THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN SOLID AND STRUCTURAL MECHANICS

System identification of large-scale linear and nonlinear structural dynamic models

VAHID YAGHOUBI

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY

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VAHID YAGHOUBI
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Department of Applied Mechanics
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: +46 (0)31-772 1000

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Abstract

System identification is a powerful technique to build a model from measurement data by using methods from different fields such as stochastic inference, optimization and linear algebra. It consists of three steps: collecting data, constructing a mathematical model and estimating its parameters. The available data often do not contain enough information or contain too much noise to enable an estimation of all uncertain model parameters with a good-enough precision. These are examples of challenges in the field of system identification. To construct a mathematical model, one should decide upon a model structure and then estimate its associated parameters. This model structure could be built with clear physical interpretation of its parameters like a parameterized finite element model, or be built just to fit to test data like general state-space or modal model. Each such model class has its own identification challenges. For the former, the complexity of finite element models can create an obstacle because of their time-consuming simulation. Furthermore, if a linear model does not represent the data with reasonable accuracy, nonlinear models need to be engaged and their modeling and parameterization impose even bigger challenges. For the latter, selecting a proper model order is a challenge and the physical relevance of identified states is an important issue. Deciding upon the physical relevance of states is presently a highly judgmental task and to instead do a classification based on physical relevance in an automated fashion is a formidable challenge. In-depth studies of such modeling and computational challenges are presented here and proper tools are suggested. They specifically target problems encountered in identification of large-scale linear and nonlinear structures. An experimental design strategy is proposed to increase the information content of test data for linear structures. By combining some new correlation metrics with a bootstrap data resampling technique, an automated procedure is developed that gives a proper model order that represent test data. The procedure's focus is on the physical relevance of identified states and on uncertainty quantification of parameter estimates. A method for stochastic parameter calibration of linear finite element models is developed by using a damping equalization method. Bootstrapping is used also here to estimate the uncertainty on the model parameters and response predictions. For identification of nonlinear systems a method is developed in which the information content of the data is increased by incorporating multiple harmonics of the response spectra. The parameter uncertainty is here estimated by employing a cross-validation technique. A fast higher-order time-integration method is developed which combines the well-known pseudo-force method with exponential time-integration methods. High-order-hold interpolation schemes are derived to increase the methods stability. As an alternative, to speed up the computations for large-scale linear models, a surrogate model for frequency response functions is developed based on sparse Polynomial Chaos Expansion.

Keywords: System identification, Uncertainty quantification, Bootstrapping, Polynomial chaos expansion, Surrogate modeling, Exponential integration, Finite element model

to my beloved family and in memory of my grandfather

Preface

The work presented here has been carried out at the department of Applied Mechanics at Chalmers University of Technology between 2011 and 2016. This project has been partially funded as a part of *Swedish Aviation Engineering Program (NFFP5)*. I would like to express my sincere appreciation to those who have contributed to this thesis.

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Gothenburg, May 2016
Vahid Yaghoubi

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A	M. K. Vakilzadeh, V. Yaghoubi, T. McKelvey, T. Abrahamsson, and L. Ljung. Experiment Design for Improved Frequency Domain Subspace System Identification of Continuous-Time Systems. IFAC-PapersOnLine 48 (2015), 886–891
Paper B	V. Yaghoubi and T. Abrahamsson. The Modal Observability Correlation as a Modal Correlation Metric. *Topics in Modal Analysis, Volume 7*. Springer, 2014, pp. 487–494
Paper C	V. Yaghoubi, M. K. Vakilzadeh, and T. Abrahamsson. Automated Modal Parameter Estimation Using Correlation Analysis and Bootstrap sampling. *submitted for international journal publication* (2016)
Paper D	M. K. Vakilzadeh, V. Yaghoubi, A. T. Johansson, and T. Abrahamsson. Stochastic Finite Element Model Calibration Based on Frequency Responses and Bootstrap Sampling. *submitted for international journal publication* (2016)
Paper E	V. Yaghoubi, S. Marelli, B. Sudret, and T. Abrahamsson. Sparse Polynomial Chaos Expansions of Frequency Response Functions Using Stochastic Frequency Transformation. *submitted for international journal publication* (2016)
Paper F	V. Yaghoubi, T. Abrahamsson, and E. A. Johnson. An Efficient Exponential Predictor-Corrector Time Integration Method for Structures with Local Nonlinearity. *submitted for international journal publication* (2015)
Paper G	Y. Chen, V. Yaghoubi, A. Linderholt, and T. Abrahamsson. Informative Data for Model Calibration of Locally Nonlinear Structures based on Multi-Harmonic Frequency Responses. *submitted for international journal publication* (2015)

Appended papers were prepared in collaboration with co-authors. The thesis author of was responseible for major progress of the works presented in **Papers A-F**. This includes planning the paper, taking part in developing theories and their numerical implementation, carrying out the numerical simulation and writing the papers. In **Paper G**, the thesis author has contributed to developing theory, carrying out the numerical implementation and simulation.

CONTENTS

Part I
Extended Summary

1 Motivation and background

Modeling and simulation (M&S) play important roles in the analysis, design and optimization of engineering systems. It targets deriving models which are accurate enough for their intended purpose that can be used to predict the systems' behavior under different conditions.

In general there are two distinctively different ways of arriving at models for engineering systems: by black-box or by white-box modeling. Black-box models are determined from flexible model structures that are made to match measured data as well as possible. Common such models for dynamic systems are state-space, ARX, ARMAX and modal models. White-box models are the result of diligent and extensive physical modeling from first principles. The resulting model could for example be given as a system of partial differential equations (PDEs). In order to obtain its time responses by simulation, the PDEs are most often spatially discretized by using the finite element method (FEM). This yields a system of ordinary differential equations (ODE) in time by which the simulation can be carried out by using time-integration methods.

In a case when the model's response is compared with test results, it is often found that the nominal model does not give the required accuracy to predict the structural response to an excitation. That is mainly due to (i) modeling errors which are the result of model simplification, discretization, etc., (ii) parameter errors which refer to inaccurate setting of model parameters and (iii) measurement inaccuracies which can be due to the effect of noise and measurement errors.

