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Accuracy Improvement in Least-Squares Estimation with Harmonic Regressor: New Preconditioning and Correction Methods

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***Abstract*— Numerical aspects of least squares estimation have not been sufficiently studied in the literature. In particular, information matrix has a large condition number for systems with harmonic regressor in the initial steps of RLS (Recursive Least Squares) estimation. A large condition number indicates invertibility problems and necessitates the development of new algorithms with improved accuracy of estimation. Symmetric and positive definite information matrix is presented in a block diagonal form in this paper using transformation, which involves the Schur complement. Block diagonal sub-matrices have significantly smaller condition numbers and therefore can be easily inverted, forming a preconditioner for a large scale system. High order algorithms with controllable accuracy are used for solving least squares estimation problem. The second part of the paper is devoted to the performance improvement in classical RLS algorithm, which represents a feedforward estimation procedure with error accumulation. Two correction feedback terms originated from combined high order algorithms are introduced for performance improvement in classical RLS algorithms. Simulation results show significant performance improvement of modified algorithm compared to classical RLS algorithm in the presence of roundoff errors.**

Keywords: Recursive Least-Squares Estimation with High Accuracy, Persistence of Excitation & Positive Definite Matrices

I. Introduction

RLS (Recursive Least Squares) algorithms are widely used in many applications such as adaptive control, signal processing, system identification and many others [1], [2]. Round-off and truncation errors have a direct impact on the accuracy of RLS estimation. This is a main obstacle to real-time implementation of RLS algorithms and motivation to use methods from matrix analysis [3] for solving algebraic equations in order to improve the performance of estimation. This necessitates also the development of modified RLS algorithm, where the estimates are corrected using correction terms originated from solution of algebraic equations for preventing error propagation.

This paper is divided in two large parts, where the first part is devoted to the solution of algebraic equations with positive definite information matrix. Classical RLS algorithm is modified for performance improvement in the second part of the paper.

RLS algorithm is a recursive realization of the solution of the algebraic equation (1) which is defined as follows:

$$H_i \theta_i = b_i \tag{1}$$

$$H_i = \lambda_0 \Big[\sum_{j=1}^{i-1} \varphi_j \ \varphi_j^T \Big] + \varphi_i \ \varphi_i^T \tag{2}$$

$$b_i = \lambda_0 \Big[\sum_{j=1}^{i-1} \varphi_j \ y_j \Big] + \varphi_i \ y_i \tag{3}$$

where symmetric matrix H_i is called information matrix, b_i is the vector that contains measured signal y_i, and $0 < \lambda_0 < 1$ is a forgetting factor, $i = 1, 2, ...$ is the step number.

Harmonic regressor φ_i [4] contains trigonometric functions at different frequencies q_p, $p = 1, 2, ..., r$: $\varphi_i^T = [1 \ cos(q_1 i) \ sin(q_1 i) \ cos(q_2 i) \ sin(q_2 i) \ ... \ cos(q_r i) \ sin(q_r i)]$.

Equation (1) can be solved with respect to the vector of estimated parameters θ_i in each step i with pre-specified accuracy. For solving equation (1) different algorithms can be used, depending on the properties of the information matrix H_i. These properties depend, in turn on a step number i. For a certain step number the matrix becomes a full rank matrix and invertible. This matrix is a positive definite matrix in all subsequent steps since harmonic regressor is persistently exciting [1], [2], [4], [5]. For a sufficiently large i this matrix becomes an SDD (Strictly Diagonally Dominant) matrix [6]. This case is studied sufficiently in [7]. This paper is devoted to the case where the information matrix H_i is a positive definite, but not an SDD matrix. All the properties of information matrix H_i are used in this paper: symmetry, positive definiteness (persistence of excitation) and the structure (2).

Notice that the condition of persistence of excitation is utilized in this paper via application of suitable methods from matrix analysis to positive definite matrices. This is an alternative to the classical way of utilization the excitation property in RLS estimation [1], [2], [8].

Information matrix has a large condition number[1] for systems with harmonic regressor in the initial steps of estimation. A large condition number indicates possible problems with invertibility of this matrix, especially for a large number of frequencies. Therefore the development of new algorithms for solution of the equation (1) in the initial steps of estimation is required.

Information matrix is presented in a block diagonal form in this paper using transformation, which involves the Schur

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[1]Condition number is the ratio, where the largest singular value of the matrix is divided by the smallest one. The condition number is a measure of sensitivity of the matrix to numerical operations.

