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In this paper we study a novel approach to the design of a robust switched controller for continuous-time systems described by a novel robust plant model using quadratic stability and multi parameter dependent quadratic stability approaches. In the proposed design procedure with an output feedback a novel quadratic cost function is proposed which allows to obtain different performance dependence on the working points. Finally a numerical examples are investigated.

Key words: Switched system, robust controller, output feedback, quadratic stability, multi parameter dependent quadratic stability.

1. Introduction

The topic of robust hybrid systems has attracted considerable attention in the past decades. Wherever continuous and discrete dynamics interact, a hybrid system arises. The main motivation for studying hybrid systems comes from the two facts:

- hybrid systems have numerous application in the real world, and
- in a real control, there are dynamical systems that cannot be stabilized by any continuous static (dynamic) output state controller but a stabilizing hybrid control scheme can be found.

There are several approaches to model hybrid systems [1], [2], [3]. In the references authors consider a discrete event system and continuous dynamics modeled by differential or difference equation. Such models are used to formulate a general stability condition for hybrid systems. In this paper, we consider the class of hybrid systems known as the switched systems [4], [5], [6]. There are at last two approaches to stability analysis and controller synthesis of switched systems. The quadratic stability approach with a com-

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mon Lyapunov function gives stability of closed-loop switched systems under an arbitrary switching law, and a multiple Lyapunov function which is less conservative. The survey of the present status of switched systems can be consulted in the excellent paper and book of [7] and [1]. The research of switched system control design mainly focuses on the case when a plant is modeled by discrete-time systems. There are only a few references which deal with the case when the plant is modeled as a continuous-time system [8], [9].

In this paper a novel switched controller design procedure is obtained in the form of bilinear matrix inequality (BMI), using a new uncertain model of switched continuous-time linear systems and a multi parameter dependent Lyapunov function. In the proposed approach there is no need to use the notion of the 'dwell time' [8], [9] which significantly complicates the robust switched controller design procedure.

Organization of the paper is as follows. Section 2 includes problem formulation of robust switched controller design and some preliminaries. In sec. 3 sufficient stability conditions in the form of BMI are given for the design of a robust switched controller. In sec. 4 the obtained results are illustrated by real examples.

2. Preliminaries and problem formulation

Let us consider a class of uncertain linear continuous-time switched systems as follows

$$\sum\sigma: \quad \begin{aligned} \dot{x}(t) &= A(\xi,\theta)x(t) + B(\xi,\theta)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, $y(t) \in \mathbb{R}^l$ is the output vector of the system to be controlled, $\sigma \in S$ is an arbitrary switching algorithm, p is the number of switched plant models and

$$\begin{aligned} A(\xi,\theta) &= A_0(\xi) + \sum_{j=1}^{p} A_j(\xi)\theta_j \quad \in \mathbb{R}^{n\times n} \\ B(\xi,\theta) &= B_0(\xi) + \sum_{j=1}^{p} B_j(\xi)\theta_j \quad \in \mathbb{R}^{n\times m} \end{aligned} \tag{2}$$

where

$$\sum_{j=1}^{p}\theta_j = 1,\, \theta_j \in \langle 0\; 1\rangle \in \Omega_\theta,\, \sum_{j=1}^{p}\dot{\theta}_j = 0,\; j = 1,2,\ldots p$$

are switching parameters. $\theta_j = 1$, when σ_j is an active plant mode, otherwise $\theta_j = 0$. Matrices $A_j(\xi)$, $B_j(\xi)$, $j = 0,1,\ldots p$ belong to the convex set: a polytope with N vertices that can be formally defined as

$$\begin{aligned} \Omega = \quad & \{A_j(\xi), B_j(\xi) = \sum_{i=1}^{N}\left(A_{ji}, B_{ji}\right)\xi_i,\; j = 0,1,\ldots p, \\ & \sum_{i=1}^{N}\xi_i = 1,\; \xi_i \geq 0,\; \xi_i \in \Omega_\xi,\; \sum_{i=1}^{N}\dot{\xi}_i = 0\}. \end{aligned} \tag{3}$$

There are two possibilities for the switching parameters $\theta_j,\ j=1,2,\dots p$:
(a) rates of change of the switching parameters are infinite, or
(b) rates of change of switching parameters θ_j are finite.

