Design Methodology for Evaluation of Global Stability in Structural Systems

Master’s thesis in Structural Engineering and Building Technology

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Cover:
Deformed shape of studied tall structural system by FEM Design

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ABSTRACT

When choosing a structural system for a building it is important to consider stability issues. Stability need to be evaluated whether the project is a high rise building or a smaller residential building. These stability effects include overturning, sliding, accidental action, dynamic effects and global buckling effects. The aim of this thesis is cover the issues of global stability to ensure safe structures in the future due to their increase in slenderness. The main intent is to present methods that a general contractor can use to evaluate global stability in buildings. After conducting interviews and performing calculations on two stabilizing systems, a list of checks could be proposed. The first check is the equilibrium of the building, which includes uplift, sliding and overturning. After this the global stability should be analyzed to decide if the stabilizing system is mobilizing enough stiffness to not experience significant global second order effects. This is evaluated with simplified Eurocode checks, the Vianello method or through the use of a linear buckling analysis. If the critical buckling load is less than 10 times the applied vertical design load then second order effects need to be included in the structural analysis. This analysis can be done by increasing the loads by a factor or performing a nonlinear analysis. After the second order forces are determined members need to be checked for their own stability. Final checks include robustness check, serviceability check and dynamic stability check. A conclusion made was also that low-rise buildings can experience buckling phenomena if their stabilizing system is slender enough. This was evident in the evaluation of a low-rise building with low stiffness.

Keywords: Global system buckling, Vianello method, Linear buckling analysis, Structural system, Stability, Buckling length, FEM-Design
# CONTENTS

Abstract i

Contents iii

Preface vii

Notations ix

1 Introduction 1

1.1 Background ......................................................... 1
1.2 Aim ................................................................. 1
1.3 Objectives ........................................................... 1
1.4 Limitations .......................................................... 2
1.5 Method ............................................................... 2

2 Global stability 3

2.1 Safety in systems .................................................... 3
2.2 Overturning and sliding ............................................. 5
2.3 Uplifting .............................................................. 6
2.4 Accidental actions ................................................... 7
2.5 Dynamic stability ................................................... 9
2.6 Global system buckling ............................................. 10
2.6.1 Deflection ......................................................... 11
2.7 Member buckling .................................................... 11
2.8 Additional stability effects ........................................ 11

3 External Actions 12

3.1 Wind loads .......................................................... 12
3.1.1 Wind loads in Eurocode ......................................... 12
3.2 Unintended inclination .............................................. 12
3.2.1 Unintended inclination in Eurocode .......................... 13
3.3 Seismic effects ...................................................... 14
3.3.1 Seismic effects in Eurocode .................................. 14
3.4 The P-delta effect .................................................... 14
3.4.1 Seconds order effects in Eurocode ........................... 15
3.5 Designed inclination ............................................... 15
3.6 Thermal actions .................................................... 15
9.2 Evaluation of idealized buildings ................................................................. 53
9.2.1 Eurocode checks with help of equivalent column ..................................... 53
9.2.2 Vianello method ...................................................................................... 54
9.2.3 Linear buckling analysis .......................................................................... 55
9.2.4 Comparing the methods ........................................................................... 55
9.2.5 Dimensioning and member checks .......................................................... 56

10 Conclusions and recommendations ............................................................... 57
10.1 Conclusions .............................................................................................. 57
10.2 Recommendations ..................................................................................... 58
10.3 Further studies ........................................................................................... 58

References ......................................................................................................... 61

Appendix A Interview questions in Swedish ....................................................... 63
Appendix B Interviews ....................................................................................... 64
Appendix C Hand calculations .......................................................................... 72
Appendix D FEM-design calculations ................................................................. 76
PREFACE

The idea for this thesis came from Carl Jonsson at Skanska Sverige, Teknik. We want to thank him together with Carl Larsson for being our supervisors at Skanska AB and making this cooperation between Skanska and Chalmers University of Technology possible. This thesis is written for the Division of Structural Engineering at the Department of Architecture and Civil Engineering at Chalmers University of Technology. We also want to thank Helena Burstrand Knutsson for accommodating us at Skanska’s offices in Gothenburg and allowing us to interact with the technical team, who’s help have been of tremendous value to us.

We want to thank our examiner Filip Nilenius for taking the time to evaluate our work. We also want to thank our Chalmers teachers Karl-Gunnar Olsson, Björn Engström and Mohammad Al-Emrani for helping us develop the thesis and guiding us in our research. We also want to thank our opponents, Priyanka Das and Karin Cajmatz, for taking the time to read and comment on our work.

Finally, we want to thank our families and friends for supporting us through our studies at Chalmers.

Göteborg June 2017

AbdulRaheem Alsofi & Andreas Grahn
Notations

Greek letters
\( \alpha \) imperfection factor depending on buckling curve
\( \alpha_h \) reduction factor for length or height
\( \alpha_m \) reduction factor for number of members
\( \alpha_{cr} \) safety factor against global steel buckling
\( \chi \) reduction factor for relevant buckling mode
\( \chi_{LT} \) reduction factor for lateral torsional buckling mode
\( \Delta T_m \) difference between outer and inner surface temperature
\( \Delta T_p \) difference in temperature between parts in the structure
\( \Delta T_u \) difference between average temperature and initial temperature
\( \delta \) deflection
\( \delta_{H. Ed} \) horizontal displacement of a point on the top story related to bottom story
\( \gamma_{M1} \) partial factor steel
\( \gamma_{ME} \) partial factor concrete
\( \lambda \) non dimensional slenderness
\( \lambda_{LBA} \) diagonal matrix of eigenvalues
\( \lambda_{lim} \) slenderness limit
\( \omega \) coefficient for concrete cracking
\( \omega_m \) mechanical reinforcement ratio
\( \phi_d \) design value for structure-ground interface friction angle
\( \phi_{1,x} \) fundamental along wind modal shape
\( \Psi \) matrix of corresponding eigenvector
\( \rho \) air density
\( \sigma_{dev} \) characteristic along-wind acceleration at height \( z \)
\( \theta_0 \) basic value for unintended inclination
\( \theta_i \) initial sway inclination
\( \theta_m \) rotation for bending moment
\( \varphi_{ef} \) effective creep ratio
\( \xi \) coefficient depending on number of storys, variation of stiffness, rigidity of base restraint and load distribution.
\( \xi_n \) coefficient dependent on number of storys

Roman lower case letters
\( a_{max} \) peak acceleration
\( b \) width of structure
\( c_e(z) \) exposure factor
\( c_f \) force coefficient
\( c_{u,d} \) design value for undrained shear strength
\( d_i \) bay length
\( e \) distance from gravity center to leeward edge
\( e_0 \) eccentricity due to initial bow imperfection
eccentricity due to unintended inclination
compressive strength of concrete
allowable stress in the vertical concrete reinforcing
nominal value of the yield strength
point of load application for horizontal load
radius of gyration
equivalent stiffness of object
coefficient for second order effects
rotational stiffness
base restraint
peak factor
critical load factor according to Vianello
effective length
length for unintended inclination
turbulence intensity at height \( z = z_s \)
mass of colliding object
along wind fundamental equivalent mass
relative force
number of members
number of storys
ratio of cross-sectional area of vertical reinforcing steel to gross area of column
basic velocity pressure
peak velocity
factored vertical imposed load of story
load vector
moment ratio between first order end moments
up-crossing frequency
object velocity at impact
mean wind velocity at height \( z = z_s \)
curvature for Vianello method
deformation capacity
assumed deflection curve
-calculated deflection curve
height
reference height
shear stiffness from beams
shear stiffness from columns
shear stiffness
total base area under compression
gross cross-sectional area of column
$A_{\text{eff}}$  effective area  \\
$E_s$  modulus of elasticity for reinforcement  \\
$E_{cd}$  design value of modulus of elasticity of concrete  \\
$F$  maximum force  \\
$F_0$  plastic strength of structure  \\
$F_{H,0Ed}$  first order horizontal forces due to wind and imperfection  \\
$F_{H,Ed}$  fictitious magnified horizontal force through simplified method  \\
$F_{v,BB}$  critical buckling load concrete without significant shear deformation  \\
$F_{v,BS}$  total shear stiffness of bracing units  \\
$F_{v,B}$  critical buckling load concrete with significant shear deformation  \\
$F_{v,Ed}$  total vertical load  \\
$G$  weight of the building  \\
$G(r)$  geometric stiffness due to load vector $r$  \\
$G_{\text{dst,d}}$  destabilising permanent actions for uplift  \\
$G_{\text{stb,d}}$  stabilising permanent actions for uplift  \\
$H$  horizontal forces acting on the building  \\
$H_d$  applied base shear  \\
$H_i$  transverse force from unintended inclination  \\
$H_{Ed}$  total horizontal load (story shear)  \\
$I_c$  second moment of area (uncracked concrete section) of bracing members  \\
$I_s$  second moment of area of reinforcement  \\
$K$  stiffness matrix  \\
$K_c$  factor for effects of cracking and creep  \\
$K_s$  factor for contribution of reinforcement  \\
$K_x$  non-dimensional coefficient  \\
$L$  total height of building  \\
$L_B$  span of bracing system  \\
$L_i$  story height  \\
$L_{cr}$  buckling length  \\
$M$  moment  \\
$N$  axial load  \\
$N_a$  longitudinal force component  \\
$N_b$  longitudinal force component  \\
$N_{cr,B}$  buckling load for bending of equivalent column  \\
$N_{cr,K}$  buckling load for equivalent column with spring  \\
$N_{cr,S}$  buckling load for shear of equivalent column  \\
$N_{cr}$  buckling load  \\
$N_{Ed}$  design compressive force  \\
$P$  total allowable axial load  \\
$P_F$  concentrated load at floor level  \\
$P_T$  concentrated load at the top  \\
$P_{cr}$  buckling load from linear buckling analysis  \\
$P_{v,i}$  factored dead load of story  \\
$Q_{\text{dst,d}}$  destabilising variable actions for uplift
\( R \) square root of resonant response
\( R_{d,s} \) shear resistance
\( R_{d,up} \) additional resistance to uplift
\( R_{pd} \) passive pressures
\( S \) safety against overturning
\( T \) averaging time for mean wind velocity
\( V_{Ed} \) design vertical load (story thrust)
\( W_y \) section modulus
1 Introduction

1.1 Background

When it comes to choosing a structural system for a building project there is a long process and different criteria that the designer has to prioritize. Stability is a very common issue that appears in many structural systems whether the project is a high rise building or a smaller residential building. These stability effects include overturning, sliding, accidental action, dynamic effects and global buckling effects.

In Gothenburg today the most focus is put into designing the building from a geotechnical stability perspective. This is due to the poor strength properties of the clay which is often the soil that has to be built on. Most of the buildings are in an international perceptive considered low and believed to be safe from additional stability requirements. This typical mindset that global buckling behavior can only be seen in tall buildings could be deceiving since buildings are becoming more slender. It is, therefore, important to remember that the buckling phenomena is not limited to tall buildings and should be studied even for shorter buildings. Stability problems can be solved by different approaches and it is not always clear what approach is most accurate or relevant. The Eurocode have several approaches to determine if second order effects, i.e global buckling behavior, is required to be covered. The methods are based on simplified criteria and particular constants, which are not explained in the code. It is important to understand where these values come from and what stability safety factors are sufficient when designing a system. With the integration of Finite element software packages, it has become easier to conduct large-scale analysis. However, with less user input the engineers do not know the significance of the safety values or what they are derived from.

1.2 Aim

The aim of this thesis is to cover the issues of global stability to ensure safe structures in the future due their increase in slenderness. The main intent is to present methods that a general contractor can use to evaluate global stability in buildings. The method developed for global stability is intended to be based on research and experience from structural engineers.

1.3 Objectives

One objective is to provide a checklist that structural engineers could follow to take the most important stability factors into consideration. The methods should handle the choice of material properties, geometry, load combinations and safety factors. Another objective is to test a modern FEM software package, Strusoft FEM-Design, and compare the results with hand calculations.
1.4 Limitations

The thesis is limited to look at stability problems associated with the structural system regardless of geotechnical effects. The boundary conditions of the columns and walls in our experiment examples are assumed to be fully fixed. Soil sub-grade modules and stiffness are assumed to be equal to infinity. Furthermore, the methods presented in this thesis are general methods for all structural materials. However, Eurocode regulations that have been presented only apply for concrete and steel bracing systems. The performed experiments were exclusive for concrete bracing elements and did not take temperature and shrinkage effects into consideration. They also relied on a simple creep model, which was assumed and not calculated. The experiment was carried on a symmetrical simple architectural layout with varying structural stiffness to study the global buckling phenomena. Other stability problems such as overturning and sliding were not included in our experiment since these subjects are connected to the geotechnical discipline. It is worth mentioning that the type of connection between beams and columns play a significant role in the structural behaviour of a building. And since the focus of this thesis is put on the stabilizing system of the structure, the connections between the bar elements are assumed to have zero stiffness on rotational degrees of freedom, i.e hinged.

1.5 Method

The first chapters of the thesis are based on research from structural journals, conference reports, and published research. After the introduction, the first matter handled are the global stability issues to give the reader background in stability and design requirements. Then the external actions are introduced and explained with the help of Eurocode guidelines. With the issues in mind, the stabilizing systems in buildings are then explained to introduce ways to create stiffness to solve those stability problems. Evaluation of buckling stability is examined separately from the other stability issues and the focus is in explaining the methods that can be used to determine if there are global stability effects. Ways of determining the second order effects are also presented. Furthermore, the methods used when studying the structural stability members are explained. Additional stability effects in terms of overturning, sliding, accidental action and dynamic stability are then handled to cover all the possible stability issues with buildings. Modeling choices are then briefly discussed to show the importance of the choices made when creating a structural model, such as geometry, material, and boundary conditions.

After this theoretical base, the structural engineers view on structural stability is presented through interviews with main structural engineers at Skanska AB and with teachers from Chalmers University of Technology. The interviews are conducted with carefully developed questions to guide the interviews. The methods presented about global buckling stability are then evaluated with two idealized buildings to show the pros and cons with the different methods. Finally, a checklist can be established by combining the information gained through research together with the results from the interviews and the idealized building examples.
2 Global stability

When designing a structure there are several effects that need to be taken into consideration. A building has to be able to transfer loads to the ground, this includes both vertical forces from the structural system and interior, and also lateral loads (Gardner, 2014). A structural system can experience several different types of instability and can only be considered stable if all levels of stability can be maintained. Instability can appear locally in an element or globally in a larger context such as a frame or entire building. The actions in the structure causing instability can be both vertical and horizontal, these forces will be described in the next chapter. For the structure to be able to transfer these loads there needs to be a continuous connection between the elements. The pathway taken by the load to the foundation is most often referred to as the load path. This chapter will handle how to ensure a secure load path and what other requirements and conditions need to be met to ensure a stable structure.

2.1 Safety in systems

One of the first priorities of any structural designer is to design a safe structure. To ensure this, the capacity of the individual elements to carry the load is often the first requirement to be studied. This is important since a failure of members can cause the entire building to collapse. However, it is not always the cross-sectional capacities that cause failure in a member. Geometrical limitations can cause a member to buckle before the full cross-section can be utilized (Eurocode 2, 2004). In steel design, members are classified to clarify how much of their capacity can be utilized before the member experience instability. The instability can be caused by local plate buckling in flanges or webs or the entire member can buckle.

So to be able to describe a system as a safe system, all the risks that arise from the environment of the construction together with the load paths and material properties must be evaluated (Gardner, 2014). This is in fact very complicated and in many cases can be impossible. There are several different factors that have a significant effect on the safety of the structural system. Some of these factors can include effects from accidental loadings, such as an explosion, or even a perturbation in the system itself due to misinterpretation of the construction drawings in the detailing or erection processes. Other factors include the risk of overturning, sliding, uplifting or global buckling of the structure. Also, the building structural behavior in form of acceleration and deflections can limit the safety of the building. Safety is usually sought by multiplying load values with numerical factors to either increase the assumed characteristic loads in the calculation or to reduce the defined material capacities.

Every action needs to be able to be transferred to the ground through at least one load path. In structural design however it is important to include redundancy in the system to ensure stability in case of accidental load where an element is eliminated. This is further discussed in Chapter 2.4. Several loads can share the same load paths or part of the path. These paths can be forced to take specific routes by including or removing certain members, which can improve stability by decreasing load taken by sensitive members. In the case of lateral forces, the members taking the load can include the facades, wind posts, horizontal structural members, such as beams and slabs, vertical structural members, such as columns and walls, and finally the foundation. The connections between these members also need to be able to transfer load between the elements.
The horizontal members can transfer lateral load through two systems, these include diaphragms and triangulated bracing, see Figure 2.1. The responsibility of these systems is to transfer the load to the vertical members. Concrete slabs, metal and plywood sheets can work as a diaphragm, which functions as a deep, thin beam (APA, 2007). The panels work as a web resisting the shear forces, while the edges work as the flanges resisting bending forces. Another system is horizontal triangulated bracing, which is usually arranged in a truss format, which translates through tension and compression (Gardner, 2014).

