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A mixed-integer optimization solver

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Abstract

Nowdays, the public transport relies heavily on fossil fuels, which leads to global warming and harmful emission to the air. Electrification of the public transportation has become a trend in recent years, aiming to decrease the demand of fossil fuel for people's daily mobility. A great number of studies has been conducted to using electricity to benefit the bus transportation. One of the main challenges is to optimize the distribution of fast charging infrastructures and the size of battery installed in the electric buses.

In this thesis, an optimization method based on the Mixed-integer linear programming (MILP) is developed to explore the possibility of the most cost-effective supply system for the electric bus networks. The challenge of developing the method is to translate a physical bus network to MILP format. It is unlikely that the charger planning and the battery sizing have a general optimal solution for all bus networks since each bus network has its own characteristics which result in different objective functions and constraints. Thus, the developed optimization method is intended to be a useful tool as it allows a quick analysis of the supply system planning for each new bus line being electrified.

The developed method is also tested on a simple bus network, and it is shown how the optimal charging infrastructures vary depending on for instance cost of batteries and number of buses used on the bus line. For the investigated bus line it is found that end-stop charging on one or both endstops is the most cost effective option, for a wide range of battery prices and for bus lines with four or more buses per bus line.

Keywords: Fast Charging, Electric Bus, City Bus Network, Public Transportation, Cost Optimization.

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Yan Huang, Sweden, September 2016

Contents

Notation

Specific Sets

$\mathbb{R}$	Real numbers.
$\mathbb{R}_+$	Nonnegative real numbers.
$\mathbb{Z}$	Integers.
$\mathbb{Z}_+$	Nonnegative integers.

Parameters

C_{ch}	Construction cost (SEK) for building a charger.
C_{trans}	Construction cost (SEK) for building a shared charger at a transfer stop.
C_{bat}	Cost (SEK/kWh) for the batteries.
E_c	Energy consumption (kWh) for travelling between two adjacent stop.
E_r	Recharged energy (kWh) at the stop.
E_b	Battery energy level (kWh) before the bus is recharged at the stop.
E_a	Battery energy level (kWh) after the bus is recharged at the stop.
SoC_{lb}	Absolute lower bound of battery usage.
SoC_{ub}	Absolute upper bound of battery usage.
P_{trans}	Predefined charging power (kW) of the shared charger.
P_{max_1}	Peak power limit for the second segment in the piecewise linear charger cost model.
P_{max_2}	Peak power limit for the third segment in the piecewise linear charger cost model.
a_0, a_1, a_2, a_3	Cost coefficients of the piecewise linear charger cost model. a_0, a_1 are used when the charging power of a charger is between 0 to P_{max_1}. And a_2, a_3 are used when the charging power of a charger is between P_{max_1} to P_{max_2}.
T_d	Dwell time at the stop.

Variables

x_i	Charging power (kW) of a charger on stop i.
z_{trans_j}	Binary variable showing if there is a shared charger on transfer stop j.
x'_i	Transformation variable used for linearization of the charger cost function.
x''_i	Transformation variable used for linearization of the charger cost function.
x_{bat_b}	Capacity (kWh) of the batteries on bus line b.
x_{init_b}	Initial energy (kWh) of the batteries on bus line b.
t_{trans_j}	Charging time (h) of a shared charger on transfer stop j.
$\lambda_{1_i}, \lambda_{2_i}, \lambda_{3_i}$	Binary variables used to build the piecewise linear charger cost model.

Definition

stops	Individual locations where buses pick up or drop off passengers.
transfer stop	Transfer points between bus lines.
bus line	Transit routes. A route is a group of stops that passengers can get to.
trips	Trips for each bus line. A trip is a sequence of two or more stops that occurs at specific time.
dwell times	Time duration that a bus stays in individual stops for each trip.

Chapter 1

Introduction

1.1 Background

In recent years, the dwindling fossil fuel resource and its environment problem has drew public attention worldwide. The increasing environmental awareness urges people to seek a sustainable way for the public transportation. The electric bus system which applies the environmental friendly energy sources is a developing trend to reduce the dependency on fossil fuel. The electric bus is capable of serving the existing schedules and route operated by their conventional counterparts. Due to the continuous growth in fast charging technology, the electric buses can be recharged by using high charging power during the loading and off-loading of the passengers at the bus stops [1]. This enables electric bus to carry a rather small battery and operate endlessly without returning to the depot [2]. The fast-charging concept contributes to a more feasible and less costly charging system compared to the overnight charging. However, the fast-charging concept highly demands the efficient charging location planning for two main reasons. One is that the number of chargers has substantial effect on the investment cost, another is that a charging system is required to guarantee the energy supply to fulfill the bus operation even under demanding circumstances.

1.2 Related Work

Recent literature on electrification of city bus network has paid attention to solve the location and sizing problems of electric bus. These studies are mainly based on two optimization techniques: Genetic Algorithm (GA) and Integer Linear Programming (ILP). Genetic Algorithms (GAs) are adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics [3]. GA is able to offer significant benefits in searching a large state-space, multi-modal state-space, or n-dimensional surface. In 2013, Chun et al. proposed an algorithm dedicated to the placement optimization of electric vehicles public charging stations in a given distribution system [4]. The traffic load varying with the daily time and stochastic charging conditions including starting time, period and charging power are considered as constraints. Similar study conducted by Mehar et al. introducing a GA model that identifies the optimal locations involving the infrastructure investment transportation cost [5]. Furthermore, Pazouki et al. [6] and an Yan et al. [7] proposed placement methods based on GA taking power grid impact into account. The charging stations in relation to the power less and voltage drops are considered together with the traffic network to explore the possibility of optimal charging locations.

Besides Genetic Algorithms, Integer Linear Programming (or Mixed-integer Linear Programming) is another useful tool which is employed in this project. ILP is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers [8]. Due to its simplicity and flexibility, ILP is widely used in different areas for solving complex management problems. A mixed-integer linear programming model is developed by Andrews et al. to place the efficient charging station infrastructure for enhancing the electric vehicle integration though decreasing the traveling distance to charging station [9]. Based on the research of location optimization for electric vehicle, Kunith et al. developed a model using Mixed-integer linear programming for planning and optimization of a fast-charging electric bus system. The study contributes to plan a multi-charging station infrastructure for innovative electric bus systems following a cost optimization approach [10]. The continuous work carried on by Kunith et al. in 2016 inspired a power-switching charging station to approximate the non-linear function of the battery's charging behavior [11]. However, the above existing researches working on optimal planning of charging infrastructure haven't investigated that the number of charging infrastructure is subject to the battery capacity of electric bus. A possible trade off between the size of battery and the number of charging infrastructures is most likely to be needed to minimize the total cost. Moreover, the cost of fast-charging infrastructure has a nonlinear relationship with its equipped charging power. A complex model in MILP to character the cost of charger has been missing from the above studies.

In this project, a piecewise linear charger cost model is used to approximate the nonlinear relationship between the cost and the charging power of a charger. And the optimization method is intend to highlight the trade off between planning charger and battery from economic perspective.

1.3 Purpose and Objective

The purpose of the project is to develop an analysis tool for planning the supply system of future public transport. The cost-optimized bus system solution is expected to have a positive influence on stimulating the development of sustainable transport.

Based on the existing work and the approach introduced in [11], this project is aiming to use mixed-integer linear programming developing an optimization method to minimize the total cost of the supply system for city bus networks. The method determines not only the efficient charging locations and respective charging power of chargers but also minimum required battery capacity of each bus for a bus network while the existing schedules and bus network are respected.

1.4 Thesis Outline

The thesis is structured as following:

The thesis starts with an introduction to the optimization method in chapter 2. In particular, the background knowledge about the mixed integer linear programming is presented at the beginning. It is followed by a introduction of the piecewise linear charger cost model which is responsible for defining the nonlinear relationship between the cost and charging power of the charger. Finally, the objective function and various linear constraints are presented to explain how the optimization method is designed.

Chapter 3 illustrates how the physical bus network is translated to MILP format. An example of the problem formulation is presented as well.

An example of a bus network is presented in chapter 4 to investigate the optimal combination of charging system and battery under different conditions.

In chapter 5, the optimizations of the bus network in different scenarios are performed. These scenarios are used to test different parameters for battery price, allowed battery usage, peak charging power of charger, and dwell time. The possible trade off between the cost of chargers and the cost of batteries is explored from different optimizations. Furthermore, the sensitivity analysis is used to identify which factors have significant influence on the optimal cost.

The thesis ends up in chapter 6 which includes the conclusions and future works.

Chapter 2

Methodology

In the following chapter the bus system is modelled as a mixed-integer linear programming problem which is able to capture the main features of a bus network. By modelling the bus network, operational and technical constraints, the optimization method can find an economical combination of charger distribution and battery size of each bus in the bus network.