The switched output feedback control algorithm is

$$u(t) = F(\theta)y(t) = F(\theta)Cx(t) \tag{4}$$

where

$$F(\theta) = F_0 + \sum_{j=1}^{p} F_j\theta_j.$$

The structure of matrices $F_j,\ j=0,1,\dots p$ can be defined by the designer. For the closed-loop system from (1) and (4) one obtains

$$\dot{x}(t)\big(A(\xi,\theta)+B(.,.)F(\theta)C\big)x(t) = A_c(\xi,\theta)x(t). \tag{5}$$

To asses the system performance we consider a novel positive definite quadratic cost function which allows to prescribe the performance for each mode $\sigma \in S$

$$J = \int_0^\infty J(t)dt = \int_0^\infty \left(x^T Q(\theta)x + u^T Ru\right)dt \tag{6}$$

where $Q(\theta) = Q_0 + \sum_{j=1}^{p} Q_j\theta_j$.

Definition 1 *Consider the stable closed-loop system* (5). *If there exist a control law* u^* (4) *and a positive scalar* J^* *such that the closed-loop system* (5) *is stable and the value of the closed-loop cost function* (6) *satisfies* $J \le J^*$, *then* J^* *is said to be the guaranteed cost and* u^* *is said to be the guaranteed cost control law for the system* (5).

Lemma 1 [10] *Consider the closed-loop switched system* (5) *with the control algorithm* (4). *Control algorithm* (4) *is the guaranteed cost algorithm if there exists a positive scalar* ε *such that for the time derivative of the positive definite Lyapunov function* $V(\xi,\theta)$ *(case b) the following condition holds*

$$B_e = \max_u \left\{ \frac{\partial V(\cdot)}{\partial x} A_c(\xi,\theta) + J(t) \right\} \le -\varepsilon x^T x \tag{7}$$

when $\varepsilon \to 0$.

Note, that for the case *a*, the Lyapunov function has the form $V(\xi)$ – quadratic stability with respect to the switching parameters θ.

3. Robust switched controller design

In this section two methods of robust switched controller design are described. The first method is connected with the notion of quadratic stability with respect to the switched parameters θ (case *a*), where we assume that the rate of θ change is infinite. In the references concerning the switched controller design, the authors refer to the case where switching can occur immediately. In some real cases the switching signal rate of change is finite, that is $|\dot{\theta}| < \infty$. This assumption will be used in the second approach. In the proposed design procedure of the robust switched controller there is no need to use the notion of the 'dwell time' [8], [9]. In continuous time switching systems, the dwell time complicates significantly the robust switched controller design procedure.

3.1. Robust quadratic stability approach

In this part we will assume that the rate of θ change is infinite. The proposed method is based on the notion of quadratic stability. Let us assume that the Lyapunov function is in the form

$$V(\xi) = x^T P(\xi) x \tag{8}$$

where

$$P(\xi) = \sum_{i=1}^{N} P_i \xi_i.$$

Firstly let derivative (8) and obtain

$$\frac{dV(\xi)}{dt} = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} \begin{bmatrix} 0 & P(\xi) \\ P(\xi) & \dot{P}(\xi) \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{9}$$

where

$$\dot{P}(\xi) = \sum_{i=1}^{N} P_i \dot{\xi}_i \le \sum_{i=1}^{N} P_i \delta_i,\ \delta_i = \max(\dot{\xi}_i).$$

The term $\dot{P}(\xi)$ in (9) can be used in the case if one wants to take into account the rate of system parameter changes. For the closed-loop system (5) one obtains

$$\left(2N_1\dot{x} + 2N_2 x\right)^T \left(\dot{x} - A_c(\xi,\theta)\right) = 0 \tag{10}$$

where $N_1, N_2 \in \mathbb{R}^{n\times n}$ are auxiliary matrices. Substituting (10), (9) and (6) to (7) we have

$$B_e = z^T W(\xi) z < 0 \tag{11}$$

where $z^T = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix}$, $W = \left\{ W_{ij}(\xi,\theta)_{2\times 2} \right\}$ and

$$\begin{aligned}
W_{11}(\xi,\theta) &= N_1^T + N_1 \\
W_{22}(\xi,\theta) &= -N_2^T A_c(\xi,\theta) - A_c(\xi,\theta)^T N_2 + \dot{P}(\xi) + Q(\theta) \\
&\quad + C^T F(\theta)^T R F(\theta) C \\
W_{12}(\xi,\theta) &= -N_1^T A_c(\xi,\theta) + N_2 + P(\xi) = W_{12}(\xi,\theta)^T.
\end{aligned}$$

