29th Nordic Seminar on Computational Mechanics
NSCM-29
R. Larsson (Ed.)
2016

RESONANT ELECTROMAGNETIC SHUNT DAMPING OF FLEXIBLE STRUCTURES

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Key words: Structural dynamics, Modal analysis, Electromagnetic shunt damping.

Summary. Electromagnetic transducers convert mechanical energy to electrical energy and vice versa. Effective passive vibration damping of flexible structures can therefore be introduced by shunting with an accurately calibrated resonant electrical network that contains a capacitor to create the desired resonance and a resistor to dissipate the correct amount of vibration energy. The modal interaction with residual vibration forms not targeted by the resonant shunt is represented by supplemental flexibility and inertia terms. This leads to modified calibration formulae that maintain the desired damping performance in the case of flexible structures with substantial modal interaction.

1 ELECTROMAGNETIC TRANSDUCER

The force f exerted by an electromagnetic transducer is proportional to the coil current I. When introducing the motor constant K the transducer force is given as[1]

$$f = KI \tag{1}$$

The current I in the electromagnetic transducer is governed by the representative electrical network model in Fig. 1(a), with an alternating voltage source

$$V_{\mathrm{em}} = i\omega K u \tag{2}$$

proportional to the velocity amplitude $i\omega u$ of the structure at transducer location. The dynamics of an electromagnetic transducer are commonly modeled by the voltage source

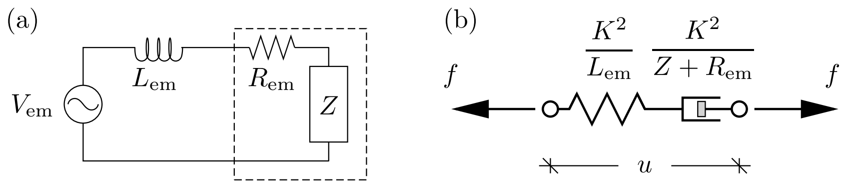

Figure 1: Electrical representation of an electromagnetic transducer with shunt $Z(\omega)$ (a) and equivalent mechanical model with stiffness and frequency dependent dashpot placed in series (b).

$V_{\rm em}$ placed in series with both a resistor $R_{\rm em}$ and an inductor $L_{\rm em}$. Conservation of energy in the network in Fig. 1(a) is secured by Kirchhoff's voltage law, which leads to the apparent mechanical force-displacement relation

$$u = \left(\frac{L_{\rm em}}{K^2} + \frac{Z(\omega) + R_{\rm em}}{i\omega K^2} \right) f \tag{3}$$

This relation identifies the two terms as a spring with stiffness $K^2/L_{\rm em}$ placed in series with a dashpot with viscous parameter $K^2/(Z(\omega) + R_{\rm em})$, see Fig. 1(b).

2 MODAL ANALYSIS

The governing equation of motion for the flexible structure can be written as

$$\big(-\omega^2 \mathbf{M} + \mathbf{K} \big) \mathbf{u} + \mathbf{w} f = \mathbf{f}_e \tag{4}$$

where the transducer force f is applied to the structure via the connectivity vector $\mathbf{w}$. Thus, the transducer force is energetically conjugated to the previously introduced transducer displacement $u = \mathbf{w}^T \mathbf{u}$. Typically, shunt damping is calibrated based on an assumed single mode response $u = u_r$, where u_r is the modal coordinate associated with the resonant mode shape vector $\mathbf{u}_r$, normalized to unity at transducer location ($\mathbf{w}^T \mathbf{u}_r = 1$). However, this simplified representation neglects any modal interaction with the other (residual) vibration modes. Therefore, the modal transducer displacement u_r is augmented by terms representing flexibility and inertia contributions from the residual modes[2],

$$u_r = u + \left(\nu_r' - \frac{\omega_r^2}{\omega^2} \nu_r'' \right) \frac{f}{k_r} \tag{5}$$