![Figure 2.1: Diaphragm and horizontal bracing (Gardner, 2014).](image)

The vertical members need to be placed in a way to provide stability in two directions and also resist torsional forces. The members can be supported with a vertical bracing system or by shear walls. The shear wall functions like a vertical cantilevered diaphragm, where load at the top is transferred to the bottom edge (APA, 2007). This shear transfer causes a rotation which needs to be resisted with ties. The vertical bracing system is arranged similarly to the horizontal system in a truss format (Gardner, 2007). These bracings systems are further explained in Chapter 4.1 and 4.3.
2.2 Overturning and sliding

The horizontal loads on a building will generate both horizontal and vertical reactions in the foundation due to its eccentricity. The foundation will have to resist an overturning moment and a sliding action, see Figure 2.2. If the building is standing on solid ground without piles it will resist this overturning moment by a moment created by structures weight acting at the bearing edge (Hambly, 1990). If the building is, however, standing on soft soil then the ground can overturn because sway generated by elevated weight. The friction in the soil need to be able to resist the shear forces transferred by the foundation, otherwise the foundation will slide on top of the soil.

Figure 2.2: Lateral loads resulting in overturning and sliding effect

Figure 2.3 shows a real life example of such failure where the 13-story apartment building collapsed killing one worker with just enough room to escape a domino series of progressive collapse of the nearby buildings.

Figure 2.3: Overturning collapse of building in Shanghai (The Wall Street Journal Jun, 2009)
2.3 Uplifting

Uplifting phenomena is also a form of instability that occurs especially in small structures that have an underground part located in areas where there is a high water table level (Gardner 2014). For instance, underground water tanks or structures comprised of relatively big submerged compartments can be critical. An example of an uplifting failure occurred in a new restaurant building located on the waterfront of Jeddah city in KSA 2013, see Figure 2.4. Unlike most structural problems, uplifting failure occurs when there are not enough loads to resist the hydraulic pressure, the weight of the removed water (Archimedes’ buoyancy principle).

In the case of the restaurant, it was due to the rise of the water table level after casting the water tank retaining walls. The workers stopped the dewatering pumps and backfilled the foundation around the water tank which was empty at that time. It is worth mentioning that this failure would not have occurred if the tank was full of water, or the top roof slab was cast at the time, which would add enough load on the foundation to keep the structure stable in its place.

As it might go against the typical mindset of the structural designer, in this case, the loads have a favorable action and the stability check should be performed assuming the minimum loads existing on the structure. Of course taking the construction process and all the stages of the field works into consideration when performing the structural calculations.
2.4 Accidental actions

Buildings are sometimes exposed to actions that are not considered “normal”. These could include explosions, vehicle collision and consequences from human error (Eurocode, 2002). To make sure that these actions do not lead to detrimental consequences, the Eurocode has tried to develop strategies for identified accidental actions. The actions that should be taken into account are dependent on the probability of them occurring, their consequences, the public perception, the measures that need to be taken and the level of acceptable risk (Eurocode 1, 2006). In reality, one cannot prevent all the risk associated with erecting a building. A certain level of risk needs to be determined by weighing the cost versus the consequences. Local damages, for example, may be accepted if the stability of the whole structure is not threatened.

Some of the strategies to reduce the risk of accidental action include preventing or reducing action from occurring, protecting the building from the effects of the action or ensuring that the building can still function after the action. Several approaches can be taken to ensure that the building can survive the action. By including sufficient redundancy in the structure, alternative load paths can take the load if a bearing member is destroyed. Another approach is to provide satisfactory ductility in members to be able to absorb the energy from impact without leading to failure. The final way is to design key elements to be able to withstand the accidental impact and only allow nonbearing members to be eliminated.

Fire safety is one accidental action that becomes especially important in taller buildings due to the more severe consequences it can cause. Regulations need to be stricter since the evacuation is more complicated. This can limit the structural system and also the materials used. Since evacuation is a major issue it could be appropriate to have elevators in a protected structural core and use materials that are less affected by the elevated temperatures. It could also be useful to have separated fire cells to limit fire from spreading.

Some examples of system perturbation that can occur in structures which are not covered in the Eurocode’s numerical safety factors are the misinterpretations that can take place due to miscommunication between the people who are working in the supply chain of design and detailing in a construction project. An example of this situation could be when construction workers separate the reinforcement bars of the stabilizing core of the building to increase production speed. The core element assumed in the design calculation shall take the role of the main stabilizing system as a whole unit “pier section”, see Figure 2.5. Separating the reinforcement bars of such a structural element will introduce shear discontinuities in the unit that would change the behavior of the whole system.
Figure 2.5: An example of perturbation in the system due to misinterpretation of the design drawings.

Armer (1992) suggested the concept of robust design to overcome such problems that are not covered in the code numerical safety factors. The first noticeable quality of such systems is the unique ability to take force paths in multiple independent routs, the so-called system redundancy.
2.5 Dynamic stability

Even if a building is able to withstand all the loads, there could still be issues concerning the serviceability. One of the requirements of a building is to fulfill comfort demands needed to provide an enjoyable experience in the building. Wind loads can induce a motion and deflections that are unsatisfactory for the comfort condition (Johann et al., 2015). The horizontal deflection can also lead to other structural issues which will be discussed separately in Section 2.6.1. The tenants of a building do not want to perceive the movement of the building or hear creaking noises. The most common way to assess the comfort in the structure is to assess the buildings acceleration and frequency (Smith & Coull, 1991).

Acceleration can be defined either by peak acceleration or normalized root-mean-square (RMS) values (Setareh, 2010). The peak acceleration at different frequencies can be used as an indicator for human motion thresholds, see Table 2.1. This is more often applied to forces that have a short duration.

Table 2.1: Human perception of acceleration (Smith & Coull, 1991).

<table>
<thead>
<tr>
<th>Acceleration $[m/s^2]$</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.05</td>
<td>Humans cannot perceive motion</td>
</tr>
<tr>
<td>0.05-0.10</td>
<td>Sensitive people can perceive motion</td>
</tr>
<tr>
<td></td>
<td>Hanging objects may move slightly</td>
</tr>
<tr>
<td>0.10-0.25</td>
<td>Majority of people will perceive motion</td>
</tr>
<tr>
<td></td>
<td>Level of motion may effect desk work</td>
</tr>
<tr>
<td></td>
<td>Long term exposure may produce motion sickness</td>
</tr>
<tr>
<td>0.25-0.40</td>
<td>Desk-work becomes difficult or almost impossible</td>
</tr>
<tr>
<td></td>
<td>Ambulation still possible</td>
</tr>
<tr>
<td>0.40-0.50</td>
<td>People strongly perceive motion</td>
</tr>
<tr>
<td></td>
<td>Difficult to walk naturally</td>
</tr>
<tr>
<td></td>
<td>Standing people may lose balance</td>
</tr>
<tr>
<td>0.50-0.60</td>
<td>Most people cannot tolerate the motion and are unable to walk naturally</td>
</tr>
<tr>
<td>0.60-0.70</td>
<td>People cannot tolerate the motion</td>
</tr>
<tr>
<td>&gt;0.85</td>
<td>Objects begin to fall and people may be injured</td>
</tr>
</tbody>
</table>

However, when evaluating continuous disturbance RMS is said to be the better indicator. The longer the duration of the vibrations the lower tolerance becomes. Eurocode has methods for calculating peak acceleration but does not provide any regulations on these values and it is up to the designer to satisfy the comfort requirements. The Council on Tall Buildings and Urban Habitat committee have summarized the relation between peak acceleration, RMS acceleration, and frequencies (Boggs, 1995). Their relation is based on the research performed by Goto Kojima and Yamada and is shown in Figure 2.6.
The figure shows that the frequency also plays a part in the human response. Certain frequencies are more sensitive to acceleration causing a human response. For inability to walk the peak acceleration only have to be $0.5 \, \text{m/s}^2$ for the frequency 0.25 Hz, while the threshold is at almost $0.6 \, \text{m/s}^2$ for 0.18 Hz and even $1 \, \text{m/s}^2$ for 0.8 Hz. It is therefore also important to understand the frequencies when designing for dynamic acceleration.

### 2.6 Global system buckling

An entire building can buckle as a whole due to lacking stiffness created by the vertical members on each level of the building. The vertical elements need to ensure stability on each level of the building (Gardner, 2014). Even if lateral load accumulate and get larger the further down in the building a failure could develop on any level. It is important to make sure that there are no weak spots in the building, i.e. soft stories. It is often suggested that the practice is to have a fairly constant utilization of the resistance on each story. This issue of global buckling is handled in Eurocode by studying the elastic buckling load or through second order analysis. This will be further discussed in Chapter 5. An additional way to indicate instability can be through measurements of the deflections. Abnormal deflections could signal...
lacking global stability.

2.6.1 Deflection

The limits of building’s maximum deflection are both a comfort requirement and a structural requirement. There is no actual explicit requirement that sets a limit to how much a building can be allowed to bow out, but there are limits of each floor which in practice creates an overall deflection requirement. According to the National Annex from the UK, each story have a requirement of $L_i/300$, which also limits the building to a maximum deflection of $L/300$ (BSI, 2008). According to Smith and Coull (1991) this value, also called drift index can, however, vary from $L/1000$ to $L/200$ depending on national codes. The deflection is an indicator of the effects of lateral actions, such as global buckling. A deflection can also lead to additional effects. One of these effects is when vertical loads act on these displacements (Hoenderkamp, 2002). The eccentricities of the vertical loads will cause additional bending moments on the structure, which can cause global failure.

2.7 Member buckling

Instability can also occur in individual members in a system. This can cause large deformations and redistribution of loads. If a member becomes unstable it will no longer be able to carry load meaning other members will need to be able to carry an additional load. This can cause other members to fail, due to the increase in load, until the structure itself is unstable. There are different types of buckling instability that can occur in members. These include flexural buckling, torsional buckling, lateral torsional buckling and plate buckling (Eurocode 3, 2005). Flexural buckling is caused by compressive stresses and can cause members to deflect perpendicularly to loading in the weak axis. Plate buckling can occur in steel members where plates have been welded together to form a cross-section. Compressive stresses cause buckling of the plates in either the webs or the flanges. Lateral torsion buckling is caused by bending stresses and results in a deflection of flanges due to compression. Finally, torsional buckling arises due to lacking torsional stiffness.

2.8 Additional stability effects

There are further effects in buildings that can cause lacking stability, such as thermal action, creep and moisture. Thermal expansion and contraction can either cause deformations or stresses dependent on the degree members are restrained. These additional effects can cause the structure to become unstable. The main components that affect the distribution of temperature are solar radiation, humidity, wind speed and changes in air temperature in shade (Radovanović et al, 2015). The impact of the actions depends on the orientation of the building, the climate, the mass of the structure, its geometry and the properties of the used materials. When designing a structure it is, therefore, important to include the effects of the thermal action. This can be designed against by using appropriate expansion joints to allow for controlled expansion and contraction. In concrete structures, moisture can cause shrinkage which results in additional forces on the system. The system also experiences creep which lowers the stiffness of the system resulting in more vulnerability to instability phenomena.
3 External Actions

To be able to determine what type of structural stability is the most crucial, the loads on the building need to be estimated. The vertical loads on a building include the weight of the building, imposed load and snow load. For instability, however, the lateral loads are more critical and include wind loads, the load due to unintended inclination, the load due to design inclination and seismic effects. These lateral loads will be explained further in this chapter, while vertical actions and soil pressures are not included in the scope of this report.

3.1 Wind loads

The wind is a movement of air caused by differences in pressure. The wind naturally fluctuates with time and acts on the external surfaces of a building (Eurocode 1, 2005). It may also act on internal surfaces if they are exposed or if external walls can be penetrated. The pressure differences create normal forces on the individual components. The wind action is a dynamic load that has a dynamic effect on the objects that are subjected to it. Yet it is usually transformed into a static action for simplification. Nonetheless, this transformation has limits and some structures, depending on their shape complicity or importance, must be checked on wind actions by wind tunnel method, which is not tackled in this report. The wind loads calculated according to Eurocode are characteristic values, which are values from wind velocity and velocity pressure based on mean return period of 50 years.

3.1.1 Wind loads in Eurocode

The structural response of the building under wind loads depends on the size of it, its shape and the dynamic properties of the structure (Eurocode 1, 2005). In Eurocode, this is checked by calculating a peak velocity pressure, $q_p$, at a reference height. The peak velocity pressure depends on the wind climate, the terrain roughness and orography, and a reference height. The peak velocity is calculated with:

$$q_p = c_e(z) \ast q_b \quad (3.1)$$

where the input $c_e(z)$ is based on terrain and reference height while $q_b$ is based on the basic velocity pressure. The wind pressure on a surface or a structural part can then be found by multiplying the peak velocity pressure with a pressure coefficient, while wind forces on structures are found by multiplying the peak velocity with a structural factor, reference area, and a force coefficient.

3.2 Unintended inclination

When creating structural elements there are always going to be deviations from the desired characteristics. It is impossible to completely follow the theoretical dimension (Lorentsen et al., 1997). The difference can be due to geometrical imperfection, such as an undesired curvature, which could lead to uneven distribution of load also reducing the buckling capacity. The deviations could also be in the form of material properties. In a steel beam, for example, there could be residual stresses created due to welding. These stresses can limit the carrying capacity of the beam. To be able to handle these imperfections,
adjustments to the applied loads have to be made. Depending on the construction there are different tolerances for the deviations. The tolerances can be applied both on individual elements and on the structure as a whole.

### 3.2.1 Unintended inclination in Eurocode

According to Eurocode, the imperfections should be taken into account in the ultimate limit states in persistent and accidental design situations, but not in serviceability (Eurocode 2, 2004). For both concrete and steel structures initial sway imperfection is represented by an inclination \( \theta_i \).

\[
\theta_i = \theta_0 \alpha_h \alpha_m
\]  

(3.2)

\( \theta_0 \) is the basic value, \( \alpha_h \) is the reduction factor for length or height and \( \alpha_m \) is the reduction factor for the number of members. The value of \( \theta_0 \) may be found in the National Annex, but the recommended value is 1/200. The other parameters can be calculated as:

\[
\alpha_h = \frac{2}{\sqrt{l_h}} ; \quad \frac{2}{3} \leq \alpha_h \leq 1
\]  

(3.3)

\[
\alpha_m = \sqrt{0.5(1 + 1/n_m)}
\]  

(3.4)

The definition of \( l_h \) and \( n_m \) depends on the effect considered. If an isolated member is studied then \( l_h \) is the length of the member and \( n_m \) is equal to one. If a bracing system is studied \( l_h \) is the height of the building and \( n_m \) is the number of vertical members contributing to resist the horizontal force on the bracing system. If a floor is studied, \( l_h \) is story height and \( n_m \) is the number of vertical elements contributing to resist horizontal force on the floor. For individual members, the effect can be transferred into an eccentricity \( e_i \) or a transverse force, \( H_i \).

\[
e_i = \theta_i l_0 / 2
\]  

(3.5)

\[
H_i = \begin{cases} 
\theta_i N & \text{unbraced system} \\
2 \theta_i N & \text{braced system}
\end{cases}
\]  

(3.6)

\( N \) is the axial load and \( l_0 \) is the effective length. For a structure, the effect is always represented by the transverse force. The effect is calculated differently for bracing systems, floor diaphragms, and roof diaphragms:

\[
H_i = \begin{cases} 
\theta_i (N_b - N_a) & \text{bracing system} \\
\theta_i (N_b - N_a)/2 & \text{floor diaphragm} \\
\theta_i N_a & \text{roof diaphragm}
\end{cases}
\]  

(3.7)

\( N_a \) and \( N_b \) are longitudinal forces contributing to \( H \). As a simplified alternative for walls and isolated columns in braced systems, an eccentricity \( e_i = l_0 / 400 \) may be used to cover implications related to normal execution deviations. When it comes to steel there are also relative initial local bow imperfections of members for flexural buckling (Eurocode 3, 2005). The imperfection is decided by an eccentricity over the length of the member. The relationship may be defined in the National Annex and there are recommended values in Eurocode. The effects of bow imperfections also have to be considered on a
bracing system. It is included by means of an equivalent geometric imperfection of the members to be restrained, in the form of an initial bow imperfection.

\[ e_0 = \alpha_m \frac{L_B}{500} \] (3.8)

where \( L_B \) is the span of the bracing system and \( \alpha_m \) is the reduction factor for number of members (see equation 3.4)

### 3.3 Seismic effects

Buildings in seismically active zones may also experience effects from earthquakes, which are created when tectonic plates in the earth’s crust interact (Taranath, 2010). The result is waves created in the crust which can destroy structural elements and cause the collapse of entire buildings. The initial quake can cause the destruction of important members and then aftershocks can re-shake the structure causing additional failure. One especially destructive effect due to the shaking is lateral loads on the structure. This load is introduced due to buildings resistance to the oscillation of the foundation. For a structure to be seismically resistant it must be able to resist the lateral forces, this is done through inelastic action including plastic deformations of the structure. The shaking can also generate vertical load, but these forces seldom have major effects on the vertical load carrying system. Accurate earthquake analysis still a difficult subject due to the varying nature of the problem. Several diverse aspects require different knowledge and the cooperation between disciplines is hard to achieve. The design process is however constantly improving and becoming more accurate.

