Hybrid Cooperative Positioning in Harsh Environments

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***Abstract*—Hybrid cooperative positioning involves the combination of satellite pseudoranges with measurements based on terrestrial radio signals, with the aim to improve both coverage and accuracy. This paper presents a theoretical analysis of hybrid cooperative positioning in low signal-to-noise ratio environments, explicitly accounting for scenarios where receivers can receive signals but cannot decode navigation messages. We propose an extended pseudorange model, suitable for evaluation of measurements made by the receiver in these scenarios, as well as extended cooperation across agents attempting to enable satellite navigation functionality of receivers operating below signal-to-noise ratio threshold required for proper message decoding. The advantages of such extended cooperation are illustrated on a realistic scenario by means of the Cramér-Rao lower bound.**

I. Introduction

Hybrid cooperative positioning networks [1] consist of communicating agents, which are equipped with two positioning technologies: a receiver for global navigation satellite systems (GNSS) and a terrestrial wireless ranging system, such as ultra-wide bandwidth (UWB) communication, capable of measuring distances with respect to neighboring agents (peers). Fusion of information from these two loosely coupled technologies has been shown to improve localization accuracy and availability [2], in particular for harsh, GNSS-challenged environments. To model GNSS availability, [2] relied on a model whereby an agent could either receive and decode a given satellite's signal, or receive no signal at all.

In practice, reception of GNSS signals is more complex, since they are affected by multipath and attenuation. In harsh environments, the signals are typically received with low signal-to-noise ratio (SNR), rendering decoding of the navigation messages impossible, but may still allow for proper tracking of ranging pseudorandom codes [3]. In such scenarios, the GNSS receiver loses the capability of estimating its position due to the lack of timing information, ephemeris, and other important parameters included in the navigation message. This information can be provided through an assisted global positioning system (AGPS), over a separate communication channel [4], and is relatively robust to timing errors [5].

In this paper, we consider the problem of hybrid cooperative positioning, with loose coupling between GNSS and UWB, for scenarios where AGPS is not available. We analyze how cooperation can be harnessed when some agents cannot decode navigation messages. The specific contributions of our work are:

- we propose modification of the conventional pseudorange model to properly address GNSS receivers operating in adverse SNR conditions;
- we propose an extended cooperative mode, allowing agents to exploit satellite positioning if decoding of navigation messages fails due to low SNR; and
- we provide fundamental insight about the benefits of extended cooperation through a Fisher information matrix (FIM) analysis.

Simulation results based on experimental data confirm the potential benefits of our proposed approach.

II. System model

Let $\mathcal{M}$ be a set of M agents located at positions $\mathbf{p}_i \in \mathbb{R}^D$ and let $\mathcal{S}$ be a set of S satellites with positions $\mathbf{s}_k \in \mathbb{R}^D$, where D represents the dimension of the position vector (typically $D = 2$ or $D = 3$). Each agent i can communicate with peer agents from a subset $\mathcal{M}_i \subseteq \mathcal{M}$, perform ranging with respect to them, and receive navigation signals from GNSS satellites grouped in a subset $\mathcal{S}_i \subseteq \mathcal{S}$. To reflect realistic[1] fusion between the GNSS and terrestrial wireless ranging system, we consider them to be loosely coupled. Each agent collects two types of positioning measurements [2]. First of all, agent i obtains *pseudorange measurements* ρ_{ik} with respect to satellite $k \in \mathcal{S}_i$, which is conventionally expressed as

$$\rho_{ik} = \|\mathbf{p}_i - \mathbf{s}_k\| + b_i + w_{ik}, \tag{1}$$

where b_i is a bias term that describes the difference between the receiver and system clocks, expressed in meters. The noise term w_{ik} is modeled as a zero mean Gaussian, $w_{ik} \sim \mathcal{N}(0, \sigma_{ik}^2)$. Secondly, agent i obtains *range measurements* r_{ij} with respect to agent $j \in \mathcal{M}_i$, modeled as

$$r_{ij} = d_{ij} + n_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\| + n_{ij}, \tag{2}$$

where d_{ij} is distance between agents, and $n_{ij} \sim \mathcal{N}(0, \tilde{\sigma}_{ij}^2)$ represents[2] measurement noise, with $\tilde{\sigma}_{ij}^2 \ll \sigma_{ik}^2, \forall(j,k)$ [8].

Different signals received by agents from navigation satellites exhibit different quality. For the purpose of the analysis,

[1]Currently available GNSS mass-market integrated circuits do not provide an interface required to implement tightly coupled systems.

[2]It is worth noting that both the range and pseudorange error statistics generally depend on environment (for example outdoor, indoor, or urban canyon), resulting in non-Gaussian errors [6], [7]. Nevertheless, for analytical tractability we will assume Gaussian errors.

the quality of signals is quantified using the carrier-to-noise density ratio (C/N_0, expressed in dB-Hz), defined as the useful signal power C over the single-sided power spectral density of thermal noise N_0.

Our goal is to evaluate and compare the positioning accuracies of agents in two cooperative modes. In the *classical cooperative mode*, agents cooperate by exchanging information about their positions and by performing ranging measurements. In the *extended cooperative mode*, agents additionally activate satellite navigation functionality of their peers who are unable of decoding the navigation messages themselves due to low SNR (i.e., low C/N_0).

