MIMO Linear Precoder Design with Non-Ideal Transmitters

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Ayça Özçelikkale, Tomas McKelvey, Mats Viberg

***Abstract*—We investigate the linear precoder design problem for multiple-input multiple-output (MIMO) channels under non-ideal transmitter hardware. We consider two different non-ideal hardware models: i) an additive noise model in which the level of the noise at an antenna is proportional to the signal power at that antenna, ii) an additive precoder error model. We focus on the problem of minimizing mean-square error at the receiver under transmit power constraints at the transmitter. For the first hardware impairment model, this scenario leads to a non-convex formulation for which we propose a block-coordinate descent technique. The proposed method has a convergence guarantee and provides rank-constrained solutions. For the second model, analytical expressions for the optimum designs are provided. We compare the performance of our hardware impairment aware designs with that of designs developed with ideal hardware assumptions. Our results suggest that significant gains can be obtained by the proposed designs for sufficiently high signal-to-noise ratio values.**

***Index Terms*— non-ideal hardware, hardware impairment, robust precoder design.**

I. Introduction

Multiple-input multiple-output systems offer significant increases in the capacity of wireless channels in rich scattering environments [1], [2]. An important practical issue in MIMO communications is the design of precoders and receiver filters, which have been successfully used to improve the performance of MIMO systems [3–5].

In practice, MIMO systems are affected by various hardware impairments including phase-noise, IQ-imbalance, amplifier non-linearities [6–8]. The impact of some of these distortions can be partially compensated using compensation algorithms at the receiver or calibration methods at the transmitter, but nevertheless residual transmitter impairments still remains effective [6], [7]. Although these residual transmitter impairments are known to affect the performance of communication systems [6–11], this point has been mostly overlooked in the case of optimization of linear precoder design. Previous work on communication system design under residual transmitter impairments have mostly focused on channel capacity as the performance metric [8–11]. On the other hand, although robust solutions for linear precoder design have been studied for various scenarios, these works typically focus on the uncertainty due to partially known channel state [12–14].

Here we focus on the robust linear precoder design for a transmitter with non-ideal hardware. To capture the effect of the residual hardware impairments, we consider two different models. The first one is an additive noise model where the noise has a special covariance structure, which is validated with the experiments [6], [7] and supported by analytical arguments [8]. The second one is an alternative additive precoder error model. We focus on the problem of minimizing mean-square error at the receiver under transmit power constraints at the transmitter. We consider statistically robust designs that provide performance guarantees on average. Our results illustrate that when the channel signal-to-noise ratio (SNR) is high, significant gains can be obtained by the proposed hardware impairment aware designs compared to non-robust designs for both models. These results also suggest that hardware impairments at the levels considered in these experiments, which are chosen to be consistent with the standards, will only be crucial when operating at considerably high SNR values.

The rest of the paper is organized as follows. In Section II, the system model and the additive noise model is described. The precoder optimization problem under this model is investigated in Section III. In Section IV, we discuss the alternative precoder error model. In Section V, performance of our designs are illustrated. Finally, the paper is concluded in Section VI.

Notation: The complex conjugate transpose and transpose of a matrix A are denoted by A^{H} and A^{T}, respectively. The ith row jth column element of a matrix A is denoted by $[A]_{ij}$. For $A \in \mathbb{C}^{n\times n}$, $\mathrm{diag}(A)$ denotes the diagonal matrix formed with $[A]_{11}, \ldots, [A]_{nn}$ on the main diagonal. Frobenius norm is denoted by $||A||= (\mathrm{tr}[AA^{\mathrm{H}}])^{1/2}$. I denotes the identity matrix with the suitable dimensions. Positive semi-definite ordering is denoted by $\succeq$. An optimal value of an optimization variable A is denoted by A^*. For a scalar a, $(a)^+$ is defined as $(a)^+ = \max(a, 0)$. $\mathbb{E}[.]$, and $\mathrm{tr}[.]$ denote the expectation and trace operators, respectively.

