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***Abstract*—Achievable information rates (AIRs) are calculated for optical fiber systems employing soft-decision and hard-decision nonbinary codes. We show that, despite the lower decoding complexity, hard-decision AIRs approach soft-decision AIRs for high spectral efficiencies and long transmission distances.**

I. Introduction

Next generation spectrally-efficient optical fiber systems will require the combination of forward error correction (FEC) and high-order modulation formats. The joint design of the FEC encoder and the bit-to-symbol mapper is known as coded modulation (CM) [1]. Achievable information rates (AIRs) of CM schemes have been recently studied in the context of optical fiber communication in [2]–[7]. In these works, following the "mismatched decoder" principle [8], different *variants* of the mutual information (MI) [9], [10] were used.

Since most of the complexity of the system lies in the CM decoder structure, it is particularly important to quantify the impact of the CM decoder implementation on the AIRs. In [2], AIRs for hard-decision (HD) and soft-decision (SD) decoders were compared for modulation formats such as PM-16QAM and PM-64QAM and a fixed transmission distance. In [6], the bit-wise MI was used to calculate AIRs for SD binary decoders. Recently, in [5], the conventional symbol-wise (SW) MI was used to estimate AIRs for SD nonbinary FEC schemes.

HD decoders represent a low complexity alternative to SD decoders. However, AIRs for HD nonbinary codes have not been explored yet in the context of optical fiber communications. In this paper, we calculate AIRs for HD nonbinary decoders in optical fiber systems. We compare these AIRs with the ones of SD decoders over different transmission distances and for two QAM modulation formats with high spectral efficiency (SE): PM-64QAM and PM-256QAM.

II. AIRs for nonbinary HD decoders

A CM encoder maps a sequence of N_b information bits into a sequence of N_s coded symbols at a rate $R = \frac{N_b}{N_s}$ bit/symbol. When nonbinary FEC is used (see left-hand side of Fig. 1), the CM encoding typically involves 2 steps: a memoryless bit-to-symbol mapping and a nonbinary FEC encoder. When SD decoding is employed (top right corner of Fig. 1), SW log-likelihoods (LLs) are calculated and passed to an SD nonbinary FEC decoder. The SD MI

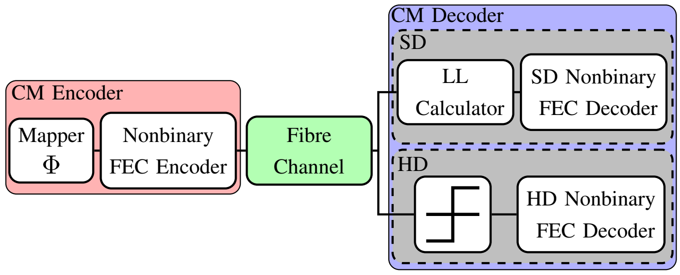

Fig. 1. Schematic diagram of a system using SD/HD nonbinary CM schemes.

$$I_{\text{SD}} = \log_2 M + \frac{1}{M}\sum_{i=1}^{M}\int_{\mathbb{C}} p_{Y|X}(y|x_i)\log_2\frac{p_{Y|X}(y|x_i)}{\sum_{n=1}^{M} p_{Y|X}(y|x_n)}dy, \quad (1)$$

where M is the constellation cardinality and $p_{Y|X}(y|x)$ is the channel law, represents an AIR for the optical fiber channel when SD decoders are adopted [11, Sec. IV].

When HD decoding is adopted (bottom right corner of Fig. 1), a multi-level threshold device produces symbol hard decisions that are fed to an HD nonbinary FEC decoder. In this case, the channel "seen" by the FEC decoder is a discrete-input discrete-output M-ary channel, characterised by symbol transition probabilities p_{ij}. The MI of this channel is therefore

$$I_{\text{HD}} = \log_2 M + \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{M} p_{ij}\log_2\frac{p_{ij}}{\sum_{n=1}^{M} p_{nj}}, \quad (2)$$

and it is an AIR when nonbinary HD FEC decoders are adopted [9, Sec. 16]. In the following, we compare (1) and (2) for split-step Fourier simulations of an optical fiber transmission. A lower bound on (1) is calculated based on the mismatched decoder principle [2], [8], whereas (2) is calculated based on Monte Carlo estimation of the probabilities p_{ij}.

