Optimal state based control of LTI systems over unreliable communication channels

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EXTENDED ABSTRACT

With the increasing use of wireless communication in networked control systems, the issue of control over unreliable channels has risen into prominence. As controllers, sensors and actuators often are positioned on different locations, it can be difficult or expensive to create reliable communication links between the components. Therefore, the question of control over lossy networks is one of increasing importance. Some methods to optimize control algorithms over lossy channels are discussed in Imer et al. (2006); Hadjicostis and Touri (2002); Schenato et al. (2007).

One solution to the issue of lossy channels is to use tree codes to characterize the submission of the data from the controller to the actuators, and from the sensors to the controller, as discussed in Sukhavasi and Hassibi (2011, 2012). By utilizing tree codes one can turn a lossy channel into a channel with a random delay. This delay will not be bounded, but follows a probability function that depends on the lossyness of the network. The problem that will be examined here has lossy channels between the controller and the actuators that are modified using tree codes into channels with a random delay. An optimal solution is derived for finite horizon discrete hold-input LQG control for this case. This solution is compared with standard LQG control in simulations, which demonstrate that a significant improvement in the cost can be achieved when the probability of delay is high.

Problem formulation

The plant considered is assumed to be an LTI system, with a certain probability of package loss between the controller and the actuator. Using tree codes this is converted to a system where communication between the controller and the actuator is subject to a random delay. The assumed probability function for this delay is

$$P(d) = (1-\alpha)\alpha^d, \tag{1}$$

where α is a positive constant assumed to be known (it depends on the probability of package loss from the original channel) and d is the number of samples the package is delayed.

The delays between consecutive steps are assumed to be independent. From this and (1), the probability that the latest control signal that has arrived is the signal sent i time units before can be derived to be

$$\begin{aligned} P_d(i) &= (1-\alpha^{(i+1)})\alpha^{\sum_{j=1}^{i} j} \\ &= (1-\alpha^{(i+1)})\alpha^{(\frac{i+i^2}{2})}. \end{aligned} \tag{2}$$

The LTI plant is described by

$$x_{k+1} = Ax_k + Bu_{ak} + w_k, \tag{3}$$

where $x \in R^n$ is the state vector, $w \in R^n$ is white Gaussian noise, and $u_{ak} \in R^m$ is the control signal applied by the actuator at time k (for hold-input control this will be the latest control signal that has arrived).

The goal is to design a controller that minimizes the quadratic criterion

$$\begin{aligned} J_N = \min_{u_c} E \Bigg[\sum_{i=0}^{N} u_{ai}^T Q u_{ai} + \sum_{i=0}^{N} x_i^T R x_i \\ + x_{N+1}^T S_{N+1} x_{N+1} \Bigg], \end{aligned} \tag{4}$$

where u_c being the control signal sent from the controller, Q is a positive definite symmetric matrix and R and S_{N+1} are positive semi-definite symmetric matrices.

Results

An optimal solution to (4) has been found using dynamic programming (Bengtsson et al., Submitted) . This gave a optimal u_{ci} which does not only depend on the state x_i, but also on the previous control signals $u_{ci-1} \dots u_{c0}$. To test the derived LQG solution a simple system was simulated:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w,$$

which can be regarded as a simplified linear model of an inverted pendulum, with x_1 being the pendulum's angular velocity, x_2 being the pendulum's angle, and w being zero mean process noise with a variance $\sigma_w^2 = \frac{1}{64}$. It is assumed that the states can be measured directly without noise.

The system was discretized using a sampling time of 0.2. For the simulations $Q = 1$, $R = S_{N+1} = I_{2\times 2}$ and $N = 300$. We compare our derived optimal LQG controller

⋆ This work was partly financed by Caltech in a graduate study exchange program and partly by VINNOVA through the PiiA-project MoveDynE.

with the standard LQG controller determined for the same cost matrices (which does not compensate for the lossyness of the channel). The results of a simulation when $\alpha = 0.85$ can be seen in Figure 1.

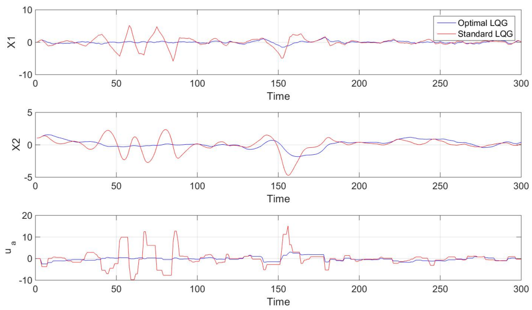

Fig. 1. Comparing the optimal LQG solution proposed here with the standard LQG solution for $\alpha = 0.85$.

Another way to compare the solutions is to compare the cost according to (4). This was done by simulating the system and calculating the total costs for both cases. These costs were then normalized by dividing each of them by the largest of the two costs. This as the system may exhibit unstable behavior in some iterations, which would dominate the costs of the iterations with stable behavior. Normalizing the costs alleviates this. The system was then simulated 1000 times from which a mean cost was calculated. A comparison between the optimal LQG solution and the standard LQG solution using this method is presented in Table 1.

Table 1. Cost function for comparing the optimal solution with the standard solution

α	Optimal LQG Cost	Standard LQG Cost
0.1	0.9966	0.9983
0.5	0.9446	0.9960
0.7	0.7909	0.9967
0.85	0.2457	0.9973
0.9	0.0276	0.9994

The same test was then done for a stable system (randomly generated):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.25 & -0.47 \\ 0.12 & -0.43 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -0.26 \\ 0.29 \end{bmatrix} u + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

using the same parameters as before except with a $Q = 0.01$ and with $\sigma_{w_1}^2 = \sigma_{w_2}^2 = 16$. The results of this can be seen in Table 2.

Table 2. Cost functions for control of the stable system

α	Optimal LQG Cost	Standard LQG Cost
0.1	0.9981	0.9994
0.5	0.9575	0.9994
0.7	0.8283	0.9998
0.85	0.36	1
0.9	0.0896	1
0.95	0.00064	1

From Table 1 and Table 2, the benefits of the proposed control algorithm are clear. While for channels where the probability of delay is small, the benefit is marginal. However once the probability for delay is sufficiently large, the control algorithm fairs far better than treating the system as a normal LQG problem.

Conclusion

This paper presents a solution to how to optimize LQG control when tree codes are used to submit data between the controller and the actuator. That is to say, there is a random unbounded delay between the controller and the actuator. The optimal LQG controller is derived for the TCP-like case under the assumption that the states can be measured directly without noise.

When the probability of delay is high, the optimal LQG controller fairs far better than the standard LQG controller. This allows one to control systems where standard LQG control would be insufficient to stabilize the system.

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