Multidimensional Modulation Formats for Coherent Optical Communications

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ABSTRACT

Coherent optical communication systems applying modulation formats with a dimensionality of four or higher are investigated and compared to systems using conventional formats. Higher dimensionality can be achieved by applying modulation over more than one polarization, time-slot, wavelength, mode or core. Both uncoded systems and systems applying forward-error correction (FEC) coding are studied in terms of spectral efficiency and sensitivity. It is shown that increasing the dimensionality for a constant spectral efficiency improves the sensitivity substantially if no coding is applied, whereas the corresponding gains generally are much smaller in FEC-coded systems.

Keywords: Modulation formats, multidimensional modulation formats, asymptotic power efficiency, power efficiency, coherent optical communication systems.

1. INTRODUCTION

The coherent receiver has enabled high spectral efficiency (SE) fiber optical transmission system using modulation formats such as polarization-multiplexed quadrature phase shift keying (PM-QPSK) and M-ary quadrature amplitude modulation (PM-MQAM). Systems applying these modulation formats are to a large extent possible due to the use of digital signal processing (DSP)[1,2] to track the carrier phase, enabling the use of a free-running local-oscillator (LO) as reference. Further, the DSP enables tracking the rotation of the polarization state, arising from the random birefringence of the fiber. For this, access to the full four-dimensional (4D) optical signal, formed by the amplitude, phase and the two polarizations, is needed in the receiver. This has opened up for the use of modulation formats utilizing the full 4D signal space rather than treating the polarization states as independent channels. The research on 4D modulation formats for fiber-optical communication systems was initialized in 2009 by the introduction of POL-QAM[3] and polarization-switched QPSK (PS-QPSK).[4]

2. METRICS FOR COMPARING MODULATION FORMATS

Comparing different modulation formats in a general sense is difficult as the performance is dependent on system parameters such as the transmission link and the forward-error correction (FEC) schemes that are used. In this paper, we will compare the modulation formats assuming an additive white Gaussian noise (AWGN) channel and using the constellation figure of merit (CFM), SE and mutual information (MI) as figures of merit. We note that modulation formats can also be designed for increased nonlinear tolerance, however as this is a property that is hard to generalize, we will not attempt to address it in this paper. Examples of such formats are an eight dimensional (8D) format designed for dispersion-managed links,[5] the phase-conjugated twin waves,[6] and the not-so-practical but interesting from a theoretical point of view, satellite constellations.[7]

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The SE of an N-dimensional modulation format is given by

$$\mathrm{SE} = \frac{\log_2(M)}{N/2}, \tag{1}$$

where M is the number of N-dimensional constellation points. Note that the factor 1/2 normalizes the SE to "dimension pair", i.e. to the two dimensions of one polarization. Hence, the unit of the SE is bits/symbol/dimension-pair (bit/2D). The CFM is given by

$$\mathrm{CFM} = \frac{d_{\min}^2 N}{2E_\mathrm{s}}, \tag{2}$$

where $d_{\min}$ is the minimum Euclidean distance between any constellation points and E_s is the average symbol energy over N dimensions.[8] The CFM gives the sensitivity at asymptotically high SNR. Hence, CFM is a good measure for systems without FEC or systems using hard-decision (HD) FEC operating at very low pre-FEC BER targets. The CFM is an appropriate measure when modulation formats are compared at the same symbol rate. This should be differentiated from the commonly used asymptotic power efficiency, which is an appropriate measure when formats are compared at the same bit rate. The SE and CFM are easy-to-use measures in the sense that they only depend on the signal geometry, and exclude properties such as bit-to-symbol mapping, the nonlinear fiber channel, and choice of FEC code/decoder.

Today's coherent optical fiber communication systems rely on FEC. Up until recently, the implemented schemes typically use HD coding schemes based on for instance Reed-Solomon[9] or BCH[10] codes. At HD pre-FEC BER targets (typically around BER $\approx 10^{-3}$),[9] the CFM is no longer a good measure. At this point, the sensitivity typically has to be found by Monte-Carlo simulations, assuming an AWGN channel. At this BER, the bit-to-symbol mapping also has to be considered and for many of the multidimensional modulation formats, Gray-coding is not possible and heuristic methods have to be applied for finding a suitable bit-to-symbol map.

