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SORM-BASED RBDO BY USING RBFN WITH A PRIORI BIAS

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Summary. The main contributions of this work concern reliability based design optimization (RBDO) and radial basis function networks (RBFN). A new approach for RBFN is suggested. Instead of defining the bias posteriori by adding extra orthogonality constraints, we suggest to simply add the bias a priori as a regression model of the sampling set. In such manner, the global behavior is captured with the bias and the local behavior is tuned in by the radial basis functions. A sequential linear programming (SLP) approach for RBDO, using first order (FORM) as well as second order reliability methods (SORM) for estimating probability of failure, is also derived and implemented. The derivation is performed by introducing intermediate variables defined by the iso-probabilistic transformation. Furthermore, the Taylor expansions of the reliability constraints are done at the most probable point (MPP) according to the Hasofer-Lind approach. The MPP is found by a Newton method using an in-exact Jacobian for variables with normal, lognormal, Gumbel, gamma and Weibull distributions. In such manner, a linear programming problem is established which is solved in sequence until convergence. The SORM-based approach is then obtained by correcting the target of reliability using three different formulas: Breitung, Hohenbichler and Tvedt. A number of established benchmarks are solved using the proposed approaches for several variables, a large number of constraints and high values on the targets of reliability. For instance, a most well-known problem of two variables and 3 constraints are generalized to 50 variables, with the five different distributions treated simultaneously, and 75 constraints. The benchmarks are also solved by performing design of experiments (DoE) and setting up corresponding RBFNs. Different strategies for DoE are also studied, e.g. Halton, Hammersley, S-optimal and successive screening sampling. In addition, we set up RBDO of a barrier by performing nonlinear explicit finite element analysis and adopting our SORM-based approach and the new RBFN with a priori bias. The results demonstrate that the implemented methodology performs most accurately and efficient.

1 RBDO, RBFN and DoE

Let us consider a RBDO problem for one objective $f = f(\boldsymbol{X})$ and a constraint $g = g(\boldsymbol{X})$, where $\boldsymbol{X}$ is considered to be a vector of N_{VAR} uncorrelated random variables with mean values μ_i which are collected in $\boldsymbol{\mu}$. The distribution of each variable is defined by a probability density function $\rho_i = \rho_i(x;\boldsymbol{\theta}_i)$, where $\boldsymbol{\theta}_i = \boldsymbol{\theta}_i(\mu_i)$ represents distribution parameters that depend on the mean value. The corresponding cumulative distribution function is defined by

$$F_i(x;\boldsymbol{\theta}_i) = \int_{-\infty}^{x} \rho_i \,\mathrm{d}x. \tag{1}$$

Our RBDO problem reads

$$\begin{cases} \min\limits_{\boldsymbol{\mu}} & \mathrm{E}[f(\boldsymbol{X})] \\ \text{s.t.} & \Pr[g(\boldsymbol{X}) \leq 0] \geq P_s, \end{cases} \tag{2}$$

where $\mathrm{E}[\cdot]$ designates the expected value of the function f, $\Pr[\cdot]$ is the probability of the constraint $g \leq 0$ being true and P_s is the target of reliability that must be satisfied.

We assume that the objective and the constraint are given as RBFNs. A radial basis function network of ingoing variables x_i, collected in $\boldsymbol{x}$, can be written as

$$f(\boldsymbol{x}) = \sum_{i=1}^{N_\Phi} \Phi_i(\boldsymbol{x})\alpha_i + b, \tag{3}$$

where $f = f(\boldsymbol{x})$ is the outgoing response of the network, $\Phi_i = \Phi_i(\boldsymbol{x})$ represents the radial basis functions, N_Φ is the number of radial basis functions, α_i are weights and b is a bias. Examples of popular radial basis functions are

$$\Phi_i(r) = \exp(-\theta_i r^2), \tag{4a}$$

$$\Phi_i(r) = \begin{cases} r^k & \text{if } k > 0 \text{ is odd} \\ r^k \log(r) & \text{if } k > 0 \text{ is even}, \end{cases} \tag{4b}$$