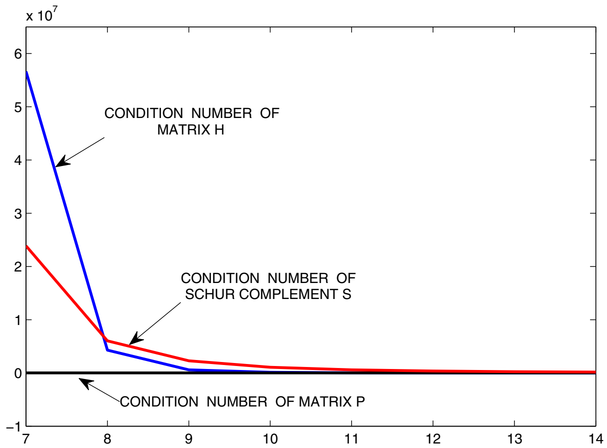


Fig. 1. Condition numbers of the matrices H, P and S are plotted with blue, black and red lines respectively for the case of three frequencies, and the size of the matrix H is 7×7, where the condition number of matrix H was divided by 10^5. The condition numbers are plotted as a function of the window size.

complement. Block diagonal sub-matrices have significantly smaller condition numbers and therefore can be easily inverted. Combined high order algorithms proposed in [7] are used for solving equation (1) as soon as a suitable preconditioner is found.

Algorithms proposed in this paper for positive definite information matrix H_i together with algorithms developed in [7] for SDD information matrix provide a complete solution of least squares estimation problem with controllable accuracy for systems with harmonic regressor.

The second part of the paper is devoted to the performance improvement in classical RLS algorithm. This algorithm is a recursive realization of (1) and represents a feedforward estimation procedure, which is initialized once. This feedforward procedure suffers from the error propagation problem. Notice that equation (1) provides information about deviations of estimated parameters from their true values. This information is not accounted in classical RLS algorithm that necessitates modification for preventing error propagation. Two correction terms originated from combined high order algorithms [7] are introduced in classical algorithm for performance improvement. The gain matrix calculated in RLS algorithms is used as preconditioner (a priori estimate) in the term that corrects estimate of the inverse of the information matrix H_i. The second correction term uses recursive estimate provided by RLS algorithm as a starting point (again as a priori estimate) and provides a high order feedback loop driven by the deviation in the equation (1). Two parameters (the orders of high order algorithm) can be used to control the accuracy of modified RLS algorithm. Simulation results show significant performance improvement of modified algorithm compared to classical RLS in the presence of roundoff errors.

II. High Order Algorithms for Inversion of Symmetric and Positive Definite Matrix

High order algorithms previously developed for inversion of SDD information matrix [7] are modified in this Section for inversion of symmetric positive definite matrices. A positive definite matrix is scaled first so that the spectral radius of the scaled matrix is less than one. High order algorithms described in [7] are applied to scaled matrix for inversion. This procedure is associated with two Lemmas presented below. The first Lemma is a simple modification of Lemma 5.1.1 in [9] (see also the initial ideas in [10]) for systems with harmonic regressor.

Lemma 1. For a given symmetric positive definite matrix H the spectral radius ρ of the matrix $I - H/\alpha$, where I is the identity matrix, is less than one, $\rho(I - H/\alpha) < 1$ provided that $\alpha = \|H\|_\infty/2 + \epsilon$, where $\|\cdot\|_\infty$ is the maximum absolute row sum norm, and ϵ is a small positive number.

Proof. All the eigenvalues of the matrix H/α are positive and less than two according to the Gershgorin circle theorem. Therefore all the eigenvalues of the matrix $I - H/\alpha$ are located in the open interval between minus one and one, and the spectral radius of this matrix is less than one.

The second Lemma represents a modification of high order algorithms [7] for symmetric positive definite matrices.

Lemma 2. The following algorithm of order $m = 2, 3, ...$

$$L_m = \sum_{d=0}^{m-2} F_{k-1}^d \tag{4}$$

$$G_k = G_{k-1} + F_{k-1} L_m G_{k-1}, \qquad G_0 = I \tag{5}$$

$$F_k = I - G_k \tilde{H}, \qquad \tilde{H} = \frac{H}{\alpha} \tag{6}$$

provides an estimate of the inverse of a positive definite and symmetric matrix H, i.e. $\lim\limits_{k\to\infty} \frac{G_k}{\alpha} \to H^{-1}$, where α is defined in Lemma 1.