III. Pseudorange availability in low SNR

In this section, we will analyze the model (1) in some depth, shedding light on different impairments that were ignored in prior works [2], [9]. Based on our analysis, we describe our extended model in Section III-C.

A. Pseudorange measurement

To estimate its location, a conventional GNSS receiver solves a set of non-linear equations based on the pseudorange measurements (1). These measurements are obtained as follows. In accordance with [10], the uncorrected pseudorange measured by agent i towards satellite k is

$$\rho'_{ik}(t) = c\left\{t_i^r(t) - t_k^s(t-\tau_{ik})\right\} + \epsilon_{ik}(t) + w_{ik}(t), \qquad (3)$$

where c is the speed of the light; τ_{ik} represents the propagation delay of the signal traveling between the satellite and agent; t is system time, while $t_i^r(t)$ and $t_k^s(t-\tau_{ik})$ represent GNSS receiver time at the moment of signal reception and satellite time at the instant of transmission (also called transmit time), respectively; $\epsilon_{ik}(t)$ models additional error effects, such as time delays caused by signal propagation through the ionosphere and troposphere as well as relativistic effects; and $w_{ik}(t)$ accounts for a measurement error given mainly by thermal noise and multipath.

Relationships between receiver clock and system time as well as satellite clock and system time, respectively, are

$$t_i^r(t) = t + \delta_i^r(t), \quad t_k^s(t-\tau_{ik}) = t - \tau_{ik} + \delta_k^s(t-\tau_{ik}), \quad (4)$$

where $\delta_i^r(t)$ and $\delta_k^s(t-\tau_{ik})$ are the receiver and satellite clock biases, respectively. Substituting (4) into (3), and compensating for $\epsilon_{ik}(t)$ and $\delta_k^s(t-\tau_{ik})$, which are assumed to be known, leads to the corrected pseudorange model (1), equivalently expressed as $\rho_{ik}(t) = c\tau_{ik} + c\delta_i^r(t) + w_{ik}(t)$.

It is clear that the receiver needs to determine $t_k^s(t-\tau_{ik})$ in order to compute the pseudorange. How this is achieved will be detailed in the next section.

B. Determining the transmit time

1) Satellite navigation messages and codes: The satellites continuously transmit periodic pseudorandom codes that are modulated by navigation messages [11]. Fig. 1 illustrates the beginning of the code periods (of duration T_P seconds) by narrow vertical lines, seen by a receiver with respect to two satellites. A code period is marked periodically by a time tag[3], expressed in the system time and transmitted by all satellites simultaneously, say at a time t_TX. Tags, as seen by the receiver, are depicted by bold vertical lines in Fig. 1.

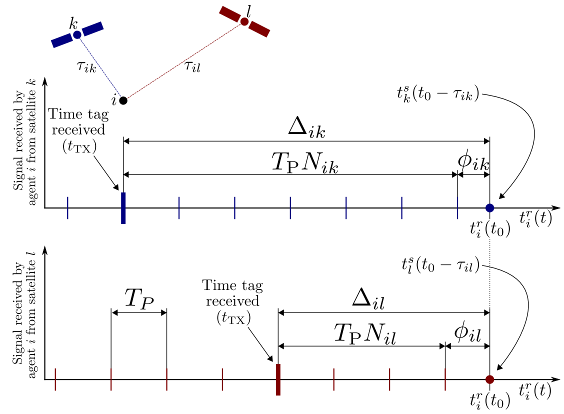


Fig. 1: An example of forming pseudoranges by a GNSS receiver. The beginnings of received code periods and time tags are depicted by narrow and bold vertical lines, respectively. Typically, the receiver performs estimation of all pseudoranges at the same measurement time instant $t_i^r(t_0)$. The transmit times $t_k^s(t_0-\tau_{ik})$, corresponding to this instant, are calculated by (5). For example, in case of GPS L1 C/A signal, $T_\mathrm{P} = 1\,\mathrm{ms}$ and the time tags are transmitted every 6 s.

GNSS receivers usually estimate the pseudoranges from all satellites at the same time instant, say $t_i^r(t_0)$. The receiver thus needs to determine the satellite time $t_k^s(t_0-\tau_{ik})$ when the signal corresponding to $t_i^r(t_0)$ was actually transmitted, for all visible satellites k. From Fig. 1, we see that

$$t_k^s(t_0-\tau_{ik}) = t_\mathrm{TX} + \Delta_{ik} = t_\mathrm{TX} + T_\mathrm{P}N_{ik} + \phi_{ik}, \qquad (5)$$

where N_{ik} is the integer number of full code periods from the moment the time tag was received, up to the measurement time $t_i^r(t_0)$, and ϕ_{ik} is the remaining fractional delay up $t_i^r(t_0)$. The receiver has now access to $t_i^r(t_0)$ and $t_k^s(t_0-\tau_{ik})$, $\forall k$, which it can substitute in (3), if it can determine t_TX, N_{ik}, and ϕ_{ik}.