Because of the quadratic stability $\theta_j\theta_k = 0,\ j \neq k$, the entries of matrix $W(\xi,\theta)$ can be simplified as follows

$$\begin{aligned}
W_{11}(\xi,\theta) &= N_1^T + N_1 \\
W_{22}(\xi,\theta) &= -N_2^T\Bigg(A_{c0}(\xi) + \sum_{j=1}^{p} A_{cj}(\xi)\theta_j + \\
&\quad + \sum_{k=1}^{p} B_k(\xi) F_k C \theta_k^2 \Bigg) - \Bigg(A_{c0}(\xi) \\
&\quad + \sum_{j=1}^{p} A_{cj}(\xi)\theta_j + \sum_{k=1}^{p} B_k(\xi) F_k C \theta_k^2 \Bigg)^T N_2 + \\
&\quad + \dot{P}(\xi) + Q_0 + \sum_{j=1}^{p} Q_j \theta_j + C^T F_0^T R F_0 C + \\
&\quad + \sum_{j=1}^{p} C^T \left(F_0^T R F_j + F_j^T R F_0 \right) C \theta_j + \\
&\quad + \sum_{k=1}^{p} C^T F_k^T R F_k C \theta_k^2 \\
W_{12}(\xi,\theta) &= -N_1^T\Bigg(A_{c0}(\xi) + \sum_{j=1}^{p} A_{cj}(\xi)\theta_j + \\
&\quad + \sum_{k=1}^{p} B_k(\xi) F_k C \theta_k^2 \Bigg) + N_2 + P(\xi).
\end{aligned}$$

The first main result concerning the quadratic stability approach is summarized in the following theorem.

Theorem 1 *Closed-loop switched system* (5) *is robust quadratically stable with guaranteed cost for all* $\sigma \in S$, $\xi \in \Omega_\xi$ *if it satisfies*

(a)

$$W(\xi,\theta) = W_0(\xi) + \sum_{j=1}^{p} W_j(\xi)\theta_j + \sum_{k=1}^{p} W_k(\xi)\theta_k^2 \tag{12}$$

(b)

$$W_{kk}(\xi) \geq 0 \tag{13}$$

where

$$W_0(\xi)=\begin{bmatrix} W_{011} & W_{012} \\ W_{012}^T & W_{022} \end{bmatrix}, W_j(\xi)=\begin{bmatrix} W_{j11} & W_{j12} \\ W_{j12}^T & W_{j22} \end{bmatrix}$$

$$\begin{aligned}
W_{011} &= N_1^T + N_1 \\
W_{012} &= -N_1^T A_{c0}(\xi) + N_2 + P(\xi) \\
W_{022} &= -N_2^T A_{c0}(\xi) - A_{c0}(\xi)^T N_2 + \dot{P}(\xi) + Q_0 + \\
&\quad +C^T F_0^T R F_0 C \\
W_{j11} &= 0 \\
W_{j12} &= -N_1^T A_{cj}(\xi) \\
W_{j22} &= -N_2^T A_{cj}(\xi) - A_{cj}(\xi)^T N_2 + Q_j + \\
&\quad +C^T \left(F_0^T R F_j + F_j^T R F_0\right) C.
\end{aligned}$$

Note that if $W(\xi,\theta)$ is convex with respect to $\theta_k, k=1,2,\dots p$, then it is negative definite in the defined hyper rectangle θ,ξ, if and only if takes negative values at the all vertices of θ. $W(\xi,\theta)$ is linear with respect to uncertain parameter $\xi_i, i=1,2,\dots N$ therefore inequalities in theorem 1 for each $\sigma \in S$ split to 2 N inequalities, $W(\theta)$. Proof immediately follows from the above discussion.

3.2. Robust multi parameter dependent quadratic stability approach

In this subsection we assume the realistic case where the switching signal rate of change is finite,

- for the switching parameters it holds

$$\sum_{j=1}^{p}\theta_j = 1,\ \theta_j \in \langle 0, 1\rangle \in \Omega_\theta \tag{14}$$

- the rate of switching parameter change $\dot{\theta}_j$ is well defined and satisfies

$$\dot{\theta}_j \in \Omega_t = \left\{ \dot{\theta}_j \in \langle \underline{\dot{\theta}}_j\ \overline{\dot{\theta}}_j \rangle, \sum_{j=1}^{p} \dot{\theta}_j = 0 \right\} \tag{15}$$

- the stable steady state points for all switching parameters θ_j are equal to zero or one.

