On Achievable Information Rates for Coherent Fiber-Optic Systems with Hard Decision Decoding

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Abstract *We analyze achievable information rates (AIRs) for coherent fiber-optic systems with hard decision decoding and show that binary codes achieve higher rates than nonbinary codes, which translates into larger transmission reach. This result is confirmed by simulation of staircase codes.*

Introduction

To cope with the increasing demands for higher spectral efficiencies, coded modulation (CM) has become essential in fiber-optic systems. To achieve high spectral efficiencies with low complexity, staircase codes[1] decoded using hard decision decoding (HDD) have been proposed in combination with higher order modulation as a very powerful CM scheme that yields very large net coding gains. Achievable information rates (AIRs) for HDD were considered in[2], where it was concluded that symbol-wise decoders (suitable for nonbinary codes) yield higher AIRs compared to bit-wise decoders (suitable for binary codes). However, the HDD considered in[2] corresponds to a decoder that exploits soft information. Indeed, the decoder is of high complexity and it is not the low-complexity standard hard decision decoding (sHDD) used to decode, e.g., staircase codes.

In this paper, we elaborate further on the AIRs of CM with HDD. In particular, we consider a CM scheme that uses the sHDD (based on the Hamming distance metric) and compute AIRs for both bit-wise and symbol-wise decoders. We show that for sHDD, the AIRs of binary codes are significantly higher than those of nonbinary codes. Therefore, for spectrally-efficient fiber-optic systems with sHDD, binary codes should be preferred. We also consider binary and nonbinary staircase codes and plot their asymptotic performance. The results confirm the conclusion arising from the analysis of the AIRs. Finally, we compute AIRs for a wavelength division multiplexing (WDM) system using the Gaussian noise model[3] which show that for sHDD, binary codes achieve larger transmission reach than nonbinary codes.

System model

The CM system is shown in Fig. 1. The information word $\boldsymbol{u}$ is encoded into codeword $\boldsymbol{x}$ using either a binary or a nonbinary code. The encoded symbols are then mapped to the symbols of a higher order constellation $\mathcal{X}$ through a mapper Φ. The modulated symbols are transmitted over a fiber-optic channel (FOC). The fiber-optic channel has memory and the exact closed-form channel law is still unknown. Therefore, to compute AIRs, we resort to the mismatch decoding framework[4], where the actual channel law is substituted by an auxiliary channel law. For long-haul uncompensated coherent fiber-optic systems, the fiber-optic channel is dominated by amplified spontaneous emission noise, and the channel law can be accurately modeled by memoryless additive white Gaussian noise (AWGN), a model known as the GN model in the literature[3].

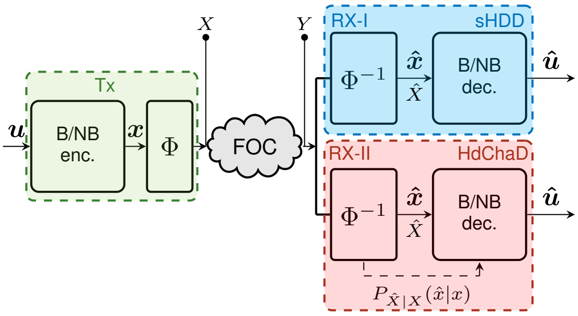

Fig. 1: System model of the CM using binary (B) and nonbinary (NB) codes for the HdChaD and sHDD systems.

For HDD, the demapper Φ^{-1} detects the received noisy symbols to the nearest constellation points and feeds the decoder with the corresponding hard-detected symbols (symbol-wise decoder) or the string of bits corresponding to the hard-detected symbol (bit-wise decoder). We denote the sequence of hard-detected symbols by $\hat{\boldsymbol{x}}$. We also denote by X, Y, and $\hat{X}$ the random variables corresponding to a single transmitted symbol, received noisy symbol, and hard-detected symbol, respectively. The detector takes a symbol-wise hard decision, $\hat{x} = \arg\max_{x\in\mathcal{X}} P_{Y|X}(y|x)$ where $\mathcal{X}$ is the alphabet of the transmitted symbols, i.e., the constellation imposed by the modulation, of cardinality $|\mathcal{X}| = M$, and $P_{Y|X}(y|x)$ is the channel conditional probability distribution, which is assumed to be Gaussian. The hard detector transforms the channel into an M-ary input, M-ary output discrete memoryless channel with transition probabilities $P_{\hat{X}|X}(\hat{x}|x)$.

We consider two different decoders. The first decoder, considered in[2], uses the input $\hat{\boldsymbol{x}}$ and also the channel transition probabilities $P_{\hat{X}|X}(\hat{x}|x)$ in the decoding procedure. Thus, the decoder exploits soft information, which has to be tuned depending on the channel signal-to-noise ratio (SNR), and hence it is not low-complexity. We call this decoder hard detector channel-aware decoder (HdChaD), which is shown with red color in Fig. 1. We also consider the low-complexity sHDD (the one used to decode BCH codes, RS codes, and staircase codes), shown with blue color in Fig. 1, which uses only the sequence of hard-detected symbols $\hat{\boldsymbol{x}}$.