#### 3.3.1 Seismic effects in Eurocode

According to Eurocode 8, structures in seismic regions need be designed and constructed with an acceptable level of reliability against seismic loads (2004). The code set two distinct demands including a no-collapse requirement and a damage limitation requirement. The no-collapse requirement states that the structure must be designed in a way so that it will not experience a local or global collapse in the event of a seismic event. It needs to sustain its structural integrity and load-bearing capacity after the event. For the damage limitation requirement, the structure needs to be designed so that the cost of an event does not drastically exceed what could have cost to secure it from damage. The tolerances for the no-collapse requirement and for the damage limitation requirement are either set by the National Authorities or are based on the consequences of failure. Earthquake zones are non-existent in Sweden (Bödvarsson et al., 2006). Therefore, structural engineers are not normally required to follow seismic design guidelines in Eurocode 8. However, for facilities that deal with radioactive materials, there is an exception due to the risk of radioactive contamination. The checks can also be requested by the client.

### 3.4 The P-delta effect

If vertical and lateral forces are studied separately the interaction between the effects are not included (Smith & Coull, 1991). The horizontal loading causes a deflection, or a delta, which is then loaded with vertical load causing an additional moment on the structure, see Figure 3.1. This effect is hence known as the P-Delta effect.
These effects are often small when studying typical building, but can become more significant in high-rise buildings that are abnormally flexible. The effect can also be seen in torsion, where displacements cause misalignment that induces additional torque. The P-delta effect is often also referred to as a second order effect, where first order is studying effects separately and second order includes the interaction.

3.4.1 Seconds order effects in Eurocode

According to Eurocode 2, second order effects must be evaluated when they can have an effect on the overall stability of a structure (2004). If effects are less than 10 percent of the first order the effects may be ignored. If it is taken into account, the equilibrium and resistance should be checked in the deformed state, including effects of cracking, non-linear material properties and creep. Geometrical imperfections, soil interaction, adjacent member interaction and biaxial bending should also be included when deemed necessary.

3.5 Designed inclination

Not all the designed structures or buildings have to be standing vertically straight. In some situations, the design requires members to be inclined to fit with the interior of the building due to architectural requirement. This inclination much like unintended inclination causes a lateral action on the member. These inclinations has no special treatment in Eurocode, however, the code principles and the previously described effects such as unintended inclination and in some cases P-delta analysis regulations hold true for this type of structures.

3.6 Thermal actions

According to Eurocode the effects of thermal actions need to be considered if there is a possibility that the ultimate or serviceability limit states are exceeded due to thermal movement or stresses (Eurocode 1, 2003). Then evaluating the effect it is important to utilize regional data and experiences since the climates vary depending on geographic location. When determining the effects consideration should be made for the variation in shade air temperature, solar radiation and created effects from heating or other processes. The temperature is defined using a uniform temperature $\Delta T_u$, the difference between average temperature and initial temperature, a linearly varying temperature $\Delta T_m$, the difference between outer and inner surfaces, and a temperature difference $\Delta T_p$, the differences between different parts in the structure. If these values cannot be determined from regional data there are guidelines in the Eurocode. To determine thermal effects, the coefficient of linear expansion of the materials should be used.
4 Components of stabilizing system in buildings

The need for a secure stabilizing system has been established in the previous chapter, where load effects need to be properly transferred to the foundation. But in the eyes of the building administrator, the stabilizing system is a requirement that cost money, is in their way in terms of installation, takes precious utilizing space and limits the design of the building (Lorentsen et al. 1997). It is therefore of utmost importance to have a close cooperation between the structural engineers, architect and installation designers to make sure that all demands are satisfied. There are several different ways to achieve a stable system which relies on different stabilizing schemes. This chapter aims to introduce some of the different systems that can be used to establish a stiff and functioning building. The choice of the stabilizing method is dependent on the height of the building, its primary activities, means of erecting and occurrence of horizontal loads (wind and earthquake).

4.1 Truss system

A truss system is a beam-column structure braced by diagonal members that are designed to resist lateral loads (Smith & Coull, 1991). There are several different ways to arrange the truss members, which can be seen in Figure 4.1

This type of bracing is often associated with steel structures since the diagonals are generally exposed to tension forces. Double diagonals can be used in a concrete design where loads are instead taken by compression. The advantage of using a truss system is that it can be replicated for several floors and creates a relatively stiff structure with little additional material usage. The disadvantage of the system is
however that connections can be expensive to produce and the members sometimes limit the placement of windows and doors. The main objective of the truss system is to transfer the lateral loads to the foundation. The loads will generate a shear and a bending deflection which needs to be resisted. The force paths generated is illustrated in Figure 4.2.

![Figure 4.2: Different structural behaviour of a bracing truss unit subjected to lateral load; (a) Flexural deformation; (b) Shear deformation; (c) Combined flexural and shear deformation (Smith and Coull, 1991).](image)

The higher the building the more dominant the bending or flexural deformation becomes. This bracing system is though more typically used in low-rise buildings where the shear deflection is the main mode. There have nonetheless been recent development with high rise buildings where bracing have even been exposed to the exterior to create a modern appearance.

### 4.2 Frame system

A frame system is a beam-column structure with moment resisting connections (Smith & Coull, 1991). The ability to take lateral loads are dependent on the stiffness created between the columns, beams and the connections. This system is unlike the truss system instead it is more suited for concrete rather than steel due to the ability to connect the structural members. A moment stiff connection in steel can be hard to establish and is generally quite expensive. The main advantage of the frame system is that it allows for creative freedom when it comes to designing space and placing doors and windows. The disadvantage is that the frames cannot be reproduced due to increasing shear force towards the base. With taller buildings, the sizes of the columns and beams become too large to be economical. According to Smith and Coull, rigid frames are only considered cost-effective up to about 25 stories.
Just as the truss system, the frame system needs to take shear and bending deflection. The shear is resisted by the columns and the beams. The bending moment is taken through extension and shortening of the columns on each side of the structure. The forces and deformations caused by the external shear are shown in Figure 4.3a and the ones caused by moment in Figure 4.3b.

Figure 4.3: Effects of lateral loads on frame system; (a) Forces and deformations caused by external shear (b) Forces and deformations caused by external moment (Modified from Smith & Coull, 1991).

The lateral displacement is ultimately composed of three things; beam bending, column bending and axial deformations of the columns. These are often studied separately and then added to get the final behavior of the structure. There are variations of frame system such as the slab-frame system, where a flat slab behaves as a beam and takes shear and bending moments (Taranath, 2010). This system has...
particular issues with shear stress concentrations at the column-slab connection. This can be solved using shear reinforcement or by adding more concrete volume over the columns in the form of columns capitals and drop panels.

### 4.3 Shear walls

A typical structural system for residential buildings is a shear wall-braced structure. The structure is laterally fortified by individual shear walls, combined wall units or a combination of both. The main task of the walls is to transfer lateral loads down to the foundation through flexural action, see Figure 4.4.

![Figure 4.4: Structural response of shear walls](image)

The main reason why shear wall-braced structures are suitable for residential buildings is that the system creates partitions which enclose space (Smith and Coull, 1991). These spaces can be utilized for apartments or in the case of hotels, hotel rooms. The main advantage is that the system performs well with regard to sound and fire insulation. Shear walls can act as dividers for fire cells and the thick walls provide sound insulation. This need to divide space can also be considered a disadvantage if a large space is instead required. The system is said to be economical up to about 35 stories. This height could be increased if the walls are combined with another stabilizing system such as a frame system.

### 4.4 Central core

Shear walls can also be combined in a way that makes it a core supported structural system, see Figure 4.5. This is usually when shear walls are cast around elevator shafts to form a core, which can resist lateral, vertical and torsional forces (Taranath, 2010). The number of cores can vary based on the nature of the structure. The advantages of this system are mostly architectural, giving the possibility to have a column-free exterior at ground level (Smith & Coull, 1991). This system is however not efficient when resisting lateral forces and is often paired with other structural systems to create sufficient resistance.
Figure 4.5: Structural response of core system created by shear walls with inertia in both horizontal directions

4.5 Tubular system

A tubular system is developed to act as a large cantilever to the ground. The main idea is to place the main carrying structural members in the exterior (Smith & Coull, 1991). The stiffness of the structure is maximized because of the utilized distance from the far edge of the boundary of the building to the gravity center of the horizontal layout plan. There are three main types of tube systems, these include framed tubes, bundled tubes, and trussed tubes.

The framed tube structure in the simplest form is created by four stiff panels connected to form a tube, see Figure 4.6. The panels are created by columns and beams on each floor of the building. The reason for placing all of the load carrying members at the exterior is to create a building cross-section with high rotational inertia. Due to the uniformity of this structure one advantage is to be able to reproduce a large number of the elements in a factory and then transport to site. Another advantage is the freedom to decorate the interior since there are no horizontal load bearing members present there. Since the floor slabs only transmit the lateral load to vertical members and are otherwise not active in the lateral resistance they can also be replicated on each floor. One disadvantage with this structure is that the potential stiffness can not be fully utilized because of shear lag. The shear lag causes bending of floor slabs and deformations in structural components, thus reducing the stiffness of the cross-section. Also due to the members being placed on the exterior, it limits the placement of openings.
Figure 4.6: Structural response of a frame tube system.

A bundled tube structure works to reduce the shear lag experienced in a building by introducing panels along the width of the building. These panels work as webs to prevent shear deformation and preserve the cross-section of the building. This system makes it possible to build a higher building, but it comes at the cost of the interior freedom.

The final system is the trussed tube system which replaces the exterior columns with truss members. This eliminates reduction due to racking and shear lag and makes the building act more like a cantilevered tube. This again frees up space in the interior but introduces more complications. This system requires a large number of joints and have a significantly worse vertical loading response if compared to the exterior columns. The diagonal members require a much larger cross-sectional area which would increase the cost. In practice, it is better to include diagonal bracing members in a frame system than to completely remove all the exterior columns.

### 4.6 Outrigger-braced system

Outrigger-braced systems are known to be among the most efficient bracing systems. As mentioned how tubular systems are characterized with its great stiffness due to its maximum structural depth, yet, it has an unfavorable effect from the architectural point of view, where it can limit the possibility for openings in the building facades which can give a negative effect on the indoor functionality and overall aesthetics. Outriggers, however, gives a similar effect to tubular in regards to utilizing the max structural depth of the building footprint. A truss or a deep solid girder spanning out from the core shear walls all the way out to the facades connecting all the boundary columns and forming a stiff cap that is usually occupying a depth of two-five stories (Smith & Coull, 1991). When the lateral loads hit one face of the building this stiff cap will utilize the columns that are located on that face as well as the columns that are located in the rare opposite side to give a force couple of tension and compression forming a resistant moment to the lateral action, see Figure 4.7.
The main idea is to increase the lateral stiffness by increasing the structural depth of the central core and the section of the outrigger boundary columns which are not necessarily on the boundary of the building. This system can still be efficient even if the utilized columns are located in the inner but relatively far from the central core in both directions. And it has been implemented in buildings that are around 60 stories high. However, according to Smith and Coull, it can be utilized for even taller buildings.

### 4.7 Guidelines when choosing a structural system

As mentioned before the choice of the stabilizing method is dependent on the height of the building, its primary activities, means of erecting and occurrence of horizontal loads. One stabilizing system is not superior to the other, they all have their benefits and their limitations. The researchers Lorentsen, Petersson and Sundquist (1997) have developed a guideline for choosing a structural system based on the height of the building, see Table 4.1. This system was first developed by American structural engineer Fazlur Kahn, but then further modified to follow current use.
Table 4.1: Recommended maximum stories for use of different stabilizing systems (Modified from Lorentsen, Petersson and Sundquist, 1997).

<table>
<thead>
<tr>
<th>Maximum stories</th>
<th>Structural system</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>A framework of fully fixed columns and beams with moment resisting joints.</td>
</tr>
<tr>
<td>25</td>
<td>A system of pinned columns and beams connected to a central core (either a concrete core or vertical trusses).</td>
</tr>
<tr>
<td>40</td>
<td>Fully or partially fixed columns and beams connected to a central core.</td>
</tr>
<tr>
<td>60</td>
<td>Fully or partially fixed columns and beams connected to a central core with additional horizontal trusses in the middle and the top of the building.</td>
</tr>
<tr>
<td>80</td>
<td>A framework composed of façade columns that are united to act as a rectangular tube restrained in the foundation.</td>
</tr>
<tr>
<td>100</td>
<td>Use of combined framework and truss system in the façade walls. The façades functions as a rectangular tube restrained in the foundation.</td>
</tr>
<tr>
<td>110</td>
<td>A system composed of several rectangular tubes forcing the inner columns to interact with the façades. Each tube is ended at different height in the building so that the influence of the wind loads is reduced to the minimum.</td>
</tr>
<tr>
<td>120</td>
<td>A truss system applied on the outside of a tube system, reinforcing the external tubes.</td>
</tr>
</tbody>
</table>
5 Evaluation of buckling stability

It is important to distinguish between buckling problems in general. The term “buckling” is widely used by structural engineers to describe an instability structural failure in a single member or in a system under compressive loads. This type of failure is not desired because it hinders the usage of the full sectional capacity of the material and fails due to mostly geometrical parameters. The term “local member buckling” is used to indicate a buckling failure in a single structural member of a certain system, such as a local plate buckling in the top flange of a steel I-beam. On the other hand, “global member buckling” is used to describe a global buckling failure in a member of a system. This includes a system composed of multiple welded plates in a certain geometry forming a single column member or buckling of a single compressed member in a truss bracing unit. Finally, the term “global system buckling” is used to describe a buckling failure of a whole structural system, such as a total bracing system that is composed of multiple members joined together in order to transfer the horizontal actions down to the foundation. Some may refer to buckling in a column as a global Euler buckling problem and some engineers prefer to call it “local system buckling failure”.

Design and analysis approaches may differ between codes and different designers. Yet the finished product shall always be the same, it is always a structural element section with a certain geometry and material. This structural element can be a column, beam, slab or a shear-wall. Nevertheless, it is always designed to withstand certain stresses acting on its cross section, and these stresses are a result of section forces: normal, shear and moment. According to Eurocode, all compressed sections must be checked for instability phenomena to make sure that the section will be able to act against these load effects with enough material utilization (Eurocode 3, 2005). The capacity of a compressed member is checked with respect to its geometry and boundary conditions with Euler buckling equations. Instead of reducing the material utilization it is possible to take instability phenomena into consideration with additional second order effects, which is a force acting on the deflection induced by transverse loads, intended inclination, unintended inclination or initial imperfections. In fact, Eurocode is recognized to be one of the first standers to introduce the concept of imperfections on isolated members to account for these additional effects (Fattorini et al, 1998), see Figure 5.1a. These imperfections can also be applied to the structure on a global level either in the form of initial deflection or an additional horizontal load.

The global bracing system in some cases can be sensitive to second order effects. Some codes classify systems to a "sway system", where second order effects due to the introduced imperfections have a significant effect on isolated members or a "non-sway systems" where the additional second orders effects on isolated members due to the introduced global imperfections have no significant effect. The importance of global system buckling arises in slender systems where the members are subjected to two kinds of second order effects. First from buckling of the member itself and second from buckling of the global system, see Figure 5.1b.
Global system buckling

Global system buckling is commonly described to be a fairly complicated subject. Most structural designers encounter uncertainties when handling this issue and not long time ago engineers utilized what is known today to be invalid approaches to evaluate bracing system buckling and its sensitivity to second order effects. With increasing heights and material optimization, buildings are becoming more slender which have created an increased need to study global instability effects. As explained in Chapter 3.4, P-delta effects can cause additional moments on the structure which can lead to failure. Second order effects may be ignored if they are less than 10 percent of corresponding first order effect (Eurocode, 2002). To check if these effects are significant enough to be included without actually having to do a second order analysis the Eurocode utilizes the elastic buckling load. Depending on the values of the elastic buckling load second order effects may be disregarded or indirectly taking into account through simplified methods. The method for calculating the buckling load and taking second order effects into account differs for steel and concrete systems and will, therefore, be presented separately.