II. System Model

A. Channel Model

In the narrow-band and stationary scenario we focus on, the multi-antenna transmitter transfers the message to the receiver according to

$$y = Hx + w \tag{1}$$

where $H \in \mathbb{C}^{n_r\times n_t}$ represents the channel gain from the transmitter to the receiver. Zero-mean complex proper Gaussian random vector $w \in \mathbb{C}^{n_r\times 1} \sim \mathcal{CN}(0, K_w)$, $K_w = \mathbb{E}[ww^{\mathrm{H}}]$ denotes the noise.

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B. Precoding at the Transmitter with Non-Ideal Hardware

With an ideal transmitter, the channel input x can be expressed as

$$x = A_o s. \tag{2}$$

Here the zero mean complex proper Gaussian random vector $s \in \mathbb{C}^{n_s}$, $s \sim \mathcal{CN}(0, K_s)$, $K_s = I$ denotes the data and $A_o \in \mathbb{C}^{n_t \times n_s}$ denotes the linear precoder.

Here we are interested in the effect of non-ideal hardware at the transmitter. Using the residual hardware impairment model from [8–11], the channel input is given as

$$x = A_o s + v. \tag{3}$$

Here $v \in \mathbb{C}^{n_t}$, $v \sim \mathcal{CN}(0, K_v)$ denotes the residual hardware impairments that remain effective after utilizing compensation algorithms and/or calibration. The Gaussian assumption on the noise is supported by experiments (see for instance [6, Fig.7]) as well as by the central limit theorem and the fact that this term models the overall effect of various different hardware impairments [6–8]. The covariance of v is given as [6–8]

$$K_v = \alpha_v \operatorname{diag}(A_o A_o^{\mathrm{H}}). \tag{4}$$

Here the level of noise at an antenna is proportional to the signal power at that antenna. This property is verified by experiments [6], [7] and the resulting model has been utilized to study performance of various multiple antenna systems with hardware impairments [8–11].

The constant $\alpha_v \geq 0$ determines the quality of the hardware. As α_v increases, the quality of the hardware decreases. Here the distortion noise v is assumed to be statistically independent of the signal s due to the usage of impairment compensation algorithms [6], [8]. We note that in contrast to w, v emerges as colored and channel dependent noise at the receiver. Moreover, its statistics depend on the precoder A_o which will be optimized.

A commonly used practical quality measure for non-ideal hardware is the error vector magnitude (EVM) [15]. The scaling factor α_v relates to EVM as follows

$$\mathrm{EVM} = \sqrt{\frac{\mathbb{E}[||v||^2]}{\mathbb{E}[||A_o s||^2]}} = \sqrt{\alpha_v} \tag{5}$$

For comparison with the model in (3), we also consider an alternative hardware impairment model with additive precoder error. This alternative model is discussed in Section IV.

C. Signal Recovery

Upon receiving y, the receiver forms an estimate of s. The associated mean-square error can be expressed as

$$\varepsilon(A_o, B) = \mathbb{E}[||s - By||^2], \tag{6}$$

where B represents the linear estimator adopted by the receiver. We note that receiver filters based on mean-square error have been used to improve performance of various MIMO systems, for instance by providing a reasonably accurate alternative for preprocessing of coded data symbols [3], [4]. An optimum B can be found as [16, Ch2]

$$\begin{aligned} B^* &= K_{sy} K_y^{-1} \qquad\qquad (7a)\\ &= A_o^{\mathrm{H}} H^{\mathrm{H}} \left(H A_o A_o^{\mathrm{H}} H^{\mathrm{H}} + \alpha_v H \operatorname{diag}(A_o A_o^{\mathrm{H}}) H^{\mathrm{H}} + K_w \right)^{-1} \end{aligned}$$