A procedure to deal with various sources of uncertainties and other deficiencies during the development of a valid mathematical model is shown in Figure 1.1. Different aspects of this procedure are treated in this thesis. Model verification, as the first step of the procedure, is dealing with modeling errors. These errors could be introduced at two stages. The first stage is when the mathematical model is approximated by a numerical model. This could introduce errors such as discretization errors and convergence errors in iterative computational schemes. The second stage is when the numerical model is implementing in the computer program. This could introduce errors as the consequence of programming mistakes or algorithm inconsistencies. The procedure engaged to eliminate the errors at the first and second stages are called *calculation verification* and *code verification*, respectively.

To tackle errors of type (ii) and (iii), a hybrid category of modeling should be employed which is called grey-box modeling. This is a parameterized white-box model with unknown or uncertain parameter setting which requires to be adjusted for increasing the model fit to test data. The associated parameterization should ideally be performed based on good physical insight.

Estimation of parameters in black-box and grey-box models is the focus of the field of system identification. The uncertainty of the estimated parameters as a consequence of measurement noise and errors can also be estimated in the process. This uncertainty estimation can be viewed as an overlap between system identification and uncertainty quantification. When experimental data are available, these uncertainty estimations can be done by employing statistical inference methods, linear algebra and optimization algorithms. Figure 1.2 shows a flowchart of system identification methods. In brief, after collecting data from a system, a suitable model set should

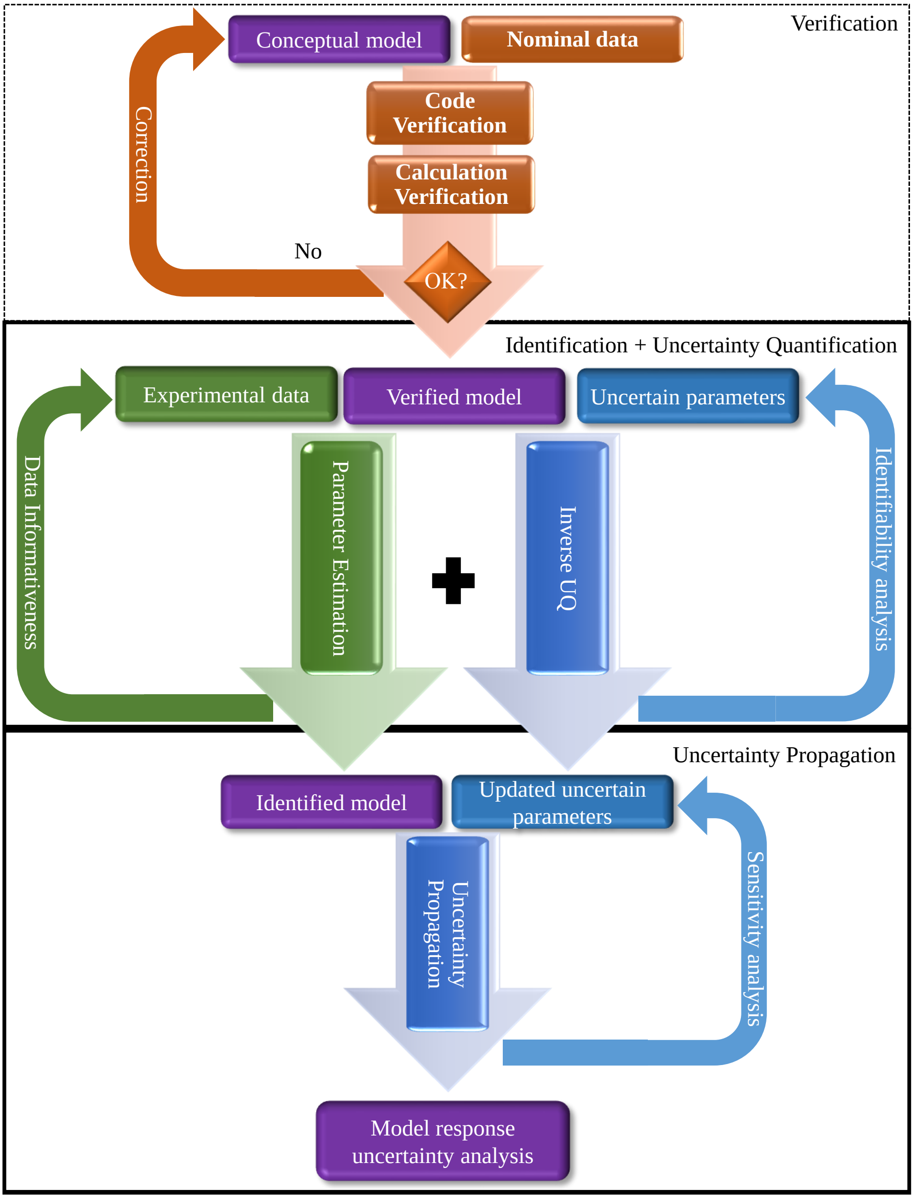

Figure 1.1: *Model development procedure: verification, identification and uncertainty quantification.*

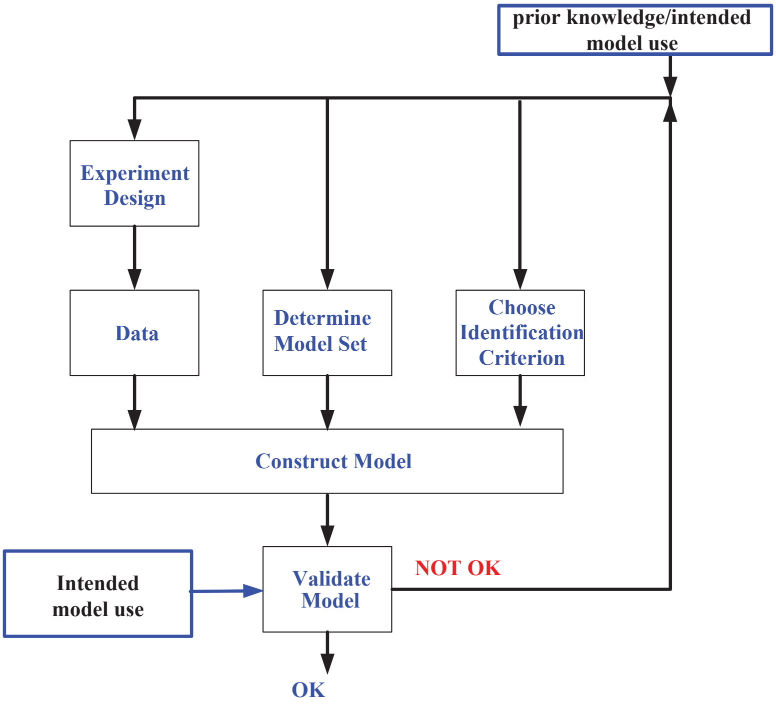

Figure 1.2: *System identification procedure [74, 49].*

be selected and then, using a proper identification algorithm, its associated parameters could be estimated.

