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STEADY-STATE CRACK GROWTH IN RATE-SENSITIVE SINGLE CRYSTALS

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Summary. The characteristics of the active plastic zone surrounding a crack growing in a single crystal (FCC, BCC, and HCP) at constant velocity is investigated for Mode I loading under plane strain assumptions. The framework builds upon a steady-state relation bringing the desired solution out in a frame translating with the crack tip. In the study, the shielding of the crack tip that follows from plastic slip is investigated by adopting the SSV-model. High resolution plots of the plastic zones are obtained and a detailed study confirms the existence of analytically determined velocity discontinuities from the literature. The plastic zone is found to be smallest for the FCC structure and largest for the HCP structure, which is also reflected in the shielding ratio, where FCC crystals show the smallest shielding and HCP the largest shielding.

1 Introduction

The active plastic zone in the vicinity of a crack tip has direct influence on the fracture toughness of a material. The plastic zone essentially acts as a shield against the elastic far field by dissipating energy through plastic slip. The objective of the study is to investigate quasi-static crack growth in rate-sensitive single crystals (FCC, BCC, HCP) under Mode I loading. The investigation is divided into two parts. In the first part, the characteristics of the plastic zone surrounding the crack tip are investigated. The goal is to validate the numerics by comparing to the work of Rice[1] and Rice et. al.[2], but also to bring new information regarding the plastic zones as the true steady-state solutions are obtained within the presented framework. In the second part of the analysis, the macroscopic crack tip shielding under the assumption of cleavage cracking is investigated. The analysis of shielding ratios relies on the SSV-model (Suo et. al.[3]), which facilitates a fracture criterion based on the J-integral.

2 Modelling Framework

The study is conducted under small scale yielding and treated as quasi-static. A Mode I far field loading is applied on the outer boundary (illustrated in Fig. 1) according to the modified boundary layer formulation (scaled by the stress intensity factor, K_I). By introducing a reference plastic zone size parameter, R_0, depending on a reference stress intensity factor, K_0, as $R_0 = (K_0/\tau_0)^2$ with $K_0 = \sqrt{(E\Gamma_{tip})/(1-\nu^2)}$ the energy release rate at the crack tip, Γ_{tip}, can be used in a local linear elastic fracture criterion facilitated by modelling the SSV domain described by Suo et. al.[3].

Since the material model is based on an elastic visco-plasticity theory, the active plastic zone that engulfs the crack tip must be specified. The quantity utilized is based on the absolute value of the accumulated slip rate, $\dot{\Lambda} = \sum_\alpha |\dot{\gamma}^{(\alpha)}|$.

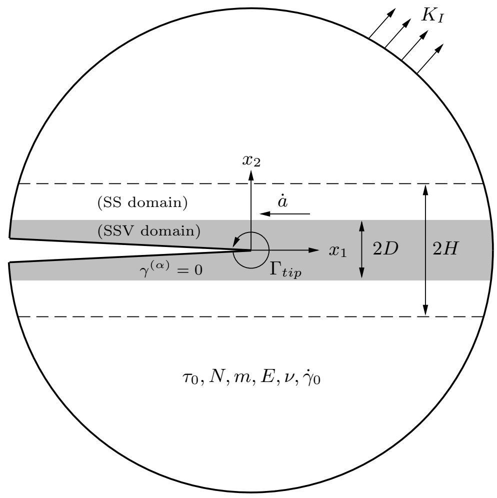


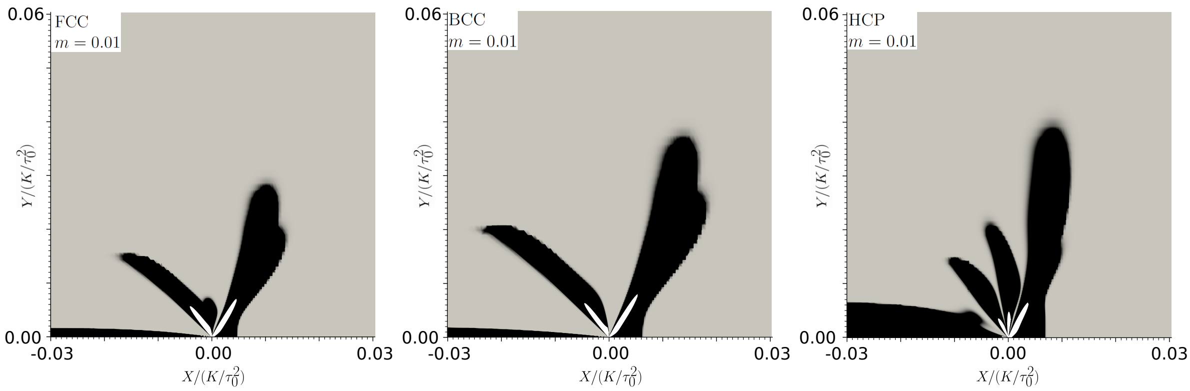
Figure 1: Mode I crack growth at steady-state in crystal plastic material. The SSV domain provide an elastic strip embedded in the steady-state domain (SS domain).

