

# PLATE TEARING UNDER MIXED MODE LOADING

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**Summary.** Cohesive-zone finite element modeling is often the technique of choice when dealing with extensive crack growth in large-scale ductile sheet metal structures. Shell elements with in-plane dimensions much larger than the plate thickness are typically employed to discretize the structure, and thus the mesh cannot accurately capture the localization process that precedes ductile failure. To fertilize accurate predictions of such sheet tearing, the energy dissipated during localization must, therefore, be accounted for in the cohesive traction-separation law. The fact is that the local thinning that takes place in front of an advancing crack can significantly enhance the crack growth resistance as the energy going into thinning the sheet typically dominates the total fracture energy. This has been investigated in great details for the case of pure Mode I tearing and both the energy dissipation, peak stress, and shape of the cohesive traction-separation law have been laid out. In a similar fashion, the present study resolves the sequence of failure details related to steady-state sheet tearing under mixed mode loading by employing the micro-mechanics based Gurson model. But, the fracture process in front of an advancing crack is here approximated by a 2D plane strain finite element model to facilitate a comprehensive parameter study to evaluate the mixed Mode I-Mode III load case.

## 1 INTRODUCTION

Large-scale plate structures are widely used in the automotive, aeroplane, and ship industry. These large plates are signified by having an out-of-plane dimension much smaller than the in-plane dimensions. In traditional finite element analysis of such large structures, the element choice is often shell elements. However, this type of element cannot handle crack initiation and growth alone, and thus the engineering approach to deal with this problem is to embed cohesive elements in the original mesh. Cohesive elements are defined by the peak stress together with either the energy release or the critical separation.

The analysis in this work is inspired from Nielsen and Hutchinson<sup>1</sup>, where the cohesive energy for steady-state crack propagation through a large-scale plate structure subjected

to pure Mode I loading was targeted. Nielsen and Hutchinson exploited a 2D plane strain set-up for a cross-section of the sheet, as illustrated in Figure 1, where different stages are depicted. Firstly, the plate is loaded and “necking” has initiated followed by local thinning (Figure 1A-B). The shear localization hereafter develops (Figure 1C) and finally, material separation is inevitable in the shape of a slant fracture (Figure 1D).

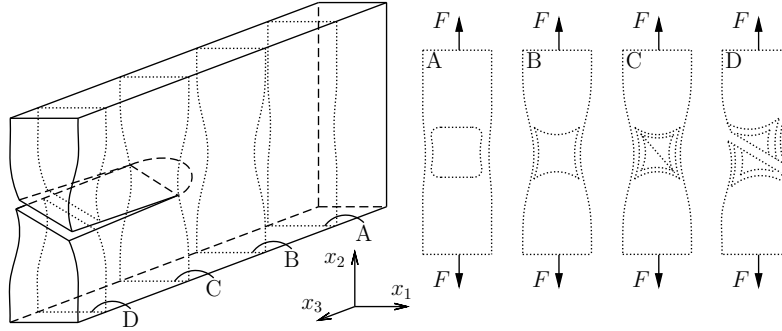


Figure 1: Graphical explanation of the problem considered.

The questions to be answered in the present study is, however, can a similarly detailed insight into the cohesive relation for mixed Mode I-Mode III loading be obtained, e.g. when going from pure tension to combined shear and tension. Moreover, is the widely accepted rotational sweep of the cohesive zone relation representing reality?

## 2 PROBLEM FORMULATION

The problem considered in this analysis is a cross-section of the plate in the  $x_1$ - $x_2$ -plane, where plane strain is assumed, see Figure 1. The Mode I-Mode III is treated with a 2D model and is depicted in Figure 2(a). An imperfection zone is embedded in a  $45^\circ$  angle across the center of the plate in order to trigger the slant fracture. The yield stress in the imperfection zone is lowered by 0.1% and the width of the zone is  $W_0/10$ . Further, the plate section has an aspect ratio of  $L_0/W_0 = 3$ .

