Transformation Optics Approach for Goos-Hänchen Shift Enhancement at Metamaterial Interfaces

Lieve Lambrechts[a], Vincent Ginis[a], Jan Danckaert[a] and Philippe Tassin[b,a]

[a] Applied Physics Research Group, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussel, Belgium

[b] Department of Physics, Chalmers University, SE-412 96, Göteborg, Sweden

ABSTRACT

Since its first observation in 1947, the Goos-Hänchen effect—an electromagnetic wave phenomenon where a totally reflected beam with finite cross section undergoes a lateral displacement from its position predicted by geometric optics—has been extensively investigated for various types of optical media such as dielectrics, metals and photonic crystals. Given their huge potential for guiding and sensing applications, the search for giant and tunable Goos-Hänchen shifts is still an open question in the field of optics and photonics. Metamaterials allow for unprecedented control over electromagnetic properties and thus provide an interesting platform in this quest for Goos-Hänchen shift enhancement. Over the last few years, the Goos-Hänchen effect has been investigated for specific metamaterial interfaces including graphene-on-dielectric surfaces, negative index materials and epsilon-near-zero materials. In this contribution, we generalize the approach for the investigation of the Goos-Hänchen effect based on the geometric formalism of transformation optics. Although this metamaterial design methodology is generally applied to manipulate the propagation of light through continuous media, we show how it can also be used to describe the reflections arising at the interface between a vacuum region and a transformed region with a metamaterial implementation. Furthermore, we establish an analytical model that relates the magnitude of the Goos-Hänchen shift to the underlying geometry of the transformed medium. This model shows how the dependence of the Goos-Hänchen shift on geometric parameters can be used to dramatically enhance the size of the shift by an appropriate choice of permittivity and permeability tensors. Numerical simulations of a beam with spatial Gaussian profile incident upon metamaterial interfaces verify the model and firmly establish a novel route towards Goos-Hänchen shift engineering using transformation optics.

Keywords: Metamaterials, metamaterial interfaces, metasurfaces, transformations optics, Gaussian beam displacement, Goos-Hänchen effect, Goos-Hänchen shift

1. INTRODUCTION

In 1968, Victor Veselago first introduced the notion of negative index materials, exhibiting simultaneous negative permittivity and permeability resulting in a negative refractive index as a theoretical artefact.[1] Contrary to materials readily found in nature, such materials would imply refraction to the opposite side of the normal, known as negative refraction. The theoretical work of John Pendry published in 1996 and 1999 demonstrated how synthetic materials could be designed exhibiting both negative permittivity and negative permeability using metallic microstructures, thus making the theoretical prediction of Veselago a possible reality.[2,3]

Not long after, in 2000, the first left-handed 'metamaterial' was experimentally demonstrated by David Smith: a composite medium consisting of a periodic array of interspaced conducting nonmagnetic split-ring resonators and continuous wires generating negative effective permeability and negative effective permittivity, respectively.[4] Such an artificial left-handed medium would exhibit unique electromagnetic properties not found in nature.[5,6] Veselago already realised that refraction to the opposite side of the normal implies the possiblity of constructing a flat lens. This was further explored by Pendry later that year, who realised that Veselago's lens could not only focus light, but would allow imaging beyond the diffraction limit.[7] This spiked the interest of many research groups, intent on experimental verification of negative refracting media in quest for the perfect lens. A first

Further author information: (Send correspondence to Lieve Lambrechts)
Lieve Lambrechts: E-mail: lieve.lambrechts@vub.ac.be, Telephone: +32 2629 3653

successful experiment displaying negative refraction was set up by Richard Shelby and his colleagues in 2001.[8] The experiment was later repeated and its results confirmed by Andrew Houck and his team at MIT.[9] Figure 1 shows the constructed metamaterial, implementing structured arrays of split-ring resonators, which give rise to a negative effective permeability and straight wires, which provide negative effective permittivity.