2.1 Mixed-Integer Linear Programming

Mixed-integer Linear programming (MILP) is a general framework for solving optimization problems involving both discrete and continuous variables with linear relationships. The use of integer variables greatly expands the scope of useful optimization problems that can be defined and solved compared to basic linear programming [12]. A special case of MILP is when the discrete variables represent alternatives. Such variables are called 0-1, or binary integer variables and can be used to model yes or no decisions [13], for instance whether to build a plant or buy a piece of equipment. A MILP in standard form is expressed as:

$$\begin{aligned}
\min \quad & C^T \begin{bmatrix} x \\ z \end{bmatrix} \\
\text{s.t.} \quad & A \begin{bmatrix} x \\ z \end{bmatrix} \leq b \\
& Aeq \begin{bmatrix} x \\ z \end{bmatrix} = beq \\
& x \geq 0, z \geq 0 \\
& x \in \mathbb{R} \\
& z \in \mathbb{Z}
\end{aligned} \tag{2.1}$$

Where x is the solution vector of non-negative real numbers, z is the solution vector of non-negative integers, C is the coefficient vector representing the linear objective function, A is the linear inequality constraint matrix, b is the linear inequality constraint vector, Aeq is the linear equality constraint matrix, beq is the linear equality constraint vector.

2.1.1 Branch and bound

The classical approach to solve MILPs is the branch-and-bound (BB or B&B) method [14]. Branch-and-Bound is a general search method for solving constraint optimization. When the branch and bound approach is applied to an integer programming problem, it is used in conjunction with the normal non-integer solution approach [15].

The branch and bound algorithm is illustrated as follows where S is set of pending problems and Z is the best cost found so far.

Algorithm 1 Branch and Bound

```
Initial inputs: S := P_0, Z := +∞
while S ≠ ∅ do
    remove P from S; solve LP(P);            ▷ LP is linear programming function
    if LP(P) is feasible then
        Let β be basic solution obtained after solving LP(P);
        if β satisfies integrality constraints then
            if β is optimal for LP(P) then
                if cost(β)<Z then store β; update Z;
                end if
            else return UNBOUNDED
            end if
        else
            if β is optimal for LP(P) ∧ P can be pruned then continue
                Let x_j be integer variable such that β_j ∉ ℤ;
                S := S ∪ {P ∧ x_j ≤ ⌊β_j⌋, P ∧ x_j ≥ ⌈β_j⌉};
            end if
        end if
    end if
    return Z
end while
```

In algorithm 1, the branch-and-bound approach automatically selects one of the nodes P in set S and attempts to solve the LP relaxation of that subproblem. The relaxation might be infeasible, in which case the subproblem is dropped. If the subproblem can be solved and the solution is integer feasible, then its objective value provides an upper bound for Z in the minimization problem; if the solution is not integer feasible, then it defines two new subproblems. Branching continues in this manner until there is no node in set S. At this point the best integer solution found is an optimal solution for MILP. If no integer solution has been found, then MILP is integer infeasible.

It is important to realize that mixed-integer linear programs are non-deterministic polynomial-time hard (NP-hard) [16]. Roughly speaking, this implies that the effort required to solve a mixed-integer linear programming problem grows exponentially with the size of the problem. Although the branch-and-bound approach is unlikely to create every single possible node, the need to explore even a small fraction of the potential number of nodes for a large problem can be resource-intensive [17].

2.2 Piecewise Linear Charger Cost Model

It is worth to mention that the cost for a charger has a nonlinear relationship with the charging power. It can as an example look like the cost function illustrated in Figure 2.1. The nonlinear model consists of a (0,0) point and a nonlinear curve. At the (0,0) point, no cost is required as no charger is built at the stop. Once a charger is required, it is inevitable to pay a rather large amount of construction cost. In this example 1 million SEK is demanded even if the charging power is very low. Increase of the charging power contributes to the cost growth with a decreasing rate as the blue line shown. However, it is noticed that the nonlinear cost model can not be directly modelled in MILP since the objective function and constraints must be strictly linear. Another way to model the relationship between the charging power and charger cost is therefore required.

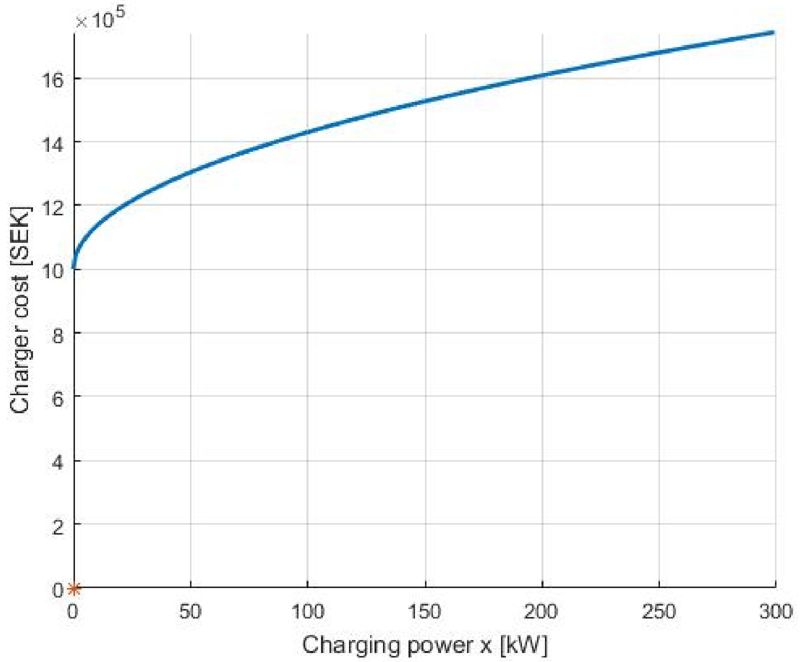

Figure 2.1: The nonlinear charger cost model

Here a linearized model is introduced to approximate the nonlinear model as shown in equation 2.2.

$$C_{ch} = \begin{cases} 0 & x = 0 \\ a_0 + a_1\, x & 0 < x \leq P_{max_1} \\ a_2 + a_3\, x & P_{max_1} < x \leq P_{max_2} \end{cases} \tag{2.2}$$

Similar to the nonlinear cost model, the piecewise linear model is made up of a (0,0) point and two linear curves with different gradients. By assigning the suitable coefficients, the piecewise linear model is able to reflect main features of the nonlinear cost model in Figure 2.2. Where $P_{max_1} = 30kW$ (purple dash line), $P_{max_2} = 300kW$ (orange dash line), $a_0 = 1 \times 10^6 SEK$, $a_1 = 1 \times 10^4 SEK/kW$, $a_2 = 1.24 \times 10^6 SEK$, $a_3 = 2 \times 10^3 SEK/kW$.

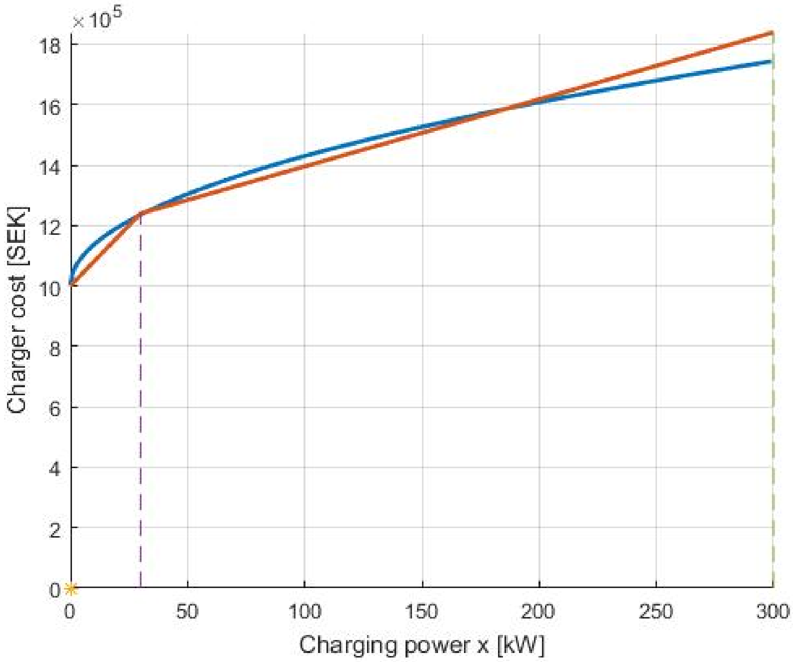

Figure 2.2: The piecewise linear charger cost model

In order to enable MILP to handle this cost model, three binary variables are used to guarantee the cost can be switched between three segments. These binary variables are like a three-phase switch which allows one phase to be 1 at a time. The piecewise linear charger cost model can be expressed by substituting the binary variables λ, the coefficients a_0, a_2 and the slope a_1, a_3.

$$C_{ch} = 0 \cdot \lambda_1 + (a_0 + a_1 x)\lambda_2 + (a_2 + a_3 x)\lambda_3 \tag{2.3}$$

- When $x = 0$, only λ_1 is allowed to be 1, it results in $C_{ch} = 0$.
- When $0 < x \leq P_{max_1}$, only λ_2 is allowed to be 1, it results in $C_{ch} = a_0 + a_1 x$.
- When $P_{max_1} < x \leq P_{max_2}$, only λ_3 is allowed to be 1, it results in $C_{ch} = a_2 + a_3 x$.

Notice that the more binary variables we use, the more accurate the cost model can be. However, it has to be emphasized that increasing size of MILP problem will result in exponential growth of calculation.

