MODELING THE MECHANICAL BEHAVIOR OF ADIPOSE TISSUE

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Summary. In this study a nonlinear viscoelastic model for adipose tissue based on the Reese-Govindjee formulation is presented. The model is formulated in a large strain framework and is applied for 3D finite element simulation of rheometer experiment.

1 INTRODUCTION

Recent statistical studies on vehicle collision data shows a strong correlation between increased risk of abdominal injury and obesity. In order to deal with this issue, understanding the mechanism of injury and how loads transfer to the internal organs is crucial. Thus, a deeper knowledge and insight into the constitutive behavior of human fat tissue, more especially white adipose tissue (WAT) found in bulk mass in abdomen, is needed to investigate the mechanism of injury. There are few studies found for characterizing the mechanical properties of adipose tissue and even less studies in large deformation and high strain rates conditions. A study by Comley and Fleck on adipose tissue in wide strain range showed stiffening of tissue in high rates more than three order of magnitude which suggests high viscoelastic properties of adipose tissue. The high deformation-rate dependency of adipose tissue stiffness was also observed by Gefen and Haberman.

Biofidelity is crucial for the constitutive modeling of adipose tissue. It has to represent tissue mechanical behavior under high strain rates and large strain levels, a situation usually occurs in vehicle collisions, in order to be used in human body models. Until today, however, no biofidelic finite element (FE) model is available for this tissue for a wide range of loading rates representative for impact. In this contribution a nonlinear viscoelastic model based on the Reese-Govindjee formulation is presented to describe the tissue response under impact loading. The model is formulated in a large strain framework with multiplicative decomposition of deformation gradient tensor and considers a non-linear evolution law for viscos strains of the material along with its non-linear elastic behavior. Then, since rheological experiments are often used to characterize viscoelastic properties of materials, 3D finite element simulation of rotational rheometer test is simulated. The objective of this simulation is, a) to verify the capability of the model to represent tissue response at wide range of frequencies and b) to study 3D shear mechanism and wave propagation of adipose tissue which have an important role to
understand transfer of impact loads and related injuries. Ultimately, the simulation model will be used to compare experimental data with model prediction, in order to verify structural assumption of the rheometer experiments as well as to carry out model calibration.

2 FINITE VISCOELASTIC MATERIAL MODEL

The Generalized Maxwell model extended to large strain is used as finite viscoelastic model. Based on Reese and Govindjee model for viscoelastic materials, deformation gradient tensor \( F \) is split multiplicatively into an elastic part, \( F_e \), and an inelastic part, \( F_v \):

\[
F = F_e F_v
\]  

(1)

The total strain energy for the Maxwell model with \( N \) chains corresponds to the viscoelastic response is given as:

\[
\Psi = \Psi^E(F) + \sum_{k=1}^{N} \Psi^{NEQ(k)}(F_e)
\]  

(2)

And second Piola-Kirchhoff stress then can be computed as:

\[
S = S^E + S^{NEQ}
\]  

(3)

The evolution law for the viscoelastic strain is given as:

\[
\dot{\varepsilon}^v = \lambda \frac{\partial \varepsilon_{eq}}{\partial \tau}, \quad \lambda = \eta(\tau_{eq})/t_r
\]  

(4)

Where \( t_r \) is relaxation time, \( M \) is Mandel stress and \( \tau_{eq} \) equivalent Mandel stress. A nonlinear evolution law is defined by \( \eta(\tau_{eq}) \) based on Norton law. Implicit backward Euler integration method is employed for the time integration of the evolution law.

2.1 Finite element simulation of rheometer test

Due to cylindrical shape of adipose sample in rotational rheometer experiment it is modeled in cylindrical coordinates. Even though displacement in \( \theta \) direction is nonzero i.e. \( u_{\theta} \neq 0 \) (because of rotation of sample), it can still be modeled in 2 dimension. It is assumed that there is no change in the sample with respect to \( \theta \) direction, i.e. \( \partial(\bullet)/\partial \theta = 0 \). Hence 4-node quadrilateral finite element with three degree of freedom in each node (Figure 1) can thoroughly represent the sample.

Figure 1: four node quadrilateral element used in cylindrical coordinates
The following discretization is applied for the displacement field:

\[
\begin{align*}
    u_r &= \sum_{i=1}^{4} N_i(r, z) u^i_r \\
    u_z &= \sum_{i=1}^{4} N_i(r, z) u^i_z \\
    u_\theta &= \sum_{i=1}^{4} N_i(r, z) u^i_\theta
\end{align*}
\] (5)

Finally the momentum balance equations is formulated in total Lagrange formulation and then coded in MATLAB. The effect of inertia is also considered in the simulation. Composite implicit time integration procedure\(^6\) which is combination of three point backward Euler integration method and trapezoidal rule, is employed for the time integration.

3 RESULT AND DISCUSSION

The rotational rheometer test in\(^2\) is simulated in wide frequency range of 10-1000 rad/s. In such an experiment, adipose tissue response is usually analyzed with Fourier transform and only the first mode corresponding to the quasi-static response is taken as sample response to rheometer test. To investigate accuracy of this assumption, both quasi-static and dynamic response is modeled and compared in Figure 2.

![Figure 2: Comparison of quasi-static and dynamic response of adipose tissue at different frequencies, at \(\omega=40\) (left), \(\omega=400\) (middle) and \(\omega=1000\) (right)](image)

As it is seen at low frequency (\(\omega=40\) rad/s) the quasi-static and the dynamic response are overlapped. As frequency increases (\(\omega=400\) rad/s) the dynamic response deviates a little from the equilibrium behavior showing that the effect of inertia has become larger. However neglecting the inertia effect does not influence the accuracy of rheometer test so much. At very high frequencies (\(\omega=1000\) rad/s) the inertia forces are comparable with the internal forces and cannot be ignored. Therefore accuracy of rheometer test is limited for frequencies up till few hundreds and above that the quasi-static assumption is not a good assumption.

Height to diameter (h/d) relation of the sample is increased to double and its effect is investigated. Figure 3 shows that increasing the sample length will results in activating inertia effects. Therefore even at this frequency (\(\omega=400\) rad/s) quasi-static assumption is a not good assumption whereas for half of this sample length it was a good assumption.
It is shown that accuracy of rheometer test is limited to frequency level and sample size\(^2,7\). This model was able to appropriately address this issue. Hence the presented model is capable to qualitatively show adipose tissue behavior from low to high strain rates. Moreover 3D computational modeling of rheometer test gives more physical insight into shear mechanism of rheometer test and wave propagation through the sample. Also limitations on structural assumption of rheometer test can be investigated further by this model.

REFERENCES