2) Acquisition of N_{ik} and ϕ_{ik}: To estimate the fractional delays ϕ_{ik} from a weak received signal, a GNSS receiver must accumulate energy of ranging signals over a long observation interval[4] [3]. In contrast, to determine the number of periods N_{ik}, the receiver needs to *decode the navigation message* and read the time tag. However, decoding the message might be unfeasible at low SNR[5], even when ϕ_{ik} can be determined. Hence, there is a gap between C/N_0 values required to obtain

[3]For example, in GPS and Galileo systems, the time tag is known as Z-count and Time Of Week, respectively.

[4]To provide some tracking limits in terms of C/N_0, GNSS receivers exploiting so called vector tracking loops architecture are reported to operate down to 16 dB-Hz even under significant dynamic conditions [12]. In case of high sensitivity receivers, it was practically shown in [13], that it is possible to track GPS L1 C/A signals as low as 10 dB-Hz under low dynamic conditions.

[5]To illustrate this on an example, the GPS L1 C/A signal is considered. It utilizes binary phase shift keying (BPKS) modulation with bit duration equal to $T_b = 20\,\mathrm{ms}$. According to [11], a quasi error-free data decoding performance is achieved when the bit error rate is at level of 10^{-5}. To provide this performance, the uncoded BPSK requires $E_b/N_0 = 10\,\mathrm{dB}$. Hence, the C/N_0 threshold, denoted as Γ_msg, is expressed as $\Gamma_\mathrm{msg} = E_b/N_0 - 10\log_{10}(T_b) \approx 27\,\mathrm{dB\text{-}Hz}$. Below it, the navigation messages cannot be decoded properly.

ϕ_{ik} and those needed to obtain N_{ik}. This is not problematic for a receiver operating under nominal outdoor conditions, which are typically $C/N_0 \geq 44\,\text{dB-Hz}$. However, in indoors scenarios, C/N_0 can drop down from $35\,\text{dB-Hz}$ near windows to 10–25 dB-Hz deeper indoors [3].

To capture these different conditions, we introduce Γ_{msg} as the minimal C/N_0 needed to decode the navigation message, and $\Gamma_{\text{floor}} < \Gamma_{\text{msg}}$ be a C/N_0 threshold below which the corresponding satellites are not considered for calculation of positioning solution due to significant noise degradation.

C. Extended pseudorange model

The model in (1) does not reflect the fact that pseudorange measurements might be unavailable or only partially available in low SNR conditions. The extension of the model we propose is as follows. When $(C/N_0)_{ik} < \Gamma_{\text{floor}}$, ρ_{ik} is not available. When $(C/N_0)_{ik} \geq \Gamma_{\text{msg}}$, ρ_{ik} is available in the form (1), since both N_{ik} and ϕ_{ik} can be determined. When $(C/N_0)_{ik} \in [\Gamma_{\text{floor}}, \Gamma_{\text{msg}})$, we have

$$\rho_{ik} = \|\mathbf{p}_i - \mathbf{s}_k\| + b_i + w_{ik} + c\, m_{ik} T_{\text{P}}, \tag{6}$$

where $m_{ik} \in \mathbb{Z}$ reflects the absence of knowledge regarding N_{ik}. This extended measurement model leads to new modes of cooperation.

IV. Cooperation: Principle and Analysis

In this section, we describe a new way of cooperation among agents and provide a Fisher information analysis.

A. Classification of agents

The set of agents $\mathcal{M}$ is partitioned as follows: $\mathcal{M}^{\text{floor}}$ is the set of agents that receive up to D satellites with C/N_0 above Γ_{floor}. These agents cannot estimate their positions using GNSS at all. Second, $\mathcal{M}^{\text{frac}}$ is the set of agents receiving at least $D+1$ satellites above Γ_{floor}, but D or fewer above Γ_{msg}. Thus, agents in $\mathcal{M}^{\text{frac}}$ are capable of measuring fractional delays ϕ_{ik}, but not the integer number of code periods N_{ik} for satellites below Γ_{msg}. Finally, $\mathcal{M}^{\text{full}}$ is the set of agents receiving at least $D+1$ satellites above Γ_{msg}. Hence, these agents are able to determine their positions based solely on GNSS. Clearly, $\mathcal{M}^{\text{floor}} \cup \mathcal{M}^{\text{frac}} \cup \mathcal{M}^{\text{full}} = \mathcal{M}$.

Recall that due to the assumption of loose coupling between GNSS receiver and wireless ranging system, pseudoranges and ranges cannot be directly combined to estimate the position of agent.

B. Modes of cooperation

We consider two modes of cooperation. In the *classical cooperative mode*, agents exchange information about their positions and perform ranging measurements with respect to their peers. Only agents in $\mathcal{M}^{\text{full}}$ are able to compute their position to start the cooperation process, while agents in $\mathcal{M}^{\text{floor}} \cup \mathcal{M}^{\text{frac}}$ are unable to exploit GNSS signals at all. In the *extended cooperative mode*, agents perform classical cooperation, but in addition agents in $\mathcal{M}^{\text{frac}}$ can utilize information regarding N_{ik} from neighboring agents in $\mathcal{M}^{\text{full}}$ in order to harness GNSS signals.