The linear approximation is however not finished yet since there are two nonlinear terms, $\lambda_2 \cdot x$ and $\lambda_3 \cdot x$, hiding in equation 2.3. In order to make the equation linear, the below variable transformation is applied to eliminate the nonlinear terms.

$$\lambda_2 \cdot x \iff x'$$
$$\lambda_3 \cdot x \iff x''$$
$$x \iff 0 \cdot \lambda_1 + x' + x''$$

With the help of the variable transformation, the piecewise linear charger cost model is finally capable of being implemented in MILP format. The piecewise linear charger cost model with binary variables and transformed variables becomes

$$C_{ch} = \begin{cases} 0 \cdot \lambda_1 & x' = 0 \wedge x'' = 0 \\ a_0\,\lambda_2 + a_1\,x' & 0 \leq x' \leq P_{max_1}\,\lambda_2 \\ a_2\,\lambda_3 + a_3\,x'' & P_{max_1}\,\lambda_3 \leq x'' \leq P_{max_2}\,\lambda_3 \\ \lambda_1 + \lambda_2 + \lambda_3 = 1 & \lambda_1, \lambda_2, \lambda_3 \in \{0, 1\} \end{cases} \tag{2.4}$$

2.3 Optimization Method

2.3.1 Objective function

The charging infrastructures and battery of the buses play the most essential roles in the cost of an electric bus system. An objective function for a bus system with b bus lines therefore can be expressed as the sum of the cost for the chargers at normal stops (C_{ch}) plus the cost for the shared chargers at transfer stops (C_{trans}) plus the cost for bus batteries (C_{bat}).

$$\min \quad \sum_i C_{ch}\, x_i + \sum_j C_{trans}\, z_{trans_j} + \sum_b C_{bat}\, N_{bus_b}\, x_{bat_b} \tag{2.5}$$

Note that the bus system has N_{bus} identical buses serving on each bus line. The optimization variable $x_i \in \mathbb{R}_+$ indicates the optimal charging power of the charger placed at stop i, and no charger will be built at stop i when $x_i = 0$. The binary variable z_{trans_j} identifies whether it is cost-saving to build a shared charger at transfer stop j. The battery capacity $x_{bat_b} \in \mathbb{R}_+$ is another optimization variable having a significant impact on the objective function.

As discussed in previous section, the piecewise linear charger cost model in equation 2.4 can be applied in the objective function to enhance the usefulness of the optimization method. Therefore, a more complex objective function is obtained to solve the optimization problem.

$$\min \quad \sum_i (a_0\, \lambda_{2_i} + a_1\, x'_i + a_2\, \lambda_{3_i} + a_3\, x''_i) + \sum_j C_{trans}\, z_{trans_j} + \sum_b C_{bat}\, N_{bus_b}\, x_{bat_b} \tag{2.6}$$

where $x'_i, x''_i \in \mathbb{R}_+$ are the transformed variables and the binary variables $\lambda_{1_i}, \lambda_{2_i}, \lambda_{3_i}$ are used for switching the cost segments of the charger.

2.3.2 Linear constraints

Numerous requirements in terms of the battery energy, operation and so on are required to guarantee the bus system is able to follow the daily bus time table. The requirements can be translated to following linear equality and inequality constraints.

The technical constraints are needed to ensure the piecewise linear cost model for chargers to switch correctly between the different linear segments. At stop i, the binary variables $\lambda_{1_i,2_i,3_i}$ and transformed variables x'_i, x''_i have to fulfill the following linear relationships.

$$0 \le x'_i \le P_{max_1}\, \lambda_{2_i} \tag{2.7}$$

$$P_{max_1}\, \lambda_{3_i} \le x''_i \le P_{max_2}\, \lambda_{3_i} \tag{2.8}$$

$$\lambda_{1_i} + \lambda_{2_i} + \lambda_{3_i} = 1 \tag{2.9}$$

Furthermore, numerous energy constraints are inevitable to reflect the network layout and the charging procedure. Since the energy consumption for traveling and the dwell time at each stop are known inputs to the optimization, the battery energy at each stop can be calculated.

At stop i or transfer stop j for each trip n and for each bus line b, the energy E_r recharged at different type of stops is the multiplication of the charging time and the charging power. If the current stop is transfer stop j, then the recharged energy $E_{r_{j,n,b}}$ equals to the real variable $t_{j,n,b}$ of the shared charger j times the constant charging power P_{trans}. Otherwise, the recharged energy $E_{r_{i,n,b}}$ at current stop i equals to the constant dwell time $T_{d_{i,n,b}}$ times the optimized charging power $(x'_i + x''_i)$ of charger i.

$$E_{r_{i,n,b}} = T_{d_{i,n,b}} \, (x'_i + x''_i) \tag{2.10}$$

$$E_{r_{j,n,b}} = P_{trans} \, t_{j,n,b} \tag{2.11}$$

The battery energy $E_{a_{k,n,b}}$ after the bus is charged up at current stop k is equal to the energy level at the previous stop ($E_{a_{k-1,n,b}}$) plus the energy recharged at the current stop ($E_{r_{k,n,b}}$) minus the energy consumption for traveling from the previous stop ($E_{c_{k,n,b}}$). On the other hand, the battery energy $E_{b_{k,n,b}}$ before the bus is charged up at current stop k is equal to the energy level at the previous stop ($E_{a_{k-1,n,b}}$) minus the consumption for traveling from the previous stop ($E_{c_{k,n,b}}$).

$$E_{a_{k,n,b}} = E_{a_{k-1,n,b}} + E_{r_{k,n,b}} - E_{c_{k,n,b}} \tag{2.12}$$

$$E_{b_{k,n,b}} = E_{a_{k-1,n,b}} - E_{c_{k,n,b}} \tag{2.13}$$

$$E_{b_{1,1,b}} = x_{init_b} \tag{2.14}$$

Where $k \in \{k | k = i \vee k = j\}$ is the suffix of the stop sequence of daily transit. Note that the first element of the E_b is the initial battery energy recharged over night at the depot. The pre-charged energy $x_{init_b} \in \mathbb{R}_+$ has a substantial impact on the charger requirement at the first stop. For instance, the first stop charging is required but the battery is fully recharged over night. In this case, the battery is forced to increase the size in order to make up the absence of recharged energy from the first charger. Thus, an optimization of the initial energy stored in batteries is needed in order to allow the optimization to put charger also on the first bus stop on the bus line.

It is obvious that the battery energy at each stop has to be maintained within the battery usage boundary. Before the bus arrives at the current stop k, the energy $E_{b_{k,n,b}}$ stored in the battery must never be lower than the security margin SoC_{lb} of battery x_b. And the energy $E_{a_{k,n,b}}$ must never exceed the maximum usage margin SoC_{ub} of battery x_b when the bus has finished charging at current stop k.

$$E_{b_{k,n,b}} \geq SoC_{lb} \cdot x_{bat_b} \tag{2.15}$$

$$E_{a_{k,n,b}} \leq SoC_{ub} \cdot x_{bat_b} \tag{2.16}$$

In order to allow readers a better understand of the energy calculations, an energy profile of several stops is illustrated in Figure 2.3. It can be seen that only bus stop 5 has a charger in this trip. The energy drop between stops is caused by the energy consumption for bus traveling. It can also be observed in the figure that all the battery energy point must be between the lower and upper bounds for battery usage.

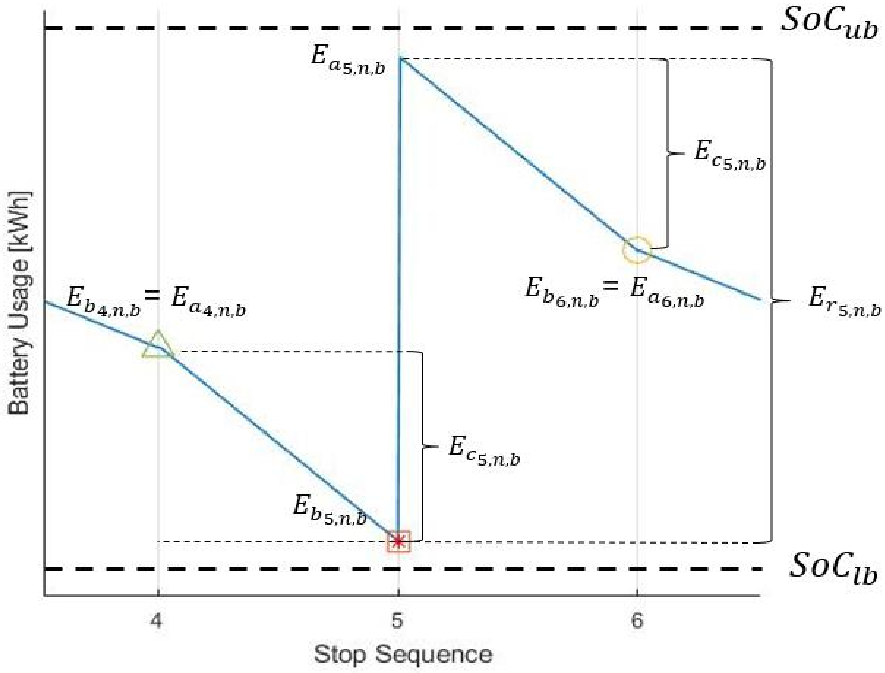

Figure 2.3: An example of the energy calculations

The constraints derived so far are insufficient to make the solutions a convex set. One more constraint is required to restrict the charging time at each transfer stop.

$$\frac{t_{j,n,b}}{T_{d_{j,n,b}}} \leq z_{trans_j} \tag{2.17}$$

In equation 2.17, the constraint reflects the charging time $t_{j,n,b}$ must not be longer than the dwell time $T_{d_{j,n,b}}$ at the transfer stop j for the trip n and for the bus line b. If the optimization says the battery has to be recharged at the current transfer stop j, the charging time $t_{j,n,b}$ becomes greater than 0 and the the binary variable z_{trans_j} is set to 1. Otherwise the binary variable remains 0 and no charger will be constructed at the transfer stop j.

