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# Calculation of noise barrier performance in a turbulent atmosphere by using substitute sources above the barrier

## Abstract

The paper presents a model that can be used for calculating the sound reduction by a noise barrier in a turbulent atmosphere. The field due to the acoustic source is substituted by a distribution of sources above the barrier (here called substitute sources). The mean power at the receiver is calculated as line-of-sight propagation through a turbulent atmosphere from all substitute sources using a mutual coherence function. In this study the strengths of the substitute sources are calculated using the Kirchhoff approximation. The calculated results show good over-all agreement with those from using a parabolic equation method (PE).

## 1. Introduction

Screens and buildings along the roadside are used as noise barriers for reducing the traffic noise in residential areas. For a good prediction of the performance of noise barriers, the non-homogeneous nature of the outdoor air is needed to be taken into account [1, 2]. Also for the similar problem where upward refraction causes the acoustic shadowing, it has been shown that almost all of the acoustic power deep inside the shadow-zone is due to the atmospheric turbulence (e.g. [3, 4]).

In terms of physical modelling, the problem situation with a noise barrier in an outdoor environment can be seen as consisting of two interacting processes: diffraction (due to the barrier) and sound propagation in an inhomogeneous medium. A direct numerical solution of the whole problem would in general be very expensive computationally (using e.g. a finite element method), and therefore a model is preferable where the two processes can be separated to some extent, without too large approximations. Two previously used models are a parabolic equation (PE) approach [2] and one based on the scattering cross-section for an inhomogeneous atmosphere [1, 5, 2]. In this paper a prediction model is presented that is based on the Rayleigh integral. The model is not limited to low angles, as the PE is. Moreover, it does not demand a step-

wise solution over distance, as the PE does, but could directly produce a result for a given geometry and frequency, and also it is numerically faster than the PE. Compared to the scattering cross-section calculations it is much slower, but is not limited by the single-scattering approximation.

The approach is that the field at a receiver, due to a source, can be described as a superposition of fields from a distribution of sources on a surface located between the source and the receiver. The surface will here be called the substitute surface, and the sources on it substitute sources. (See Figure 1.)

If the substitute surface is located between the barrier and the receiver, there will be a free path from all substitute sources to the receiver, and the calculation of the sound propagation along the free path is possible for various types of inhomogeneous atmosphere. Here, a mutual coherence function for a turbulent atmosphere is applied.

In this model the turbulent atmosphere is assumed to cause an increased noise level behind the barrier due to a decorrelation of the contributions from the substitute sources. This implies that, in the absence of turbulence, the contributions from the substitute sources must be interfering negatively.

The strengths of the substitute sources can, as a first approximation, be calculated as for a barrier in a homogeneous atmosphere. This approximation would be acceptable for weak inhomogeneity (a weak turbulence) and if the distance from the source to the barrier is short compared to the total source-receiver distance.

The numerical results are compared to those from a PE method. In the presented model, the ground surface used for the PE calculations is taken into account according to known results for sound propagation near ground in a turbulent atmosphere [6, 7, 8, 9]. Moreover, due to the comparison, the computations are made for a few single frequencies but for many source-receiver distances.

## 2. Theory

The theoretical tools needed for the model can be seen to consist of the following. First, the strengths of the substitute sources need to be determined, i.e. the normal velocity of the sound field at the substitute surface is needed as the source distribution for the Rayleigh integral. In this study the normal velocity is approximated by the free field due to the source, i.e. the Kirchhoff approximation, and the normal velocity due to the introduction of the barrier is neglected. Second, the expected power at the receiver of the sum of the waves propagated through the turbulent atmosphere from all the substitute sources

need to be estimated. This is done by calculating the mutual coherence between all substitute sources, using the so-called mutual coherence function (MCF), or transverse coherence function, for a random medium.

The theoretical description of the problem is held for three-dimensional situations. The numerical results presented here are, however, for two-dimensional situations, and the necessary modifications of the theory are shown. The three-dimensional calculations are computationally much more expensive, and therefore this first study of the model is held in two dimensions. Moreover, the PE method used for the comparison assumes that the medium and the boundary conditions vary only in two dimensions, fulfilling axial symmetry.

### 2.1. Use of the Rayleigh integral

If the substitute surface (denoted  $S$ ) is a plane and the particle velocity  $v_n$  normal to the plain is known, then the monopole source strengths of the substitute sources are known, and the response  $p$  at the receiver position  $\mathbf{x}_R$  can be calculated as a Rayleigh integral:

$$p(\mathbf{x}) = \frac{j\omega\rho_0}{2\pi} \int_S v_n(\mathbf{x}_S) G(\mathbf{x}_S|\mathbf{x}_R) dS. \quad (1)$$

In equation (1)  $\mathbf{x}_S$  is a point on the surface  $S$ ,  $\omega$  the angular frequency of a time-oscillation  $e^{j\omega t}$  with time  $t$ ,  $\rho_0$  the medium density, and  $G$  a Green's function. For a homogeneous free space (in three dimensions) the Green's function can be written

$$G(\mathbf{x}_S|\mathbf{x}_R) = \frac{e^{-jkR}}{R}, \quad (2)$$

where  $R$  is the distance between  $\mathbf{x}_S$  and  $\mathbf{x}_R$  and  $k$  is the wave number  $k = \omega/c$ , where  $c$  is the sound speed. Instead of the free space Green's function (equation 2), some other Green's function can be used if it suits the situation better. For instance, a sound speed gradient that causes a curving of the sound paths can be described by an appropriate Green's function, obtained either analytically or numerically.

