Nonbinary Staircase Codes for Spectrally and Energy Efficient Fiber-Optic Systems

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Alireza Sheikh[1], **Alexandre Graell i Amat**[1], **and Magnus Karlsson**[2]
[1]*Department of Signals and Systems,*
[2]*Department of Microtechnology and Nanoscience,*
Chalmers University of Technology, SE-412 96 Gothenburg, Sweden
asheikh@chalmers.se

Abstract: We consider the design of nonbinary staircase codes with higher order modulation for spectrally and energy efficient fiber-optic systems. We optimize the code parameters based on density evolution.

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1. Introduction

The quest for spectrally efficient fiber-optic communication systems that perform close to the theoretical limits has spurred a great deal of research on forward error correction (FEC) codes in combination with higher order modulations. One of the most promising FEC schemes are staircase codes, introduced by Smith *et al.* in [1]. Staircase codes are decoded using iterative hard decision decoding (HDD) with bounded-distance decoding (BDD) and provide a 0.42 dB net coding gain over the best proposed code from the ITU-T G.975.1 recommendation. As compared to low-density parity-check (LDPC) codes with soft decision decoding (SDD), staircase codes yield much lower implementation complexity with considerably decreased power consumption [2], yet with astonishing performance. The staircase codes introduced in [1] are binary codes that use Bose-Chaudhuri-Hocquenghem (BCH) codes as component codes.

One of the most popular ways to achieve high spectral efficiency is to combine a binary code with a higher order modulation using the bit-interleaved coded (BICM) modulation principle. However, it has been recently shown in [3] that for HDD, binary codes may be unsuitable for spectrally-efficient, long-haul systems and that nonbinary codes offer better performance. Indeed, it was shown in [3] that nonbinary codes with HDD can achieve almost the same information rates as the power hungry SDD bit-wise decoders for higher order modulation.

In this paper, motivated by these results and the energy efficiency of HDD compared to SDD, we consider nonbinary staircase codes with (nonbinary) Reed-Solomon (RS) codes as component codes in combination with a higher order modulation. In particular, we consider a fiber-optic channel dominated by amplified spontaneous emission (ASE) noise, which can be modeled as an additive white Gaussian noise (AWGN) channel using the GN model [4]. Nonbinary staircase codes can be seen as a subclass of the nonbinary spatially-coupled generalized LDPC (SC-GLDPC) code ensemble. This allows us to optimize the parameters of nonbinary staircase codes using density evolution (DE), as proposed in [5] for binary staircase codes. This design approach is analytical and dramatically decreases the optimization time compared to a brute-force approach based on simulations [5]. We model the nonbinary transmission channel as a q-ary symmetric channel (QSC). One can see the QSC as an auxiliary channel for the true AWGN channel which is used for code optimization. We then adapt the DE for the binary SC-GLDPC code ensemble over the binary symmetric channel [6] to the nonbinary SC-GLDPC code ensemble over the QSC, rigorously accounting for decoding miscorrections. Using the derived DE we optimize the code parameters of nonbinary staircase codes with overheads (OHs) ranging from 6.25 % to 33.3 % to minimize the gap to capacity for the QSC. Finally, by means of simulations, we show that the optimized codes perform also the best on the AWGN channel, and also compare favorably to nonbinary product codes.

2. Nonbinary Staircase Codes

Let $\mathscr{C}$ be a shortened RS code constructed over the Galois field GF(2^v) with (even) block length $n = 2^v - 1 - s$, where s is the shortening length, error correcting capability t, and information length $k = 2^v - 2t - 1 - s$. A shortened RS code is thus completely specified by the parameters (v, t, s). A nonbinary staircase code is defined as the set of all matrices $\mathbf{B}_{\mathrm{i}}^{\mathrm{m}\times\mathrm{m}}$, $\mathrm{i} = 1, 2, \ldots$, such that each row of the matrix $[\mathbf{B}_{\mathrm{i-1}}^{\mathrm{T}}, \mathbf{B}_{\mathrm{i}}]$ is a valid codeword in $\mathscr{C}$. $\mathbf{B}_0$ is initialized to the all-zero matrix and each component code spreads out in two consecutive blocks, i.e., $m = n/2$.

