29th Nordic Seminar on Computational Mechanics
NSCM-29
R. Larsson (Ed.)
2016

MATHEMATICAL MODELING OF ELASTOTONOMETRY OF AN EYE

SVETLANA M. BAUER, EKATERINA O. BODENKOVA

Saint Petersburg State University
Universitetskii pr 28, Saint-Petersburg, Russia, 198504
e-mail: s.bauer@spbu.ru

Key words: Elastotonometry of a human eye, the Maklakoff tonometer, pathologies, FEM package Ansys.

Summary. Elastotonometry of a human eye and particularly the causes of pathologies are studied. Measurement of IOP with the Maklakoff tonometer is considered as a contact problem and simulated numerically with software package Ansys. Cornea and sclera are assumed to be transversely isotropic, the cornea is treated as a non-uniform shell. The influence of various parameters of the cornea and sclera on tonometric pressure and dependence of elastotonometric pressure on a load are analyzed.

1 INTRODUCTION

Elastotonometry of a human eye and some causes of pathologies are studied. Elastotonometry is a method of intraocular pressure measurement with Maklkoff's applanation tonometers of different weights. This method was proposed by V.P. Filatov and S.F. Kalfa as a new method of an early glaucoma diagnosis. In elasotonometry the weights of tonometers 5, 7.5, 10 and 15g are used. The results of measurements with different weights are presented in the form of a XY-plot (so called "elastic curve") with the masses of the tonometers on the abscissa-axis and tonometric intraocular pressure on the ordinate axis. For a normal eye this curve increases monotonically (Fig.1). But sometimes it's not true and the tonometric pressure for weight 7.5 g is larger than tonometric pressure for weight 10 g (Fig.2). Elastotonometry permits to reveal both existence of biomechanical abnormalities and type of abnormality. It is known, for example, that the elastic curve for an eye with glaucoma lies above the curve for a normal eye.

2 MATHEMATICAL MODEL

Firstly, the problem of static loading of the eye cornea by a flat stamp modeling an applanation tonometer is solved analytically. The eye is treated as the composition of a soft shell (cornea) and an elastic shell (sclera). The shell equilibrium equations and geometrical relations are used[1]. This simple model can explain the change of the form of the elastic curve under glaucoma, when the rigidity of the sclera increases. But in fact the cornea is not absolutely soft and uniform shell[2].

Measurement of IOP with the Maklakoff tonometer is simulated also numerically with FEM package Ansys. Materials are assumed to be transversely isotropic; the cornea is treated as a non-uniform shell. The influence of various parameters of the cornea and sclera on tonometric pressure and dependence of elastotonometric pressure on a load are analyzed.

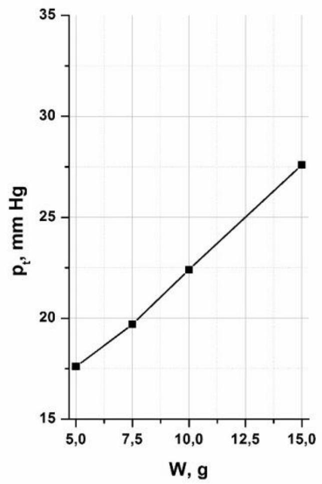


Figure 1: Elastic curve for a normal eye

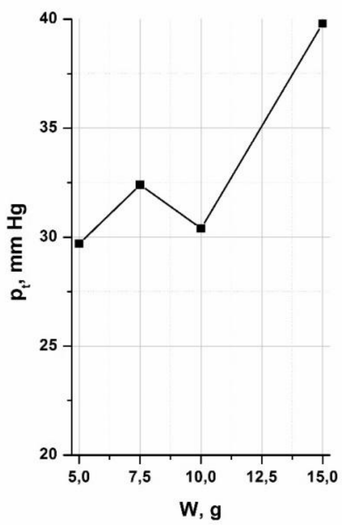


Figure 2: Elastic curve for an abnormal eye

3 THE INFLUENCE OF GEOMETRIC PARAMETERS

Firstly, the influence of geometry of eye on tonometric pressure is studied. This question is relevant because the eye diseases are spread. Ellipsoidal shape of the eye is common (e.g. myopia, hypermetropia).

