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REACHING THE RATE-INDEPENDENT LIMIT IN VISCO-PLASTICITY WITHOUT GOING THERE: A NUMERICAL "TRICK" AND AN EXPERIMENTAL CHALLENGE

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Summary. A numerical method to attain rate-independent results from a visco-plastic formulation, but without pushing the numerics toward this cumbersome limit, is presented. The trick is to exploit the characteristic time introduced through the constitutive material model to constitute a characteristic velocity at which the material response becomes rate-independent. The method apply to multiple problems of practical engineering interest and is, here, demonstrated for cell model calculations.

1 INTRODUCTION

Visco-plastic constitutive models are used widely in the analysis of engineering structures, and numerous researchers frequently employ such frameworks to develop new material models and numerical procedures. Visco-plastic models are particularly interesting when corresponding rate-independent frameworks are too cumbersome to treat - or simply not exist. Thus, focus is often not aimed at the material rate-sensitivity, but rather is the rate-independent results approximated by pushing the models toward this limit. Approaching the limit, however, comes with the prize of strong non-linearity and consequently unstable numerics. The present study compiles recent year work where a new approach to reaching rate-independent results has emerged. The trick, here, is that the rate-independent results are brought out without ever pushing the model toward the strong non-linearity. The new approach boils down to exploiting the characteristic time introduced through the constitutive material model which, in combination with the characteristic length scale of the problem at hand, defines a characteristic velocity. The neat thing is that specific macroscopic quantities (that obey certain monotonic behavior with the rate-sensitivity) will remain constant, regardless of the chosen rate-sensitivity, when a deformation rate is imposed according to the characteristic velocity. That is, the rate-independent results for this specific macroscopic quantity are achieved for a numerically stable setting where rate-sensitivity prevails. This despite that the microscopic field may not be the same for the two solutions. The numerical procedure is readily laid out for a homogeneous stress/strain field, but what is intriguing is the fact that it also holds for highly none-homogeneous fields and much complicated deformations. For example, the characteristic velocity has been identified and exploited for problems such as crack propagation (both isotropic[1] and single crystals[2] at all scales), sheet rolling[3], wire drawing[4], and voided unit cell calculations (see Section 2) - both in conventional finite element frameworks and specialized steady-state models. The exploitation of the characteristic velocity remains, however, for now to be a numerical "trick" and experiments that can support its existence remain to the challenge.

2 PROBLEM FORMULATION AND MODEL DESCRIPTION

In the following, the characteristic velocity is demonstrated for a voided cell model (see Fig. 1a), where the matrix obeys a visco-plastic material law. Here, taking as off-set a full 3D framework based on the Fleck and Willis[5] visco-plastic strain gradient theory and a small strain/deflection assumption. The adopted visco-plastic power-law reads;

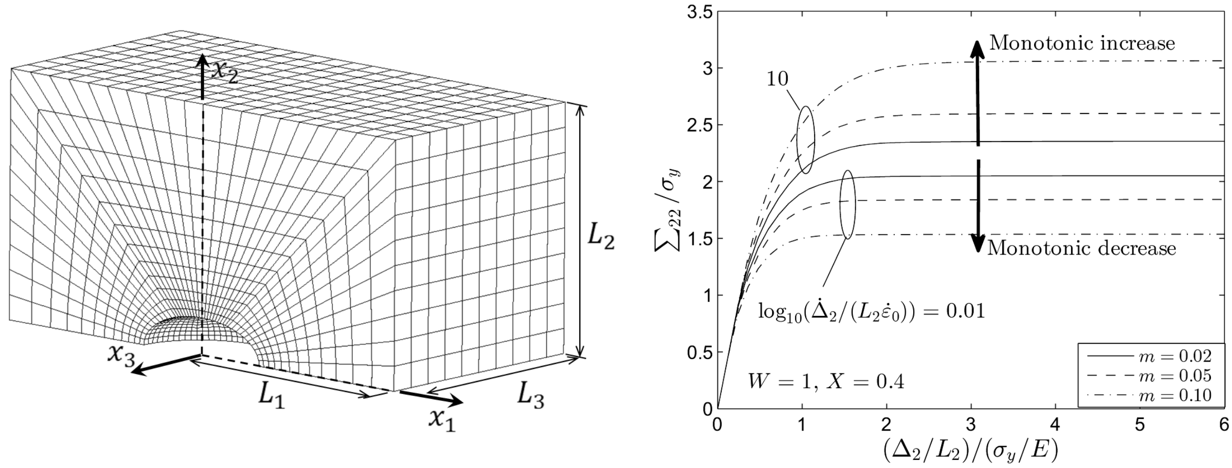
$$\dot{\varepsilon}_e^p = \dot{\varepsilon}_0 \left(\frac{\sigma_C}{g(E^p)} \right)^{1/m} \tag{1}$$

where σ_C and E^p are the gradient enriched effective stress and strain quantities, respectively, m is the strain rate-sensitivity, and $\dot{\varepsilon}_0$ is the reference strain rate. The latter is of key importance to the following approach to the rate-independent material response.