The force f is here normalized by the modal stiffness k_r and the corresponding residual mode coefficients inside the parenthesis are then determined as

$$\nu_r' = k_r \mathbf{w}^{\rm T} \mathbf{K}_r^{-1} \mathbf{w} - 1 + \nu_r'' \quad , \quad \nu_r'' = k_r \mathbf{w}^{\rm T} \mathbf{K}_r^{-1} \mathbf{K} \mathbf{K}_r^{-1} \mathbf{w} - k_r \mathbf{w}^{\rm T} \mathbf{K}_r^{-1} \mathbf{w} \tag{6}$$

where the modified stiffness matrix $\mathbf{K}_r$ is shifted by the inertia contained in the resonant vibration mode,

$$\mathbf{K}_r = \mathbf{K} - \omega_r^2 \left(\mathbf{M} - \frac{(\mathbf{M}\mathbf{u}_r)(\mathbf{M}\mathbf{u}_r)^{\rm T}}{\mathbf{u}_r^{\rm T} \mathbf{M} \mathbf{u}_r} \right) \tag{7}$$

It is observed that the resonant mode dynamics are not altered by this modification of the stiffness matrix, as shown by the modal stiffness $\mathbf{u}_r^T \mathbf{K}_r \mathbf{u}_r = \mathbf{u}_r^T \mathbf{K} \mathbf{u}_r = k_r$.

The modal force-displacement relation is obtained by eliminating the actual transducer displacement u in (3) by the augmented representation in (5),

$$u_r = \left(\frac{k_r}{K^2} L_{\rm em} + \nu_r' + \frac{k_r}{K^2} \frac{Z(\omega) + R_{\rm em}}{i\omega} - \frac{\omega_r^2}{\omega^2} \nu_r'' \right) \frac{f}{k_r} \tag{8}$$

Here the first two terms represent a spring, while the last term introduces inertia by an inerter element. The remaining term in the parenthesis contains the shunt impedance $Z(\omega)$, which in the following section is chosen as a classic series RC-network.

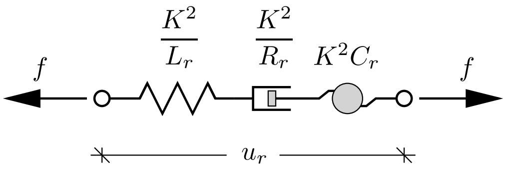

Figure 2: Equivalent vibration absorber with residual mode corrections.

3 DESIGN OF RESONANT SHUNT

It follows from Fig. 1(a) that the coil inductance and resistance appear in series with the shunt, and furthermore the residual mode contributions in (3) also appear as additive terms in a flexibility formulation, whereby they represent equivalent mechanical components connected in series. Thus, a series RC shunt is now introduced because the coil and residual mode contributions can then be fully absorbed and single mode calibration principles may be used directly to determined the optimal shunt tuning. In a series RC shunt the supplemental resistance R_s and capacitance C_s form the impedance function

$$Z(\omega) = R_\mathrm{s} + \frac{1}{i\omega C_\mathrm{s}} \tag{9}$$

By substitution into (8) a modal flexibility relation is obtained for a mechanical element with a spring, viscous dashpot and inerter placed in series, see Fig. 2. It is given as

$$u_r = \frac{1}{K^2}\left(L_r + \frac{R_r}{i\omega} - \frac{1}{\omega^2 C_r} \right) f \tag{10}$$

introducing the modal inductance L_r, resistance R_r and capacitance C_r,

$$\frac{L_r k_r}{K^2} = \frac{L_\mathrm{em} k_r}{K^2} + \nu_r' \quad , \quad R_r = R_\mathrm{em} + R_\mathrm{s} \quad , \quad \frac{k_r}{K^2 C_r} = \frac{k_r}{K^2 C_\mathrm{s}} + \omega_r^2 \nu_r'' \tag{11}$$

The stiffness, damping and inertance of the equivalent mechanical vibration absorber in Fig. 2 can now be determined by the readers most preferred calibration principle for an assumed single-mode system and subsequently applied to the flexible structure because the interaction with residual modes is explicitly represented by ν_r' and ν_r'' in the modal inductance L_r and capacitance C_r, respectively.

