THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Bounds on the maximum coding rate of multiple-access channels and feedback channels

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To whomsoever it may concern

Abstract

We provide upper and lower bounds on the coding rate of multiple-access channels (MACs) and feedback channels. Traditional MACs have been extensively studied under the assumption of availability of perfect channel state information (CSI). In Paper A we relax this assumption for a Rayleigh block-fading MAC and provide bounds on the sum-rate capacity. The upper bound relies on a dual formula for channel capacity and on the assumption that the users can cooperate perfectly. The lower bound is derived assuming a noncooperative scenario where each user employs unitary space-time modulation (independently from the other users). Numerical results show that the gap between the upper and the lower bound is small already at moderate SNR values.

Motivated by the growth of machine-type communication, in Paper B we present a finite-blocklength analysis of the throughput and the average delay achievable in a wireless system where (i) several uncoordinated users transmit short coded packets, (ii) interference is treated as noise, and (iii) 1-bit feedback from the intended receivers enables the use of a simple automatic repeat request protocol. Our analysis exploits the recent results on the characterization of the maximum coding rate at finite blocklength and finite block-error probability by Polyanskiy, Poor, and Verdú (2010), and by Yang *et al.* (2014). For a given number of information bits, we determine the coded-packet size that maximizes the per-user throughput and minimizes the average delay. Finally, in Paper C, we present nonasymptotic achievability and converse bounds on the maximum coding rate (for a fixed average error probability and a fixed average blocklength) of variable-length full-feedback (VLF) and variable-length stop-feedback (VLSF) codes operating over a binary erasure channel (BEC). For the VLF setup, the achievability bound relies on a scheme that maps each message onto a variable-length Huffman codeword and then repeats each bit of the codeword until it is received correctly. The converse bound is inspired by the meta-converse framework by Polyanskiy, Poor, and Verdú (2010) and relies on binary sequential hypothesis testing. For the case of zero error probability, our achievability and converse bounds match. For the VLSF case, we provide achievability bounds that exploit the following feature of BEC: the decoder can assess the correctness of its estimate by verifying whether the chosen codeword is the only one that is compatible with the erasure pattern.

Keywords: Shannon capacity, block-fading channel, multiple-access channel, Gaussian collision channel, finite blocklength, quasi-static fading, full feedback, stop feedback

List of Included Publications

This thesis is based on the following publications:

[A] R. Devassy, G. Durisi, J. Östman, W. Yang, T. Eftimov, and Z. Utkovski, "Finite-SNR bounds on the sum-rate capacity of Rayleigh block-fading multiple-access channels with no a priori CSI," *IEEE Trans. Commun.*, vol. 63, no. 10, pp. 3621–3632, Oct. 2015.

[B] R. Devassy, G. Durisi, P. Popovski, and E. G. Ström, "Finite-blocklength analysis of the ARQ-protocol throughput over the Gaussian collision channel," in *Int. Symp. Commun., Cont., Signal Process. (ISCCSP)*, Athens, Greece, May 2014, invited paper, pp. 173–177.

[C] R. Devassy, G. Durisi, B. Lindqvist, W. Yang, and M. Dalai, "Nonasymptotic coding-rate bounds for binary erasure channels with feedback," *IEEE Inf. Theory Workshop (ITW)*, submitted for publication.

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Acronyms

ARQ:	Automatic repeat request
AWGN:	Additive white Gaussian noise
BEC:	Binary erasure channel
CoMP:	Coordinated multi-point
CSI:	Channel state information
i.i.d.:	Independent and identically distributed
MAC:	Multiple-access channel
MIMO:	Multiple-input multiple-output
SNR:	Signal-to-noise ratio
VLF:	Variable-length feedback
VLSF:	Variable-length stop-feedback