The model is either chosen to represent a linear or nonlinear view on reality. Linear models are dominating the representation of the global structural dynamics behavior of complex mechanical systems. This is due to the fact that linear models are computationally efficient and have simple input-output relationships which most often offer good enough insight into the systems' dynamics. However, many dynamic systems behave somewhat nonlinear during their normal operations and all structures show nonlinear characteristics when subjected to extreme loading conditions. If significant nonlinear characteristics are found from testing of the system, a linear model may be judged being insufficient to represent the structural behavior and a nonlinear model has to be engaged. This frequently occurs for mechanical systems that include various sources of nonlinearity such as joints with gap and dry friction and structural parts subjected to large deformations. The validity of a model is crucial for most applications. It might be found that a linear model is not good enough (and thus falsified) for the intended use and nonlinear effects need to be brought into the modeling. Today's demand for high fidelity models often calls for modeling of these nonlinear effects properly. This motivates that much effort is spent on nonlinear system identification in industry and academia [43, 81]. Another possible validation finding is that a linear model structure is good enough, but the model parameters need to be calibrated.

As shown in Figure 1.1, the final step of developing a mathematical model is to estimate the uncertainty on the model's response prediction. This is an aspect of the uncertainty quantification which is often called uncertainty propagation. In general, this procedure requires a lot of model evaluations which means a heavy computational burden for large-scale dynamic system. This motivates the necessity of fast simulation methods.

2 Research challenges in system identification

The inverse nature of system identification imposes several challenges. These include selecting a proper model structure and describing its uncertain parameters appropriately. These also include the parameters identifiability issue which concerns whether the parameters can be estimated with small uncertainty or not. Identification obstacles can be handled before conducting tests to make the test informative with respect to the most influential parameters. This can be done by finding the most effective locations of sensors and actuators and finding the best type of measurement conditions which results in the estimation of the parameters with minimum level of uncertainty.

Furthermore, most identification algorithms are based on minimizing a deviation metric that describes the difference between the measure data and the model's response. This is called a cost function. In order to have fast convergence in such minimization, these methods often use the information of the gradient and Hessian of the cost function. Since the cost functions might be quite irregular and can contain several local minima, the minimizers may get stuck into these local minima but to which one, depends on start condition of the minimization algorithm [72, 62, 27]. This is another major challenge in such identification methods.

For black-box system identification, several methods have been developed to construct a mathematical model with little user interaction [51, 58, 75]. One central problem here is to select the complexity of the model. This is an art that is more-or-less left to the user to decide upon. If the model shows linear time invariant (LTI) behavior, this reduces to selecting a proper model order. In the context of system identification, there exists an extensive literature for order estimation of linear dynamical models, *e.g.* the Singular Value Criterion (SVC) [7] and the Akaike Information Criterion (AIC) [3]. These criteria mostly target the prediction capability of the model. However, in the context of structural dynamics, in addition to that, the physical relevance of the parameters is another important factor which should be considered. Since distinguishing between physical and noise states is a highly judgmental task, user expertise plays a significant role in it. One major challenge is to automate this procedure.

For nonlinear system identification more parameters are normally required to substantially increase the model's capability of representing the real structural behavior as compared to a corresponding linear model. Therefore, a set of candidate parameterized properties that affect the nonlinear behavior have to be added. The selection of such candidates is a challenging task which must be done based on insight into the physics which governs the behavior of the system at hand.

In addition, the trend towards working with more detailed models of large-scale engineering systems is continuing but long simulation time is one of the major complexity limiting factors. This is specially valid for nonlinear systems. Although the computational power of modern computers grows very fast, the increasing model complexity, more precise description of model properties and more detailed representation of the system geometry result in considerable execution time and memory usage for the computer simulations. Therefore, efficient simulation tools are practical necessities. The situation become more challenging when it comes to system identification process because it normally involves a large number of model evaluations.

3 Research aim and scope

Given input and output test data of a dynamic system, the overall goal of this work is to develop a *fast* and *robust* framework to identify a model that give a similar input to output relation. These objectives for the black-box and grey-box system identification are treated in this thesis.

3.1 Black-box system identification

In black-box system identification, a model structure should be selected and then, as the identification part, its parameters are estimated from the available test data. The major objective of black-box identification here is to develop a fast and robust framework for the identification of a proper mathematical model from one single noisy measurement dataset. Moreover, the framework should give confidence interval for the estimated parameters. Here, the framework is called *fast* because it avoids high-dimensional optimization in the overall procedure and is called *robust* since it is rigorous, independent of threshold values, rules-of-thumb and user interaction. It is thus autonomous and its sensitivity to measurement noise is small. If a modal model is selected as the model structure, this task will be referred to as an automated modal parameter estimation. If on the other hand, a state-space model structure is selected, this addresses the proper model order selection by focusing on the physical relevance of the associated parameters, see **Paper C**. These were topics of several other recent works [76, 39]. However, most of the presented methods were not fast, *i.e.* their algorithm included high-dimensional optimization algorithms [63, 76, 77], or robust, they are not fully automatic and need user decisions.

3.2 Grey-box system identification

A grey-box model has a known structure but with unknown parameter setting. Giving good estimation of these settings when test data is available is the focus of grey-box system identification efforts. To get it *fast* for large-scale dynamic structures, the method requires fast forward simulations and to get it *robust*, the method should work well in presence of realistic levels of measurement noise [16].