Proof. The matrix L_m can be presented in the following form $L_m = (G_{k-1}\tilde{H})^{-1} - (G_{k-1}\tilde{H})^{-1} F_{k-1}^{m-1}$. Substitution of L_m in (5) yields $F_k = F_{k-1}^m$ and $F_k = F_0^{m^k}$, where $k = 1, 2, 3, ...$. According to Lemma 1 there exists $\alpha > 0$ such that $\rho(F_0) < 1$, where $F_0 = I - \tilde{H}$ and $\tilde{H} = \frac{H}{\alpha}$. Moreover, the 2-norm of F_0 is also less than one, $\|F_0\|_2 < 1$. Then the error F_k converges to zero, $\lim\limits_{k\to\infty} F_k \to 0$ and therefore the matrix $\frac{G_k}{\alpha}$ provides an estimate of H^{-1}.

III. Accuracy Improvement of the Inversion of the Positive Definite Matrix via Partitioning Method

Information matrix is presented in a block diagonal form in this Section using transformation, which involves the Schur complement, aiming for reduction of large condition number in the initial steps of estimation. Block diagonal sub-matrices have significantly smaller condition numbers and therefore can be easily inverted. Symmetric and positive definite matrix

H can be partitioned as follows [3]:

$$H = \begin{bmatrix} P & B \\ B^T & C \end{bmatrix}$$

where P and C are square. This matrix can be transformed to block-diagonal form using the following transformation matrix

$$T = \begin{bmatrix} I & 0 \\ X^T & I \end{bmatrix}$$

where $X = -P^{-1}B$ and I is identity matrix, and

$$H_T = T\ H\ T^T = \begin{bmatrix} P & 0 \\ 0 & S \end{bmatrix}$$

where $S = C - B^T P^{-1} B > 0$ is the Schur complement of $P > 0$. Approximate inverse matrix of the positive definite and symmetric matrix H can be used as a preconditioner. Minimal eigenvalue of the matrix $\frac{H}{\alpha}$, where α is chosen according to Lemma 1 is too close to zero in the initial steps of estimation. Therefore the spectral radius of $I - \frac{H}{\alpha}$ is too close to one, that has direct impact on the convergence rate. Block diagonal decomposition shown above can be used for the convergence rate and accuracy improvement. In other words the condition number of the matrix H is large for a small window size, which means that the matrix is almost non-invertible. However, this matrix can be inverted with a very low accuracy. Transformation of this matrix to two matrices of the reduced sizes results also in the reduction of the condition numbers of these matrices [11]. Condition numbers of the matrices H, P and S are plotted in Figure 1 for the case of three frequencies, where the condition number of matrix H was divided by 10^5. The Figure shows a significant reduction of the condition number due to the block diagonal decomposition. This implies significant improvement in the accuracy and convergence rate of the matrix inversion algorithm. Approximate inverse $\hat{H}^{-1}$ as a preconditioner G_0 is calculated as follows:

$$G_0 = \hat{H}^{-1} = \begin{bmatrix} I & \hat{X} \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{P}^{-1} & 0 \\ 0 & \hat{S}^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ \hat{X}^T & I \end{bmatrix}$$

where $\hat{X} = -\hat{P}^{-1}B$, $\hat{S} = C - B^T \hat{P}^{-1} B$, and $\hat{P}^{-1}$ is an estimate of the inverse of the matrix $P = P^T > 0$.

A. Two Stage Preconditioning

Preconditioner G_0 is calculated in two stages via sequential application of the matrix inversion algorithm described in Lemma 2. Estimate $\hat{P}^{-1}$ of the inverse of matrix P is calculated in the first stage with a reasonable accuracy. Estimate of the inverse of the Schur complement S and the preconditioning matrix G_0 are calculated in the second stage and the norm $\|I - G_0 H\|_2$ is used as an output variable.

Efficiency of the proposed method is illustrated in Figure 2, where two error norms $\|I - G_0 H\|_2$ for different G_0 as functions of the order of the inversion algorithm and the step number are compared. The norm plotted in the right subplot shows direct application of the matrix inversion algorithm described in Lemma 2 to inversion of matrix H. The left subplot shows the case where matrix G_0 is calculated using two stage preconditioning method with the same order and the number of steps for calculation of $\hat{P}^{-1}$ and $\hat{S}^{-1}$.

Two stage preconditioning method provides better performance with approximately the same computational burden. This method can be extended to multilevel preconditioning method, where the matrices P and S are decomposed further using the same technique [9],[12],[13]. However, accuracy of multilevel method is limited by the inversion accuracy at the initial level.