If it is assumed that the barrier has a hard plane surface toward the receiver, then the surface of integration  $S$  can be placed so that it coincides with the barrier's surface toward the receiver, as shown in Figure 1. This can lead to a simplified problem since the particle velocity is zero on the hard barrier surface.

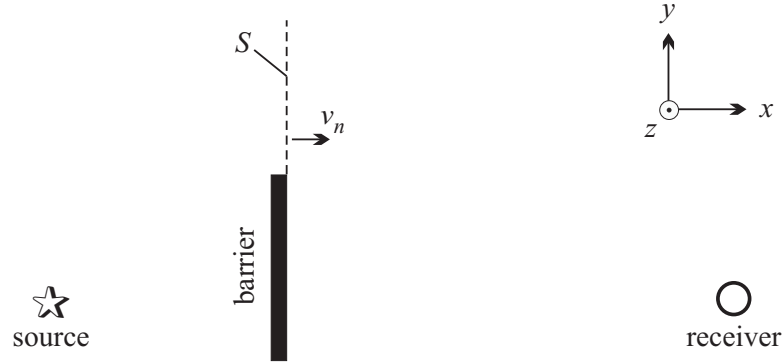


Figure 1. Geometrical situation with source, barrier, receiver, and the substitute surface  $S$ .

The normal velocity  $v_n$  on the surface  $S$  can be seen as consisting of two parts: the free field contribution  $v_{n0}$ , set to zero on the barrier, and the contribution due to the diffraction from the barrier  $v_{nd}$

$$v_n = v_{n0} + v_{nd}. \quad (3)$$

The free field velocity contribution  $v_{n0}$  can be calculated from the free field pressure  $p_0$ , as

$$v_{n0} = \frac{-1}{j\omega\rho_0} \nabla p_0 \cdot \mathbf{n}, \quad (4)$$

where  $\mathbf{n}$  is the unit vector normal to the surface  $S$ . The free field pressure  $p_0$  can be written

$$p_0(\mathbf{x}_S) = \frac{e^{-jkR_0}}{R_0}, \quad (5)$$

where  $R_0$  is the distance from the source to the point  $\mathbf{x}_S$  on the surface  $S$ .

It could be noted that the Rayleigh integral can be formulated in the time domain, which could be an interesting alternative approach (e.g. [10, pp. 409-410]).

In general, the free field velocity contribution  $v_{n0}$  can be obtained straightforwardly, while the diffraction contribution  $v_{nd}$  is more complicated to obtain. The model with the substitute surface, without the turbulence, allows for a study of the error due to omitting  $v_{nd}$  (see following Subsection). This approximation  $v_n = v_{n0}$ , the Kirchhoff approximation, is not necessary for the model, but is used for simplification reasons for the calculations in this paper.

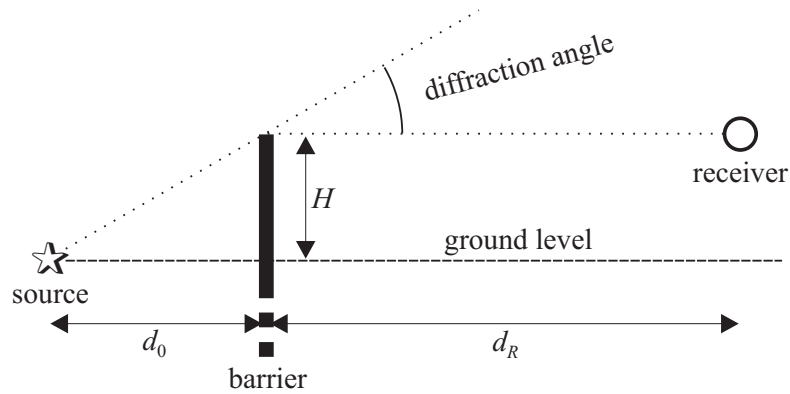


Figure 2. Geometry for the parameter study.

## 2.2. The Kirchhoff approximation

To estimate the limitations in geometry and in frequency of the Kirchhoff approximation, a brief parameter study is performed, where the results are compared to those from using the uniform theory of diffraction (UTD) [11, 12]. (For the situations studied here, UTD provides very accurate results, which in this comparison can be seen as the true results.)

