

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Methods for Direct Optimal Control of Multibody Systems

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Abstract

In this thesis we present practical tools and techniques to numerically solve optimal control problems involving multibody systems. The driving application for the thesis has been developing motion planning tools for industrial robots and digital humans. For solution of constrained optimal control problems in real world applications, direct methods are by far the most widespread, and successfully used techniques. In direct methods, the optimal control problem is transcribed into a nonlinear program, which can be solved using standard nonlinear programming algorithms. Evaluation of the equations of motion, and derivatives thereof, are the most expensive operations in using direct methods for optimal control of multibody systems. We propose two methods to reduce this cost when using variational integrators to discretize the dynamics. First, we present efficient algorithms for computation of the residual of the constrained discrete Euler-Lagrange equations of motion for tree structured, rigid multibody systems. In particular, we present new recursive formulas for computing sensitivities of the kinetic energy. This enables us to solve the inverse dynamics problem of the discrete system with linear computational complexity. The resulting algorithms are easy to implement, and can naturally be applied to a very broad class of multibody systems by imposing constraints on the coordinates by means of Lagrange multipliers. Second, we show how to speed up the derivative computations by exploiting separability of the discrete equations of motion. Exploiting the structure of the dynamics is particularly important when applying the direct optimal control methods to digital humans, where the state space is substantially larger than for an industrial robot. Another problem addressed in the thesis is incorporation of collision avoidance in direct optimal control methods. While conceptually straightforward, in general, collision avoidance results in a large number of constraints, or non-smooth constraint functions. To avoid these issues, we propose a new scheme based on iterative refinement of a collision free trajectory, and approximation of obstacles in configuration space.

Keywords: Optimal control, multibody dynamics, discrete mechanics, robotics, motion planning, digital human modeling.

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Staffan Björkenstam
Göteborg, October 2016

List of publications

This thesis is based on the following appended papers:

- Paper 1.** Staffan Björkenstam, Daniel Gleeson, Robert Bohlin, Johan S. Carlson, and Bengt Lennartson. *Energy efficient and collision free motion of industrial robots using optimal control*. Proceedings of 9th IEEE International Conference on Automation Science and Engineering, pp. 510-515, Madison, USA, 2013.
- Paper 2.** Staffan Björkenstam, Johan S. Carlson, and Bengt Lennartson. *Exploiting sparsity in the discrete mechanics and optimal control method with application to human motion planning*. Proceedings of 11th IEEE International Conference on Automation Science and Engineering, pp. 769-774, Göteborg, Sweden, 2015.
- Paper 3.** Staffan Björkenstam, Sigrid Leyendecker, Johan S. Carlson, and Bengt Lennartson. *Inverse dynamics for discrete mechanics of multibody systems with application to direct optimal control*. Submitted to ASME Journal of Computational and Nonlinear Dynamics (2016).

Other relevant publications co-authored by Staffan Björkenstam:

- Staffan Björkenstam, Domenico Spensieri, Johan S Carlson, Robert Bohlin, Daniel Gleeson. *Efficient sequencing of industrial robots through optimal control*. Proceedings of 5th CIRP Conference on Assembly Technologies and Systems, pp 194-199, 2014.
- Daniel Gleeson, Staffan Björkenstam, Robert Bohlin, Johan S Carlson, Bengt Lennartson. *Optimizing robot trajectories for automatic robot code generation*. Proceedings of 11th IEEE International Conference on Automation Science and Engineering, pp 495-500, 2015
- Staffan Björkenstam, Niclas Delfs, Johan S Carlson, Robert Bohlin, Bengt Lennartson. *Enhancing digital human motion planning of assembly tasks through dynamics and optimal control*. Proceedings of 6th CIRP Conference on Assembly Technologies and Systems, pp 20-25, 2016.
- Daniel Gleeson, Staffan Björkenstam, Robert Bohlin, Johan S Carlson, Bengt Lennartson. *Towards Energy Optimization Using Trajectory Smoothing and Automatic Code Generation for Robotic Assembly*. Proceedings of 6th CIRP Conference on Assembly Technologies and Systems, pp 20-25, 2016.

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Part I

Introductory chapters

Chapter 1

Introduction

Most classical control techniques for robotics are based on the idea to use feedback to override the dynamics of the system, and cancel out nonlinearities. However, to achieve high performance, the control system should take advantage of the dynamics, not cancel it out. Optimal control is a systematic way to create such controllers.

In this thesis we will apply optimal control techniques to both industrial robots and digital humans. In industrial robotics we are interested in finding the motions, which complete the primary task, and at the same time minimizes a performance index, such as, for example, the energy consumption. For digital humans we are interested in automatically generating realistic motions, which can be used in automatic assessment tools to evaluate workplace ergonomics.

The use of optimal control methods in biomechanical modeling, is well justified in that the sensorimotor system is a product of evolution, a process whose limit is optimality (Todorov 2004). Optimal control has been the single the most successful model of biomechanical movement, and has been used to explain several behavioral observations (Todorov and Jordan 2002). Furthermore, optimal control techniques have been successfully applied to human motion planning and biomechanical systems (Pandy et al. 1992; Lo and Metaxas 1999; Xiang et al. 2010; Maas and Leyendecker 2013), and offers an appealing approach to model and solve complex tasks. Most alternative control methods need a detailed description of how the goal should be accomplished. Optimal control methods, on the other hand, only require a performance criterion that describes the goal, and then finds the motion details automatically by searching for a control input that achieves the best possible performance.

Research questions

The main research questions are:

- How can discrete mechanics be efficiently used in direct optimal control methods, applied to high dimensional multibody systems?
- What is the best way to incorporate collision avoidance in direct optimal control methods?

Contributions

The main contributions of this thesis can be summarized as follows:

- New algorithms to compute the residual of the discrete Euler-Lagrange equations for tree structured, rigid multibody systems, with linear computational complexity.
- A scheme to exploit the separability of the discrete Euler-Lagrange equations of motion.
- A new way to incorporate collision avoidance in direct optimal control methods, based on iterative refinement of a collision free trajectory, and approximation of obstacles in configuration space.

Outline of the thesis

The thesis is outlined as follows. In Chapter 2, we present background on numerical methods of optimal control. Chapter 3 gives a brief introduction to classical mechanics. In Chapter 4, we narrow our scope to mechanical systems of rigid bodies. In particular, we discuss mechanical systems where at least part of the system can be modeled as a kinematic tree of rigid bodies. Chapter 5 introduces discrete mechanics, a discrete analog of the theory presented in Chapter 3. Discrete mechanics results in robust integrators, and naturally extend to mechanical systems with holonomic constraints. Next, in Chapter 6, we take a step back and describe the applications, which has motivated this thesis. Finally, Chapter 7 summarizes the appended papers, and Chapter 8 concludes the thesis with a discussion on possible future work.