Two stage preconditioning method proposed above can also be applied for the case where matrices H, P and S are SDD matrices [14]. The performance of the method is illustrated in Figure 3 for SDD matrices. Two error norms $\|I - G_0 H\|_\infty$ for different G_0 as functions of the order of the inversion algorithm and the step number are compared. The matrix G_0 is calculated in the same ways as for positive definite matrices. Diagonal matrices (with inverses of the diagonal elements of each SDD matrix on diagonals) are used as preconditioners instead of the parameter α as it is described in [7]. Figure 3 shows that the difference in performance for the case of partitioning and straightforward calculation of the inverse of SDD matrix H is not significant. Comparison of Figure 2 and Figure 3 shows that strict diagonal dominance of information matrix is stronger property than the positive definiteness. The diagonal matrix with inverted diagonal elements of information matrix on the diagonal is better preconditioning than the method described in Lemma 1 for positive definite matrices.

IV. High Order Algorithms for Calculation of the Parameter Vector

Lemma 3. The following combined algorithm of orders $m = 2, 3, ...$ and $n = 1, 2, ...$

$$F_{k-1} = I - G_{k-1} H \tag{7}$$

$$L_m = \sum_{d=0}^{m-2} F_{k-1}^d \tag{8}$$

$$G_k = G_{k-1} + F_{k-1} L_m G_{k-1} \tag{9}$$

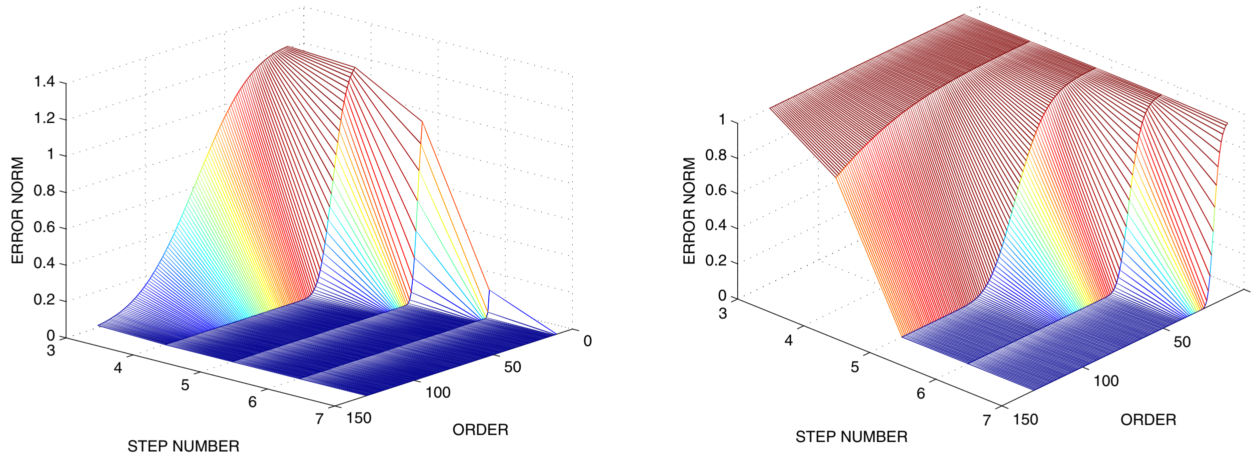
$$F_k = I - G_k H \tag{10}$$

$$\Gamma_n = \sum_{d=0}^{n-1} F_k^d \tag{11}$$

$$\vartheta_k = \vartheta_{k-1} - \Gamma_n\ G_k \left\{ H \vartheta_{k-1} - b \right\} \tag{12}$$