Staircase codes can be seen as a particular code in the ensemble of SC-GLDPC codes proposed in [6], by considering nonbinary component codes. The SC-GLDPC code ensemble is defined by the parameters $(\mathscr{C}, u, L, w)$, where L is the

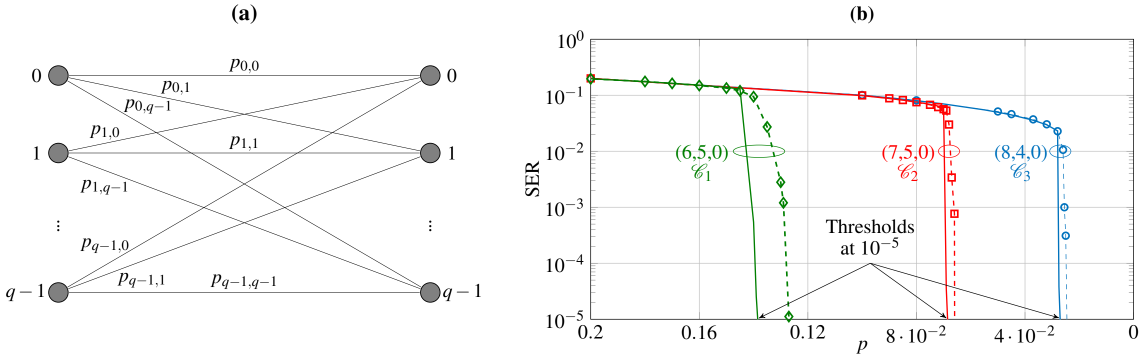

Figure 1. (a) The q-ary channel, (b) DE (solid line) and simulation results (dashed line) for the QSC for different (v,t,s) parameters.

number of spatial positions, u is the number of the constraint nodes in each spatial position, and w is the coupling width. In particular, it can be shown that staircase codes are contained in the ensemble for $u = m = n/2$ (each component code is spread out in two consecutive blocks), $w = 2$ (each coded symbol is protected by two component codes), and $L \to \infty$.

Staircase codes can be decoded using a sliding window decoder. A window of length W at the i-th stage of the decoding comprises the blocks $\mathbf{B}_{\mathrm{i}}, \mathbf{B}_{\mathrm{i+1}}, \ldots, \mathbf{B}_{\mathrm{i+W-1}}$ and after ℓ decoding iterations within the window, the block $\mathbf{B}_{\mathrm{i}}$ is decoded and the window slides one block, i.e., new window comprises the blocks $\mathbf{B}_{\mathrm{i+1}}, \ldots, \mathbf{B}_{\mathrm{i+W}}$ (see [1, Sec. IV]).

3. Density evolution

DE is a method to predict the performance of graph-based codes when the block length goes to infinity. Assume that the coded symbols are mapped to a QAM constellation with the same size of the code construction field, i.e., the coded symbols from GF(q) are mapped to a q-QAM constellation. The optical channel can be modeled as an AWGN channel using the GN model under certain conditions [4]. Therefore, we assume that the symbols are transmitted through an AWGN channel. At the receiver, HDD is applied, i.e., the received symbol is mapped to the nearest constellation point. One can see the nonbinary-input AWGN channel with HDD as a q-ary channel, shown in Fig. 1(a). The simplest q-ary channel is the QSC, where symbol i is received correctly with probability $p_{i,i} = 1 - p$ for $i = 0, 1, \ldots, q-1$, and it is mistaken onto any other symbol with the same probability, i.e., $p_{i,j} = \frac{p}{q-1}$ for $i \neq j$, $i, j = 0, 1, \ldots, q-1$. We remark that the AWGN channel deviates from the QSC as q increases. However, one can interpret the QSC as an auxiliary channel for the true AWGN channel, which simplifies the code design. To restrict the complexity of the decoder, we consider BDD of the component codes, meaning that the component code decoder decodes only if the number of errors is less than t [6, Sec. II. A]. In [6] a DE analysis was presented for binary SC-GLDPC codes with BCH component codes over the binary symmetric channel. Here, we adapt this analysis to nonbinary SC-GLDPC codes using RS component codes over the QSC. We use the weight spectrum of the RS code [7, Eq. 8] in [6, Eqs. 2-5] and adapt these equations to account for the probability of miscorrection of the RS component codes and to the use of window decoding of the staircase code.

Example: To show the effectiveness of the DE analysis, in Fig. 1(b) we simulate the symbol error rate (SER) of three staircase codes with three different RS codes as component codes over the QSC and compare it to the performance predicted by the DE analysis. $\mathscr{C}_1$, $\mathscr{C}_2$, and $\mathscr{C}_3$ are RS codes with $(v,t,s) = (6,5,0)$, $(v,t,s) = (7,5,0)$, and $(v,t,s) = (8,4,0)$, respectively. For the simulations, we used the sliding-window decoding with $W = 7$ and $\ell = 8$. For the DE the code ensembles $(\mathscr{C}_i, \infty, 20, 2)$ for $i = 1,2,3$ with the same W and ℓ are considered. The decoding thresholds for $\mathscr{C}_1$, $\mathscr{C}_2$, and $\mathscr{C}_3$ to achieve an SER of 10^{-5} are 0.138, 0.064, and 0.027, respectively. As can be seen, the DE gives predicts well the SER where the staircase code performance curve "bends" into the characteristic waterfall behavior and the prediction becomes better as the block length increases.