For the first derivative of the Lyapunov function

$$V(\xi,\theta) = x^T P(\xi,\theta) x \tag{16}$$

where $P(\xi,\theta)=P_0(\xi)+\sum_{j=1}^{p}P_j(\xi)\theta_j$ and $P_i(\xi)=\sum_{j=0}^{N}P_{ij}\xi_j$ one obtains

$$\frac{dV(\cdot)}{dt}=\begin{bmatrix}\dot{x}^T & x^T\end{bmatrix}\begin{bmatrix}0 & P(\xi,\theta)\\ P(\xi,\theta) & \dot{P}(\xi,\theta)\end{bmatrix}\begin{bmatrix}\dot{x}\\ x\end{bmatrix} \tag{17}$$

where

$$\begin{aligned}\dot{P}(\xi,\theta) &= \sum_{i=1}^{N}\left[\left(P_{0i}+\sum_{j=1}^{p}P_{ij}\theta_j\right)\dot{\xi}_i+\sum_{j=1}^{p}P_{ij}\dot{\theta}_j\right]\xi_i=\\ &=\sum_{i=1}^{p}\left[P_{0i}\dot{\xi}_i+\sum_{j=1}^{p}P_{ij}\dot{\theta}_j+\sum_{j=1}^{p}P_{ij}\dot{\xi}_i\theta_j\right]\xi_i.\end{aligned}$$

Substituting (10), (17) and (6) to (7) one obtains the sufficient stability conditions for robust multi parameter dependent quadratic stability of closed-loop switched systems for all $\sigma\in S$ and $\xi\in\Omega_s$ as follows.

Theorem 2 *Closed-loop system* (5) *is robust multi parameter dependent quadratic stable with guaranteed cost if there exists a positive definite Lyapunov matrix* $P(\xi,\theta)>0$*, auxiliary matrices* N_1, N_2*, and gain matrices* (4) *such that for an arbitrary switching algorithm* $\sigma\in S$ *and system uncertainty* $\xi\in\Omega_s$ *the following holds*

(a)

$$\begin{aligned}U(\xi,\theta)= \quad & U_0(\xi)+\sum_{j=1}^{p}U_j(\xi)\theta_j+\sum_{j=1}^{p}\sum_{k>j}^{p}U_{jk}(\xi)\theta_j\theta_k\\ &+\sum_{k=1}^{p}U_{kk}(\xi)\theta_k^2<0\end{aligned} \tag{18}$$

(b)

$$U_{kk}(\xi)\geq 0,\ k=1,2,\dots p \tag{19}$$

where

$$\begin{aligned}U_0(\xi) &= \begin{bmatrix}U_{011} & U_{012}\\ U_{012}^T & U_{022}\end{bmatrix}, \quad U_j(\xi) = \begin{bmatrix}U_{j11} & U_{j12}\\ U_{j12}^T & U_{j22}\end{bmatrix},\\ U_{jk}(\xi) &= \begin{bmatrix}U_{jk11} & U_{jk12}\\ U_{jk12}^T & U_{jk22}\end{bmatrix}, \quad U_{kk}(\xi) = \begin{bmatrix}U_{kk11} & U_{kk12}\\ U_{kk12}^T & U_{kk22}\end{bmatrix}\end{aligned}$$

$$
\begin{aligned}
U_{011} &= -N_1^T + N_2 \\
U_{012} &= -N_1^T A_{c0}(\xi) + N_2 + P_0(\xi) \\
U_{022} &= -N_2^T A_{c0}(\xi) - A_{c0}(\xi)^T N_2 + C^T F_0^T R F_0 C + \\
&\quad + \sum_{i=1}^{N} \left(P_{0i}\dot{\xi}_i + \sum_{j=1}^{p} P_{ij}\dot{\theta}_j \right) \xi_i + Q_0 \\
U_{j11} &= U_{jk11} = U_{kk11} = 0 \\
U_{j12} &= -N_1^T A_{cj}(\xi) + P_j(\xi) \\
U_{j22} &= -N_2^T A_{cj} - A_{cj}^T N_2 + Q_j \sum_{i=1}^{N} P_{ij}\dot{\xi}_i \xi_i \\
&\quad + C^T \left(F_0^T R F_k + F_k^T R F_j \right) C \\
U_{jk12} &= -N_1^T A_{cjk}(\xi) \\
U_{jk22} &= -N_2^T A_{cjk}(\xi) - A_{cjk}(\xi)^T N_2 + \\
&\quad + C^T \left(F_j^T R F_k + F_k^T R F_j \right) C \\
U_{kk12} &= -N_1^T A_{ckk}(\xi) \\
U_{kk22} &= -N_2^T A_{ckk}(\xi) - A_{ckk}(\xi)^T N_2 + C^T F_k^T R F_k C.
\end{aligned}
$$