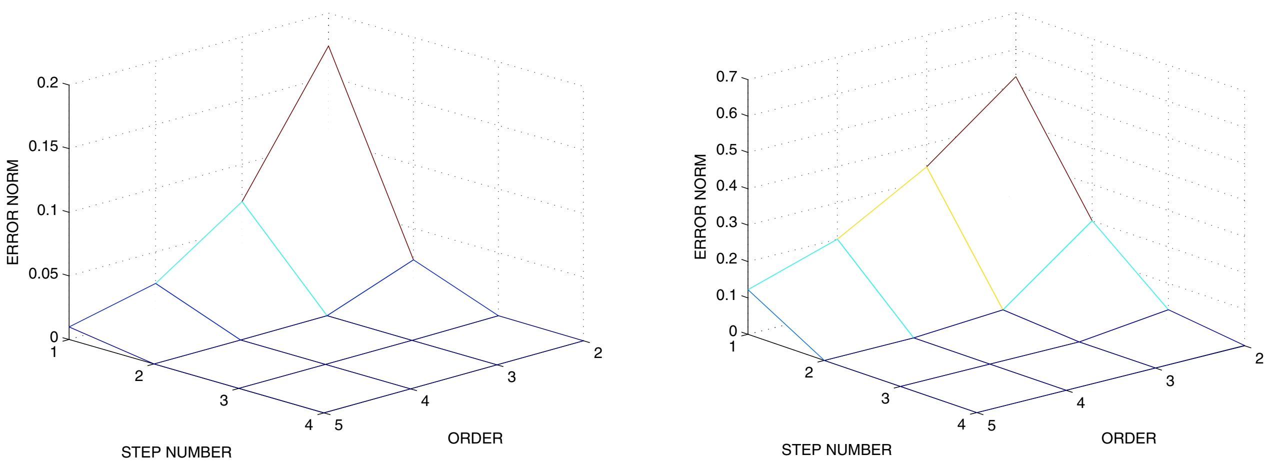
provides an estimate ϑ_k of the parameter vector θ_i such that $\lim_{k \to \infty} \vartheta_k \to \theta_i$, if $\|I - G_0 H\| < 1$, and G_0 is chosen in Section III-A. The index i is dropped in H and b for simplicity. The proof of this Lemma is presented in [7].

Remark. The order n should be chosen higher than the order m. The matrix Γ_n in (11) can be partitioned in this case as follows: $\Gamma_n = \sum_{d=0}^{n-1} F_k^d = Q_k + \sum_{d=m-1}^{n-1} F_k^d$, where $Q_k = \sum_{d=0}^{m-2} F_k^d$ is the part of the matrix Γ_n associated with the gain matrix L_m defined in (8), and $k = 1, 2, ...$. The



(a) (b)

Fig. 2. The 2-norm $\|I - G_0 H\|_2$ where matrix G_0 is calculated using two stage preconditioning method with the same order and the number of steps for calculation of $\hat{P}^{-1}$ and $\hat{S}^{-1}$ is plotted in Subplot (a).
The norm plotted in the Subplot (b) shows direct application of the matrix inversion algorithm described in Lemma 2 to inversion of matrix H.



(a) (b)

Fig. 3. The norm $\|I - G_0 H\|_\infty$ where matrix G_0 is calculated using two stage preconditioning method with the same order and the number of steps for calculation of $\hat{P}^{-1}$ and $\hat{S}^{-1}$ is plotted in Subplot (a). The matrices P and S are SDD matrices.
The norm plotted in the Subplot (b) shows direct application of the matrix inversion algorithm described in [7] to inversion of SDD matrix H.

gain matrix L_m is equal to Q_{k-1}, $L_m = Q_{k-1} = \sum_{d=0}^{m-2} F_{k-1}^d$ and $Q_0 = \sum_{d=0}^{m-2} F_0^d$. The computational burden is reduced in this case since the common part is calculated once for both matrices Γ_n and L_m.

The performance of the algorithm (7) - (12) is illustrated in Figure 4, where the error norm $\|\vartheta_k - \theta_i\|$ is plotted as a function of order n and step number $k = 1, 2, \ldots$ for fixed order m. The matrix H is the matrix with large condition number and the preconditioning matrix G_0 is chosen so that $\|I - G_0 H\|_2$ is sufficiently close to one.

V. Modification of Classical RLS Algorithm: Preventing Error Propagation

Preconditioning methods described above use symmetry and positive definiteness of the information matrix only. Structural property (2) can also be used for calculating preconditioner using matrix inversion relation. This idea was implemented in the RLS estimation (see [1], [2] and references therein). RLS solution of (1) with $R_i = H_i^{-1}$ can be written as follows:

$$\hat{\theta}_i = \hat{\theta}_{i-1} + \frac{R_{i-1}\varphi_i}{\lambda_0 + \varphi_i^T R_{i-1}\varphi_i}(y_i - \hat{\theta}_{i-1}^T \varphi_i) \quad (13)$$