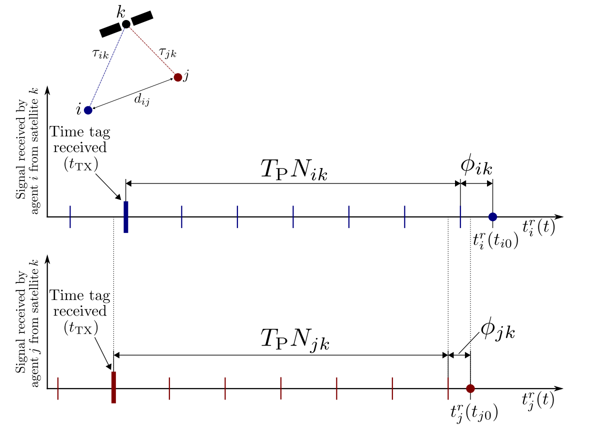


Fig. 2: An illustration of timely synchronized pseudorange measurements. If both agents make measurements within the interval of code period, then $N_{ik} = N_{jk}$ or $|N_{ik} - N_{jk}| = 1$.

The extended cooperative mode involves an agent $i \in \mathcal{M}^{\text{full}}$ helping another agent $j \in \mathcal{M}^{\text{frac}}$ to determine N_{jk} for $k \in \mathcal{S}_i \cap \mathcal{S}_j$. Under the condition that $d_{ij} \ll c T_{\text{P}}$ (i.e., agents are within 300 km of each other) and $|t_i^r(t_{i0}) - t_i^r(t_{j0})| < T_{\text{P}}$ (i.e., agents pre-agree to determine pseudoranges roughly at the same time), it follows that either $N_{ik} = N_{jk}$ or $N_{ik} = N_{jk}+1$ or $N_{ik} = N_{jk}-1$. The situation is depicted in Fig. 2. Agent i can send N_{ik} to agent j, who can locally determine the correct value of N_{jk} through a simple consistency verification. Note that agent i can also provide agent j with additional navigation information, e.g., satellite clock drifts, which only change slowly in time and thus require no tight time synchronization. An alternative implementation of extended cooperation, which does not require tight synchronization, can treat the problem differently as localization with integer ambiguity [5]. It exploits the fact that there is a unique combination of receiver position and time for which the measured fractional delays ϕ_{ik} could have been observed. If agent $i \in \mathcal{M}^{\text{full}}$ provides a rough position and time reference to agent $j \in \mathcal{M}^{\text{frac}}$, agent j can determine m_{jk} and thus estimate its position, using the method from [5].

However, for the purpose of this paper, both approaches lead to the same goal: to *activate* GNSS functionality of $\mathcal{M}^{\text{frac}}$ agents.

C. Fisher information analysis

1) General formulation from [2]: The hybrid cooperative network leads to a FIM $\mathbf{F}$ consisting of a cooperative and non-cooperative contribution, i.e., $\mathbf{F} = \mathbf{F}_{\text{coop}} + \mathbf{F}_{\text{non-coop}}$ [2]. The cooperative part, which is a result of ranging among agents, can be expressed as

$$\mathbf{F}_{\text{coop}} = \begin{pmatrix} \mathbf{F}'_1 & \mathbf{0} & \mathbf{K}_{12} & \mathbf{0} & \cdots & \mathbf{K}_{1M} & \mathbf{0} \\ \mathbf{0}^T & 0 & \mathbf{0}^T & 0 & & \mathbf{0}^T & 0 \\ \mathbf{K}_{21} & \mathbf{0} & \mathbf{F}'_2 & \mathbf{0} & & & \\ \mathbf{0}^T & 0 & \mathbf{0}^T & 0 & & & \\ \vdots & & & & \ddots & & \\ \mathbf{K}_{M1} & \mathbf{0} & & & & \mathbf{F}'_M & \mathbf{0} \\ \mathbf{0}' & 0 & & & & \mathbf{0}^T & 0 \end{pmatrix}, \tag{7}$$

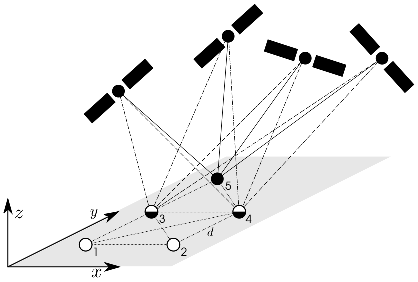


Fig. 3: Scenario 1 with 4 satellites and 5 agents.

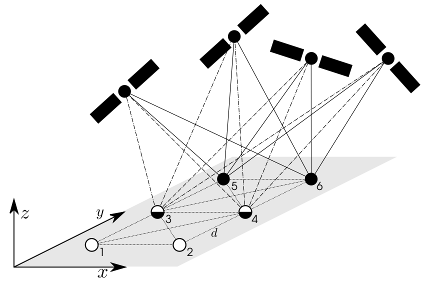


Fig. 4: Scenario 2 with 4 satellites and 6 agents.

which is a $(D+1)M \times (D+1)M$ positive semidefinite matrix, with elements

$$\mathbf{F}'_i = \sum_{j \in \mathcal{M}_i} \frac{1}{\tilde{\sigma}_{ij}^2} \mathbf{u}_{ji} \mathbf{u}_{ji}^T, \tag{8}$$

$$\mathbf{K}_{ij} = \begin{cases} -\frac{1}{\tilde{\sigma}_{ij}^2} \mathbf{u}_{ji} \mathbf{u}_{ji}^T, & \text{if } j \in \mathcal{M}_i, \\ \mathbf{0}, & \text{otherwise,} \end{cases} \tag{9}$$

where $\mathbf{0}$ is vector or matrix of appropriate size consisting only of zeros, and $\mathbf{u}_{ji} = (\mathbf{p}_j - \mathbf{p}_i)/\|\mathbf{p}_j - \mathbf{p}_i\|$ is a unit vector pointing from agent i to agent j.