Note that $U(\xi,\theta)$ is linear with respect to uncertain parameter ξ, therefore inequalities (18), (19) for all $\sigma \in S$ split to $2N$ inequalities. If the solution of (18) is feasible, the designed robust switched controller for all $\sigma \in S$ ensures multi parameter-dependent quadratic stability with a guaranteed cost for the rate of switching parameter changes given by (15).

4. Examples

The first example has been used in [11]. Consider a simplified manual transmission model

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= [-a(x_2) + u]/(1+v)
\end{aligned}
\tag{20}
$$

where x_1 is the ground speed, x_2 is the acceleration, $u \in \langle 0,1 \rangle$ is the throttle position, and $v \in \{1,2,3,4\}$ is the gear shift position. The function $a(\cdot)$ is positive for positive argument. Model (20) can be transformed to the form

$$
\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & \dfrac{-a}{1+v} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{1+v} \end{bmatrix}}_{B} u. \tag{21}
$$

Substituting $a = 1.9$ and $v = [1,2,3,4]$ we can transform (21) to the form (1). With randomly generated uncertainty the system matrices are:

$$
\begin{aligned}
A_{0|1} &= \begin{bmatrix} 0 & 1 \\ 0 & -0.5979 \end{bmatrix}, & A_{0|2} &= \begin{bmatrix} 0 & 0 \\ 0 & -0.6218 \end{bmatrix} \\
A_{1|1} &= \begin{bmatrix} 0 & 1 \\ 0 & -0.3336 \end{bmatrix}, & A_{1|2} &= \begin{bmatrix} 0 & 0 \\ 0 & -0.3472 \end{bmatrix} \\
A_{2|1} &= \begin{bmatrix} 0 & 1 \\ 0 & -0.0233 \end{bmatrix}, & A_{2|2} &= \begin{bmatrix} 0 & 0 \\ 0 & -0.0242 \end{bmatrix} \\
A_{3|1} &= \begin{bmatrix} 0 & 1 \\ 0 & 0.1319 \end{bmatrix}, & A_{3|2} &= \begin{bmatrix} 0 & 0 \\ 0 & 0.1373 \end{bmatrix} \\
A_{4|1} &= \begin{bmatrix} 0 & 1 \\ 0 & 0.2250 \end{bmatrix}, & A_{4|2} &= \begin{bmatrix} 0 & 0 \\ 0 & 0.2342 \end{bmatrix} \\
B_{0|1} &= \begin{bmatrix} 0 \\ 0.3144 \end{bmatrix}, & B_{0|2} &= \begin{bmatrix} 0 \\ 0.3272 \end{bmatrix} \\
B_{1|1} &= \begin{bmatrix} 0 \\ 0.1756 \end{bmatrix}, & B_{1|2} &= \begin{bmatrix} 0 \\ 0.1828 \end{bmatrix} \\
B_{2|1} &= \begin{bmatrix} 0 \\ 0.0123 \end{bmatrix}, & B_{2|2} &= \begin{bmatrix} 0 \\ 0.0128 \end{bmatrix} \\
B_{3|1} &= \begin{bmatrix} 0 \\ -0.0694 \end{bmatrix}, & B_{3|2} &= \begin{bmatrix} 0 \\ -0.0722 \end{bmatrix} \\
B_{4|1} &= \begin{bmatrix} 0 \\ -0.1184 \end{bmatrix}, & B_{4|2} &= \begin{bmatrix} 0 \\ -0.1232 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned}
$$

Using theorems 1 and 2 with weighting matrices $Q = qI$, $q = 0.1$, $R = rI$, $r = 1$ and $\rho_U = 1\times 10^5$, $\rho_L = 1\times 10^{-5}$, $\dot{\theta}_i = 5\,s^{-1}$ we obtain the switched controller in the form (3) having the parameters depending on the cases discussed:

Case of robust quadratic stability (*Theorem* 1)

$$
\begin{aligned}
F_0 &= [\ \ 0.9654 \quad 0.1449\ ] \\
F_1 &= [\ -1.9410 \quad -1.8581\ ] \\
F_2 &= [\ -1.9408 \quad -1.8579\ ] \\
F_3 &= [\ -1.9405 \quad -1.8576\ ] \\
F_4 &= [\ -1.9409 \quad -1.8580\ ].
\end{aligned} \tag{22}
$$

Case of robust multi parameter-dependent quadratic stability (*Theorem* 2)

$$\begin{array}{lcl} F_0 & = & [\ 0.4491 \quad 0.4268\ ] \\ F_1 & = & [\ -1.4232 \quad -2.1633\ ] \\ F_2 & = & [\ -1.4143 \quad -2.1504\ ] \\ F_3 & = & [\ -1.4219 \quad -2.1615\ ] \\ F_4 & = & [\ -1.4234 \quad -2.1637\ ]. \end{array} \tag{23}$$

In the simulations we switched the gear shift as follows: $v=1\ if\ x_1 \in \langle 0, 0.3\rangle$, $v=2\ if\ x_1 \in (0.3, 0.6\rangle$, $v=3\ if\ x_1 \in (0.6, 0.8\rangle$ and $v=4\ if\ x_1 \in (0.8, \infty)$, and the switching rate of v is established with $\dot{\theta}_i = 5\ s^{-1}$. The simulation results (Fig. 1, 2, 3 and 4) prove that the theorems hold, and we can see that the controller output in both cases achieves the maximal/minimal values ($u \in \langle 0, 1\rangle$).

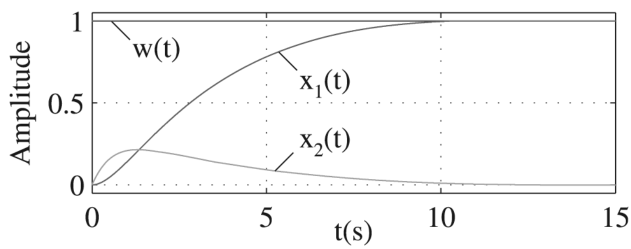

Figure 1: Simulation results $w(t), y(t)$ with switched controller (22) – QS

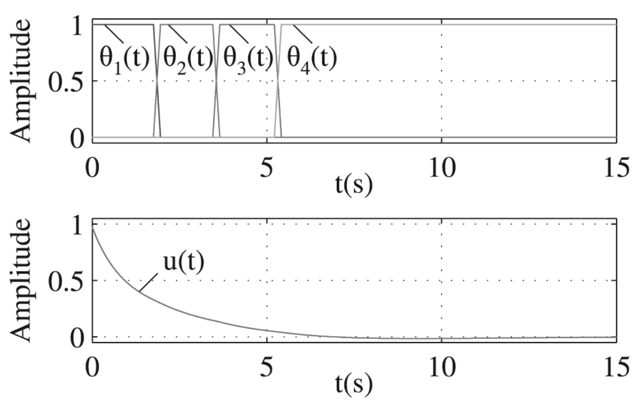

Figure 2: Calculated switching parameters $\theta(t)$ and the controller output with switched controller (22) – QS

The second example shows one of the possible applications of our approach to the robust switched controller design. Control systems over data networks are commonly referred to as networked control systems (NCSs). For the NCSs, the sampled data and controller signals are transmitted through a network. As the result, it leads to a network-induced delay in a networked control closed-loop system. The existence of such a kind of delay in a network-based control loop can induce instability or poor performance of control systems.