We note that due to the Gaussian distribution and the statistical independence assumptions on the relevant signals, $B\,y$ gives the minimum mean-square error (MMSE) estimation of s. The resulting MMSE can be expressed as

$$\begin{aligned} \varepsilon(A_o) &= \operatorname{tr}[K_s - K_{sy} K_y^{-1} K_{sy}^{\mathrm{H}}] && (8a)\\ &= n_s - \operatorname{tr}[A_o^{\mathrm{H}} H^{\mathrm{H}} (H A_o A_o^{\mathrm{H}} H^{\mathrm{H}} + K_{\bar{w}})^{-1} H A_o] && (8b)\\ &= \operatorname{tr}[(I + A_o^{\mathrm{H}} H^{\mathrm{H}} K_{\bar{w}}^{-1} H A_o)^{-1}] && (8c) \end{aligned}$$

where (8c) follows from Sherman-Morrison-Woodbury identity [17] and

$$K_{\bar{w}} = \alpha_v H \operatorname{diag}(A_o A_o^{\mathrm{H}}) H^{\mathrm{H}} + K_w \tag{9}$$

denotes the covariance of the effective noise at the receiver, i.e. $\bar{w} = Hv + w$.

III. Linear Precoder Design

Our aim is to find the robust precoder design that minimizes the MMSE under hardware impairments. We consider our designs under the following power constraint at the transmitter

$$\mathbb{E}[||A_o s||^2] = \operatorname{tr}[A_o A_o^{\mathrm{H}}] \leq P, \qquad P > 0. \tag{10}$$

Here the power constraint is given in terms of $A_o s$ instead of $A_o s + v$, since the former is the variable we have control over. For the former, the power constraint is considered as a constraint on the design whereas for the latter it is considered as a contraint at the output of the antenna system. Nevertheless, (10) can be equivalently expressed as a power constraint on $A_o s + v$ as follows

$$\begin{aligned} \mathbb{E}[||A_o s + v||^2] &= \operatorname{tr}[A_o A_o^{\mathrm{H}}] + \alpha_v \operatorname{tr}[\operatorname{diag}(A_o A_o^{\mathrm{H}})] && (11)\\ &\leq (1 + \alpha_v) P. && (12) \end{aligned}$$

We are interested in the following precoder design problem

$$\begin{aligned} (P1) \qquad & \min_{A_o} \quad \varepsilon(A_o) && (13a)\\ & \text{s.t.} \quad \operatorname{tr}[A_o A_o^{\mathrm{H}}] \leq P. && (13b) \end{aligned}$$

where $\varepsilon(A_o)$ is as defined in (8). We note that this formulation investigates *statistically* robust designs that provide performance guarantees *on average* as opposed to robust design approaches based on outage or worst-case performance.

Here $\varepsilon(A_o)$ is not a convex function of A_o. Although an optimal solution to the precoding problem can be constructed for the case with $\alpha_v = 0$ [3], [5], these results do not immediately generalize to (13).

It is possible to rewrite Problem P1 in terms of a new variable $R_{A_o} = A_o A_o^{\mathrm{H}} \succeq 0$. However, such a formulation in general does not lead to a convex optimization problem. In particular, using $\operatorname{tr}[AB] = \operatorname{tr}[BA]$, (8b) can be expressed as

$$\begin{aligned} \varepsilon^R(R_{A_o}) = n_s - \operatorname{tr}[(H R_{A_o} H^{\mathrm{H}} + \alpha_v H \operatorname{diag}(R_{A_o}) H^{\mathrm{H}} \\ + K_w)^{-1} H R_{A_o} H^{\mathrm{H}}]. \end{aligned} \tag{14}$$

Hence Problem P1 can be written as

$$\begin{aligned} (\bar{P}1) \qquad & \min_{R_{A_o} \succeq 0} \quad \varepsilon^R(R_{A_o}) && (15a)\\ & \text{s.t.} \quad \operatorname{tr}[R_{A_o}] \leq P && (15b)\\ & \qquad \operatorname{rank}(R_{A_o}) \leq n_s. && (15c) \end{aligned}$$

Here the rank constraint in (15c) ensures that Problem P1 and Problem $\bar{P}1$ are equivalent, so that an optimal $A_o \in \mathbb{C}^{n_t \times n_s}$ can be always found from an optimal $R_{A_o} \in \mathbb{C}^{n_t \times n_t}$. This rank-condition forms a non-convex constraint when $n_s < n_t$ (otherwise it is trivial in the sense that an optimal $A_o \in \mathbb{C}^{n_t \times n_s}$ can be always found from an optimal $R_{A_o} \in \mathbb{C}^{n_t \times n_t}$). Hence in general writing the problem in terms of R_{A_o} does not result in a convex formulation.