Contents

Part I

Overview

CHAPTER 1

Introduction

We live in an era of ever-exploding amount of information, where communication technologies are often pushed to its limits. Market leaders in the communication sector predicts an astounding ten-fold increase in the amount of mobile data and around a two-fold increase in the number of smartphone users by the end of 2020 [1, 2]. The need of large-scale densification of mobile broadband infrastructure is evident from these predictions. Several technologies have been proposed to deliver this increasing data demands like coordinated multi-point (CoMP) [3], network multiple-input multiple output (MIMO) [4], and interference alignment [5]. However, the theoretically predicted throughput increase has not been visible in experimental demonstrations [3, 6]. One potential reason for the disagreement between theoretical analysis and experiments is the assumption of perfect channel state information (CSI) being available. Often in practical systems pilots are used to estimate the channel coefficients. Studying the *Shannon capacity* [7] with no assumption of perfect CSI can help us quantify the cost of acquiring CSI. Some earlier works, e.g. [8, 9] present an asymptotic analysis (asymptotic in the signal-to-noise ratio (SNR)) of the Shannon capacity of point-to-point multiple-antenna setting under the assumption of no *a priori* CSI. We continue in this line of work and consider a multiple-access channel (MAC) where two or more noncooperating users communicate with a single receiver. This scenario is relevant for the uplink of wireless cellular networks, where the users may be mobile terminals and the receiver may be a cellular base station. We model the fading process using the so-called Rayleigh block-fading model [10, 11]. In this thesis we present finite-SNR upper and lower bounds on the sum-rate capacity—the fundamental limit on the sum of coding rates of all users—of the Rayleigh block-fading MAC under the assumption of no *a priori* CSI.

Along with the exponential increase in the mobile data traffic, the data in [1, 2] also show a similar trend for machine-type communication. Machine-type communication is the key enabler for a whole new set of applications like traffic safety, traffic efficiency, smart grid, e-health, and efficient industrial communications [12]. A common feature of these applications is that they often require the transmission of short packets (no more than hundreds of bits), which need to be correctly decoded at the intended receiver within stringent latency requirements. Designing wireless communication systems able to support such services is challenging because most of the results available within the field of wireless communication theory are asymptotic in the packet length. Indeed, the classic performance metric used in wireless communication theory, i.e., Shannon capacity, which is the largest data rate at which reliable communication (i.e., communication with arbitrarily low error probability) is possible, is an asymptotic performance measure (asymptotic in the allowed packet length). In the emerging machine-type communication based applications, the transmitted packets are short, and hence, channel capacity might be a poor benchmark. In this scenario, a more suitable performance metric is instead the *maximum achievable rate* at a given packet length and packet error probability. The computation of the maximum achievable rate for discrete channels has been proven to be an NP-hard problem [13]. Nevertheless, easy-to-compute bounds for various channels with positive capacity have been developed in [14–16]. In this thesis, using the aforementioned bounds, we provide a preliminary investigation on the trade-off between packet length and throughput for a simple system, where several *uncoordinated* users transmit short coded packets using frequency-hopping and a simple automatic repeat request (ARQ) protocol. This setup is particularly relevant for machine-type communication systems involving a very large number of devices.

The simplistic model we used to study machine-type communication systems assumes a 1-bit feedback mechanism, namely the simple ARQ. One generalization of this setup is to assume hybrid ARQ where when the receiver indicates a decoding failure, instead of repeating the same codeword, the transmitter sends additional coded bits. Most of the studies in the literature for this setup like [17, 18] are tailored towards traditional wireless communication systems where one can assume suitably large packet lengths. Motivated by surge of machine-type communication systems, we aim to study the throughput of a hybrid-ARQ systems for point-to-point communication links with feedback after every channel use. Point-to-point communication with an instantaneous and error-free feedback link have been studied extensively in the literature. In the pioneering work [19], Shannon showed that a fixed-blocklength full-feedback setup—where the channel output is sent to the transmitter through the feedback link—offers no increase in capacity compared to not having feedback. However, if the use of variable-length codes is permitted, the availability of full feedback turns out to be beneficial. Burnashev [20] derived the reliability function for the case when full feedback is available and variable-length feedback (VLF) codes are used, for all rates between zero and capacity. Furthermore, this full-feedback reliability function is strictly greater than the reliability function for no feedback. There have been many works on the asymptotic characterization (average number of channel uses being very large) of full feedback systems [21–23]. Note that hybrid-ARQ system cannot be modeled effectively by using the full feedback assumption. Polyanskiy *et al.* [24] obtained nonasymptotic (finite average number of channel uses) bounds showing that with VLF codes one can approach capacity faster than fixed-blocklength codes. Interestingly, the achievability bound used in [6] to prove this result is actually based on variable-length stop-feedback (VLSF) codes. In the VLSF setup, the feedback link is used by the receiver only to send a single bit, indicating to stop the transmission of the current message. This setup models exactly hybrid ARQ systems. In this thesis, we study the throughput of hybrid ARQ systems with finite average blocklength (number of channel uses). We begin our analysis by assuming one among the simplest channel models, i.e., the binary erasure channel (BEC). We provide achievability and converse bounds for the minimum average blocklength for a given number of codewords and maximum allowed probability of error for both VLF and VLSF codes.

