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A STOCHASTIC MODEL WHICH PREDICTS THE STRENGTH OF GLASS PLATES IN BENDING

DAVID T. KINSELLA[*] AND KENT PERSSON[†]

[*]Division of Structural Mechanics
Faculty of Engineering LTH
Lund University, P.O. Box 118, 221 00 Lund
e-mail: david.kinsella@construction.lth.se, web page: http://www.byggmek.lth.se/

[†]Division of Structural Mechanics
Faculty of Engineering LTH
Lund University, P.O. Box 118, 221 00 Lund
e-mail: kent.persson@construction.lth.se, web page: http://www.byggmek.lth.se/

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Summary. When ordinary float glass plates are tested experimentally in common setups the observed failure loads vary substantially which poses a challenge to the safe and rational design of glass structures. The Weibull distribution is deduced from the weakest-link theory which à priori would make its application to portray the brittle strength of glass appear consistent. As an alternative it is possible to simulate the failure stress in an ordinary float glass plate in a stochastic manner. This approach opens up for the possibility of implementing a surface flaws condition more in line with recent research in glass technology. The method of implementation relies on computations in which the FE-software ABAQUS/CAE is integrated with MATLAB. The nonlinear mechanical response of a glass plate in bending is coupled with a set of stochastically defined surface flaws which are spatially distributed according to given rules. The results show that it is possible to implement the method with reasonable speed and efficiency. Further, the results are in good agreement with empirical data on annealed glass strength.

1 BACKGROUND

Load-bearing glass components enable outstanding structures with remarkable properties such as light transmission and slim components. Light, in particular, allows for the architectural space to appear. Glass is increasingly being applied to structures. Appraising the strength of a glass component is key to its safe and due application. According to an hypothesis formulated by Griffith[1] in 1920, fracture is initiated at microscopic and submicroscopic surface flaws. Moreover, it has been shown that the fracture stress of glass is highly dependent upon the loading time until failure due to an hypothetical thermally activated process which takes place at the crack tip and which causes subcritical crack growth. Experiments carried out on annealed glass plates in various types of bending show that the short-term strength usually ranges from about 20 MPa to 200 MPa. The coefficient of variation is frequently as large as 30-50%. Glass is amorphous and behaves perfectly brittle. Weibull[2] applied the weakest-link theory to a homogeneous solid in three dimensions and proposed the following statistical distribution function to model fracture data of a brittle solid, viz.

$$P(X \leq x) = 1 - e^{-\left(\frac{x}{k}\right)^m} \tag{1}$$

where k and m are parameters and X is the random variable for the strength. Later, Jayatilaka and Trustrum[3] demonstrated that the function in equation (1) can be deduced from the weakest-link theory if it is assumed that the surface flaws belong to a single population whose flaw depth distribution behaves like a power-law in the tail. Hertzian indentation tests[4] appear to give some credence to this hypothesis. Hence there exists sound, physical arguments for the portrayal of glass strength using Weibull's model. Today it is ordinary practice to fit a Weibull distribution to experimental data but all too often the resulting fit is poor and there is evidence that a normal and lognormal distribution are at least as good at explaining empirical data. From a theoretical standpoint however, the adoption of a Weibull model is far more consistent than any adoption of a normal or lognormal model due to its deduction from the weakest-link theory.

2 A REFINED SURFACE FLAWS CONCEPT

It is a challenge to qualify the surface flaws condition in glass due to the very lack of empirical data. Nonetheless, results from Hertzian indentation tests and measurements with optical scanning techniques indicate that the surface flaws might be separated into two populations. The first population represents the effects of large flaws which originate from surface scratching, impact and machining damage, etc. These flaws number about a handful on an as-received surface area measuring 200x200 mm^2. as indicated in a recent study[5]. The second population represents the effects of microscopic and submicroscopic flaws which arise during glass formation and manufacture, e.g. when a microscopic dust grain from the atmosphere or a glass splinter from the cutting of sheet glass gets in between the contact surfaces and causes friction. The combination of these two populations has been suggested previously in the literature and represents a refinement over the hypothetical flaws condition assumed by Jayatilaka and Trustrum[3]. The refined concept however, calls out for a novel method of implementation.