The following results were obtained: when the radius of curvature of the cornea is increased, the contact area of eye and tonometer also increases, but the pressure tonometric decreases. Thus the change of the shape and size of the eyes affects the behavior of elastic curve.

4 THE INFLUENCE OF RIGIDITY OF THE SCLERA AND CORNEA

Next, the influence of rigidity of the sclera and cornea on tonometric pressure is analyzed. The rigidity of sclera and cornea can be increased in glaucoma, so tangential elastic modules are varied. For certain parameters there is a sloping plot, or fracture in elastic curve.

Sclera: $E_\varphi = E_\vartheta$, MPa	3	5	10	15	20	30
Cornea: $E_\varphi = E_\vartheta$, MPa	0.5	1	1.5	3	4	5

Table 1: Tangential elastic modules of sclera and cornea

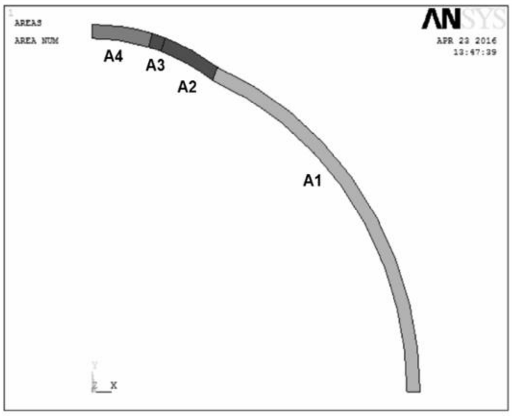
5 THE INFLUENCE OF INHOMOGENEITY OF THE CORNEA

The influence of inhomogeneity of the cornea on tonometric pressure is studied. Inhomogeneity of the cornea could increase in such diseases as marginal keratitis and dystrophia corneal marginalis ectatica. These diseases are characterized by the development of punctate opacities in the cornea and ingrowth of blood vessels into it, followed by thinning.

This fact is simulated by adding soft inserts of different sizes in the cornea. Elastic modules of insertion: $\boldsymbol{E_\varphi} = \boldsymbol{E_\theta} = 0.3$ MPa. The values of tangential elastic modules of the cornea and sclera were varied in the same range (Table).

In all cases there is fracture in elastic curve. When size of insert is decreases, there is large pressure drop on the last step of loading. Indeed, when the load reaches the end of the insertion, the contact area grows slowly, and tonometric pressure increases strongly.

Thus, the present model confirms that the deviation in Kalfa's elastic curve may be due to corneal pathology



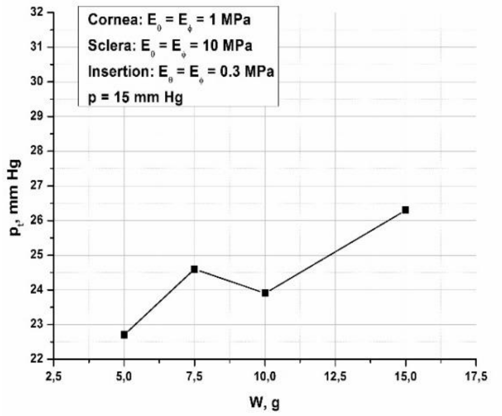


Figure 3 - 4: Mathematical model of the eye with a soft insert in the cornea (area A3) of average size. The dependence of tonometric pressure of the weight of the load for the model

Thus, the present model confirms that the deviation in Kalfa's elastic curve may be due to corneal pathology.

6 CONCLUSIONS

The results show that both the parameters of the cornea and the parameters of the sclera effect on the tonometric pressure. Parameters of the sclera especially affect, when loads of more than 5 grams are used. Fractures of elastic curve may be associated with the tightening of the sclera, which may occur in glaucoma. It may also be due to corneal pathologies: when the cornea becomes particularly weak around sclera, for example, in dystrophy corneal marginalis ectatica.

REFERENCES

[1] S.M. Bauer, G.A. Lyubimov, P. E. Tovsyik, "Mathematical Modelling of Maklakoff's Method for Measuring the Intraocular Pressure" // *Fluid Dynamics*, 2005, Vol.40, N. 1, pp. 20-33

[2] E. N. Iomdina "Mechanical Properties of the Human Eye Tissues" in Modern Problems of Biomechanics, Vol.11, pp. 183-200, Moscow University Press, Moscow, (in Russian) 2006