AIRs for HdChaD

The AIR of the symbol-wise HdChaD (HdChaD-SW) system is[2]

$$I_{\text{HdChaD}-\text{SW}} = \mathbb{E}\left[\log_2\left(\frac{P_{\hat{X}|X}(\hat{X}|X)}{\frac{1}{M}\sum_{x\in\mathcal{X}} P_{\hat{X}|X}(\hat{X}|x)}\right)\right], \tag{1}$$

where M is the modulation order and the expectation is over $X, \hat{X}$. Let m denote the number of bits used to label each constellation symbol. The AIR of the bit-wise HdChaD (HdChaD-BW) system is[2]

$$I_{\text{HdChaD}-\text{BW}} = m - \sum_{i=1}^{m} h_{\text{b}}(\epsilon_i), \tag{2}$$

where ϵ_i is the channel error probability of the *i*th bit in the label. We remark that (1) and (2) are called the hard decision symbol-wise and hard decision bit-wise rates in[2]. However, this decoder is not a standard HD decoder, in the sense that as can be seen from (1) and (2) the decoding metric still embeds soft information. In fact, the use of the channel transition probabilities $P_{\hat{X}|X}(\hat{x}|x)$ implies that the decoder exploits soft information. In this sense, the detector can be seen as a soft detector with a (coarse) quantization.

AIRs for sHDD

For sHDD, the decoder uses the Hamming distance metric to decode the received noisy vector. The decoded codeword is $\hat{\boldsymbol{x}}_{\text{sHHD}} = \underset{\boldsymbol{x}\in\mathcal{C}}{\arg\min}\, d_{\text{H}}(\hat{\boldsymbol{x}}, \boldsymbol{x})$ where $\mathcal{C}$ is the code, $\hat{\boldsymbol{x}}$ is the hard-detected codeword, and $d_{\text{H}}(\hat{\boldsymbol{x}}, \boldsymbol{x}) \triangleq |\{i|\hat{x}_i \neq x_i\}|$ is the Hamming distance. One can use the Hamming distance metric to find the mismatch metric[4] corresponding to binary and nonbinary codes and then use it to find the corresponding AIRs.

Proposition 1 *An AIR for the symbol-wise sHDD system with nonbinary codes is*

$$I_{\text{sHHD}-\text{SW}} = \log_2 M - h_{\text{b}}(\delta) - \delta\log_2(M-1), \tag{3}$$

where $h_{\text{b}}(\delta) = -\delta\log_2\delta - (1-\delta)\log_2(1-\delta)$ *is the binary entropy function,* δ *being the (uncoded) symbol error probability, i.e.,* $\delta = \Pr\{\hat{X} \neq X\}$.

Note that (3) is the capacity of a q-ary symmetric channel (QSC) with symbol error probability δ. In fact, the sHDD transforms the AWGN channel into a QSC from the decoder perspective.

Proposition 2 *An AIR for the bit-wise sHDD system with binary codes is*

$$I_{\text{sHHD}-\text{BW}} = m[1 - h_{\text{b}}(\bar{\epsilon})], \tag{4}$$

where $\bar{\epsilon} = \frac{1}{m}\sum_{i=1}^{m}\epsilon_i$.

Numerical results

In this section, we evaluate the AIRs derived in the previous sections. In Fig. 2 and Fig. 3 we plot the AIRs for 16-QAM and 64-QAM, receptively, as a function of the SNR (E_s/N_0). For the sake of comparison, we also depict the AIR for soft decision decoding (SDD) which can be seen as an upper bound on the AIRs of the HDD system. As can be seen, for HdChaD, symbol-wise decoding leads to higher AIRs compared to the bit-wise decoding, leading to the conclusion in[2] that "HD binary decoders are shown to be unsuitable for spectrally-efficient, long-haul systems". However, if the sHDD is considered, the bit-wise decoding yields significantly higher AIRs than the symbol-wise decoding. Thus, in practice (i.e., for the conventional low-complexity HDD), binary codes/decoders are more spectrally-efficient than their nonbinary counterparts and should be preferred. In the figures, we also plot the asymptotic performance of binary and nonbinary staircase codes computed using the density evolution in[5,6]. In particular, we consider staircase codes with overheads 33.33%, 25%, 20%, 11.11%, 9.09% and 8.04%. As can be seen, the performance of binary and nonbinary staircase codes are in agree-

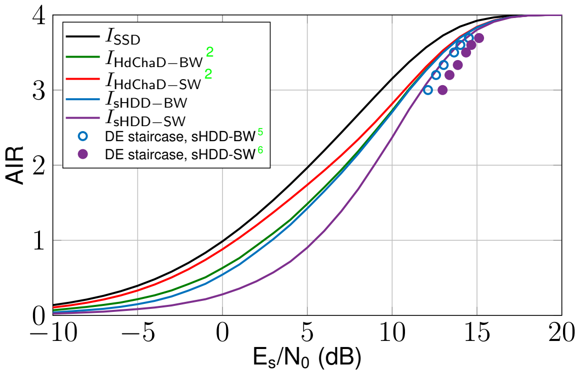

Fig. 2: AIRs for 16-QAM.