A suitable compromise between large modal damping and effective frequency response mitigation is obtained by the so-called balanced calibration with equal modal damping, previously used for tuning of mechanical vibration absorbers[2] and piezoelectric shunt damping[3]. For this calibration procedure a desired damping ratio ζ_des can be introduced to define a modal coupling coefficient $\kappa_r \simeq 8\zeta_\mathrm{des}^2$. The modal inductance, resistance and capacitance are then determined by the calibration formulae derived for an assumed single-mode structure. In normalized form they are obtained as

$$\frac{K^2}{k_r}\frac{1}{L_r} = \kappa_r \quad , \quad \frac{K^2}{k_r}\frac{\omega_r}{R_r} = \sqrt{\tfrac{1}{2}\kappa_r} \quad , \quad \frac{K^2}{k_r} C_r \omega_r^2 = \kappa_r \tag{12}$$

The actual values L_em, R_s and C_s are then finally determined by (11).

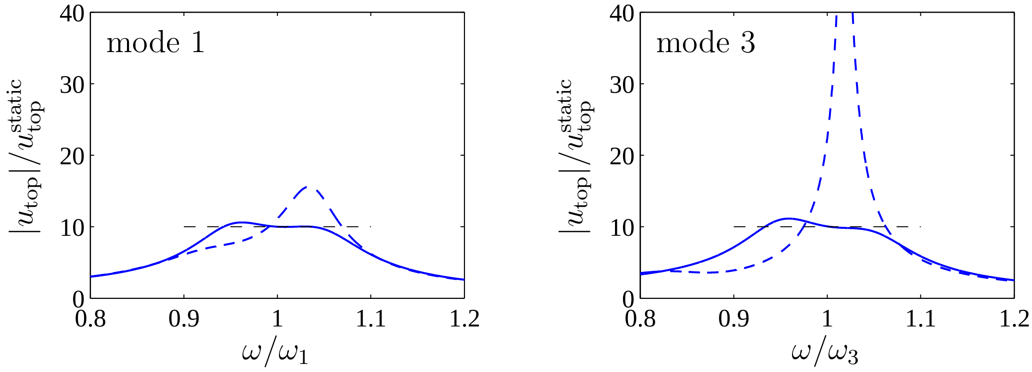

Figure 3: Dynamic amplification curves for modes 1 and 3 with $\zeta_{\text{des}} = 0.05$.

4 NUMERICAL EXAMPLE

An electromagnetic transducer with a series RC-shunt is now placed between the two bottom floors of a ten-storey shear frame structure[2]. For modes $r = 1$ and 3 the residual mode correction factors are $\{\nu_r', \nu_r'\} = \{4.78, 0.13\}$ and $\{30.03, 9.05\}$, respectively. The desired damping ratio is $\zeta_{\text{des}} = 0.05$, corresponding to a dynamic amplification factor $DAF = 10$. A complex modal analysis shows that the two damping ratios associated with the targeted vibration form is $\zeta_1 = \{0.0500, 0.0499\}$ and $\zeta_3 = \{0.0505, 0.0494\}$, while the equal damping property is lost when the residual mode correction is not included: $\zeta_1^0 = \{0.0293, 0.0660\}$ and $\zeta_3^0 = \{0.0071, 0.0668\}$ when $\nu_r' = 0$ and $\nu_r'' = 0$. Figure 3 shows the dynamic amplification curves for harmonic loading of the flexible shear frame structure targeting modes 1 and 3. The amplitude curves obtained by the corrected calibration formulae (solid line) recover the desired flat plateau and a maximum $DAF \simeq 10$, while the maximum DAF for $r = 1$ and 3 without correction is 15.6 and 62.1, respectively. Thus, the residual mode correction, represented by the two parameters ν_r' and ν_r'', is important to include when the desired damping level and harmonic response amplitude is obtained for a flexible structure with multiple vibration modes.

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