Calculation of crack width and crack spacing

Ingemar Löfgren
Thomas Concrete Group
E-mail: ingemar.lofgren@tcg.nu

ABSTRACT

The present paper discusses crack propagation and special attention is given to how the combined effect of reinforcement and fibre bridging influences the crack spacing and width in the serviceability limit state. Two analytical approaches, for calculating the crack spacing and crack width, are presented. The first model is a modification of the conventional crack spacing model presented in Eurocode 2 and is valid for the case when cracking is caused by an external load. The second model, which is based on a bond-slip relationship and a compatibility requirement, is valid for cracking caused by restraint stresses. Moreover, in the paper some examples are provided of how the models can be used.

Key words: Fibre-reinforced concrete, Cracking, Restraint, Serviceability, Shrinkage.

1. INTRODUCTION

Concrete has a low tensile strength and tensile strain capacity and cracking is initiated at a tensile strain of about 0.1 mm/m which can be compared to the drying shrinkage of concrete of about 0.6 to 0.8 mm/m. Hence, cracks are almost unavoidable and reinforcement is needed to control the behaviour after cracking and to limit crack widths. Large crack widths are not aesthetic but may also lead to accelerated reinforcement corrosion in severe environments, leakage in water-retaining/resisting structures, insanitary conditions, or obstructions and interruptions in production processes. Cracking may be caused by external applied forces, imposed deformations, by shrinkage or thermal strains which are externally and/or internally restrained, or by a combination of these. When cracking is caused by an external applied force the crack width, if sufficient amount of reinforcement is added, will depend on the applied force. However, if cracking is caused by an imposed deformation the force in the member depends on the actual stiffness and the crack width on the number of cracked formed. However, most codes do not distinguish between these two cases. Furthermore, for structures having both fibre- and bar reinforcement there exist almost no guidelines exists for structural engineers.

2. THE CRACKING PROCESS

The cracking process differs depending on whether it is caused by an external load, imposed deformation or restrained shrinkage, see Figure 1. When cracking is caused by an external load the reinforcement is usually designed such that it is able to transfer the load after cracking without yielding. For this case the load will cause an immediate cracking process where several cracks are formed and which are relatively uniformly distributed. For this type of situation the standard method in Eurocode 2 can be used to determine the minimum reinforcement and for estimating the crack spacing and crack width. For a member with combined reinforcement (fibre- and bar reinforcement) this approach has to be modified. When the cracking is caused by an imposed deformation a different behaviour can be observed. When a crack is formed this is accompanied by a sudden drop in the force N and the stiffness of the element also decreases. For a new crack to be formed the deformation has to be increased so that the force N again reach the critical value ($N > N_{cr}$). However, the force depends on the stiffness of the member and if this is low a large deformation may be required before a new crack can be formed, compare (b-1) and (b-2) in Figure 1, and this results in fewer but larger cracks. For this type of cracking process the standard approach for determining crack spacing and crack width cannot be used.

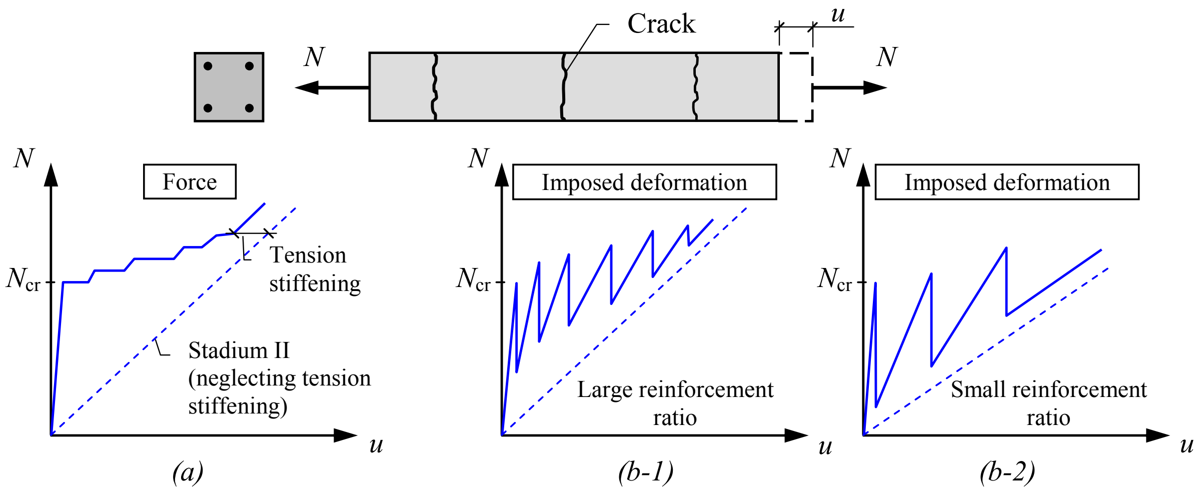


Figure 1. A reinforced concrete member subjected to: (a) axial force; (b) imposed deformation, (b-1) with a large reinforcement ratio and (b-2) with a small reinforcement ratio. Based on Ghali et al [1].