III. Results and discussion

The system under investigation consists of multiple spans of 80 km of standard single-mode fiber, with erbium-doped fiber

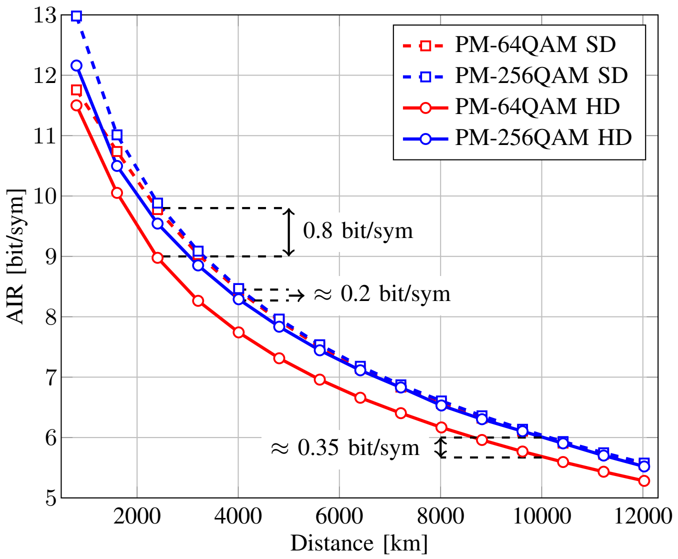

Fig. 2. AIRs vs. distance with nonbinary FEC and either SD or HD decoders, for PM-64QAM (red) and PM-256QAM (blue).

amplifiers to fully compensate for the span losses. Five quasi-Nyquist shaped 32 GBaud channels with 33 GHz spacing are transmitted through the fiber channel. At the receiver, electronic dispersion compensation is performed followed by a matched filter to detect the central channel. 2^{18} transmitted symbols are used for the calculation of the MIs. For each transmission distance, the transmitted power per channel is varied until the optimum AIR for each decoder structure was found.

In Fig. 2, SD (dashed lines) and HD (solid lines) AIRs vs. transmission distance are shown for PM-64QAM (red curves) and PM-256QAM (blue curves). For PM-256QAM, the AIRs for HD decoders closely approach the SD AIRs for long and ultra-long transmission distances. In particular, for distances larger than 4000 km, the AIR differences between the two decoder implementations are smaller than 0.2 bit/sym. In this scenario, HD decoding appears to be an attractive alternative to SD decoding due to the lower complexity and negligible loss in performance.

When PM-64QAM is used, and for distances longer than 2000 km, SD decoders show AIRs comparable to those obtained for PM-256QAM with both SD and HD decoding. However, as shown in Fig. 2, switching from an SD to an HD decoder for PM-64QAM can result in AIR penalties of up to 0.8 bit/sym (at approximately 2000 km distance). This penalty reduces to less than 0.35 bit/sym for ultra-long transmission distances ($\geq$10000 km).

For transmission distances longer than 2000 km, comparable AIRs can be achieved by either adopting a low order modulation format (PM-64QAM) and an SD decoder or by using a high order modulation format (PM-256QAM) with an HD decoder. For shorter transmission distances ($\leq$ 2000 km), however, SD decoders still outperform the HD ones by more than 0.5 bit/sym.

In conclusion, as the cardinality of the modulation increases and for medium/long distances, most of the transmitted information can be preserved using a nonbinary decoder, whether or not the soft information is used. However, for short transmission distances, soft information can still be exploited to obtain significant AIR gains.

IV. Conclusions

The AIRs of a communication system significantly depends on the specific CM scheme adopted. We have calculated AIRs of HD nonbinary-coded optical fiber systems using PM-64QAM and PM-256QAM and we have compared them to the SD case. While SD decoders perform significantly better for short/medium distances, the potential of HD nonbinary CM schemes can be unleashed using high nominal SE modulation formats in the ultra-long haul transmission regime. As an example, we have shown that the performance of HD nonbinary decoders using PM-256QAM is comparable to SD nonbinary decoders regardless of the modulation format used. For the same AIR, complexity can be traded between FEC decoder implementation (HD as opposed to SD) and modulation format (PM-256QAM as opposed to PM-64QAM). Further investigation is required as to how closely pragmatic HD nonbinary codes can approach the presented information rates.

Acknowledgments

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