Chapter 2

Direct optimal control

Numerical optimal control is a large research field, and many different methods exist (Betts 1998). However, for solution of constrained optimal control problems in real world applications, direct methods are by far the most widespread, and successfully used techniques (Diehl et al. 2006).

2.1 Background

The objective of optimal control theory is to determine the control input for a dynamical system, such that it satisfies constraints placed on the system, and at the same time minimizes some performance criterion. Mathematically, a typical optimal control problem can be stated as follows: Find the control vector $\mathbf{u}(t)$ which minimizes the cost functional

$$J = \chi(\mathbf{x}(t_s), t_s, \mathbf{x}(t_f), t_f) + \int_{t_s}^{t_f} \mathcal{L}(\mathbf{x}(t), \mathbf{u}(t), t) dt, \quad (2.1a)$$

while satisfying

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t), \quad (2.1b)$$

$$\mathbf{g}(\mathbf{x}(t), \mathbf{u}(t), t) \geq 0, \quad (2.1c)$$

$$\mathbf{H}(\mathbf{x}(t_s), t_s, \mathbf{x}(t_f), t_f) = 0, \quad (2.1d)$$

for $t \in [t_s, t_f]$, where $\mathbf{x}(t)$ is the state vector of the dynamical system. The cost functional (2.1a) contains two terms: the function χ which accounts for costs associated with the initial and terminal state, and the integral of the control Lagrangian, \mathcal{L} , which describes costs incurred along the trajectory. The differential equations (2.1b) are known as the state equations, and describe the dynamics of the system. Constraints on the state and the control along the trajectory are included in the path constraints (2.1c), while (2.1d) contain the boundary conditions.

Optimal control theory has its roots in calculus of variations, and can be traced back to giants such as Bernolli, Newton, Leibniz, Euler, and Lagrange, see (Sussmann and Willems 1997) for a historic prescriptive on optimal control. Modern optimal control grew into its own branch of applied mathematics in the 1950s. The rapid

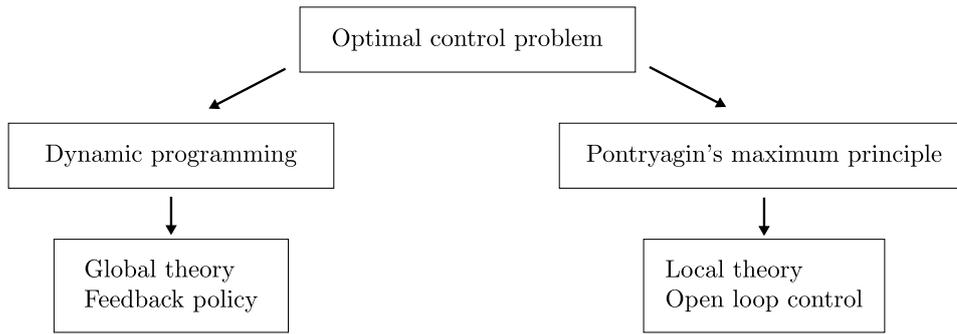


Figure 2.1: Dynamic Programming provides an optimal feedback policy, whereas the Maximum Principle only gives necessary conditions for a trajectory to be optimal

development in the 50s, fueled by the space race between the United States and the Soviet Union, was mainly the result of two very important discoveries. Namely, Dynamic Programming developed by Richard Bellman in the United States, and the Maximum Principle developed by Lev Pontryagin in the Soviet Union. The nature of the two principles is quite different, see Figure 2.1. While the Maximum Principle gives necessary conditions for a trajectory to be optimal, Dynamic Programming provides a necessary and sufficient condition for an optimum, but this condition must be satisfied over the whole state space.

In continuous time, Dynamic Programming leads to a partial differential equation known as the Hamilton-Jacobi-Bellman (HJB) equation. Unfortunately solving the HJB-equation is notoriously hard, and in practice it can only be solved analytically if the optimal control problem has a very special structure, such as, for example, the linear-quadratic optimal control problem where the HJB-equations yield Riccati-equations, which in turn give the Linear-Quadratic Regulator. Another possibility, is to search for approximate solutions of the HJB-equation by discretizing both the state and the control space, and then use Dynamic Programming to find the optimal solution to this discrete problem. However, this approach suffers from the so called curse of dimensionality, which renders it impractical even for systems of moderate size (Kirk 2012).

If we lower our ambitions, and focus on finding a local solution to the optimal control problem, then we can instead use Pontryagin's maximum principle. Numerical methods to find solutions to PMP are usually divided into indirect methods and direct methods. Indirect methods use PMP to derive the continuous optimality condition, which are then discretized and solved numerically. Indirect methods have the nice feature that they by construction are consistent with the PMP, but suffer from practical issues, such as how to start the iteration, convergence, and handling of inequality constraints. In direct methods, the optimal control problem is directly discretized, resulting in a nonlinear optimization problem. With the significant progress in large-scale computations and sparse nonlinear programming algorithms, direct methods have become very popular. By varying the choice of discretization scheme, many different direct optimal control methods can be constructed. One such method that has gained popularity, and has been used successfully for example by

NASA in space missions (Kang and Bedrossian 2007), is the pseudospectral method, see (Elnagar et al. 1995; Ross and Fahroo 2003). In pseudospectral methods the continuous functions are approximated by polynomials which collocate the differential equations and the constraints at a certain set of points. Different pseudospectral methods use different sets, two common choices are the Gauss-Legendre points and the Gauss-Lobatto points. Another successful method based on collocation, primarily used in robotics, is DIRCOL (O. V. Stryk 1993; O. v. Stryk and Schlemmer 1994). If the dynamical system under consideration is Hamiltonian, then it is also possible to use a variational integrator (J. E. Marsden and West 2001) to approximate the dynamics. This gives rise to the discrete mechanics and optimal control (DMOC) method (Sina Ober-Blöbaum et al. 2011). There has been an extensive research on using discrete mechanics and variational integrators in optimal control (S. Leyendecker, S. Ober-Blöbaum, et al. 2010). This is mainly due to the excellent numerical properties observed when variational integrators are used, which are largely attributed to the fact that the discrete approximations retain fundamental characteristics of the continuous system. In practice, this structure preservation leads to very stable integration schemes, enabling the use of large time steps. This is also true when integrating systems with holonomic constraints (Wendlandt and J. E. Marsden 1997; S. Leyendecker, J. Marsden, et al. 2008), making discrete mechanics well suited for multibody dynamics. A comparison between the implicit midpoint rule and a second order DMOC-method applied to an optimal control problem can be found in (Junge et al. 2005).