Compared to plain concrete (i.e. without fibres) fibre-reinforced concrete exhibits the ability to transfer tensile stresses also after cracking, see Figure 2. This material property is referred to as the residual tensile strength or, for describing the whole curve, the stress-crack opening relationship (σ-w relationship). The residual tensile strength increases with increased fibre dosage but is also influenced by the type of fibre (e.g. slenderness, geometry, material, etc.)

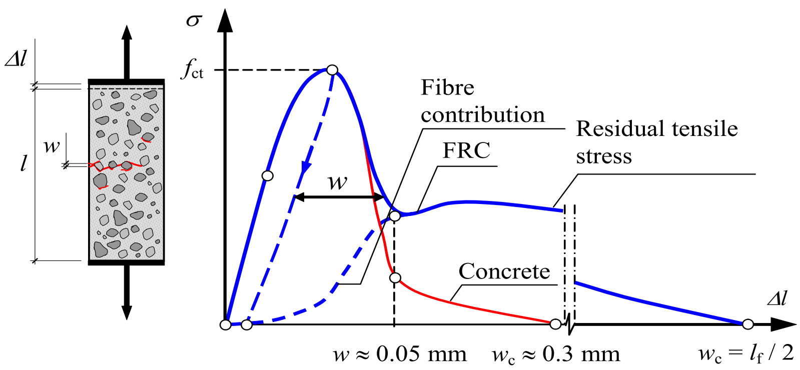


Figure 2. Schematic description of the fracture behaviour of fibre-reinforced concrete (FRC).

3. FORCE INDUCED CRACKING

The crack spacing in reinforced concrete structures (without fibres) can be calculated using the following expression presented in Eurocode 2:

$$s_{r.\max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{s,eff}} \text{ [mm]} \qquad (1)$$

where:
c is the concrete cover
ϕ is the bar diameter
$\rho_{s,ff}$ is the effective reinforcement ratio, $\rho_{s,eff} = A_s / A_{c,eff}$ and $A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement
k_1 = 0.8 for high bond bars and 1.6 for bars with an effectively plain surface
k_2 = 0.5 for bending, 1.0 for pure tension or $(\varepsilon_1 + \varepsilon_2)/(2 \cdot \varepsilon_1)$ for eccentric tension
k_3 = 3.4
k_4 = 0.425

For a section with combined reinforcement a similar expression, which takes into account the contribution from the fibre reinforcement, can be derived. Consider a reinforced tension rod loaded with the crack load, N_{cr}, according to Figure 3. The rod is reinforced with a centrally placed reinforcement bar, with an area of A_s, and fibres. The force equilibrium in the region between two cracks with the maximum crack distance $s_{r,max} = 2 \cdot l_{t,max}$ is analysed, see Figure 3.

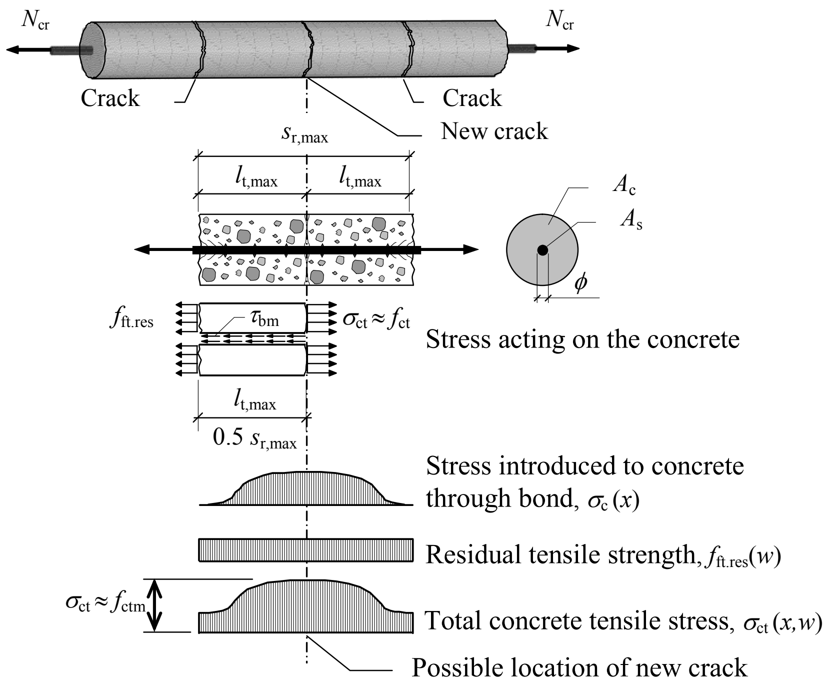


Figure 3. Equilibrium of forces for a tension rod.

