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ON MULTI-OBJECTIVE TOPOLOGY OPTIMIZATION AND TRACING OF PARETO-OPTIMAL STRUCTURES

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Summary. In this paper an sequential linear programming (SLP) algorithm for solving multi-objective topology optimization problems have been implemented and numerical results for a design domain have been calculated. Examples shows that the SLP algorithm performs similar to the single objective optimality criteria algorithm but also that some further work is needed.

1 INTRODUCTION

Topology optimization, a subfield of structural optimization, can be seen as finding the best connectivity between loads and supports with respect to desired performance quantified by objective functions[3,8]. Topology optimization can e.g. be applied in order to find the shape of a beam or the reinforcement layout in a concrete slab. However, classical topology optimization considers only one objective while in many practical situations there are several conflicting objectives to consider. In this work we present a formulation and solution approach for finding the Pareto-optimal solutions of multi-objective topology optimization problems.

2 PROBLEM FORMULATION

In the context of a finite element discretization the distribution of material is assumed to be constant within each element, thus the vector of design variables $\boldsymbol{\rho}$ with components ρ_e the material at element e. A continuous interpolation formulation approach for intermediate values of ρ_e is used. The problem can be written as:

$$\begin{cases} \min\limits_{\boldsymbol{\rho}} & (V(\boldsymbol{\rho}), C(\boldsymbol{\rho}), -\lambda_1(\boldsymbol{\rho})) \\ \text{subject to} & \mathbf{0} \leq \boldsymbol{\rho} \leq \mathbf{1} \end{cases} . \tag{1}$$

The considered objectives are: minimization of volume $V(\boldsymbol{\rho}) = \sum_{e=1}^{n_{el}} \rho_e V_e^0$; minimization of compliance C under static loads, which includes solving $\mathbf{K}(\boldsymbol{\rho})\mathbf{u}\{\boldsymbol{\rho}\} = \mathbf{f}(\boldsymbol{\rho})$ where $\mathbf{K}$, $\mathbf{u}$ and $\mathbf{f}$ are the stiffness matrix, displacement vector and load vector respectively, given the design vector $\boldsymbol{\rho}$; and maximization of the fundamental eigenvalue λ_1 of the structure under free vibration, wich includes solving the eignevalue problem $\left[\mathbf{K}^{\text{free}}(\boldsymbol{\rho}) - \lambda_1 \mathbf{M}^{\text{free}}(\boldsymbol{\rho})\right] \boldsymbol{\phi}_1^{\text{free}} = \mathbf{0}$, where $\mathbf{M}$ and $\boldsymbol{\phi}_1$ are the mass matrix and eigenvector respectively. Hence, C and λ_1 are ultimately functions of $\boldsymbol{\rho}$, so-called nested formulation. A complete formulation of the objective functions and their derivatives are given in[1].

The multi-objective problem in eq. (1) is recast into a single objective optimization problem using compromise programming[2]. Introducing weights w_i operating on each objective, allowing us to trace the Pareto-optimal frontier, and making the objectives dimensionless and scaled in order to ensure the objectives have the same magnitude, eq. (1) can be written as:

$$\begin{cases} \min\limits_{\boldsymbol{\rho}} & l_p(\boldsymbol{\rho}) = \left[\sum_{i=1}^{n_f} w_i^{p_n} \left(\frac{f_i(\boldsymbol{\rho}) - \min f_i}{\max f_i - \min f_i}\right)^{p_n}\right]^{(1/p_n)} \\ \text{subject to} & \mathbf{0} \leq \boldsymbol{\rho} \leq \mathbf{1} \end{cases}, \tag{2}$$

$l_p(\boldsymbol{\rho})$ is the *compromise function*, $f_i(\boldsymbol{\rho})$ denotes objective function i and p_n is the P-norm. Note that the the scaling parameters, $\min f_i$ and $\max f_i$, of this method have a large impact on the scope of the resulting Pareto-optimal solutions[1].

3 SOLVING THE PROBLEM

As a result of the finite element discretization with choosing the design variables associated with the elements, numerical oscillations in the solution may occur causing patterns similar to checkerboards[3]. Such structures can not transfer any load. These issues can be resolved by several means[4] of which the application of a filter is a common choice. A mesh independent density filter[5] along with a continuation method[4] operating on the filter radius is used. The continuation method is based on the idea of gradually going from an artificial convex problem to the non convex problem in a number of steps.

Integer solutions are encouraged for by applying the RAMP[3] interpolation scheme such that $E_{\min} \leq E_e(\rho_e) \leq E_e^0$ where $E_{\min}$ is a very small stiffness introduced in order to avoid singularities when $\rho_e = 0$ and E_e^0 is the stiffness of the base material. Both schemes penalize intermediate solutions. The optimization problem is solved using a sequential linear programming (SLP) algorithm[6].

The method has been implemented in Matlab and been used in order to solve a numerical example: the support structure of a bridge deck, see Figure 1. Further examples are presented in[1]. Some of the obtained structures are presented in Figure 2. In[1] a comparison of the results with single objective topology optimization results based on the optimality criteria (OC) method[3,7,8] is presented. Since the results are not as clear integer-solutions as desired, it seems that the SLP algorithm is not capable of solving the presented problem in a satisfying way.