5.1.1 Concrete systems

There are several ways to check if second order effects due to global behavior are required to be included in the analysis of a concrete system. These methods include the Equivalent column method, Vianellos method, Linear buckling analysis or simplified criteria in the Eurocode.
Eurocode checks

For a building, global second order effects can be ignored according to Eurocode if the total vertical load $F_{V,Ed}$ fulfills the following criteria (Eurocode 2, 2004):

$$F_{V,Ed} \leq k_1 \frac{n_s}{n_s + 1.6} \frac{\sum E_{cd} I_c}{L^2}$$  \hspace{1cm} (5.1)

$L$ is the total height of the building above a level of moment restraint, $n_s$ is the number of stories, $E_{cd}$ is the design value of the modulus of elasticity of concrete and $I_c$ is the second moment of area (uncracked concrete section) of bracing members. The value of $k_1$ may be found in National Annex, but the recommended value is 0.31. This simplification is only valid if loads on each story is close to the same, the structure is reasonably symmetrical and the stiffness is reasonably constant. The bracing members also need to be fixed at the base to prevent rotation. The structure should also not be experiencing torsional instability or global shear deformations for this simplification to be appropriate.

If this criterion is not met, the Eurocode have an additional criterion that takes global bending and shear deformations into account. In the case of no significant shear deformation present (tall buildings) the critical buckling load is $F_{v,BB}$. If the section shows to have a cracking behavior under the ULS loads the constant $\omega$ is equal to 0.4. Otherwise, it can be set to 0.8 to increase the material stiffness contribution since no cracking behavior is manifested under the ultimate loads.

$$F_{v,BB} = \xi \frac{\sum \omega E_{cd} I_c}{L^2}$$ \hspace{1cm} (5.2)

$$\xi = 7.8 \frac{n_s}{n_s + 1.6} \frac{1}{1 + 0.7k_m}$$ \hspace{1cm} (5.3)

$$k_m = \frac{\theta_m E I}{M L_B}$$ \hspace{1cm} (5.4)

$\xi$ is a coefficient depending on number of stories, variation of stiffness, rigidity of base restraint and load distribution. The base restraint, $k_m$, is dependent on rotation for bending moment, $\theta_m$, the stiffness and the height of the bracing unit. This relationship is only valid to be used if the structure has a constant EI and loading along the whole height of the bracing system.

In the case where significant shear deformation is present the critical buckling load is calculated according to the following expression:

$$F_{v,B} = \frac{F_{v,BB}}{1 + \frac{F_{v,BB}}{F_{v,BS}}}$$ \hspace{1cm} (5.5)

where $F_{v,BB}$ is calculated according to the presented path for the first case where the system has no significant shear deformation and $F_{v,BS}$ is the total shear stiffness of bracing units. Finally after calculating the critical buckling load $F_{v,B}$ or $F_{v,BB}$ the code presented the following criteria to allow considering the structure to be safe without carrying on a second order analysis.

$$F_{v,B} \geq 10 F_{v,Ed} \quad \text{with significant shear deformation}$$ \hspace{1cm} (5.6)

$$F_{v,BB} \geq 10 F_{v,Ed} \quad \text{without significant shear deformation}$$ \hspace{1cm} (5.7)
Equivalent column method

The global buckling load proposed in the Eurocode have been developed through the use of the equivalent column method. This section aims to show where the equations from the Eurocode are derived from. The global buckling load for a simple building structure can be approximated with the buckling load for equivalent columns with uniform stiffnesses (Zalka, 2000). These columns may have either a bending stiffness, a shear stiffness or both (Kollar, 2008). The buckling loads of columns with concentrated loads on the top with either bending deformation, shear deformation or supported by a spring are presented below.

\[ N_{cr,B} = \frac{\pi^2 EI}{4L^2} \quad \text{bending} \]  
\[ N_{cr,S} = \hat{S} \quad \text{shear} \]  
\[ N_{cr,K} = \frac{k_o}{L} = \frac{M/\theta}{L} \quad \text{spring} \]

For shear walls the shear stiffness is the cross-sectional area times the shear modulus, \( G \), while for simple frame systems the shear contribution is summarized separately from beams, \( \hat{S}_b \), and columns, \( \hat{S}_c \), and then combined.

\[ \hat{S} = \frac{1}{\hat{S}_b} + \frac{1}{\hat{S}_c} \]
\[ \hat{S}_b = \sum \frac{12EI_{bi}}{d_iL_i} \]
\[ \hat{S}_c = \sum \frac{\pi^2 EI_{ci}}{L_i^2} \]

(5.11)

For the beams the \( d_i \) is the bay length. The buckling loads of columns with continuous loads along the height is the same for shear deformation but differs for bending and the spring.

\[ N_{cr,B} = \frac{7.837EI}{L^2} \quad \text{bending} \]  
\[ N_{cr,K} = \frac{2k_o}{L} \quad \text{spring} \]

(5.12) (5.13)

The buckling loads are approximated by using summation theorems. Three of these include the Dunkerley, the Föppl and the Southwell summation. The summation schemes are presented in Figure 5.2.

<table>
<thead>
<tr>
<th>Dunkerley summation</th>
<th>Föppl summation</th>
<th>Southwell summation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{\alpha_{cr,1}} + \frac{1}{\alpha_{cr,2}} )</td>
<td>( \frac{1}{N_{cr,1}} + \frac{1}{N_{cr,2}} )</td>
<td>( N_{cr} = N_{cr,1} + N_{cr,2} )</td>
</tr>
</tbody>
</table>

Figure 5.2: Summation Theorems (Kollar, 2008)
Using Föppl summation the influence of modification of rotation at the base becomes:

\[
N_{cr} = \left( \frac{1}{N_{cr,B}} + \frac{1}{2k_w/L} \right)^{-1} = \frac{N_{cr,B}}{1 + \frac{3.9EI}{k_wL}}
\]  

(5.14)

If shear is instead considered together with bending it becomes:

\[
N_{cr} = \left( \frac{1}{N_{cr,B}} + \frac{1}{\hat{S}} \right)^{-1} = \frac{N_{cr,B}}{1 + \frac{N_{cr,B}}{\hat{S}}}
\]  

(5.15)

Both expressions above are given in Eurocode, but the rotational influence had a 0.7, instead of 3.9. This difference is unknown, but Professor Kollar at Budapest University of Technology and Economics believes that this is a misprint in Eurocode. The shear calculations have been shown to have an error up to 40 percent and an alternative approximation was made by Hegedus and Kollar, with a 0.6 multiplier before \( N_{cr,B}/\hat{S} \). Eurocode suggests that when there are several bracing elements the shear stiffness is calculated as the sum of their shear stiffnesses. This summation however may lead to unconservative results if the shear and bending stiffness ratios of the elements are different. A more conservative solution is to use the Southwell’s summation and add the buckling loads together rather than the stiffnesses. In the analysis, a uniformly distributed load along the height have been assumed but in reality, most of the loads are acting at the levels of the floors. The distributed load is therefore replaced by concentrated loads, which is then divided into a uniform load and a concentrated load on the top. The effect of this is a factor \( \xi_n \), which is dependent on the number of stories.

\[
\xi_n = \frac{n_s}{n_s + 1.59(2\frac{P_T}{P_F} - 1)}
\]  

(5.16)

\( P_T \) is the concentrated load at the top and \( P_F \) is concentrated load at floor level. This expression is given in Eurocode with \( P_T = P_F \). The application of this expression is recommended only if \( P_T > P_F/2 \). When the deformation is dictated by shear, bending deformation is insignificant, which means that the buckling load is no longer affected by the position of the load thus the number of stories has no effect. When all of the effects are summarized we get the following equations:

\[
N_{cr} = N_{cr,S} \left( 1 + \frac{N_{cr,S}}{\xi_n \frac{2k_w}{L}} \right)^{-1}
\]  

(5.17)

\[
N_{cr,S} = \min \left\{ N_{cr,n} \left( 1 + 0.6 \frac{N_{cr,n}}{\hat{S}} \right)^{-1} \right\}
\]  

(5.18)

\[
N_{cr,n} = 7.8 \xi_n \frac{EI}{L^2}
\]  

(5.19)

These equations have been simplified in the Eurocode into \( F_{v, BB} \) and \( F_{v, B} \). The limitations of the equivalent column methods should also be kept in mind when applying the Eurocode checks.
The Vianello method

The Vianello method can also be used to calculate the global critical load for a structure with varying stiffness. The method is focused on the possible scenarios for a centrally compressed column that is subjected to additional actions (Petersson and Sundquist, 2000). After the external forces are removed there are three possible cases can occur. The deformation can either go back, increase or remain. To calculate the global buckling load a deflection curve, \( y_a \), due to external load is chosen and assumed to remain after load release. With the input of this deflection curve, a moment and then a curvature, \( y'' \), can be calculated, which is then integrated twice to get a new deflection curve, \( y_c \). If the normal force is equal to the buckling load the new deflection curve should be the same. The critical load factor, \( k_v \), then becomes the relationship between the assumed and calculated deflection times the number of stories.

\[
y'' = -\frac{M}{EI} = \frac{N_{cr}y_a}{EI} \tag{5.20}
\]

\[
k_v = n \frac{\sum y_a}{\sum y_c} \tag{5.21}
\]

\[
N_{cr} = k_v \frac{EI}{L^2} \tag{5.22}
\]

If the critical load is 10 times larger than the applied load, then second order analysis can be disregarded according to the criteria in the Eurocode for \( F_{v,BB} \).

Linear buckling analysis

The most accurate method to calculate the critical elastic buckling load is the use of Linear buckling analysis. It predicts the theoretical buckling load of an ideal linear elastic structure by solving an eigenvalue problem (CSi, 2015). The structural eigenvalues are computed from constraints and loading conditions. With the solution of eigenvalues and eigenvectors, the buckling loads can be determined with their associated mode shape. This is the shape the structure takes under buckling. The eigenvalue problem is defined as:

\[
[K - \lambda_{LBA} G(r)] \Psi = 0 \tag{5.23}
\]

where \( K \) is the stiffness matrix, \( G(r) \) is the geometric stiffness due to load vector \( r \), \( \lambda_{LBA} \) is the diagonal matrix of eigenvalues and \( \Psi \) is a matrix of corresponding eigenvectors. A buckling load can be calculated by applying the eigenvalue to the reference load \( r \).

\[
P_{cr} = \lambda_{LBA} r \tag{5.24}
\]

In a real structure, imperfections and nonlinear behavior keep the system from achieving this theoretical buckling strength, leading Eigenvalue analysis to over-predict buckling load. The buckling load is only used to indicate if the second order effects are significant enough, by comparing it to the applied load. If the buckling load is 10 times larger than the applied load then there is no need to include second order effects in the analysis of the global system.
5.1.2 Steel systems

There are several ways to check if second order effects due to global behavior are required to be included in the analysis of a steel system. These methods include Eurocode check, National codes and Linear buckling analysis. In steel systems, the terms sway and non-sway system indicate if second order effects should be considered.

Eurocode checks

For steel structures, the Code have adopted an approach, Horne’s method, that calculates a ratio directly using the resulting horizontal deformation (sway) as an input to estimate it (Eurocode 3, 2005).

\[
\alpha_{cr} = \frac{H_{Ed}}{V_{Ed}} \frac{L_i}{\delta_{H.Ed}}
\]

(5.25)

\(H_{Ed}\) is the total horizontal load (story shear), \(V_{Ed}\) is the design vertical load (story thrust) and \(\delta_{H.Ed}\) is the horizontal displacement of a point on the top story related to the bottom story. Each story, i.e frame, need to be checked to see if the system is non-sway, \(\alpha_{cr} \leq 10\). The system is considered a non-sway system if every story is non-sway. If even a single frame is sway then the building need to be checked with second order effects.

The expression (5.24) is not applicable if the non-dimensional slenderness, \(\lambda\), of hinged beams or rafters in the system is bigger than the following value:

\[
\lambda \geq 0.3 \sqrt{\frac{A f_y}{N_{Ed}}}
\]

(5.26)

\(f_y\) is the nominal value of the yield strength and \(N_{Ed}\) is the design compressive force. It is worth mentioning that the code does not give a specific method to calculate the value of the horizontal displacement. However, some reliable methods are presented in the following chapters of this report, and an experiment will be carried on later to compare the results between them.

National codes

The British Code BS 5950: part 1 presents a simple clear criterion to differ between sway and non-sway systems. The main parameter that was used in the British code was the deflection of the top of each story in reference to its bottom level. The deflection \(\delta\) under a known and clearly defined horizontal force \(F_{nh,i}\) must be checked if it satisfies the expression (5.28) otherwise, the frame is to be considered as a sway frame and the second order effects resulting from the global buckling have to be taken into account.

\[
F_{nh,i} = \frac{P_{v,i} + q_{v,i}}{200} \leq \frac{L_i}{2000}
\]

(5.27)

(5.28)

\(P_{v,i}\) is the factored dead load of the story (i) and \(q_{v,i}\) is the factored vertical imposed load of story (i). It is observed that the given criterion in the British code has a clear definition of the horizontal force on each
story. This is different compared with e.g. the French code, see equation 5.29, where the load case is not specified. In the British code, a system is always the same for all load cases while in other code it might be classified as a "sway" for certain cases and "non-sway" for others.

$$\delta \leq \frac{L}{200}$$

That is not the only difference between the two standards. In the French code, $\delta$ is not a limit for a single story horizontal displacement but is a limit for the whole frame horizontal translation which is not a help to understand its structural behavior (Fattorini et al., 1998). Furthermore, the French standard does not provide any kind of methodology for the analysis of systems that does not fulfill the criterion (5.29). The same thing can be said about the Italian standard CNR and the Swiss code. The American standard LRFD mentioned that a major difference between braced and unbraced frames is the value of a $K$ factor for the vertically compresses members. Yet, it has a limited indication of analysis and design tools that can be used for calculating the second order effects on sway frames. It is worth mentioning that the methodologies for sway frames presented in the British code are quite similar to those presented in Eurocode and they were named in the British code as the following: Extended simple design, which is similar to the "sway mode buckling length" presented in EC3. Amplified sway method and this method happens to be similar to "amplified sway moments" method in EC3.

**Linear buckling analysis**

Linear buckling analysis can also be used for steel structures similar to the method described under concrete systems. The method calculates a critical elastic buckling load by solving an eigenvalue problem. This load can be used to determine if the system is a sway or a non-sway system. If the buckling load is more than 10 times the applied load then the system is a sway system and second order effects need to be considered. Else the second order effects can be neglected and a first order global analysis is sufficient to design dimension of the members.

### 5.1.3 Global second order effects

The global second order effects can be taken into account by conducting a second order analysis, where all imperfections are taken into account. There are however simplified methods for both steel and concrete that indirectly takes second order effects into account.

**Simplified method concrete**

The global second order effects can be taken into account by analyzing the structure with fictitious magnified horizontal forces (Eurocode 2, 2004).

$$F_{H,Ed} = \frac{F_{H,0Ed}}{1 - F_{v,Ed}/F_{v,B}}$$  \hspace{1cm} (5.30)

$F_{H,0Ed}$ is the first order horizontal forces due to wind and imperfection. The buckling load $F_{v,B}$ is calculated according to the previous chapter. In this case, however, nominal stiffness values should be used, including the effect of creep.
The nominal stiffness can be calculated with:

\[
EI = K_c E_{cd} I_c + K_s E_s I_s \tag{5.31}
\]

where \(K_c\) is a factor for effects of cracking and creep, \(K_s\) is a factor for the contribution of reinforcement, \(E_s\) is the modulus of elasticity for reinforcement, and \(I_s\) is the second moment of area of reinforcement. If \(A_s/A_c\) is greater than 0.02 then \(K_s\) and \(K_c\) can be calculated with:

\[
K_s = 1 \tag{5.32}
\]

\[
K_c = k_a k_b / (1 + \varphi_{ef}) \tag{5.33}
\]

\[
k_a = \sqrt{f_{ck}/20} \text{ [MPa]} \tag{5.34}
\]

\[
k_b = 0.30 n \leq 0.20 \tag{5.35}
\]

where \(n\) is the relative force \(N_{Ed}/(A_c f_{cd})\), \(\varphi_{ef}\) is the effective creep ratio, and \(f_{ck}\) is the characteristic strength of the concrete.