3 A NEW METHOD

A new method to model the strength of glass plates in bending is being developed using ideas put forward by Yankelevsky[6]. The plate surface area which is under tension is subdivided into small regions. Depending on which surface flaws concept that is adopted, a set of cracks with random depths are sampled and uniformly distributed among the regions. From one simulation to another the sampled cracks will vary in a stochastic manner. The plate is subjected to an applied loading in small increments. The stresses which arise at the middle of each region as the load is increased are reinforced by the random crack which is present there. The stresses near the crack tip are represented by the stress intensity factor which depends on the mode of crack opening. The mode I stress intensity factor is

$$K_I = Y \cdot \sigma\sqrt{\pi a} \tag{2}$$

where Y is a geometry factor, σ is the remote stress which acts perpendicularly to the crack plane and a is the crack depth. For a half-penny shaped surface crack in a semi-infinite specimen the geometry factor provided by Tada et al[7] is $Y = 0.722$. Irwin's[8] criterion for brittle fracture states that

$$K_I \geq K_{Ic} \tag{3}$$

assuming mode I crack opening where K_{Ic} is the fracture toughness of glass which amounts to about 0.75 MPa$\sqrt{\mathrm{m}}$. In each loading increment, the regions are searched through and a verification is performed to determine whether or not the fracture toughness has been exceeded anywhere. By the weakest-link theory, global failure occurs as soon as any one region fails. At that point in the loading history, the failure stresses and the failure location are readily identified. By running a simulation in series, it is possible to produce a synthetic sample of the apparent strength of a glass plate.

4 IMPLEMENTATION

The new method and the surface flaws concept are implemented numerically. First the stresses due to plate bending are calculated in a finite element analysis by means of the software ABAQUS/CAE. The stresses at the middle of each subdivided region are extracted and inserted into the software MATLAB. The subdivision of the surface area into planar regions depends on the new method and is not the same as the finite element mesh. Using MATLAB it is possible to draw a random sample of surface flaws which are represented by the crack depth and by the orientation of the crack plane. The random flaws are assigned to the regions and paired up with the local stresses. The first region that fails is identified and the maximum principal stress at this region is recorded.

5 PRELIMINARY RESULTS

Investigations are under way into simulating the stochastic strength of glass plates which measure roughly 200x200x6 mm^3 and which are loaded in coaxial double ring bending. The nonlinear mechanical response of a glass plate in coaxial double ring bending has previously been simulated using first-order plate theory or the finite element method with shell elements. In order to predict the mechanical response in greater detail, however, a finite element model is constructed using a mixture of linear hexahedral and tetrahedral elements for the glass part and rigid analytical shells for the loading and reaction ring parts. Once the stresses have been calculated they may be extracted using a Python-script and inserted into MATLAB. If one assumes a single surface flaws population with a Pareto distribution for the flaw depth one might observe samples of the fracture stress and location as indicated in figure 1. The stochastic model parameters have to be calibrated. This is done using stastitics in order to provide an optimal fit with experimental data. According to preliminary results, the stochastic model is able to predict the fracture stress associated with a data sample which otherwise is not modelled properly by a two-parameter Weibull distribution.

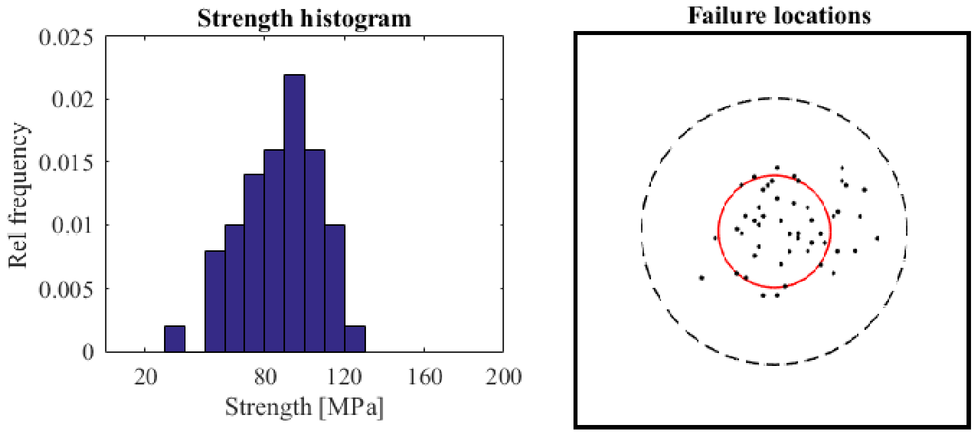

Figure 1: Normalized histogram of the simulated strength and failure locations.

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