1.1 Scope of Thesis

The aim of this thesis is to present bounds on the coding rate of MAC and feedback channels. We consider the Rayleigh block-fading MAC and provide finite-SNR upper and lower bounds on the sum-rate capacity. These bounds are presented in Paper A. In Paper B, we provide a preliminary investigation on the trade-off between packet length and coding rate for simple ARQ protocol over the Gaussian collision channel. In Paper C, we lay the foundation for studying the coding rate of hybrid ARQ systems with finite average delay (number of channel uses or blocklength). We present achievability and converse bounds for the minimum average blocklength for a given number of codewords and maximum allowed probability of error for both VLF and VLSF codes, assuming the underlying forward communication channel to be the BEC.

1.2 Organization of Thesis

The thesis is organized as follows: we introduce the problem setting for MAC in Chapter 2; then we define the relevant quantities of interest related to feedback channels in Chapter 3; in Chapter 4 we provide a brief overview of our contributions in the attached papers.

1.3 Notation

This section describes the notation used in Part I of this thesis. Uppercase letters denote matrices, lowercase letters designate scalars, and boldface letters denote random quantities. Uppercase calligraphic letters denote sets and the $n$fold Cartesian product of a set $\mathcal{X}$ is denoted by $\mathcal{X}^n$. The set of complex number is denoted by $\mathbb{C}$, and $\mathbb{C}^{m\times n}$ stands for the set of matrices having m rows, n columns, and entries from $\mathbb{C}$. The trace and the Hermitian transpose of a matrix $\mathbf{A}$ are denoted by $\mathbb{T}\mathrm{r}\{\mathbf{A}\}$ and $\mathbf{A}^\dagger$, respectively. With $\mathbb{E}[\,\cdot\,]$ we denote expectation and $I(\mathbf{x};\mathbf{y})$ stands for the mutual information between the random variables $\mathbf{x}$ and $\mathbf{y}$. We use $\mathcal{CN}(0,\sigma^2)$ to denote a circularly symmetric complex Gaussian random variable with zero mean and variance σ^2.

CHAPTER 2

Multiple-Access Channels

In this chapter we introduce the concepts essential in understanding our contributions in Paper A. Specifically, we will define the sum-rate capacity of a Rayleigh block-fading MAC with no *a priori* channel state information (CSI).

2.1 Rayleigh Block-Fading Multiple-Access Channel

The MAC models a scenario where two or more noncooperating users communicate with a single receiver. We consider the setup where neither the users nor the receiver have *a priori* information on the realization of the fading process (no *a priori* CSI). We shall focus on the so-called Rayleigh block-fading model [10, 11]. The two key features of this model are that (i) the fading coefficients associated to the channels between each transmit and receive antenna pair are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables; (ii) each fading coefficient remains constant over t_c channel uses before changing to a new independent realization. The parameter t_c, which is the ratio between the channel coherence time and the symbol duration, will be referred to in this thesis as *coherence interval.*

We consider n_u users communicating with a receiver having n_r antennas. We assume that each user is equipped with one or more antennas and denote by n_i the number of antennas at user i, $i = 1, \ldots, n_\mathrm{u}$. The received signal over an arbitrary coherence interval $\mathbf{Y} \in \mathbb{C}^{n_\mathrm{r} \times t_\mathrm{c}}$ can be compactly written in matrix notation as follows:

$$\mathbf{Y} = \sum_{i=1}^{n_\mathrm{u}} \mathbf{S}_i \mathbf{X}_i + \mathbf{W}. \tag{2.1}$$