One thing which needs to be emphasized is that the recharged energy at a certain bus stop is equal to the product of the charging time and the charger power. However, since MILP cannot handle the product of two variables, it is only possible to optimize either the charging time, and then the charging power must be constant, or to optimize the charger power and then the charging time has to be constant (Note that two multiplied variables can not be optimized at the same time since MILP is a strictly linear solver). In order to achieve different amounts of recharged energy for different bus lines at a shared charger, the charging time for the charger is optimized instead of optimizing the charging power. The reason is that the charging time is independent of the charger cost. Varying charging time according to the different bus line results in less effort on enlarging size of MILP problem. On the contrary, using charging power as the variable at a transfer stop will lead to different construction costs for one shared charger. In this case, more binary variables and transformation variables are therefore required to decide which charging power of the shared charger contributes to the optimal energy profile and cost.

Chapter 3

Modelling Bus Network in MILP

Building the mathematical model for the physical bus network is one of the challenges in this project. In this chapter, MILP concepts are used to describe the real bus network.

3.1 Mathematical Model Formulation

It is important to note that MILP must model time as discrete steps. However, the time steps don't have to be constant, instead we can model only the time instances when the energy in the battery can be increased. This means that we only model every time the bus stays at a bus stop as one time step, irrespective of the varying time it takes to drive between the different bus stops. Therefore the specific time for the bus arriving and leaving the stops has not factored in the mathematical model. Instead, the daily time table is translated to the stops in sequence equipped with the duration of bus staying in the stop.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & SoC_{lb} & -SoC_{ub} \\
-T_{d_{1,1,b}} & -T_{d_{1,1,b}} & 0 & 0 & SoC_{lb} & -SoC_{ub} \\
-T_{d_{1,1,b}} & -T_{d_{1,1,b}} & -T_{d_{2,1,b}} & -T_{d_{2,1,b}} & SoC_{lb} & -SoC_{ub} \\
-\sum_{n=1}^{2} T_{d_{1,n,b}} & -\sum_{n=1}^{2} T_{d_{1,n,b}} & -T_{d_{2,1,b}} & -T_{d_{2,1,b}} & SoC_{lb} & -SoC_{ub} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\sum_n T_{d_{1,n,b}} & -\sum_n T_{d_{1,n,b}} & -\sum_n T_{d_{2,n,b}} & -\sum_n T_{d_{2,n,b}} & SoC_{lb} & -SoC_{ub} \\
T_{d_{1,1,b}} & T_{d_{1,1,b}} & 0 & 0 & -SoC_{lb} & SoC_{ub} \\
T_{d_{1,1,b}} & T_{d_{1,1,b}} & T_{d_{2,1,b}} & T_{d_{2,1,b}} & -SoC_{lb} & SoC_{ub} \\
\sum_{n=1}^{2} T_{d_{1,n,b}} & \sum_{n=1}^{2} T_{d_{1,n,b}} & T_{d_{2,1,b}} & T_{d_{2,1,b}} & -SoC_{lb} & SoC_{ub} \\
\sum_{n=1}^{2} T_{d_{1,n,b}} & \sum_{n=1}^{2} T_{d_{1,n,b}} & \sum_{n=1}^{2} T_{d_{2,n,b}} & \sum_{n=1}^{2} T_{d_{2,n,b}} & -SoC_{lb} & SoC_{ub} \\
\sum_{n=1}^{3} T_{d_{1,n,b}} & \sum_{n=1}^{3} T_{d_{1,n,b}} & \sum_{n=1}^{2} T_{d_{2,n,b}} & \sum_{n=1}^{2} T_{d_{2,n,b}} & -SoC_{lb} & SoC_{ub} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\sum_n T_{d_{1,n,b}} & \sum_n T_{d_{1,n,b}} & \sum_n T_{d_{2,n,b}} & \sum_n T_{d_{2,n,b}} & -SoC_{lb} & SoC_{ub}
\end{bmatrix}
\begin{bmatrix}
x'_1 \\ x''_1 \\ x'_2 \\ x''_2 \\ x_{bat} \\ x_{init}
\end{bmatrix}
\leq
\begin{bmatrix}
-E_{c_{1,1,b}} \\
-\sum_{i=1,2} E_{c_{i,1,b}} \\
-\sum_{i=1,2,1} E_{c_{i,2,b}} \\
-\sum_{i=1,2,1,2} E_{c_{i,2,b}} \\
\vdots \\
-\sum_{i,n} E_{c_{i,n,b}} \\
E_{c_{1,1,b}} \\
\sum_{i=1,2} E_{c_{i,1,b}} \\
\sum_{i=1,2,1} E_{c_{i,2,b}} \\
\sum_{i=1,2,1,2} E_{c_{i,2,b}} \\
\sum_{i=1,2,1,2,1} E_{c_{i,3,b}} \\
\vdots \\
\sum_{i,n} E_{c_{i,n,b}}
\end{bmatrix}
$$

Figure 3.1: The bus system in MILP format

For each bus serving a bus line, all stops are required to be visited in sequence to fulfill the daily time table. And each stop on different trip can be considered as a node which captures the energy left in battery of the bus. The node contains the information of the energy consumption between two adjacent stops, dwell time for the bus, allowed peak power for potential charger and so on. The Energy calculation for each node can be represented as inequality linear constraints in MILP. Finally, the bus network layout, operating schedule and battery energy profile are reflected in the linear constraints.

The mathematical model for a two-stop bus line is presented in Figure 3.1. It is noticed that each inequality linear constraint describes the battery energy at the node. The left matrix contains the accumulated working time (T_d) for each charger and the allowed boundary for battery usage (SoC_{lb}/SoC_{ub}). On the other hand, the energy that the bus has consumed from the initial stop of the day makes up the right vector. Furthermore, the linear constrains in blue color expresses the energy level after the bus is charged up at current stop, which must never be higher than the maximum margin of the battery according to the equation 2.12 and 2.16, while the red constraints represent that the battery energy before being charged up at the current stop must never be lower than the minimum usage boundary (equation 2.13 and 2.15).

Note that in order to fulfill the algebra criteria, two identical columns in the left matrix are applied to matching the transformed variables x_i' and x_i''.

Chapter 4

Application Case

An example bus network will be presented in this chapter in order to investigate what is the optimal combination of battery and charging system under different conditions.

4.1 2-line Bus Network

The **Bus Network** consists of two simplified bus lines. Each bus line covers two terminal stops and one middle stop. One of terminal stops is a transfer stop shared between the two bus lines. Note that the stop in between has two potential charging spot for outward and return direction. In order to distinguish them in each trip, the stop is split into two individual stops along the bus line implying respective direction as shown in Figure 4.1.

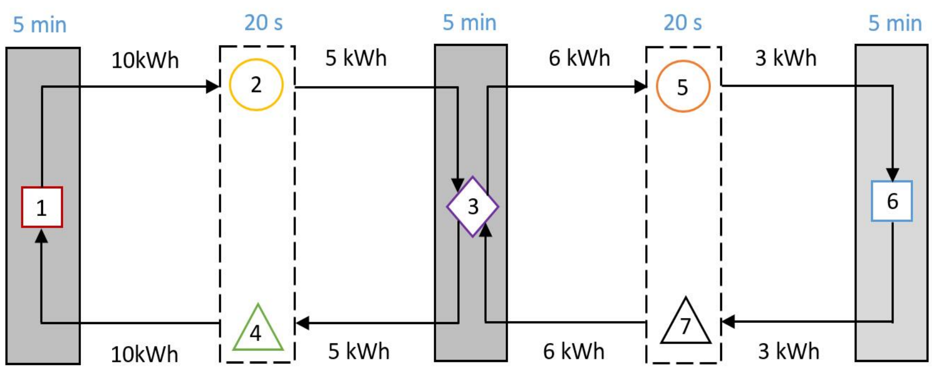

Figure 4.1: 2-line Bus Network

A **Bus Trip** is a sequence of two or more stops that occurs at specific time. In each bus route two service trips are represented according to the outward and return trip between terminal stops. In order to deal with peak transit demand, 4 buses and 16 times round trips with identical length related to daily bus schedule of real transport system are simulated for each bus line.

The **Energy Consumption** for a stop is the energy required from the previous stop on the trip. The energy consumption for each stop is illustrated in Table 4.1. During a whole day a total 470 kWh and 285 kWh of energy is consumed, for bus line 1 and 2 respectively. These daily energy consumptions correspond to 16 round trips of each bus line. According to the 1.6 kWh/km of energy demand per distance, the electric bus system satisfies transit demand of 300 km/day and 180 km/day. Since the energy consumption from/to depot has not been addressed in the bus system, the bus sets out at the terminal stop directly without any extra energy consumption. However, the model would allow the depots to be included. They can just be modelled as some more bus stops. Apart from the energy consumption from/to depot, the influence of traffic volume and auxiliary heating should be considered in the future works.