In the parameter study a thin hard barrier with height  $H = 0.7, 1.4, 2.8, 4.2,$  or  $5.7$  m is placed at a horizontal distance of  $d_0 = 20$  m from the source and  $d_R = 20$  m from the receiver. The source is placed at ground level and the receiver is placed at the height of the screen edge. The different diffraction angles are then  $2^\circ, 4^\circ, 8^\circ, 12^\circ,$  and  $16^\circ$ . See Figure (2). No reflections from a ground surface are considered and the results are presented as the sound pressure level relative to free field. The calculations with the Kirchhoff approximation are made for a coherent line source, i.e. a two-dimensional situation (see Section 3). (The corresponding three-dimensional results deviate at most about 0.3 dB, which is attained only for the largest diffraction angle  $16^\circ$  at the lowest frequencies.)

The Kirchhoff approximation is valid when the distances from source and receiver to the screen are large compared to the height of the screen, i.e. for small diffraction angles. It should be noted that, strictly, this only holds for a semi-infinite screen. In real cases the field diffracted at the screen edge might be reflected in a ground surface and diffracted again at the edge, and thereby influence the field at the receiver. These higher-order diffraction terms increase in strength when the screen height is reduced. Therefore, the error when using the Kirchhoff approximation or the UTD for a screen on ground can be substantial for very low screens in comparison to the acoustic wavelength.

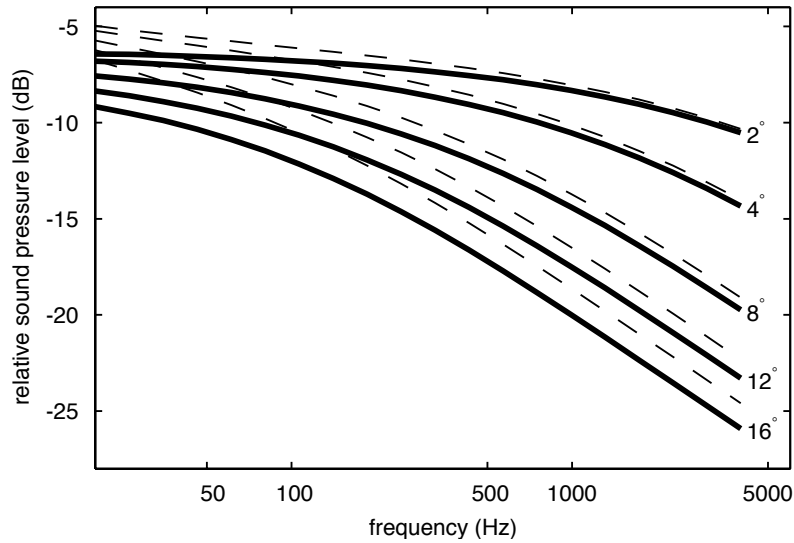


Figure 3. Results of applying the Kirchhoff approximation (solid) compared to UTD (dashed) for different diffraction angles.

The error due to the Kirchhoff approximation depends mainly on frequency and diffraction angle. For the lower frequencies the error increases with decreasing frequency; whereas, for higher frequencies, the error is very weakly dependent on frequency, as can be seen in the calculated results, Figure (3). To have an error smaller than 1 dB, at the higher frequencies, the diffraction angle must be smaller than about  $12^\circ$ , as can be seen in Figure 3. Higher frequencies mean that the source and the receiver are far away from the screen edge in comparison to the wavelength  $\lambda$  (i.e.  $R_0, R \gg \lambda$ ).

In this situation we have a distance to the screen edge from the source or the receiver that is about 20 m. The frequency can be normalised with respect to this distance, since the important parameter (except from the diffraction angle) is the number of wavelength from the source (or the receiver) to the screen edge. For instance, at the frequency 500 Hz we have a wavelength of about 0.7 m, and the number of wavelengths to the receiver is  $f_{\text{norm}} = 20/0.7 \approx 30$ . In conclusion one can say that if the normalised frequency  $f_{\text{norm}}$  is larger than about 30 (500 Hz for the geometries used here) and the diffraction angle is smaller than about  $12^\circ$ , the error is smaller than 1 dB, for situations similar to the ones studied here.

Furthermore, when the Kirchhoff approximation is valid, a change of the acoustic properties of the barrier surfaces will be without effect, since the free field contribution will be unchanged. Also, changing a thin screen into a wedge will have no effect.

The reasoning above explains the applicability of one-way PE methods to situations with low barriers. In these implementations the PE method calculates wave propagation in one direction (outward from the source) and a barrier is modelled by setting the pressure field equal to zero at the location of the barrier [13]. The free field above the barrier is calculated correctly, and as long as the Kirchhoff approximation is valid, the free field will produce the correct result at the receiver. Consequently, when the Kirchhoff approximation is not valid one would assume that the one-way PE method is not valid either, when including a barrier.