A relaxation of Problem P1 can be formed by lifting the rank constraint, i.e. omitting (15c) in Problem $\bar{P}1$. Nevertheless, in general this relaxation is not tight. To see this, let us consider the special case with $\alpha_v = 0$. As P increases, the rank of optimal R_{A_o} typically increases (depending on eigenvalues of $H^\dagger H$) [3], [5, Table 3.1]. Hence the relaxation will give solutions with full rank (i.e. n_t) under relatively high values of P. On the other hand, admissable solutions for Problem P1 can be only found from optimal R_{A_o} if it satisfies (15c).

Looking at the expression for $\varepsilon(A_o)$ in (8c), the effect of residual transmitter distortion is seen to enter into the error expression through $K_{\bar{w}}$, the covariance matrix of the effective noise at the receiver. $K_{\bar{w}}$ in general depends on A_o, the precoder to be optimized, which makes this optimization problem particularly challenging to solve.

A. Precoder Design with Fixed Receiver Filter

In order to propose a design for Problem P1, we first consider the case where the receiver uses a fixed estimation filter. More precisely, we consider the following problem

$$
\begin{aligned}
(P2) \qquad & \min_{A_o} \quad \mathbb{E}[||s - By||^2] && (16a)\\
& \text{s.t.} \quad \operatorname{tr}[A_o A_o^{\mathrm{H}}] \leq P. && (16b)
\end{aligned}
$$

For a given B, the mean-square error in (6) can be written as

$$
\begin{aligned}
\varepsilon(A_o, B) &= \mathbb{E}[||s - B(HA_o s + Hv + w)||^2] && (17a)\\
&= \mathbb{E}[||(I - BHA_o)s||^2] + \mathbb{E}[||B(Hv + w)||^2] && (17b)\\
&= ||I - BHA_o||^2 + \operatorname{tr}[B(HK_vH^{\mathrm{H}} + K_w)B^{\mathrm{H}}] && (17c)\\
&= \operatorname{tr}[A_0^{\mathrm{H}} H^{\mathrm{H}} B^{\mathrm{H}} BHA_0] - 2\operatorname{Re}[\operatorname{tr}[BHA_0]] \\
&\quad + n_s + \alpha_v \operatorname{tr}[BH \operatorname{diag}(A_o A_o^{\mathrm{H}}) H^{\mathrm{H}} B^{\mathrm{H}}] \\
&\quad + \operatorname{tr}[BK_w B^{\mathrm{H}}] && (17d)
\end{aligned}
$$

where $\operatorname{Re}[z]$ denotes the real part of $z \in \mathbb{C}$. We note that

$$
\begin{aligned}
\operatorname{tr}[M \operatorname{diag}(A_o A_o^{\mathrm{H}}) M^{\mathrm{H}}] &= \operatorname{tr}[\operatorname{diag}(A_o A_o^{\mathrm{H}}) M^{\mathrm{H}} M]\\
&= \operatorname{tr}[A_o A_o^{\mathrm{H}} \operatorname{diag}(M^{\mathrm{H}} M)]\\
&= \operatorname{tr}[A_o^{\mathrm{H}} \operatorname{diag}(M^{\mathrm{H}} M) A_o]
\end{aligned}
$$

where M is a matrix of appropriate dimensions. Hence the terms in (17d) that include $\operatorname{diag}(A_o A_o^{\mathrm{H}})$ can be also expressed as convex quadratic functions of A_o.

Hence the objective function of Problem P2, i.e. (17d), is a convex quadratic function of A_o. Similarly the transmitter power constraint, (10), bounds a convex quadratic function of A_o from above, hence forms a convex constraint. As a result, the optimization formulation in (16) is convex, and it can be solved by standard numerical optimization tools, such as SDPT3, SeDuMi and CVX [18–20].

Algorithm 1 Algorithm for Problem P1

Initialize:
Set $A_o^0 = I$.
Using A_o^0 and (7a), find B^0. Let i=1.
repeat
Using B^{i-1}, solve (16) for A_o^i.
Using A_o^i and (7a), find B^i.
Using A_o^i,B^i and (17d), find the error e^i.
until $(e^{i-1} - e^i \leq \epsilon)$ *// The stopping criterion is met.*
Output: A_o^i.