**Simplified method steel**

The second order effects in steel structures can be taken into account by magnifying the applied horizontal load, the equivalent loads due to imperfection and other possible sway effects (Eurocode 3, 2005). This provided that the \(\alpha_{cr}\) is above 3.

\[
F_{H,Ed} = \frac{F_{H,0Ed}}{1 - \frac{1}{\alpha_{cr}}} \tag{5.36}
\]

For basic cases, the global effects can be taken into account in member checks by using appropriate buckling lengths according to global buckling mode of the structure. This method is often referred to as the sway buckling lengths method.

**Nonlinear buckling analysis**

The most accurate method is to perform a second order analysis with a nonlinear buckling analysis. Loading is increased incrementally until a small change in load level causes a large change in displacement. This condition indicates that a structure has become unstable (CSi Knowledge base, 2014). The method is a static method but it takes nonlinear effects into account such as material and geometrical nonlinearities, geometric imperfections and load perturbations. To initiate the appropriate buckling mode either a small load or initial imperfection needs to be applied. This method can be used to check that the maximum stressed member is below its full capacity at the applied design load. If this is the case then the structure can be considered to be safe.
5.2 Member buckling

Buckling of members is often more discussed in steel construction since there is a possibility of elastic buckling behavior. The buckling behavior, however, is a second order effect which causes additional moment which affects the cross-section. This effect can be seen in concrete as well but it always leads to failure since concrete cannot plasticize, and will instead crack and reduce the cross-sectional capacity. Since the material behaviors are different for the materials they will be handled separately.

5.2.1 Steel

Buckling in steel can be investigated either with buckling curves or second order analysis of the members. In Eurocode, the buckling curve method works by reducing the cross-sectional capacity of a member with a buckling factor, $\chi$ (Eurocode 3, 2005). There are three types of buckling that can occur flexural, torsional, lateral torsional or local plate buckling. If the second order effects and the imperfections are introduced in the global analysis then there is no need to check the individual members for these instability modes. If they are not totally accounted for then they have to be checked with the moments and forces from the global analysis, including global imperfections and second order effects. In the case of sway buckling length method then the members have to be checked with global buckling critical lengths and not member critical length. This can however only be done in basic cases.

If a member needs to be checked for flexural buckling then it should be verified by comparing the design buckling resistance $N_{b,Rd}$ to the design value for the compression force $N_{Ed}$. The buckling resistance is calculated with:

$$N_{b,Rd} = \frac{\chi Af_y}{\gamma_{M1}}$$

for Class 1, 2 and 3 cross-sections (5.37)

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$$

for Class 4 cross-sections (5.38)

where $\chi$ is the reduction factor for relevant buckling mode, and $A_{eff}$ is the effective area. The reduction factor is determined by:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \tilde{\lambda}^2}}$$

$$\phi = 0.5 \left[ 1 + \alpha (\tilde{\lambda} - 0.2) + \tilde{\lambda}^2 \right]$$

$$\tilde{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i} \frac{1}{\tilde{\lambda}_1}$$

for Class 1, 2 and 3 cross-sections (5.41)

$$\tilde{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \frac{L_{cr}}{i} \frac{\sqrt{A_{eff}}}{\tilde{\lambda}_1}$$

for Class 4 cross-sections (5.42)

$$\tilde{\lambda}_1 = \pi \sqrt{\frac{E}{f_y}}$$

(5.43)
where $L_{cr}$ is the buckling length, $i$ is the radius of gyration and $\alpha$ is an imperfection factor depending on buckling curve. If the either the torsional or torsional-flexural buckling load is less than flexural buckling load then $N_{cr}$ is replaced with their load.

A laterally restrained member subjected to bending also needs to be checked against lateral-torsional buckling, where the design value of moment $M_{Ed}$ is compared to the design buckling resistance $M_{b,Rd}$:

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{Y_{M1}}$$  \hspace{1cm} (5.44)

$W_y$ is the section modulus and is given by:

$$W_{pl,y} \text{ (plastic)} \quad \text{for Class 1 and 2 cross-sections}$$  \hspace{1cm} (5.45)

$$W_{el,y} \text{ (elastic)} \quad \text{for Class 3 cross-sections}$$  \hspace{1cm} (5.46)

$$W_{eff,y} \text{ (effective)} \quad \text{for Class 4 cross-sections}$$  \hspace{1cm} (5.47)

The reduction factor $\chi_{LT}$ is calculated with:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}}$$  \hspace{1cm} (5.48)

$$\phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} (\lambda_{LT}^2 - 0.2) + \lambda_{LT}^2 \right]$$  \hspace{1cm} (5.49)

$$\lambda_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$  \hspace{1cm} (5.50)

where $M_{cr}$ is the elastic critical moment for lateral-torsional buckling and $\alpha_{LT}$ is an imperfection factor depending on buckling curve for lateral torsional buckling.

Plate buckling needs to be checked for members in cross-section class 4. The resistance of the members can then be determined using effective areas. If only hot-rolled sections are used then there are no members in class 4 and plate buckling can be neglected. The method for determining effective areas is not in the scope of this thesis.

Members can also be checked using second order analysis on the member level. In that case, an initial imperfection can be used to check second order effects. This is a more general method to treat instability and can be used under any loading and support condition. An initial imperfection is applied and then a design moment is decided considering second order effects. This is then combined together with stresses from axial load to determine if stresses are exceeded for the member.

$$M_{Ed} = N_{Ed} \left( \frac{N_{cr}}{N_{cr} - N_{Ed}} e_{0.d} \right)$$  \hspace{1cm} (5.51)

$$\frac{M_{Ed}}{W_y} + \frac{N_{cr}}{A} \leq \frac{f_y}{Y_{M1}}$$  \hspace{1cm} (5.52)
5.2.2 Concrete

In a concrete member, second order effects can be ignored if slenderness is below a certain value $\lambda_{\text{lim}}$ (Eurocode 2, 2004). This value can be found in National Annex, but is recommended to be defined as:

$$\lambda_{\text{lim}} = 20 \sqrt{1 + 2\omega_m \frac{1.7 - r_m}{1 + 0.2 \varphi_{ef}}} \sqrt{n}$$

(5.53)

where $\omega_m$ is the mechanical reinforcement ratio $A_s f_{yd} / A_c f_{cd}$ and $r_m$ is the moment ratio between first order end moments. In early design when the mechanical reinforcement ratio, moment ratio and the effective creep is not known the slenderness limit can be calculated with $\lambda_{\text{lim}} = 10.78 / \sqrt{n}$. The members slenderness is defined as:

$$\lambda = \frac{l_0}{l}$$

(5.54)

where $l_0$ is the effective length. If the criteria is not met, i.e the slenderness is greater than the limit, then a second order analysis needs to be conducted. Then initial imperfection for the system and the members need to be included.

5.3 Cross section dimensioning

The engineering takes a leading role during the stage of preliminary dimensioning of cross-sections. The initial approach in cross-sectional dimensioning is different from one engineer to another. Some engineers can use their experience together with simple measuring tools to determine the force paths and then calculate the corresponding needed initial sectional dimensions. Other engineers use more accurate computer-based approaches to determine the force paths. In some FEM programs such as ETABS, there is a command that can be used to iterate chosen sections for a certain structure. The program can carry out a full iteration process to assume a certain force path and corresponding internal forces together with an initial optimized sectional dimensioning. Furthermore, a list of certain sections can be given to the program to choose from, which comes very handy. Other approaches can incorporate engineering experience into the computer program.

After preliminary dimensioning, the final stresses for the different load cases need to be analyzed for the structure. Technological development has given more tools to engineers to help analyze and calculate the most accurate values of the resulting internal forces in a certain structural system. The methods can be either direct or indirect. The traditional approach was to use an indirect method where a linear analysis is performed with initial imperfection. In these cases, the global and local P-delta effects are ignored. With the approximate behavior of the system, the second order effects are determined to be significant or not, see Chapter 5.1. If they are then they are included by the simplified methods explained in the Chapter 5.1.3 or not then they can be ignored. The other method is to use a nonlinear analysis from the beginning and includes the imperfections as well as the global and local P-delta effects. This method captures the real behavior of the system and only section capacity checks are need. Member buckling checks can then be ignored.
There are advantages and disadvantages with the two methods. The indirect analysis is simpler and
requires less computational effort, while the direct analysis captures the real behavior and does not
require member instability checks. The choice of the method could be based on the slenderness of the
members and the system. If the members or system are slender and do experience second order effects it
would be appropriate to invest the effort to perform a non-linear analysis while if the system is bulky
then a first order analysis will save time and effort.

5.3.1 Cross-section dimensioning for stabilizing system

Dimensioning the stabilizing system can be a complicated process taking into consideration that it is
subjected to moment and normal force at the same time. Additionally, buckling of the system also needs
to be taken into consideration. It is not clear in Codes what the value of the buckling length be for a
member of the stabilizing system e.g. a shear wall should be. Considering that the critical buckling force
can be obtained from one of the methods mentioned above, then the buckling length can be obtained
using the same relationship with the the critical load according to the equation (5.55).

\[ l_0 = \pi \sqrt{EI / N_{cr}} \] (5.55)

This value of \( l_0 \) can be used when dimensioning the shear walls and when calculating the reinforcement. It
can also be used as an input in FEM design software when carrying on a calculation of wall reinforcement
of the program see, Figure 5.3.

![Input of the buckling length in shear walls design parameters](image)

It is worth mentioning that these relationships are not confirmed with any academic reference. An
experiment was however carried out calculating the reinforcement of the walls and the output was within
acceptable limits.
6 Evaluation of stability considering other phenomena

Additional instability phenomena that have to be checked for the structure includes overturning, sliding and uplift which are effects that can compromise the equilibrium of the building and cause movement. Other effects include dynamic stability, which is a serviceability requirement and accidental actions, which handles robustness and resistance against accidental loads.

6.1 Overturning and sliding

If the building is assumed to be standing on a non-deformable soil and the structure is considered rigid then safety against overturning can be calculated by comparing the moments created by self-weight against moments from lateral forces (Merrill and Ambrose, 1990). The building would try to rotate about the edge on the leeward side, see Figure 6.1.

The safety against overturning, $S$ can be represented with:

$$S = \frac{G e}{H h} \quad (6.1)$$

where $G$ is the weight of the building, $e$ is the distance from gravity center to leeward edge, $H$ is the horizontal forces acting on the building and $h$ is the point of load application for the horizontal load. The resistance can be enhanced by anchoring the structure to the ground. The weight of the soil over the footings can be used to increase the weight of the building in the calculation. One way to evaluate the overturning resistance in finite element modeling is by controlling the resulted stresses under the foundation, where a positive stress value indicates an overturning case of failure.

To prevent sliding the shear strength of the material under the foundation need to be more than the applied base shear, $H_d$ resulting from lateral loads. According to Eurocode 7, the shear force can be
compared to the sliding resistance through (2004):

\[ H_d \leq R_{d,s} + R_{pd} \]  

(6.2)

\( R_{d,s} \) is shear resistance and \( R_{pd} \) is passive pressures. The shear resistance can be calculated for a drained or undrained case.

\[ R_{d,s} = \begin{cases} V_{Ed} \tan \phi_d, & \text{drained} \\ A_c c_{u,d}, & \text{undrained} \end{cases} \]  

(6.3)

\( V_{Ed} \) is the design value of vertical load, \( \phi_d \) is the design value for structure-ground interface friction angle, \( A_c \) is the total base area under compression and \( c_{u,d} \) is design value for undrained shear strength.

There are also FEM programs that can be integrated that follow EC7 and can calculate soil bearing, sliding and settlement. These programs can either use 2D or 3D soil models depending on the complexity of the foundation. Further considerations of geotechnical stability are not discussed in this thesis.

### 6.2 Uplifting

The check for uplift is made in Eurocode 7 by comparing the destabilizing actions, water pressures and upward forces, to the stabilizing actions, self-weight of building and soil, and the resistances, friction forces and anchorage (2004).

\[ G_{dst,d} + Q_{dst,d} \leq G_{stb,d} + R_{d,up} \]  

(6.4)

\( G_{dst,d} \) is destabilizing permanent actions for uplift, \( Q_{dst,d} \) is destabilizing variable actions for uplift, \( G_{stb,d} \) is stabilizing permanent actions for uplift and \( R_{d,up} \) is additional resistance to uplift. These additional resistance to uplift forces may be achieved in various ways. Measures include increasing the weight of the building, anchoring the structure and decreasing the water pressures by draining the surrounding soil. Finite element modeling can be utilized to verify that piles do not fail due to the resulting uplift force. Furthermore, if the interaction between the structure and the soil modeled properly, by using a partial fixation boundary condition that has a defined stiffness in all the degrees of freedom, then the absence of positive translation in the foundation nodes can confirm stability against uplifting.

### 6.3 Accidental actions

The Eurocode uses consequence classes to differentiate efforts required to ensure sufficient safety against accidental actions (2002). There are three defined classes CC3, CC2, and CC1, which indicate high, medium and low consequence of failure for building or work. The highest consequence class indicates very great economic, social or environmental impact and is applicable to public buildings with a high consequence of failure. The medium class instead indicate considerable consequence and is applicable to residential and office buildings, or public buildings with a low consequence of failure. The lowest class indicate small or negligible consequence and is suited to agricultural buildings, greenhouses or buildings where people rarely enter such as storage units.
To adapt the requirements to the consequences, the classes are used to define the needed checks against accidental actions. For class 1, no specific consideration is necessary for accidental actions except to ensure that the robustness and stability rules are followed (Eurocode 1, 2006). For class 2 additional requirements can include either a simplified analysis with static equivalent actions or demonstration of sufficient design and detailing. For the highest class, there can be requirements for dynamic analyses, non-linear models, and interaction between the load and the structure depending on the situation.

6.3.1 Robustness design

For robustness design, there are suggestions made in Annex A in the Eurocode 1 (2006). To further divide the required actions class 2 is divided into two, one upper risk and one lower risk. The upper risk includes hotels, offices and residential buildings greater than 4 stories and public buildings in class 2 with some exceptions. The lower includes the rest of the building defined for class 2. The adoption of the strategies should provide enough robustness for the building to experience localized failure without unreasonable collapse.

For buildings in consequence class 1, no further consideration for robustness is necessary provided that the building has been designed adequately against normal use. For lower class 2 buildings additional horizontal ties or anchorage floors to walls should be provided. In the upper-class vertical ties should also be added. The building should also be checked so that the removal of any supporting column, beam supporting a column or wall does not result in an unstable structure or that local damage is not above limits. If the removal leads to failure the individual element need to be designed to resist any relevant accidental load. In the case of a class 3 building, a risk assessment must be made to take both expected and not expected risks into account. This risk assessment method is described in Annex B, but will not be further discussed in this thesis.

6.3.2 Impact analysis

Buildings can endure action from impact through either dynamic effects, nonlinear material behavior or both (Eurocode, 2006). A dynamic analysis can be performed to study the dynamic effects of an impact and determine the resistance of the structure. More complex models can integrate non-linear material behavior in the analysis, but the Eurocode only deal with the dynamic effects. An impact can either be considered hard, where the energy is consumed by the impacting body or soft when the structure deforms to absorb the impact energy. For a hard impact, the building is assumed to be rigid and that the impacting object deforms during the collision. The maximum force, $F$ is then given by:

$$ F = v_i \sqrt{k \ m} $$

(6.5)

where $v_i$ is the object velocity at impact, $k$ is the equivalent stiffness of object and $m$ is the mass of the colliding object. A simplified analysis can be carried out using standard values to ensure safety against hard impact. The equivalent static loads can be found in Eurocode dependent on the category of traffic, see Table 5.1. Forces are for example assumed higher in the case of motorways than for roads in urban areas. The forces are defined in terms of the direction of normal travel and perpendicular to the direction of normal travel. The placement of the force depends on the type of vehicle considered.
Table 6.1: Equivalent static loads for different category of traffic

<table>
<thead>
<tr>
<th>Category of traffic</th>
<th>Force $F_{dx}$ [kN]</th>
<th>Force $F_{dy}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorways and country national and main roads</td>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td>Country roads in rural area</td>
<td>750</td>
<td>375</td>
</tr>
<tr>
<td>Roads in urban area</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>Courtyards and parking garages with access to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cars</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Lorries</td>
<td>150</td>
<td>75</td>
</tr>
</tbody>
</table>

If the structure is instead designed to absorb the impact, the ductility needs to be checked so that plastic deformations can take the kinetic energy from the colliding object. The requirement is checked through the following expression:

$$\frac{1}{2} m v_i^2 \leq F_0 y_0$$  \hspace{1cm} (6.6)

where $F_0$ is the plastic strength of the structure and $y_0$ is the deformation capacity. Methods to calculate plastic strength and deformation capacity are not handled in this thesis.