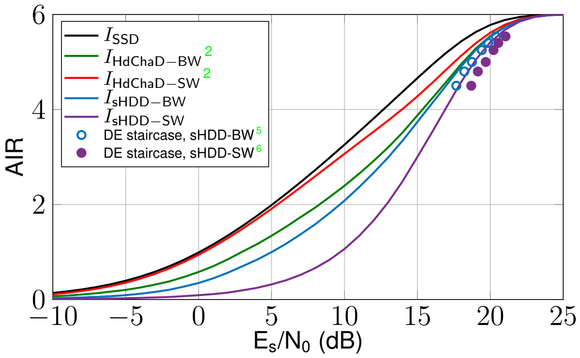

Fig. 3: AIRs for 64-QAM.

ment with the derived AIRs and binary staircase codes achieve better performance than nonbinary staircase codes. We also used the GN model to compute the AIRs for different fiber lengths. In particular, we consider a WDM system with 81 channels, each with a data rate of 28 Gbaud and 80 km span length, a fiber with $\alpha = 0.2$ dB/km, $D = 16$ ps/nm/km, and $\gamma = 1.3$ (w.km)$^{-1}$, optical center wavelength $\lambda = 1550$ nm, and lump amplification with $n_{\mathrm{sp}} = 2$. We remark that for each fiber length we have computed the optimum transmitted power and used it for the numerical results. Moreover, we depict only the AIRs of the middle channel where the interference is the highest as a pessimistic approximation of the AIRs for the other channels. In Fig. 4 and Fig. 5, we depict the AIRs of SDD, HdChaD and sHDD for different fiber length. As can be observed, for HdChaD, symbol-wise decoding (i.e., nonbinary codes) yield a reach enhancement compared to bit-wise decoding (i.e., binary codes). In contrast, for sHDD the bit-wise decoder achieves a significantly larger transmission reach compared to the symbol-wise decoder. For example, for 3 bit/symbol and 16-QAM, the binary code achieves a 555 km reach enhancement in comparison to the nonbinary code. Furthermore, for 4 bit/symbol and 64-QAM, the binary code achieves a 293 km reach enhancement in comparison to the nonbinary code.

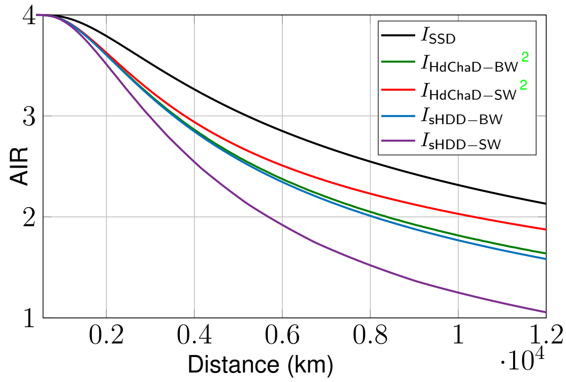

Fig. 4: AIRs for 16-QAM modulation versus fiber length.

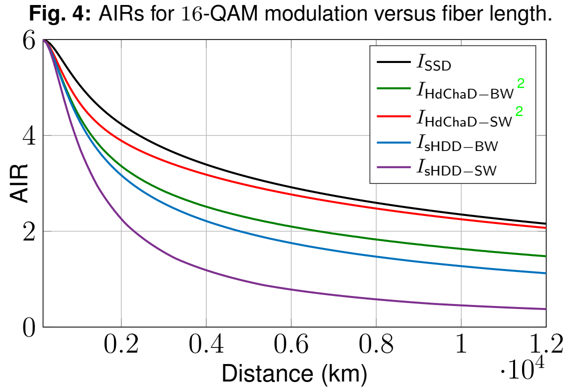

Fig. 5: AIRs for 64-QAM modulation versus fiber length.

Conclusion

We analyzed the AIRs of CM with HDD and showed that if the standard low-complexity HDD is used, binary codes yield significantly higher AIRs than nonbinary codes. Simulation results of binary and nonbinary codes confirmed these findings. Thus, an important outcome of this paper is that binary codes should be the preferred coding scheme to achieve higher spectral efficiencies in coherent fiber-optic systems.

Acknowledgment

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