29th Nordic Seminar on Computational Mechanics
NSCM-29
R. Larsson (Ed.)
2016

GRADIENT BASED POST-BUCKLING OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES USING KOITER'S METHOD

SØREN R. D. HENRICHSEN*, **PAUL M. WEAVER**†, **ESBEN LINDGAARD*** **AND ERIK LUND***

*Department of Mechanical and Manufacturing Engineering
Aalborg University
Fibigerstræde 16, 9220 Aalborg East, Denmark
e-mail: srh,elo,el@m-tech.aau.dk, web page: http://www.m-tech.aau.dk/

†Advanced Composites Centre for Innovation and Science, Dep. Aerospace Engineering
University of Bristol, United Kingdom
University Walk, Bristol, BS8 1TR, United Kingdom
e-mail: Paul.Weaver@bristol.ac.uk - Web page: http://www.bristol.ac.uk

Key words: Asymptotic Post-buckling, Composite Structures, Composite Optimization, Koiter's Method, Post-buckling Optimization.

Summary. The paper presents a gradient based approach to optimization of the post-buckling response of thin-walled laminated composite structures subjected to compressive loads. The post-buckling analysis is based on Koiter's asymptotic method and design sensitivities of the Koiter factors are derived using the direct differentiation method. The proposed optimization formulations are demonstrated on a square laminated composite plate and a curved panel where the post-buckling stability is optimized.

1 INTRODUCTION

One of the main advantages of fiber reinforced laminated composite structures is the possibility of tailoring the structural response by designing the layup. These structures are typically thin-walled structures prone to buckling, when compressed, and they often have a design load close to the buckling load. In these cases the performance of the structure in the post-buckling regime is of interest, and it is the aim of this work to develop gradient based optimization methods for improving the post-buckling performance. The paper presents Koiter's asymptotic analysis method applied together with an outline of the sensitivity analysis and the optimization formulations developed. The approach is documented by fiber angle optimization examples, see further details in Henrichsen *et al.*[1].

2 ANALYSIS OF POST-BUCKLING RESPONSE

Detailed analysis of the post-buckling response is normally computationally expensive and challenging due to the nonlinearities involved. In most cases a path following solution algorithm like the arc length method is used to trace the equilibrium curve. In this work we would like to extract the most important information about the post-buckling response and reduce the complexity of the analysis, and thus Koiter's asymptotic analysis method has been applied, see references[2,3,4]. Such asymptotic methods substitute the full and complex response by a series of simpler problems[5]. Recently it has been demonstrated how Koiter analysis can be combined with the Differential Quadrature Method[6].

The initial post-buckling response after the bifurcation load has been reached is estimated using a two-term expansion in order to extract information about the type of buckling i.e., symmetric/asymmetric, and the stability of the post-buckling response. If the response further in the post-buckling regime is of interest, more terms are needed in the expansion. Assuming that the load and deformation for the distinct critical bifurcation point ^{c} are known, the initial post-buckling response is represented by a Taylor-like expansion, where the expanded load factor λ, displacements $\boldsymbol{u}$, strains $\boldsymbol{\epsilon}$, and stresses $\boldsymbol{\sigma}$ are extrapolated into the post-buckling regime as

$$\lambda = \lambda_c + a\lambda_c\xi + b\lambda_c\xi^2 + c\lambda_c\xi^3 + \dots \tag{1}$$

$$\boldsymbol{u} = {}^0\boldsymbol{u}\lambda + {}^1\boldsymbol{u}\xi + {}^2\boldsymbol{u}\xi^2 + {}^3\boldsymbol{u}\xi^3 + \dots \tag{2}$$

$$\boldsymbol{\epsilon} = {}^0\boldsymbol{\epsilon}\lambda + {}^1\boldsymbol{\epsilon}\xi + {}^2\boldsymbol{\epsilon}\xi^2 + {}^3\boldsymbol{\epsilon}\xi^3 + \dots \tag{3}$$

$$\boldsymbol{\sigma} = {}^0\boldsymbol{\sigma}\lambda + {}^1\boldsymbol{\sigma}\xi + {}^2\boldsymbol{\sigma}\xi^2 + {}^3\boldsymbol{\sigma}\xi^3 + \dots \tag{4}$$

Here a superscript ^{0} defines a pre-buckling quantity, whereas all higher quantities are related to the post-buckling state. λ is the post-buckling load factor normalized with respect to the applied load, λ_c is the critical buckling load factor, a, b, and c are the first three Koiter factors which are non-dimensional, ${}^0\boldsymbol{u}$ is the pre-buckling displacement field, ${}^1\boldsymbol{u}$ through ${}^3\boldsymbol{u}$ are the post-buckling displacement fields, and ξ is the perturbation variable. ${}^{0\dots3}\boldsymbol{\epsilon}$ and ${}^{0\dots3}\boldsymbol{\sigma}$ are the expanded strains and stresses with ^{0} being the pre-buckling quantities. The expansions for λ, $\boldsymbol{u}$, $\boldsymbol{\epsilon}$, and $\boldsymbol{\sigma}$ are assumed to be valid asymptotically as $\xi \to 0$. A general finite element implementation based on 9-node equivalent single layer isoparametric shell elements using first order shear deformation theory is applied.

3 SENSITIVITY ANALYSIS AND OPTIMIZATION FORMULATIONS

The objective is to maximize the post-buckling stability of laminated composite structures by gradient-based optimization techniques. Thus, the design sensitivities of the Koiter factors and the critical buckling load factor are needed, and these are obtained using the direct differentiation approach[1].