At the crack the fibre reinforced concrete transfers a stress $f_{ft.res}$. At the midpoint between the two cracks the concrete is about to crack and the stress is thus $\sigma_{ct} \approx f_{ctm}$. The increase of stress is a result of stresses being transferred from the reinforcement to the concrete through bond. The bond stress τ_b varies along the transmission length and has an average value of τ_{bm} which can be calculated as:

$$\tau_{bm} = \frac{\int_0^{l_{t,max}} \tau_b(x)dx}{l_{t,max}} \tag{2}$$

If the tension rod is cut in the middle between the two cracks and along the interface between the reinforcement and concrete the following equilibrium condition can be formulated:

$$\tau_{bm} \cdot \pi \cdot \phi(0.5 \cdot s_{r,max}) + f_{ft.res} \cdot A_c = f_{ctm} \cdot A_c \tag{3}$$

The concrete gross cross-sectional area can be formulated as:

$$A_c = A_s \cdot \frac{A_c}{A_s} = \frac{A_s}{\rho_s} \tag{4}$$

with ρ_s = reinforcement ratio

Inserted in (3) gives

$$\tau_{bm} \cdot \pi \cdot \phi(0.5 \cdot s_{r,max}) = \frac{\pi\phi^2}{4\rho_s}\left(f_{ctm} - f_{ft.res}\right) \tag{6}$$

$$\Rightarrow s_{r,max} = \frac{1}{2} \cdot \frac{\left(f_{ctm} - f_{ft.res}\right)}{\tau_{bm}} \cdot \frac{\phi}{\rho_s} \tag{7}$$

The minimum crack spacing is equal to half the maximum crack spacing. Accordingly, the minimum crack spacing can be calculated as:

$$s_{r,min} = \frac{1}{4} \cdot \frac{\left(f_{ctm} - f_{ft.res}\right)}{\tau_{bm}} \cdot \frac{\phi}{\rho_s} \tag{8}$$

The average crack spacing during the crack formation can be estimated as the average value of (7) and (8) which gives (in Eurocode 2 it is assumed that $s_{r,max} = 1.7 \times s_{r,m}$):

$$s_{rm} = \frac{3}{8} \cdot \frac{\left(f_{ctm} - f_{ft.res}\right)}{\tau_{bm}} \cdot \frac{\phi}{\rho_s} \tag{9}$$

The stress transfer from the reinforcement to the surrounding concrete depends partly on the surface properties of the reinforcement and partly on the properties of the concrete. Based on experimental results, it has been found that the average bond stress can be calculated as:

$$\tau_{bm} = \frac{3}{2 \cdot k_1} \cdot f_{ctm} \tag{10}$$

If the expression for the average bond stress is introduced into (9), the following expression is obtained for the crack spacing of a tension rod:

$$s_{rm} = 0.25 \cdot k_1 \frac{\left(f_{ctm} - f_{ft.res}\right)}{f_{ct}} \cdot \frac{\phi}{\rho_s} \text{ [mm]} \tag{11}$$

$$s_{rm} = 0.25 \cdot k_1 \left(1 - \frac{f_{ft.res}}{f_{ctm}}\right) \cdot \frac{\phi}{\rho_s} \text{ [mm]} \tag{12}$$

The conclusion is that for calculating the crack spacing the basic formula as suggested in Eurocode 2 can be used but it has to be modified with the relationship between the residual tensile strength and the tensile strength with the introduced variable as follows:

$$k_5 = \left(1 - \frac{f_{ft.res}}{f_{ctm}}\right) \tag{13}$$

If the effect of concrete cover, the spacing of the reinforcement, and type of loading (tension or flexural) the following expression can be used to calculate the crack spacing:

$$s_{r,\max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot \frac{\phi}{\rho_{s,eff}} \text{ [mm]} \tag{14}$$

$$s_{r,average} = \frac{1}{1.7} \cdot \left(k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot \frac{\phi}{\rho_{s,eff}}\right) \text{ [mm]} \tag{15}$$

3.1 Example

In order to investigate the proposed crack spacing formula full-scale beams were casted and tested. The experimental program consisted of five series (three beams in each series) with different fibre dosage and type and amount of reinforcement, sees Table 1. The full-scale beams were simply supported with 1800 mm span and subjected to a four-point load, see Figure 4. The full details of the experiments can be found in Gustafsson and Karlsson [2].

Table 1. Test series without and with fibre reinforcement (type Dramix® RC-65/35 from Bekaert) and amount of conventional reinforcement.