$$\Phi_i(r) = \sqrt{1 + \theta_i r^2}, \tag{4c}$$

$$\Phi_i(r) = \frac{1}{\sqrt{1 + \theta_i r^2}}, \tag{4d}$$

where θ_i represents given parameters and

$$r = \sqrt{(\boldsymbol{x} - \boldsymbol{c}_i)^T(\boldsymbol{x} - \boldsymbol{c}_i)} \tag{5}$$

is the radial distance. In the latter expression, $\boldsymbol{c}_i$ is the center point for each radial basis function.

The RBFN are fitted to DoEs established by Halton, Hammersley, S-optimal and/or successive screening sampling. The Halton sequence and the Hammersley sequence are two examples of sparse uniform samplings generated by quasi-random sequences. Let p_1, p_2,...,p_D represent a sequence of prime numbers, where D is the dimension of a Halton point defined by

$$\boldsymbol{x}_{\text{Hal}} = \boldsymbol{x}_{\text{Hal}}(k) = \{\Phi(k,p_1), \Phi(k,p_2), \ldots, \Phi(k,p_D)\} \tag{6}$$

for a non-negative integer k. Furthermore, for any prime number p,

$$\Phi(k,p) = \frac{a_0}{p} + \frac{a_1}{p^2} + \ldots + \frac{a_M}{p^{M+1}}, \tag{7}$$

where the integers a_0, a_1,..., a_M are obtained from the fact that k can be represented as

$$k = a_0 + a_1 p + a p^2 + \ldots + a_M p^M. \tag{8}$$

A quasi-random set of N Halton points is now simply obtained by taking a sequence of Halton points in (6) for $k = 0$, 1, 2,..., $N-1$. By defining the Hammersley point as

$$\boldsymbol{x}_{\text{Ham}} = \boldsymbol{x}_{\text{Ham}}(k) = \left\{\frac{k}{N}, \Phi(k,p_1), \Phi(k,p_2), \ldots, \Phi(k,p_{D-1})\right\}, \tag{9}$$

we can easily generate a set of Hammersley sampling points in a similar way as for the Halton set.

2 A NUMERICAL BENCHMARK

Let us solve a most established RBDO benchmark with the algorithm implemented in this work. The RBDO benchmark reads

$$\begin{cases} \min\limits_{\mu_i} & \mathrm{E}[X_1 + X_2] \\ \text{s.t.} & \begin{cases} \Pr[20 - X_1^2 X_2 \leq 0] \geq P_s \\ \Pr[1 - \dfrac{(X_1 + X_2 - 5)^2}{30} - \dfrac{(X_1 - X_2 - 12)^2}{120} \leq 0] \geq P_s \\ \Pr[X_1^2 + 8X_2 - 75 \leq 0] \geq P_s, \end{cases} \end{cases} \tag{10}$$

where $\mathrm{VAR}[X_i] = 0.3^2$. Typically, P_s is chosen to be $\Phi(3)$ ≈0.9987 and the distribution is the normal one. Here, we solve (10) for different values on the target of reliability $P_s = \{0.99, 0.999, 0.9999, 0.99999\}$ and five different distributions, i.e. normal, lognormal, Gumbel, gamma and Weibull. The corresponding target reliability indices are $\beta_t = \{2.33, 3.09, 3.72, 4.26\}$. The two latter targets are considerable higher than what is typically reported. The algorithm performs also well for even higher targets such as e.g. $P_s = 99.9999\%$. The solutions are plotted in Figure 1 together with MCS-based scatter plots. The problem in (10) has also been expanded to 50 variables and 75 constraints and solved succesfully for different distributions simultaneously.

