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INSTABILITY INVESTIGATION FOR THIN SPHERICAL MEMBRANES WITH CONTACTS

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Summary. A thin spherical membrane placed on the horizontal plane was investigated in the current work. The sphere was considered as without or with vertical contacts. Different boundary conditions were applied based on the problem settings. Multi-parametric problems were defined and stability conclusions were made using generalized eigenvalue analyses. Both limit points and bifurcation points were isolated. Different orders of penalty functions were compared and showed no effects on non-critical paths but gave effects on critical paths. Numerical results showed that contact conditions would introduce bifurcations and secondary paths, completely dependent on the discretization. The contact induced bifurcation points were isolated and the secondary branches were followed using generalized path following approach.

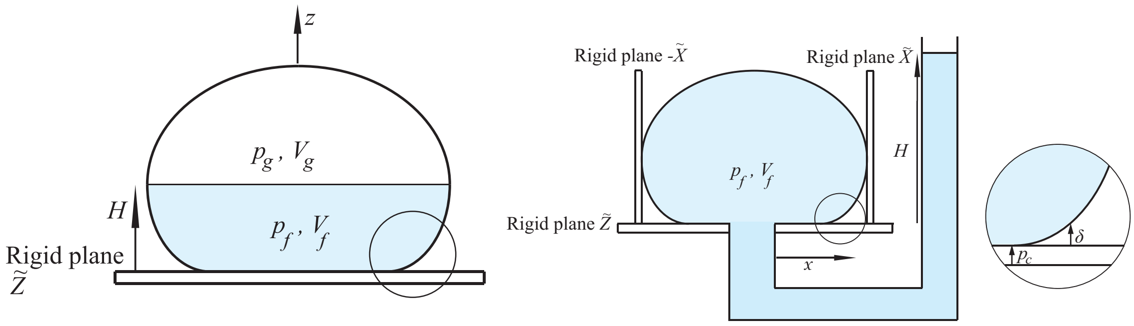
1 Introduction

Contacts between membranes and substrates are of importance in many fields, and the studies on them are of interest. Discussion about the contact formulations have been presented before[1]. Various methods have been developed to implement the contact conditions. Due to the extra constraints, the stability analysis in the context of the unilateral frictionless contacts would be interesting to investigate[2]. The present work focused on evaluating the stability for a membrane structure with contacts.

2 Mathematical models

2.1 Problem description

In this work, a spherical membrane with uniform thickness and resting on a horizontal rigid and friction-less plane was studied, Figure 1. The initial radius is denoted as r_o, thickness as t, and its shear modulus in a hyper-elastic material model as μ. It was firstly to be assumed inflated by gas and fluid, 1(a), where gas pressure was considered as a uniform over-pressure, and fluid pressure as a linearly distributed pressure in gravity direction. Five degrees of freedom were constrained.



(a) (b)

Figure 1: Deformed spheres filled with gas and fluid, without or with vertical contacts $\tilde{X}$, and placed on the rigid plane $\tilde{Z}$. Notations p_g, p_f, V_g and V_f are used to define the pressures and volumes of gas and fluid, respectively. Contact condition is described using contact pressure p_c and distance between membrane and rigid plane δ.

Then, a fluid-filled sphere was assumed to contact with two vertical rigid and frictionless planes, subfigure 1(b). A certain area at bottom of the sphere was completely fixed at the rigid plane $\tilde{Z}$, to avoid translation and rotation motions. The position of two vertical planes $\tilde{X}$ was assumed as constant in the present work, but can also be introduced as variable by adding an augmenting equation.

In this work, the spherical membrane was considered as isotropic, incompressible and hyper-elastic, and the two-parameter Mooney-Rivlin model with the ratio k was used to give the material description. The contact condition was defined point-wisely, and implemented using different orders of penalty functions.

2.2 The multi-parametric setting

The basic equilibrium equations were formulated based on the discretized residual equations:

$$\boldsymbol{F}(\boldsymbol{u}, \boldsymbol{\Lambda}) \equiv \boldsymbol{f}(\boldsymbol{u}) - \boldsymbol{f}_c(\boldsymbol{u}) - \boldsymbol{p}(\boldsymbol{u}, \boldsymbol{\Lambda}) = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{F}, \boldsymbol{f}, \boldsymbol{f}_c$ are defined as vectors of residual, internal, contact, external forces in the global degrees of freedom. Notation $\boldsymbol{\Lambda}$ is used for the loading parameters, which includes gas pressure p_g, fluid height and volume H, V_f for the first problem setting, and the position of the vertical planes $\tilde{X}$ for the second setting. Additionally, the contact force vector $\boldsymbol{f}_c$ is constructed as:

$$\boldsymbol{f}_c = \boldsymbol{f}_z + \boldsymbol{f}_x \tag{2}$$

where notations denote the horizontal and vertical rigid contacts. For the gas-fluid inflated sphere, Equ. 1(a), $\boldsymbol{f}_x = \mathbf{0}$. A generalized equilibrium equations was formulated:

$$\boldsymbol{G}(\boldsymbol{z}) \equiv \boldsymbol{G}(\boldsymbol{u}, \boldsymbol{\Lambda}) \equiv \begin{pmatrix} \boldsymbol{F}(\boldsymbol{u}, \boldsymbol{\Lambda}) \\ \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{\Lambda}) \end{pmatrix} = \mathbf{0} \tag{3}$$

where $\boldsymbol{g}$ is defined as a set of augmenting equations for imposing the constraints on the chosen parameters $\boldsymbol{\Lambda}$, for instance, the constraint on the fluid volume.