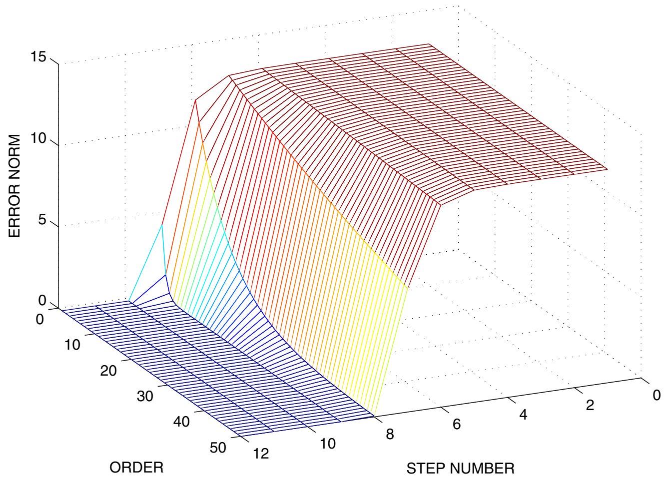


Fig. 4. The error norm $\|\vartheta_k - \theta_i\|$ is plotted as a function of order n and step number $k = 1, 2, \ldots$ for fixed order $m = 10$. The matrix H is the matrix with a large condition number and the preconditioning matrix G_0 is chosen so that $\|I - G_0 H\|_2$ is sufficiently close to one.

$$R_i = \frac{1}{\lambda_0}\left[R_{i-1} - \frac{R_{i-1}\varphi_i\varphi_i^T R_{i-1}}{(\lambda_0 + \varphi_i^T R_{i-1}\varphi_i)}\right] \quad (14)$$

where $\hat{\theta}_i$ is an estimate of the vector of true parameters θ_i, $i = 1, 2, \ldots$. The gain matrix is initialized to the inverse matrix $\hat{H}_0^{-1}$, $R_0 = \hat{H}_0^{-1}$, where $i = 0$ is referred to a step, where information matrix gets a full rank. The inverse matrix $\hat{H}_0^{-1}$ is calculated using matrix inversion algorithm described in Lemma 1 with preconditioning technique proposed in Section III, and $\hat{\theta}_0 = R_0 b_0$. The matrix R_0 should be calculated with a very high accuracy since accuracy deteriorates in the subsequent steps. Direct application of RLS algorithms (13) - (14) has the following obstacles: (1) slow convergence of matrix inversion algorithm for calculation of $\hat{H}_0^{-1}$ due to high condition number and (2) slow convergence of estimated parameters and accuracy deterioration in the presence of roundoff errors.

A. RLS and High Order Algorithms: Win to Win Combination

Algorithms (13) - (14) and (7) - (12) can be combined aiming to improve convergence rate and robustness with respect to roundoff errors. Estimate of the parameter vector $\hat{\theta}_i$ can be used as initial condition for (12) and the matrix R_i can be used as a preconditioner. Combined algorithms can be written in the following form:

$$\delta_{i-1} = \underbrace{\hat{\theta}_{i-1} + \frac{R_{i-1}\varphi_i}{\lambda_0 + \varphi_i^T R_{i-1}\varphi_i}(y_i - \hat{\theta}_{i-1}^T\varphi_i)}_{RLS\ algorithm} \quad (15)$$

$$G_{i-1} = \underbrace{\frac{1}{\lambda_0}\left[R_{i-1} - \frac{R_{i-1}\varphi_i\varphi_i^T R_{i-1}}{(\lambda_0 + \varphi_i^T R_{i-1}\varphi_i)}\right]}_{RLS\ algorithm} \quad (16)$$

$$F_{i-1} = \underbrace{I - G_{i-1}H_i}_{initial\ inversion\ error} \quad (17)$$

$$L_m = \sum_{d=0}^{m-2} F_{i-1}^d \quad (18)$$

$$G_i = G_{i-1} + F_{i-1}L_m G_{i-1} \quad (19)$$

$$F_i = \underbrace{I - G_i H_i}_{final\ inversion\ error} \quad (20)$$

$$\Gamma_n = \sum_{d=0}^{n-1} F_i^d \quad (21)$$

$$\delta_i = \delta_{i-1} - \Gamma_n\, G_i\, \underbrace{\{H_i\delta_{i-1} - b_i\}}_{estimation\ error} \quad (22)$$

$$\hat{\theta}_i = \delta_i \quad (23)$$

$$R_i = G_i \quad (24)$$

Algorithm is initialized in the same way as the algorithm (13) - (14).