### 2.3. Artificial damping of the substitute sources

When implementing the model, a finite substitute surface  $S$  is needed. If then the size of the surface  $S$  is varied (or if the source or the receiver is moved), the error due to the finite surface shows an oscillatory pattern, corresponding to the Fresnel-zones. The introduction of an artificial damping of the substitute sources leads to weaker oscillations and thereby a smaller surface is needed (see Figure 4). Here, the calculations are made for a three-dimensional situation without a barrier, where the substitute surface is placed midrange between the point source and the receiver. The substitute surface is circular and its radius  $r_S$  is varied from 0 to 16 m. The frequency is 500 Hz. The thin line shows the result without artificial damping. For the thick line the damping starts at  $r_S = d_0 = 4$  m and the damping factor is  $\exp[-2(r_S/d_0 - 1)]$ .

One can see in Figure 4 that it suffices to extend the radius  $r_S$  of the substitute surface out to about the total source-receiver distance; for  $r_S \geq 8$  m the error is smaller than 10%. When the substitute surface is moved closer to either the source or the receiver its radius can be decreased, with maintained accuracy. In the discretisation a distance of  $\lambda/5$  between the substitute sources is usually enough, but for this plot  $\lambda/20$  is used to show a smoother result. (In this example a discretisation is used only in radial direction; in circumferential direction a continuous source is used.)

### 2.4. Influence of a turbulent atmosphere

There will be line-of-sight propagation from the substitute sources on the surface  $S$  to the receiver, that is, no barriers or other obstacles are shielding the sound propagation. The subject of line-of-sight propagation in a turbulent atmosphere has been studied extensively (e.g. [14, 15, 16, 17, 18]), and the theoretical results most useful here deal with the correlation between acoustic pressure signals that are received at different positions but are originating from a single



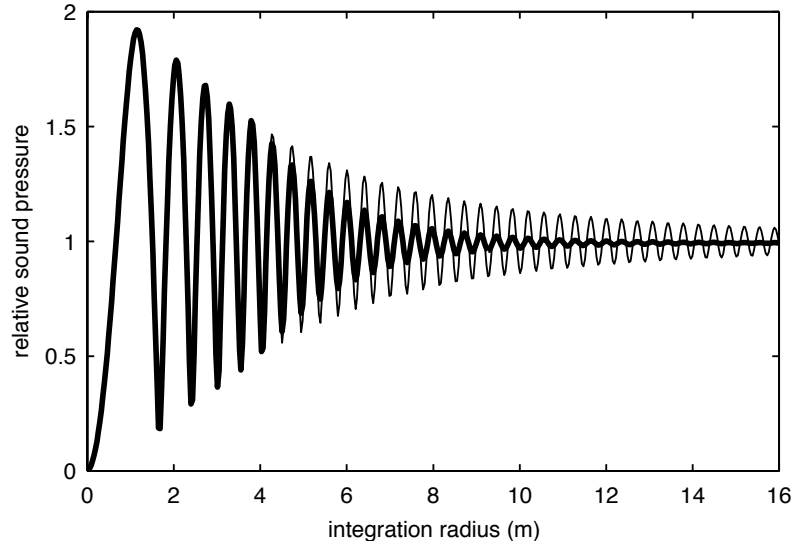


Figure 4. Solution with and without artificial damping of the substitute sources, plotted as relative sound pressure versus integration radius,  $r_S$ .

point monopole source. These theoretical results can be applied to the reciprocal problem at hand: the correlation between signals coming from different sources to one receiver position. The correlation between signals from different sources is used to calculate the mean square pressure amplitude for single frequencies. The correlation between two source signals is usually described by the mutual coherence function  $\Gamma$ .

Let us start with a homogeneous free space and the pressure  $p$  due to two point sources as

$$p = p_1 e^{j\omega t} + p_2 e^{j\omega t}, \quad (6)$$

where  $p_1$  and  $p_2$  are the complex pressure amplitudes due to source 1 and source 2. Then we introduce small fluctuations in the refractive index or the velocity of the medium that are zero mean in space and time, and vary slowly with time compared to the time period of the sound. For this situation the long-term average of the square of the pressure amplitude can be computed according to e.g. [8] or [19], as

$$\langle |p|^2 \rangle = |p_1|^2 + |p_2|^2 + 2|p_1 p_2| \cos \left[ \arg \left( \frac{p_2}{p_1} \right) \right] \Gamma_{12}, \quad (7)$$

where  $0 \leq \Gamma_{12} \leq 1$ . For  $N$  sources it could be formulated as [19]

$$\langle |p|^2 \rangle = \sum_{i=1}^N |p_i|^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N |p_i p_j| \cos \left[ \arg \left( \frac{p_j}{p_i} \right) \right] \Gamma_{ij}. \quad (8)$$