In a variety of application fields grey-box models that give plausible representation of reality have been built up over decades of studies. They are often large, detailed and complex and thus time consuming for simulation. Since identification often involves many repeated simulations, fast simulation of those is a key. To this end two different strategies can be devised: (i) Speed up the simulation by using fast time-integration method like the one presented in **Paper F**. (ii) Speed up the simulation by the use of approximants, for instance by using surrogate models, like the one described in **Paper E**.

The models used usually contains many uncertain parameters and test data are rarely rich enough to allow reliable identification of all these parameters. Moreover, the test data collected from the associated system is also usually contaminated with measurement noise. Consequently a robust identification procedure is a necessity. This can be achieved by (i) analyzing the identifiability of the parameters and evaluating the information content of the data employed for estimation of the parameters, see **Paper G**, or (ii) using the given noise information in the cost function, see **Paper D**.

4 Research methodology

To perform a system identification, a set of the model's responses should be selected to compare with the corresponding test data. These responses could be represented as time, frequency or modal domain data. To analyze the behavior of a linear dynamic system, the frequency response function (FRF) is important because it provides information over a given frequency range of interest with a clear physical interpretation. This is the main reason for the focus on the FRF in different areas of research such as model updating [32, 87], vibration control [15, 53] and system identification [13, 85]. The research presented here also focus on FRF data.

The FRF is often measured through vibration testing and can be used for identification of black-box or grey-box dynamic models. For this purpose, a parameterized model is required. In black-box system identification, the parameters are viewed as vehicles for adjusting the fit to the data and does not reflect any physical interpretation in the system [49]. These parameters depend on the selected model structure and its complexity. The choice of an appropriate model structure is thus a very basic and important choice when performing black-box system identification. Examples of such model structures for dynamic systems are state-space and modal models. Both these are used in this thesis. The modal model is the most common model in the community of structural dynamics and its associated parameters includes modal frequency, modal damping and modal vectors. The state-space models are selected due to the following reasons: (i) they can represent multi-input, multi-output (MIMO) systems equally convenient as single-input single-output (SISO) systems and (ii) they can easily be expanded to include more complex physical behavior, *e.g.* nonlinear models. The parameters of a state-space model are the elements of some canonical form of its associated matrices. It should be emphasized that for LTI systems, the modal models and state-space models are interchangeable with a unique mapping.

The required complexity of the selected model is a crucial aspect. It needs to be determined if the test data can be represented by a linear model or whether more advanced models are required. The determination of model complexity is one of the challenges in most identification algorithms. Much work has been addressed this issue in the context of both control theory and structural dynamics. This is elaborated upon in Section 4.1.

In grey-box system identification, the models typically reflect the underlying physics or mechanistic understanding of the system. Parameterized finite element models are examples of such models. Their parameters usually have physical interpretation such as:

- parameters describing the geometry of the system, like cross-section area and length of structural members
- parameters describing the material constitutive law, like Young's moduli, Poison's ratio and yield stress.
- parameters describing the loading of the systems, like applied pressure and enfourced ground motion

To estimate the best setting of these parameters, a proper estimation method should be selected. Well-known estimation methods are the maximum likelihood estimation (MLE), the maximum *a posterior* (MAP) and the state-space subspace-based (S4) estimation methods [49, 59].

Given uncertainties in the test data, the parameters' uncertainties can be estimated by statistical methods. This gives an overlap between system identification and uncertainty quantification (UQ). In general, there are two major types of uncertainty quantification which can be inferred from the following citation borrowed from [73] about the definition and reasoning behind the UQ: "Quantification of Uncertainty is driven by the identification, characterization, and quantification of the uncertainties that appear in the code predictions of "Best Estimate" calculations. The thrust of Best Estimate plus Uncertainty (BE+U) is that prediction is probabilistic precisely because of our inability to complete V&V [Verification and Validation] in some definitive sense and because of uncertainties intrinsic to complex modeling activities". This means that one should estimate the uncertainty bound of the parameters at the Best Estimate, as the first type of UQ, and then propagate the estimated parameters through the model to establish credible bounds on model's prediction on an intended application in a regime of interest [64], as the second type of UQ.

The first type is referred to as inverse uncertainty quantification and its target is to estimate the covariance matrix of the parameters. This can be done using the inverse of the Fisher information matrix (FIM) to provide the Cramér-Rao lower bound [41] on the estimated parameters, or by performing cross-validation checking, *e.g.* by k-fold or leave-one-out sampling methods. Bootstrapping is an alternative method that could be used for this purpose. This will be discussed in more detail in Section 4.2.1.

The second type is uncertainty propagation. Techniques for uncertainty propagation can generally be classified as intrusive or non-intrusive [47]. In the intrusive approaches the governing equations of the mathematical models are modified such that one explicit function relates the stochastic properties of the system responses to the system's parameters. The perturbation method, the Karhunen-Loeve expansion and intrusive polynomial chaos expansion are some tools used for this purpose. In contrast, non-intrusive approaches use already existing deterministic codes to estimate uncertainty by evaluation at sample points given by parameters' assumed probability distribution functions (PDFs). The Monte Carlo methods and non-intrusive polynomial chaos expansion are examples of these methods.

Besides that, the uncertainty propagation can be categorized based on its target [70] as:

- *Second moment methods*, these methods mostly focus on the central part of the system responses PDF *i.e.* mean and standard deviation of the system response. Examples of such methods are *perturbation methods* and *quadrature methods*. These methods are limited to studies of the first two statistical moments of the response.

- *Reliability methods*, these methods mostly focus on the tail of the system response PDF to evaluate the probability that the response exceeds a prescribed threshold. First-order reliability method*FORM*, Second-order reliability method*SORM*, *importance sampling* and *subset simulation* are examples of such methods [70, 5].

- *Whole PDF*, these methods are interested in the entire system response PDF. *Monte Carlo Simulation* (MCS) is the basic tool for this purpose. Another approach is based on *spectral methods* [82, 26].

Both system identification and uncertainty quantification include a lot of model evaluations. If a large-scale or nonlinear structure is of interest, the forward simulation will require extensive computational resources the computational efficiency of the used methods become an inevitable ingredient. This will be discussed deeper in Section 4.2.2.

