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VERIFICATION OF FINITE ELEMENT METHODS FOR STIFFENED SPHERICAL SHELLS

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Summary. We analyze different shell deformation states with typical finite element software used in structural engineering. The analysis is carried out in the context of the so called Girkmann problem which involves a stiffened spherical shell. The accuracy of the simplest triangular and quadrilateral shell elements are compared to semi-analytical and numerical reference values available in the literature.

1 INTRODUCTION

The modelling assumptions of conventional shell finite elements employed in industrial software extend beyond the limits of mathematical convergence theory. In particular, the connection between the discretization and the actual differential operators of the mathematical structural model is obscure[1]. Moreover, connection of shell elements to other types of structural or solid elements requires special care. While modern technology may enable the use of solid elements instead of shell elements in a variety of situations, post-processing of stress resultants from 3D stress field from a solid element model can be difficult. Such stress resultants are useful, or even required, e.g. in the design of reinforced concrete structures.

In this work, we analyze and compare the accuracy of typical lowest-order triangular and quadrilateral shell finite elements in context of a model problem for concrete structures[2]. While bending-dominated deformations associated to the so called simple edge effect can be approximated reasonably with both formulations, membrane-dominated deformations are difficult to approximate with low-order elements.

2 PROBLEM STATEMENT

The structure to be analyzed consists of a spherical dome made of reinforced concrete, assumed to be homogenous, isotropic and linearly elastic with the Young's modulus E=20.59 GPa and the Poisson's ratio υ=0.

The shell thickness is h=60 mm and the crown radius ρ_0=15.00 m. The shell is connected to a stiffening ring at the meridional angle α=40° and $r_0=\rho_0/\sin\alpha$=23.34 m is the radius of the midsurface of the spherical shell. The dimensions of the ring are: a=0.60 m, b=0.50 m. The notation is shown in Fig. 1.

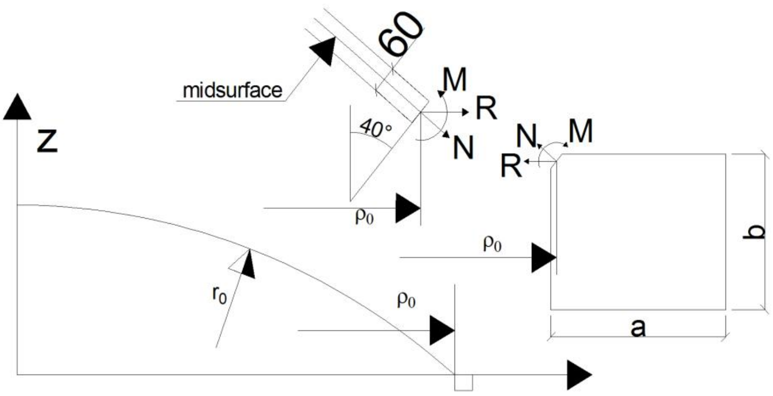

Figure 1: The problem geometry : Cross-section of the rotational shell and foot ring beam.

3 NUMERICAL RESULTS AND CONCLUSIONS

The structural response of the spherical shell is analyzed by studying the following load cases (Fig. 2):

Case 1: Self weight of shell structure and support force N on the edge
Case 2: R=1 N/m
Case 3: M=1 Nm/m
Case 4: Self weight of shell structure and shell is supported on edge with foot ring beam. Foot ring beam is assumed to be weightless.

The support force N in the skew force splitting becomes determined by static equilibrium:

$$N = -\frac{g r_0}{1+\cos\alpha} = -\frac{32{,}69\,{}^{kN}\!/_{m^3} \times 0{,}06m \times 23{,}34m}{1+\cos 40°} = -25{,}92 kN/m \tag{1}$$

where $g=Fh$ and F=32690 N/m^3 is the weight of the concrete and cladding.

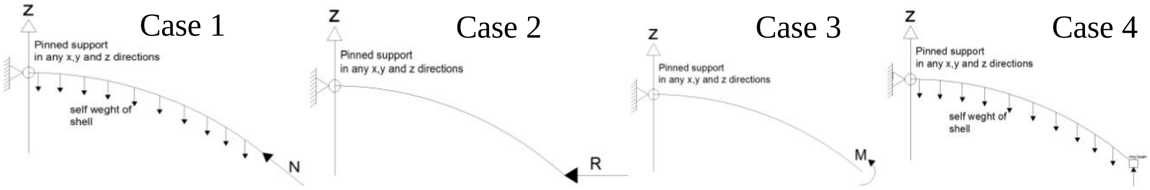

Figure 2: Specification of the load cases.

Each load case is analyzed by using triangular and quadrilateral elements as shown in Fig. 3. The comparison is performed by recording the value of the horizontal displacement and normal rotation at the junction. The results are shown in Table 1 and normalized against the reference values[3] of Table 2.

Good accuracy is achieved in Cases 2 and 3 for both type of elements whereas the accuracy of displacements is poor in Case 1. The problem may be caused by numerical instability of the lowest-order elements in the membrane-dominated case but the phenomenon requires further investigation[3].

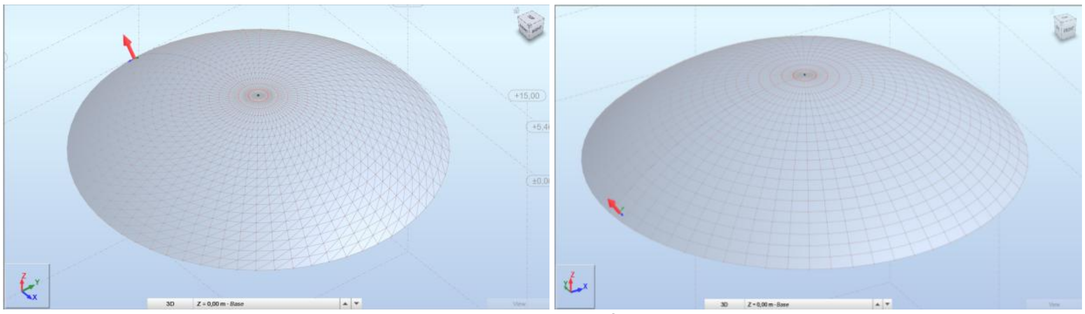

Figure 3: Triangular and quadrilateral finite element meshes.

		Triangular	Quadrilateral
Case 1	E*U	0.76	0.75
	E*ψ	3.23	3.30
Case 2	E*U/R	0.98	1.00
	E*ψ/R	0.96	1.00
Case 3	E*U/M	0.96	1.00
	E*ψ/M	0.98	1.00

Table 1 : Horizontal displacement and normal rotation at the junction in the different cases.

Case 1	E*U	2300000 N/m
	E*ψ	933800 N/m^2
Case 2	E*U/R	8345
	E*ψ/R	-14770 1/m
Case 3	E*U/M	14770 1/m
	E*ψ/M	-51130 1/m^2

Table 2: Reference values for the compliances in SI-units from [2].

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