2.2 Nonlinear programming

Numerical optimization is a key component in direct methods for optimal control. A typical nonlinear program, corresponding to a transcribed optimal control problem, can be stated as:

$$\begin{aligned} & \underset{\mathbf{y} \in \mathbb{R}^n}{\text{minimize}} && \mathcal{F}(\mathbf{y}) \\ & \text{subject to} && \mathbf{c}(\mathbf{y}) = \mathbf{0}, \\ & && \mathbf{h}(\mathbf{y}) \leq \mathbf{0}, \end{aligned} \tag{2.2}$$

where $\mathcal{F} : \mathbb{R}^n \rightarrow \mathbb{R}$, $\mathbf{c} : \mathbb{R}^n \rightarrow \mathbb{R}^{m_c}$, and $\mathbf{h} : \mathbb{R}^n \rightarrow \mathbb{R}^{m_h}$ are twice continuously differentiable functions. We will denote constraints for which equality holds at a given point \mathbf{y} , as the active constraints at \mathbf{y} . Introducing multipliers $\boldsymbol{\mu} \in \mathbb{R}^{m_c}$ and $\boldsymbol{\nu} \in \mathbb{R}^{m_h}$ we define the Lagrangian function of the optimization problem as

$$\mathcal{L}(\mathbf{y}, \boldsymbol{\mu}, \boldsymbol{\nu}) = \mathcal{F}(\mathbf{y}) + \boldsymbol{\mu}^T \mathbf{c}(\mathbf{y}) + \boldsymbol{\nu}^T \mathbf{h}(\mathbf{y}).$$

The first order necessary conditions, known as the Karush-Kuhn-Tucker (KKT) conditions, for an optimal point can then be stated as: If \mathbf{y}^* is a regular point and a

local optimizer of (2.2) then there exist multipliers $\boldsymbol{\mu}^*$, $\boldsymbol{\nu}^*$ such that

$$\nabla_y \mathcal{L}(\mathbf{y}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*) = \mathbf{0} \quad (\text{Optimality}) \quad (2.3a)$$

$$\mathbf{c}(\mathbf{y}^*) = \mathbf{0}, \quad (\text{Primal feasibility}) \quad (2.3b)$$

$$\mathbf{h}(\mathbf{y}^*) \leq \mathbf{0},$$

$$\boldsymbol{\nu}^* \geq \mathbf{0}, \quad (\text{Dual feasibility}) \quad (2.3c)$$

$$\nu_i^* h_i(\mathbf{y}^*) = 0, \text{ for } i = 1, \dots, m_h. \quad (\text{Complementarity}) \quad (2.3d)$$

A constraint qualification commonly used in practice, to assure that \mathbf{y}^* is a regular point of (2.2), is the Linear Independence Constraint Qualification (LICQ), which states that gradients of the active constraints at \mathbf{y}^* should be linearly independent. If the nonlinear programming problem is a convex problem, then the KKT conditions are not only necessary conditions for a local optimum, but also sufficient conditions for a global optimum. In general, we can not make this assumption. However, we can give the following second order sufficient condition for a local optimum: Assume that \mathbf{y}^* is a regular point of (2.2) satisfying the KKT-conditions with multipliers $(\boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$, and \mathbf{Z} is a matrix spanning the nullspace of the active constraints at \mathbf{y}^* , then if $\mathbf{Z}^T \nabla_y^2 \mathcal{L}(\mathbf{y}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*) \mathbf{Z}$ is a positive definite matrix, \mathbf{y}^* is a local minimizer of (2.2). The matrix $\nabla_y^2 \mathcal{L}$ plays an important role in optimization algorithms, and is known as the Hessian of the Lagrangian.

If care is taken in the discretization of the optimal control problem, multipliers of the nonlinear program converges to the costate, and the Karush-Kuhn-Tucker conditions of the nonlinear program are really a well behaved discrete version of Pontryagin's Maximum Principle, see (Benson 2005; Sina Ober-Blöbaum et al. 2011).

Two of the most successful methods for large scale nonlinear programming are sequential quadratic programming and interior point methods. Both methods are based on searching for solutions to the Karush-Kuhn-Tucker conditions (2.3). As the name suggests the SQP-method attempts to solves the nonlinear program by solving a sequence of quadratic approximations. Interior point methods, on the other hand, introduce a regularization parameter in the complementarity condition (2.3d), and solve a sequence of the regularized KKT conditions, with decreasing values of the parameter, using Newton's method. See (Nocedal and Wright 2006) for details on these methods. Since these are local methods, convergence to a solution is not guaranteed, and can be highly dependent on the starting point. Hence, if possible, a good initial guess of the solution should be provided to the solver.

In our implementation we use the interior point solver IPOPT (Wächter and Biegler 2006). In order to achieve rapid convergence to a solution, gradients of the constraints and the objective, as well as the Hessian of the Lagrangian should be provided to the solver. Since these matrices originate from a transcribed optimal control problem, and in particular a transcribed optimal control problem where the dynamical system is a multibody system, there is structure which should be exploited. This is the subject of Paper 2, where sparse finite differencing is used to approximate the derivatives, and the partial separability of the discrete equations of motion is exploited to reduce the number of directions needed for the derivative calculations. In Paper 3, we instead use algorithmic differentiation to evaluate the derivatives.

Derivatives evaluated using algorithmic differentiation are exact to floating point precision, which has proven to greatly improve convergence on some of our problems, compared to finite difference approximation. Furthermore, the methods to exploit separability, from Paper 2, are also applicable when using algorithmic differentiation.

2.3 Summary

Direct methods of optimal control transcribes the continuous optimal control problem into a finite dimensional optimization problem, which can be solved using a variety of well developed nonlinear programming solvers. In order to use these solvers the objective function and the constraint functions of the nonlinear program must be available for evaluation. Furthermore, to achieve high performance, the solvers should also be supplied with derivative information. For the problems under consideration in this thesis, these functions contain quantities involving the dynamics of the multibody systems. Hence, much of the focus in this thesis is on exploiting the structure of multibody systems, in order to efficiently apply direct optimal control methods.

Chapter 3

Classical mechanics

Here we give a brief introduction to classical mechanics. This will be useful later, when we derive the equations of motion of kinematic trees in Chapter 4, and in particular, in Chapter 5, when we introduce discrete mechanics.