From equation (8) it is possible to see how an expression for the case of a continuous source distribution can be written:

$$\langle |p|^2 \rangle = \left\langle \int \int p(x) p^*(x') dx dx' \right\rangle = \int \int |p(x) p(x')| \cos \left[ \arg \left( \frac{p(x')}{p(x)} \right) \right] \Gamma(x, x') dx dx', \quad (9)$$

where  $x$  and  $x'$  are positions on the substitute surface, and where  $*$  stands for the complex conjugate. This integral expression (9) is basically the same as in [20], where the effect of turbulence on the sound from a line source is studied. (In [20] the formulation is for the mean acoustic power which gives a factor  $1/2$  extra.) If there would be a homogeneous atmosphere,  $\Gamma \equiv 1$ , equation (9) could be seen as the same as the square of the Rayleigh integral in equation (1).

Now we can calculate the influence of a turbulent atmosphere for our case where the effect of a barrier is modelled by a distribution of substitute sources on a surface  $S$ . If the strength of the substitute sources is described by  $v_n$  as in equation (1), we get

$$\langle |p|^2 \rangle = \left( \frac{\omega \rho_0}{2\pi} \right)^2 \times \int_S \int_S |v_n G v'_n G'| \cos \left[ \arg \left( \frac{v'_n G'}{v_n G} \right) \right] \Gamma dS dS', \quad (10)$$

where  $p = p(\mathbf{x}_R)$ ,  $v_n = v_n(\mathbf{x}_S)$ ,  $v'_n = v_n(\mathbf{x}'_S)$ ,  $G = G(\mathbf{x}_S | \mathbf{x}_R)$ ,  $G' = G(\mathbf{x}'_S | \mathbf{x}_R)$ ,  $\Gamma = \Gamma(\mathbf{x}_S, \mathbf{x}'_S)$ , and  $dS'$  refers to  $\mathbf{x}'_S$  and  $dS$  refers to  $\mathbf{x}_S$ .

For the free space Green's function according to equation (2) we can rewrite equation (10) as

$$\langle |p|^2 \rangle = \left( \frac{\omega \rho_0}{2\pi} \right)^2 \times \int_S \int_S \left| \frac{v_n v'_n}{R R'} \right| \cos \left[ k[R' - R] + \arg \left( \frac{v'_n}{v_n} \right) \right] \Gamma dS dS', \quad (11)$$

where  $R = |\mathbf{x}_S - \mathbf{x}_R|$  and  $R' = |\mathbf{x}'_S - \mathbf{x}_R|$ .

For the description of the turbulence, a homogeneous and isotropic turbulence is assumed, that is, the fluctuations are assumed to follow the same statis-

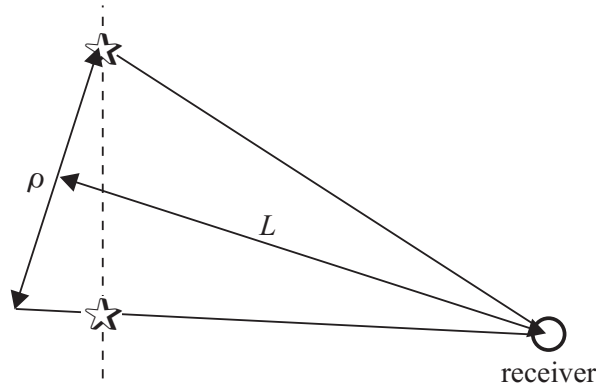


Figure 5. Longitudinal ( $L$ ) and transversal ( $\rho$ ) distance for two sources and one receiver.

tics in all points and in all directions. The turbulence is described by a fluctuating part  $\mu$  of the index of refraction  $n = \langle n \rangle + \mu$ , where  $\mu$  here is taken to follow a Gaussian correlation function

$$\langle \mu(\mathbf{x} + \mathbf{s})\mu(\mathbf{x}) \rangle = \mu_0^2 \exp(-s^2/l^2), \quad (12)$$

where  $\mathbf{x}$  and  $\mathbf{s}$  are two vectors in space,  $s = |\mathbf{s}|$ ,  $\mu_0$  the standard deviation of  $\mu$ , and  $l$  the correlation length. Other turbulence models than the Gaussian could be used, and this would then lead to different mutual coherence functions.

Temperature and velocity fluctuations affect the sound field in different ways. The mutual coherence function for velocity fluctuations is deduced in [21, 18], where also the older result for Gaussian temperature fluctuations is shown. Here, only temperature fluctuations are considered, but to include other medium fluctuations should not be a problem.

If we assume that the turbulence can be described by the Gaussian correlation function (12) we can write the mutual coherence function as [22, Eq. (11)]

$$\Gamma(L, \rho) = \exp \left[ -\sqrt{\pi} \mu_0^2 k^2 l L \left( 1 - \frac{\Phi(\rho/l)}{\rho/l} \right) \right], \quad (13)$$

where  $\rho$  is the transversal distance between the sources,  $L$  is the longitudinal distance to the receiver, and  $\Phi(\rho/l) = \int_0^{\rho/l} \exp(-u^2) du$ .