The **Dwell Time** represents the duration that a bus arrives at and departs from individual stops for each trip. According to the Table 4.1, dwell time of 5 minutes is predefined for terminal stops to ensure sufficient charging energy and the buses can stay in the normal stops for 20 seconds.

Table 4.1: Energy consumption and dwell time for each stop

	Line 1		**Line 2**	
	Energy Demand	*Dwell Time*	*Energy Demand*	*Dwell Time*
Stop 1	10 kWh	5 $mins$	-	-
Stop 2	10 kWh	20 s	-	-
Stop 3	5 kWh	5 $mins$	6 kWh	5 $mins$
Stop 4	5 kWh	20 s	-	-
Stop 5	-	-	6 kWh	20 s
Stop 6	-	-	3 kWh	5 $mins$
Stop 7	-	-	3 kWh	20 s

The **Cost parameters** involve the investment of the chargers and the price of batteries. The cost parameters of the chargers presented in Table 4.2 are available from [18]. In addition, a fixed charging power amounting to 300 kW can be supplied in potential transfer stop. In this case, 1.84 MSEK will be invested if planning a shared charger on the transfer stop benefits the optimal cost. As for the price of battery, 15000 SEK/kWh, it comes from electric bus system available on global market [19].

Table 4.2: Cost parameters for the piecewise linear charger cost model

Parameter		Parameter	
P_{max_1}	30 kW	P_{max_2}	300 kW
a_0	1×10^6 SEK	a_1	1×10^4 $kSEK/kW$
a_2	1.24×10^6 SEK	a_3	2×10^3 $MSEK$

The **Battery Usage Boundary** defines a allowed region of battery to charge and discharge. Many studies reveal that the depth of discharge (DoD) is correlated to capacity fade [20]. Small depth of discharge (DoD) makes the battery last many more cycles than big DoD cycles. Thus, in order to reduce the capacity fading process and extend the longevity of the battery, one of effective ways is to restrict battery staying within a security range [21]. 30% to 70% SoC is the predefined security range in this case.

Chapter 5

Results and Analysis

In this project an optimization method for the supply systems of a electric city bus network is developed and applied to identify the required chargers and batteries of the specific bus network. The example bus network introduced in previous chapter will be a test case for the optimization method. Optimizations of original example network and different scenarios reflecting other conditions will be performed. The obtained results will be illustrated and analyzed in this chapter.

5.1 Influence of Battery Price

The battery price is one of the substantial factors when it comes to the cost of the electric bus system. By varying the battery price, the possible trade-off relationship between the chargers and battery sizing will be investigated in this section. In order to make it easier to illustrate the result, line 1 of the 2-line bus network will be investigated individually as shown in Figure 5.1. The terminal stop 3 is therefore not modelled as a transfer stop but instead modelled as all the other stops along the bus line.

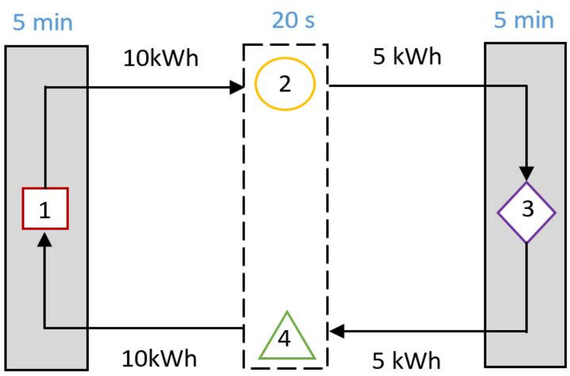

Figure 5.1: The layout and operating requirements of individual bus line

Where □ is the initial stop of each round trip; ○ is a stop on the outward trip; ◊ is the terminal stop; △ is a stop on the return trip.

The individual bus line inherits the characteristic of the line 1 in example bus network. The parameters of individual bus line are shown in Table 5.1.

Table 5.1: Individual Bus Line Scenario

Line 1 Scenario	
Maximum Charging Power	300 kW
Allowed Battery Usage	30%–70% SoC
Dwell Time at stops (except terminal stops)	20 s
Dwell Time at terminal stops	5 min
Number of Buses	4
Number of Round Trip	16 $trip/day$
Length of Round Trip	18.75 km
Energy Demand	1.6 kWh/km

5.1.1 Variation of battery price

The optimization is carried out several times with varying the battery price from zero to infinity SEK/kWh. Depending on the battery price, four different optimal solutions are obtained.

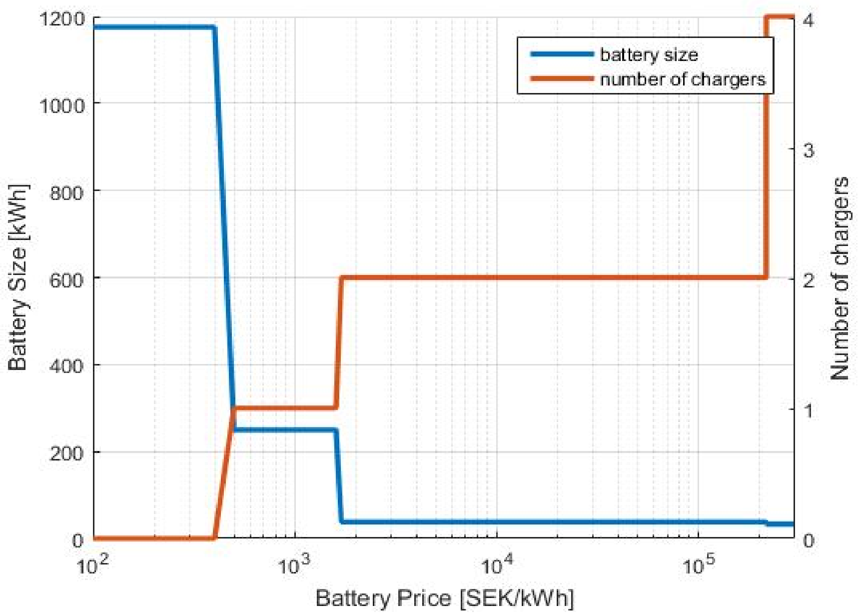

Figure 5.2: The variation of the battery size and the number of chargers

Figure 5.2 illustrates how the optimization method deals with the increasing battery price. Once the cost of the batteries reaches 400 SEK/kWh, the battery size is reduced dramatically to 250 kW and then in two more steps decreases to 37.5 kWh at 1800 SEK/kWh, and finally downs to 33 kWh at 216000 SEK/kWh. The number of chargers on the other hand increases gradually. The two-charger solution which has a wide price range is likely a reasonable solution. As a reference, it can be said that battery prices for commercial vehicles today range from about 3000 SEK/kWh up to 15000 SEK/kWh. Future projections for batteries to electric vehicles point towards prices as low as 1000 SEK/kWh, but it is uncertain if battery price for commercial vehicles will get that low. For today's battery prices the two-charger solution is the best, and if prices continue to drop significantly the one charger solution may be a future option (for this bus line). It also can be seen that the four-charger solution is optimal only when the battery is extremely expensive, like more than 0.2 MSEK/kWh. Therefore, it is not a feasible solution in real life. Each optimal solution at the particular range of the battery price will be discussed from the energy aspect as follow:

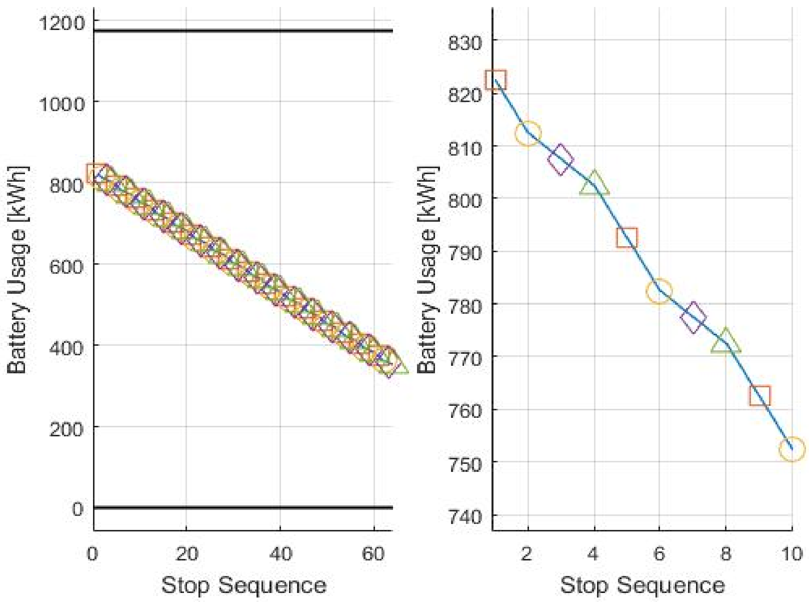

Figure 5.3: The energy profile of cheap battery and zoom-in figure

Solution 1: As it can be seen from Figure 5.3, a extremely huge battery is used when the battery price is set to 400 SEK/kWh or lower. The zoom-in figure shows that no chargers are necessary along the bus line, which is similar to the over-night charging bus. The size of the battery is mainly determined by the energy consumption. 470 kWh usage of battery satisfies up to 300 km daily transit demand. Since only 40% of the battery capacity is allowed to be used, a battery of 1175 kWh is required.