3.1 Equations of motion

The Euler-Lagrange equations of motion for a mechanical system with configuration $\mathbf{q} \in Q \subseteq \mathbb{R}^{n_q}$ and kinetic energy $T(\mathbf{q}, \dot{\mathbf{q}})$ can be stated as

$$\frac{\partial T}{\partial \mathbf{q}} - \frac{d}{dt} \frac{\partial T}{\partial \dot{\mathbf{q}}} + \mathbf{F} = \mathbf{0}, \quad (3.1)$$

where \mathbf{F} is the generalized force acting on the system.

We now introduce the Lagrangian as the difference between kinetic and potential energy, $L = T - V$. We also assume that the potential energy depends only on the configuration and not the velocity i.e. $V = V(\mathbf{q})$. The Euler-Lagrange equations can then be written as

$$\frac{\partial L}{\partial \mathbf{q}} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} + \mathbf{F} = \mathbf{0}, \quad (3.2)$$

where

$$\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial T}{\partial \dot{\mathbf{q}}}$$

and

$$\frac{\partial L}{\partial \mathbf{q}} = \frac{\partial T}{\partial \mathbf{q}} - \frac{\partial V}{\partial \mathbf{q}}.$$

Note that, while \mathbf{F} is still a generalized force acting on the system, it is different from \mathbf{F} in (3.1), since it no longer includes the contribution from the potential V .

3.2 The principle of stationary action

The motion of a mechanical system can also be described using a variational principle. Consider the mechanical system specified by a configuration manifold $Q \subseteq \mathbb{R}^{n_q}$ and

a Lagrangian $L : TQ \rightarrow \mathbb{R}$, where TQ is the tangent bundle of the configuration manifold. Furthermore, let $U \in \mathbb{R}^{n_u}$ be the set of admissible controls and $\mathbf{F} : TQ \times U \rightarrow T^*Q$ the external force acting on the system, where T^*Q is the cotangent bundle of the configuration manifold. Then an extended version of Hamilton's Principle, sometimes referred to as the Lagrange-d'Alembert principle, states that a trajectory of the system will satisfy

$$\delta \int_{t_1}^{t_2} L(\mathbf{q}(t), \dot{\mathbf{q}}(t)) dt + \int_{t_1}^{t_2} \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \mathbf{u}(t)) \cdot \delta \mathbf{q} dt = 0 \quad (3.3)$$

where variations are taken with respect to \mathbf{q} , with fixed endpoints. It is easily checked that integration by parts and the fundamental lemma of calculus of variations give us back the Euler-Lagrange equations of motion (3.2).

3.3 Constrained mechanical systems

It is not always practical, or even possible, to come up with an appropriate set of independent coordinates for a complicated mechanical system. Therefore, we will now extend the mechanical system to include constraints on the coordinates using Lagrange multipliers.

Again, consider the mechanical system specified by a configuration manifold $Q \subseteq \mathbb{R}^{n_q}$ and a Lagrangian $L : TQ \rightarrow \mathbb{R}$. Furthermore, let $U \in \mathbb{R}^{n_u}$ be the set of admissible controls and $\mathbf{F} : TQ \times U \rightarrow T^*Q$ the external force acting on the system. Now, suppose the motion of the system is also constrained by the equation $\phi(\mathbf{q}) = \mathbf{0} \in \mathbb{R}^m$ to lie in the constraint manifold $C = \phi^{-1}(\mathbf{0}) \subset Q$. Introducing multipliers $\boldsymbol{\lambda}(t) \in \mathbb{R}^m$, the Lagrange-d'Alembert principle states that trajectories of the system satisfy

$$\delta \int_{t_1}^{t_2} L(\mathbf{q}(t), \dot{\mathbf{q}}(t)) + \boldsymbol{\lambda}(t)^T \phi(\mathbf{q}(t)) dt + \int_{t_1}^{t_2} \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \mathbf{u}(t)) \cdot \delta \mathbf{q} dt = 0 \quad (3.4)$$

where variations are taken with respect to both \mathbf{q} , fixed at the endpoints, and $\boldsymbol{\lambda}$. Integration by parts and the fundamental lemma of calculus of variations give the following differential algebraic equations, known as the constrained Euler-Lagrange equations of motion:

$$\frac{\partial L}{\partial \mathbf{q}}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) - \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) + \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \mathbf{u}(t)) + \boldsymbol{\lambda}(t)^T \boldsymbol{\Phi}(\mathbf{q}(t)) = \mathbf{0}, \quad (3.5a)$$

$$\phi(\mathbf{q}(t)) = \mathbf{0}, \quad (3.5b)$$

where $\boldsymbol{\Phi}$ denotes the Jacobian of the constraint function.

3.4 Summary

The variational principle (3.4) gives a compact and elegant representation of the motion of a mechanical system. However, it also has practical implications on the

numerics. This is the topic of Chapter 5, where we define a discrete analog of (3.4). The equations of motion derived from this discrete variational principle preserves characteristics of the continuous system, which discretizations of (3.5) not necessarily do.

Chapter 4

Rigid body dynamics

In this thesis we restrict ourselves to multibody systems of interconnected rigid bodies. In particular, we are interested in mechanical systems where at least part of the system can be modeled as a kinematic tree of rigid bodies. A kinematic tree is a collection of bodies, connected by kinematic joints in a topological tree structure. A wide range of mechanical systems, such as industrial robots and the human body, can be effectively modeled as kinematic trees.

4.1 Spatial vector algebra

Here we give a brief introduction to spatial vector algebra, a compact and efficient way to derive and implement rigid body dynamics algorithms. We keep this section short, and refer interested readers to (Featherstone 2010a; Featherstone 2010b; Featherstone 2008) for more detail.

In spatial vector algebra, vectors are six-dimensional and belong either to the motion vector space V_M or its dual, the force vector space V_F . We use hats to distinguish spatial vectors from regular three dimensional vectors. Furthermore, we follow the convention that the angular part comes before the linear part. For example, in a particular coordinate system, the spatial velocity vector of a rigid body can be written as

$$\hat{\mathbf{v}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix},$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity of the rigid body, and $\mathbf{v} \in \mathbb{R}^3$ is the linear velocity of the body-fixed point that currently coincides with the origin. Similarly, the spatial force acting on a rigid body has the following coordinates

$$\hat{\mathbf{f}} = \begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{f} \end{bmatrix},$$

where $\boldsymbol{\tau} \in \mathbb{R}^3$ is the total torque about the origin, and $\mathbf{f} \in \mathbb{R}^3$ is the force acting along a line passing through the origin. Furthermore, the spatial momentum of a rigid body is defined as

$$\hat{\mathbf{p}} = \mathbf{I}\hat{\mathbf{v}} \in V_F, \tag{4.1}$$

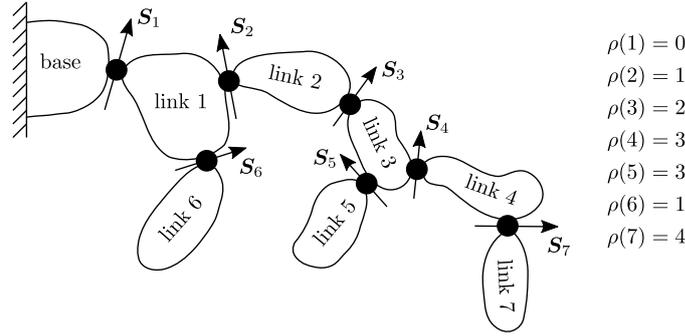


Figure 4.1: Example of a kinematic tree with $N_B = 7$. For link 3 we have $\kappa(3) = \{1, 2, 3\}$, $\nu(3) = \{3, 4, 5, 7\}$, and $\mu(3) = \{4, 5\}$.

