which can be used

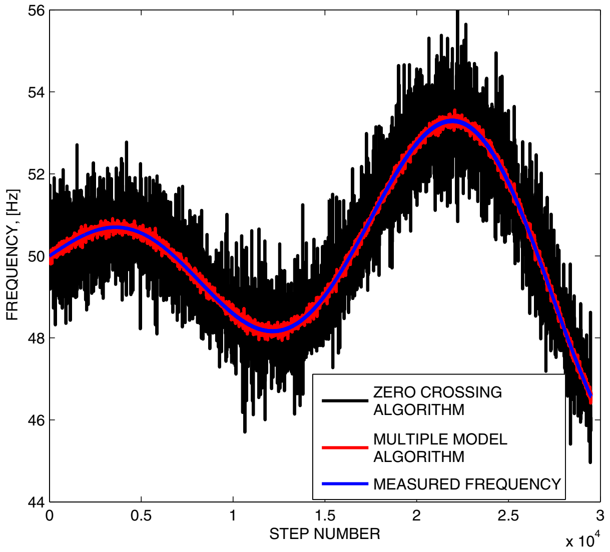


FIG 2. The signal with time varying frequency and the second and the fifth harmonic was processed for frequency estimation by classical zero crossing algorithm, plotted with a black line and algorithm with multiple model, plotted with a red line. Actual frequency of the signal is plotted with a blue line.

as a preconditioner for high order algorithms that solve the equation $A\theta = b$ with respect to the parameter vector θ, see Section 2.1.

Double-sided algorithm of order $h = 1, 2, ...$ which can be derived from the matrix inversion lemma or from splitting (14) is presented as follows:

$$G_k = (S^{-1}D)^h G_{k-1}(DS^{-1})^h + \{\sum_{j=0}^{2h-1}(S^{-1}D)^j\}S^{-1} \quad (15)$$

where G_k is an estimate of A^{-1}, the spectral radius $\rho(S^{-1}D) < 1$, $k = 1, 2, ...$, and the matrix S is easy invertible matrix (or the matrix whose inverse is known). The matrix G_0 is arbitrary and can be calculated via Neumann series as follows: $G_0 = \{\sum_{j=0}^{2h-1}(S^{-1}D)^j)\}S^{-1}$ since the following relations hold:

$$\sum_{j=0}^{h-1}(S^{-1}D)^j[S^{-1} + S^{-1}DS^{-1}](DS^{-1})^j$$
$$= \{\sum_{j=0}^{2h-1}(S^{-1}D)^j\}S^{-1} = (I - (S^{-1}D)^{2h})A^{-1}$$

and G_0 is close to A^{-1} for a sufficiently large h. Notice that the sum $\sum_{j=0}^{h-1}(S^{-1}D)^j[S^{-1} + S^{-1}DS^{-1}](DS^{-1})^j$ is computationally efficient lower-order representation of $\{\sum_{j=0}^{2h-1}(S^{-1}D)^j)\}S^{-1}$, see Stotsky (2016).

Splitting (14) can also be written in the following form:

$$I - S^{-1}A = S^{-1}D \quad (16)$$

where I is the identity matrix, and $\rho(S^{-1}D) < 1$ for symmetric and positive definite matrices A and S, provided that $2S - A$ is a positive definite matrix, which imposes restriction on the choice of S and D, Horn et al. (1985). Notice that the matrix S^{-1} can be used as a preconditioner for high order algorithms described in Isaacson et al. (1966) and Stotsky (2014), (2015) provided that $2S > A$. Algorithm (15) has the following error model:

$$G_k - A^{-1} = (S^{-1}D)^{hk}\{G_0 - A^{-1}\}(DS^{-1})^{hk} \quad (17)$$

The convergence rate increases with the order h, and it is determined by the eigenvalues of the matrix $S^{-1}D$.