The normalized capacity of the QSC with parameter p is $C_{\mathrm{QSC}}(p,q) = 1 - \frac{h_{\mathrm{b}}(p) + p\log_2(q-1)}{\log_2(q)}$ where h_{b} is the binary entropy function [8, Sec. I]. For a staircase code with given parameters (v,t,s), the gap to the capacity of the QSC can be predicted using the DE described above. For a given OH, we optimize the code parameters to minimize the gap to capacity.

Table 1. Optimized nonbinary staircase code parameters for different OHs

OH (%)	6.25	6.67	7.14	7.69	8.33	9.09	10.00	11.11	12.50	14.29	16.67	20.00	25.00	33.33
v	8	8	8	8	8	8	8	7	7	7	7	7	6	6
t	3	3	4	4	4	5	5	3	3	3	4	5	3	3
s	51	63	15	31	47	15	35	7	19	31	15	7	3	15

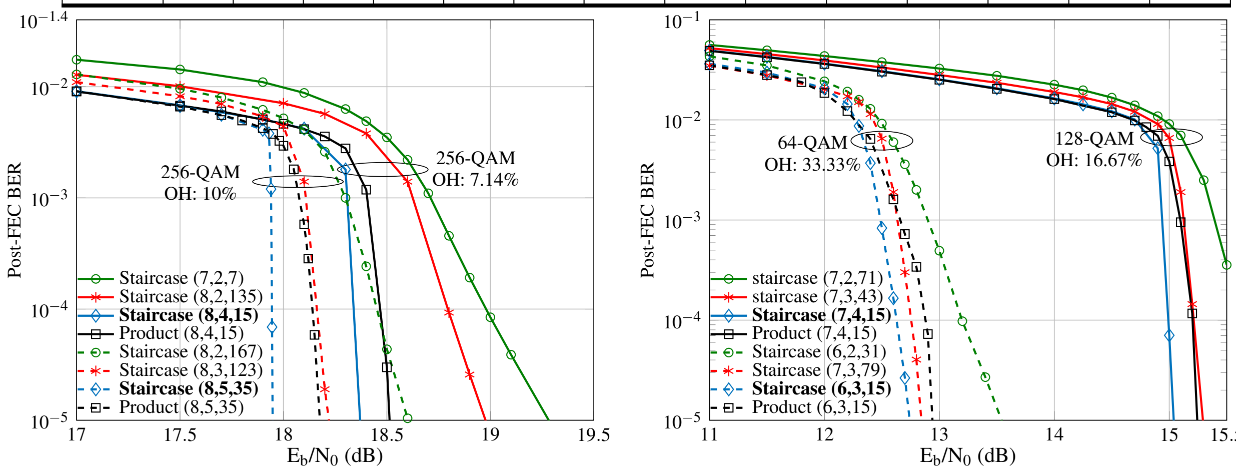

Figure 2. BER performance of the optimized staircase codes, other staircase code candidates, and product codes on the AWGN channel for OHs 7.14%, 10% (left figure) and 16.67% , 33.33% (right figure). The optimized staricase codes are shown with the bold legend.

4. Results and Discussion

We assume RS codes with parameters $v \in \{4,5,6,7,8\}$ and $t \in \{2,3,4,5\}$ as the search space for designing nonbinary staircase codes with $\text{OH} \in \{1/i,\ i : 3,4,\dots,16\}$. For a staircase code with rate R ($\text{OH} = 1/R - 1$), one can easily show that the shortening parameter of the RS code is $s = 2^v - 1 - \frac{4t}{1-R}$, rounded to the nearest integer. We performed the DE for an SC-GLDPC ensemble with $W = 7$ and $\ell = 8$ and selected the code that yields the minimum gap to the capacity of the QSC at a BER 10^{-15}. Table 1 summarizes the parameters of the best nonbinary staircase codes for different OHs. To show the effectiveness of our design method based on the QSC for the true AWGN channel, in Fig. 2 we give simulation results for the optimized staircase codes and some other staircase code candidates for OH $= 7.14\%$, OH $= 10\%$, OH $= 16.67\%$, and OH $= 33.33\%$. In the simulations, we used the standard QAM rectangular constellations. As can be seen, despite the fact that the codes are optimized for the auxiliary QSC, the optimized codes also achieve the best performance for the true AWGN channel compared to the other (non-optimized) staircase codes of the same rate. In the figure, we also compare the performance with that of product codes with RS component codes and the same OH. It is shown that the optimized staircase codes yield better performance.

5. Conclusion

We addressed the design of nonbinary staircase codes for spectrally efficient fiber-optic communications. Since they are decoded using HDD, nonbinary staircase codes can yield lower power consumption as compared to codes using SDD [2]. We optimized the staircase codes parameters based on DE over the QSC, which is used as an auxiliary channel of the AWGN channel. By means of simulations, we have shown that the optimized codes achieve superior performance than other candidates for the AWGN channel and also perform better than nonbinary product codes.

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