In modern coherent systems, soft-decision (SD) FEC schemes using for instance low-density parity check (LDPC) codes[11] and turbo-product codes (TPC)[12] are applied. For SD decoding, there is no longer any relation between the BER before and after decoding, which can be understood from the facts that the decoder works on soft information, typically as log-likelihood ratios (LLRs), and that information is lost once decisions are made on the received symbols.[13,14] It should be noted that in the fiber optical research community today, the pre-FEC BER is still often used as a figure of merit, even when SD coding schemes are assumed.

The FEC encoders and decoders are most often omitted in experiments since off-line processing is used and it is simply not feasible to sample enough data for proper statistics at error-free conditions (typically BER $< 10^{-15}$). For these systems, an estimate of the achievable information rate using the MI is a more accurate measure.[13,15] In this paper, we will assume the channel to be AWGN in order to calculate the MI for different modulation formats. For memoryless decoders, assuming an independent and identically distributed Gaussian channel has been shown in experiments to be a good estimate for channels without inline dispersion compensation[16,17] which justifies the AWGN assumption.

Assuming an N-dimensional channel input $\boldsymbol{X}$ drawn from the constellation that is used with uniform probability, the MI $I(\boldsymbol{X};\boldsymbol{Y})$ is given as

$$I(X;Y) \triangleq \mathbb{E}\left[\log_2 \frac{p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{Y}|\boldsymbol{X})}{p_{\boldsymbol{Y}}(\boldsymbol{Y})}\right], \tag{3}$$

where $\boldsymbol{Y}$ is the the N-dimensional channel output, $p_{\boldsymbol{Y}|\boldsymbol{X}}$ is the channel transition distribution, $p_{\boldsymbol{Y}}$ the channel output distribution and the expectation is taken over both $\boldsymbol{X}$ and $\boldsymbol{Y}$.[15,18] To find the MI of the constellations discussed in this paper, we use Monte Carlo estimations over K symbols as

$$I(\boldsymbol{X};\boldsymbol{Y}) \approx \frac{1}{K}\sum_{i=1}^{K} \log_2 \frac{p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}_i|\boldsymbol{x}_i)}{p_{\boldsymbol{Y}}(\boldsymbol{y}_i)}, \tag{4}$$

where the accuracy increases with K.

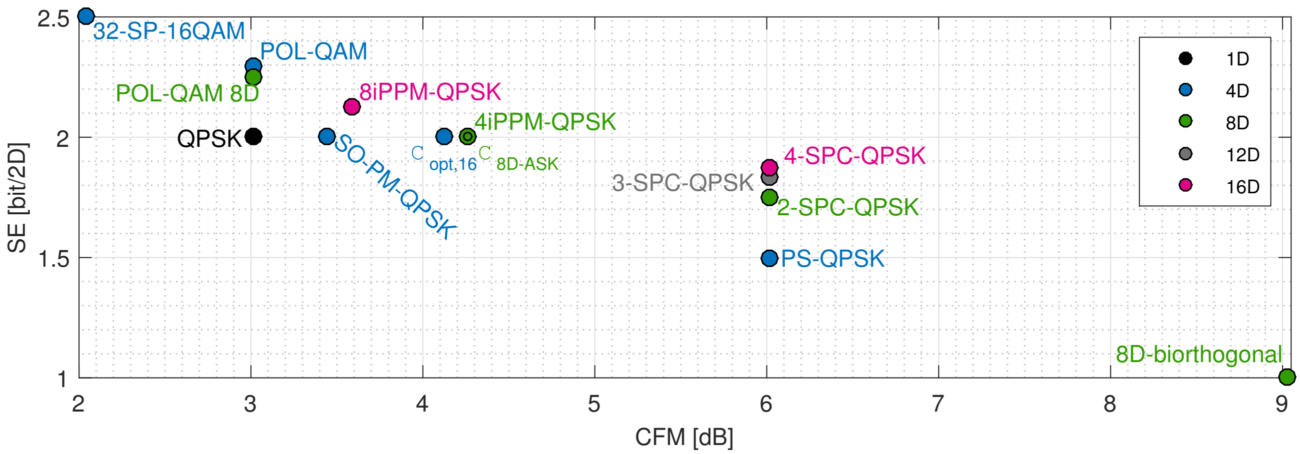

Figure 1. Spectral efficiency versus CFM for formats comparable to QPSK. Colors indicate the lowest number of dimensions in which the formats can be expressed. Figure adapted from.[19]

3. MODULATION FORMATS COMPARABLE TO PM-QPSK

PM-QPSK is the most commonly used modulation format for coherent communication systems. The reasons for this is that it is easy to implement, using binary driving signals and the receiver DSP based on the constant modulus algorithm and Viterbi-Viterbi phase tracking[1] can be implemented with low complexity. Further, the achievable transmission reach easily covers transoceanic distances. In this section, we compare multidimensional formats which have an SE comparable to PM-QPSK. The SE and CFM for these formats are plotted in Fig. 1. More spectrally efficient formats are covered in the next section. QPSK has a SE of 2 bit/2D and an CFM of 3.01 dB.

