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BENCHMARK COMPUTATIONS WITH HIGH-ORDER SHELL FINITE ELEMENTS

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Summary. We compare the relative accuracy and efficiency of high-order shell formulations and state of the art linear triangular shell elements in challenging benchmark tests featuring different shell deformation types.

1 INTRODUCTION

In the finite element modelling of shell structures parametric error growth, or locking, is detected for various shell deformation types. The numerical phenomenon is especially harmful in the standard lowest-order ($p = 1$) finite element approximation and significant mesh over-refinement is sometimes needed to compensate for the effect. However, the level of error amplification does not depend on the degree of the approximating polynomials used in the FE approximation and considerably milder mesh over-refinement is needed at higher values of p.

Another long standing approach to modelling of thin structures is the derivation of special low-order (linear/bilinear) formulations that would avoid the parametric error growth once and for all. For shells, the ultimate dream element is yet to be found but there exist reduced-strain formulations that work very well on rectangular meshes, at least[1].

2 MODEL SPECIFICATIONS

We base our calculations to Reissner-Mindlin type shell models formulated using curvilinear coordinates on the shell middle surface. Consequently, the shell displacement field is then assumed to be of the form

$$\vec{U}(x,y) = (u_\lambda(x,y) + \zeta\theta_\lambda(x,y))\vec{e}^{\lambda}(x,y) + w(x,y)\vec{n}(x,y), \tag{1}$$

where $\boldsymbol{u} = (u_1, u_2)$ are the tangential displacements, w is the transverse deflection and $\theta = (\theta_1, \theta_2)$ are the angles of rotation of the middle surface normal vector $\vec{n}(x, y)$.

Two model variants are considered. For the p-version, we use a geometrically exact model based on a global chart $(x, y) \mapsto \boldsymbol{r}(x, y)$ whereas for the h-version we use a collection of discrete charts $(x, y) \mapsto \boldsymbol{r}_K(x, y)$ corresponding to each element of the mesh, see Figure 1. It should be mentioned that the discrete charts are never explicitly constructed but referred to only implicitly in order to calculate geometric curvatures over each element[1].

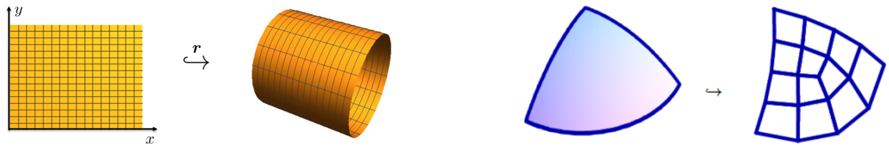

Figure 1: Coordinate charts used for the p-version and the h-version, respectively.

3 NUMERICAL RESULTS AND CONCLUSIONS

As an example we show some numerical results obtained for the classical benchmark problems involving a closed circular cylindrical shell and a closed doubly curved hyperboloid. The shells are loaded by a self-equilibrating external pressure which is assumed axially constant but varies sinusoidally in the angular direction, see[2,3]. When the ends of the shells are left free from kinematic constraints, pure bending occurs and this is the most severe case concerning the problem of membrane locking.

Figures 2 and 3 compare the accuracy of *reduced-strain triangular shell elements* similar to the quadrilateral elements introduced in[1] to geometrically exact p-method. In these cases, the accuracy of the specialized linear elements is comparable to the accuracy of the quadratic elements based on standard variational principles. In fact, the linear method appears to be locking-free when applied to the cylindrical shell. However, for general geometries, sufficiently high polynomial order ($p \geq 4$) is required to achieve accurate results for small values of the thickness.