4.1 On black-box system identification

Over the last few decades, a lot of effort has been put to develop efficient algorithms for identification of the modal parameters using time or frequency domain [39, 40]. A central problem in most of these algorithms is to determine a good estimate of a proper model order. In addition, the physical relevance of the individual modes of the identified model is of importance. Therefore, the common practice is to identify a model with an order higher than needed to ensure that all physical modes present in the considered frequency band are captured [77, 78]. However, this inevitably results in the appearance of the so-called noise modes in the identified model, *i.e.* modes which are present in the model due to measurement noise or computational imprecision but have no relevance to the physics of the tested system. Various tools have been developed in the literature to detect and eliminate such noise modes from a model. The most widespread tool is undoubtedly the so-called stabilization diagram [29, 30].

In recent years, many attempts have been made to automate the interpretation of stabilization diagram, or the modal parameter estimation procedure in general [55, 61]. Owing to the fact that analyzing the stabilization diagram reduces to finding the modes with similar properties, the majority of automation strategies borrow methods from statistical machine learning [34], such as Support Vector Machine (SVM) [30] and clustering algorithms [63, 78, 76].

However, most of these methods are not robust in the sense that they require user-defined parameters or thresholds. To the authors knowledge, the only exceptions are the algorithms proposed by Vanlanduit *et al.* [76], Reynder *et al.* [63], and Verboven *et al.* [78, 77]. Nevertheless, their methods are not fast. This means that they involve high-dimensional optimization procedures, either for their estimation [76, 77] or clustering algorithm [63], which deteriorates their performance both in terms of the computational efficiency and convergence.

Paper C presents an algorithm for fast and robust modal parameter estimation in which the only required optimization is performed in a three-dimensional space. To this end, for the estimation, a S4-based identification method is employed and for the clustering, a correlation-based strategy is used.

The S4 identification algorithm is selected for the estimation because (i) there is no need for an explicit model parametrization (ii) the elegance and computational efficiency of the algorithm which is due to avoiding high-dimensional optimizations. This method relies on QR and singular value decompositions which are well-understood techniques from numerical linear algebra. However, although this identification algorithm has been proved to be theoretically consistent [51], in presence of noise and under certain conditions it loses the consistency which leads to bias in the estimated parameters. Several approaches have been developed in the literature to solve this issue. For more discussion see **Paper A**.

The correlation-based clustering is selected due to its non-iterative nature. One major challenge to tackle is the *spatial aliasing* phenomenon. It occurs when few sensors are engaged in the experimental determination of modal vectors and results in high correlation values between two modal vectors that correspond to different resonance frequencies. Another challenge is the nonunique eigenvectors of coalescent modes which becomes a crucial issue in correlation analysis as well as in automated modal parameter estimation algorithm. In **Paper B** two correlation metrics are described which can treat these issues simultaneously. They are further used in **Paper C** for an automated modal parameter estimation.

4.2 On grey-box system identification

The grey-box system identification described here combines a parameterized FE model of a structure with test data to identify its unknown parameters. Therefore, it requires a fast simulation method for FE analysis as well as a proper identification method. The fast simulation is especially important when a large-scale and/or nonlinear system is considered.

4.2.1 Robust system identification

Frequency response data are commonly used for the FE model calibration. This is due to the fact that the information can be collected in wide frequency range and the calibration problem becomes overdetermined due to the availability of FRF data from MIMO experiments [25] that provide a rich dataset. One specific problem here is that the system's frequency response is strongly affected by damping and the damping is notoriously difficult to model using first principles. The damping is thus most often described as modal damping in calibration exercises. However, it is then important that the correct modes from the model and the test data get matched together. This task, called mode pairing, is very tedious and delicate. Recently a new method, called damping equalization, has been proposed to avoid the mode pairing for deterministic parameter estimation [1]. The damping equalization is achieved by imposing the same modal damping on all experimentally found system modes and making proper adjustment of the test data. That technique is used here.

Spectral input and output data can also be used for nonlinear system identification. For a nonlinear system, the stationary frequency response (if such exists) is multi-harmonic and load dependent. The reason is that when a nonlinear system excited by a loading with one single frequency components, it reacts such that its response contains different harmonics of that frequency [81, 56, 2]. Therefore, the multi-harmonic FRF should be considered. This is done in **Paper G**.

Since test data are often corrupted by measurement noise, the point estimation of the parameters can be affected and are often subject to bias and variance. The statistics for the estimated parameters is thus of great interest [31, 67]. To this end, the statistical problem can be formulated in two distinct ways based on either the Bayesian or the Frequentist paradigm. The Bayesian approach views probability as a measure of relative plausibility of different possibilities conditional on available information [8]. In contrast, the Frequentist approach assigns a probability measure to an inferred value of a parameter setting through the notion of repetition [19]. The Maximum Likelihood Estimator (MLE) is a frequently used estimator in the Frequentist literature. It gives the parameter setting by maximizing the associated likelihood function [59]. If the model's parameters are identifiable [49], the distribution of the MLE can be asymptotically approximated by a Gaussian distribution whose covariance matrix is the inverse of the FIM given by the negative Hessian of the logorithm of the likelihood function [49]. This can be also carried out by performing cross-validation methods, *e.g.* by using k-fold or leave-one-out sampling methods.

Bootstrapping is an alternative approach to quantify the uncertainty in the model parameters. The idea behind bootstrapping is to generate several datasets by repeated random sample the observed data and then, perform repeated parameter estimations for each individual bootstrap dataset [34, 20]. Bootstrapping results in a number of samples from the model parameters distribution which can be used to quantify the degree of confidence in the parameter estimates.

To evaluate the uncertainty of the model response prediction, it is of special interest to propagate the estimated model parameter's uncertainty through forward simulation [50, 54]. For this purpose the most straightforward approach is to use the assumed asymptotic distribution (Gaussian) for the parameter's uncertainty and then, directly use samples of the distribution. Bootstrapping is a well-known approach to assess the credibility of the model predictions. In particular, this method is of interest when the goal is to quantify the prediction error and its variability for model responses for which there exist no measured data [18]. To this end, several bootstrap rules rule have been developed which basically keep track of how well a model evaluated at an individual sample from the distribution of the parameter estimates predicts the response of interest at data points that are not included in the associated bootstrap dataset [18]. One such methods is the 0.632 bootstrap. For discussion on the available method and the proposed methodology of using bootstrapping method for uncertainty quantification of linear structures see **Paper D** and for using cross-validation technique for a nonlinear structure see **Paper G**.