PS-QPSK has been shown to be the most power efficient format in four dimensions.[4,20] The format is either given by $\mathcal{X}_{\mathrm{PS-QPSK}} = (\pm1, \pm1, 0, 0), (0, 0, \pm1, \pm1)$ or it can be seen as a single-parity check (SPC) code on the PM-QPSK symbol alphabet. These two realizations corresponds to a 45° polarization rotation of the constellation.[20] Instead of using the two polarization states to achieve the four dimensions, two consecutive time-slots can be used forming 2-ary pulse position modulation QPSK (2PPM-QPSK) which, for a memoryless AWGN channel, is an identical format to PS-QPSK.[21] PS-QPSK has an SE of 1.5 bit/2D and a CFM of 6.02 dB. Demonstrated in experiments, PS-QPSK has been shown to achieve longer transmission distance than PM-QPSK by between 21%[22] and 30 %[23] in WDM systems with the two formats operating at the same bitrate. In Fig. 2, PS-QPSK is compared to QPSK in terms of achievable information rate as a function of SNR. As seen, PS-QPSK has a lower achievable rate for any given SNR which can be understood from the fact that PS-QPSK is a subset of the PM-QPSK symbol alphabet.

POL-QAM is another 4D format that uses six states of polarizations (SOPs) with four QPSK phase states each to transmit data.[3] This format has an SE of 2.29 bit/2D and a CFM of 3.01 dB.[4] Hence, POL-QAM can increase the SE over QPSK without a reduction in CFM. Experimental realizations of this format have largely been limited by penalties from the increased complexity of the transmitter although it has been shown that using the extra SE for stronger FEC coding is promising.[24] Since POL-QAM transmits 24 symbols, it is not possible to map an integer number of bits per symbol. Instead bits are mapped onto two consecutive symbols in time, which is discussed in.[4] Since this method does not use all combination of symbols, the SE is slightly reduced with a maintained CFM as shown in Fig. 1 where this method is denoted POL-QAM 8D. Comparing POL-QAM to QPSK in terms of achievable information rate, Fig. 2, it can be seen that the extra SE available with POL-QAM could be used for stronger FEC coding. In addition, QPSK is a subset of POL-QAM and, following the same argument as for PS-QPSK, POL-QAM always has a higher achievable rate.

The format 32-SP-QAM is a 4D modulation format based on subsequent set-partitioning on a PM-16QAM constellation.[25] This modulation format has an increased SE to 2.5 bit/2D over QPSK at the cost of roughly 1 dB lower CFM. The 32-SP-QAM format has been experimentally realized with low implementation penalty