2) Impact of extended cooperation on $\mathbf{F}_{\text{non-coop}}$*:* Now we focus on the non-cooperative part, which is outcome of capability of agents to estimate their positions using GNSS receivers. We introduce

$$\mathcal{S}_i^{(\Gamma)} = \{k \in \mathcal{S}_i : (C/N_0)_{ik} \geq \Gamma\} \tag{10}$$

as the set of satellites from which the agent i receives signals with C/N_0 values equal or higher Γ.

Since agents make independent GNSS observations, $\mathbf{F}_{\text{non-coop}}$ is a block-diagonal matrix, where the block corresponding to agent i is of the form

$$\mathbf{F}_i = \sum_{k \in \mathcal{S}_i^{(\Gamma)}} \frac{1}{\sigma_{ik}^2} \begin{pmatrix} \mathbf{q}_{ki} \mathbf{q}_{ki}^T & -\mathbf{q}_{ki} \\ -\mathbf{q}_{ki}^{\mathrm{T}} & 1 \end{pmatrix}. \tag{11}$$

where $\mathbf{q}_{ki} = (\mathbf{s}_k - \mathbf{p}_i)/\|\mathbf{s}_k - \mathbf{p}_i\|$ is a unit vector pointing from agent i to satellite k.

Under classic cooperation, only agents in $\mathcal{M}^{\text{full}}$ contribute into $\mathbf{F}_{\text{non-coop}}$, whereas in extended mode, the matrix is created based on agents from $\mathcal{M}^{\text{full}}$ as well as $\mathcal{M}^{\text{frac}}$. In turn, this allows us to express

$$\mathbf{F}_{\text{non-coop}} = \mathbf{F}_{\text{non-coop}}^{\text{full}} + \alpha \mathbf{F}_{\text{non-coop}}^{\text{frac}}, \tag{12}$$

in which the evaluation of $\mathbf{F}_{\text{non-coop}}^{\text{full}}$ is done using (11) for agents only from $\mathcal{M}^{\text{full}}$ and $\Gamma = \Gamma_{\text{msg}}$ and, similarly, $\mathbf{F}_{\text{non-coop}}^{\text{frac}}$ is calculated for agents only from $\mathcal{M}^{\text{frac}}$ and $\Gamma = \Gamma_{\text{floor}}$. The parameter $\alpha \in \{0, 1\}$ controls the calculation of non-cooperative part of FIM. For $\alpha = 0$, classical cooperation is performed, while for $\alpha = 1$, extended cooperation is used.

It is worth noting that the definition (12) avoids artificial increasing of the FIM in a case of insufficient number of satellites operating above a threshold. For example, assume the classical cooperative mode and an agent with only 1 satellite above Γ_{msg}. The definition excludes this particular satellite from the FIM since the agent is not able to exploit the corresponding positioning information due to the loose coupling of the GNSS receiver and wireless ranging system.

3) Cramér-Rao lower bound: The Cramér-Rao lower bound (CRLB) is derived from the inverse of the FIM [14]. Let $\mathbf{C} = \mathbf{F}^{-1}$ and let $\mathbf{C}_i$ be a $(D+1) \times (D+1)$ sub-matrix lying on diagonal of $\mathbf{C}$ and corresponding to agent i. In case of $D = 3$, four diagonal elements of $\mathbf{C}_i$ express the bounds on variances of parameters $\mathbf{p}_i = [x_i, y_i, z_i]^T$ and b_i. For purpose of our analysis, the bound of the standard deviation of the horizontal (i.e., *x*-*y* plane) estimation error is defined as[6]

$$\sigma_i^{(h)} = \sqrt{\mathbf{C}_i[1,1] + \mathbf{C}_i[2,2]}. \tag{13}$$

An average bound on the standard deviation of the horizontal error over all agents is introduced as

$$\bar{\sigma}^{(h)} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (\sigma_i^{(h)})^2}. \tag{14}$$

Both $\sigma_i^{(h)}$ and $\bar{\sigma}^{(h)}$ are expressed in meters. Let $\sigma_i^{(h,\alpha=1)}$ and $\sigma_i^{(h,\alpha=0)}$ be standard deviations of horizontal estimation errors in case of extended cooperative mode ($\alpha = 1$) and classical cooperative mode ($\alpha = 0$), respectively. Finally, $\bar{\sigma}^{(h,\alpha=1)}$ and $\bar{\sigma}^{(h,\alpha=0)}$ are defined similarly.