Here, $\mathbf{X}_i \in \mathbb{C}^{n_i \times t_\mathrm{c}}$ denotes the signal transmitted by user i during the coherence interval, and the matrix $\mathbf{S}_i \in \mathbb{C}^{n_\mathrm{r} \times n_i}$ contains the fading coefficients associated with the channels between each transmit antenna of user i and the receive antennas, within the coherence interval. From the Rayleigh block-fading model, it follows that $\mathbf{S}_i$ has i.i.d. $\mathcal{CN}(0,1)$ entries and that the channel matrices $\{\mathbf{S}_i\}_{i=1}^{n_\mathrm{u}}$ are independent. Finally, the matrix $\mathbf{W} \in \mathbb{C}^{n_\mathrm{r} \times t_\mathrm{c}}$, whose entries are i.i.d. $\mathcal{CN}(0,1)$-distributed, denotes the additive noise. Let

$$n_\mathrm{t} = \sum_{i=1}^{n_\mathrm{u}} n_i \tag{2.2}$$

be the total number of transmit antennas. We can rewrite (2.1) as

$$\mathbf{Y} = \mathbf{S}\mathbf{X} + \mathbf{W} \tag{2.3}$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 & \cdots & \mathbf{S}_{n_\mathrm{u}} \end{bmatrix} \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}} \tag{2.4}$$

and

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_{n_\mathrm{u}} \end{bmatrix} \in \mathbb{C}^{n_\mathrm{t} \times t_\mathrm{c}}. \tag{2.5}$$

We assume that $\mathbf{W}$ and $\mathbf{S}$ are independent, and that their probability law does not depend on $\mathbf{X}$. We also assume that $t_\mathrm{c} \geq \max(n_\mathrm{t}, n_\mathrm{r})$ and focus on the no *a priori* CSI scenario where neither the transmitter nor the receiver have prior knowledge of matrix $\mathbf{S}$.

2.2 Sum-rate Capacity

The sum-rate capacity of the Rayleigh block-fading MAC in (2.3) is given by

$$\mathcal{C}(\rho) = \frac{1}{t_\mathrm{c}} \sup I(\mathbf{X}; \mathbf{Y}) \tag{2.6}$$

where the supremum is over all probability distributions on $\mathbf{X}$ for which $\{\mathbf{X}_i\}_{i=1}^{n_\mathrm{u}}$ are independent and the per-user power constraint

$$\mathbb{E}\Big[\mathrm{Tr}\{\mathbf{X}_i \mathbf{X}_i^\dagger\}\Big] \leq \frac{t_\mathrm{c} n_i \rho}{n_\mathrm{t}},\ i = 1, 2, \ldots, n_\mathrm{u} \tag{2.7}$$

is satisfied. Here, ρ can be thought of as the total energy per channel use available over all users. The particular form of the average power constraint in (2.7) allows all users to transmit at the same average power per antenna. In Paper A we present nonasymptotic (finite ρ) bounds for the sum-rate capacity $\mathcal{C}(\rho)$ of the Rayleigh block-fading MAC in (2.3).

CHAPTER 3

Feedback Channels

Two papers in this thesis, namely Paper B and Paper C, that focus on feedback channels. This chapter introduces the problem setting in both of these papers. In Paper B we examine the Gaussian collision channel with feedback and an overview of the same is provided in Section 3.1. The BEC with two kinds of feedback (full and stop feedback), which is the subject matter of Paper C, is presented in Section 3.2.

3.1 Gaussian Collision Channel with Feedback

We consider a wireless communication system where several *uncoordinated* users transmit short coded packets using frequency hopping and a simple ARQ protocol. This setup is closely related to the one known in the literature as *slotted Gaussian collision channel with feedback* [17, 25]. Unlike the asymptotic—infinite packet length—analysis of the system presented in [17], in Paper B we provide a finite packet length analysis by utilizing the recent finite-blocklength information-theoretic results presented in [14–16]. We consider both the case when the channel among each users is impaired by additive Gaussian noise only, and the quasi-static fading case, where the fading gains are random but stay constant over the duration of each packet. In Section 3.1.1, we define the system model and state our assumptions; and in Section 3.1.2 we give the expressions for throughput and delay of the system.