4. INSTABILITY FOR ILL-CONDITIONED MATRICES

Consider the following algorithm of order $m = 2, 3, ...$

$$F_k = I - G_k\frac{A}{\alpha}, \qquad G_0 = I \quad (18)$$

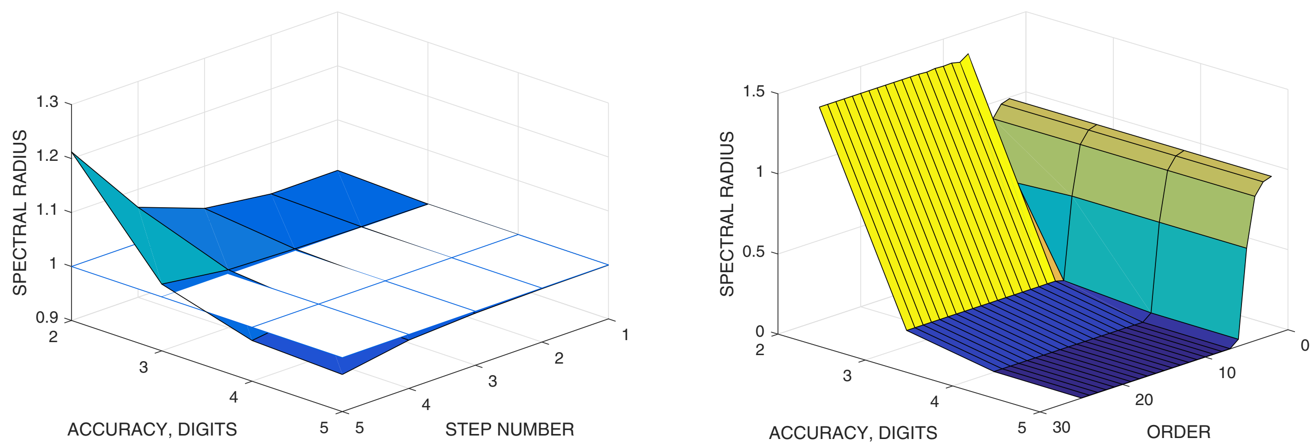
$$G_k = \{\sum_{d=0}^{m-1} F_{k-1}^d\}\, G_{k-1} \quad (19)$$

$$= G_{k-1} + F_{k-1}\,\{\sum_{d=0}^{m-2} F_{k-1}^d\}\, G_{k-1} \quad (20)$$

where algorithm (20), described in Stotsky (2015) is lower order realization of algorithm (19), described in Isaacson et al. (1966). The algorithm provides an estimate of the inverse of a positive definite and symmetric matrix A and $\lim_{k\to\infty}\frac{G_k}{\alpha} \to A^{-1}$ since $F_k = F_0^{m^k}$, where $\alpha = \|A\|_\infty/2 + \epsilon$, $\epsilon > 0$ and $\rho(F_0) = \rho(I - A/\alpha) < 1$, $k = 1, 2, 3, ...$.

Algorithm (18) - (20) provides accurate estimate of the inverse if computational accuracy is high. However, the algorithm accumulates roundoff errors in finite precision implementation environment. For example, minimal eigenvalue of the matrix $\frac{A}{\alpha}$, where $\alpha = \|A\|_\infty/2 + \epsilon$ is too close to zero for ill-conditioned matrices. Therefore the spectral radius of $I - A/\alpha$ is too close to one, and roundoff error in calculation of the scaling factor α have a serious impact on the algorithm performance and may even result in numerical instability. The problem is illustrated in Figure 3, where instability is present for matrix inversion with limited computational accuracy. Subplots (a) and (b) in Figure 3 show the spectral radius of the matrix $F_k = I - G_k\frac{A}{\alpha}$ for algorithm (18)-(20) for ill-conditioned information matrix A of the system with harmonic regressor with three frequencies. All the variables in the algorithm (18)-(20) are rounded to $2, ..., 5$-digit accuracy. The spectral radius as a function of a step number is plotted in Subplot (a) for the third order algorithm, $m = 3$. The spectral radius as a function of the order of the algorithm is plotted in Subplot (b) for one step, $k = 1$, which corresponds to Neumann series.