Equations (15) and (16) represent classical RLS algorithm with such intermediate variables as the matrix G_{i-1} and the vector δ_{i-1}. Equations (17) - (22) represent the correction term (for further processing of intermediate variables) originated from combined high order algorithm (7) - (12) with one step. Algorithm (15) - (24) has two outputs: the vector of estimated parameters $\hat{\theta}_i$ and improved estimate G_i of the inverse of matrix H_i. Two feedback loops driven by the inversion error $I - G_i H_i$ and parameter estimation error $H_i\delta_{i-1} - b_i$ were incorporated into the classical RLS algorithm for stopping errors from propagating to the next step. The orders n and m, which control the accuracy may vary with step number, providing different estimation performance. Accuracy of the matrix inversion algorithm can be estimated using the error model $F_i = F_{i-1}^m$, where $\|F_{i-1}\| < 1$ and $\|F_i\| << \|F_{i-1}\|$ for a sufficiently high order m. Accuracy of the parameter estimation can be evaluated using the error model $\tilde{\delta}_i = F_i^n\tilde{\delta}_{i-1}$, where $\tilde{\delta}_i = \delta_i - \theta_i$, $\tilde{\delta}_{i-1} = \delta_{i-1} - \theta_i$, and $\theta_i = H_i^{-1}b_i$. Output variable δ_i provides better estimate of θ_i than intermediate variable δ_{i-1}, which is calculated using classical RLS in (15) i.e., $\|\tilde{\delta}_i\| << \|\tilde{\delta}_{i-1}\|$ for a sufficiently high order n. Estimation performance provided initially by classical RLS algorithm is improved by high order algorithm, which makes RLS algorithm more robust.

Performance of the algorithm (15) - (24) is illustrated in Figure 5 for the system with three frequencies in the presence of roundoff errors, where all the variables were rounded to two digits. The first subplot shows three estimated parameters of classical RLS algorithm (13) - (14), plotted with dashdot lines and modified algorithm (15) - (24) plotted with solid lines of the same colors. Actual parameters are plotted with dotted lines. The norm $\|I - R_i H_i\|_\infty$ is plotted with a solid black line for classical RLS algorithm (13) - (14) in the second subplot of Figure 5. Inversion errors were introduced in initialization of the algorithms. Notice that this norm exceeds one very quickly due to roundoff errors in equation (14), which represents evolution of the gain matrix. Roundoff errors have impact on this matrix in the first hand. Nevertheless the parameters converge slowly to their true values in the classical RLS algorithm due to the fact that the algorithm is stable, if the matrix R_i^{-1} is positive definite (see RLS stability Lemma in [8]). The properties of RLS algorithm are similar to the properties of Kaczmarz projection algorithm in this case. Convergence rate improvement of modified algorithms (15) - (24) is significant as it is shown in the first subplot of Figure 5. The correction terms (17) - (22) make the algorithm closer to classical RLS algorithm, where the parameters converge in one step when information matrix becomes a full rank matrix.

The norm $\|I - G_i H_i\|_\infty$ is plotted with dashed green line in the second subplot, and shows performance improvement due to the correction term (17) - (20), which prevents propagation of the inversion error in the presence of significant roundoff errors since $\|F_i\|_\infty << \|F_{i-1}\|_\infty$.

The norm $\|I - G_i H_i\|_\infty$ which is plotted with dashed green line in the second subplot can be compared to the norm $\|I - D_i^{-1} H_i\|_\infty$ plotted with dashdot blue line, where the diagonal matrix D_i contains diagonal elements of matrix H_i aiming to find the best preconditioner for the case where H_i is an SDD matrix. The matrix H_i becomes an SDD matrix, when blue line crosses red line. The matrix G_i is better preconditioner in the initial steps of estimation, and D_i (which can easily be calculated) can be used as a preconditioner when the information matrix becomes an SDD matrix for reduction of the computational burden. Notice that reduction of forgetting factor λ_0 makes the norm $\|I - G_i H_i\|_\infty$ significantly less than the norm $\|I - D_i^{-1} H_i\|_\infty$.

Finally, the number of steps in the high order part (17) - (22) of the algorithm (15) - (24) can be easily increased to further improve accuracy of estimation.

VI. Conclusion

This paper shows that the accuracy of RLS estimation can be improved using methods from matrix analysis [3]. Positive definiteness of information matrix associated with persistently exciting harmonic regressor [1], [2], [4], [5] is used in this paper for design of new two stage preconditioning method. Moreover, classical RLS algorithms [1], [2] are modified for prevention of error propagation via introduction of feedback terms originated from combined high order algorithms [7].

The results are especially relevant for processing of periodic sequences with non-stationary parameters estimated in moving windows of small sizes. The results are also applicable for other types of regressors.