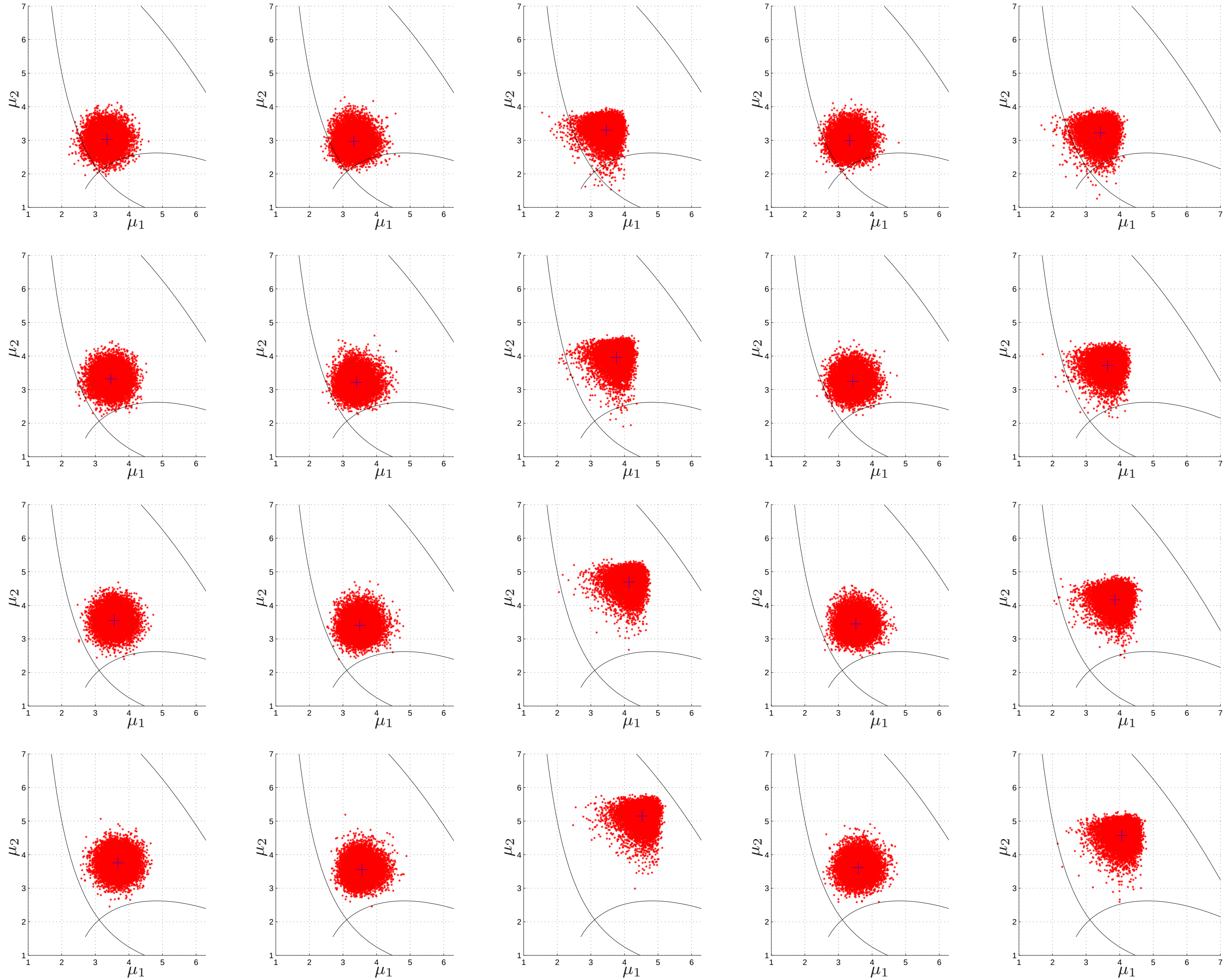

Figure 1: *Solutions and scatter plots for the analytical example. Column 1: normal, column 2: lognormal, column 3: Gumbel, column 4: gamma and column 5: Weibull. Row 1:* $P_s = 0.99$, *…, row 4:* $P_s = 0.9999$.