where \mathbf{I} is the 6×6 spatial inertia matrix. To be consistent with the notation used in (Featherstone 2008), we treat both force and motion vectors as column vectors.

Using spatial vector algebra Newton's and Euler's equations of motion of a rigid body can be written compactly as

$$\hat{\mathbf{f}} = \frac{d}{dt}\hat{\mathbf{p}} = \mathbf{I}\hat{\mathbf{a}} + \hat{\mathbf{v}} \times^* \mathbf{I}\hat{\mathbf{v}} \quad (4.2)$$

where $\hat{\mathbf{f}}$ is the net spatial force acting on the body, $\hat{\mathbf{a}} = d\hat{\mathbf{v}}/dt$ is the spatial acceleration, and \times^* is the spatial cross product operator between motion vectors and force vectors.

4.2 Kinematic trees

A kinematic tree structure of N_B rigid bodies can be modeled by a base link, with index 0, fixed relative to a global inertial reference system, and a set of N_B links, with indices $1, \dots, N_B$, connected by N_B joints such that joint i connects link i to its parent, link $\rho(i)$. Without loss of generality, we choose indices such that $\rho(i) < i$, for $i = 1, \dots, N_B$. Furthermore, from $\{\rho(i)\}_{i=1}^{N_B}$ it is possible to define the support, child, and subtree sets. The support set, $\kappa(i)$, is the set of links on the path from link i to the base, whereas the child set, $\mu(i)$, is the set of children of link i , and finally the subtree set, $\nu(i)$, is the set of links in the subtree starting at link i . An example of a kinematic tree can be seen in Figure 4.1.

The configuration of the entire kinematic tree is represented by the coordinate vector

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_{N_B} \end{bmatrix} \in \mathbb{R}^{n_q},$$

where $\mathbf{q}_i \in \mathbb{R}^{d_i}$ is the coordinate of joint i . For our applications, we are interested in holonomic joints without explicit time dependence, where the spatial velocity across joint i can be written as $\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{\rho(i)} = \mathbf{S}_i \dot{\mathbf{q}}_i$, where $\hat{\mathbf{v}}_i$ is the spatial velocity of link i , relative to the inertial frame, and $\mathbf{S}_i(\mathbf{q}_i) \in \mathbb{R}^{6 \times d_i}$ is the motion subspace of joint i .

This gives us the following recursive formula for computing the spatial velocity of link i :

$$\hat{\mathbf{v}}_i = \hat{\mathbf{v}}_{\rho(i)} + \mathbf{S}_i \dot{\mathbf{q}}_i, \quad (4.3)$$

or equivalently

$$\hat{\mathbf{v}}_k = \sum_{j \in \kappa(k)} \mathbf{S}_j \dot{\mathbf{q}}_j.$$

Thus, for a given state, $(\mathbf{q}, \dot{\mathbf{q}})$, we can compute the spatial velocity of each body recursively using (4.3), starting from $\hat{\mathbf{v}}_0 = \mathbf{0}$.

The equations of motion of a kinematic tree

The kinetic energy of a rigid body is $\frac{1}{2} \hat{\mathbf{v}}^T \mathbf{I} \hat{\mathbf{v}}$ where $\hat{\mathbf{v}}$ is the spatial velocity of the body and \mathbf{I} is the 6×6 spatial inertia matrix. The total kinetic energy, T , of a kinematic tree is then simply the sum of the kinetic energy of each body in the tree T_k for $k = 1, 2 \dots N_B$:

$$T = \sum_{k=1}^{N_B} T_k = \frac{1}{2} \sum_{k=1}^{N_B} \hat{\mathbf{v}}_k^T \mathbf{I}_k \hat{\mathbf{v}}_k = \frac{1}{2} \sum_{k=1}^{N_B} \sum_{i \in \kappa(k)} \sum_{j \in \kappa(k)} \dot{\mathbf{q}}_i^T \mathbf{S}_i^T \mathbf{I}_k \mathbf{S}_j \dot{\mathbf{q}}_j = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}} \quad (4.4)$$

where $\mathbf{M}(\mathbf{q})$ is the $n \times n$ mass matrix, also known as the joint space inertia matrix. Plugging this expression into the Euler-Lagrange equations (3.1) we get the following familiar equations of motion:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\mathbf{q}}} - \frac{\partial T}{\partial \mathbf{q}} = \ddot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{F}, \quad (4.5)$$

where $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the Coriolis and centrifugal contribution. Furthermore, if the mass matrix is constant (4.5) simplifies to Newton's equations of motion:

$$\ddot{\mathbf{q}}^T \mathbf{M} = \mathbf{F}.$$

Solving the equations of motion (4.5) for the acceleration, $\ddot{\mathbf{q}}$, is known as the forward dynamics problem, while solving for \mathbf{F} is called the inverse dynamics problem. There exist efficient recursive algorithms for both these problems (Featherstone 2008). For optimal control purposes we want to evaluate the residual of (4.5). Hence, we are primarily interested in the inverse dynamics problem, which can be efficiently solved using the recursive Newton-Euler algorithm.

The recursive Newton-Euler algorithm

The recursive Newton-Euler algorithm (Luh et al. 1980; Featherstone and Orin 2000) is a particularly efficient way to compute the force \mathbf{F} in (4.5), i.e. solve the inverse dynamics problem, when the system has a tree topology.

The algorithm can be divided into the following three steps:

1. Calculate the velocity and acceleration of each body in the tree.

2. Calculate the force required to produce these accelerations.
3. Calculate the forces transmitted across the joints from forces acting on the bodies.