Daigle *et al.* [22] concluded that for long ranges the amplitude fluctuations saturate and then equation (13) gives a too small coherence. The amplitude fluctuations are estimated to saturate roughly when  $(\sqrt{\pi}/2)\mu_0^2 k^2 l L \geq 1$  [22], which, in the situations studied here, only would be the case for the highest frequencies and the longest distances. Equation (13) is therefore used throughout this study.

When deducing equation (13) the two source positions are assumed to be at equal distance from the receiver, and  $L$  the distance from the receiver to the midpoint between the two sources. Here, the two sources are not necessarily at equal distance from the receiver and therefore some modification must be made. In this study the longest of the two distances determines the value of  $L$ , as shown in Figure 5. The transversal distance  $\rho$  forms the base of a triangle where the two other sides are of equal length and whose common angle have bisector  $L$ . (For the two-dimensional situations studied here, however, the results are not so sensitive to the exact choice of  $L$  and  $\rho$ . For instance, a numerical test showed negligible difference between  $L$  being chosen as the longest or the shortest of the two source-receiver distances. This is probably due to the fact that the most important contributions come from sources that are low in comparison to their distance to the receiver. In that case, the relative changes in  $L$  and  $\rho$  are small when the choice of  $L$  is shifted.)

The mutual coherence function can be deduced via the parabolic equation and the Markov approximation [17]. Other methods than by using the parabolic equation can be applied [23], but, in any case, it is assumed that the transversal distance  $\rho$  is small compared to the longitudinal distance  $L$ . This approximation leads to an overestimation of the mutual coherence function. Here, the transversal distances are about equal to the longitudinal distances only for the shorter ranges studied; for the longer ranges the transversal distances are much smaller than the longitudinal distances, and therefore it assumed here that the results are correct at the longer ranges. Concerning the range of acoustic wavelengths where the theory is valid, it can be concluded that no restriction due to the turbulent scales is necessary [8].

If a ground surface is introduced, there will be direct and ground reflected waves whose mutual coherence need to be estimated. The situation with a single source above a hard ground has been studied experimentally [6] and theoretically [7, 8, 9]. A good estimate of the mutual coherence function is by taking the maximum separation distance  $h$  between the two paths as  $\rho$  in equation (13) [9]. For the coherence between the direct wave from one source and the ground reflected wave from another source, the transversal separation between the two sources is added to  $h$  to produce the  $\rho$  used in equation (13).

### 3. Implementation

In the numerical study the geometry is two-dimensional, and the surface integrals in equation (9) can be reduced to line integrals in the vertical direction  $y$ . Also the turbulence is assumed to be two-dimensional. For the mutual coher-

ence function (equation 13), the values of the input parameters are found from the projections on the vertical  $xy$ -plane. This is in accordance with Salomons' analysis [9] for a turbulence that is rotationally symmetrical around the vertical axis through the source.

The acoustic pressure  $p_0$  due to a coherent line source is calculated as the far-field approximation

$$p_0 = \frac{Q\sqrt{2}}{\sqrt{\pi k R}} e^{-j(kR - \pi/4)}, \quad (14)$$

where  $Q$  is a source strength.

The Kirchhoff approximation is used, so that the normal velocity is calculated according to equation (4), with  $p_0$  according to equation (14).

The results are compared with those from using a PE method [2]. In all calculations the turbulence was modelled by a Gaussian correlation function (12) with  $\mu_0^2 = 3 \cdot 10^{-6}$  and  $l = 1.1$  m. The source was located on the surface of a hard ground and the receiver at the height of the screen edge  $H$ . For the calculations at 500 Hz, two different source-screen distances were used,  $d_0 = 100$  and 200 m, and the screen height  $H$  was either 10 or 20 m. The calculations at  $f = 1000$  Hz were made only for  $d_0 = 100$  m and  $H = 10$  m. The mean acoustic power was calculated for receiver distances up to 1000 m from the source. In the PE calculations, 50 realisations of the turbulent atmosphere were used for the estimation of the mean power. (The finite number of averages leads to an error in the solution, which would decrease for a larger number of averages and then give smoother results.)

The maximum height  $y_{\max}$  used for the substitute sources was 250 m for  $d_0 = 100$  m and 450 m for  $d_0 = 200$ . These values of  $d_0$  were found by numerical tests in the absence of turbulence.

Artificial damping was introduced in the strength of the substitute sources to decrease the small oscillations in the solution when the receiver distance is varied. (See Subsection 2.3 and Figure 4). The damping was chosen to start at  $y = y_{\max}/2$ , and the strengths of the substitute sources above that were multiplied by the factor  $\exp(-a * (y - y_{\max}/2))$ , with  $a = 0.05 \text{ m}^{-1}$ . For the discretisation, a horizontal distance between the substitute sources of  $\lambda/5$  was used.