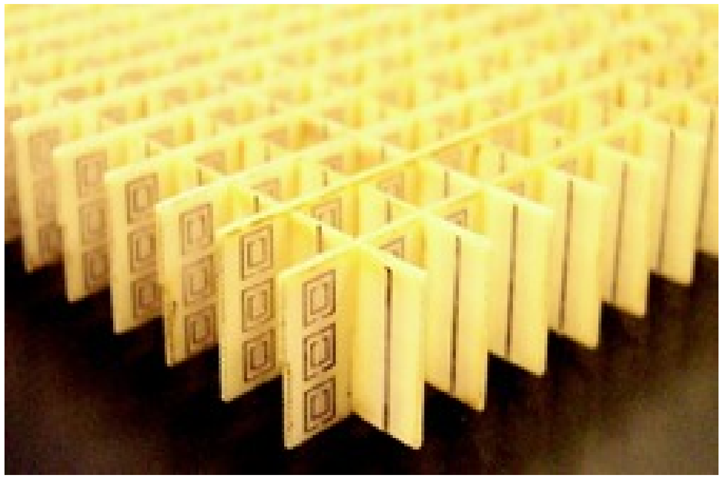

Figure 1: Photograph of the metamaterial structure used in 2003 by Houck *et al.* to demonstrate negative refraction. Negative effective permeability is provided by the periodic array of square split-ring resonators, whereas the arrays of straight wires provide negative effective permittivity. Reprinted figure with permission from Ref. 7. Copyright (2003) by the American Physical Society.

In the following decade, further research showed—what is probably just a first glimpse of—the wide range of designer electromagnetic response made possible by the use of artificially constructed metamaterials.[10–12] By building a material consisting of structured arrays of subwavelength resonant structures, which emulate the role of atoms in matter, the material will behave as a homogenous medium described by an effective refractive index. By engineering the properties of the individual building blocks as well as their periodic arrangement throughout the material, one can achieve a desired electromagnetic response.

2. TRANSFORMATION OPTICS

To design metamaterials exhibiting a specific electromagnetic response, one needs a design procedure that can translate the desired effect on electromagnetic waves into the material parameters needed for the metamaterial medium. Transformation optics is a convenient tool in this regard. Since its theoretical conception in 2006 transformation optics has already shown great versatility in controlling optical properties. At first, transformation optics has allowed for the design of metamaterial devices that modify the trajectory of light, such as beam benders, beam splitters and invisibility cloaks.[13–18] Furthermore, several applications have been devised which manipulate electromagnetic fields up to an unprecedented level, going far beyond the modification of light ray trajectories.[19–30]

In 2006 Ulf Leonhardt and John Pendry independently came to the realisation that the form invariance of Maxwell's equations under coordinate transformations can be used to identify an equivalence between a medium with traditional electromagnetic material parameters, expressed on the background of a curved coordinate system, and a material with exotic material parameters, expressed in a traditional (Cartesian) coordinate system.[31,32]

Following Einstein's general relativity, we know that the presence of large masses introduces curvature in the fabric of spacetime. In this curved spacetime, light rays will propagate along the curved coordinate lines or geodesics. This bending of light bears an analogy to the bending of light that occurs when light propagates through a medium with an inhomogeneous refractive index profile. In this case, light will choose a path of extremal optical path length according to Fermat's principle, resulting in a curved trajectory through the inhomogeneous medium.

This analogy becomes more apparent when considering Maxwell's equations describing the propagation of light in both cases. First,

$$\pm[i,j,k]\frac{\partial E_k}{\partial x^j} = -\frac{\partial \sqrt{g} g^{ij} \mu_0 H_j}{\partial t},$$

$$\pm[i,j,k]\frac{\partial \mu_0 H_k}{\partial x^j} = \varepsilon_0 \mu_0 \frac{\partial \left(\sqrt{g} g^{ij} E_j\right)}{\partial t}$$

show Maxwell's equations in empty space, expressed on the background of a curved coordinate system specified by the metric tensor g_{ij}. Here, $g = \det(g_{ij})$ and $[i,j,k]$ is the permutation symbol, defined by $[i,j,k] = 1$ if (ijk) is an even permutation of (123), $[ijk] = 1$ if (ijk) is an odd permutation of (123), and $[ijk] = 0$ for all other combinations of the triplet (ijk). Second,

$$[i,j,k]\frac{\partial E_k}{\partial x^j} = -\frac{\partial \mu_0 \mu^{ij} H_j}{\partial t},$$

$$[i,j,k]\frac{\partial \mu_0 H_k}{\partial x^j} = \varepsilon_0 \mu_0 \frac{\partial \left(\varepsilon^{ij} E_j\right)}{\partial t}$$

describe Maxwell's equations in a specific material with electric permittivity and magnetic permeability tensors ε^{ij} and μ^{ij}, expressed in a regular Cartesian coordinate system.