Solution 2: For a range of battery price from 500 to 1700 SEK/kWh, we obtain a one-charger solution. With the increase of the battery price, the huge battery is no longer optimal. An effective way to reduce the battery size is to build only one charger at the initial stop as Figure 5.4 shown. The 16 times with the end-stop charging result in an almost 4 times smaller battery. It is noteworthy that the savings that the reduction of the battery leads to, is only sufficient to pay for one charger. That is why there is still a 5 kWh energy drop in battery after each round trip, as the maximum charger power has been set to 300 kW and the bus only has

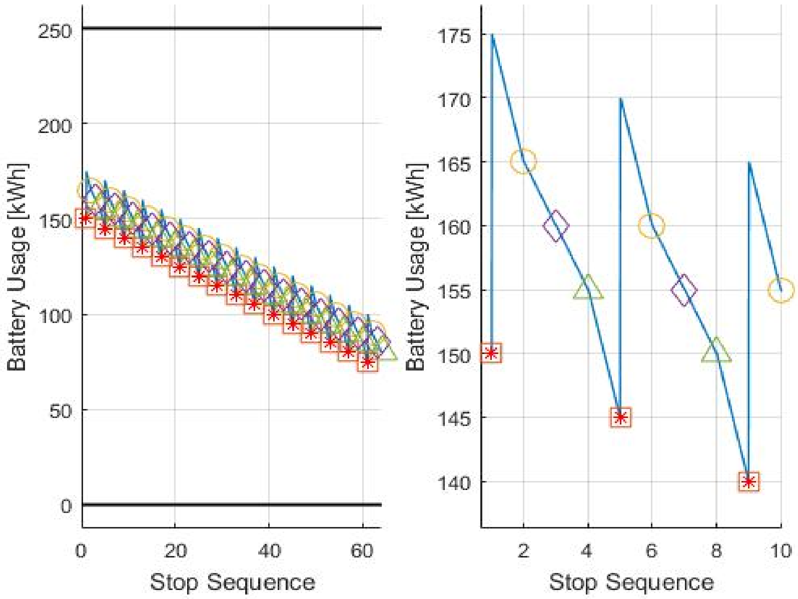

Figure 5.4: The energy profile of inexpensive battery and zoom-in figure

5 minute dwell time.

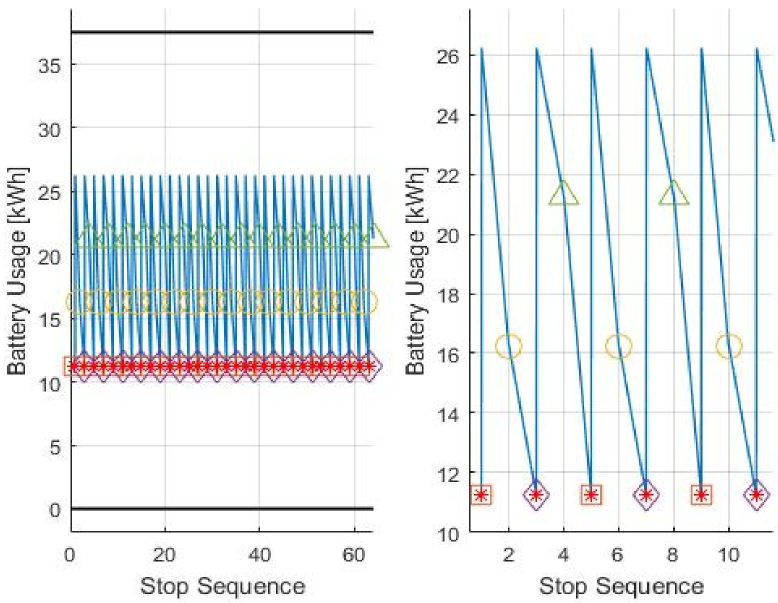

Figure 5.5: The energy profile of expensive battery and zoom-in figure

Solution 3: The effect of even more expensive batteries (1800 – 215900 SEK/kWh) leads to a two-charger solution. In this case, the energy drop as mentioned above is eliminated to maintain a small battery. The energy curve in Figure 5.5 becomes cyclic and stable since supplied charging energy satisfies the energy requirement of each round trip. Both terminal stops are chosen to feed the battery and a rather small battery of 37.5 kWh is used.

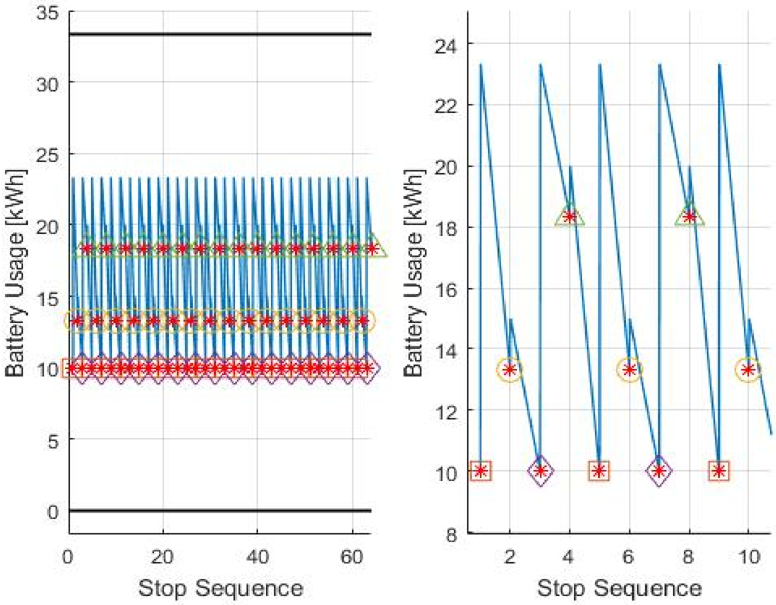

Figure 5.6: The energy profile of extremely expensive battery and zoom-in figure

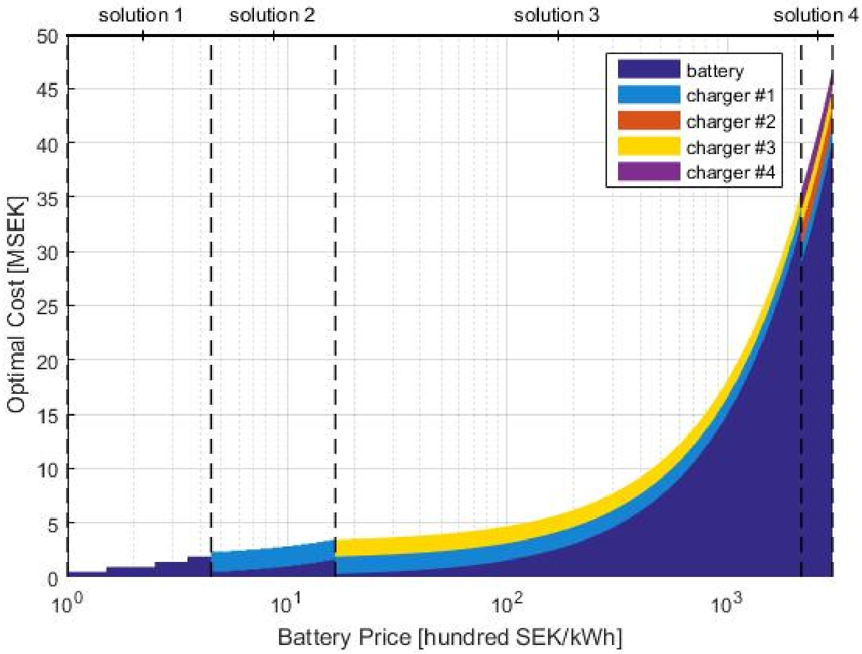

Figure 5.7: The optimal cost with the increasing battery price

Solution 4: The size of battery will finally converge to 33.3 kWh fed by four chargers as the battery price is 216000 SEK/kWh or higher. It can be observed from Figure 5.6 that the size of battery is determined by the highest energy consumption between two adjacent stops. Obviously the battery size can not become any smaller, otherwise the bus will run out of the stored energy and stop somewhere in the middle of a trip.

In addition to the energy profile of the optimal solutions, it is also interesting to analyse the trade off between charger planning and battery sizing from the cost aspect. The optimal cost for each particular battery price can be observed in Figure 5.7. Once additional chargers are introduced to the bus system, the battery cost will be decreased substantially. However, the increasing price of battery will eventually result in a higher batteries cost. It is noticed that many existing electric bus lines in real life use end-stop charging, which has a low cost of batteries.