B. Joint Precoder and Receiver Filter Design

We now consider Problem P1 in (13). We rewrite it equivalently as follows

$$
\begin{aligned}
(P1) \qquad & \min_{A_o, B} \quad \varepsilon(A_o, B) && (18a)\\
& \text{s.t.} \quad \operatorname{tr}[A_o A_o^{\mathrm{H}}] \leq P. && (18b)
\end{aligned}
$$

To find a design for (13), we propose a block-coordinate descent approach. This method is summarized in Algorithm I. Here we take turns in fixing A and B. For the fixed B step, an optimal solution for A can be found using (16). For the fixed A step, an optimal B is found using (7a). We note that due to non-convexity of (13), we cannot provide any guarantees for the global optimality of the solutions provided by Algorithm 1. Nevertheless, we observe that the method is guaranteed to converge as follows:

Lemma 3.1: *The sequence $\{\varepsilon(A_o^i)\}_{i \in \mathbb{N}}$ converges monotonically.*

Proof: The objective function is bounded from below. In both fixed A_o, and fixed B steps, convex functions are minimized over convex domains and these sub-problems are solvable provided $P > 0$. Hence by [21, Thm. 4.5], $\{\varepsilon(A_o^i)\}_{i \in \mathbb{N}}$ converges monotonically.

IV. An Alternative Non-Ideal Hardware Model

For comparison purposes, we now discuss an alternative hardware impairment model with additive precoder error. Now the channel input is modelled as

$$
x = As = (A_o + A_d)s, \tag{19}
$$

where A_o denotes the designed linear precoder and A_d denotes the additional term due to non-ideal hardware. Here we design A_o and attempt to use it at the transmitter, but non-ideal hardware introduces an additional term and $A_o + A_d$ is realized instead. Discussion of such implementation errors in an optimization setting where the design variable is implemented with an additive error term can be found in [22], [23].

Here s, w, w_E and A_d are assumed to be statistically independent. The elements of A_d are modelled as i.i.d. complex proper Gaussian variables with $[A_d]_{i,j} \sim \mathcal{CN}(0, \sigma_a^2)$. The Gaussian assumption on A_d is again supported by the central limit theorem and the fact that this term models the aggregate effect of impairments in various components used in the precoder realization.

In the rest of the section, we discuss the relationship between this additive precoder error model and the previous additive noise model under a linear receiver filtering scheme.

The performance criterion at the receiver is again the average mean-square error

$$\varepsilon^p(A_o, B) = \mathbb{E}_{\mathrm{s},\mathrm{A_d}}[||s - By||^2] \tag{20}$$

Here the subscripts s and A_d denote the expectation with respect to random signals (including the noise) and the random component of the precoder, respectively. Due to the presence of the random matrix A_d, which is multiplied by the data vector s, this performance criterion does not correspond to the MMSE estimation of s. It rather gives the linear minimum mean-square error (LMMSE) estimate, which gives the minimum mean-square error achievable by a linear estimator.

For a given B, the mean-square error at the receiver can be written as

$$\begin{aligned}\varepsilon^p(A_o, B) &= \mathbb{E}_{s,A_d}[||s - By||^2] \\ &= ||I - BHA_o||^2 + \mathbb{E}_{A_d}[||BHA_d||^2] + \mathrm{tr}[BK_w B^{\mathrm{H}}] \\ &= \mathrm{tr}[A_o^{\mathrm{H}} H^{\mathrm{H}} B^{\mathrm{H}} BHA_o] - 2\,\mathrm{Re}[\mathrm{tr}[BHA_o]] \\ &\quad + n_s + n_s\sigma_a^2\,\mathrm{tr}[BHH^{\mathrm{H}}B^{\mathrm{H}}] + \mathrm{tr}[BK_w B^{\mathrm{H}}]\end{aligned} \tag{21}$$

where we have used statistical independence of s, w and A_d and the fact that

$$\mathbb{E}_{A_d}[||MA_d||^2] = n_s\sigma_a^2\ \mathrm{tr}[MM^{\mathrm{H}}] \tag{22}$$

for a deterministic matrix M of appropriate dimensions.