4.2.2 Fast forward simulation

To identify a model for a large-scale dynamic system, long overall simulation time is normally one of the major obstacles. It occurs due to two reasons: (i) each forward simulation in itself is time-consuming and (ii) a lot of evaluations are required by simulations due to iterative nature of identification. In the literature, model reduction [44, 60], efficient time-integration [84, 6, 48] and parallel simulation methods [71, 86] are different strategies that address the first issue. In this context, an efficient time-integration method has been developed for nonlinear structures to reduce simulation time.

There are three important aspects in simulating a nonlinear structure. The first is that sufficiently accurate predictions/simulations of the nonlinear structure's response should be obtained. Then, the efficiency of the simulation required to be such that one may obtain results fast enough for convenience. Last but not least, is the issue of numerical stability of the simulation method, *e.g.* see **Paper F**.

One well-established method that considers the effect of structural system nonlinearity is the pseudo-force method [33]. In this method, the nonlinear effects are considered as being external forces. Felippa and Park [21] used this method to treat the nonlinearity in structural dynamics. Lin et al. [46] presented an iterative pseudo-force method for second-order systems to accommodate non-proportional damping in structures. As higher order ODEs can always be recast into first order form, suitable numerical integration schemes like different Runge-Kutta methods can be used to find system responses. Although these integration methods are well-established, a significant effort is still being expended for nonlinear systems to find methods that integrate the nonlinear equations efficiently and accurately. Exponential time integration methods belongs to a class of methods that attempt to solve a first-order ODE system by use of a semi-analytic approach [14, 23, 35]. These methods are thus fast. This motivates researchers to reintroduce these methods in some areas for fast and accurate simulation of large scale circuits [79, 90], and for nonlinear [89] or stiff systems [52]. Since these methods use the exact analytic solutions for certain type of stimuli, the local truncation error that is introduced by most other numerical methods is not a problem. Hence for the free decay initial-value problem of a physically stable linear system it is A-*stable* and accurate, independently of the chosen length of time-step. The approximations are introduced to the ODE solutions when transient response under general loading conditions is of interest. In such cases, exponential integration with time-stepping schemes based on hold interpolation of the loading can be done with different hold orders. The zero-order hold, the first-order hold and the triangular hold schemes are well established. To increase the stability and accuracy of the results, higher-order hold and, in particular, the second-order hold have been introduced [88, 17]. For more discussions on the available methods and the proposed methodology see **Paper F**.

Stochastic model reduction [4] and surrogate modeling [22] are two methods employed to replace a computationally expensive model with an approximant that can reproduce the essential features faster. Of interest here are surrogate models. In analogy with the methods of uncertainty propagation, they can be created intrusively or non-intrusively. Perturbation method [66] is an example of intrusive approach. This is a classical method for uncertainty propagation purpose but it is only accurate when the parameters has small coefficients of variation (COV), say COV<5%. An alternative method is the intrusive polynomial chaos expansion methods [26]. It was first introduced for Gaussian input random variables [80], and then extended to the other types of random variables leading to generalized polynomial chaos methods [83, 68].

The non-intrusive methods can be categorized as being either regression methods [11, 9] or projection methods [28, 45]. Kriging [24, 38] and non-intrusive PCE [10] or combinations thereof [42, 65] are examples of non-intrusive approaches. The major drawback of the PCE methods, both intrusive and non-intrusive, is the large number of unknown coefficients that need to be computed in problems with large parameter spaces. This is referred to as the curse of dimensionality [69]. Sparse [12], adaptive [11] and adaptive sparse [10] polynomial chaos expansions have been developed to dramatically reduce the computational cost for this type of problems.

Very few recent papers address the direct implementation of PCE to represent frequency responses of systems [57, 36, 37]. They have indicated that PCE does not converge at the peaks even with very high-order PCE for FRFs of very trivial cases. For deeper discussion on this topic and a proposed methodology to alleviate this problem see **Paper E**.

5 Thesis contribution

The significant parts of the thesis contributes to the black-box and grey-box system identification of large-scale linear/nonlinear dynamic systems. They are summarized at the following.

5.1 Automated modal parameter estimation

The major contribution here is the development and validation of a automated modal parameter estimation algorithm such that the following apply: (i) it involves a system identification algorithm which allows for fast and robust identification of MIMO systems of a given order, (ii) it avoids high-dimensional optimization, (iii) it provides uncertainty bounds on the estimated modal parameters and (iv) it needs no user-specified parameters or thresholds.

The keys to success of this method are recently developed correlation metrics which help to improve the performance of the engaged correlation-based clustering algorithms. This improvement is done by augmenting the eigenvalue information with the eigenvector into a correlation analysis, like the one often performed by use of the Modal Assurance Criteria (MAC). This correlation is called Modal Observability Correlation (MOC) and is presented in **Paper B**. It can treat the spatial aliasing phenomenon. Moreover, the major problem of nonunique eigenvector of the coalescent eigenvalues are addressed by projecting these eigenvector to a subspace with fixed and orthogonal basis. This subspace can be obtained by applying QR-factorization to the spatial information of the loading.

To do the identification, a S4 identification algorithm is employed. Another contribution in this part is an experimental design to make the S4 identification algorithm more consistent and reduce the bias in the estimated parameters. This is described in **Paper A**.

5.2 Fast simulation methods for large-scale systems

One contribution here is the development of a surrogate model for the FRF of a linear system by using the sparse PCE, see **Paper E**. To this end, there are two major difficulties which are addressed in this work. (i) The shift of eigenfrequencies due to parameters variation, (ii) the non-smooth behavior of the FRFs. The method of stochastic frequency transformation has been developed in which those problems are addressed. To make the method work for systems with a large parameter space, a sparse and adaptive PCE has been used.

The next contribution is the development a predictor-corrector exponential time-integration method to simulate nonlinear structures fast and accurate. This conditionally stable method is based on the well-known concept of pseudo-force for nonlinear simulation adapted for exponential time-integration methods. To increase the domain of stability, a second-order hold interpolation has been developed. It was shown that it contributes positively to accuracy and efficiency. The method is applied to a complex nonlinear structure, see **Paper F** for more discussions. Some successful attempts have been made to extend the concept for parallel simulation method in [86].