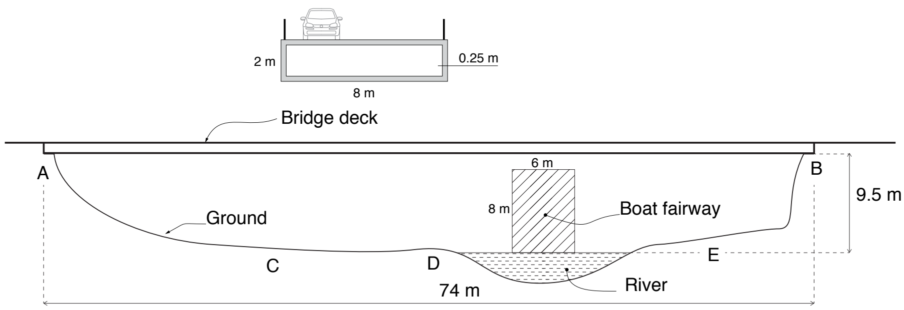

Figure 1: Bridge example configuration. Supports allowed at points A-E; no structure allowed in the river and in the boat fairway.

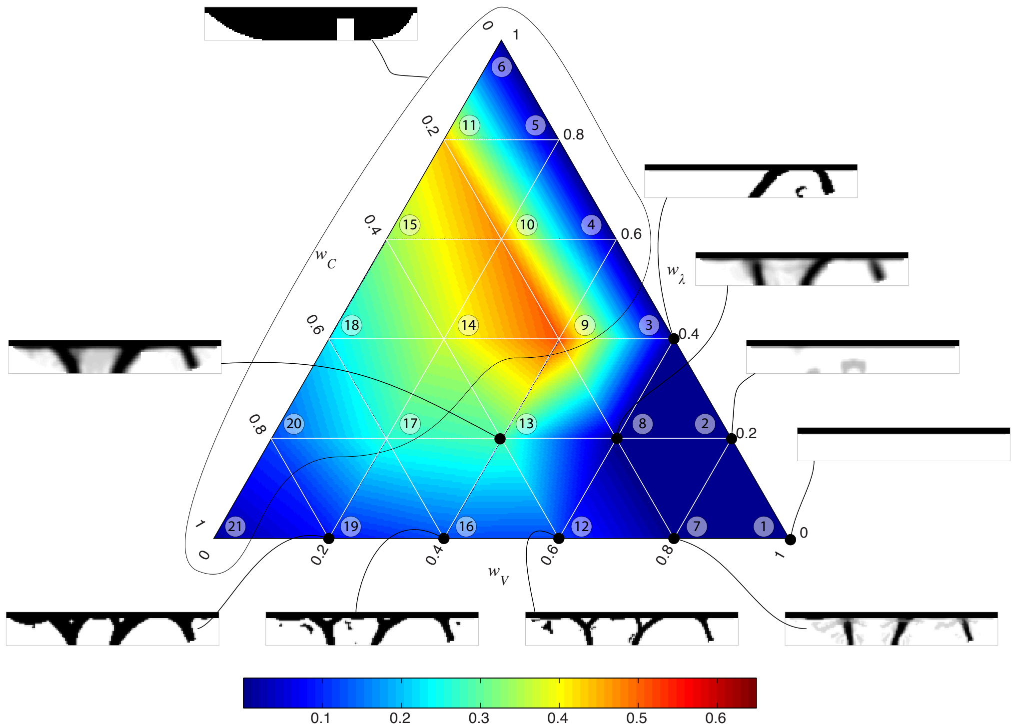

Figure 2: Compromise value $l_p(\boldsymbol{\rho}^*)$ and Pareto-optimal solutions. Weight point numbers k within circles. Here the RAMP interpolation scheme is used with $q = 20$ and the initial guess set to $\boldsymbol{\rho}^{(k),(0)} = \mathbf{1}, \forall k$. The continuation method is applied on the density filter with $r_{\min} = [5, 4, 3, 2, 1] \times 0.6$. Note that weight points 4-6, 9-11, 14-15, 17-18 and 20-21 yields the same result for this configuration.

4 DISCUSSION

As seen, the SLP algorithm dose not yield valid results. Due to the nature of the compromise function $l_p(\boldsymbol{\rho})$, objectives that are relatively flat around the minima point may cause problem for the algorithm to handle and it will terminate without finding the optimum. Regarding the considered objectives, it is evident that the compliance objective (load dependent) and the eigenvalue objective (load independent) in the general case yields completely different structures.

5 FUTURE WORK

To rule out if there are more suitable solution methods for the problem presented, other algorithms than the SLP should be tested. Further more, the scaling of the objectives, and especially the Eigenvalue, could be studied more thoroughly in order to clarify the impact on the found solution. Multiplicity of eigenvalues, i.e. structures that gives several mode shapes for the same eigenvalue ($\lambda_{1,2}, \phi_1, \phi_2$) should be investigated, especially when dealing with symmetrical design domains. The occurrence of such eigenvalues may lead to poor convergence in the optimization process. Finally, in order to make real use of the results, multiple load cases and three dimensional design domains should be considered.

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