The unit cell consists of a rectangular block of matrix material that surrounds a spheroidal void with aspect ratio, $W = R_2/R_1$ (and $R_1 = R_3$), and a relative void spacing $\chi = R_1/L_1$. Here, R_i and L_i are the void radii in the three directions and the dimensions of the unit cell, respectively. The boundary and loading conditions on the cell are adopted from Tekoğlu et al.[6] such that the average volume stress components; $\sum_{11}$, $\sum_{22}$, and $\sum_{33}$ can be prescribed by the macroscopic straining; $E_{ii} = U_i/(2L_i)$, where U_i are the normal displacements at the unit cell boundaries. Shearing of the unit cell is omitted in the present study. All FE calculations are preformed with 20-node 3D elements using reduced Gauss integration ($2 \times 2 \times 2$) for the displacement field, whereas 8-node 3D elements with corresponding Gauss integration are used for the plastic strain rate field.

3 RESULTS AND DISCUSSION

By omitting strain hardening, the load-deflection curves develop as depicted in Fig. 1b. Results are, here, shown for the case of a spherical void ($W = 1$) and a relative void spacing of $\chi = 0.4$, for three different rate-sensitivities ($m = [0.01, 0.05, 0.1]$). By loading the unit cell at two distinct deformation rates ($\log_{10}(\dot{\Delta}_2/L_2\dot{\varepsilon}_0) = [0.1, 10]$), it is found that the average stress level increases monotonically with increasing rate-sensitivity at low deformation rate and vice versa at high rates. This has to do with stress build-up/relaxation,



(a) (b)

Figure 1: a) Unit cell model and a typical mesh ($W = 0.5$), b) Average volume stress vs. deformation.

and a similar monotonic behavior with the rate-sensitivity can be observed of a wide range of problems that exploit the visco-plastic relation in Eq. (1). As a consequence of the monotonic behavior one can identify a characteristic velocity at which the response becomes independent of the rate-sensitivity, and then directly extract the striven results by imposing this rate along with any rate-sensitivity. Thus, without pushing the numerics to the unstable limit of $m \to 0$. For example, take the stress plateau reached in the individual load curves from Fig. 1b, and display the stress as function of the deformation rate (see Fig. 2). Clearly, by keeping all other parameters constant the curves intersect in one point which uniquely defines the characteristic velocity for this parameter set.

In Fig. 2, the stress plateau obtained at the characteristic deformation rate is directly comparable to the coalescence stress predicted by the so-called Thomason criterion (see Tekoğlu et al.[6]) shown as the thick solid (blue) curve. It is, here, worth noticing that the utilized Thomason criterion is based on a J2-flow (rate-independent) theory, and yet it matches almost perfectly to the value obtained at the characteristic deformation rate - regardless of the rate-sensitivity, m.

4 CONCLUDING REMARKS

Despite the lack of experimental evidence for the characteristic velocity, it facilitates a new approach to attain rate-independent responses of metallic materials characterized by the visco-plastic relation in Eq. (1). This procedure can be summarized in three steps: (i) ensure monotonic behavior with rate-sensitivity, (ii) perform sets of calculations for two rate-sensitivities, m, and plot the quantity of interest vs. deformation rate, (iii) identify the intersection point of the two curves which then reflects the rate-independent response.

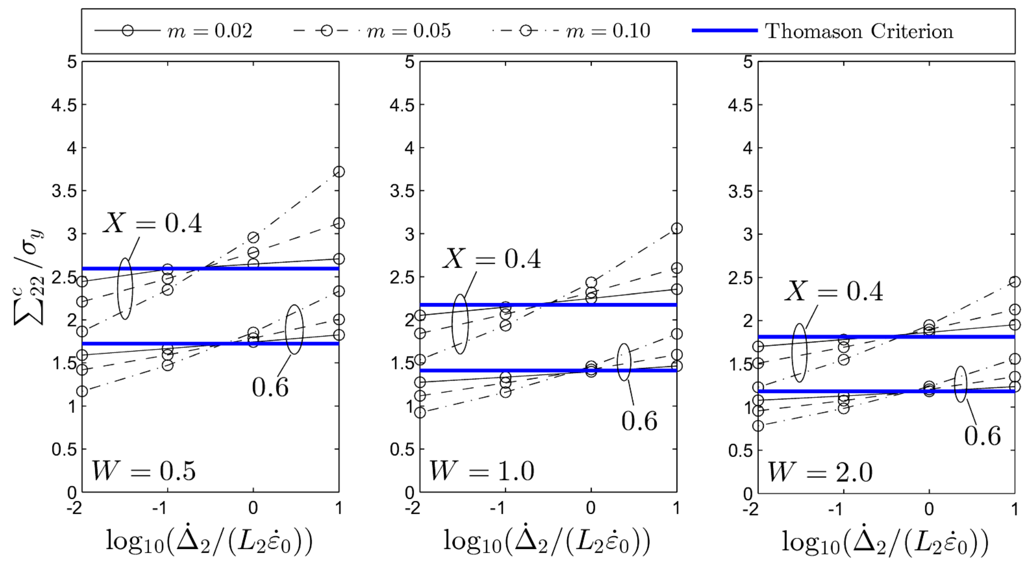


Figure 2: Average stress plateau vs. deformation rate with comparison to the (rate-independent) Thomason criterion marked by the thick solid (blue) line.

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