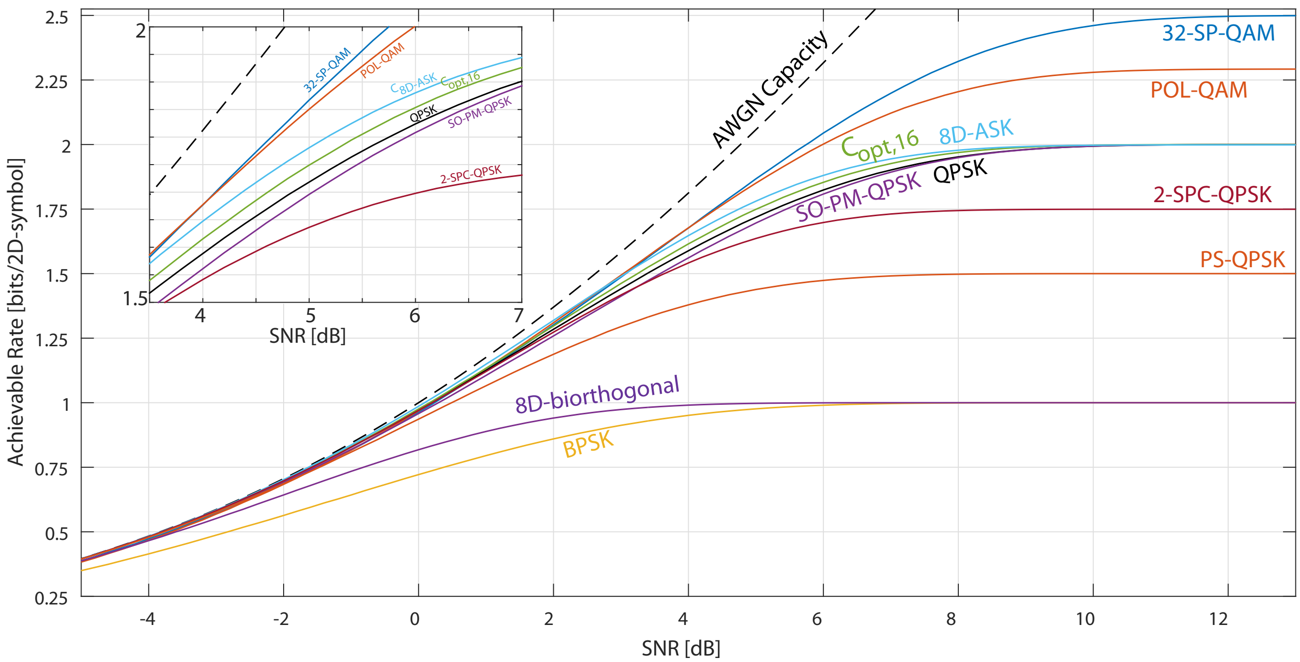

Figure 2. Achievable information rate as a function of SNR for formats comparable to QPSK. Colors indicate the lowest dimensionality that the formats can be expressed in. Figure adapted from.[19]

at 28 Gbaud[26] although simulations with an LDPC coding scheme show very similar performance as QPSK.[27] Subset-optimized-PM-QPSK (SO-PM-QPSK) is a 4D format that optimizes the amplitude ratio between the even and odd set-partitioned subsets of the PM-QPSK constellation and it has the same SE as QPSK but the CFM is increased by 0.44 dB.[28] However, compared in terms of MI in Fig. 2, it is clear that QPSK is a better choice when strong FEC coding is considered as SO-PM-QPSK has lower achievable rate compared to QPSK.

The format $C_{opt,16}$ is the best known 16-point constellation in terms of CFM in four dimensions.[29] It has the same SE as QPSK but an increased CFM by roughly 1.11 dB. This format has been experimentally realized in transmission.[30] This format has also been studied in terms of MI and generalized mutual information (GMI)[31] where it was shown that in terms of MI, this format has a higher achievable rate than QPSK which is also seen in Fig. 2. However, in terms of GMI, which gives a good estimate of the achievable rate in the context of bit-wise decoders, this format was shown to have a significantly lower performance compared to QPSK.[31]

Optimizing the modulation formats in a higher dimensional space allows for further improvements. One such family of modulation formats is SPC-coded QPSK, originally investigated for modulation over spatial superchannels in multicore fiber transmission,[32] which decreases the loss in SE of the SPC by sharing the parity bit over more dimensions. For a single-core system, this bit could be shared over an increasing number of timeslots. The CFM and SE for such formats are shown in Fig. 1, over 2, 3 and 4 cores/timeslot denoted 2-SPC-QPSK, 3-SPC-QPSK, and 4-SPC-QPSK respectively. Note that 1-ary SPC corresponds to PS-QPSK and if the SPC is applied over an infinite number of dimensions, the SE approaches that of QPSK but the CFM approaches a 3 dB increase over QPSK. In Fig. 2, the achievable information rate for 2-SPC-QPSK is plotted as a function of SNR. This format is a subset of QPSK in eight dimensions and hence always has a lower achievable rate than QPSK. However, in systems limited to a low overhead of the FEC, this modulation format could offer a reasonable tradeoff between SE and sensitivity in-between PS-QPSK and QPSK.