Series	Fibre dosage [vol-%] and [kg/m^3]	Reinforcement Number and diameter [mm]	Number of beams
1	V_f= 0 % (0 kg/m^3)	3 ϕ8	3
2	V_f= 0.5 % (39.3 kg/m^3)	3 ϕ8	3
3	V_f= 0.25 % (19.6 kg/ m^3)	3 ϕ6	3
4	V_f= 0.5 % (39.3 kg/ m^3)	3 ϕ6	3
5	V_f= 0.75 % (58.9 kg/ m^3)	3 ϕ6	3

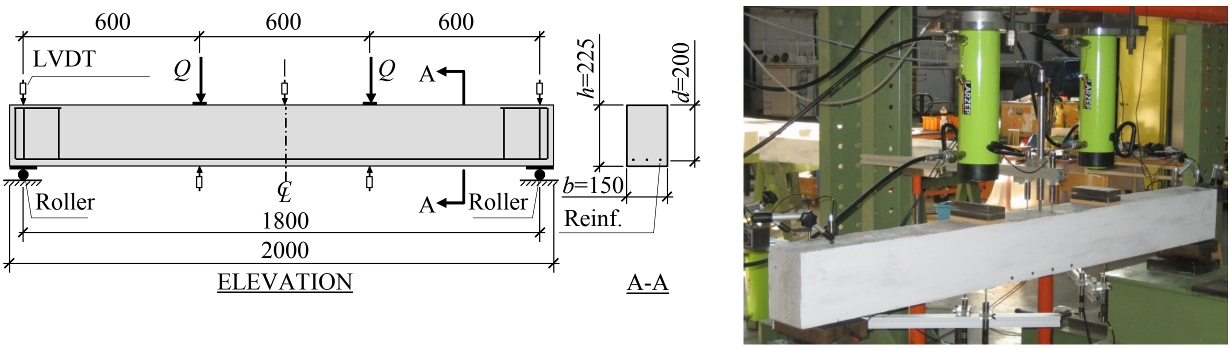
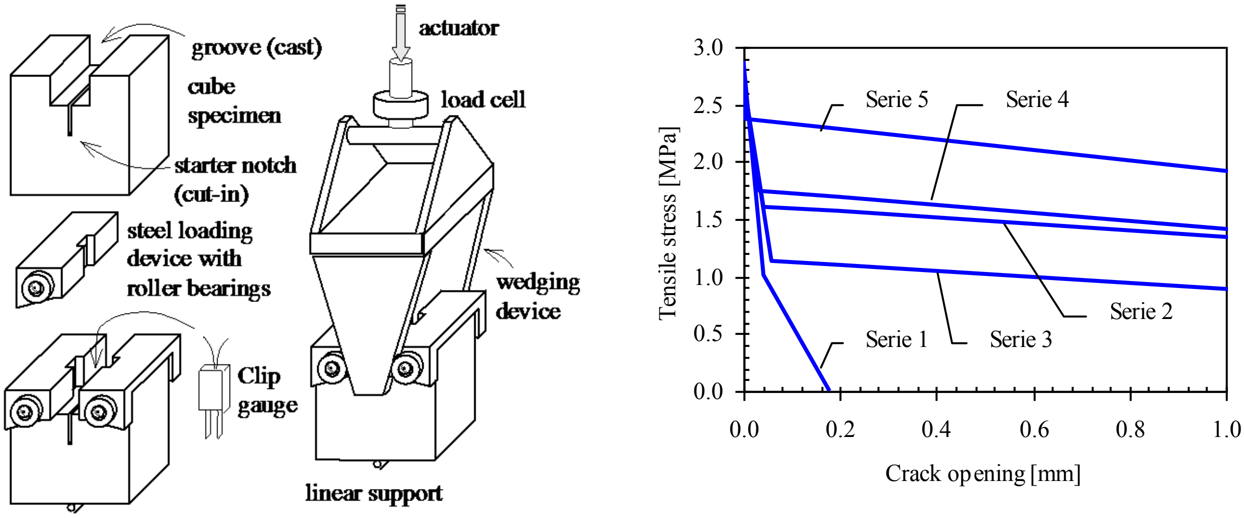


Figure 4. Test set-up (full-scale beams).

In addition to the full-scale beams wedge-splitting test (WST) were conducted, see NT-BUILD 511 [3], and in order to determine the residual tensile strength inverse analyses were carried out, see Löfgren [4]. In Figure 5(a) the WST-method is outlined and in Figure 5(b) the stress-crack opening relationships can be seen.



(a) (b)

Figure 5. (a) Schematic description of the WST-method. (b) Obtained stress-crack opening relationships.

In Figure 6 the calculated crack spacing is compared with the crack spacing obtained in the experiments. In addition, a comparison is also made with the proposal according to RILEM TC 162-TDF [5], where the crack spacing is calculated as:

$$s_{rm} = \left(50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi_b}{\rho_r} \right) \left(\frac{50}{l_f / \phi_f} \right) \qquad (16)$$

As can be seen in Figure 6, the RILEM proposal does not consider the effect of increased fibre content but whereas the proposal according to equation 15 takes into account the residual tensile strength of the fibre-reinforced concrete and thus are able to predict that the crack spacing decreases with increased fibre content, or with increased fibre slenderness as this also increases the residual tensile strength.