### 6.4 Dynamic stability

Even if the Eurocodes does not provide any guidelines on limiting values for peak acceleration in buildings, it does provide equations for calculating the along-wind peak acceleration (Eurocode 1, 2005). The acceleration is calculated through the use of the peak factor, $k_p$, which is the ratio of the maximum acceleration to its standard value. The peak acceleration can, therefore, be calculated as:

$$a_{max}(z) = k_p \sigma_{dev}(z)$$  \hspace{1cm} (6.7)

The peak factor is calculated with the up-crossing frequency, $v$, and the averaging time for mean wind velocity, $T = 600$ seconds.

$$k_p = \max \left\{ \frac{\sqrt{2 \ln(vT)}}{3} + \frac{0.6}{\sqrt{2 \ln(vT)}} \right\}$$  \hspace{1cm} (6.8)

The standard deviation of the characteristic along-wind acceleration at height $z$ is calculated with:

$$\sigma_{dev}(z) = \frac{c_f \rho b l_v(z_s) v_m^2(z_s)}{m_{1,x}} R K_x \phi_{1,x}(z)$$  \hspace{1cm} (6.9)

where $c_f$ is the force coefficient, $\rho$ is the air density, $b$ is the width of the structure, $z_s$ is the reference height, $l_v(z_s)$ is the turbulence intensity at height $z = z_s$ above ground, $v_m(z_s)$ is the mean wind velocity, $m_{1,x}$ is the along wind fundamental equivalent mass, $R$ is the square root of resonant response, $K_x$ is the non-dimensional coefficient and $\phi_{1,x}(z)$ is the fundamental along-wind modal shape.
7 Modeling choices

The model created to design for stability is always a simulation and can never truly represent reality. There are several assumptions and decisions that have to be made by the engineers to produce a model that can be evaluated that is sufficiently close to reality. The interaction between structural elements, their stiffness, and their possible translation or rotation are some of the modeling choices that have to be made. The choice of geometry, material properties and theory will be discussed in this section. The additional choices required when modeling with finite elements will also be mentioned.

7.1 Geometry and boundary conditions

Depending on which method is followed to model the structural system, there is a series of dissections the engineer must take while modeling the chosen geometry. Depending on the level of detailing the geometry can be reduced due to symmetry or simplified to decrease the complexity of calculations. With the improvements in computer software and hardware more detailing can be included in finite element models. For hand calculations, however, there is a need for an understanding of the structural components and the force path of the chosen system to be able to simplify geometry in a suitable way. A whole structure can be modeled as one stand alone element with a certain cross-sectional properties and stiffness. The model would be based on a large number of assumptions but can be valuable if compared to a more complex simulation.

In addition to geometry, one of the most important choices that the structural designer makes is the connectivity types between different elements. These choices together with the estimated cross sections of the elements will determine a force path through the structural system. Differences in connection stiffness can have a significant effect on the structural response of a building. It is therefore of utmost importance that engineers have knowledge about the interaction between members and have detailed connections between them.

Another important aspect when modeling is to consider the boundary conditions, i.e the response at external elements. When analyzing a structure it is important not to forget the modeling of soil and the interaction between the foundation and Earth. This can play a significant role in the resulting response. This can be very hard to model accurately, in fact, the geological properties of the soil alone has an error of 30 percent (Lorentsen et al. 1997). The designer engineer often chooses to model the soil as full restraining boundary condition based on his personal assumption that the soil provides a zero deformation boundary of the system. While modeling the soil as a partial fixation boundary condition seems to give a more realistic behavior of the response. Springs with a certain stiffness k can simulate the soil response. This stiffness can be calculated through a stiffness relationship with an input of a calculated soil subgrade reaction based on Winkler model (Gerolymos & Gazetas, 2005).
7.2 Material properties

Materials are usually modeled as linearly elastic with a Youngs modulus, defined as the slope of the relation between stress and strain under tension or compression load, even if materials such as concrete are often nonlinear (Rombach, 2011). Exact models are often too complex and time-consuming to justify their use. This can, however, mean that the models ignore stiffness reduction due to cracking or yielding. This is dealt with in the Eurocode by reducing stiffness if cracking behavior is predicted in concrete members. Steel, however, is assumed as a complete linear elastic material and is allowed to use the full value of the elastic stiffness when carrying on an elastic global analysis. Nonlinear behavior can, however, more easily be modeled when using finite elements. Factors such as temperature, humidity, and age can be taken into account. It is important however to remember that all material values are based on probability and can never be truly represented. The structural engineer can never have full control of the quality of construction and need to design with safety margins.

7.3 Theory

To estimate the behavior of structural members several different aspects need to be taken into account. To be able to describe the link between the load, the stiffness of the members and the deflection several relationships need to be combined. These include kinematics, constitutive relationship, equilibrium, and resultants. The simplest application of this is through Euler-Bernoulli beam theory, with the assumption of linear elastic material (Timoshenko & Gere, 1961). A differential equation can be derived by combining the relationships. The basic assumption with beam theory is that the cross-section remains intact throughout loading without a change in size or form. This results in the simplified kinematic relationship between the curvature, $K$, and the deflection $w$:

$$ K = \frac{d w}{d x} $$ (7.1)

Due to the assumption of linear elasticity the stress, $\sigma_x$, can be expressed in terms of a Young’s modulus, $E$, times the strain, $\varepsilon_x$.

$$ \sigma_x(x, y) = E \varepsilon_x(x, y) $$ (7.2)

The resultants in terms of moment, $M$, and shear, $V$, can be expressed as:

$$ M(x) = \int \int y \sigma(x, y) \, dy \, dz $$ (7.3)

$$ V(x) = \int \int \sigma_{xy}(x, y) \, dy \, dz $$ (7.4)

Finally equilibrium needs to be satisfied. This creates a relationship between moment, shear force and load, $q$.

$$ \frac{d M}{d x} = V $$ (7.5)

$$ \frac{d V}{d x} = -q $$ (7.6)
If they are combined:

\[
\frac{d^2 M}{dx^2} = -q
\]  
(7.7)

\[
(7.8)
\]

If above assumptions are combined the differential equation for a Euler-Bernoulli beam is derived.

\[
\frac{d^2}{dx^2} \left[ EI \frac{d^2 w}{dx^2} \right] = q
\]  
(7.9)

This can be solved with FEM, which is a tool used to solve differential equations. This theory is applied to beam elements but there are other theories utilized. These include plate theories such as Mindlin and Kirchhoff. These theories are created to allow engineers to model reality. The structural members will not act like the theory in reality but is in many cases assumed to be close enough.

### 7.4 Finite element modelling

The Finite element method is a tool used to evaluate a structure based on differential equations derived through simplifications (Rombach, 2011). It is a capable tool but it also comes with limitations. Users of FEM software assumes that the programs are free from error but this is in reality not true. Engineers need to be cautious when utilizing the tool and always remember that the program only follows assumptions made by the user. The program should never replace the experience of structural engineers and the understanding of the behavior of the structure. The method can be used to increase the accuracy of the calculation but is always dependent on the accuracy of the assumptions. Additionally, computers also have a limited accuracy.

The studied structure is essentially subdivided into a finite number of elements, hence the name. This is where most errors are made in the model. It is important that the size of the elements are small enough to capture the behavior of important regions. This has become easier with the advancement of computers, since calculation execute faster and more elements can be analyzed in the same amount compared to before. The elements are also solved using form function, which can cause problems if different types of elements are combined and form function are not compatible.

The occurrence of singularities, or infinite stress and internal forces, is caused by simplifications and assumptions of elements behavior. This is only a problem that only can occur in a numerical model and not happen in reality. In reality, there would be yielding or cracking of the material if the stress becomes too large. This issue is generated because of inconsistency in use of material behavior together with loading or supports. It is, therefore, important to consider how to apply boundary conditions and loads. In the cause of uniform force, the loads are applied as equivalent loads to the nodes. If there is not a significant number of elements the shear force can be overestimated since the load is not distributed enough.

It is also important to know what element to choose, such as beam, plate, shell or continuum element. Beams and columns are mostly chosen to be modeled as 1D elements that have certain cross-sectional
properties. While shear walls and slabs are usually modeled as 2D area members with a certain thickness. Depending on what kind of results are sought, in some occasions, a beam with its reinforcement is modeled as a 3D solid element. Furthermore, element type can have an important role in how the load is distributed as well.

With the increasing complexity and size of the models, it has also become more difficult to catch mistakes. The sheer amount of data can cause the user to overlook a potential mistake in modeling. Even if the program supplies warnings to try to catch these types of issues, there can still be errors that are not realized by the engineer. The program should, for instance, identify the occurrence of kinematic effects, but this can be unnoticed. Some programs may also fix the kinematic degrees of freedom, which can further conceal the problem.
8 Structural engineer’s view on structural stability

Since there are several different choices that have to be made when studying the stability of a building, the results of analysis can differ between engineers. This chapter intends to investigate how different engineer’s view structural stability. This will be realized through interviews with experienced structural engineers at Swedish construction companies. The interviews will be conducted in Swedish and then summarized in English. The interviews will be recorded to better capture the views of the participants. The interviewees will have the possibility to verify the summary of the interviews so that the answers from the participants does not become distorted when summarized and translated into English. The interviews will be semi-formal with prepared questions but will allow for additional questions that arise during the interviews. The prepared questions will also be sent to the interviewees before to give them time to prepare.

8.1 Questions for structural engineers

The following questions will be asked during the interviews. The questions have been translated to Swedish, see Appendix A, to facilitate better interviews since the mother tongue of the engineers is Swedish. Additional questions may arise during the interviews.

Background information:

- What is your field of expertise?
- How many years have you been active in construction?
- What is your current title and what company do you work for?
- What is your educational background?

Instability in buildings:

- What different types of instability do you consider relevant to calculate? (Overturning and sliding / Uplifting / Accidental action / Dynamic stability / Global buckling / Local stability)
- What about (remaining instability issues)?
- Who is responsible for stability checks?
- What is your experience dealing with these phenomena?

Method when analyzing stability:

- In what order do you look at stability?
- What stabilizing systems do you use? Why? Which of them do you use the most?
- How do you compare with old projects?
• What tools and methods do you use when checking stability?
• What modeling choices do you make in regards to (material, geometry, actions)
• What is the output of these methods? Safety factors? Critical load? Deformation?
• What are the acceptable limits of the safety factor and slenderness and why?
• Are there other checks you consider regarding the stability of the system? What are they?
• In case the system shows a stability failure, how do you solve it?
• Do you have to show calculations to someone? (authorities)

Additional effects

• How do you consider accidental loads?
• Do you consider expansion effects?
• What role does moisture shrinkage, creep has on the stability of the structure and how to handle it?
• What are the dimensional limits?

8.2 Results from interviews

After reaching out to several engineers within Skanska and other construction companies only two were able to participate in the interviews. Through previous connections to teachers at Chalmers, it was also possible to get interviews with two distinguished teachers, one within concrete and one within steel, at Chalmers technical university. Even though the sample size is small it still gives the perspective of structural engineers both in an academical and an industry setting. The summary of each interview is presented in Appendix B.

According to the interviews, the first thing you have to look at the building’s overall stability by looking at wind loads and unintended inclination acting on the building. For overturning the equilibrium for the unfavorable load case for uplift is used and for the dimensioning of the foundation piles, the structural load case used since the self-weight is otherwise underestimated. After this is done there needs to be enough stabilizing units to take the load and a stabilizing system that can transfer the loads from the facades and floors to the walls that act as stabilizing units. For normal buildings, the structural engineers usually work with an already created architectural drawing and try to implement a structural system. Checks for torsion and deflection is also necessary to ensure that the building has enough stiffness. When it came to accidental actions they referred to the Eurocode. The engineers interviewed typically worked at an early stage and did not take second order effects into account. The responsibility to dimension the stabilizing system was most often given to the stabilizing system supplier. They did not know exactly what programs they used but believes that they do take second order effects into account since they often discuss the second order moments.
The engineers typically worked with office buildings were in almost all cases used a core for stability, with shear walls in the facade. They can differ in widths and height, but the core systems are well known. The layout is often the same with openings in the ends to have channels for installations, etc. In a new project, they would compare the moments in the structure between old and new projects. If it is similar then they know that they are within reasonable limits.

The more pressing issue seemed to be to stabilize the foundation and global system buckling was not perceived to be an issue. They said that they typically do not take second order effects into account because they rarely work with tall buildings. This was interesting since buckling can take place in a short building if it is slender enough. It was difficult to get answers when it came to tools and models for evaluating stability since they mostly referred to the system suppliers for the responsibility and that they often worked in early stages of the projects. They did, however, have suggestions in how you could improve the stability of the structural system. One suggestion was to increase the number of walls or the length of the walls. It would also be helpful to increase the thickness of the walls or the concrete quality.

When asking about any requirements to show calculation they both referred to EKS-10. Where the authorities have restricted the codes and will demand more in the future. Many years ago it was common to ask a building panel for permission to build, but this was later abandoned and it became the developer’s responsibility. This change did not lower the standard of calculations but decreased the amount that was properly documented. Attention was brought to this to the authorities and they have implemented stricter regulations for documentation of dimensioning starting next year. The authorities still do not check anything and it is still the structural engineers that are responsible for dimensioning the building properly.

The interviews with the teachers gave a valuable explanation of the code and also gave an academical perspective. Just like the engineers at Skanska the teachers agreed that they first step is to check equilibrium. The interviews gave insight in the differences between a concrete and a steel system, but also pointed out similarities between them. Björn Engström, a teacher in concrete systems, suggest that it is not reasonable to design systems that do not experience significant second order effects. Rather that designing slender systems and taking the second order into account is a much more reasonable approach due to the reduction of material. Mohammad Al-Emrani also pointed out the possible issues with construction loads for steel members being transported.
9 Evaluation of methods for global buckling stability

As explained in the earlier chapter there are several different methods that can be used to evaluate global instability from buckling. This chapter aims to compare the methods with the help of idealized buildings. The buildings will be dimensioned to take vertical loads and will later be checked with the methods for global buckling behavior.

9.1 Definition of idealized buildings

To evaluate how good the different methods are to determine critical buckling load for different types of buildings two idealized buildings are created, one tall and one short. These buildings will be evaluated with the different methods and the results will later be compared. To ensure that one building is experiencing flexural behavior a ratio 5:1, height over width, will be used. To make sure that the buildings members are of appropriate size a preliminary dimensioning will be performed. Initially, however, the architectural layout and structural system need to be determined.

9.1.1 Architectural layout tall building

The intended architectural function of the building is to serve in the commercial sector. The use of the facades for bracing structural elements is allowed to a certain limit. Yet, a minimum of 10 meters open spans is desired in the internal layouts, giving the possibility for more flexibility in placing the partitioning walls for each floor. The story height of the building is 3.3 meters and the total height of the building is 100 meters. The footprint of the building is a square shape with a dimension of 20 meters. The following Figure 9.1 shows a conceptional architectural layout with a section elevation of the whole structure.
9.1.2 Structural layout tall building

The structural material that shall be used is cast in situ reinforced concrete C30/37. The choice of this material is coming from the fact that it is one of the most commonly used structural materials. The facades are utilized to place shear walls as bracing elements and so is the elevator shaft and the staircase. The thickness of the walls ranges from 0.5m in the bottom stories to 0.25m at the top stories. An open span of 10 meters was respected in placing the vertical elements, to render the flexibility in partitioning the interior layout of each floor. See Figure 9.2 for a preliminary structural layout.

A 25cm thick post-tensioned concrete slab is assumed. The thickness is based on the thickness curve
shown in Figure 9.3 (Ritz et al., 1985). No drop beams are placed in the interior considering the low clearance of the story height. The foundation of the building and all elements under the ground level are out of the scope in this report.