V. Numerical results

A. Simulation setup

We have evaluated the performance of the proposed extended cooperation in a number of scenarios. For each of the scenarios, we set $\tilde{\sigma}_{ij} = 0.3\,\mathrm{m}, \forall(i,j)$ and $\sigma_{ik} = \sigma_{\text{sat}\to\text{agt}}, \forall(i,k)$[7]. The positions of satellites are taken from an publicly available SP3 file[8]. In the graphical illustrations, two agents are connected by an edge provided that they are capable to communicate and perform ranging among themselves (similar for connections between an agent and a satellite). Finally, the sets $\mathcal{M}^{\text{full}}$, $\mathcal{M}^{\text{frac}}$, and $\mathcal{M}^{\text{floor}}$ are denoted by symbols ●, ◒ and ○, respectively.

[6] An extension of the analysis to cover other parameters is straightforward.

[7] This implies that variance of w_{ik} does not vary with C/N_0. This is over-simplified assumption, which we consider only for sake of clarity. Note that the presented framework and analysis can be applied to arbitrary individual variances. In general, the variance of w_{ik} depends on C/N_0 and several other aspects, such as integration time and filtering of delay measurements. To justify the constant value of $\sigma_{\text{sat}\to\text{agt}}$ for all agents, we can also argue that two agents receiving the same satellite navigation signal with different C/N_0 values, can theoretically achieve the same pseudorange errors by, for example, selecting different integration times.

[8] Available at ftp://igscb.jpl.nasa.gov/igscb/product/1802/igr18020.sp3.Z.

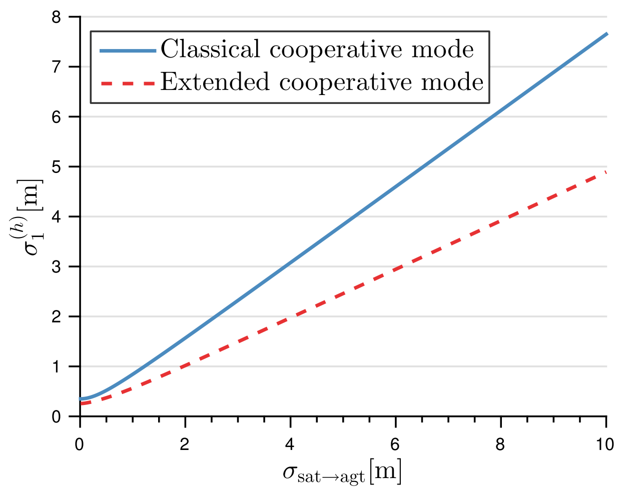


Fig. 5: An analysis of scenario 2, demonstrating the gains of extended cooperation as a function of the pseudorange standard deviation.

B. Scenario 1: Improvement in availabilty

A first scenario is depicted in Fig. 3, in which the distance between agents d is equal to $20\,\mathrm{m}$. If $\mathcal{M}^{\text{frac}}$ agents 3 and 4 are not activated (i.e., they are not able to determine their positions using GNSS), only agent 5 can estimate its absolute position. In this case, $\mathbf{F}$ is not invertible and hence no conclusion can be drawn about positioning accuracies of agents 1 to 4. However, if the $\mathcal{M}^{\text{frac}}$ agents are activated by means of the extended cooperation with the agent 5, then all the agents can localize themselves absolutely in the space. Thus, in this case, the activation of $\mathcal{M}^{\text{frac}}$ agents is critical in order to allow the others to even determine their absolute positions. Hence, the extended cooperative mode improves availability of the positioning information.

C. Scenario 2: Improvement in accuracy

To observe an improvement in positioning accuracies caused by activation of $\mathcal{M}^{\text{frac}}$ agents, the example is extended by additional agent 6, as illustrated in Fig. 4. In this case, even if $\mathcal{M}^{\text{frac}}$ agents are not activated, $\mathbf{F}$ is invertible. In Fig. 5 the standard deviation of horizontal positioning error of agent 1 is depicted as a function of standard deviation $\sigma_{\text{sat}\to\text{agt}}$ for classical as well as extended cooperative modes. The accuracy improvement in this example is above 55% for practically reasonable values of $\sigma_{\text{sat}\to\text{agt}}$.

D. Scenario 3: Large scale scenario

To analyze the positioning accuracies improvements in a real scenario, GPS data collected during a measurement campaign in an office environment at Chalmers University of Technology are used. Particularly, during this campaign, an office environment was charted in detail by means of collection of various positioning data provided by an u-blox 6 GNSS receiver. The data was measured at locations defined by a dense square grid with size $1.5\,\mathrm{m}$.