In addition to instability which occurs for two and three digit accuracy there is even deterioration of the algorithm performance for better computational accuracy, associated with slower convergence and lower estimation accuracy. Problems associated with instability and performance deterioration are usually more pronounced for Neumann



(a) (b)

FIG 3. Subplots (a) and (b) show the spectral radius of the error matrix $F_k = I - G_k \dfrac{A}{\alpha}$ for algorithm (18)-(20) for ill-conditioned information matrix A of the system with harmonic regressor with three frequencies. All the variables in the algorithm are rounded to $2, ..., 5$-digit accuracy. The spectral radius as a function of a step number is plotted in Subplot (a) for the third order algorithm. The spectral radius is equal to one on a white surface. The spectral radius as a function of the order of the algorithm is plotted in Subplot (b) for one step, $k = 1$, which corresponds to Neumann series.

series of high order, see Subplot (b) in Figure 3 rather than for Newton-Schulz algorithm, see Subplot (a) in the same Figure. Therefore Neuman series of high order are not implementable in finite precision environment due to sensitivity to roundoff errors.

Robust inversion algorithms for positive definite ill-conditioned matrices, which are suitable for implementation in finite precision environment should be designed. The algorithms should provide the best possible accuracy in the presence of roundoff errors.

New algorithms could be associated with Neuman series and should be more robust with respect to roundoff errors and error accumulation. Algorithm (18)-(20) should be modified for better tolerance of the scaling factor α with respect to roundoff errors. New robust algorithms should also prevent error accumulation. The matrix can be inverted by parts via application of a sequential procedure (without error accumulation) with large stability margins and the same inversion accuracy in each step.

5. STEPWISE SPLITTING METHOD

Suppose that a positive definite and symmetric matrix initially split as follows, $A = S_1 - D_1$, where S_1 is a positive definite and symmetric matrix and D_1 is a symmetric matrix, where S_1^{-1} is known. Such splitting occurs in recursive calculation of information matrix in least-squares method for example. The matrix A can be further recursively split as follows:

$$A = S_i - D_i \tag{21}$$

$$\beta_i = ||S_i^{-1} D_i||_\infty + \varepsilon \tag{22}$$

$$S_{i+1} = S_i - \frac{1}{\beta_i} D_i \tag{23}$$

$$D_{i+1} = S_{i+1} - A = \frac{\beta_i - 1}{\beta_i} D_i \tag{24}$$

$$||I - S_i^{-1} S_{i+1}||_\infty = \frac{||S_i^{-1} D_i||_\infty}{\beta_i} < 1 \tag{25}$$

$$A = S_{i+1} - D_{i+1} \tag{26}$$

where ε is a small positive number, $i = 1, 2, ...i_*$, and $i = i + 1$ while $\beta_i > 1$, and the norm $|| \cdot ||_\infty$ is defined as the maximum absolute row sum norm. Inverse of S_{i+1} is calculated in each step using algorithm (18)-(20) with preconditioning matrix S_i^{-1}(instead of I/α), which satisfies inequality (25).

The choice of β_i in (22) guarantees convergence and robustness via the condition (25), and minimizes the number of steps of the algorithm.

Algorithm is robust with respect to numerical inaccuracies, associated with roundoff errors and error accumulation. The spectral radius $\rho(S_i^{-1} D_i)$ is the lower bound for maximum absolute row sum norm, defined in (22) guarantees robustness with respect to errors in calculation of the scaling factor β_i. A proper choice of ε associated with a trade-off between robustness and additional computational effort is also a tool for robustness improvement.

Finally, stepwise accumulation of numerical errors is prevented via proper calculation of $D_{i+1} = S_{i+1} - A$ in (24) taking into account that the matrix A should remain the same in each step i.

