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THE GEOMETRIC NONLINEAR GENERALIZED BRAZIER EFFECT - A LINEARIZED FINITE ELEMENT BASED SOLUTION

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Summary. A thin walled circular tubular cross section under large bending moments ovalized and this is often denoted as the Brazier effect. This effect can also be seen in other types of profiles such as wind turbine blades. The paper shows that torsional moment gives a similar effect and this has been denoted the generalized Brazier effect. The original work of Brazier dealt with very large deformations that changed the cross section significantly and hereby also the bending moment of inertia and the bending moment capacity.

In this paper the aim is to describe the Brazier effect for smaller deformation not taking into account the change in moment of inertia. However, the generalized Brazier effect gives additional stresses directed perpendicular to the beam axis. In composite structures these extra stresses may influence the fatigue life significantly.

The paper demonstrates a linearized method to solve the complex non-linear geometric problem with a high accuracy. This is of importance in simulations of wind turbine blades, where the wind load simulations are based on small Finite Element models based on beam type elements in order to be realistic. The linearized solution exploits that the generalized Brazier effect is a local effect not influencing the overall mechanical behavior of the structure significantly. The offset is a nonlinear geometric beam-type Finite Element calculation, which takes into account the large displacements and rotations. The beam-type model defines the stresses which mainly are in the direction of the beam axis. The generalized Brazier effect is calculated as a linear load case based on these stresses.

1 INTRODUCTION

The Brazier effect,[1] , causes ovalization of thin-walled circular tube under bending, and this effect can also be seen in other types of thin-walled profiles,[2]. Reference [3] presented a more direct way of calculating the stresses due to the Brazier effect. The idea is to analyze a cross-section subjected to bending moments. In a large displacement theory the normal stresses from the bending moment will have a component directed inward which causes the ovalization. These inward directed stresses will create stresses perpendicular to the beam axis. A similar effect from the torsional moment has been identified and the combined action are denoted the generalized Brazier effect. The generalized Brazier effect can be implemented very directly in a Finite Element code, and basically it is an extra load case taking into account the local nonlinear geometric effects from the generalized Brazier effect. The additional load case uses the normal stresses in the beam axis direction from the first calculation. The initial load case can come from a non-linear geometric beam-type analysis, but in this context we for convenience use a linear analysis on a solid/shell model. The applications and accuracy of the proposed method have been demonstrated in an accompanying paper.

2 THEORY FOR THE GENERALIZED BRAZIER EFFECT

Brazier, [1] describes the ovalization of a cylindrical tube under pure bending. The effect can also been seen in other profiles, [2], and in [3] a more general computational approach was devised.

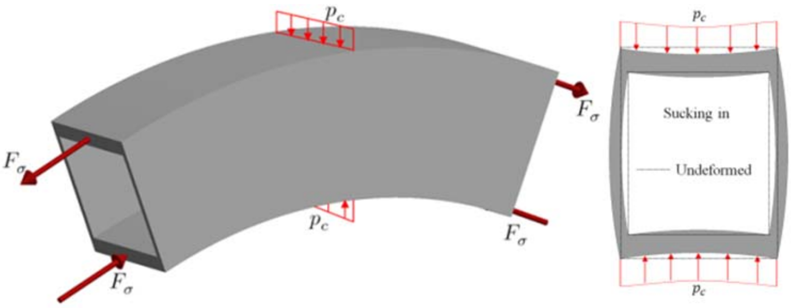

Figure 1: Illustrates a square tube applied to pure bending and Brazier forces are included.

The Brazier effect is due to the normal stresses, and combined with large rotations they will have a component directed inward. This was in [2] denoted the crushing pressure, and a little misleading explained as an external force. In [3] the inward stress component was described as a volume load acting in each fiber. In a Finite Element this contribution is calculated very directly.

With the axial nodal forces and the deformed rotation of an element it is possible to determine the volume forces denoted the Brazier forces. The equations (1),(2),(3) are valid for 8-noded solid elements, and similar equations can be established for other element types. The first step is to calculate the average stress in the element expressed as the consistent element forces, F_e. By retrieving the normal vector $(\overline{n}_1, \overline{n}_2)$ for the surfaces of an element, the inward directed vector $\overline{n}_e$ can be calculated. The volume force is calculated as the scalar product of the element force and the inward directed force,[4].

$$\frac{\sum_{n=1}^{8} F_{1,n} - \sum_{n=1}^{8} F_{2,n}}{2} = F_e \tag{1}$$

$$\overline{n}_e = \overline{n}_1 + \overline{n}_2 \tag{2}$$

$$\overline{F}_{e,Brazier} = F_e\, \overline{n}_e \tag{3}$$

In cylindrical tubes there is no extra effect from large rotations causing torsional moments. However, in general profiles such as a square thin-walled cross section under pure torsion the cross section will deform as illustrated in Figure 2.