The office floorplan can be seen from Fig. 6, which depicts the number of satellites with $C/N_0 \geq \Gamma_{\text{msg}} = 27\,\text{dB-Hz}$ in each point of the measurement grid. The gray area inside the figure represents a courtyard. 50 agents are randomly placed

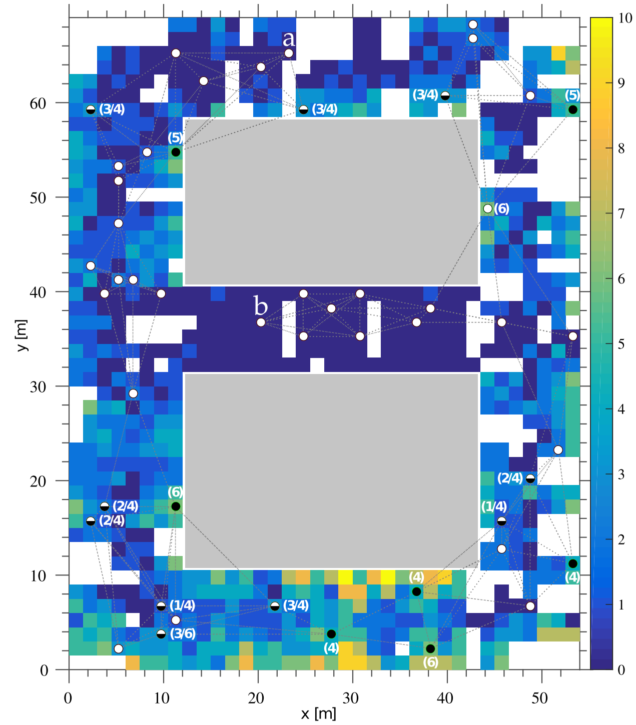
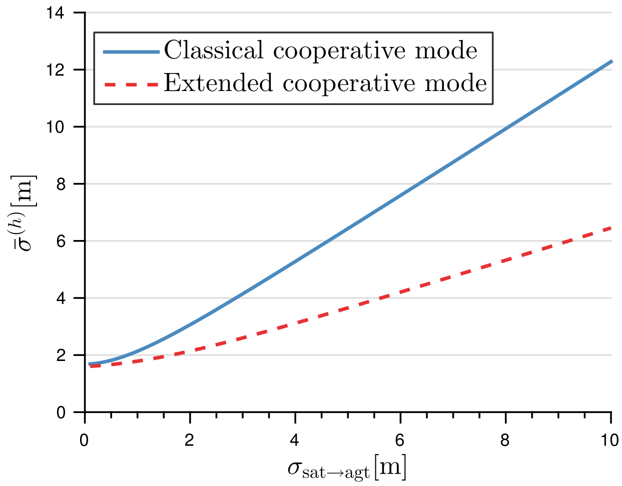


Fig. 6: Number of satellites with $C/N_0 \geq 27\,\text{dB-Hz}$ and random placement of agents. The first number in a bracket located close to an agent stands for the number of satellites for which $C/N_0 \geq \Gamma_{\text{msg}}$, and, if applicable, the second number determines the number of satellites with $C/N_0 \geq \Gamma_{\text{floor}}$.

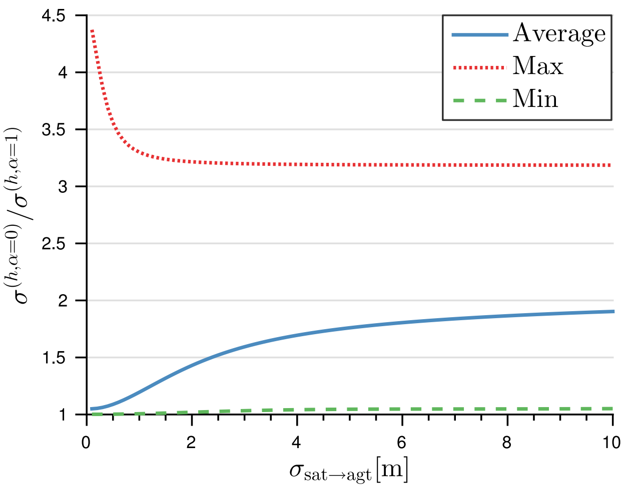
within the environment. Based on the number of available satellites, the agents are graphically distinguished into the three sets. The agents perform ranging towards several surrounding agents, whose average number is around 6.

In this particular example, it is assumed that the activation of $\mathcal{M}^{\text{frac}}$ agents makes available satellites with $C/N_0 \geq \Gamma_{\text{floor}} = 24\,\text{dB-Hz}$, i.e., only $3\,\text{dB}$ below the limit needed for decoding of GPS L1 C/A navigation messages.

Fig. 7a depicts the average standard deviation of horizontal positioning error over all considered agents for both cooperative modes. The ratio of the average characteristics from Fig. 7a is shown in Fig. 7b by a solid line. Fig. 7b also depicts the ratio of standard deviations for the agents whose positioning accuracies are improved maximally as well as minimally. The maximum and minimum improvement occurs for the agent denoted in Fig. 6 as *a* and *b*, respectively. The maximum improvement due to activation of $\mathcal{M}^{\text{frac}}$ agents is, in this particular scenario, significant: more than three times lower standard deviation of horizontal positioning error for practically reasonable values of $\sigma_{\text{sat}\to\text{agt}}$. This is due to a favorable location of agent *a*, close to two agents in $\mathcal{M}^{\text{frac}}$ and an agent in $\mathcal{M}^{\text{full}}$, which ensures high increase in corresponding part of FIM after activation of the $\mathcal{M}^{\text{frac}}$ agents. On the other hand, agent *b* is only connected to agents in $\mathcal{M}^{\text{floor}}$. Therefore, the activation of agents in $\mathcal{M}^{\text{frac}}$ has no significant impact on the positioning error of agent *b*. The empirical cumulative distribution function $\mathrm{CDF}(\sigma^{(h)})$ for this scenario at $\sigma_{\text{sat}\to\text{agt}} = 5\,\mathrm{m}$ is depicted in Fig. 8. For example, it demonstrates that in case of classical cooperation, there are



(a)



(b)

Fig. 7: An analysis of the office environment scenario.