The formula for updating the link velocities is already given by (4.3). Differentiating (4.3) yields

$$\hat{\mathbf{a}}_i = \hat{\mathbf{a}}_{\rho(i)} + \mathbf{S}_i \ddot{\mathbf{q}}_i + \dot{\mathbf{S}}_i \dot{\mathbf{q}}_i, \quad (4.6)$$

for the spatial acceleration of link i . The Newton-Euler equations of rigid body motion (4.2) states that

$$\hat{\mathbf{f}}_i^B = \mathbf{I}_i \hat{\mathbf{a}}_i + \hat{\mathbf{v}}_i \times^* \mathbf{I}_i \hat{\mathbf{v}}_i, \quad (4.7)$$

where $\hat{\mathbf{f}}_i^B$ is the net force acting on link i . The spatial force required at joint i , can then be computed as

$$\hat{\mathbf{f}}_i = \hat{\mathbf{f}}_i^B + \sum_{j \in \mu(i)} \hat{\mathbf{f}}_j. \quad (4.8)$$

Using (4.8), we can compute the force transmitted across each joint, working our way from the leaves to the base of the tree. Finally, the generalized force $\mathbf{F}_i \in \mathbb{R}^{d_i}$, required at joint i to produce $\ddot{\mathbf{q}}$, is given by

$$\mathbf{F}_i = \hat{\mathbf{f}}_i^T \mathbf{S}_i. \quad (4.9)$$

4.3 Summary

The systems under consideration in this thesis can be represented as tree structures of rigid bodies. For these systems there exist efficient recursive algorithms to compute the state, and to evaluate the equations of motion. For optimal control purposes, we are primarily interested in the inverse dynamics problem.

Chapter 5

Discrete mechanics

Analogous to Hamilton's principle in continuous mechanics (3.3), the discrete equations of motion, known as the discrete Euler-Lagrange equations, can be derived from a corresponding discrete variational principle. Integrators derived in this way are symplectic-momentum preserving, and exhibit excellent energy behavior (J. E. Marsden and West 2001). In practice, this structure preservation leads to very stable integration schemes, enabling the use of large time steps. This is also true when integrating systems with holonomic constraints (Wendlandt and J. E. Marsden 1997; S. Leyendecker, J. Marsden, et al. 2008), making discrete mechanics well suited for multibody dynamics.

5.1 The discrete Euler-Lagrange equations

The key idea of variational integrators is to directly approximate the variational principle (3.4) rather than the differential equations (3.5). We begin by discretizing $\mathbf{q}(t)$ in $[t_1, t_2]$ using a fixed time step $h = (t_2 - t_1)/N$, such that $\mathbf{q}^{(k)}$ is an approximation of $\mathbf{q}(t_1 + kh)$ for $k = 0, \dots, N$. Furthermore, the control is discretized such that $\mathbf{u}^{(k)}$ is an approximation of $\mathbf{u}(t_1 + (k + \frac{1}{2})h)$ for $k = 0, \dots, N - 1$. We then replace the continuous state space, TQ , with the discrete state space, $Q \times Q$, and construct a discrete Lagrangian $L_d : Q \times Q \rightarrow \mathbb{R}$ such that

$$L_d(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}) \approx \int_{t_1+kh}^{t_1+(k+1)h} L(\mathbf{q}(t), \dot{\mathbf{q}}(t)) dt. \quad (5.1)$$

The virtual work of the external force, \mathbf{F} , is approximated as

$$\begin{aligned} \sum_{k=0}^{N-1} \left(\mathbf{F}_d^-(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}, \mathbf{u}^{(k)}) \cdot \delta \mathbf{q}^{(k)} + \mathbf{F}_d^+(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}, \mathbf{u}^{(k)}) \cdot \delta \mathbf{q}^{(k+1)} \right) \\ \approx \int_{t_1}^{t_2} \mathbf{F}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \mathbf{u}(t)) \cdot \delta \mathbf{q} dt, \quad (5.2) \end{aligned}$$

where \mathbf{F}_d^+ and \mathbf{F}_d^- are the left and right discrete forces. Introducing the discrete Lagrange multipliers, $\boldsymbol{\lambda}^{(k)}$ for $k = 0, \dots, N$, a discrete variational principle, corresponding to (3.4), can be formulated as (S. Leyendecker, J. Marsden, et al. 2008)

$$\begin{aligned} \delta \sum_{k=0}^{N-1} \left(L_d(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}) + \frac{1}{2} \boldsymbol{\phi}^T(\mathbf{q}^{(k)}) \boldsymbol{\lambda}^{(k)} + \frac{1}{2} \boldsymbol{\phi}^T(\mathbf{q}^{(k+1)}) \boldsymbol{\lambda}^{(k+1)} \right) \\ + \sum_{k=0}^{N-1} \left(\mathbf{F}_d^-(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}, \mathbf{u}^{(k)}) \cdot \delta \mathbf{q}^{(k)} + \mathbf{F}_d^+(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}, \mathbf{u}^{(k)}) \cdot \delta \mathbf{q}^{(k+1)} \right) = 0 \end{aligned} \quad (5.3)$$

for all variations $\delta \boldsymbol{\lambda}^{(k)}$ and $\delta \mathbf{q}^{(k)}$, with $\delta \mathbf{q}^{(0)} = \delta \mathbf{q}^{(N)} = \mathbf{0}$. This principle is equivalent to the discrete constrained Euler-Lagrange equations:

$$\begin{aligned} D_2 L_d(\mathbf{q}^{(k-1)}, \mathbf{q}^{(k)}) + D_1 L_d(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}) + \mathbf{F}_d^+(\mathbf{q}^{(k-1)}, \mathbf{q}^{(k)}, \mathbf{u}^{(k-1)}) \\ + \mathbf{F}_d^-(\mathbf{q}^{(k)}, \mathbf{q}^{(k+1)}, \mathbf{u}^{(k)}) + (\boldsymbol{\lambda}^{(k)})^T \boldsymbol{\Phi}(\mathbf{q}^{(k)}) = \mathbf{0}, \end{aligned} \quad (5.4a)$$

$$\boldsymbol{\phi}(\mathbf{q}^{(k+1)}) = \mathbf{0}, \quad (5.4b)$$

where $D_1 L_d$ and $D_2 L_d$ are the partial derivatives with respect to the first and second argument, respectively. These equations define a variational integrator by implicitly mapping $(\mathbf{q}^{(k-1)}, \mathbf{q}^{(k)}, \mathbf{u}^{(k-1)}, \mathbf{u}^{(k)})$ to $(\mathbf{q}^{(k+1)}, \boldsymbol{\lambda}^{(k)})$.

Please refer to (J. E. Marsden and West 2001) for a thorough introduction to discrete mechanics, and (S. Leyendecker, J. Marsden, et al. 2008) for more details on variational integrators for constrained dynamical systems.