In Figure 2(b), a graphical explanation of the cohesive energy is depicted. The cohesive energy is here defined as the area under the traction-separation curve from the peak stress until slant fracture. Once the peak stress is reached, essentially all plastic deformation localize in the middle region of the plate section. The total cohesive energy,  $\Gamma_0$  has contributions from  $\Gamma_I$  and  $\Gamma_{II}$  as defined in Figure 2(b).

## 3 MODEL DESCRIPTION

The material is governed by the micro-mechanics based Gurson<sup>2</sup> model, which is defined from the following yield surface:  $\Phi = \frac{\sigma_e^2}{\sigma_M^2} + 2q_1 f^* \cosh\left(\frac{q_2}{2} \frac{\sigma_k^k}{\sigma_M}\right) - (1 + (q_1 f^*)^2)$ , where  $q_1$  and  $q_2$  is fitting parameters suggested by Tvergaard<sup>3</sup>,  $\sigma_e$  is the effective macroscopic Mises stress,  $\sigma_M$  is the stress in the matrix material and  $f^*$  is a function of the void

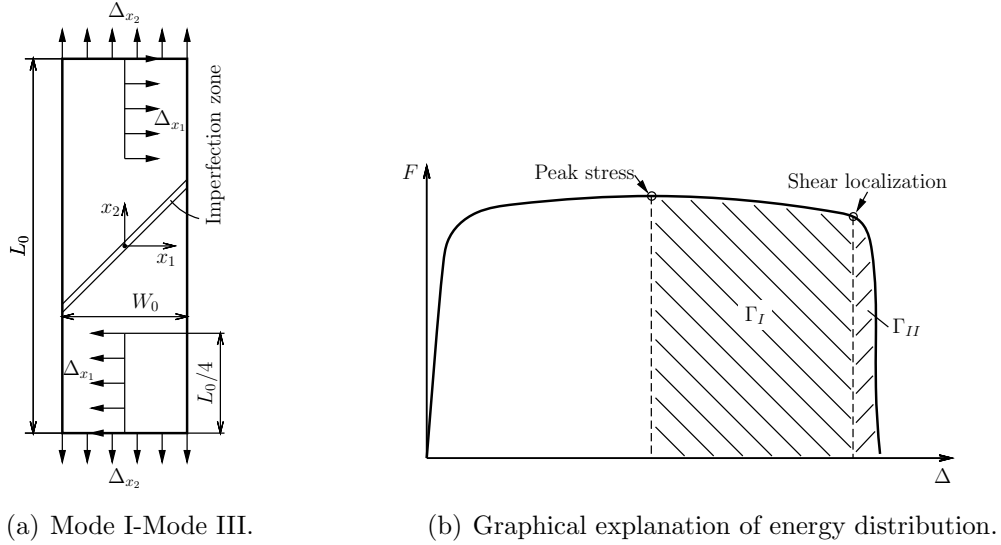


Figure 2: Illustrations of the different models employed in Abaqus.

volume fraction, that takes void coalescence into account. Nucleation of voids and the Nahshon-Hutchinson shear parameter are absent in this analysis. A critical porosity of  $f_C = 0.15$  and a final porosity of  $f_F = 0.4$  are used in the computations.

The computations are performed in the finite element software Abaqus/Explicit. The Mode I-Mode III finite element model is discretized using elements of type CPE4R (4-noded 2D element with plane strain assumption and reduced integration). A mesh convergence study has been performed to determine an adequate mesh to predict the shear localization.

## 4 RESULTS

Different ratios of the mixed loading are investigated in the range from 0% shear (corresponding to pure Mode I) to 60% shear defined from the ratio  $\Delta_{x_1}/\Delta_{x_2}$ . As a first parameter investigation, three different initial porosities,  $f_0 = 0.005$ ,  $f_0 = 0.01$  and  $f_0 = 0.02$ , are considered.