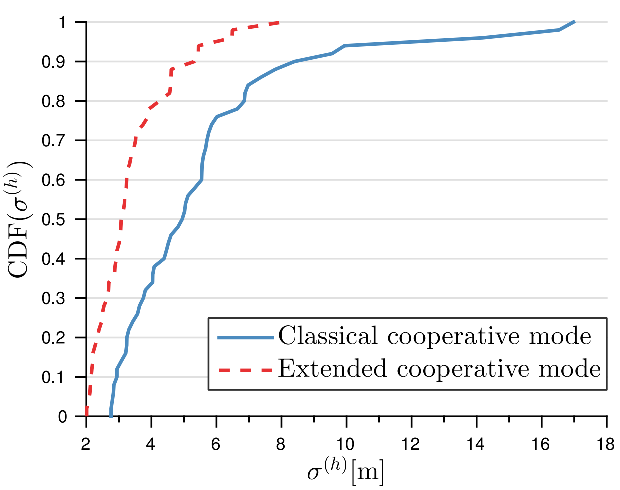


Fig. 8: Empirical cumulative distribution function for the office environment scenario, $\sigma_{\mathrm{sat}\to\mathrm{agt}} = 5\,\mathrm{m}$.

80% of agents with standard deviation of horizontal error below $6.9\,\mathrm{m}$, whereas in case of extended cooperation the same amount of agents achieves the error better than $4.5\,\mathrm{m}$.

VI. Conclusion

We have considered hybrid cooperative positioning in harsh environments based on an extended pseudorange model, which captures the fact that the pseudorange can be known by the receiver only with integer ambiguity in these environments. We have proposed an extended cooperative mode to solve this ambiguity by means of cooperation between agents. Through a Fisher information analysis we are able to quantifiably compare different cooperation approaches. Simulation results indicate benefits in terms of availability and accuracy over classical cooperation.

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References

[1] R. Garello, L. L. Presti, G. E. Corazza, and J. Samson, "Peer-to-Peer Cooperative Positioning, Part I: GNSS-Aided Acquisition," *InsideGNSS*, pp. 55–63, 2012.

[2] F. Penna, M. A. Caceres, and H. Wymeersch, "Cramér-Rao Bound for Hybrid GNSS-Terrestrial Cooperative Positioning," *IEEE Communications Letters*, vol. 14, no. 11, pp. 1005–1007, Nov. 2010.

[3] G. Seco-Granados, J. Lopez-Salcedo, D. Jimenez-Banos, and G. Lopez-Risueno, "Challenges in Indoor Global Navigation Satellite Systems: Unveiling its core features in signal processing," *IEEE Signal Processing Magazine*, vol. 29, no. 2, pp. 108–131, Mar. 2012.

[4] F. Van Diggelen, *A-GPS: Assisted GPS, GNSS, and SBAS*. Artech House, 2009.

[5] N. Sirola and J. Syrjärinne, "GPS position can be computed without the navigation data," in *Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS)*, 2002, pp. 2741–2744.

[6] A. Rabaoui, N. Viandier, J. Marais, and E. Duflos, "Using Dirichlet Process Mixtures for the Modelling of GNSS Pseudorange Errors in Urban Canyon," in *Proceedings of the 22nd International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS)*, Sep. 2009, pp. 2391–2399.

[7] S. Li, M. Hedley, and I. B. Collings, "An empirical ranging error model and efficient cooperative positioning for indoor applications," in *IEEE International Conference on Communications Workshops*, 2015.

[8] P. D. Groves, *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, Second Edition*. Artech House, 2013.

[9] M. Caceres, F. Penna, H. Wymeersch, and R. Garello, "Hybrid cooperative positioning based on distributed belief propagation," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 10, pp. 1948–1958, December 2011.

[10] P. Misra and P. Enge, *Global Positioning System: Signals, Measurements, and Performance*. Ganga-Jamuna Press, 2001.

[11] B. W. Parkinson and J. J. Spilker, *Global Positioning System: Theory and Applications, Volume 1*. American Institute of Aeronautics & Astronautics, 1996, vol. 1.

[12] M. Lashley, D. M. Bevly, and J. Y. Hung, "Performance Analysis of Vector Tracking Algorithms for Weak GPS Signals in High Dynamics," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, no. 4, pp. 661–673, Aug. 2009.

[13] P. L. Kazemi and C. O'Driscoll, "Comparison of Assisted and Stand-Alone Methods for Increasing Coherent Integration Time for Weak GPS Signal Tracking," in *Proceedings of the 21st International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2008)*, Sep. 2008, pp. 1730–1740.

[14] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation theory*. Prentice-Hall PTR, 1993.