5.2 Discrete mechanics and optimal control

In order to use the variational integrators in direct optimal control methods, we need to be able to quickly evaluate the residual of the discrete Euler-Lagrange equations (5.4). However, since the dynamics is formulated using discrete mechanics, we can not directly use the recursive Newton-Euler algorithm to compute this residual, as we do in Paper 1. Effort has been made to develop efficient inverse dynamics algorithms for discrete mechanics. For example, in (Johnson and Murphey 2009; Johnson and Murphey 2008) a caching scheme is proposed to exploit the tree structure of the mechanical system by reusing previously computed quantities from the tree. However, this approach leads to a solution of the inverse dynamics problem which has quadratic computational complexity, $O(N_B^2)$, with respect to the number of bodies, N_B , in the system. In Paper 3, we take a different approach and derive new recursive algorithms for efficient computation of the terms in the discrete Euler-Lagrange equations. The resulting algorithms are closely related to the recursive Newton-Euler algorithm, described in Section 4.2, but computes the quantities required by a variational integrator. Analogous to the recursive Newton-Euler algorithm, the new algorithms have linear time and memory complexity, which is the theoretical minimum for a sequential solution to this problem. Furthermore, the algorithms are simple to implement, and can also very easily be used together with algorithmic differentiation (Griewank and Walther 2008) to evaluate derivatives of the equations. In fact, using the adjoint mode of algorithmic differentiation it is possible to construct $O(N_B^2)$ -algorithms for both the Jacobian of the DEL-equations, and the contribution of the DEL-equations to the Hessian of the Lagrangian.

5.3 Summary

Discrete mechanics is an attractive way to formulate the discrete equations of motion for multibody systems. The discrete equations preserve characteristics of the continuous system, and naturally extend to systems with holonomic constraints. Furthermore, using the algorithms presented in Paper 3, they can also be efficiently used in direct optimal control methods.

Chapter 6

Applications

The appended manuscripts were written at Fraunhofer-Chalmers Centre while developing motion planning tools for industrial robots and digital humans. The two main areas of application, which have motivated this work, are sustainable manufacturing, and workplace ergonomics.

6.1 Sustainable manufacturing

Sustainable manufacturing is defined as “the creation of manufactured products that use processes that minimize negative environmental impacts, conserve energy and natural resources, are safe for employees, communities, and consumers and are economically sound” by The U.S. Department of Commerce 2010 (S. Department of Commerce 2010). This implies the need to consider and balance between economical, ecological and social factors to achieve a sustainable production system. A more detailed discussion on measures and awareness used at Chalmers University of Technology can be found in (Johansson et al. 2012). By using Computer Aided Engineering (CAE), physical prototypes can be replaced by simulation, new products can be introduced faster, the efficiency of the production system can be optimized using mathematical methods and algorithms, and it can be done by simulation experts and production engineers in a safe and healthy environment.

The automotive industry is an example of an equipment and energy intensive manufacturing, where up to 28% of the vehicle life cycle energy is spent during production. For example, a typical automotive car body consists of about 300 sheet metal parts, joined by about 4000 welds. Typical joining methods are spot welding, arc welding, gluing and stud welding. In car body assembly plants, the welds are distributed to several hundred industrial welding robots, which are organized in up to 100 stations. The body shop is indeed investment intense, with the robots as the main consumer of energy (Meike and Ribickis 2011). In (Almström et al. 2011) it is highlighted how utilization affects different aspects of sustainable production, the link between utilization and productivity, as well as practical considerations when improving utilization in manufacturing industry. Therefore from a sustainability perspective it is highly motivated to develop new software methods and algorithms

for further improvement of equipment utilization and energy efficiency of robotized manufacturing systems.

In (Segeborn et al. 2011) it is shown that the balancing of weld work load between the executing stations and robots has a significant influence on achievable production rate and equipment utilization. Robot line balancing is a complex problem, where a number of welding robots in a number of stations are available to execute an overall weld load. Each weld is to be assigned to a specific station and robot, such that the line cycle time is minimized. Line balancing efficiency depends on station load balancing, robot welding sequencing, path planning and effectiveness of robot coordination for collision free execution within each other's working envelopes. Robot coordination impairs cycle time by inserting waiting positions and signals into the original paths. At Volvo Cars it has been proven that by using automatic path planning and line balancing instead of standard off-line programming the cycle time in welding lines can be improved by as much as 25%. The next step for improving the automatic path planning and line balancing is to include detailed optimization of motion profiles between welds, see Paper 1 for an example of such motion profiles generated using a direct optimal control method. This choice can also be motivated from an energy efficiency aspect. Meike and Ribickis (2011) investigated different strategies to operate robots in an energy efficient way. Motion profile optimization was one of the strategies pointed out among others, such as automatic shut-down and start-up, reusing braking energy, and brake management.

6.2 Human factors and ergonomics

Although the degree of automation is increasing in manufacturing industries, many assembly operations are performed manually. To avoid injuries and to reach sustainable production of high quality, comfortable environments for the operators are vital, see (Falck, Örtengren, et al. 2010) and (Falck and Rosenqvist 2014). Poor station layouts, poor product designs or badly chosen assembly sequences are common sources leading to unfavorable poses and motions. To keep costs low, preventive actions should be taken early in a project, raising the need for feasibility and ergonomics studies in virtual environments long before physical prototypes are available.

Today, in the automotive industries, such studies are conducted to some extent. The full potential, however, is far from reached due to limited software support in terms of capability for realistic pose prediction, motion generation and collision avoidance. As a consequence, ergonomics studies are time consuming and are mostly done for static poses, not for full assembly motions. Furthermore, these ergonomic studies, even though performed by a small group of highly specialized simulation engineers, show low reproducibility within the group (Lämkuil et al. 2008).

The human body is very complex, and even highly approximate mechanical models result in systems with very large numbers of degrees of freedom. To solve optimal control problems involving these high dimensional dynamical systems, it is important to exploit the structure of the optimal control problem, as well as the dynamics. This is the main topic of Paper 2 and Paper 3. The work in these papers was done as part of the Cromm (Creation of Muscle Manikins) project (Högberg et al.