An equivalence relation can be established between these sets of equations, relating the metric tensor g_{ij} describing the curved coordinate system to electric permittivity and magnetic permeability tensors ε^{ij} and μ^{ij} as follows:

$$\varepsilon^{ij} = \mu^{ij} = \pm\sqrt{g} g^{ij}.$$

This equivalence relation can then be used to design macroscopic metamaterials that allow for enhanced manipulation of electromagnetic waves based on geometrical deformations.[33,34]

The design procedure usually consists of three steps, as visualized in Fig. 2. We start from light propagation in an empty space expressed in a Cartesian coordinate system [see Fig. 2(a)]. Subsequently, we transform the path of the light ray in a desired way, by changing the coordinate system as shown in Fig. 2(b). Using the equivalence relations of transformation optics, we then obtain the electromagnetic parameters of the specific (meta)material in which light propagates along this desired path [see Fig. 2(c)].

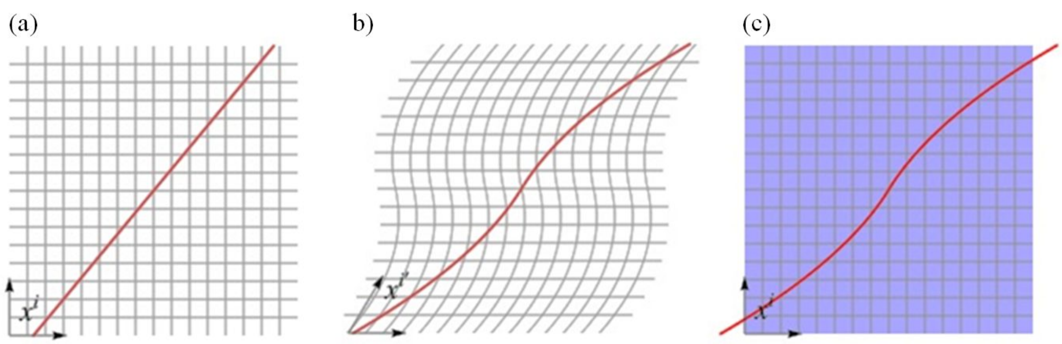

Figure 2: The design procedure of transformation optics: (a) the path of a light ray in empty space expressed in Cartesian coordinates, (b) is transformed under a coordinate transformation. (c) Using the equivalence relations, the effect of this coordinate transformation is implemented in a specific material.

This formalism can thus elegantly be used to transform the path of a light ray in a desired way and subsequently obtain a material implementation from the metric describing the performed coordinate transformation. Recently, there has been a growing interest in using metamaterial interfaces for waveguiding purposes.[35] Although transformation optics was initially developed for the smooth manipulation of light beam trajectories, we will show in this contribution that it can also be used to understand and design optical phenomena at (meta)material interfaces.

3. THE GOOS-HÄNCHEN EFFECT

The Goos-Hänchen effect is such an interface phenomenon. When an electromagnetic beam with finite cross section totally reflects at the interface between two media, the reflected beam experiences a small shift s along the interface—known as the Goos-Hänchen shift—with respect to the beam predicted by geometric optics (see Fig. 3). This effect was observed experimentally for the first time by Hermann Goos and Hilda Hänchen in 1947, employing multiple total internal reflections in a glass slab, hereby significantly increasing the extremely small shift associated with a single reflection.[36]