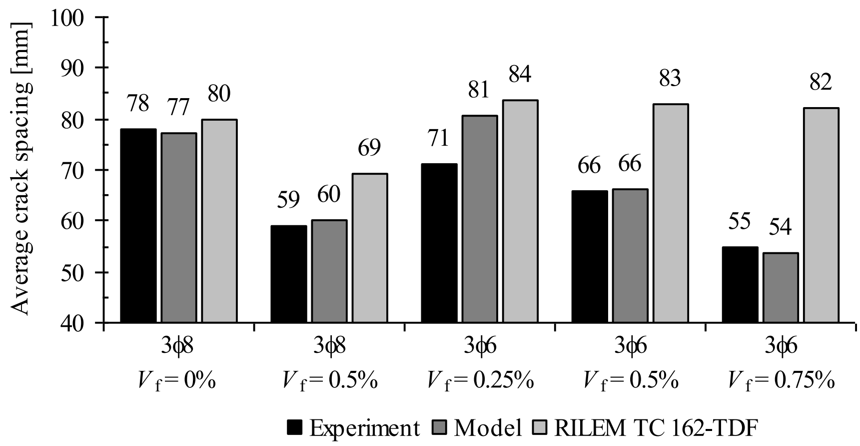


Figure 6. Comparison between calculated crack spacing and the crack spacing obtained in the experiments.

4. RESTRAINT INDUCED CRACKING

Engström [6] has proposed a model for analysing restraint induced cracking and the cracking process is analysed by modelling the cracks as non-linear springs, see Figure 7. Löfgren [7] extended the model to include the effect of fibre reinforcement.

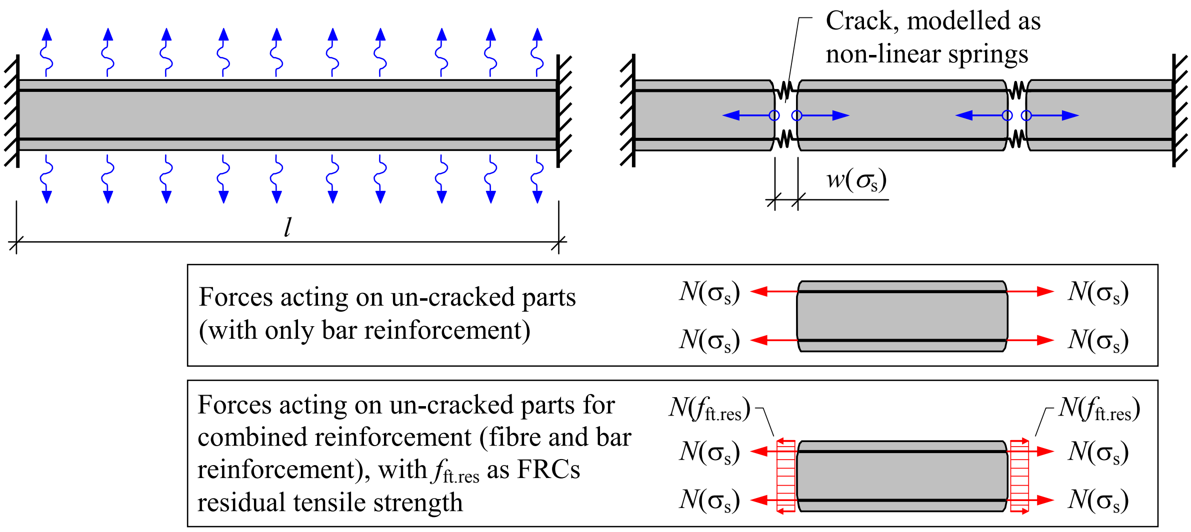


Figure 7. Model for analysing restraint induced cracking.

Engström's model is based on a bond-slip relationship which has been used to derive an analytical expression describing the crack width as a function of the reinforcement stress:

$$w(\sigma_s) = 0.42 \cdot \left(\frac{\phi \cdot \sigma_s^2}{0.22 \cdot f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_c} \cdot \frac{A_s}{A_{ef}}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\phi \text{ (with } \phi \text{ in mm)} \tag{17}$$

Where ϕ is the bar diameter, σ_s is the stress in the reinforcement, f_{cm} is the average compressive concrete strength, E_s and E_c is the modulus of elasticity of the reinforcement respectively the concrete, and A_{ef} is the effective concrete area. The effective concrete area can be calculated as $A_{ef} = b \cdot h_{ef}$, where h_{ef} is the part of the tensile zone which has the same centre of gravity as the reinforcement. The last additional term in (eqv. 17) considers the influence of a zone nearby the crack where bond is assumed to be fully broken due to radial cracks towards the free surface.

