

# NUMERICAL MODELLING OF REINFORCED POROUS MATERIALS

## NSCM-29

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**Key words:** Porous material, Reinforcement, Mechanical Properties, Numerical Model.

**Summary.** The objective of study is to present an adequate model of reinforced by fibres porous material. For this reason reinforced cellular plastic is concerned with both the elastic and strength properties of material.

## 1 INTRODUCTION

Reinforced plastics are multiphase composite materials consisting of a polymeric matrix, reinforcement and a mobile, usually gaseous phase. The derivation of a model to describe the mechanical properties of reinforced foam provides a basis for the interpretation of structure and properties relationships, which in turn may be used to derive and predict design data.

The motivation to examine reinforced plastics is specifically attributable to the following reasons: (i) to reduce the high coefficient of thermal expansion, (ii) to produce a flatter modulus temperature profile as well as higher modulus, and (iii) to reduce temperature creep.

## 2 STRUCTURE OF REINFORCED CELLULAR PLASTICS

A strut-like polyhedral structure is characteristic for the open-cell plastics. The base polymer is concentrated in struts and knots. For to six struts usually enter a knot in the plastic foams with a uniform (isotropic) structure. An additional orientation of the struts parallel to foam rise direction is observed in monotropic foam<sup>1,2</sup>.

The microstructure of reinforced foams is composed of discrete spherical bubbles in short fibre-reinforced matrix. These materials can therefore be described as tree-phase composites taking into account the relative volume fractions of the constituents. The approaches used in the analysis of cellular materials and development of methods to predict their behavior is: phenomenological models to explain stress-strain behavior of material; and the analysis of the load-deformation behavior models.

## 3 ORIENTATIONAL MODEL OF REINFORCED PLASTICS

The fibres incorporated into the matrix are coated with foam material and becomes a composite struts. These struts bridge across several cells with the individual cell struts radiating outwards. As regards deformative properties, the free-rise foams are monotropic materials with the isotropy plane perpendicular to the rise direction.

If pores in reinforced porous material have uniform distribution the coefficient of reinforcement by using cross sections of components is expressed in the form:

$$\mu_f = \frac{A_f}{A_m + A_f + A_p}, \quad (1)$$

where the indexes  $m, f$  and  $p$  refer to the matrix, fibre and pore, respectively.

Short chopped fibres usually 3-12mm in length, can be used to reinforce low-density foams. The fibres incorporated in the polymer matrix are coated with resin and becomes in essence a composite struts. Under compressive loading, the system of struts support the load until a point is reached where their critical buckling load is exceeded. The struts buckle, causing a collapse of the cellular structure. The variation of polyurethane foam strength depending on reduced density and reinforcement content is discussed in <sup>3</sup>.

Elastic modulus of short fibre reinforced porous base material can be determined by using the rule of mixtures and geometry of cubic strut model of cell. The tensor of effective constants  $A_{\alpha\beta\gamma\delta}$  of reinforced plastic characterize stiffness of homogenous material as an integral one. It connects stresses and strains averaged throughout the material:

$$\langle \sigma_{\alpha\beta} \rangle = A_{\alpha\beta\gamma\delta} \langle \varepsilon_{\gamma\delta} \rangle. \quad (2)$$

The ergodic hypothesis is assumed valid in calculations. This permits to replace an averaging of physical quantities throughout the volume of material with averaging throughout a cluster of one-type situations (an ensemble). A local model cell is obtained by cutting out a rotational ellipsoid around material knot so that a half of each reinforced strut entering the knot which belongs to the model cell (Fig. 1).