3.1.1 System model and assumptions

We assume n_u transmitter-receiver pairs operating concurrently. For the fading scenario, we consider communication over a time-frequency selective fading channel with coherence time t_c and coherence bandwidth b_c. The available bandwidth $b_\mathrm{a} > b_\mathrm{c}$ is divided into $n_f = b_\mathrm{a}/b_\mathrm{c}$ non-interfering frequency bands. For simplicity, we shall assume in the following that n_f is an integer. For each slot, each user chooses a frequency band uniformly at random and independently from the other users, and transmits over this band a coded packet consisting of n complex symbols (corresponding to n channel uses) of duration $n/b_\mathrm{c} < t_\mathrm{c}$ seconds. These assumptions guarantee that the fading channel stays constant over the duration of each coded packet. The received vector[1] $\mathbf{Y} \in \mathbb{C}^n$ corresponding to the coded packet $\mathbf{X}_1 \in \mathbb{C}^n$ transmitted by user 1 during one (arbitrary) packet transmission slot is given by

$$\mathbf{Y} = h_1\mathbf{X}_1 + \sum_s h_s\mathbf{X}_s + \mathbf{W}. \tag{3.1}$$

Here, h_s denotes the fading coefficient corresponding to user s, the index s spans the set of interfering users (i.e., users that chose the same frequency band as user 1 for transmission), and $\mathbf{W}$ models the additive noise vector, whose entries are independent and identically distributed circularly symmetric complex Gaussian random variables with unit variance. The additive white Gaussian noise (AWGN) scenario is readily obtained from (3.1) by assuming that the channel gains in (3.1) are deterministic.

At the intended receiver, which is assumed to be perfectly aware of the frequency band chosen by the corresponding transmitter, but which ignores the choice of the other (unintended) users, decoding is attempted. A binary feedback about the status of the decoding operation is sent back to the transmitter. If the feedback indicates a decoding failure, the transmitter repeats the same coded packet over the next packet transmission slot, after having selected a different frequency band. If the feedback indicates decoding success, then the next coded packet is transmitted. Each coded packet corresponds to k information bits (we assume that all users need to deliver similar payloads). Furthermore, each user maps the k information bits to n coded bits independently from the other users, i.e., no coordination among users is assumed.

To simplify the analysis, we shall also assume what follows:

(i) Each user has an infinite number of information bits to transmit (*full-buffer* assumption) and as soon as the transmission of the current packet is stopped because decoding is successful, the transmission of the next packet is started.

(ii) The feedback is instantaneous and error free.

(iii) Interference resulting from several users contending the same medium is treated as additive Gaussian noise.

[1]Note that in Section 3.1 we shall use uppercase letters to denote vectors instead of matrices.

(iv) All users transmit at the same power.

(v) The fading coefficients $\{h_s\}$ are independent and identically distributed and perfectly known to the receiver.

The assumption (iii) imply that, given the fading coefficients $\{h_s\}$, the second and third term on the right-hand-side of (3.1) can be jointly modeled as a circularly symmetric Gaussian random variable.

3.1.2 Throughput and delay

Using the renewal-reward theorem [26] we conclude that the overall throughput η of the system defined in Section 3.1.1, measured in bits per second per Hertz (or bits per channel use), corresponding to the transmission of coded packets of length n is given by

$$\eta = n_{\mathrm{u}} \frac{k}{n} \left[1 - \epsilon(n,k)\right] \tag{3.2}$$

where, $\epsilon(n,k)$ denotes the packet error rate. The corresponding average delay (measured in number of channel uses) is given by

$$\delta = \frac{n}{1-\epsilon(n,k)}. \tag{3.3}$$

This expression holds under the assumption of unlimited number of retransmissions. In Paper B we use the approximations for the minimum packet error rate as a function of the number of information bits k and the packet length n provided in [14–16] to optimize the packet length n for a fixed number of of information bits k.