The cohesive energy is depicted in Figure 3(a) for various initial porosities and for increasing ratio of  $\Delta_{x_1}/\Delta_{x_2}$ . It is evident, that the cohesive energy is not constant when considering varying amount of shearing contributions. In fact, the cohesive energy decreases somewhat drastic with an increase in the Mode III loading. In Figure 3(b), the deviation between the energy for increasing ratio of  $\Delta_{x_1}/\Delta_{x_2}$  and the pure tension load case is depicted. For the lowest initial porosity of  $f_0 = 0.005$ , the drop in the cohesive energy reaches 18%, whereas for the remaining two initial porosities (at  $f_0 = 0.01$  and  $f_0 = 0.02$ ), the deviation reaches a level at 13%. It is worth noticing that this decrease somewhat contradicts the widely used rotational sweep of the cohesive zone law to cover out-of-plane actions.

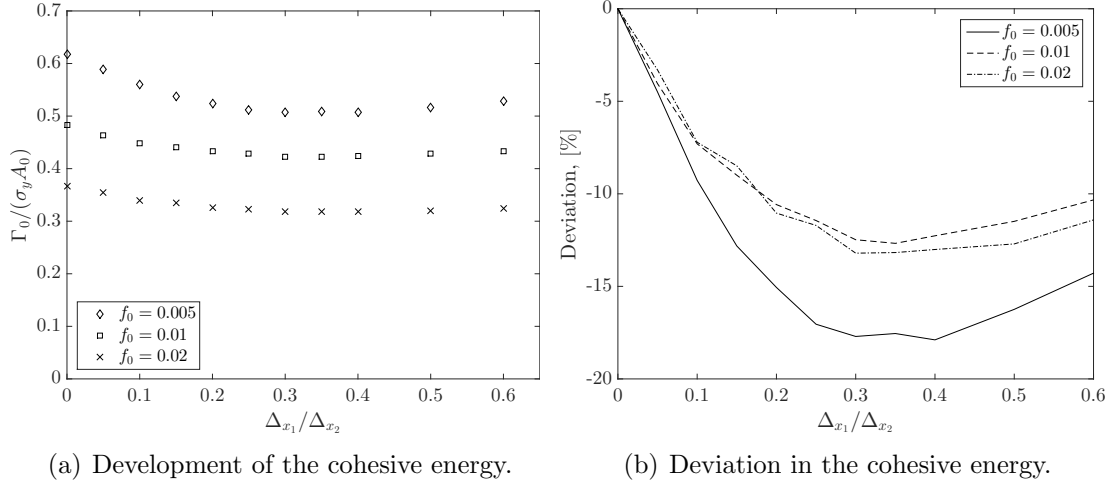


Figure 3: Mode I-Mode III analysis for varying ratios of  $\Delta x_1/\Delta x_2$ . The deviation is calculated as  $(\Gamma_{0,\Delta x_1/\Delta x_2} - \Gamma_{0,Mode I})/\Gamma_{0,Mode I}$ .

## 5 CONCLUSIONS

The cohesive zone energy is varying significantly for the mixed Mode I-Mode III load case. The drop is on the order of 15-20% depending on the material parameters. However, it shows that the widely used rotational sweep of Mode I cohesive zones can lead to underestimation of crack growth.

## 6 ACKNOWLEDGEMENTS

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## REFERENCES

- [1] Nielsen, K. L. & Hutchinson, J. W. Cohesive traction-separation laws for tearing of ductile metal plates. *International Journal of Impact Engineering* **48**, 15–23 (2012).
- [2] Gurson, A. L. Continuum theory of ductile rupture by void nucleation and growth. *Journal of Engineering Materials and Technology* 2–15 (1977).
- [3] Tvergaard, V. Material failure by void growth to coalescence. *Advances in Applied Mechanics* **27**, 83–151 (1990).