#### 4. Results – comparison with PE

The calculated results are presented in Figures 6-9 as the sound pressure level relative to the level without the screen, i.e. the negative of insertion loss. The calculations using the substitute-sources model (SSM in the legend) are made

each 25 meters in range, and the results are plotted as solid lines with circular marks. The PE calculations are plotted as solid lines. The results for a turbulent atmosphere have been plotted together with the results for a homogeneous (i.e. non-turbulent) atmosphere, and the two highest curves show the results for the turbulent atmosphere using the two different models. The dashed line shows the correct diffraction without turbulence, using UTD.

In Figures 6-9 the model using substitute sources gives results that show a good over-all agreement with the PE calculations.

At 1000 Hz, however, (Figure 9) the PE calculations show a slightly larger influence of turbulence than the substitute sources. One cause for that could be the following. In the situations without a screen, the introduction of turbulence was shown to cause the PE method to predict a sound level on ground that increased with distance relative to free field [2]. At 1000 m range the increase (above the expected 6 dB due to the hard ground) was 2 dB at 500 Hz and 6 dB at 1000 Hz. This focusing is accompanied by a decreased sound level at the height of the screen edge. At 1000 Hz this decrease was about 0.5-1 dB at the height 10 m, but at 500 Hz it was not noticeable. Since these calculations without screen were used as reference for calculating the insertion loss, it became slightly lower. This could account for a significant part of the discrepancy shown here in Figure 9. (In [2] the deviation in the PE results due to the low reference was too small to appear as interesting to further investigate in comparison to the overall differences between the PE calculations and the scattering cross-section calculations made there.)

## 5. Discussion and conclusions

The model using substitute sources, presented here, gives results that show a good over-all agreement with the PE calculations made. The substitute-sources model can therefore be concluded to have similar qualities as the PE, which has been validated by measurements for the similar situation with upward refraction and turbulence (e.g. [4, 24]). The substitute-sources model is computationally faster than the PE, and if the kirchhoff approximation is not applied, the model would not be limited to low angles, as the PE is. Compared to calculations based on the scattering cross-section, the presented model is much slower, but is not limited by the single-scattering approximation.

At 1000 Hz the PE calculations show a slightly larger influence of turbulence than by using the substitute sources. One cause for that could be that the PE method predicts a sound pressure level slightly less than the expected 6 dB

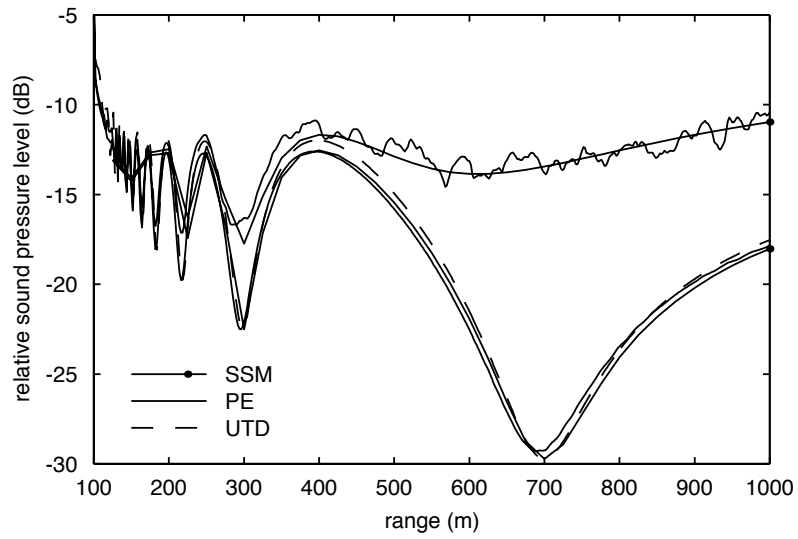


Figure 6. Relative sound pressure level at 500 Hz, for a 10 m high screen at range 100 m.