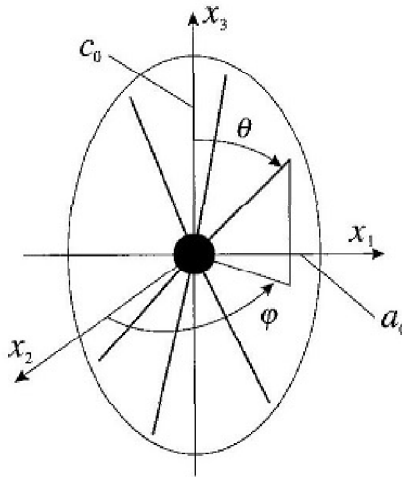


Figure 1: Ellipsoidal model cell of plastic with struts and knot;  $a_0, c_0$  – semi axis of model before deformation.

Considering the relationship for a unidirectional short-fibre representative composite

element in the direction of fibre  $x'_1$ , the elastic compliance  $S'_{1111}$  can be determined by using the law of mixtures:

$$S'_{1111} = \frac{1}{[1 + \mu_f(m_1 - 1)]E_{m,p}}, \quad (3)$$

Where in the case of anisotropic fibre  $m_1 = E_{f1} / E_{m,p}$ . The remaining compliances are determined according to reinforcement theory.

In order to determine the lower bond of material characteristics the orientational averaging according to Reuss hypothesis<sup>4,5,6,7</sup> regarding the uniformity of the stress field has to be performed using the relationship:

$$S_{\alpha\beta\gamma\delta} = \frac{1}{V^*} \int_0^\pi \int_0^{2\pi} S'_{ijkl} l_{i\alpha} l_{j\beta} l_{k\gamma} l_{l\delta} f(\theta, \varphi) \sin \theta d\theta d\varphi \quad (4)$$

( $i, j, k, l = 1, 2, 3$ ;  $\alpha, \beta, \gamma, \delta = 1, 2, 3$ ), where  $l_{i\alpha}$  are the cosines of the angles between composite axis and the axis of structural element of the given direction  $k$ ;  $S'_{ijkl}$  is the elastic compliance tensor of a unidirectional reinforced structural element. The value  $V^*$  is determined by using expression:

$$V^* = \int_0^\pi \int_0^{2\pi} f(\theta, \varphi) \sin \theta d\theta d\varphi. \quad (5)$$

In a similar way, according to Voigt method<sup>8</sup> the stiffness tensor is found using the assumption of uniformity of the strain field. In Fig. 6 the variation of elastic modulus depending on reinforcement coefficient for the case of uniform fibre distribution is shown.

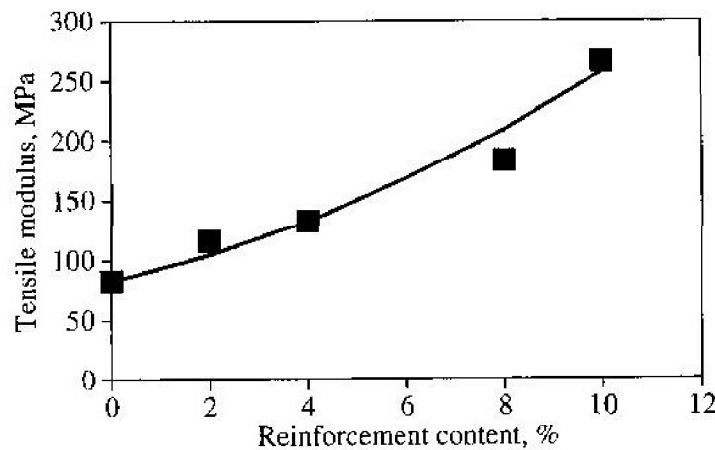


Figure 2: Tensile modulus for short-fibre reinforced cellular plastic; ■ – experiment<sup>3</sup>.

## 4 CONCLUSIONS

- Agreement of the analytical solution with the experimental data can be described as a reasonable showing for further investigations, particularly in the area of the interface between the fibre and matrix. The methods developed provide the ability predict the properties for the design of materials with required characteristics.
- The benefit of higher reinforcement content on elastic modulus is negated somewhat by the problems of short-fibre incorporation into the system.

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