3.2 Binary Erasure Channel with Feedback

In Paper C, we consider the BEC with two different feedback mechanisms, namely full feedback and stop feedback. In the full-feedback scenario, the transmitter has noiseless access to all the previously received symbols. We assume a variable-length setup where the transmitter is allowed to transmit until the receiver decides to stop and declare its estimate of the current message. Since the transmitter is aware of the channel outputs, it can stop transmission of current message when the receiver has decided to stop. We shall call the codes used in the full-feedback scenario as VLF codes. We are interested in the minimum average blocklength (number of channel uses) of VLF codes with fixed number of messages and fixed error probability.

Stop feedback refers to the setup where, through the feedback link, the receiver indicates to the transmitter whether to stop or continue transmission of the current message. This setup is also known as decision feedback and it encompasses hybrid ARQ schemes. The codes used in the stop-feedback scenario will be called as VLSF codes. Analogous to the full-feedback scenario, we are interested in the minimum average blocklength (number of channel uses) of VLSF codes under the assumption of a fixed number of messages and fixed error probability.

The two setups are formally introduced below.

3.2.1 Definition

We consider a BEC with input alphabet $\mathcal{X} = \{0, 1\}$ and output alphabet $\mathcal{Y} = \{0, \mathrm{e}, 1\}$, where e denotes an erasure. A VLF code for the BEC is defined as follows.

Definition 1: *([24, Def. 1]) An $(\ell, n_{\mathrm{m}}, \epsilon)$–VLF code, where ℓ is a positive real number, n_{m} is a positive integer, and $\epsilon \in [0, 1]$, consists of:*

1. *A random variable $\mathbf{u}$, defined on a set $\mathcal{U}$ with $|\mathcal{U}| \le 2$, whose realization is revealed to the encoder and the decoder before the start of transmission. The random variable $\mathbf{u}$ acts as common randomness and enables the use of randomized encoding and decoding strategies.*

2. *A sequence of encoders $f_n : \mathcal{U} \times \mathcal{W} \times \mathcal{Y}^{n-1} \to \mathcal{X}, n \ge 1$ that generate the channel inputs*

$$\mathbf{x}_n = f_n(\mathbf{u}, \mathbf{w}, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{n-1}). \tag{3.4}$$

   Here, $\mathbf{w}$ denotes the message, which is uniformly distributed on $\mathcal{W} = \{1, 2, \ldots, n_{\mathrm{m}}\}$. Note that the channel input at time n depends on all previous channel outputs (full feedback).

3. *A sequence of decoders $g_n : \mathcal{U} \times \mathcal{Y}^n \to \mathcal{W}$ that provide the estimate of $\mathbf{w}$ at time n.*

4. *A nonnegative integer-valued random variable τ, which is a stopping time of the filtration*

$$\mathcal{G}_n = \sigma\{\mathbf{u}, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n\} \tag{3.5}$$

   and satisfies

$$\mathbb{E}[\tau] \le \ell. \tag{3.6}$$

5. *The final estimate $\widehat{\mathbf{w}} = g_\tau(\mathbf{u}, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_\tau)$ of $\mathbf{w}$, which satisfies the error-probability constraint*

$$\Pr\{\widehat{\mathbf{w}} \ne \mathbf{w}\} \le \epsilon. \tag{3.7}$$

VLSF codes are a special case of VLF codes. The peculiarity of VLSF codes is that the sequence of encoders is not allowed to depend on the past channel outputs, i.e.,

$$f_n : \mathcal{U} \times \mathcal{W} \to \mathcal{X}, \ n \ge 1. \tag{3.8}$$

3.2.2 Coding rate and minimum average blocklength

The coding rate r of an $(\ell, n_{\mathrm{m}}, \epsilon)$–VLF code is defined as

$$r = \frac{\log_2 n_{\mathrm{m}}}{\mathbb{E}[\tau]}. \tag{3.9}$$

As pointed out earlier, VLSF codes are special case of VLF codes, and hence, the coding rate of a VLSF code is defined analogously as in (3.9).