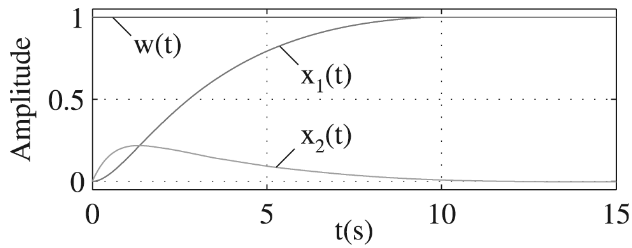

Figure 3: Simulation results $w(t), y(t)$ with switched controller (23) – MPQS

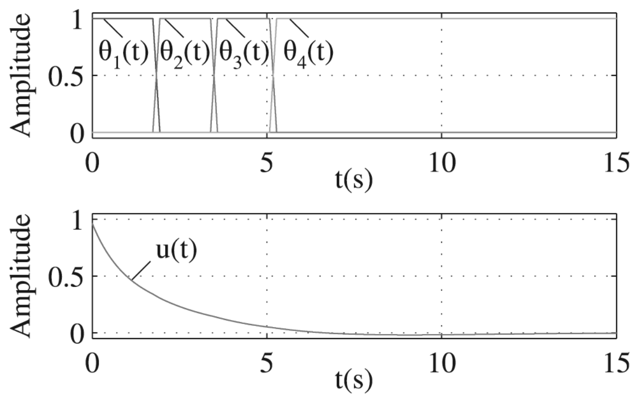

Figure 4: Calculated switching parameters $\theta(t)$ and the controller output with switched controller (23) – MPQS

Assume that a linear system with transfer function $G(s)$ is integrated to NCSs, which inevitably leads to a change in the plant transfer function $G(s)e^{-T_d s}$, where T_d is a variable plant time delay. The value of T_d depends on the load of the communication network. Assume that one can define four middle values of time delay $T_{di}, i = 1,2,3,4$ for the fourth communication network loads.

For a PI switched controller design and simulation a laboratory model of a DC-motor has been used. This model consists of two co-operating real DC servomotors. The first one serves as a motor and the second one as a generator. The mechanical interconnection is realized by an inertia load. The power supply, signal measurement and motor control are performed by motor electronics. In electronics, there is an RC component connected to the input of the motor to enable the changes of the time constant and the gain of the controlled system. System dynamics parameters can be tuned with a potentiometer.

Input voltage $u_{in}(t)$ is used as a manipulated variable within the range $0-10\,V$. The revolutions per minute converted into voltage in the range $0-10\,V$ forms the measured output variable. The output is affected by the load (perturbation), which can be set manually using the potentiometer in the range $0-10\,V$. After the DC motor system was identified the following transfer function has been obtained

$$Sys = \frac{0.0627s + 1.281}{2.081s^2 + 2.506s + 1}. \tag{24}$$

For 4 chosen middle values of the induced time delays $T_d = [0.1, 0.2, 0.3, 0.4]\,s$ and using the first order Padé approximation we computed 4 plant transfer functions which were transformed into the state space representation. The obtained 4 plant models were extended by one state for the switched PI controller design. Finally one obtains the plant models with added uncertainty in the form (1)

$$A_{0|1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4.9506 & -13.0218 & -11.5045 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{0|2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5.0006 & -13.1520 & -11.7370 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{1|1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -4.6465 & -11.5382 & -9.6791 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{1|2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -4.6465 & -11.5382 & -9.6791 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{2|1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.1981 & 0.5018 & 0.4126 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{2|2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.2021 & 0.5018 & 0.4209 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{3|1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.8001 & 4.4696 & 3.7123 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{3|2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.8181 & 4.5598 & 3.7498 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{4|1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2.5743 & 6.5218 & 5.3626 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{4|2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2.6003 & 6.5870 & 5.4709 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{0|1} = \begin{bmatrix} -0.0304 \\ 0.0493 \\ 4.1652 \\ 0 \end{bmatrix}, \qquad B_{0|2} = \begin{bmatrix} -0.0308 \\ 0.0503 \\ 4.2069 \\ 0 \end{bmatrix}$$

$$B_{1|1} = \begin{bmatrix} -0.0002 \\ 0.5775 \\ -4.5362 \\ 0 \end{bmatrix}, \qquad B_{1|2} = \begin{bmatrix} -0.0002 \\ 0.5891 \\ -4.5816 \\ 0 \end{bmatrix}$$

$$B_{2|1} = \begin{bmatrix} -0.0002 \\ -0.0259 \\ 2.1004 \\ 0 \end{bmatrix}, \qquad B_{2|2} = \begin{bmatrix} -0.0002 \\ -0.0264 \\ 2.1214 \\ 0 \end{bmatrix}$$

$$B_{3|1} = \begin{bmatrix} 0.0007 \\ -0.2265 \\ 1.59999 \\ 0 \end{bmatrix}, \quad B_{3|2} = \begin{bmatrix} 0.0008 \\ -0.2311 \\ 1.6159 \\ 0 \end{bmatrix}$$