The response during the cracking process can described with the following deformation criteria:

$$\frac{N(\sigma_s, f_{ft,res}) \cdot l}{E_c \cdot A_I} \cdot (1 + \varphi_{ef}) + n \cdot w(\sigma_s) = R \cdot \varepsilon_{cs} \cdot l \tag{18}$$

where $N(\sigma_s, f_{ft.res})$ is the force acting on un-cracked parts, l is the length of the member, $A_I = A_c + A_s(E_s/E_c - 1)$, φ_{ef} is the effective creep coefficient, n is the number of cracks and R is the degree of restraint (R=0 for no restraint and R=1 for full restraint). $N(\sigma_s, f_{ft.res})$ can be calculated as:

$$N(\sigma_s, f_{ft,res}) = \sigma_s \cdot A_s + f_{ft,res} \cdot (A_{ef} - A_s) \tag{19}$$

If $N(\sigma_s, f_{ft.res})$ is larger than the force required to initiate a new crack, N_1, more cracks will be formed. However, if it is smaller only one crack will be formed. The force required to initiate a new crack, N_1, can be calculated as:

$$N_1 = f_{ctm} \cdot \left(A_{ef} + \left(\frac{E_s}{E_c} - 1 \right) \cdot A_s \right) \tag{20}$$

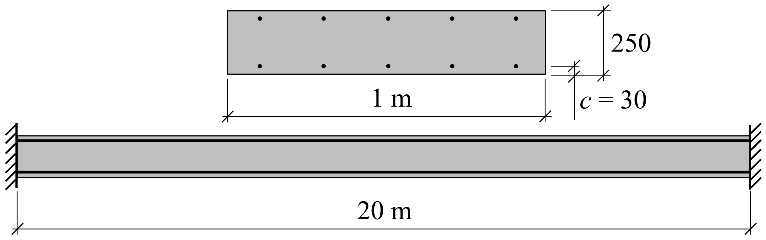
where f_{ctm} is the average tensile strength.

If $N(\sigma_s, f_{ft.res}) > N_1$ a new crack is initiated (n increases). If $N(\sigma_s, f_{ft.res}) < N_1$ the cracking process stops and the actual crack width can be determined using expression (17).

4.1 Example

In order to exemplify how the crack width depends on the residual tensile strength, the amount of reinforcement, and the bar diameter the following example has been analysed, see Figure 8.

Example:
A reinforced "slab-on-grade", 20 meter long, with full restraint (R=1).
Reinforced with ϕ 8, 10 or 12 (0.2% < ρ < 0.8%)



Material properties, concrete C30/37 (w/c ≈ 0.55):
Tensile strength: f_{ctm} = 2.9 MPa ($f_{ctk,\,0.05}$ = 2.0 MPa)
Residual tensile strength: 0 MPa < $f_{ft.res}$ < 2.5 MPa
Creep coefficient: φ_{ef} = 2.5
Concrete shrinkage: ε_{cs} = 600 10^{-6}

Figure 8. Calculation example.

Since the calculation procedure requires iterations, where the number of cracks is step-wise increased, it is better suited for computer calculations. Hence, the presented model has been implemented in a small Excel program where the calculation can be made automatically, see Figure 9.

