Abstract

Power demands for heating of buildings vary a lot over the day and between seasons. These variations cause uneven supply of heat in district heating networks, which is both inefficient and costly. The buildings that are connected to the district heating system can be potentially utilized as auxiliary thermal storages to smooth out the variations in the heat supply demand. Since these effects can be achieved at the expense of thermal comfort in buildings, reliable planning tools are needed. This work presents a handy analytical model of a building connected to the district heating that can be used to estimate both the reduction of heating power demand and the discomfort that follows. The analytical analysis that gives straightforward results for a tentative building is complemented with a numerical analysis for cases with extra constraints. The analyses show promising results for this technique.

1. Introduction

District heating is a common solution for heating of buildings in urban areas in Sweden and approximately a half of all the buildings are heated in this way. The base production of heat in the system is done primarily from waste incineration, surplus heat from industrial plants and renewable energy sources. Sometimes auxiliary heat is required to cover the peaks in the demand, which is solved by starting up extra power units, run by fossil fuels, or separate
heat storage facilities such as large hot water reservoirs. The peaks in the demand are often short-termed; they occur typically in the morning, when there is the largest demand for space heating and hot water in buildings or during very cold weather. If the peaks in the demand could be avoided, the entire district system would be more cost-efficient and with less CO₂ emissions.

The thermal mass of the heated buildings connected to the district heating network can be potentially utilized as an alternative thermal storage to smooth out the peaks in the heat demand, as presented in [1]. Since this effect can be achieved at the expense of thermal comfort in the buildings, i.e. by allowing variations of indoor temperatures, reliable planning tools are needed. This work presents a handy analytical model of a building connected to the district heating that can be used to estimate both the reduction of peaks in heat demand and a possible discomfort that follows. The analytical analysis that gives straightforward results for a tentative building is complemented with a numerical analysis for cases with extra constraints.

2. The analyzed system

The analyzed building has a hydronic heating system with radiators and feed-forward control strategy, as shown in Figure 1. The supply temperature to the radiators \( T_F \) (°C) is found as a static linear function of the outdoor air temperature \( T_e \) (°C), in which constants \( A \) (W) and \( B \) (W/K) are fixed parameters. These parameters are based on the thermal conditions; U-values, exposed areas, internal gains, and the set indoor temperature of the building. They are determined by (10), which will soon be derived. The return temperature, i.e. the water temperature coming back from the radiators, is denoted \( T_R \) (°C). The feed-forward is a common control strategy in HVAC systems [2].

\[
Q(W) \text{ is the supplied heat to the system required to keep the forward temperature equal to } T_F. \text{ The consequence for the system, when the external temperature is deliberately changed by } \Delta T_e, \text{ results in a changed heating demand of } \Delta Q(W) \text{ and a changed indoor temperature of } \Delta T_i(°C).
\]

3. Radiator heating

The heat, \( Q_r \) (W), which is released by the radiators, is estimated by a linear model:

\[
Q_r = (T_r - T_i)A_r\alpha \approx \left( \frac{T_F + T_R - T_i}{2} \right)A_r\alpha
\]

(1)

Here, \( A_r \) (m²) is the surface area of the radiators, \( T_r \) (°C) the mean radiator temperature, \( \alpha \) (W/m²K) is the total heat transfer coefficient including both long wave radiation and convection. The heat capacity of the radiator is neglected,
while the heat capacity of the water in the radiator system is treated below. The heat supplied to the radiators by water is:

\[ Q_r = (T_F - T_R)mc_f \]  

(2)

Here, \( \dot{m} \) (kg/s) is the water mass flow rate and \( c_f \) (J/kgK) the heat capacity of the fluid. Combining (1) and (2) we get:

\[ Q_r = \tilde{K} \cdot (T_F - T_i) \]

\[ \tilde{K} = \frac{A_r \alpha}{1 + \frac{A_r \alpha}{2 \dot{m} c_f}} \]

(3)

The notation \( K \) will in general denote a thermal conductance with the unit W/K.

4 Thermal model of the building

A lumped model will be assumed in the model, where \( C \) (J/K) is the total heat capacity inside of the thermally insulated building envelope. A network model [3] of the building is shown in Figure 2 (a). Here, \( Q_{int} \) (W) represents internal gains.

![Fig. 2. (a) Thermal network model of building. (b) Simplified/reduce network.](image)

The total heat loss, \( Q_v \) (W), due to transmission through the envelope and ventilation becomes:

\[ Q_v = \sum (UA + nV \rho_a c_{pa}) (T_i - T_e) = K_v (T_i - T_e) \]

(4)

Here, \( UA \) (W/K) is the U-value and area of each envelope element, \( n \) (1/s) represent the air exchange rate, \( V \) (m³) the air volume inside the building and \( \rho_a c_{pa} \) (J/m³K) the volumetric heat capacity of air. The ambient equivalent temperature \( T_a \), used in the reduced network of Fig. 2 (b) becomes, [3]:

\[ T_a = \frac{Q_{int} + \tilde{K} \cdot T_F + K_v \cdot T_e}{\tilde{K}} \]

\[ \tilde{K} = K_v + \tilde{K} \]

(5)

Using (2) and (3) the return temperature from the radiators can be determined:

\[ T_R = T_F - \frac{\tilde{K} \cdot (T_F - T_i)}{mc_f} = T_F \left( 1 - \frac{\tilde{K}}{mc_f} \right) + T_i \frac{\tilde{K}}{mc_f} \]

(6)
5. Thermal storage of the water in the radiator system

Figure 3 illustrates the combined system of building and radiator system. The latter contains a lot of water, \( V_{H_2O} \) (m\(^3\)), and represents a thermal storage of its own. In the analysis the storage is treated as an inbuilt separate water tank with heat storage capability holding the uniform temperature \( T_i \) (°C).