Combining PPM with QPSK has been shown to be an effective method of increasing the sensitivity, however it comes at the cost of a reduction in SE.[33] Another option is to use more than one pulse carrying QPSK per frame[34] to reduce the reduction in SE. Two interesting formats are inverse pulse position modulation (iPPM) in combination with QPSK with 4 or 8 pulse slots. As seen in Fig. 1, the eight dimensional format 4iPPM-QPSK has the same SE as QPSK but an increased CFM by 1.25 dB over QPSK. Using iPPM over frames of eight

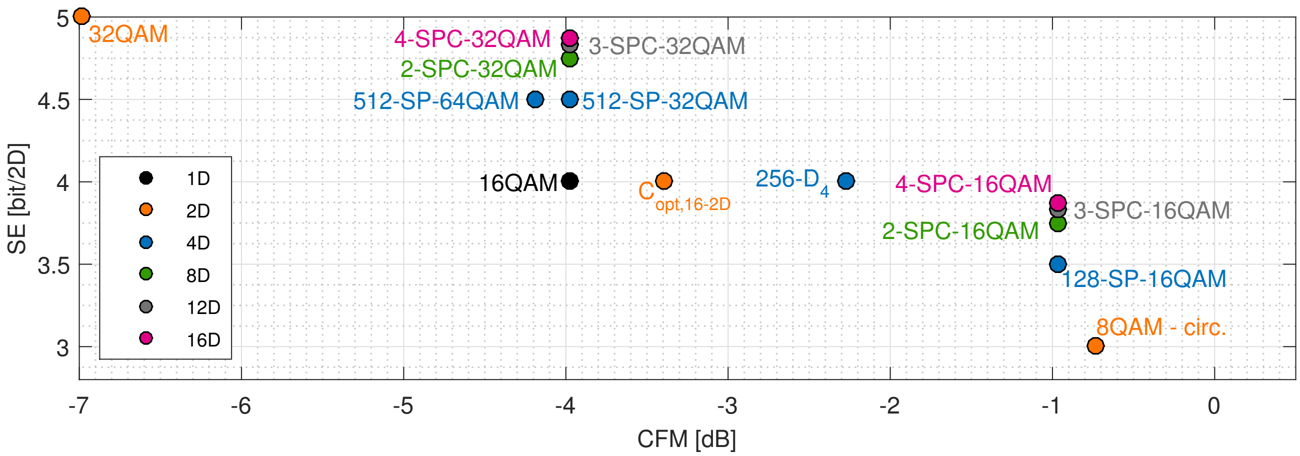

Figure 3. Spectral efficiency as a function of CFM for formats comparable to 16QAM. Figure adapted from[19]

slots, the format 8iPPM-QPSK is realized which has 0.125 bit/2D higher SE than QPSK while simultaneously having 0.58 dB higher CFM. A format with the same SE and CFM as 4iPPM-QPSK is investigated in.[35] This 8D format, here denoted as $C_{8D-APSK}$ is constructed by combining the PM-QPSK constellation with the 2PPM-QPSK constellation, where the QPSK symbols of the two sub-constellations have a $\pi/4$ phase rotation between them. The combination is done such that a PM-QPSK symbol is transmitted followed by a 2PPM-QPSK symbol, or vice versa. Although having the same SE and CFM as 4iPPM-QPSK, the $C_{8D-APSK}$ constellation has 10 nearest neighboring constellation points while 4iPPM-QPSK has 18, which could possibly indicate that $C_{8D-APSK}$ has a better performance in the low SNR region. On the other hand, 4iPPM-QPSK requires fewer levels in the transmitter driving signals than $C_{8D-APSK}$. Compared to QPSK in terms of MI, $C_{8D-APSK}$ has significantly better performance, which can be seen in Fig. 2, making this an interesting format for systems applying advanced SD FEC schemes.

In Fig. 1, the SE and CFM for biorthogonal modulation in eight dimensions (8D-biorthogonal) is plotted and as seen, this format has half the SE compared to QPSK but has a gain in CFM of roughly 6 dB over QPSK. This format has been implemented using either two wavelength channels[36] or two consecutive timeslots[37] to realize the eight dimensions. Compared at the same symbol rate this format could achieve 84 % increased transmission distance compared to QPSK in experiments.[36] Compared in terms of MI, Fig. 2, 8D-biorthogonal modulation has a significantly better performance than BPSK. Hence, it is an interesting format for systems operating at low SNRs, say around 0 dB, if the complexity of the FEC scheme is restrained such that QPSK in combination with strong coding is not an option.