A number of different optimization formulations have been implemented and investigated in order to handle both asymmetric post-buckling and symmetric post-buckling responses. The Koiter factors are used to optimize the initial post-buckling response of the structures considered, and in order to have a benchmark on the performance of the considered structures, maximization of the critical buckling load factor is also performed.

When the bifurcation is symmetric the Koiter a-factor is zero and the initial post-buckling response is governed by the Koiter b-factor. To maximize the post-buckling stability maximization of the Koiter b-factor is applied in such cases, while the absolute value of a is limited by a small number. This formulation only considers the post-buckling curvature factor i.e., the Koiter b-factor, in the objective as this factor defines the post-buckling stability. An alternative approach is to maximize the $b\lambda_c$ product, as the product is present in the expansion of the load factor.

When an asymmetric post-buckling response is present the Koiter a-factor is non-zero. Consequently the initial post-buckling response is unstable. In order to minimize the asymmetry the a-factor should be as close to zero as possible. In such cases the absolute value of the a-factor is minimized while a lower bound on b is prescribed.

4 EXAMPLES

The gradient based optimization formulations are demonstrated on two different examples i.e., a simply supported square plate and a curved panel, see Figure 1 and details in[1]. The parameterization is based on elementwise fiber angles but other laminate parameterizations are implemented within the presented framework. Only fiber angle results for the plate example are included here.

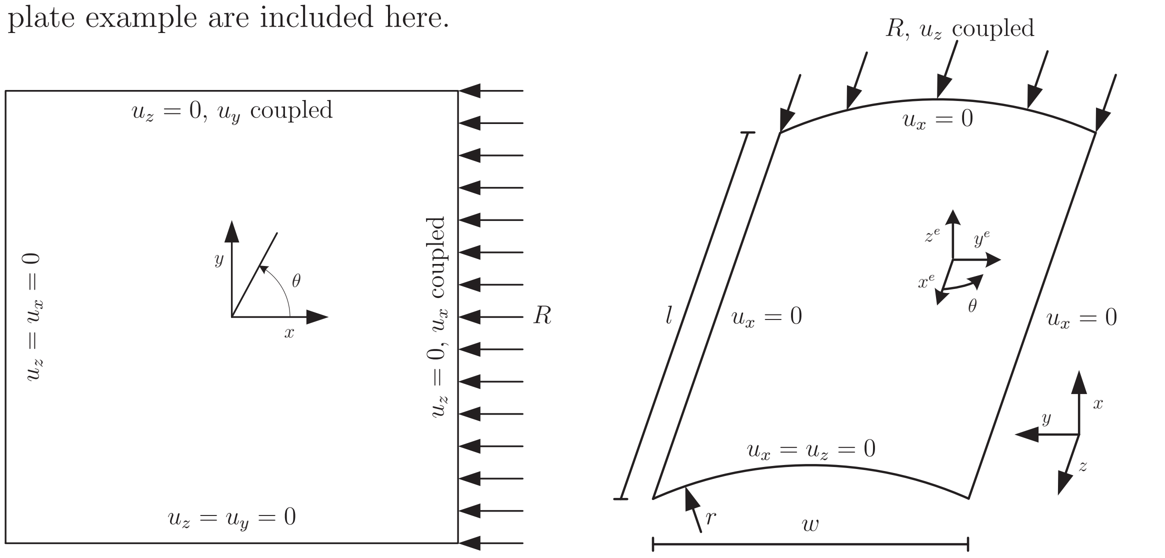

Figure 1: A simply supported single-layered square plate and a curved eight-layered panel subjected to compressive loads

The optimum fiber angles for the buckling load and post-buckling stability are shown on Figure 2. It is immediately seen that the Max λ_c case is doubly symmetric, whereas the two post-buckling optimized structures only have one symmetry.

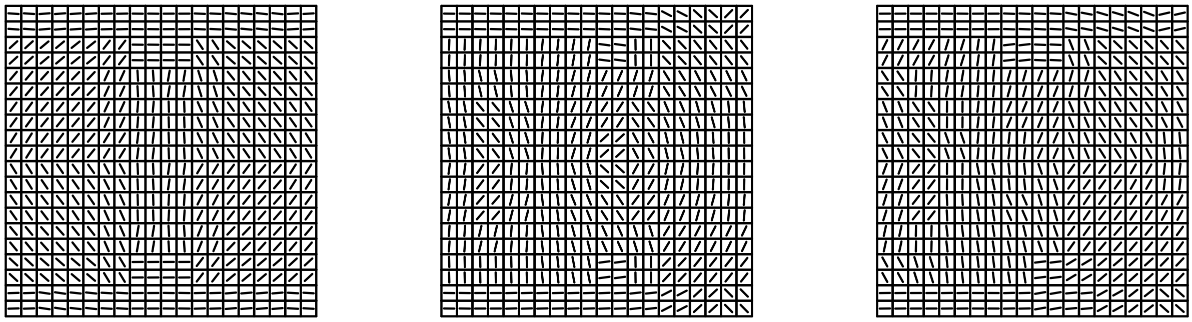

Figure 2: Fiber angle optimization results for simply supported single-layered square plate

5 CONCLUSIONS

The proposed post-buckling optimizations successfully optimize the post-buckling stability even in cases where the pre-buckling response is nonlinear, resulting in structures with increased post-buckling stability. The post-buckling optimization method operates directly on the physical phenomenon related to post-buckling stability. Consequently, the proposed method enables the possibility of obtaining a better performance of the laminated composite structure in the post-buckling regime.

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