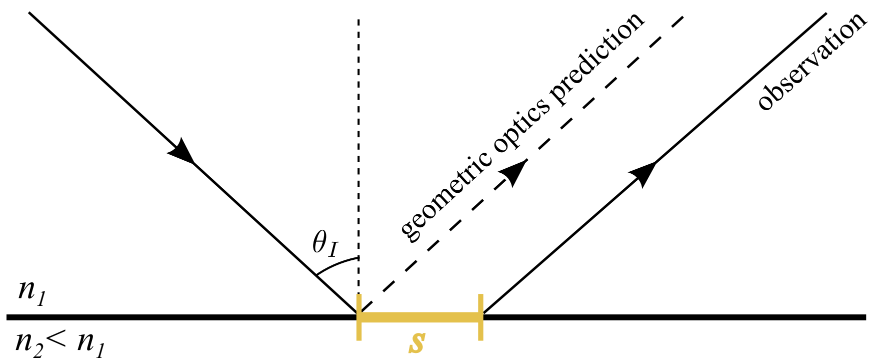

Figure 3: Electromagnetic beams with finite cross section display a lateral shift s, known as the Goos-Hänchen shift, when passing from an optically dense medium to an optically less dense medium.

A theoretical explanation of this reflection phenomenon was originally proposed by Kurt Artmann in 1948 using a stationary phase method.[37,38] Considering the incident beam as the superposition of plane waves that encounter different phase shifts upon reflection, it can be shown that the superposition of these slightly phase-shifted plane waves result in a reflected beam that is laterally displaced along the interface. If ϕ_r denotes the phase of the complex reflection coefficient r, the Goos-Hänchen shift s is given by

$$s = -\frac{\partial \phi_r}{\partial k_I^{\parallel}} = -\frac{1}{k_I \cos\theta_I}\frac{d\phi_r}{d\theta_I},$$

where $k_I^{\parallel}$ represents the component of the incident wave vector parallel to the material interface and θ_I is the angle of incidence.

Over the past few decades, the Goos-Hänchen effect has been extensively investigated among all kinds of optical media, such as dielectrics,[39–41] metals,[42,43] photonic crystals[44,45] and metamaterials such as left-handed media[46] and epsilon-near-zero metamaterials.[47] Cleverly employing various aspects of this effect has led to several applications in sensing[48] and waveguiding.[49] However, an open question remaining in this field is the finding of a scheme to enhance the magnitude of the Goos-Hänchen effect.

In this contribution, we explore a new route towards Goos-Hänchen shift tuning using the geometric paradigm of transformation optics. We illustrate how the Goos-Hänchen shift s can be related to the geometric properties of the reflecting material. This relationship can then be used to dramatically enhance Goos-Hänchen shifts, opening a route towards efficient beam manipulation.

4. GOOS-HÄNCHEN SHIFT ENHANCEMENT AT METAMATERIAL INTERFACES USING TRANSFORMATION OPTICS

Transformation optics has been extensively used to manipulate the propagation of electromagnetic beams through continuous media. In our setup, however, we use this technique to describe the physics that arises at the interface between a vacuum region that remains untransformed, and a transformed vacuum region. Applying a first-principles approach using Maxwell's equations in combination with the appropriate boundary conditions, we obtained analytical expressions for the (complex) reflection coefficients describing the scattering at the interface between an (untransformed) vacuum region and a metamaterial region implementing an isotropic stretching of the coordinates.

Using this analytical model, different reflective regimes can be distinguished, depending on the stretching parameter a of the coordinate transformation and on the angle of incidence θ_I. The derived expressions for the reflection and transmission coefficients were verified numerically by full-wave simulations using a finite-elements solver, both for TE and TM polarization.

The steep variation of the phase of the reflection coefficient with respect to the angle of incidence apparent in the regime of total reflection, i.e. for stretching parameters $a < 1$, implies the presence of a nontrivial Goos-Hänchen effect. From the analytical model, one can obtain expressions for the Goos-Hänchen shift s, depending on the angle of incidence θ_I for a stretching of the coordinates by a. For certain stretching parameters, the minimal Goos-Hänchen shift that can be obtained already exceeds one wavelength, opening up a route towards dramatic Goos-Hänchen shift enhancement.

5. CONCLUSIONS

We illustrated how the Goos-Hänchen shift can be related to the geometric properties of the reflecting metamaterial by using the geometrical formalism of transformation optics. The established relationship between the Goos-Hänchen shift s and the parameter of the coordinate stretching a can then be employed to design metamaterials exhibiting dramatically enhanced Goos-Hänchen shifts, thus opening a route towards efficient beam manipulation.

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