The columns sizes are assumed to be (90x90) cm. This size is based on the assumed load path and the estimated contribution area of the column. The vertical imposed loads on the office floor according to Eurocode is 3.0 \( kN/m^2 \) and the vertical permanent load is equal to the weight of the slab per square meters 0.25x25 = 6.25\( kN/m^2 \) adding the weight of architectural cover, wall partitions and utilities in the roof 2.75 \( kN/m^2 \). In total, the characteristic SLS combination would equal to 12 \( kN/m^2 \). The contribution area of one column is estimated to be around 16 square meters and would result in a total SLS concentrated load of 252.75 kN including the dead weight of the column itself which is equal to 60.75 kN per story. This number shall be multiplied by the number of the stories to get the total load on the bottom cross section and that would be equal to 8341 kN. The assumed compressive strength capacity of the concrete is 35 MPa and steel yield strength is 400 MPa. According to the following working stress equation (Hicks, 2010) the total allowable axial load on the column is equal to 8383.5 kN which shows that the size of the column is within the acceptable limits.

\[
P = A_g (0.25 f_c + f_s p_g)
\]

(9.1)

\( P \) is the total allowable axial load (N), \( A_g \) is the gross cross-sectional area of the column (\( mm^2 \)), \( f_c \) is the compressive strength of the concrete (MPa), \( f_s \) is the allowable stress in the vertical concrete reinforcing equal to 40 percent of minimum yield strength but not more than 207 MPa and \( p_g \) is the ratio of the cross-sectional area of vertical reinforcing steel to gross area of the column. The ratio should not be less than 0.01 or more than 0.08. The final cross-sections are shown in Figure 9.4.
9.1.3 Architectural layout short building

The short version of this building is assumed to be seven stories to simulate a typical short commercial building in the Swedish market. To make the comparison more close to reality it is assumed that the architect gives a limited space for concrete shear walls or other kinds of bracing units. Since this is a short building, the elevator shaft together with a parallel shear wall in the facade shall be satisfactory for global stability. However, more vertical supports are allowed to be used in this layout and a span of 5 meters is acceptable in this building, to not set a high demand on the concrete slab. See figure 9.5 for a more detailed description of the structure.
9.1.4 Structural layout short building

The structural system is taken from the architectural drawings and is preliminary dimensioned using the similar approach as in Chapter 9.1.2. Figures 9.6 and 9.7 below shall give a clear picture of the structure and the stabilizing system.

Figure 9.6: Preliminary structural layout

Figure 9.7: Cross-section dimensions of stabilizing unit
9.2 Evaluation of idealized buildings

The different methods will be used to determine the critical buckling load for the two idealized buildings. Both buildings assume that the stabilizing system includes the shear walls and the core, i.e. the columns are only used to transfer vertical load. Additional assumptions made and steps taken for the methods will be further explained when discussing the methods.

9.2.1 Eurocode checks with help of equivalent column

The first check proposed in the Eurocode requires a symmetrical building with loads on each story being reasonably similar. This is true however the tall building does not have a reasonably constant stiffness along the height. Therefore this check has to be done for all locations where stiffness changes. The material used is the same for all stories and both buildings. The design modulus $E_{cd}$ is derived together with a simplified model for creep. The creep can be determined more accurately in Eurocode, but for simplicity, the effective creep has been chosen as 2.5.

\[
E_{cd} = \frac{E_{cm}}{\gamma_{CE} (1 + \varphi_{ef})} = \frac{33 \text{ GPa}}{1.2 (1 + 2.5)} = 7.86 \text{ GPa}
\]  

(9.2)

Stiffness and areas were calculated using mass properties in Autocad. The ULS load on the building was calculated to 8.523 MN per story. The critical load limit was calculated according to the Eurocode guidelines introduced in Chapter 5.1.1. The calculation can be seen in Appendix C. The results from the calculation of the tall and the short building is presented in Table 9.1.

Table 9.1: Initial Eurocode check for global system buckling for idealized short and tall building [in MN]

<table>
<thead>
<tr>
<th>Building (Storys)</th>
<th>Tall (22-31)</th>
<th>Tall (13-31)</th>
<th>Tall (3-31)</th>
<th>Tall (1-31)</th>
<th>Short (1-7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{lim}$</td>
<td>1942</td>
<td>830</td>
<td>608</td>
<td>763</td>
<td>32</td>
</tr>
<tr>
<td>$F_{v,Ed}$</td>
<td>85</td>
<td>162</td>
<td>247</td>
<td>264</td>
<td>46</td>
</tr>
</tbody>
</table>

The calculations show that the tall building can be considered safe from buckling behavior and a second order analysis is not necessary for this building. It is interesting to note that the smallest margin was not at the first story but at the third. This is due to the reduction in wall thickness that starts on the third floor. It shows that reduction of stiffness should not be reduced too much lower down in the building as it can cause soft story buckling. Even if the short building it is lower it does not satisfy the load criteria. This is due to the low bending stiffness of the buildings cross-section. This building would require a second order analysis to make sure that the members are dimensioned to take the P-delta effect, which in this case is significant.

The second Eurocode check also takes shear deformations into account, see Chapter 5.1.1. The calculations can be seen in Appendix C and the result is also shown in Table 9.2. To account for cracking behavior the constant $\omega$ was determined to be 0.8 for the both buildings. This assumes that there is no cracking of the vertical stabilizing system under ULS load. The factor however still reduces stiffness for
the system to account for uncertainties. The results also display a safety factor, $\alpha_{cr}$, which is the buckling load over the applied design load.

Table 9.2: Second Eurocode check for global system buckling for idealized short and tall building

<table>
<thead>
<tr>
<th>Building (Storys)</th>
<th>Tall (22-31)</th>
<th>Tall (13-31)</th>
<th>Tall (3-31)</th>
<th>Tall (1-31)</th>
<th>Short (1-7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{lim}$ [MN]</td>
<td>986</td>
<td>541</td>
<td>429</td>
<td>542</td>
<td>20</td>
</tr>
<tr>
<td>$F_{v,Ed}$ [MN]</td>
<td>85</td>
<td>162</td>
<td>247</td>
<td>264</td>
<td>46</td>
</tr>
<tr>
<td>$\alpha_{cr}$</td>
<td>116</td>
<td>33.43</td>
<td>17.35</td>
<td>20.52</td>
<td>4.29</td>
</tr>
</tbody>
</table>

The second check has similar results as the simplified check, where the tall building is below the limit i.e a critical buckling load more than 10 times the applied design load. The short building, however, has a safety factor of 4.29 on the buckling load of which does not satisfy the criteria and must, therefore, be analyzed with second order taking the P-delta effects into account.

9.2.2 Vianello method

The Vianello method can calculate the global critical load for the tall building incorporating the varying stiffness. Initially a deflection curve, $y_a$ is chosen where the value is a factor of the maximum deflection, i.e the top is one and the bottom zero. A moment can then be calculated by multiplying the applied loads by the deflection. The curvature is then calculated by dividing the moment with EI, where the value is a factor of the maximum EI. With the curvature know the calculated deflection can be achieved by integrating twice. The assumed deflection is then compared to the calculated one. The ratio should be similar for the full deflection curve otherwise a new iteration is required. The new deflection curve can be calculated by dividing the new deflection value with the maximum calculated deflection. The new deflection can be used as an assumed deflection and the process can be iterated. If the ratio is reasonably the same for the full height then a buckling constant can be calculated by taking the sum of the assumed deflection over the calculated one times the number of stories. The calculations can be seen in Appendix C and the results are shown in Table 9.3.

Table 9.3: Vianello method for global system buckling for idealized short and tall building

<table>
<thead>
<tr>
<th>Building (Storys)</th>
<th>Tall (1-31)</th>
<th>Short (1-7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{cr}$ [MN]</td>
<td>4072</td>
<td>214</td>
</tr>
<tr>
<td>$F_{v,Ed}$ [MN]</td>
<td>264</td>
<td>46</td>
</tr>
<tr>
<td>$\alpha_{cr}$</td>
<td>15.42</td>
<td>4.65</td>
</tr>
</tbody>
</table>

The stiffness used in the method was reduced similarly to the second Eurocode check to account for cracking. This method also indicates that the tall building has sufficient stability to neglect second order effects while the short building fails the criteria and must be analyzed taking second order effects into account.
9.2.3 Linear buckling analysis

The linear buckling analysis was performed using a program called FEM-design. The building was modeled in 3D with openings and with hinged connections between all structural elements. The shear walls were considered as a continuous element to the top for both buildings. In the analysis, unintended inclination was included as a horizontal line load acting on all floors. The calculated $\Theta_i$ was 0.0025 which gives a horizontal load of 21 kN on each floor. This was then transformed into a line load, which became 1.05 kN/m. The calculations can be seen in Appendix D and the results in Table 9.4.

<table>
<thead>
<tr>
<th>Building (Storys)</th>
<th>Tall (1-31)</th>
<th>Short (1-7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{LBA}(\alpha_{cr})$</td>
<td>21.06</td>
<td>3.33</td>
</tr>
</tbody>
</table>

The buckling analysis confirms the hand calculations and suggests that the tall building can neglect second order effects while the short building needs to take second order effects into account.

9.2.4 Comparing the methods

The different methods all came to the same conclusion when it came to whether second order effects needed to be taken into account. All methods concluded that tall building could be analyzed without taking second order effects into account while the short building do have to, see summary of results in Table 9.5.

<table>
<thead>
<tr>
<th>Building</th>
<th>Tall</th>
<th>Short</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurocode ($\alpha_{cr}$)</td>
<td>17.35</td>
<td>4.29</td>
</tr>
<tr>
<td>Vianello ($\alpha_{cr}$)</td>
<td>15.42</td>
<td>4.65</td>
</tr>
<tr>
<td>Linear Buckling Analysis ($\alpha_{cr}$)</td>
<td>21.06</td>
<td>3.33</td>
</tr>
</tbody>
</table>

This conclusion is very important since buckling behavior is most often thought as a tall building phenomena and is not applicable for short buildings. This is however not the case since short building can be slender and lack stiffness from stabilizing systems. When it comes to the values of the buckling load it seems as the hand calculations give conservative results for the tall building. This is most likely due to underestimating the stiffness since reduction was done due to cracking even if the stresses indicate that there should not be any significant cracking. For the short building, however, the hand calculations were higher than the linear buckling analysis. This is most likely due to the hand calculations inability to calculate the buckling load for a system that is experiencing second order effects. If calculations show that there are significant second order effects it is suggested to investigate with more accurate methods and not rely on the simplification made for the hand calculations.
The hand calculations were quick and did give an indication if the system was experiencing significant second order effects. The linear buckling analysis should be more accurate but it took significantly more time to create a finite element model which could then be analyzed.

9.2.5 Dimensioning and member checks

After completing a global analysis is important to apply the results to the members since they are what is ultimately dimensioned to take all the loads. If the system does experience second order effects it is suggested to run a non-linear analysis which increases the load in increments. This will give larger moment in the members which need to be dimensioned for. If the effects are not significant it is still suggested to used the global buckling load to determine the effective lengths for the stabilizing members. For the shear walls, the effective length of the tall building was calculated as 132 m and 37 m for the short one. This is longer than if you consider the walls braced on each floor, but shorter if you consider the shear wall as a simple cantilever without support. The wall is connected to other member and is, therefore, semi-braced and can take more load than if it was by itself. For the non-stabilizing system, i.e. the columns, the effective length is the story height.
10 Conclusions and recommendations

10.1 Conclusions

After conducting interviews and performing calculations on two stabilizing systems, a list of checks can be established. According to the structural engineers, the first step is to check equilibrium of the building. This includes uplift, sliding and overturning. It is important to note that the most unfavorable situation can be to have a smaller partial coefficient on the dead load. When designing the foundation piles however it is important to use the structural load case, i.e the ULS load case, since the members are being dimensioned for ultimate load conditions. After equilibrium check, the global stability should be analyzed to decide if the stabilizing system is mobilizing enough stiffness to not experience significant global second order effects. This can be checked with hand calculations that simplify the building to a column such as the Eurocode checks and the Vianello method, but also with finite elements with a Linear buckling analysis. If the critical buckling load is less than 10 times the applied vertical design load then second order effects need to be included in the structural analysis. This analysis can be done by increasing the loads by a factor or performing a nonlinear analysis. After the second order forces are determined members need to be checked for their own stability. If a second order nonlinear analysis was performed this step is not necessary, but if a simplified method was used the stability check need to be performed that includes the second order effects from the global system.

One conclusion made is that short buildings can experience buckling phenomena if their stabilizing system is slender enough. The phenomena do not only occur in tall buildings. In fact, when it comes to tall buildings the architect has the stability of the structure on top of his/her design criteria, which makes it easier for the structural designer in some cases where he/she gets a suggested stabilizing system from the architect. This could minimize the time taken in finding a suitable location for the stabilizing unit. But when it comes to short commercial buildings the architect tends to not think about the stabilizing system of the structure in the early stages of the design, and leave this task to the structural designer.

After the system have been designed to handle the global buckling behavior it is also important to secure the system from accidental loads. Depending on the consequence class different criteria need to be fulfilled. These include things like robustness design and impact analysis. Even if the Eurocode does not give limitations on the peak acceleration it would be wise to make sure that the building is within reasonable limits. Finally, the building needs to be checked in Serviceability limit state to ensure the building have sufficient stiffness to ensure the building can function properly and not experience large deformations at the top stories.
10.2 Recommendations

The best practice is to give the stabilizing system and the load path of the structure a high priority in the building design criteria. This means that global stability issues should not be left to get solved in late stages of the project design process. Moreover, it is better if the structural engineer gets involved in the early design stages where he/she can help decide what stabilizing system is most suitable. The preliminary dimensions of this system can be assumed and confirmed with the architect at that stage. Vianello method or Eurocode checks can be utilized to carry out the preliminary dimensioning of the stabilizing system. In some more complicated cases, FEM tools can be utilized for the same purpose.

According to the interviews with the experts, it is better to design slender buildings and take second order effects into account than just using a bulky stabilizing system. This means that the recommended global buckling safety factor, \( \alpha_{cr} \), shall lay in the range between 3-10. The engineers at Skanska believe the value 6 is the lowest they would go with, and any value above 10 is considered to be an inefficient use of material and space. However, the most efficient value of this factor is a subject of more research and study.

In later stages of the project design and optimization of the structural system, it is recommended to make a horizontal deflection check of the building. In many cases, horizontal deflection limits can tell a lot about the stabilizing system behavior. In case the structural designer made a mistake, the deflection check is an important sensor that can raise attention to many existing problems. Nonetheless, this check is not an abiding rule and in some cases can be overruled if the designer has a valid justification.

When shear walls work as a stabilizing system, it shall be designed for the interaction between normal force and bending moment. The correct value of the buckling length, \( l_0 \), to be used was discussed in Chapter 5.3.1.

10.3 Further studies

Further studies could include evaluation of the accuracy of second order analysis with simplified methods compared to nonlinear analysis. An investigation could also be made about what safety factor \( \alpha_{cr} \) is most effective when designing a stabilizing system. The final limits could also be investigated. How low \( \alpha_{cr} \) can you design a structural system for without compromising the safety of the building. And finally what \( l_0 \) should be used when designing the structural members taking second order effects into account.
References


A Interview questions in Swedish

Bakgrundsinformation
- Vad är ditt verksamhetsområde?
- Hur många år har du varit aktiv inom konstruktion?
- Vad är din nuvarande titel inom det företag du arbetar för?
- Vad har du för utbildningsbakgrund?

Instabilitet i byggnader
- Vilka olika typer av instabilitet anser ni vara relevanta att undersöka? (Stjälpning och glidning/ Lyftkrafter från vattentryck / Olyckslaster / Dynamisk stabilitet / Global knäckning / lokal stabilitet)
- Vad om (återstående instabilitet frågor)?
- Vem är ansvarig för stabilitets kontroller?
- Vad är din erfarenhet av att hantera dessa fenomen?

Metod vid analys av stabilitet
- I vilken ordning ser du på stabilitet?
- Vilka stabiliserande system använder du? Varför? Vilken av dem använder du mest?
- Hur använder du dig av gamla projekt?
- Vilka verktyg och metoder använder du när du kontrollerar stabilitet?
- Vilka modellerings val gör du för (material, geometri, laster)
- Vad som är resultatet av dessa metoder? Säkerhetsfaktorer? Kritisk belastning? Deformation?
- Vad är de godtagbara gränserna för säkerhetsfaktorn och slankhet och varför?
- Finns det andra kontroller du bör kontrollera för systemets stabilitet? Vilka i så fall?
- Om systemet visar instabilitet, hur löser man det?
- Behöver man visa beräkningarna till någon? (myndigheterna)

Övriga effekter
- Hur utvärderar ni olyckslaster?
- Tar ni expansionseffekter i beaktning?
- Vilken effekt har krympning och krypning på byggnadens stabilitet och hur hanterar man det?
- Vad för dimensionsbegränsningar finns det?
B Interviews

Interview with Björn Engström
21 mars 2017 13:00
Raheem and Andreas

Background
Björn have been a teacher in Concrete systems at Chalmers and also become involved in general structural engineering. He has started a course called structural systems which has focused on stabilising systems for buildings. He has gained knowledge in the field by lecturing and acting as a supervisor for master thesis.

What is instability?
When discussing instability he first mentions the lack of system capacity to take horizontal forces. The system that handles the horizontal forces can also take vertical load and act as columns. If their limits are exceeded they can buckle. He mentions that 2nd order effects can increase the sectional moments for the members and cause failure. It is important that the members have sufficient carrying capacity. He also mentions that there can be interaction between members causing a joint mode. The building can also experience torsion due to interaction between vertical and horizontal load.