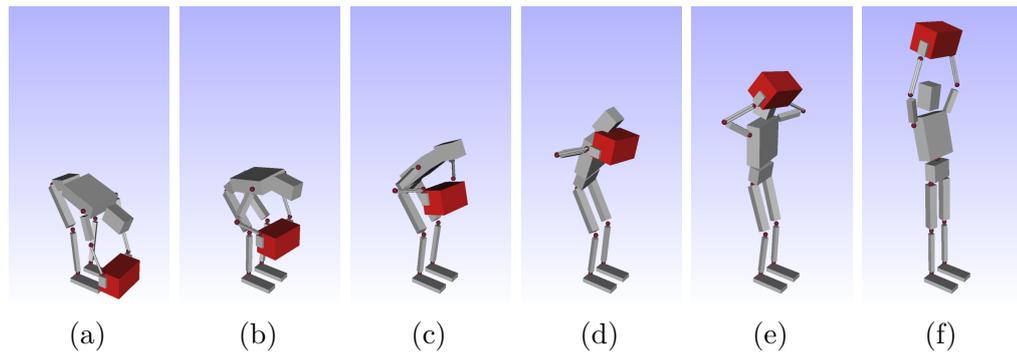


Figure 6.1: Snapshots of a motion for the digital human, whose task it is to pick up a box from the ground and lift it as high as possible, see Paper 3. Snapshots taken at (a) $t = 0.0$ s, (b) $t = 0.53$ s, (c) $t = 1.06$ s, (d) $t = 1.59$ s, (e) $t = 2.12$ s, (f) $t = 2.65$ s.

2016), which aims to extend the work presented in (Bohlin et al. 2011) and (Delfs et al. 2014) by including dynamics and optimal control. The results have been implemented in the IMMA framework (Hanson et al. 2011). Figure 6.1 shows the optimized motion of a digital human, whose task it is to pick up a box from the ground and lift it as high as possible. More detail can be found in Paper 3.

6.3 Summary

Sustainable manufacturing and workplace ergonomics are two areas of applications, which have motivated the research done in this thesis. The presented methods could, however, easily be applied to a much wider range of applications.

Chapter 7

Summary of appended papers

This thesis contains three manuscripts: (i) *Energy Efficient and Collision Free Motion of Industrial Robots using Optimal Control*, (ii) *Exploiting Sparsity in the Discrete Mechanics and Optimal Control Method with Application to Human Motion Planning*, and (iii) *Inverse Dynamics for Discrete Mechanics of Multibody Systems with Application to Direct Optimal Control*. The manuscripts share the common goal of developing practical tools and techniques to numerically solve optimal control problems involving multibody systems. The papers were written at Fraunhofer-Chalmers Centre, while developing motion planning tools for industrial robots and digital humans.

7.1 Summary of Paper 1

In a production plant for complex assembled products there could be up to several hundred of robots used for handling and joining operations. Thus, improvement in robot motions can have a huge impact on equipment utilization and energy consumption. These are two of the most important aspects of sustainability in a production system. Therefore, this paper presents an algorithm for generating efficient and collision free motion of industrial robots using path planning and direct transcription methods for numerical optimal control. As a measure of efficiency for moving between configurations we use a combination of the energy norm of the applied actuator torques and the cycle time. Velocity and torque limits are handled and modeled as hard constraints. However, more general problems can be solved by the same approach. Our novel algorithm solves the problem in three steps; (i) first a path planning algorithm calculates an initial collision free path, (ii) a convex optimal control problem is then formulated to follow this path, and finally (iii) a nonlinear optimal control problem is solved to iteratively improve the trajectory. The resulting trajectory is guaranteed to be collision free by restrictions in the configuration space, based on a local sensitivity analysis. The algorithm has been successfully applied to several industrial cases, demonstrating that the proposed method can be used effectively in practical applications.

7.2 Summary of Paper 2

The discrete equations of motion derived using a variational principle are particularly attractive to be used in numerical optimal control methods. This is mainly because: i) they exhibit excellent energy behavior, ii) they extend gracefully to systems with holonomic constraints and iii) they admit compact representation of the discrete state space. In this paper we propose the use of sparse finite differencing techniques for the Discrete Mechanics and Optimal Control method. In particular we show how to efficiently construct estimates of the Jacobian and Hessian matrices when the dynamics of the optimal control problem is discretized using a variational integrator. To demonstrate the effectiveness of this scheme we solve a human motion planning problem of an industrial assembly task, modeled as a multibody system, consisting of more than one hundred degrees of freedom.

7.3 Summary of Paper 3

In this paper we present efficient algorithms for computation of the residual of the constrained discrete Euler-Lagrange equations of motion for tree structured, rigid multibody systems. In particular, we present new recursive formulas for computing sensitivities of the kinetic energy. This enables us to solve the inverse dynamics problem of the discrete system with linear computational complexity. The resulting algorithms are easy to implement, and can naturally be applied to a very broad class of multibody systems by imposing constraints on the coordinates by means of Lagrange multipliers. Our interest in inverse dynamics is primarily to apply direct transcription optimal control methods to multibody systems. As an example application, we present a digital human motion planning problem, which we solve using the proposed method.

Chapter 8

Conclusions and future work

This thesis presents efficient and practical methods for direct optimal control of multibody systems. In Paper 2 techniques are described to increase performance of direct optimal control methods, by exploiting the structure of the discrete equations of motion. Furthermore, in Paper 3 new recursive dynamics algorithms for discrete mechanics is developed to exploit the tree structure of the mechanical system. These results make discrete mechanics an attractive formulation to be used in direct methods for optimal control of high dimensional multibody systems. Incorporating collision avoidance in optimal control methods is difficult, and in general results in a very large number of constraints, or non-smooth constraint functions. While the collision avoidance scheme presented in Paper 1 avoids these problems, it has other drawbacks. In particular, it does not give a guarantee for convergence to a local optimum. Hence, the best way to incorporate collision avoidance is still considered an open question. An interesting approach, which we plan to further explore, is to integrate collision avoidance in an active set method, as proposed in (Gerdt et al. 2012).

The methods presented in this thesis have been implemented in software tools developed at Fraunhofer-Chalmers centre. There are, however, still areas that need to be improved, and further investigated. One improvement is to extend the inverse dynamics algorithms to variational integrators on Lie groups (Bou-Rabee and J. E. Marsden 2009), which has the benefit of automatically enforcing that configurations remain within their proper group without recourse to computationally expensive rejections or constraints. Another interesting topic is transient contact between rigid bodies in an optimal control problem. One way to model contact is by adding several phases to the optimal control problem. Each phase would then have its own dynamics, dependent on the contact state, and the phases connect to each other by including appropriate boundary conditions. However, this approach has a serious drawback, since it requires knowledge of the optimal contact sequence before hand. An alternative approach would be to model contact using complementarity conditions (Stewart 2000). However, nonlinear programs with complementarity conditions fail to satisfy the linear independence constraint qualifications, and thus require special techniques to be solved (Peng et al. 2004; Posa and Tedrake 2013). Still, this could be a very powerful technique for finding motions describing, for example, locomotion (Koch and Sigrid Leyendecker 2016) and grasping, where it is

hard to know, a priori, the exact sequence of contacts.

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