5.3 Grey-box system identification

The first contribution of this part is the development of a new framework for identification and uncertainty quantification of a linear model as proposed in **Paper D**. This method consists of three steps of which the first is to replace the measured data with an identified state-space model using the algorithm proposed in **Paper C**. The dedicated frequency sampling strategy proposed in **Paper A** is used as an experimental design for state-space model identification with less bias. In the next step, the grey-box model is calibrated against this state-space model. The method is free from mode matching which is necessary for most other algorithms [1]. This damping equalization in conjunction with a logarithmic cost function results in good parameter estimates. These estimates are used to start the next step, the uncertainty quantification. To do the uncertainty quantification the bootstrapping technique is used to estimate the uncertainty in the parameters as well as uncertainty in the model response prediction.

The accuracy of the parameters are improved by employing two strategies. (i) Analysis of the identifiability of the parameters. The parameters are said to be identifiable if they can be estimated with small uncertainties. This uncertainty analysis is done by evaluating the inverse of the FIM [49]. (ii) A strategy that incorporates the measurement noise information in the cost function. This is done by weighting the deviation metric at each discrete frequency using the signal-to-noise ratio of the measured frequency responses. It contributes to the reduction of bias in the estimated parameters.

The next contribution here is to develop a framework for nonlinear system identification by the use of multi-harmonic FRFs as described in **Paper G**. The information content of the FRF data is increased by including the sub-harmonics and super-harmonics to the fundamental harmonic FRF. This information is used for the calibration process together with the identifiability analysis of the parameters to improve the accuracy of the parameter estimates. To decrease the risk that minimizer gets stuck into local minima of the cost function, the calibration optimization process is started from several points of the parameter space. These starting points are obtained by the Latin-hypercube sampling. A k-fold cross-validation is used to estimate the uncertainty of the parameters.

6 Summary of the appended papers

- **Paper A**: *Experiment Design for Improved Frequency Domain Subspace System Identification of Continuous-time Systems*

  A widely used approach for identification of LTI and MIMO systems from continuous-time frequency response data is to solve it in discrete-time domain using a subspace based identification algorithm which incorporates a bilinear transformation. However, the bilinear transformation nonlinearly maps the distribution of discrete frequencies from continuous-time domain to discrete-time domain which may make identification algorithm to be ill-conditioned. In this paper we propose a solution to get around this problem by designing a dedicated frequency sampling strategy. Promising results are obtained when the algorithm is applied to synthetic data from a 6DOF mass-spring model with low modal controllability.

- **Paper B**: *The Modal Observability Correlation as a Modal Correlation Metric*

  The historical development of the MAC originated from the need of a correlation metric for comparing experimental modal vectors estimated from measured data to eigenvectors that have been determined from finite element calculation. For systems with well separated eigenvalues with many system degrees-of-freedom (DOF) represented in the eigenvectors it is normally easy to distinguish eigenvectors associated to different eigenvalues by low MAC correlation numbers. However, for eigenvectors with a sparse DOF sampling it may be hard to distinguish between vectors by MAC correlation numbers. To reduce the problem of distinguishing between eigensolutions, this paper advocates the use of a new correlation metric based on the observability matrix of the a modally decoupled state-space realization.

- **Paper C**: *Automated Modal Parameter Estimation Using Correlation Analysis and Bootstrap Sampling*

  The estimation of modal parameters from a set of noisy measured data is a highly judgmental task, with user expertise playing a significant role in distinguishing between estimated physical and noise modes. Various methods have been developed to automate this procedure. The common approach is to identify models with different orders and cluster similar modes together. Most methods based on this approach suffer from high-dimensional optimization in either the estimation or clustering step. To overcome this problem, this study presents a novel algorithm for autonomous modal parameter estimation in which the only required optimization is performed in a three-dimensional space. To this end, an S4 identification method is employed for the estimation and a non-iterative correlation-based method is used for the clustering. This clustering is at the heart of the paper. The keys to success are correlation metrics that are able to treat problems of spatial eigenvector aliasing and nonunique eigenvectors of coalescent modes simultaneously. The algorithm commences by the identification of an excessively high-order model from measured frequency response functions. The high number of modes of this model provide bases for two subspaces: one for likely physical modes of the tested system and one for its complement. By employing the bootstrap technique, several datasets are generated from the same basic test dataset and for each of them a model is identified to form a set of models. By correlation analysis with the subspaces, the highly correlated modes of these models which appear repeatedly are clustered together and the noise modes are removed. A fuzzy c-means clustering procedure performed on a three-dimensional feature space assigns a degree of physicalness to each cluster in a final procedure step. Case studies indicate that the algorithm successfully clusters the similar modes and quantify the extent to which each cluster is physical.

- **Paper D**: *Stochastic Finite Element Model Calibration Based on Frequency Responses and Bootstrap Sampling*

  A new stochastic finite element model calibration framework for estimation of the uncertainty in model parameters and predictions from the measured frequency responses is proposed in this paper. It combines the principles of bootstrapping with the technique of FE model calibration with damping equalization. The challenge for the calibration problem is to find an initial estimate of the parameters that is reasonably close to the global minimum of the deviation between model predictions and measurement data. The idea of model calibration with damping equalization is to formulate the calibration metric as the deviation between the logarithm of the frequency responses of FE model and a test data model found from measurement. In that test data model, the same level of modal damping is imposed on all modes. This formulation gives a smooth metric with a large radius of convergence to the global minimum. In this study, practical suggestions are made to improve the performance of this calibration procedure when dealing with noisy measurements. A dedicated frequency sampling strategy is suggested for measurement of frequency responses in order to improve the estimate of a test data model. The deviation metric at each discrete frequency is weighted using the signal-to-noise ratio of the measured frequency responses. For uncertainty quantification, the experimental data is resampled using the bootstrapping approach and repeated calibration produce uncertainty bounds on the model parameters and predictions.