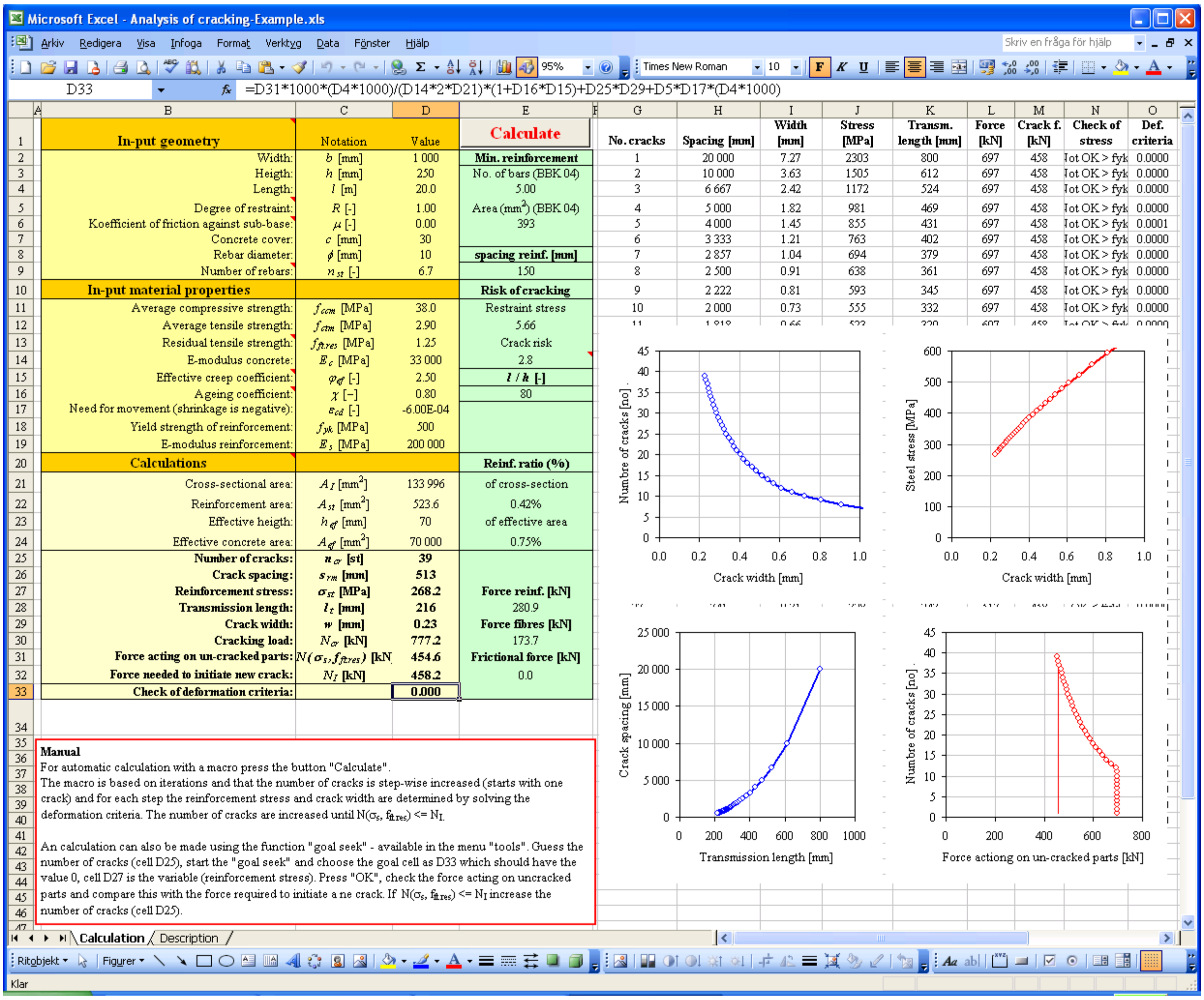


Figure 9. Calculation program in Excel.

The calculation results for the calculation example are presented in Figure 10 to Figure 12. As can be seen the crack width decreases significantly with increasing reinforcement ratio (ρ) and with increasing residual tensile strength. In addition, it can be seen that a small bar diameter is beneficial; see also Figure 13 which shows how the crack width depends on bar diameter and reinforcement stress.

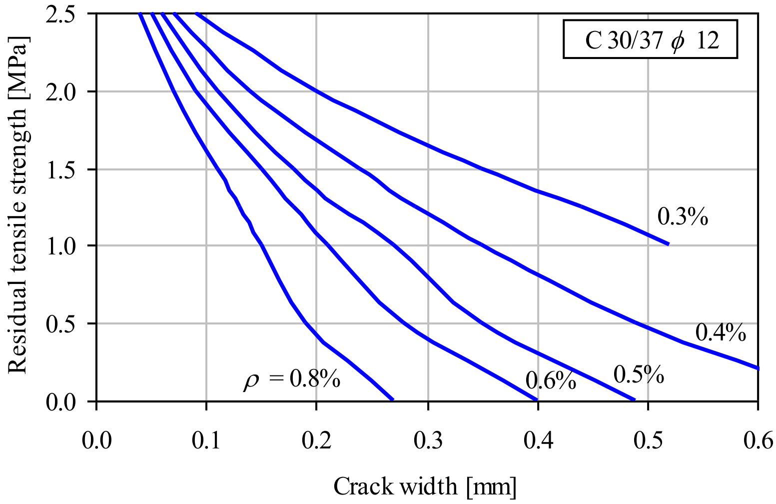


Figure 10. Influence of the residual tensile strength and reinforcement ratio (ρ) for 12 mm bar.

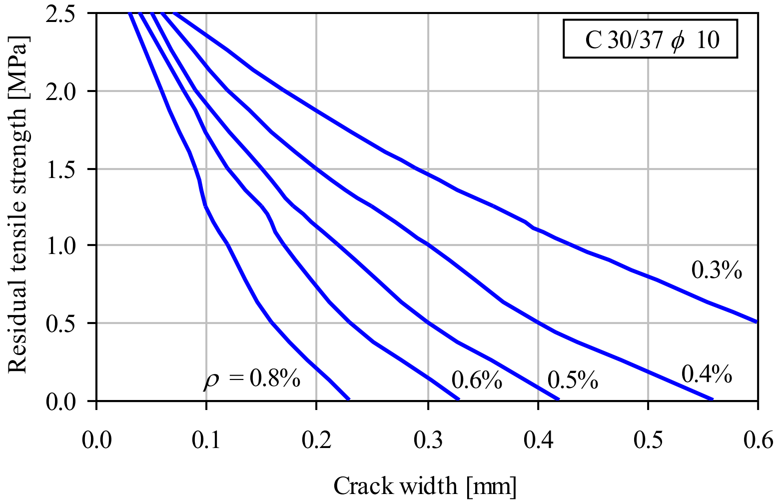


Figure 11. Influence of the residual tensile strength and reinforcement ratio (ρ) for 10 mm bar.