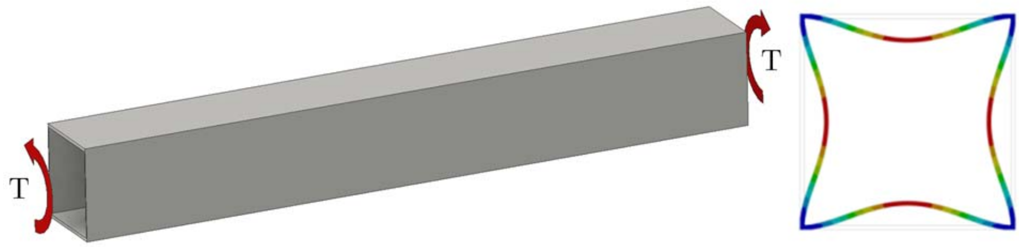

Figure 2: Deformation of the cross section due to Brazier by torsion.

The torsional and bending effect is denoted the generalized Brazier effect, and the deformation patterns have similarities. In both cases the cross section is forced inwards, and both the bending moment of inertia and the torsional moment of inertia are reduced. This nonlinear effect is not pursued in this paper, where the focus is solely on the influence on stresses.

Similar to the effect of bending the effect can be translated into volumetric loads that create a local torsional moment on the cross section. The shear stresses and deformation of the element defines the consistent nodal forces of an element. The deformation of an element under torsion gives an offset between the two *z*-planes. Offset of the sides from the shear center is seen on Figure 3

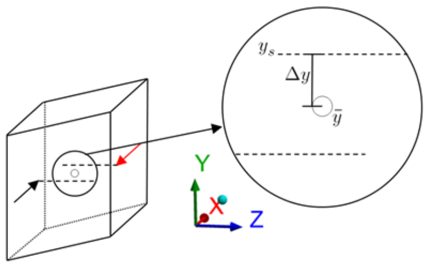

Figure 3: Offset of the *z*-planes in the *y*-direction.

By extracting the consistent nodal forces form the FE program or calculating the forces from shear stresses by equation (4)

$$F_x = \int \tau_{zx}\, dA \approx A\frac{1}{8}\sum_{n=1}^{8}\tau_{zx,n}$$
$$F_y = \int \tau_{zy}\, dA \approx A\frac{1}{8}\sum_{n=1}^{8}\tau_{zy,n} \tag{4}$$

Hereafter the local torsional moment of the element is calculated.

$$M_{z,e} = 2\left(F_x\,\Delta y + F_y\,\Delta x\right) = 4\,F\,h \tag{5}$$

In Figure 4 the graph shows the normal stress along the top flange. A linear analysis gives zero stresses, whereas a nonlinear analysis gives non-zero stresses. The green line is the full nonlinear analysis, and the blue represents the approximated method actual calculated as two linear load cases. The difference is very small.

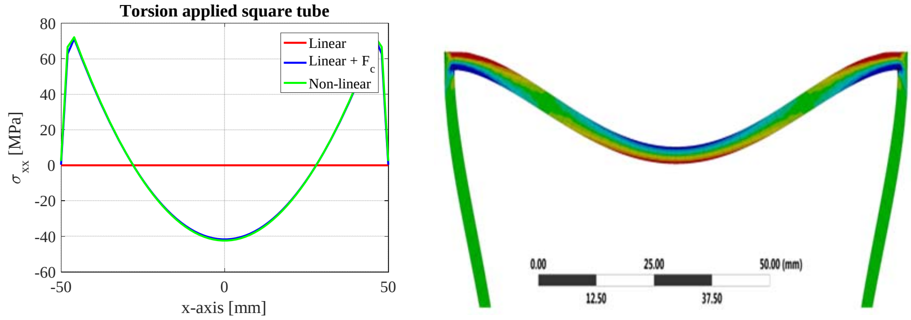

Figure 4: Stresses along the top flange.

4 CONCLUSIONS

Thin-walled beam-type structures such as wind turbine blades are influenced by large displacements and rotations which demands a full non-linear geometric analysis in order to simulate the overall behavior. The more localized nonlinear effects stemming from the generalized Brazier effect can only be calculated by large 3-D solid/shell Finite Element models. The paper proposes an approximate method allowing incorporation of the localized nonlinear effects in a beam-type Finite Element model. This will open for a more realistic estimation of fatigue stresses, which is often limiting the design life of a wind turbine blade.

REFERENCES

[1] L.G. Brazier, "On the Flexure of Thin Cylindrical Shells and other 'Thin' Sections," *Royal Society*, vol. Containing Papers of aMathematical and Physical Character, no. 116, pp. 104-114, Sep 1927.

[2] Find Mølholt Jensen, Weaver, P. M., Cecchini, L. S., Stang, H. and Nielsen, R. F., "The Brazier effect in wind turbine blades and its influence on design," *Wind Energy*, vol. 15, no. 2, pp. 319-333, 2012.

[3] Lars Damkilde. Brian Lund, "A Simplified Analysis of the Brazier Effect in Composite Beams," Aalborg Universitet, Esbjerg, 2007. ISBN: 978-19-050-8832-4.

[4] Jan A. Nikolajsen and Peter R. Lauridsen, "Failure modes caused by non-linear geometric effects in wind turbine blades," Aalborg Universitet, Esbjerg, AAUE, 2016, Master Thesis.