- **Paper E**: *Sparse Polynomial Chaos Expansions of Frequency Response Functions Using Stochastic Frequency Transformation*

  FRFs are important for assessing the behavior of stochastic linear dynamic systems. For large systems, their evaluations are time-consuming even for a single simulation. Therefore, uncertainty quantification by crude Monte-Carlo (MC) simulation is not feasible for such systems. In this paper, a surrogate model by using non-intrusive adaptive sparse PCE is proposed. In this approach, a stochastic frequency-transformation is developed to maximize the similarity between the FRFs before applying PCE. This allows using low-order PCE for each frequency. A principal component analysis is employed to reduce the number of random outputs. The proposed approach is applied to two case studies: a simple 2-DOF system and a 6-DOF system with 16 random inputs. The accuracy assessment of the results indicates that the proposed approach can predict the FRF with reasonable accuracy. Besides, it is shown that the first two moments of the FRFs obtained by the PCE converge to the reference results faster than MC methods.

- **Paper F**: *An Efficient Exponential Predictor-Corrector Time Integration Method for Structures with Local Nonlinearity*

  Simulating the nonlinear behavior of complex systems requires significant computational effort. Despite the rapid progress in computing technology, the demand is still strong for more efficient simulation methods in diverse structural dynamics fields such as nonlinear system identification and nonlinear system reliability. In addition to efficiency, algorithmic stability and accuracy must be addressed in the development of new simulation procedures. In this paper, we propose an efficient and accurate method to treat localized nonlinearities in a structure. The method is conditionally stable. The system equations are separated into a state-invariant linear part, and a state-dependent nonlinear part that is considered to be external pseudo-forces that act on the linear system. The response of the system is obtained by fixed point iterations in which the pseudo-forces are updated until convergence. Although the method works well with one time-step ahead prediction, the effect of multiple time-step ahead prediction is also investigated and shown to increase algorithm efficiency. Since the method is based on linear state-space model form similarity transformations and model reduction can be easily exploited. To perform the numerical integration, time-stepping schemes like the exponential first-order hold method can be used to take advantage of their efficiency and accuracy. To increase the accuracy and stability of the method, a second-order hold equivalent is derived and implemented. The efficiency, stability and accuracy of the method are demonstrated by numerical examples.

- **Paper G**: *Informative Data for Model Calibration of Locally Nonlinear Structures based on Multi-Harmonic Frequency Responses*

  In industry, linear FE-models commonly serve to represent the global structural behavior. However, available test data may show evidence of significant nonlinear dynamics. In such a case, an baseline linear model may be judged insufficient to represent the structure. The causes of the nonlinear characteristics may be local in nature, and the remaining part of the structure be satisfactorily represented by linear descriptions. Although a baseline linear model can then serve as a good foundation, the parameters needed to substantially increase the model's capability of representing the real structure are most likely not included in that model. Therefore, a set of candidate parameters to control the nonlinear effects has to be added. The selection of the model parameters and data for calibration form a coupled problem. An over-parameterized model for calibration results in unreliable estimates. The test should be designed such that test data could be chosen so that the expected variances of the estimated values of the chosen parameters are made small. The multi-harmonic steady-state responses due to periodic excitation are shown to contain valuable information for the calibration process of models from structures with local nonlinearities. In the paper, synthetic test data from a model that represents a nonlinear benchmark structure are used to verify the method. The model calibration and the k-fold cross-validation are based on the Levenberg-Marquardt and the Gauss-Newton minimizers, respectively. To increase the possibility of finding a global minimum, candidates for minimizer starting points are found by the Latin hypercube sampling method. The points with the smallest deviation, *i.e.* the lowest value of the objective function, is selected as a starting point for the calibration and cross-validation. The calibration result shows good agreement with the true parameter setting, and the k-fold cross validation result shows that the variance of the estimated parameters are reduced when including multi-harmonic nonlinear frequency response functions.

7 Concluding remarks and future work

The various accomplishments of this dissertation can be classified into two broad categories: (i) black-box system identification and (ii) grey-box system identification.

In black-box system identification, the objective is here to develop a framework for automated modal parameter estimation from noisy FRF data such that it can provide statistical information for the estimated parameters without the need for high dimensional optimization. For this purpose, first, two correlation metrics have been developed to treat spatial aliasing and non-unique eigenvectors of coalescent modes in modal correlation. Using these correlation metrics, a framework has been developed in which bootstrapping for sampling test data plays a significant role. A subspace-based linear algebra plays another significant role in both the estimation of modal parameters and clustering the modes with physical and noise mode behaviors.

In grey-box system identification, the major goal is to identify a large-scale nonlinear dynamic system when the test data is contaminated with measurement noise. The common approach is to first identify the underlying linear system. Then, remaining deviations between the test result and the model response can be considered as nonlinearities for which models should be identified. Based on this, a framework for stochastic calibration of linear models using FRF data is provided. Besides estimating the parameters and their associated uncertainties, the uncertainty on the model response prediction is estimated. For robustness, the given noise information is used to weight the cost function to reduce the effect of high noise-to-signal ratio. In addition, identifiability analysis of the parameters performed before the estimation process, contributes to the robustness of the method.

A nonlinear identification method has been proposed based on frequency response. In this method, the information content of the data with respect to the parameters was increased by including side harmonics of the FRF and excluding non-informative part of the measured data. The identifiability of model parameters with respect to the available data was analyzed by using the Cramér-Rao lower bound and the calibrated model was validated using a k-fold cross validation procedure.

Long simulation time could be a hinder for the use of these methods for large-scale systems. To resolve this issue, a surrogate model for the FRF of linear systems by using the PCE is developed. The major challenge of using the PCE for non-smooth system responses is treated. Sparse and adaptive methods are used to avoid the curse of dimensionality problem encountered by models with large-parameter space. Together with these, a method for fast simulation of nonlinear structures is developed. This can be considered as a combination of the pseudo-force method and the exponential time-integration method to enjoy the benefit of both methods by being accurate and efficient. It is given in one-time-step-ahead and multiple-time-step-ahead forms. The method is conditionally stable. To expand the region of stability, three different schemes of second-order hold interpolation were derived and investigated.

In this thesis, it is assumed that the model is verified and is without any modeling errors. One interesting extension of the current work is thus to take such modeling errors into account. Moreover, the selection of a proper model is an important aspect of the system identification. This is suggested as an option for future investigation.

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