We define the minimum average blocklength of VLF codes with n_{m} codewords and error probability not exceeding ϵ as follows:

$$\ell_f^*(n_{\mathrm{m}}, \epsilon) = \min\{\ell \ : \ \exists(\ell, n_{\mathrm{m}}, \epsilon)\text{–VLF code}\}. \tag{3.10}$$

Analogously, we define the minimum average blocklength of VLSF codes with n_{m} codewords and error probability not exceeding ϵ as

$$\ell_{sf}^*(n_{\mathrm{m}}, \epsilon) = \min\{\ell \ : \ \exists(\ell, n_{\mathrm{m}}, \epsilon)\text{–VLSF code}\}. \tag{3.11}$$

In Paper C, we present upper and lower bounds on both $\ell_f^*(n_{\mathrm{m}}, \epsilon)$ and $\ell_{sf}^*(n_{\mathrm{m}}, \epsilon)$.

CHAPTER 4

Contributions

The list of papers appended in this thesis and a summary of each one of them is provided in Section 4.1 below. Additional publications by the author, which are not included in this thesis, are listed in Section 4.2.

4.1 Included Publications

1. **Paper A: "Finite-SNR bounds on the sum-rate capacity of Rayleigh block-fading multiple-access channels with no *a priori* CSI"**

   We provide nonasymptotic upper and lower bounds on the sum-rate capacity of Rayleigh block-fading MACs for the set up where *a priori* channel state information is not available. The upper bound relies on a dual formula for channel capacity and on the assumption that the users can cooperate perfectly. The lower bound is derived assuming a noncooperative scenario where each user employs unitary space-time modulation (independently from the other users). Numerical results show that the gap between the upper and the lower bound is small already at moderate SNR values. This suggests that the sum-rate capacity gains obtainable through user cooperation are minimal for the scenarios considered in the paper.

2. **Paper B: "Finite-blocklength analysis of the ARQ-protocol throughput over the Gaussian collision channel"**

   We present a finite-blocklength analysis of the throughput and the average delay achievable in a wireless system where (i) several uncoordinated users transmit short coded packets, (ii) interference is treated as noise, and (iii) 1-bit feedback from the intended receivers enables the use of a simple ARQ protocol. Our analysis exploits the recent results on the characterization of the maximum coding rate at finite blocklength and finite block-error probability by Polyanskiy, Poor, and Verdú (2010), and by Yang *et al.* (2013). For a given number of information bits, we determine the coded-packet size that maximize the per-user throughput and minimize the average delay. Our numerical results indicate that, when optimal codes are used, very short coded packets (of length between 50 to 100 channel uses) yield significantly lower average delay at an almost negligible throughput loss, compared to longer coded packets.

3. **Paper C: "Nonasymptotic coding-rate bounds for binary erasure channels with feedback"**

   We present nonasymptotic achievability and converse bounds on the maximum coding rate (for a fixed average error probability and a fixed average blocklength) of VLF and VLSF codes operating over a BEC. For the VLF setup, the achievability bound relies on a scheme that maps each message onto a variable-length Huffman codeword and then repeats each bit of the codeword until it is received correctly. The converse bound is inspired by the meta-converse framework by Polyanskiy, Poor, and Verdú (2010) and relies on binary sequential hypothesis testing. For the case of zero error probability, our achievability and converse bounds match. For the VLSF case, we provide achievability bounds that exploit the following feature of BEC: the decoder can assess the correctness of its estimate by verifying whether the chosen codeword is the only one that is compatible with the erasure pattern. One of these bounds is obtained by analyzing the performance of a variable-length extension of random linear fountain codes. The gap between the VLSF achievability and the VLF converse bound, when number of messages is small, is significant: 23% for 8 messages on a BEC with erasure probability 0.5. The absence of a tight VLSF converse bound does not allow us to assess whether this gap is fundamental.

4.2 Publications Not Included

Publications by the author, which are not included in this thesis, are listed below.

1. G. Durisi, A. Tarable, C. Camarda, R. Devassy, and G. Montorsi, "Capacity bounds for MIMO microwave backhaul links affected by phase noise," *IEEE Trans. Commun.*, vol. 62, no. 3, pp. 920–929, Mar. 2014.

2. T. R. Lakshmana, A. Tölli, R. Devassy, and T. Svensson, "Precoder design with incomplete feedback for joint transmission," *IEEE Trans. Wireless Commun.*, vol. 15, no. 3, pp. 1923–1936, Mar. 2016.

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