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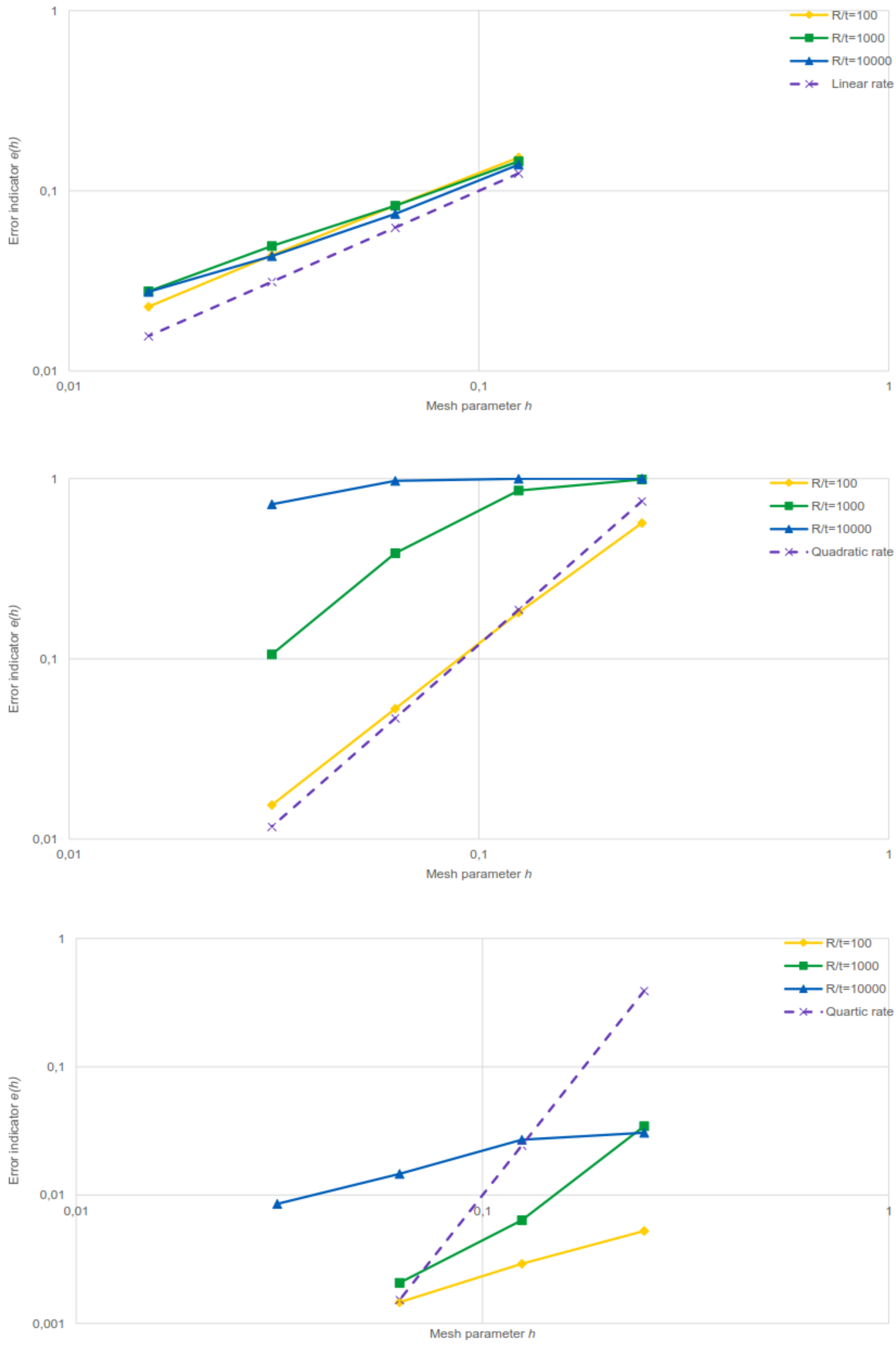

Figure 2: Cylinder with free ends: MITC3S (top) vs. Standard Quadratic (middle) vs. Standard Quartic (bottom) formulations. The error indicator is the relative error in the strain energy and it is computed for different values of the radius to thickness ratio R/t and mesh size h.