4. MODULATION FORMATS COMPARABLE TO PM-16QAM

In the previous section, formats comparable to PM-QPSK were discussed. It can be argued that these formats mostly apply to systems that require an extreme sensitivity since the transmission distances achievable with PM-QPSK are sufficient for many transoceanic applications. When a higher spectral efficiency is needed, PM-16QAM is often considered. However, the transmission distance that can be achieved with PM-16QAM is much more limited and only in combination with a concatenation of strong SD FEC schemes, transoceanic distances have been achieved.[38] For these systems, alternative modulation formats that relax the complexity of the FEC schemes or that can trade a small amount of SE for increased transmission reach are highly interesting. In this section, modulation formats are compared to 16QAM which has an SE of 4 bit/2D and a CFM of −3.98 dB. In Fig. 3 the SE as a function of CFM is plotted for several optimized modulation formats comparable to 16QAM. Also shown is conventional 32QAM which has 1 bit/2D higher SE but roughly 3 dB lower CFM and circular 8QAM which has 1 bit/2D lower SE and 3.25 dB higher CFM compared to 16QAM. In Fig. 4, the achievable information rate is plotted as a function of SNR for the modulation formats compared in this section. As seen, 32QAM has an higher achievable information rate compared to 16QAM at any SNR. However, this assumes FEC schemes with large overheads and code lengths for 32QAM which might be complex to implement.

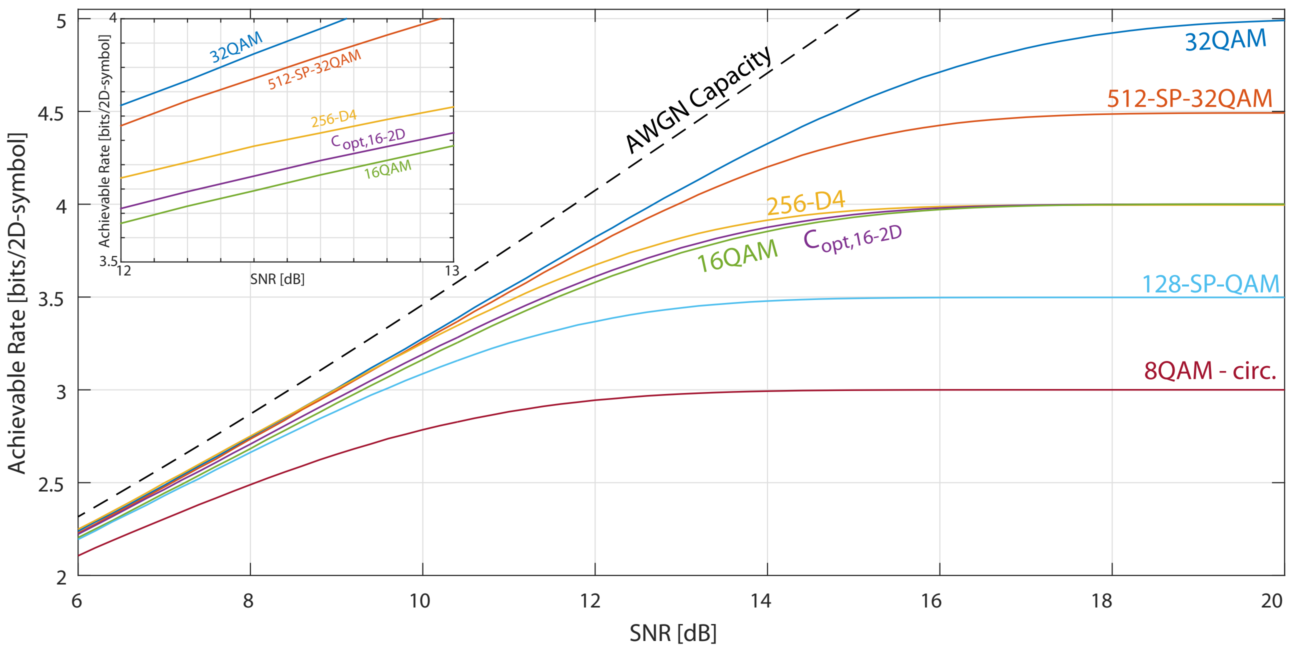

Figure 4. Achievable information rate as a function of SNR for formats comparable to 16QAM. Figure adapted from.[19]