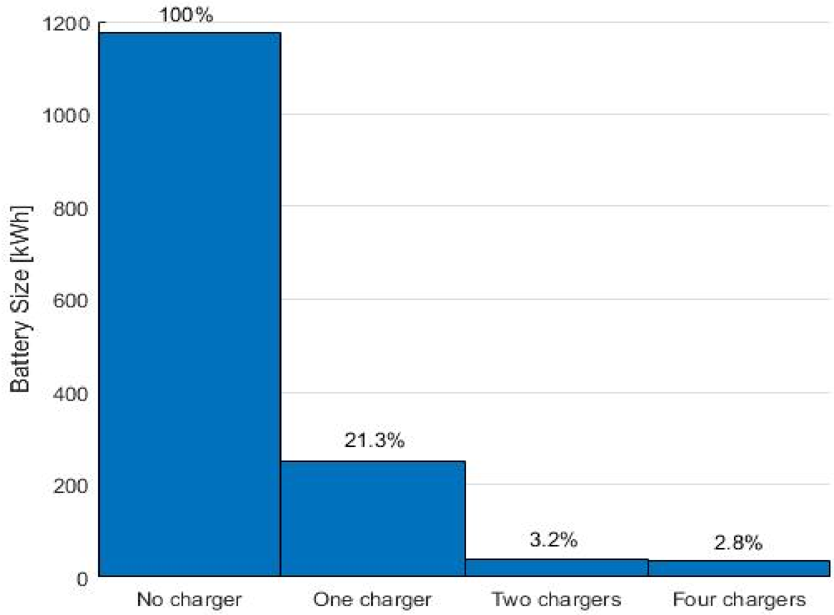

Figure 5.8: The variation of chargers and battery size between solutions

Figure 5.8 shows the variation of chargers and battery size between solutions. The battery shrinks its size by around 80% while the one-charger solution becomes optimal. By building chargers at two terminal stops, up to almost 96.8% of battery size is decreased comparing to the original huge battery of 1175 kWh. The battery size of 33.3 kWh is suggested by the solution where four chargers are built. However, it is not economical to obtain only less than 1% reduction of the battery size by paying for two more chargers. Therefore, two-charger solution is most effective to save the battery size.

5.1.2 Variation of number of buses

In fact, the price of battery can not be varied in such wide price span as discussed above. However, according to the transit demand more or less than four buses can be required on each bus line. Since the number of buses directly multiplies the battery cost in the objective function, it will influence what is the optimal charger solution. The results of the 1 bus, 4buses and 16 buses will be presented respectively.

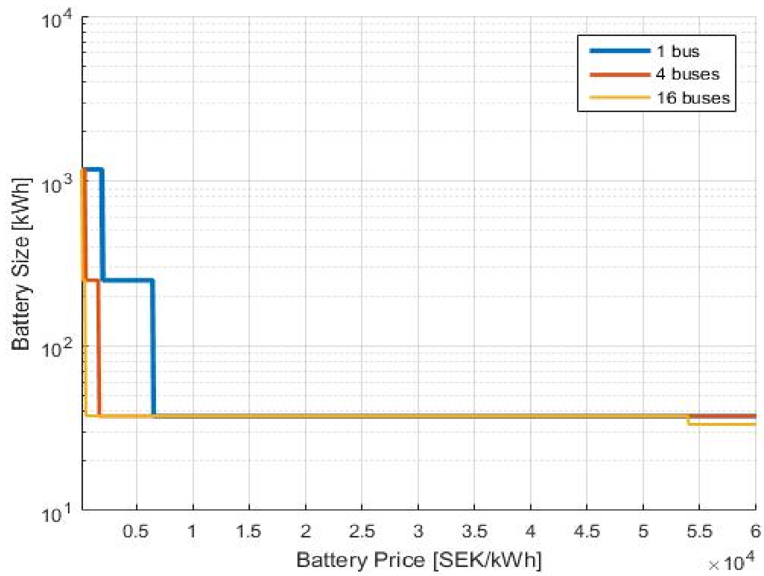

Figure 5.9: The variation in battery size for different number of buses serving the investigated bus line

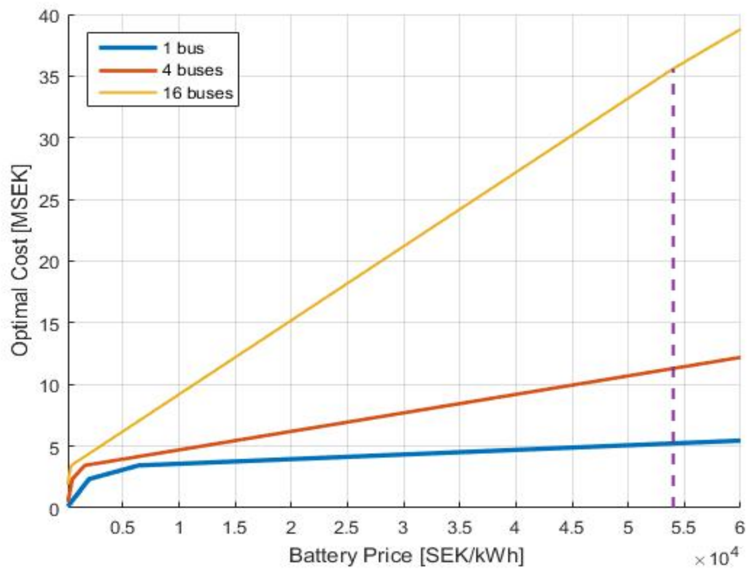

Figure 5.10: The variation in optimal cost for different number of buses

As it can be observed in Figure 5.9, the two-charger solution begins to give the optimal cost at the battery price of 6400 SEK/kWh while only one bus is running back and forth between two end stops. However, this situation arises at the battery price of 1600 SEK/kWh while 4 buses are used and 400 SEK/kWh for 16 buses. It is obvious that the price span is extended and shrank by exactly 4 times when one bus or 16 buses are used respectively. Within a price range from 6400 to 54000 SEK/kWh, the charger requirement and battery size are the same for 1, 4 and 16 buses.

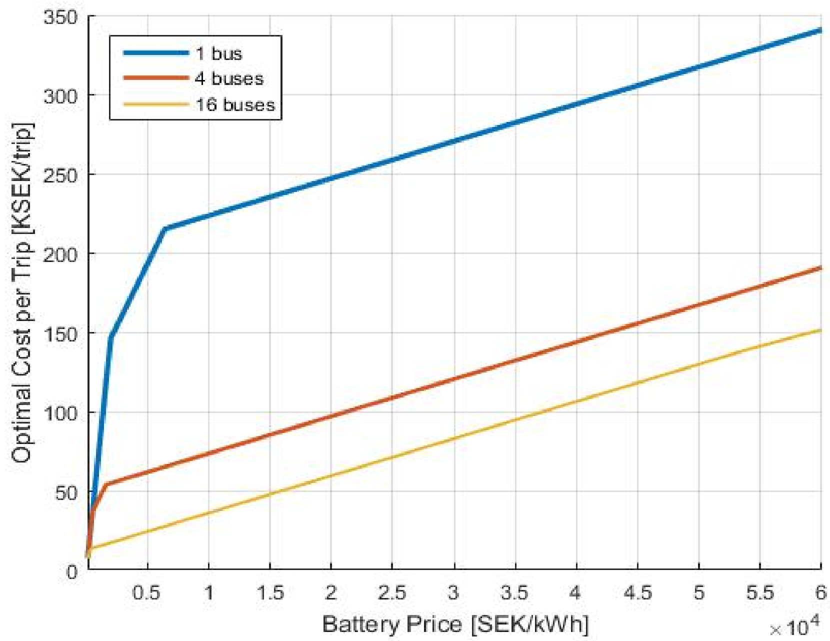

Figure 5.11: The variation of optimal cost per trip for different number of serving buses

The optimal cost is illustrated in Figure 5.10 where solutions cause the slope varying in each curve. Note that only yellow curve reaches the four-charger solution before the battery price amounts to 600000 SEK/kWh. To better understand the effect on the specific cost of operating the bus we can look at the optimal cost per trip, as shown in Figure 5.11. From that figure it is clear that the more buses are used, the less cost is spent on each round trip, event though Figure 5.10 showed that the total cost was increasing with the number of buses.

5.2 Influence of Other Factors

5.2.1 Scenario description

The diversity of operating requirements requires different value of the parameters, such as different peak charging power, different dwell time and so on. Then it is interesting to study the optimal cost according to the different parameters. A sensitivity analysis is used to identify which factors have a significant impact on the optimal cost of chargers and batteries. The main factors of the investigation are listed as follow.

- **Peak charging power**
- **Dwell time at terminal stops**
- **Allowed battery usage**
- **Battery price (number of buses)**

The analysis begins with setting up a baseline scenario which results in a reference charger solution and a reference cost. For the baseline scenario, the predefined parameters are presented in Table 5.2. All parameters are applied to the both bus lines.

Table 5.2: Baseline Scenario

Baseline Scenario	
Maximum charging power	300 kW
Allowed battery usage	30%–70% SoC
Battery price	15000 SEK/kW
Dwell time at stops (except terminal stops)	20 s
Dwell time at terminal stops	5 min
Number of buses on bus line 1	4
Number of buses on bus line 2	4

The sensitivity analysis is conducted by only one parameter variation for each optimization. And each investigating parameter is decreased and increased by 50% respectively. Therefore, in total 8 optimizations are performed.

Table 5.3: Parameter variations

Scenario variation	-50% / 50%
Maximum charging power	150 kW / 450 kW
Allowed battery usage	40%–60% SoC / 20%–80% SoC
Dwell time at terminal stops	2.5 min / 7.5 min
Battery price	7500 SEK/kW / 22500 SEK/kW

5.2.2 Results

The final results for the baseline scenario are illustrated in Figure 5.12. The selected chargers are located at terminal stop 1 and shared terminal stop 3. The charger at stop 1 supplies 180 kW of the charging power while the 300-kW charger is shared with two bus lines at stop 3. In the bus line 2, the batteries are allowed to be recharged 3.6 minutes each round trip at the transfer stop. On the other hand, two chargers are built to supply the line 1 since more energy consumption up to 470 kWh is required.