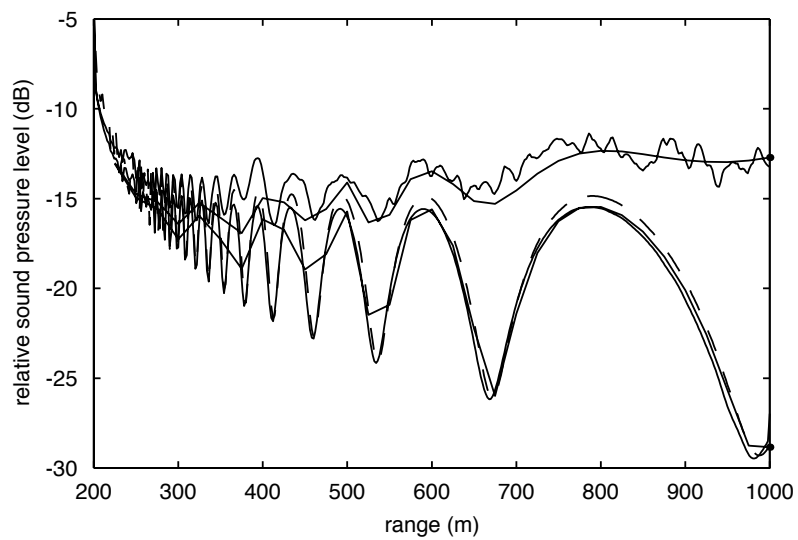


Figure 7. Relative sound pressure level at 500 Hz, for a 20 m high screen at range 200 m.

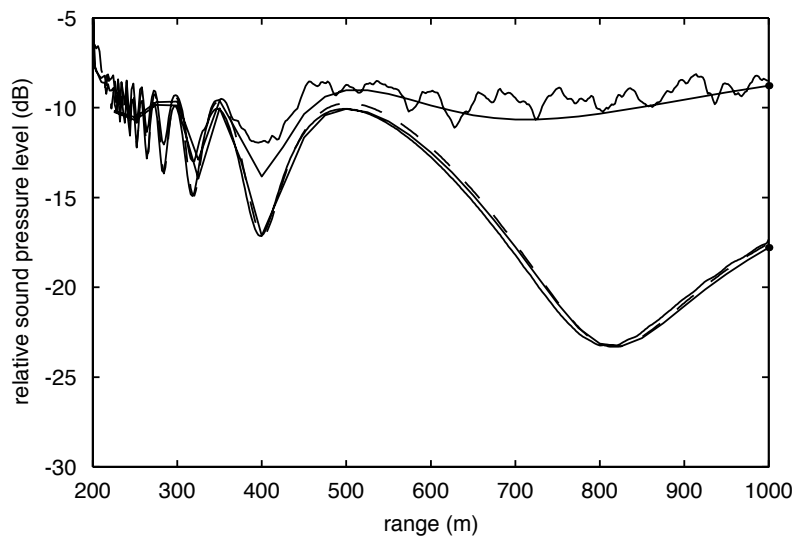


Figure 8. Relative sound pressure level at 500 Hz, for a 10 m high screen at range 200 m.

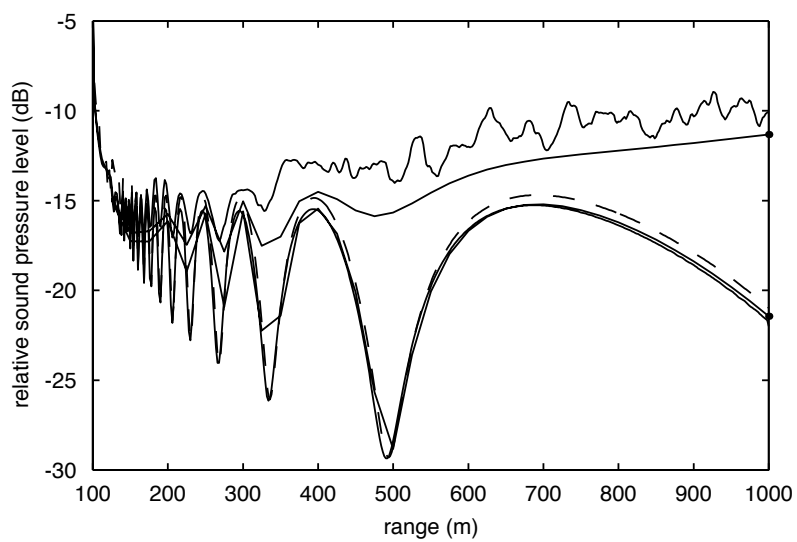


Figure 9. Relative sound pressure level at 1000 Hz, for a 10 m high screen at range 100 m.



relative to free field, for the situation with a turbulent atmosphere, a source on a hard ground, a receiver at 10 m height, and without the screen.

In this paper the turbulence is assumed to be introduced into a homogeneous free space, and it is for this situation the used mutual coherence function has been deduced. It is, however, a reasonable approximation that the mutual coherence function also can be used for a weakly modified medium, for instance as in [19] where a linear sound speed profile, with the corresponding Green's function, is considered. For a more strongly modified medium, the mutual coherence function, as well as the Green's function, could be estimated numerically. In this respect the substitute-sources model is applicable to a large variety of geometrical and atmospherical situations.

For future work it would be of interest to extend the model to three dimensions, with a point source and a three-dimensional turbulence. Also to take into account the correct diffraction above the barrier would be of interest to try, i.e. to not use the Kirchhoff approximation.

Moreover, it could be possible to include in the model a thick barrier of finite length, a finite impedance ground, a sound speed profile, and an anisotropic and inhomogeneous turbulence.

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