![Figure 3](image)

The water is flowing from the storage to the radiators. Once it has left the storage tank it is heated up, with the effect \( Q \) (W) in order to reach the demanded forward temperature \( T_F \).

\[
Q = (T_F - T_i) \cdot \dot{m}c_f
\]  

(7)

We have the following heat balance for the storage:

\[
C_r \frac{dT_i}{dt} = (T_R - T_i) \cdot \dot{m}c_f \quad C_r = V_{H_2O} \cdot \rho_{H_2O}
\]  

(8)

6. Equation for the complete system

The following equation system represents the coupled system of heat balance for the storage and the building. Equation (5, 6) and (7) have been used.

\[
\begin{align*}
C_r \frac{dT_i}{dt} &= K \cdot (T_a - T_i) = Q_{int} + K \cdot (T_F - T_i) + K_n \cdot (T_e - T_i) \\
C_r \frac{dT_i}{dt} &= (T_R - T_i) \cdot \dot{m}c_f = \bar{K} \cdot (T_i - T_F) + \dot{m}c_f \cdot (T_F - T_i)
\end{align*}
\]  

(9)

7. Balance temperature without adjusting the external temperature

The derivative of the temperatures, on the left hand side of (9) is zero, once the system is in balance. From the first equation of (9) we can determine the parameters \( A \) and \( B \) required keeping a stable indoor temperature of \( T_i \) when all other conditions are stable.

\[
T_F = T_i + \frac{K \cdot T_i - Q_{int}}{K} - \frac{K_n \cdot T_e}{K} \quad \text{thus;} \quad A = T_i + \frac{K_n T_i - Q_{int}}{K} \quad B = \frac{K_n}{K}
\]  

(10)
8. Change in indoor temperature when adjusting the external temperature

Superposition technique can be used to determine the disturbance of the system when suddenly a change in the forward temperature is initiated. In particular we can find out the change in the indoor temperature, $\Delta T_i$, and the change in the supply of heat to the system, $\Delta Q$. By fictitiously changing the outdoor temperature sent to the control system, by the amount of $\Delta T_e$, the system will respond by changing the forward temperature by $-B\Delta T_e$.

The actual outdoor temperature for this superimposed process is zero, since there is no change in the real outdoor temperature. The deviation temperature, in the room and in the storage is zero at time zero. There is no deviation in the internal gains, thus equal to zero. With all these assumptions inserted in (9) we get:

$$\begin{align*} 
\frac{d\Delta T_i}{dt} &= -\Delta T_i \frac{K}{C} + \Delta T_f \frac{\bar{K}}{C} \\
\frac{d\Delta T_e}{dt} &= \Delta T_i \frac{\bar{K}}{C} - \Delta T_f \frac{\bar{m}_c f}{C} + \Delta T_f \frac{\bar{m}_c f - \bar{K}}{C} 
\end{align*}$$

(11)

The change in the heating demand, of the building becomes:

$$\Delta Q = (\Delta T_f - \Delta T_i) \cdot \bar{m}_c f$$

(12)

The final steady-state indoor temperature, after some time, becomes:

$$\Delta T_i^s = \frac{\bar{K}}{K} \Delta T_F = \frac{1}{1 + K_{ir} / \bar{K}} \Delta T_F, \quad \Delta T_i^s = \Delta T_f \left(1 - \frac{\bar{K} - \bar{K}^2 / \bar{K}}{\bar{m}_c f} \right)$$

(13)

9. Time dependent solution for the changed indoor temperature induced by a step change in the exterior one

There is a handy solutions to the changed indoor temperature due to step-change in the exterior temperature sent to the control system.

$$\Delta T_i(t) = \Delta T_i^s \left(1 - e^{-t/t_{ci}}\right) \quad t_{ci} = C / \bar{K} \quad \Delta T_f(t) = \Delta T_F \cdot H(t)$$

(14)

Here, $H(t)$ represents the Heaviside unit step function. For the step-change solution for the storage temperature we have:

$$\Delta T_i(t) = \Delta T_i^s \left(1 - e^{-t/t_{ci}}\right) + \Delta T_F \frac{\bar{K}^2}{C_r \bar{K}} \frac{e^{-t/t_{ci}} - e^{-t/t_{ci}}}{1/t_{ci} - 1/t_{ci}} \quad t_{ci} = \frac{C_r}{\bar{c} \bar{m}}$$

(15)

10. Comparison between analytic formula and numerical model

The above presented model of the building with the radiators and the water tank, as presented by equations (3, 4), (6) and (9, 10) is developed also as a numerical model in the Matlab/Simulink environment [4, 5] and solved by using explicit time scheme and variable time steps. The advantage of the numerical model is that it allows system analysis at varying outdoor and indoor conditions, and systems constrains as shown in [6]. The verification of the numerical model against the above presented analytical solution is shown in Figure 4.
10. Discussion

The above presented result shows that the thermal mass of the heated building can be potentially utilized as an alternative thermal storage to smooth out the peaks in the heat demand. A deliberate change of the control signal by 1 °C results in a slow drop of indoor temperature and a substantial drop of the storage temperature. The former is well within the allowed comfort limits (0.2 °C) after the first 24 h, while the latter at the same time drops for about 2°C. The less heat delivery to the storage and building can be compensated later, when there is less heating demand in the system, so that the total heat delivery to the building remains the same. It is worth noting that the results are highly dependent on the building thermal characteristics but also on the exterior temperature pattern.

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References