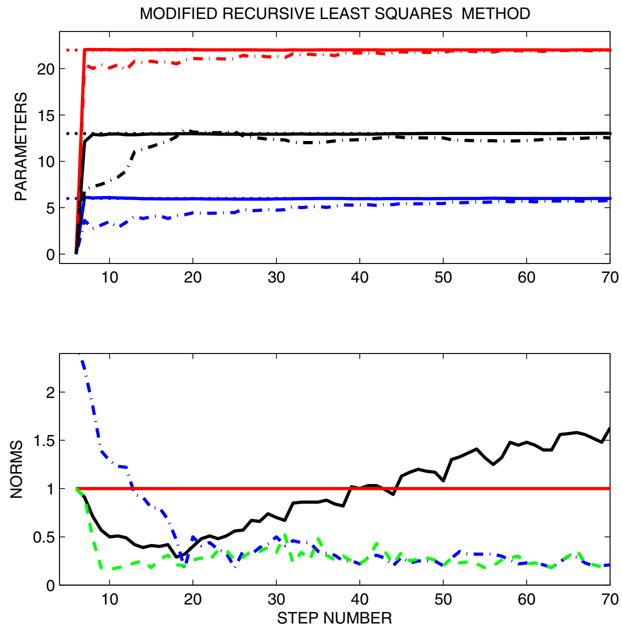


Fig. 5. Estimation of the frequency content of the signal with three frequencies with rounding of all variables to two-digit accuracy. The first subplot shows three estimated parameters of classical RLS algorithm (13) - (14), plotted with dashdot lines and modified algorithm (15) - (24) with $m = 2$ and $n = 3$ plotted with solid lines of the same colors. Actual parameters are plotted with dotted lines. The norm $\|I - R_i H_i\|_\infty$ is plotted with solid black line for classical RLS algorithm (13) - (14) in the second subplot. The norm $\|I - G_i H_i\|_\infty$ is plotted with dashed green line in the second subplot for algorithm (15) - (24) and shows performance improvement due to the correction term (17) - (20). Finally, the norm $\|I - D_i^{-1} H_i\|_\infty$ plotted with dashdot blue line where the matrix D_i contains diagonal elements of matrix H_i.

References

[1] Ljung L., System Identification: Theory for the User, Prentice-Hall, Upper Saddle River, NJ, 1999.
[2] Fomin, V. N., Fradkov A.L. and Yakubovich V.A., Adaptive Control of the Dynamic Plants, Nauka, Moscow, 1981, (in Russian).
[3] Horn R. and Johnson C., Matrix Analysis, Cambridge University Press, 1985.
[4] Bayard D., A General Theory of Linear Time-Invariant Adaptive Feedforward Systems with Harmonic Regressors, IEEE Trans. Autom. Control, vol. 45, N 11, 2000, pp. 1983-1996.
[5] Gevers M. et al., Identification and the Information Matrix: How to Get Just Sufficiently Rich ?, IEEE Trans. Autom. Control, vol. 54, N 12, 2009, pp. 2828-2840.
[6] Stotsky A., Recursive Trigonometric Interpolation Algorithms, Proc. IMechE Part I: Journal of Systems and Control Engineering, vol. 224, N 1, 2010, pp. 65-77.
[7] Stotsky A., High Order Algorithms in Robust Least-Squares Estimation with SDD Information Matrix: Redesign, Simplification and Unification, Proc. of 53-rd CDC, Dec. 15-17, 2014, USA, pp. 271-276.
[8] Stotsky A., Harmonic Regressor: Robust Solution to Least-Squares Problem, Proc. IMechE Part I: Journal of Sytems and Control Engineering, vol. 227, N 8, 2013, pp. 662-668.
[9] Chen K., Matrix Preconditioning Techniques and Applications, Cambridge University Press, Cambridge, UK, 2005.
[10] Fadeev D. and Fadeeva V., Computational Methods of Linear Algebra, Freeman, San Francisco, 1963.
[11] Mandel J., On Block Diagonal and Schur Complement Preconditioning, Numer. Math. vol. 58, 1990, pp. 79-93.
[12] Zhang J., On Preconditiong Schur Complement and Schur Complement Preconditiong, Electronic Transactions on Numerical Analysis, vol. 10, 2000, pp. 115-130.
[13] Benzi M., Preconditioning Techniques for Large Linear Systems: A Survey, Journal of Computational Physics vol. 182, 2002, pp. 418-477.
[14] Stotsky A., Blade Root Moment Sensor Failure Detection Based on Multibeam LIDAR for Fault-Tolerant Individual Pitch Control of Wind Turbines, Energy Science & Engineering, vol. 2, N 3, 2014, pp.107-115.