2.3 Solution methods

The equilibrium paths were found using the generalized path following algorithm[3]. Stability behaviors of a multi-parametric system were evaluated based on a generalized eigenvalue analysis[4]. The dependence of the instability behavior on the parameters was analyzed by performing fold line evaluations. The mechanical response to the multi-parametric loading was analyzed and visualized using the solution surface algorithm.

3 Numerical examples

A sphere with initial radius $r_o = 10$ mm, uniform thickness $t = 0.01$ mm, shear modulus $\mu = 0.4225$ MPa, with constitutive constant $k = 0.1$, and placed on a horizontal, non-friction and rigid plane was considered for the two problem settings. The density of fluid was assumed as $\rho = 10^{-6}$kg/mm^3.

3.1 A sphere rest on the horizontal rigid plane.

A sphere pressurized by gas and fluid was placed on the plane, without vertical contacts. The fluid volume inside was kept constant, and only gas was injected. The equilibrium paths are shown in Figure 2. The numerical results shows that the fixed volume constrints

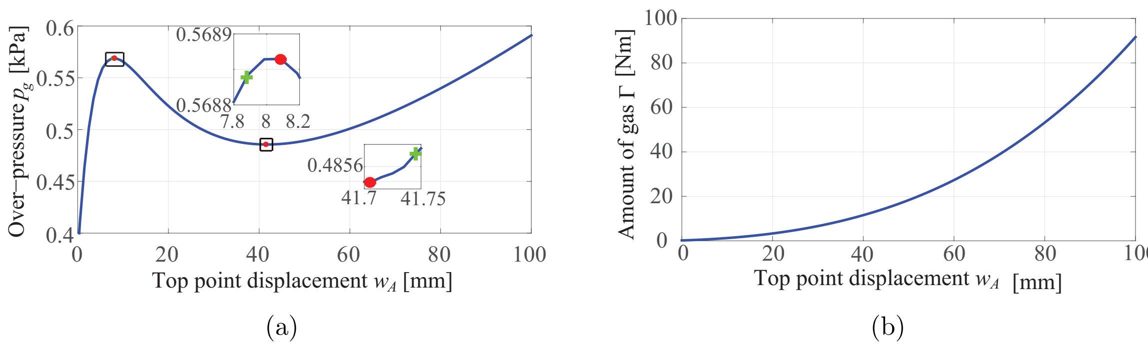
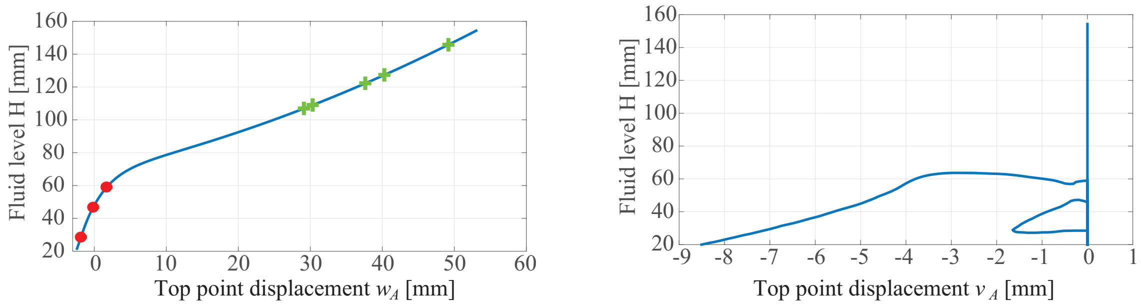


Figure 2: A sphere inflated by gas and fluid placed on the plane. The volume of fluid was constant as $V_o = 4.189 \times 10^3$mm^3. The over-pressure (a) and amount of gas (b) with respect to top point displacement were shown.

affect the stability conclusions, and it is of essence to specify the loading parameters and constraints for the stability interpretation.

3.2 A sphere contacted with vertical planes.

The sphere pressurized by fluid was placed on the rigid plane, and contacted with two vertical planes. The fluid level was considered as the main loading parameter, and several bifurcation points were isolated. Three points can be obtained from a finer meshed model, and the secondary paths are followed and shown in subfigure 3(b). However, the green bifurcation points in subfigure 3(a) can not be obtained from a finer mesh model, as these are highly dependent on the used discretization and induced by the contact condition.



(a) (b)

Figure 3: A fluid-filled sphere contacted with vertical planes. The relation between fluid level with respect to top point displacement in z (a) and y (b) directions.

4 CONCLUSIONS

The augmenting equations are essential for stability interpretations for a multi-parametric problem. The formulations for introducing the augmenting equations are important for interpreting the stability behavior. A sphere contacting with two vertical planes can bifurcate from the primary path. Some of the contact introduced bifurcations are artifacts and highly dependent on the used discretizations.

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