What order to check instability?
When discussing the order in which to study instability he said that overturning and sliding was to be check first for equilibrium. After this is complete the structural system can be evaluated to see if it can distribute horizontal loads.

What parameters should be used?
When discussing what load parameters to be used he mentioned EQU (loss of equilibrium), and STR (Internal failure of members or structure). With stiffness there are three different methods, where nominal stiffness for example modifies the EI. He believes that you have to consider phenomena that can cause second order effects. The reduction in stiffness due to creep and cracking are important to include. It is possible to use a effective modulus to reduce the EI for the building. He does not however consider shrinkage to be of importance within these calculations.

Methods can be used?
The use of different methods depend on which stage in the process you are and what type of building you are analysing. It is for example not suggested to make a detailed analysis in early stages of the design process before the stabilizing system has been chosen. If the building is a high rise it might be appropriate to model it as a fixed column. A Core can be used for smaller buildings, but high rise buildings might need more complex systems. Maybe through the use of a facade with solid walls. This can however cause difficulties with placements of windows. If several parts of a building is connected they can act together and the normal force will not be constant. For this case the Vianello method might be better suited, He briefly mentions that the Nonlinear models takes geometric nonlinearities into considerations i.e second order effects.
Checks for concrete

To check concrete systems he mentions that you have to check the slenderness of the columns and the system. If the slenderness is within limits you can disregard 2nd order effects, but you still need to dimension according the 1st order analysis. The elastic buckling parameter is only a theoretical value to indicate the presence of 2nd order effects. The failure of a member will always happen before any linear buckling behavior. It is not recommended to dimension the chosen stabilizing system in a way to avoid carrying on an analysis for the 2nd order effects, since that way of design work can result a non-economical system. It is better to make a slender building and take 2nd order effects into account. There is no actual way to check slenderness of a system as a whole, while there are methods that makes it possible only for individual columns. He says that maybe if you represent the system as an equivalent column then you can get a slenderness value of the system, but he has not done this himself. In Eurocode there are criteria to determine when 2nd order effects can be disregarded.

What is important is to make sure that the carrying capacity is larger than the load effect, to determine the 2nd order effects if needed and increase the loads. He believes it is sufficient to make sure the structure does not buckle (meaning a safety factor of 1). In high rise building you can also check stiffness though deflection of top story. One also have to make sure stiffness is enough to handle sway, to make sure there is not too much acceleration. This is important in early stages of preliminary dimensioning.

How to handle instability?

He believes that you can handle instability by increasing the number of bracing members, their placement or their dimensions. The goal is to increase the system stiffness.

Literature

He directed us to work made by Bo Westerberg, a swedish expert within the field, and a book made by Lorentsson. He also mentioned previous master thesis he had supervised.

Expansion joints

When discussing expansion joints he said that it is not in Eurocode (maybe in prefabrication chapter), but that people follow rules of thumb. He said that it depend on shape of the building and the creep and shrinkage (from construction start). It also depends on what restriction there are? If there is only one core there is no problem, since each side can expand freely. If there is two however there can be tension between them in the lower parts where it cannot bend. Cracks can occur and then tension disappears (is this an issue? Maybe not). If slabs are simply supported then there can be possibility for small movement. (Structural connections and Norwegian book for rule of thumb).
Interview with Mohammad Al-Emrani  
23 mars 2017 11:00  
Raheem and Andreas

Background  
Mohammad is currently employed as a researcher and teacher at Chalmers University of Technology. He is active in the field of steel structures and also teaches a course on the subject.

Instability  
According to Mohammad, the greatest risk of buckling for a steel structure is during construction. The reason for this is that the constriction is at this point is bare and have not yet been fully stabilized. In the finished building you will have the addition of shear walls, floors and other members that contribute to the stability of the building. In the unfinished building you only have a system of beams and column and therefore a potential risk for different types of instability. The instability can occur at the element level if the members are not braced or at the system level if the system is not braced. This type of construction instability is the most common according to his experience. The instability can occur due to that the building is not finished according or that the engineer have been more focused on the finished product and have not studied the stages of the construction process. One failure that can occur is during lifting of the beams, when the beams experience loads that are different from the final loads. It is important that the engineer know the construction methods, but this is often not the case. There are added risk if this connection is not established, since other construction methods can introduce different loads. It is not recommended to change the method at the construction site.

Tools and methods  
What method you choose to evaluate stability depends on your stills and what tools you are comfortable with. But more generally, elementary cases can be solved with simple hand calculation, such as isolated beams and columns experiencing lateral torsional and column buckling. If beams and columns fulfill the demands from the codes then you can use simple buckling curves, where it is easy to calculate slenderness and then reduce the carrying capacity. These methods assume that you have a prismatic cross section, i.e constant, and this is not always the case. There are instructional manuals on how to non-prismatic cross sections.

Structural systems can be divided into two frameworks: sway and non-sway. A sway system experience large deformations laterally and has no stabilization in this direction. A non-sway system is braced laterally and only need to be checked for Isolated member buckling. For sway systems you also have to study the whole system. You also need to check member buckling in this case too. Both of these systems can be present within a building. You have to check how much the stiffness is contributing to the sway mode. A system can also be connected to a core.

Second order effects  
Buckling can be decided from Euler buckling of individual element, where you have a normal force, shear force and moment on every member. You then use a critical length or equivalent buckling length for columns and frames. If this is done you have to conduct a stability check on every member. This is what was done back in the day. Now you can have computer programs that conduct a first order analysis.
and then checks every member for buckling.

But you can also conduct a 2nd order analysis from the start, then you do not have to check every individual member separately. This method is independent from calculating a Pcr, having a prismatic cross-section and having specific boundary conditions. This method can be used for either a frame or a full building. Regardless of your load you will always have a first buckling mode. The critical load is then when we reach the yield stress if we conduct a elastic analysis. There is a general method in the Eurocode which involves Alpha-ultimate and critical. These can be used for any system no matter how complicated. You simply conduct a buckling analysis to collect the initial imperfections and then conduct a second order analysis to get the cross-sectional forces and stresses. You get the critical load when your most loaded cross section reach yielding. In second order analysis you add the load stepwise and check if the cross sections reach yielding, if not at the applied load then you are safe.

Second order effects is only important if the construction is slender. The effects are included in the buckling curves, with eccentricities or with equivalent horizontal loads.

**Handle instability**

If the function of the building changes and loads need to be increase you can stabilize the system in different ways. The method to stabilize changes depending on what type of instability you have (sway or element level). You can either prevent deformation with bracing, increase the bending resistance or the stiffness. You decrease the Lcr by supporting or bracing columns. You can also connect members laterally to brace. You can increase stiffness by welding additional plates (an ear sometimes). You can also use carbon fiber to increase stiffness (this does not have to be welded only glued).

**Literature**

Talk to other master thesis students that have specialized in 1st vs 2nd order. General method in Eurocode. Additional codes online.
Interview with Sören Börjesson
6 April 2017 09:30
Raheem and Andreas

Sören is a structural engineer specialized in house construction and also has a role as assignment manager. He has worked at Skanska for 31 years and got his degree in civil engineering at Chalmers Technical University in Gothenburg.

According to Sören, first you have to look at the building’s overall stability by looking at wind loads and unintended inclination acting on the building. Then you need to make sure that there is enough stabilizing units to take the load and a stabilizing system that can transfer the loads from the facades and floors to the walls that acts as stabilizing units.

For normal buildings the structural engineers usually work with an already created architectural drawing and try to implement a structural system. Maybe for taller building you can start with the stabilizing system. The architect knows that there will need to be a stabilizing system and it often it ends up being the walls. In normal building up to 8 storys you typically use concrete walls in two main directions. But you can use two main types of systems: concrete walls or steel frameworks (if you do not consider small timber houses). You utilize the walls that exist, if you have appartment separating walls they will be of concrete. Most often there will be plenty of walls in one direction, then you have to try to find placements for walls in the other direction that does not interfere with the architecture. This is different for office buildings where you have large open spaces and the layout is based around the elevator shafts. Then you consider what walls you have in the elevator shafts and the stairwells, and determine if they are enough to provide stability.

He is involved in the calculation stage and gives information to the structural system supplier, who is responsible for the stability. He calculates the wind loads and the loads from unintended inclination and distributes it over the structure based on geometry and stiffness, were stiffer walls will take more load. They calculate with two perpendicular load cases, which gives a load on each wall. The walls are then dimensioned with sufficient thickness and reinforcement. For steel frames the dimensions tend to be smaller at the top and then larger towards the bottom. But the biggest issue is more often to be able to stabilize the foundation. It is often their job to stabilize the foundation of a structure stabilized with piles or a slab sitting on the ground. They get information from the geo-technicians in how to consider the ground conditions when stabilizing the foundation with piles or a flat slab. Then you have to look at the deflections at the top to see if it is ok, then we consider it acceptable lower in the structure as well. This is more noticeable in taller buildings. Second order effects are always present and are considered based on the calculation programs. In an early stage, the people who have been around for a while typically use hand calculations, while younger engineers may tend to run a finite element analysis.

But we typically do not take second order effects into account because we rarely work with tall buildings. It is when you have those where you have to take global second order effects into account. In normal building you assume that second order effects can be neglected. He had been involved in a tall building, the Svenska mässan towers. In that instance they had performed an advanced wind calculation on the tower. He was not sure exactly how it was conducted. If they have difficult circumstances they tend to
involve colleagues from bridge construction, who have more experience working with slender structures.

The authorities have restricted the codes thru EKS-10, and will demand things that have not been required before. When he started working they always had to ask a building panel for permission to build, but this was later abandoned and it became the developer’s responsibility. This change did not lower the standard of calculations, but decreased the amount that was properly documented. Attention was brought to this to the authorities and they have implemented stricter regulations for documentation of dimensioning starting next year. The authorities still does not check anything and it is still the structural engineers that are responsible for dimensioning the building properly. There is possibility to hire external inspectors to control that everything have been done according to regulations.

The ultimate limit state is used for dimensioning but the serviceability limit state is used to check deformations. You choose the Young’s modulus in the calculation programs for the different load cases. You simply choose the concrete quality in the program. Sören has calculated the buckling length as the length between storys in the weak direction, but in the strong direction he does not consider them stabilized. He has then instead considered it as a console to capture a more global buckling mode. This load case gives an increase in reinforcement that he considers reasonable. All of his colleague does not agree that this increase is necessary.

If stability is not sufficient you can increase the number of walls or the length of the walls. It be helpful to increase the thickness of the walls. You also have to check to see if the building can handle torsion. If the building is deflecting too much you also have to increase the stiffness. They do not check the deflection of every floor, but in taller building it might be required.
Interview with Kenneth Franzen
7 April 2017 09:30
Andreas

Kenneth is a structural engineer who has worked for Skanska for six years. He is a part of a department focused on house construction. He previously studied the civil engineering track “väg och vatten” and completed a master’s degree specialized in construction. He works primarily with the foundation, but looks in an early stage at deflections to make sure that they are reasonable. He then focuses on the foundation and checks risk for overturning, sliding, uplift and compression failure. He does not focus on torsion but it should have been handled previously by another engineer. He looks at the moment and checks if the loads are reasonable for compression and uplift, and also that it can take the horizontal forces and not cause sliding. He checks the equilibrium and dimensions the foundation and the foundation piles. For overturning you look at equilibrium for the unfavorable load case for uplift. On the compression side they use the structural load case since they dimension the foundation piles. In equilibrium you underestimate the self-weight with coefficients, so it is not used for compression only for uplift.

Typically two engineers work together at the office in different stages. In an early stage one looks at total stability in terms of structural system and torsion while the other one focuses on the foundation. In the systems act stage they look loosely at the stability and checks if it has a possibility to be stable. Later in the building document stage they give over the responsibility to a structural systems supplier. They are then responsible for the stability down to the foundation. Kenneth then looks at the reaction forces in the foundation, both lift, compression and horizontal forces. When you work with hand calculations you need the loads, the spans, the moments and the compressions in the concrete structure. When you dimension locally you use the design values for the material. Typically you check if the thickness can handle the compression and assume a concrete quality. You can change either the thickness or the material parameters. The most way is to change the concrete quality since the space is more valuable for the client. In Gothenburg foundation piles is almost always used and not a flat slab on the ground. The engineer checks if the pile can handle the compression and the geo-technician looks at the full picture. Piles are so commonly used since they mainly construct on clay soil.

In an early stage they check first order effects from wind load and loads from unintended inclination. They do not check second order effect and do not make any checks for acceleration. Experience from earlier projects is used. Kenneth typically works with office buildings where in almost all cases he uses a core for stability, with shear walls in the facade. They can differ in widths and height, but the core systems are well known. The layout is often the same with openings in the ends to have channels for installations, etc. Otherwise they compare the moments in the structure between old and new projects. If it is similar then they know that they are within reasonable limits. He was not sure how the structural systems supplier works, but he assumed that they take second order effects into consideration since they usually talk about them when they get the loads for the supplier. They probably use computer programs that through nonlinear analysis takes the second order effects into account.

Residential houses are often houses with apartment separating walls in concrete to fulfill fire and noise demands. In these houses there tend to be a lot of walls and stability is typically not a problem. When you work with offices the rentable space is very important to the client and they do not want the structural
engineer to reduce the space. You look at how you can stiffen the core and this is done differently depending on the facade. If you have prefabricated concrete sandwich panels then you can utilize shear walls there which contributes to increased stability. If there is a glas-facade then you could insert cross-bracing in steel. These are the method they work with initially.

In the system act stage they secure the dimensions to be able to handle the vertical load and reinforce the foundation to be able to handle loads in case of an eliminated column. There are large loads on the columns when trying to secure them. One risk mode of failure is that a column is eliminated, but the other columns need to be able to take this load. It is however not always the case that the loads increase on adjacent columns since other material parameters are used for this load case. But typically they focus on the piles, which have a higher capacity for short-term accidental load. They have a dialog with the prefab structural systems supplier, who have different choices they can make. In some projects they dimension for the full accidental load. In some cases they ask them to reduce the accidental loads through other methods, but this is not always easily if you want to optimize the building plot without making intrusion on communal ground. Early in projects they assume that the columns should be able to be eliminated and then they later have a conversation with the structural system supplier. You could dimension the columns for collision and the 100 m2 load, but these are large loads. Kenneth have not been involved in a project where they have dimensioned the essential load carrying system to take full impact. It is mostly handled by reducing risk of impact and allowing for eliminated columns. To do this they implement horizontal and vertical ties. In Eurocode 1991-1-7 they give the different ways to handle accidental load depending on the consequence class. Their office buildings are class 2 a or b.

The structural system supplier need to be able to handle expansion and creep. They go through calculations with supplier but don’t have to show authorities. But with EKS-10 they will in the future have to record assumptions, conditions and eventually attach calculations. They will still not have to do this in an early stage, but they can send them to the supplier. This demand for calculation is new and will have to be submitted to a consultation board.

The biggest issue Kenneth sees is sliding, large horizontal forces and lift associated with the central core. You can also get tension in piles to prevent house from lifting, this is created with cohesion in the piles.
## C Hand calculations

### Eurocode checks for tall building

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**Stiffness**

- **Storeys**: (1-7)
- **Number of storeys**: 7
- \( l_c \): 2.7 \( \text{m} \)
- \( A_c \): 2.64 \( \text{m}^2 \)

**Load**

- **ULS floor**: 15.57 \( \text{kN/m}^2 \)
- **ULS walls**: 272.646 \( \text{kN} \)
- **ULS total storey**: 6.500646 \( \text{MN} \)

**Eurocode Check 1**

- **Storeys**: (1-7)
- \( k_1 \): 0.31
- \( n_s \): 7.00
- \( L \): 23.10
- \( l_c \): 2.7
- \( F_{\text{lim}} \): 32
- \( F_{v Ed} \): 46

**Eurocode Check 2**

- **Storeys**: (1-7)
- \( \xi \): 6.348837209
- \( n_s \): 7
- \( L \): 23.10
- \( l_c \): 2.7
- \( w \): 0.80
- \( F_{v BB} \): 202
- \( F_{v BS} \): 8297
- \( F_{v B} \): 197
- \( F_{\text{lim}} \): 20
- \( F_{v ED} \): 46
- \( s \): 4.29
Vianello's method for tall building

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| FvED| 264  | MN    |
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### Vianello's method for tall building

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</table>

| 0.00 | 0.00 | 1.00 | 3.08 | 1 | 3.08 | 0.00 | 0.00 | 11.93 | 27.81 | 0.000 |

| Ecd  | 7857  | MPa |
| dx  | 0.03  |     |
| Iref | 2.2   | m4  |
| incl. | 0.8  |     |
| kv  | 6.73  |     |
| Ner  | 214   | MN  |
| FvED | 46    | MN  |
| s    | 4.65  |     |
D FEM-design calculations