The crack growth problem is analysed in the 2D plane strain steady-state framework suggested by Dean and Hutchinson[4], where the crack propagates at a constant velocity, $\dot{a}$. The procedure iterates directly on the stationary condition where the stress and strain field are constant to an observer that is fixed relative to the crack tip. This method is based on the steady-state condition, where a time derived quantity $\dot{f}$ in the constitutive model can be related to a spatial derivative through the velocity $\dot{a}$ ($\dot{f} = -\dot{a}\partial f/\partial x$), followed by numerical integration to determine the total quantities. The constitutive model employed is the visco-plastic power law slip rate relation, $\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \mathrm{sgn}\left(\tau^{(\alpha)}\right)\left(|\tau^{(\alpha)}|/g^{(\alpha)}\right)^{1/m}$, by Hutchinson[5].

In order to analyse the single crystal material which has a 3D crystallographic structure, in a 2D plane strain setting, it is necessary to invoke effective slip systems by combining the out-of-plane slip systems to equivalent in plane slip systems (only for the FCC and BCC). These effective systems are imposed according to the work of Rice[1].

3 Results

Comparing the active plastic zones in Fig. 2 to the analytical predictions by Rice[1] reveals a striking match in terms of the velocity discontinuities. These velocity discontinuities are presented as the white regions of concentrated plastic straining and should be located at the angles 54.7° and 125.3°. The active plastic zones also have the same features as in the numerical study of Rice et. al.[2], with even more details due to the improved numerical procedure. The development of the plastic zones also indicate that



(a) (b) (c)

Figure 2: Accumulated slip rate, $\dot{\Lambda}$, for steady-state crack growth in perfectly plastic single crystal showing the plastic region, $\dot{\Lambda}G/(\zeta\dot{\gamma}_0\tau_0) \geq 1$, (black region) and a region of highly concentrated plastic straining (discontinuities), $\dot{\Lambda}G/(\zeta\dot{\gamma}_0\tau_0) \geq 2000$, (white region) for (a) an FCC, (b) a BCC, and (c) an HCP crystal structure.

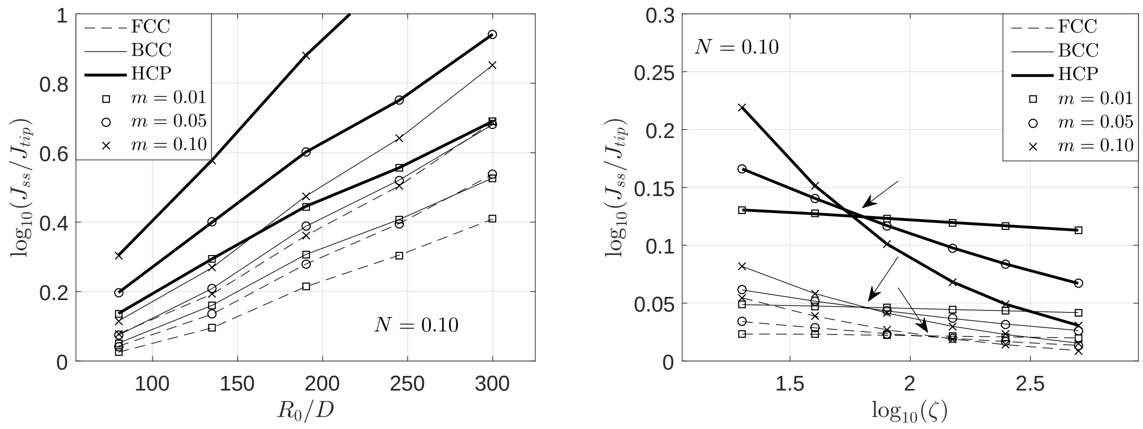
the solution has reached the actual steady-state, compared to Rice et. al.[2] which uses an incremental framework where the steady-state solution is only obtained approximately by going through a transient crack growth phase. Upon inspection of the plastic zones, it is seen that the plastic zone is smallest for the FCC and largest for the HCP structure. The plastic zones for the FCC and BCC structures are similar in shape whereas the plastic zone for the HCP crystal differs significantly.

Figure 3a presents the shielding ratio as a function of the height of the SSV-region for a quasi-statically growing crack under Mode I loading. The results are presented for both the FCC, BCC, and HCP crystal structures. Common for all crystal structures, is the increase in crack tip shielding as the SSV region becomes thinner (R_0/D increases). It is observed that the largest shielding effect is obtained for HCP consistent with the study of the plastic zone where it is seen that this structure indeed has the largest extent of the plastic zone whereas FCC has the smallest shielding ratio).

By investigating the effect of the crack velocity at different rate sensitivities, m, (see Fig. 3b) it was found that a “characteristic velocity” exists. At this velocity, the rate independent response can be determined with the rate dependent model. Thus, beneficial parameters, in terms of numerical convenience, can be chosen when studying the rate independent response with a rate dependent model (e.g. a higher rate sensitivity, m).

4 Conclusions

In accordance with Rice[1], distinct sectors that divide the domain near the crack tip have been identified for the three basic crystal structures in metals (FCC, BCC, and HCP). The size and shape of the plastic zone significantly affect the macroscopic fracture toughness of the material as investigated by applying the SSV model. The magnitude of the active plastic zone is smallest for the FCC crystal and largest for the HCP crystal. The



(a) (b)

Figure 3: Crack tip shielding ratio as a function of the (a) inverse of dislocation free region (SSV), D, ($\zeta = 10$) and (b) crack propagation velocity, ζ.

shielding ratio is smallest for the FCC crystal and largest for the HCP crystal, consistent with the magnitude of the plastic zones for the different crystal structures. The study of rate sensitivity leads to the finding of a characteristic velocity at which the shielding ratio becomes independent of the rate-sensitivity, thus allowing for studying the rate independent response with a rate dependent model.

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