By taking the derivative of (21) with respect to B, and equating to zero, an optimal linear estimator B can be found as

$$B^* = A_o^{\mathrm{H}} H^{\mathrm{H}} (HA_o A_o^{\mathrm{H}} H^{\mathrm{H}} + n_s\sigma_a^2 HH^{\mathrm{H}} + K_w)^{-1}. \tag{23}$$

The resulting mean-square error can be expressed as

$$\varepsilon^p(A_o, B^*) = (I + A_o^{\mathrm{H}} H^{\mathrm{H}} (K_{\bar{w}}^p)^{-1} HA_o)^{-1} \tag{24}$$

where we have put B^* into (21) and used Sherman-Morrison-Woodbury identity [17]. Here

$$K_{\bar{w}}^p = n_s\sigma_a^2 HH^{\mathrm{H}} + K_w\,. \tag{25}$$

The behaviour of LMMSE estimation under the additive precoder error model is quite similar to fading channel scenario where the channel consists of a known mean component and a fading component, see for instance [12]. We also observe that the general form of (21) is similar to (17d), where in both expressions the residual hardware impairments introduce an additive error term. Similarly, in both (23) and (7a), (and also in (24) and (8c)) there is an effective additional noise term which assumes different expressions under each model. Hence, although the starting points of the models are seemingly quite different, their general behaviour can be said to be in a similar form under LMMSE estimation. In Section V, we present a comparison of error performance under these two hardware impairment models.

We note that the optimum precoder for minimizing (24) under (10) can be found by utilizing the arguments used for finding the optimum precoder when there is no residual hardware impairment:

Lemma 4.1: *Let $n_s \leq \min(n_t, n_r)$. Let $G = H^{\mathrm{H}}(K_{\bar{w}}^p)^{-1}H$ has the following singular value decomposition $G = U\Lambda_G U^{\mathrm{H}}$, where $U \in \mathbb{C}^{n_t \times n_t}$ is a unitary matrix and $\Lambda_G = \mathrm{diag}(\lambda_{G,i})$, $\lambda_{G,1} \geq \lambda_{G,2}, \ldots, \lambda_{G,n_t}$. Then there is an optimum precoder A_o for minimizing $\varepsilon^p(A_o, B^*)$ in (24) with the following form:*

$$A_o = \bar{U}\Lambda^{1/2} \tag{26}$$

where $\bar{U}$ is the $n_t \times n_s$ submatrix of U where only the first n_s columns are included. $\Lambda = \mathrm{diag}(p_i)$ is the diagonal matrix with

$$p_i = \nu(\lambda_{G,i}^{-1/2} - \lambda_{G,i}^{-1})^+, \quad i = 1, \ldots, n_s \tag{27}$$

where ν is chosen so that the power constraint is satisfied with equality $\sum_{i=1}^{n_s} p_i = P$.

The proof follows from, for instance, [3], [5]. We note that the assumption $n_s \leq \min(n_t, n_r)$ is made only for convenience in presentation and an optimum solution solution can be found for all cases.

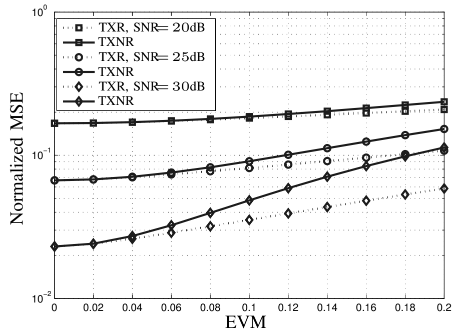

Fig. 1. Mean-square error versus EVM, additive noise model.

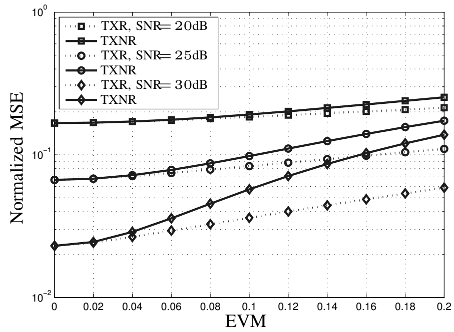

Fig. 2. Mean-square error versus EVM, additive precoder error model.