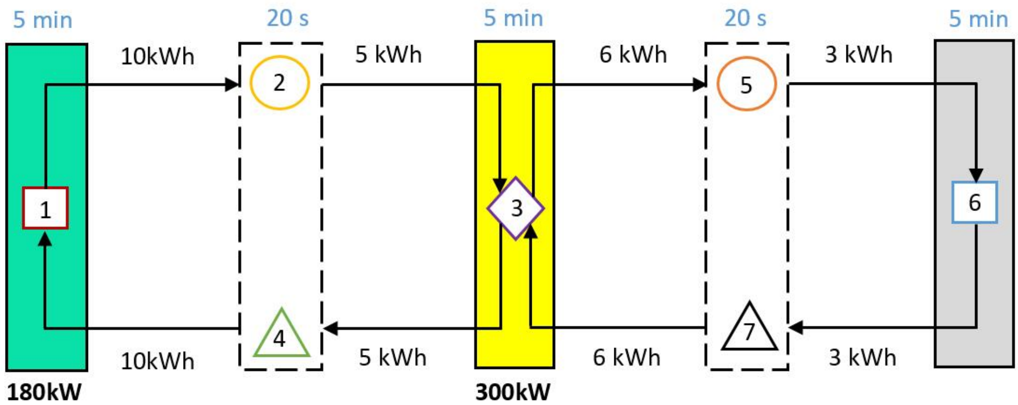

Figure 5.12: The charger requirements in baseline scenario

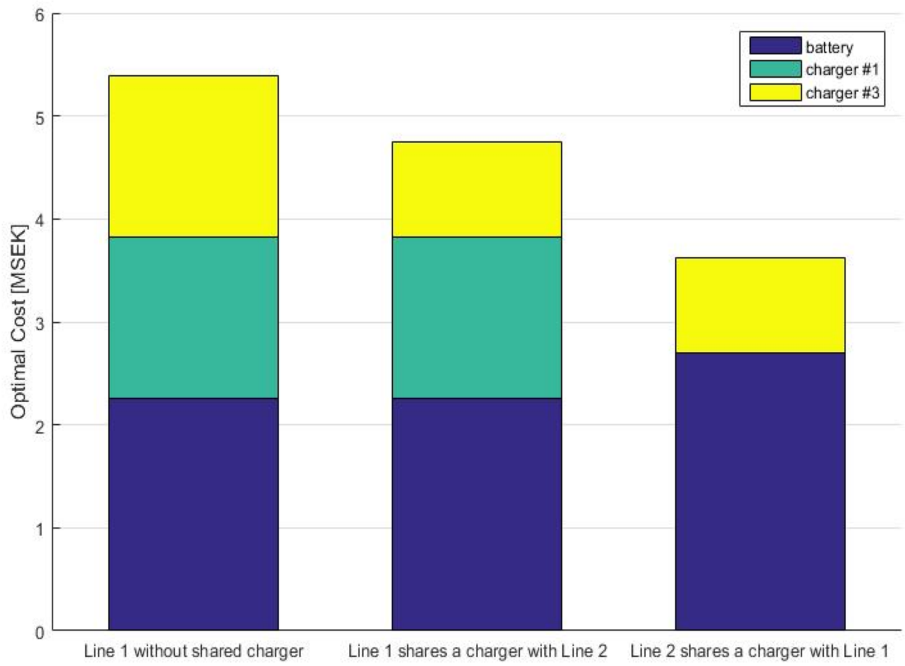

Figure 5.13: The optimal cost for individual bus line in baseline scenario (Battery cost is for four buses)

As illustrated by Figure 5.13 the optimal cost involving the charging infrastructures and the battery systems are different between the bus lines of 2-line bus network. Since the shared charger provides both bus lines with the charging energy, it can be considered that each bus line pays for the half of the expense of the shared charger. 3.6 million SEK is spent on the bus line 2 operated by 4 buses of which the cost of 45-kWh batteries makes up almost 75%. Whereas the bus line 1 contributes to 4,74 million SEK where 47% of the expense is used for the batteries with 37 kWh. The whole electric bus supply system costs 8.39 million SEK and more than 50% of the optimal cost is attributed by the batteries cost. Besides, multiple use of the charger contributes to cost savings. Compared to the individual bus line 1, the bus line 1 shared a charger with bus line 2 saves around 0.9 million SEK which is exactly the half of the cost of the shared charger.

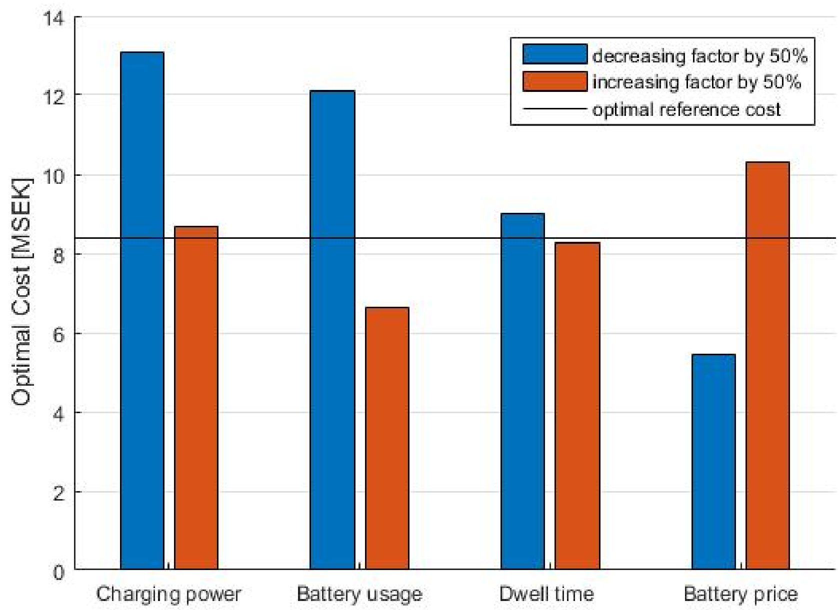

Figure 5.14: The optimal cost for different parameter variations

The summary of the optimal total cost for all scenarios is shown in Figure 5.13. It can be observed that the required charging infrastructures and batteries substantially differs between each parameter variation.

In particular, decreasing the charging power by 50% results in the most significant influence on the optimal cost. In this case, the end-stop charging is insufficient when the peak charging power for all chargers amounts to 150 kW. Therefore, the increasing charger requirements contributes to around 4.5 MSEK growth of the optimal cost. The extension of the battery usage and reduction of the battery price lead to great difference of the optimal cost as well. These two parameter variations give the increase of 3.7 MSEK and the decrease of 2.9 MSEK respectively. The variations of the dwell time at the terminal stops contribute to the lowest effect on the optimal cost. The supply system of the bus network becomes cheapest when the battery price is only half of the baseline scenario.

Chapter 6

Conclusion and Discussion

6.1 Summary

This thesis project introduces a linear optimization method for planning the most efficient supply system of a bus network based on the mixed-integer linear programming. The optimal cost involving the cost of different types of charger and the cost for bus batteries is proposed in an objective function considering the energy and the time table constraints.

To enhance the usefulness of the optimization method, a piecewise linear charger cost model has been developed, to approximate the nonlinear relationship between the cost and the charging power of a charger. By using the piecewise linear cost model, the developed method not only gives minimal cost of the supply system, but also is capable of optimizing

- placement of the chargers
- capacity of batteries for each bus line
- charging power at the stops (except the transfer stops)
- charging time at the transfer stops

A main conclusion is that the price of battery have a substantial influence on the complexity of the charging location. A specific trade off between the battery sizing and the chargers is required to minimize the investment.

Additionally, the sensitivity analysis demonstrates that the complexity of the supply system highly depends on the characteristics of the applied network. In the given bus network example, the decrease of the charging power have the most remarkable impact on the charger requirements. The battery usage margin affects the battery sizing most. Also the battery price has the potential to reduce the cost of the supply system significantly. Therefore, the economic benefits can be gained by using sensitivity analysis for complex bus networks.

The optimization method is an advanced tool for optimizing the supply system and therefore supports the evaluation of electrified bus systems. However, some limitations need to be emphasized. The power grid limitations and multiple use of existing grid infrastructures have not been considered in the method. In addition, in order to avoid the constraint violations all buses travelling in the bus line are deployed with identical daily trips.

6.2 Future Work

Some followed up works are worthwhile to be done in the future:

- Further study on the optimization method is to consider the power grid impact on a electric bus network. The power accessibility affects the complexity of charging infrastructures substantially. For instance, taking the full advantage of the existing power grid for the subway stations or other public facilities can result in a great cost saving. Also it should most of the times be avoided to install a charger on a stop which lacks the energy grid capability.

- The planning and optimization of the supply system for a real city bus network can be considered for further research. A good way to extract the public transit schedule and associated geographic information is to use the General Transit Feed Specification (GTFS) [22]. And it is convenient to apply the GTFS specification to provide schedules and geographic information to Google Maps and other Google applications that show transit information [23].

- Our continued pursuit for the optimization method is to develop the method using nonlinear techniques, for instance mixed integer nonlinear programming. The nonlinear system brings many benefits to the optimization method. In particular the nonlinear system is capable of handling the multiplication of variables, which means it allows the optimization method to give the best peak power and charging time for each charger. In addition, the accuracy of charger cost model can be enhanced by using high-order objective function.

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