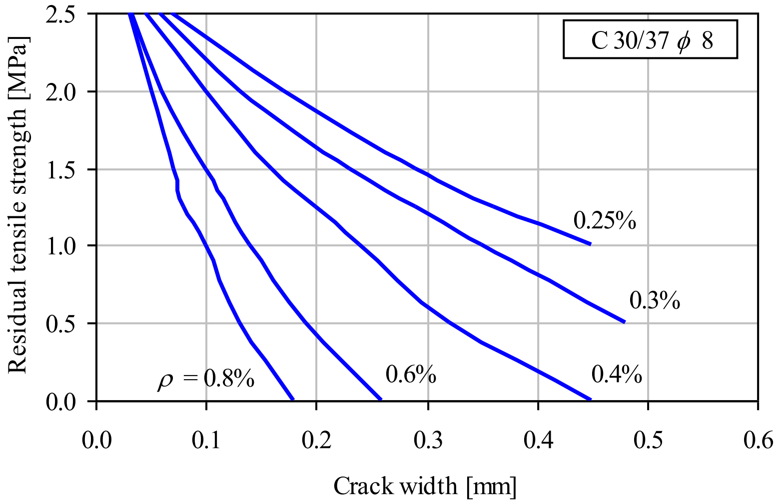


Figure 12. Influence of the residual tensile strength and reinforcement ratio (ρ) for 8 mm bar.

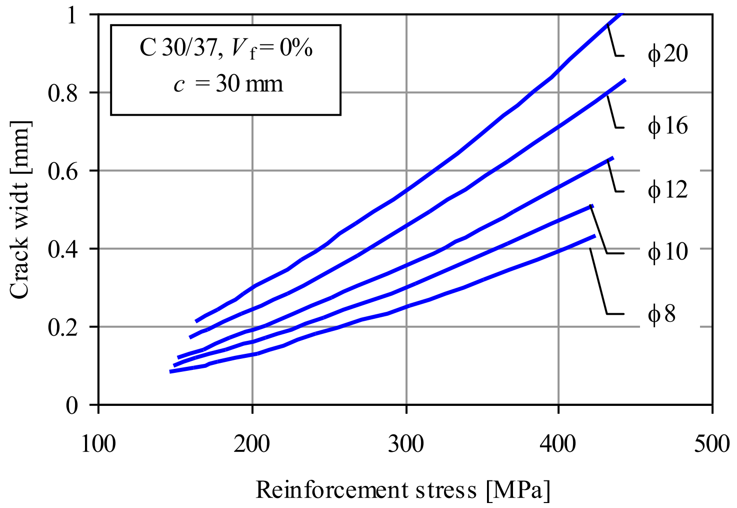


Figure 13. Influence of the bar diameter.

5. DISCUSSION AND CONCLUSIONS

In this paper two models for calculating the crack width for structures with combined reinforcement (i.e. fibre- and bar diameter) have been presented. The first model is valid for the case when cracking is caused by an external force while the second model is for structures subjected to restraint forces. In conclusion it can be said that:

- It is relatively simple to introduce the effect of fibre reinforcement (residual tensile strength) in models for force induced cracking (crack spacing and crack width).
- Restraint induced cracking, for which models currently is lacking in codes, can be analysed with the proposed model.
- The Restraint cracking model is more complicated but can easily be implemented in e.g. Excel for automatic calculations.
- Combined reinforcement (fibre- and bar reinforcement) is effective for crack control.
- However, test methods able to accurately determine the residual tensile strength (or even better the σ-w relationship) of FRC is required.

6. REFERENCES

1. Ghali, A., Favre, R. and Elbadry, M.: *Concrete Structures – Stresses and Deformations*. 3rd ed, Spon Press, London, 2002.
2. Gustafsson, M. and Karlsson, S.: *Fiberarmerade betongkonstruktioner – Analys av sprickavstånd och sprickbredd* (Fibre-reinforced concrete – Analysis of crack spacing and -width). Examensarbete 2006:105, Institutionen för bygg- och miljöteknik, Avdelningen för konstruktionsteknik, Chalmers tekniska högskola.
3. RILEM TC 162- TDF: Test and design methods for steel fibre reinforced concrete: σ-ε-Design Method Final Recommendation, (Chairlady L. Vandewalle), *Materials and Structures*, Vol. 36 October 2003, pp. 560-567.
4. NT BUILD 511: *Wedge Splitting Test method (WST) – fracture testing of fibre-reinforced concrete (Mode I)*, Nordic Innovation Centre, Oslo, 2005.
5. Löfgren, I.: *Fibre-reinforced Concrete for Industrial Construction - a fracture mechanics approach to material testing and structural analysis*. PhD-thesis, Department of Civil and

Environmental Engineering - Structural Engineering, Chalmers University of Technology. Göteborg, 2005.

6. Engström, B.: *Restraint cracking of reinforced concrete structures*. Undervisningsmaterial Institutionen för bygg- & miljöteknik, Chalmers tekniska högskola, 2006.
7. Löfgren, I.: Beräkning av sprickbredd för konstruktioner utsatta för tvångskrafter (Calculation of crack width for structures subjected to restraint forces). *Bygg & Teknik* 7/07.