$$B_{4|1} = \begin{bmatrix} -0.0002 \\ -0.3251 \\ 0.8360 \\ 0 \end{bmatrix}, \quad B_{4|2} = \begin{bmatrix} -0.0003 \\ -0.3317 \\ 0.8443 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using theorem 1 and 2 with weighting matrices $Q = qI$, $q = 0.01$, $R = rI$, $r = 3$ and $\rho_U = 10$, $\rho_L = 1\times10^{-5}$, $\dot{\theta}_i = 10\, s^{-1}$ a robust switched PI controller is obtained in the form (3):

Case of robust quadratic stability (*Theorem* 1)

BMIsolverfailed.

Case of robust multi parameter-dependent quadratic stability (*Theorem* 2)

$$\begin{aligned} F_0 &= [\ -0.2666 \quad -0.1056\ ] \\ F_1 &= [\ -0.2975 \quad -0.1423\ ] \\ F_2 &= [\ -0.0336 \quad -0.0229\ ] \\ F_3 &= [\ -0.0035 \quad -0.0061\ ] \\ F_4 &= [\ 0.1788 \quad 0.1805\ ]\times10^{-10}. \end{aligned} \tag{25}$$

Measured results on the real plant (Fig. 5, 6 and 7) confirm that theorem 2 holds and that the robust multi parameter-dependent quadratic stability is less conservative than the robust quadratic stability. In the simulation the switching algorithm (middle time delay) is shown in Fig. 7 from which the scheduled parameters were calculated.

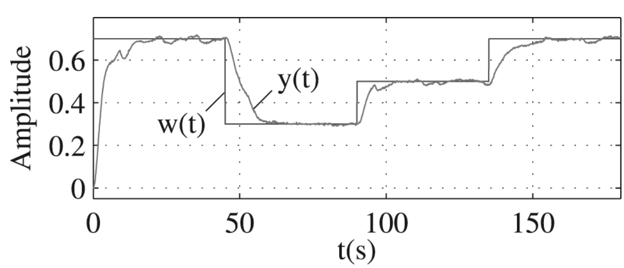

Figure 5: Measured results $w(t), y(t)$ with switched controller (25)

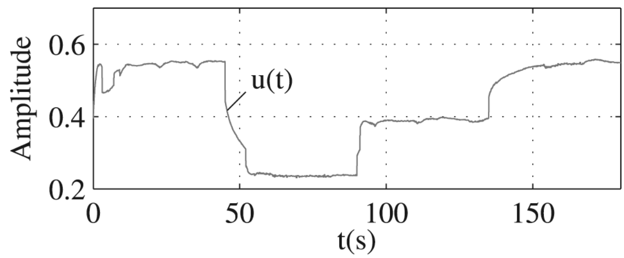

Figure 6: Switched Controller output (25)

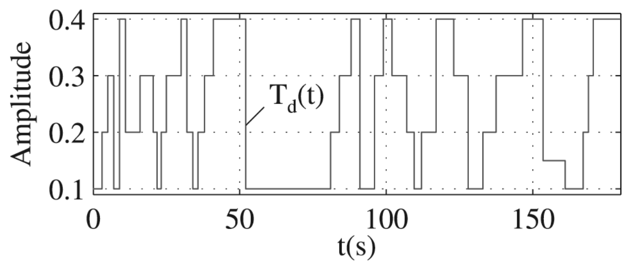

Figure 7: Time delay changes

5. Conclusion

The paper addresses the problem of the robust switched controller design which ensures the closed-loop stability and guaranteed cost for a prescribed rate of change of system switching. A novel gain-scheduling plant model is presented for linear continuous-time invariant switched systems. The first proposed method is connected with the notion of quadratic stability with respect to switched parameter θ. In this case we assume that the rate of θ change is infinite. In some real cases the rate of change of the switching signal is finite. This assumption was used in the second approach to obtain the robust switched controller design procedure. Other advantages of the proposed methods are that for the switched controller design, there is no need to use the approach of the 'dwell time', which markedly complicates the design procedure. The rate of the switching signal change can be prescribed by the designer, which opens new possibilities for practical realizations and development of new theoretical approaches. The obtained design procedure can be implemented easily to the standard LMI or BMI approaches. Numerical examples illustrate the effectiveness of the proposed approach.

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