Algorithm converges when β_i, calculated in (22) is less than one (or equal to one), and it is assigned to one $\beta_{i*} = 1$. The procedure converges to the following splitting:

$$A = S_{i*} - D_{i*} \tag{27}$$

$$||I - S_{i*}^{-1} A||_\infty = ||S_{i*}^{-1} D_{i*}||_\infty < 1 \tag{28}$$

which allows calculation of A^{-1} with preconditioning matrix S_{i*}^{-1}.

Splitting (21) - (26) facilitates calculation of the inverse of matrix A, which can be used as a preconditioner for algorithm (18)-(20).

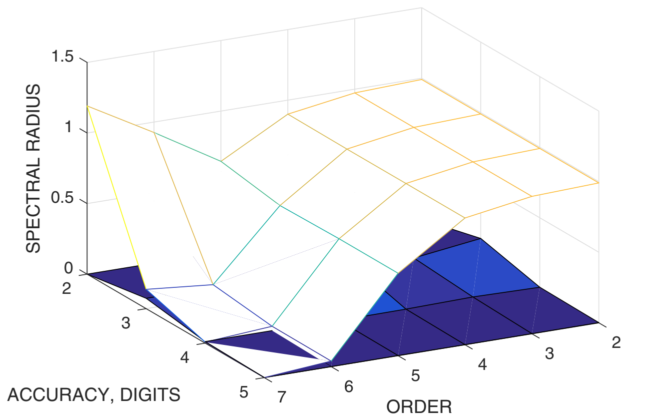


FIG 4. Comparison of two spectral radiuses as functions of the order, where the spectral radius of the error matrix $F_k = I - G_k \frac{A}{\alpha}$ for algorithm (18)-(20), $k = 1$, which corresponds to Neumann series is plotted as a white surface. The spectral radius of the error matrix $F_k = I - G_k A$ for the algorithm (21) - (26), (27)-(28) is plotted with a blue surface. Matrix inversion algorithm of the same order associated with Neumann series is applied for inversion of the matrix S_i in each step in the algorithm (21) - (26), (27)-(28). All the variables in both algorithm are rounded to $2, ..., 5$-digit accuracy.

Comparison of two spectral radiuses as functions of the order of the algorithms and digits of accuracy is plotted in Figure 4. The spectral radius of the error matrix $F_k = I - G_k \frac{A}{\alpha}$ for algorithm (18)-(20), $k = 1$, which corresponds to Neumann series is plotted as a white surface. The spectral radius of the error matrix $F_k = I - G_k A$ for the algorithm (21) - (26), (27)-(28) is plotted with a blue surface. Matrix inversion algorithm of the same order associated with Neumann series is applied for inversion of the matrix S_i in each step in the algorithm (21) - (26), (27)-(28). All the variables in both algorithm are rounded to $2, ..., 5$-digit accuracy.

6. CONCLUSION

Significant distortions associated with harmonics, noise and fast changes of the fundamental frequency of voltage and current signals are expected in future electricity networks. These disturbances will also appear around zero crossing points of the signals, deteriorating accuracy of the grid frequency estimation via classical zero crossing method.

Future frequency estimation algorithms will be model based (see Stotsky (2016) for detailed explanation), that allows complete reconstruction of the frequency contents of the signals and recovering fast frequency variations from noise contaminated signals. Such algorithm, which is based on multiple model with harmonic regressor and require minimum information about the frequency contents of signals is described in Section 2.1. Simulation results show significant improvement of estimation performance compared to classical zero crossing method and its modifications for frequency tracking.

It is also shown that instability associated with inversion of ill-conditioned information matrices in multiple model in finite precision implementation environment can be eliminated and estimation accuracy can be improved with stepwise splitting method.

Robustness of the multiple model approach with harmonic regressor with respect to measurement noises and different types of errors (modeling and roundoff errors and others) together with possibilities for accuracy improvement makes this approach more attractive in implementation compared to the methods based on estimation of the coefficients of difference equations, see for example Stotsky (2012).

The multiple model approach with harmonic regressor can be extended to the multi-frequency case, where the signal contains multiple frequency components, which are not related to each other. Accuracy of frequency estimation can be improved in this case in many other applications.

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