The 4D format 128-SP-QAM is found by applying a set-partitioning operation to the PM-16QAM constellation.[25] The set-partitioning operation can also be seen as applying a SPC, discussed in the previous section, over 7 information bits to generate the 8^{th} bit, assuming a conventional PM-16QAM transmitter is used. The 128-SP-QAM format sacrifices 0.5 bit/2D in SE to gain roughly 3 dB in CFM over 16QAM. In WDM experiment compared at the same symbol rate, 128-SP-QAM has been shown to achieve more than 69 % increased transmission distance over PM-16QAM.[39,40] The achievable information rate for 128-SP-QAM is plotted in Fig. 4 where it, compared to 16QAM, always have a lower performance. However, limited to a small overhead, 128-SP-QAM can provide a sensitivity increase over 16QAM. Compared to 8QAM, 128-SP-QAM provides a higher achievable rate and with a small overhead it is only marginally less sensitive than 8QAM. As discussed in the previous section, the SPC can be shared among several 4D symbols which was experimentally investigated for multicore fiber transmission systems in.[41] The SE and CFM for the SPC applied over 2, 3, and 4 four-dimensional symbols with either 16QAM or 32QAM are shown in Fig. 3. By sharing the parity bit, the SE can be increased over 128-SP-QAM with a maintained CFM. The same applies for the 32QAM case, where the SE is increased over 512-SP-32QAM with a maintained CFM.

Two different SP formats with SE = 4.5 bit/2D are shown in Fig. 3 where 512 points in four dimensions are derived using SP on either the PM-32QAM[42] or the PM-64QAM constellation.[43] The format based on 32QAM has slightly higher CFM and is less complex to implement since it requires a lower number of levels in the driving signals. The achievable information rate as a function of SNR for 512-SP-32QAM is plotted in Fig. 4 and as seen it provides an intermediate alternative between 16QAM and 32QAM.

The format 256-D_4 is constructed using 256 points from the D_4 lattice, which is the most dense lattice in four dimensions, which gives it the same SE as 16QAM with an increased CFM of 1.71 dB over 16QAM. This format was studied in the context of bit-wise decoding in[31] where it was shown that in terms of GMI, this format has lower achievable rates compared to 16QAM due to the infeasibility to Gray code the constellation. In,[44] this format was experimentally realized and compared to PM-16QAM. It was shown that below roughly BER $= 10^{-3}$, the 256-D_4 format achieved an increased transmission reach. However, after decoding of a TPC with a 21.3 % overhead, PM-16QAM had the longest transmission distance. Compared in terms of achievable information rate in Fig. 4, it is clear that 256-D_4 is an interesting alternative to 16QAM, provided that suitable decoding schemes with iterations between the demapper and the decoder can be implemented.

Also plotted in Fig. 3 is the SE and CFM for the best known packing in two dimensions of 16 points, here denoted as $C_{opt,16-2D}$.[45] This format has 0.58 dB increased CFM over 16QAM. However, compared to 256-D_4 which is optimized in four dimension, the $C_{opt,16-2D}$ format has 1.13 dB lower CFM, demonstrating the benefit of optimizing formats in a higher dimensional space. The same conclusion can be drawn comparing the achievable information rate in Fig. 4 of $C_{opt,16-2D}$ and 256-D_4. Optimizing the constellation in two dimensions provides a small benefit over 16QAM, but not as large as optimizing the constellation in four dimensions.

5. CONCLUSIONS

In this paper, a range of multidimensional modulation formats have been compared in terms of CFM, SE, and MI. We have presented several alternative formats with performance comparable to either QPSK or 16QAM and shown that it is possible to increase the CFM over QPSK or 16QAM without loss in SE. Further, several formats that are sacrificing SE in order to increase the CFM have been discussed. We have also shown that 8iPPM-QPSK can simultaneously increase both the SE and CFM over QPSK. The choice of modulation formats depends on several aspects such as the required SE, sensitivity, complexity of the transmitter and receiver, resolution of the analog/digital converters, the applied FEC scheme etc.

When the formats are compared in terms of MI, i.e. for systems with advanced SD FEC schemes, we have seen that different formats than in the CFM study have the best performance. For instance, SO-PM-QPSK has higher CFM than QPSK but in terms of MI it has a lower achievable information rate. Several of the discussed modulation formats can be described as subsets of other known modulation formats and these subsets always have a lower MI compared to the original format. However, if the overhead is limited, it might still be beneficial in terms of required SNR to use a subset format instead of a higher order format. If no restriction applies, the best formats in terms of MI are constructed from as large constellations as possible, for reasonable SNR values.

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