V. Numerical Results

We now illustrate the performance of the hardware impairment aware designs. In our examples, we consider the following channel model [24]

$$H = \sum_{i=1}^{L} \kappa_i a_R(\theta_{R,i}) a_T^{\mathrm{T}}(\theta_{T,i}). \tag{28}$$

Here $a_c(\theta) = [1\ e^{j2\pi d\cos(\theta)} \ldots\ e^{j2\pi(n_c-1)d\cos(\theta)}]^T$, where $c = T, R$. Here $a_T(\theta_{T,i})$ is the array steering vector at the transmitter and $a_R(\theta_{R,i})$ is the array response vector at the receiver corresponding to ith path in the channel. κ_i is the corresponding complex path amplitude. We normalize the channel matrix as $H/||H||$. The following parameters are used for the experiments: $L=2$, $\kappa_1=\kappa_2=1$, $d=0.5$, $\theta_{R,1}=\pi/6$, $\theta_{R,2}=\pi/3$, $\theta_{T,1}=\pi/4$, $\theta_{T,2}=\pi/5$. Let $n_t=3, n_r=2$, $n_s=2$, $K_w=\sigma_w^2 I$. $P=10$, $\epsilon=10^{-7}n_s$. SNR$=1/\sigma_w^2$ (dB). The error values are normalized by dividing with $n_s=\mathrm{tr}[K_s]$. The relevant convex optimization problems are solved using [18–20]. We denote the proposed hardware impairment aware designs with TXR. The performance of the designs that assume ideal hardware is shown with $TXNR$. We note designs for $TXNR$ can be found analytically [3], [5]. The performance of all designs is reported using an estimator B that is aware of the hardware impairments, making the non-robust linear precoder designs the sole ingredient that degrade the performance.

The level of hardware impairments in the two models are parameterized by two different variables; α_v and σ_a^2 in Section II and in Section IV, respectively. To relate these two parameters, we define the following parameter for the additive precoder error model

$$\alpha_a \doteq \frac{\mathbb{E}_{s,A_d}[||A_d s||^2]}{\mathbb{E}_s[||A_o s||^2]} = \frac{n_s n_t \sigma_a^2}{P} \tag{29}$$

Here we have used the fact that optimum strategies use all the available power, i.e. $\mathbb{E}_s[||A_o s||^2]=P$. We set $\alpha_a=\alpha_v$ and consider $\alpha_v\in[0,0.2^2]$ in the experiments. We note that 3GPP LTE specifies EVM$=\sqrt{\alpha_v}$ to be in the range $[0.08, 0.175]$ [15].

The trade-offs between the error and the hardware impairment levels are presented in Fig. 1 and Fig. 2, for the additive noise model of Section II and additive precoder error model of Section IV, respectively. We observe that for both models high levels of hardware impairment degrade the system performance leading to higher performance gap between robust and non-robust solutions. Comparing the results for varying SNR values, shows that this performance gap quickly diminishes when the SNR decreases. This suggests that hardware impairments at the levels considered in these experiments will only be crucial when operating at considerably high SNR values. Comparing Fig. 1 and Fig. 2 we observe that the error performances of the robust solutions under the two impairment models are very close. This is consistent with the fact that the hardware impairments levels are relatively small and they are adjusted using (29) and setting $\alpha_a=\alpha_v$. Yet it also suggests that the fact that in the first model the level of the additive noise is proportional to the signal power at that antenna may have limited effect on the performance of the robust solutions.

VI. Conclusions

Linear precoder design in MIMO systems is investigated under transmitter impairments. Our results illustrated that when the channel SNR is high enough, significant gains can be obtained by the proposed robust impairment-aware designs compared to non-robust solutions. We have considered two hardware impairment models, one of which introduces an additive noise term and the other one introduces an additive precoder error. Our numerical results suggest that although the starting point of these two impairment models are different, the error behaviour of the proposed robust solutions under these two models are quite similar for the level of hardware impairments considered. Further investigation of the relationship between these two models is considered future work.

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