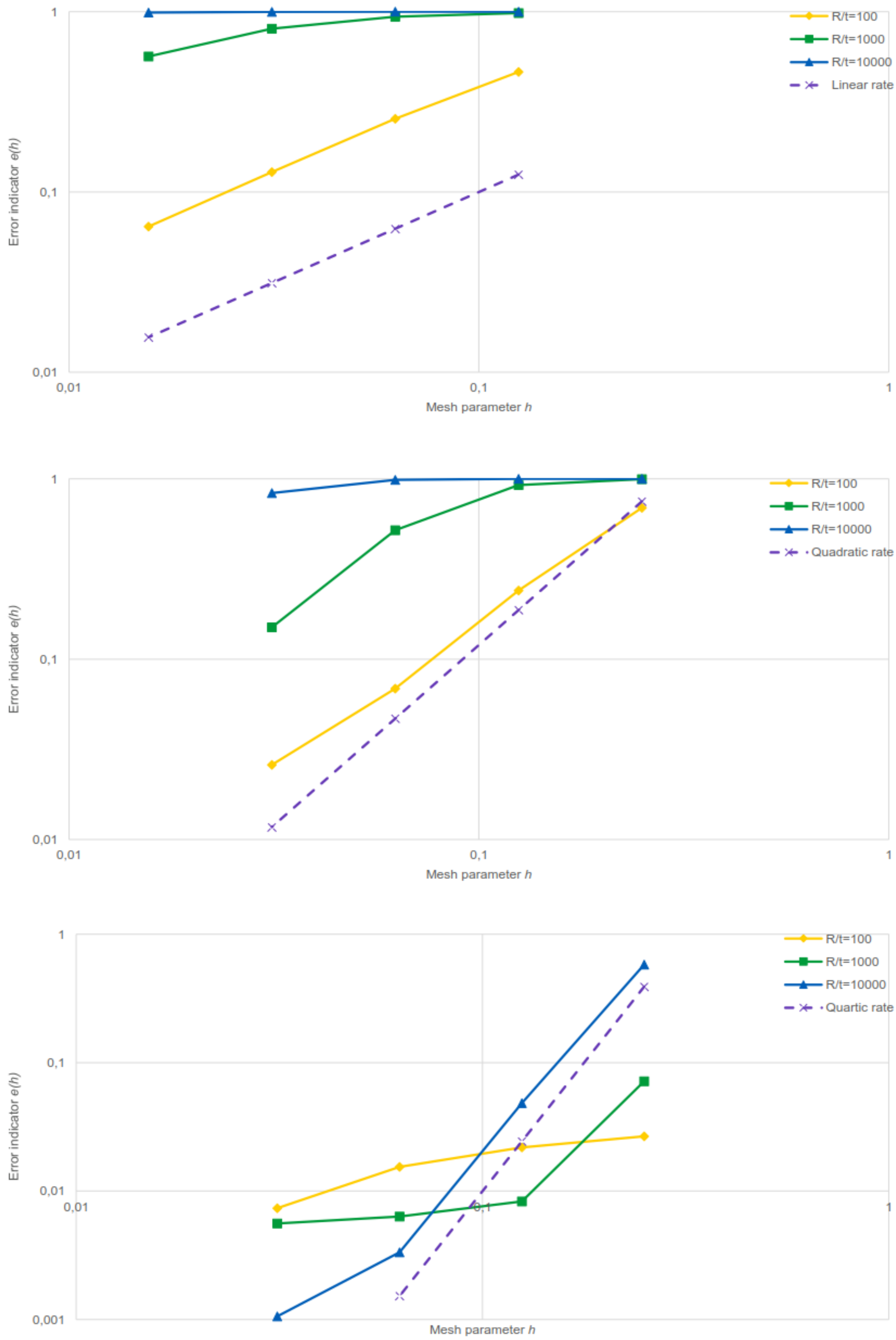

Figure 3: Hyperboloid with free ends: Stabilized MITC3S (top) vs. Standard Quadratic (middle) vs. Standard Quartic (bottom) formulations. The error indicator is the relative error in the strain energy and it is computed for different values of the radius to thickness ratio R/t and mesh size h.