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(article starts on next page)

On S-Parameter based Complex Correlation of Multi-Port Antenna

Xiaoming Chen [1] and Per-Simon Kildal [2]

[1] Qamcom Research & Technology, Gothenburg, Sweden

[2] Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden

Abstract—**In this paper, we present a S-parameter based expression for the complex correlations of multi-port antennas. The formula is tested against the correlation formula based on embedded antenna patterns and the measurement in a reverberation chamber (RC). A 4-port wideband antenna is used as an example.**

***Index Terms*—Correlation, multi-port antenna, reverberation chamber (RC).**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are becoming more and more popular due to their capability of data rate enhancement [1]. A MIMO system necessitates multi-port antennas. In practice, the space allocated for the multi-port antenna at the terminal side is usually limited. Hence, compact multi-port antenna designs are necessary. However, a drawback of using compact multi-port antennas is that the antenna correlations become inevitable [2]. One important task of the over-the-air (OTA) testing of MIMO terminals [3] is to determine the antenna correlation of the MIMO device under test.

Different methods for determining antenna correlations have been proposed for determining the antenna correlation: i) measuring the antenna in a stochastic multipath environment, e.g., reverberation chamber (RC) [4]; ii) using embedded radiation patterns [5]; iii) using S-parameters (S-par) [6]. Compared with the other two methods, the S-par method is the easiest to implement. It requires neither lots of stochastic samples nor time-consuming antenna pattern measurements. However, the S-par method is valid to antennas with little Ohmic loss. For lossy antennas, it tends to underestimate the antenna correlation [7]. Note that for antennas where the Ohmic loss resides only in the feeding cables of the antennas, one can still use the S-par method by calibrating out the cable losses; however, when the radiating elements are lossy, it is difficult to calibrate out the loss, in which case, the S-par method is not accurate [7]. Bearing this in mind, this work focuses on the S-par method and uses a multi-port antenna with little Ohmic loss as an example. In order to study the Ohmic loss effect on the correlation calculated using the S-par method, we also consider the case where Ohmic losses exist in the feeding cable of the antennas by numerically adding losses to the ports of the Bowtie antennas.

II. S-PAR BASED COMPLEX CORRELATION

Envelope correlations based on S-par for 2-port was first derived in [6]. The formula was later on extended to multi-port antennas (e.g., antennas with more than 2 ports) in [8]. The complex correlation in terms of S-par can be readily derived based on the law of power conservation [9]: For lossless antenna with N ports,

$$\mathbf{C}^H\mathbf{C}+\mathbf{S}^H\mathbf{S}=\mathbf{I} \tag{1}$$

where the superscript ^{H} is the Hermitian operator, $\mathbf{I}$ is the identity matrix, $\mathbf{S}$ is the S-parameter matrix, and the elements in matrix $\mathbf{C}$ are given as

$$[\mathbf{C}]_{mn}=\frac{\sqrt{D_mD_n}}{4\pi}\iint_{4\pi}\mathbf{g}_m^H(\Omega)\mathbf{g}_n(\Omega)d\Omega \tag{2}$$

where $\mathbf{\Omega}$ is the solid angle of arrival, $\mathbf{g}_i$ (i = 1,…, N) is the embedded far field function (a column vector with elements representing for different polarizations) at the ith antenna port, and D_i is directivity at the ith antenna port.

Assume reciprocity of the multi-port antenna network and from (1), the elements in $\mathbf{C}$ are related to the elements in $\mathbf{S}$ as

$$[\mathbf{C}]_{mn}=\sum_{i=1}^{N}[\mathbf{S}]_{mi}^*[\mathbf{S}]_{in} \tag{3}$$

where the superscript * denotes complex conjugate.

In rich isotropic multipath (RIMP) environment [3], the complex-valued correlation between the mth and nth antenna ports is [5]

$$\rho_{mn}=\frac{\iint_{4\pi}\mathbf{g}_m^H(\Omega)\mathbf{g}_n(\Omega)d\Omega}{\sqrt{\iint_{4\pi}\mathbf{g}_m^H(\Omega)\mathbf{g}_m(\Omega)d\Omega\cdot\iint_{4\pi}\mathbf{g}_n^H(\Omega)\mathbf{g}_n(\Omega)d\Omega}}. \tag{4}$$

Combining (2)-(4), and the correlation can be expressed in terms of S-parameters,

$$\rho_{ij}=\frac{-\sum_{n=1}^{N}S_{in}^*S_{nj}}{\sqrt{\prod_{m=i,j}(1-\sum_{n=1}^{N}|S_{nm}|^2)}}. \tag{5}$$

As a special case of the general expression (1), the S-par based complex correlation for a 2-port antenna is given by

$$\rho = \frac{-(S_{11}^{*}S_{12} + S_{21}^{*}S_{22})}{\sqrt{(1-|S_{11}|^2-|S_{21}|^2)(1-|S_{22}|^2-|S_{12}|^2)}} \tag{6}$$

III. MEASUREMENT AND SIMULATION

In order to verify (5), we choose the compact wideband 4-port Bowtie antenna [10] as an example for the correlation analysis. The antenna was simulated using CST, from which we extracted S-par and embedded radiation patterns. It was also measured in the Bluetest RC [11], from which we recorded the channel samples at all its 4 ports. Correlations can then be obtained from the embedded radiation patterns as well as the RC measurement. Those will be used in comparing the S-par based correlaiton expression (5).

The expressions for calculating complex correlation based on the embedded radiation patterns is given in (4), whereas the correlation measurement in RC has been described in [12]. For the sake of completeness, we briefly present it here.

The RC is basically a metal cavity with many excited modes that are stirred to create a RIMP environment [3]. The RC used in this work has a size of 1.75 × 1.25 × 1.8 m^3 and is equipped with two plate stirrers, a turntable platform (on which the antenna under test is mounted), and three fixed RC antennas. In the measurements, the platform was moved to 20 positions and at each platform position the two plates move simultaneously to 10 positions. At each stirrer position and for each of the three wall antennas a full frequency sweep was performed by a vector network analyzer (VNA). Thus, for correlation evaluation, there are 600 channel samples per frequency point. In order to improve measurement accuracy, the frequency stirring [13] (or electronic stirring [14]) technique is used. The measurement frequency ranges from 1.5 to 3 GHz with a frequency step of 1 MHz. A 10-MHz frequency stirring is used. Therefore, eventually there are 6000 channel samples for calculating the correlation. Note that due to the correlation in the spatial [15] and frequency [16] domains, the independent samples [17], [18] will be less than 6000.

Denote $\mathbf{h}_m$ as the mth sample (of the total M = 6000 samples) of the channel vector $\mathbf{h}$ of the multi-port antenna, the sample covariance matrix can be calculated as

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{m=1}^{M}\left(\mathbf{h}_m - \frac{1}{M}\sum_{n=1}^{M}\mathbf{h}_n\right)\left(\mathbf{h}_m - \frac{1}{M}\sum_{n=1}^{M}\mathbf{h}_n\right)^H . \tag{7}$$

The measured correlation coefficients in the RC can then be obtained as

$$\hat{\rho}_{mn} = \frac{[\hat{\mathbf{R}}]_{mn}}{\sqrt{[\hat{\mathbf{R}}]_{mm}[\hat{\mathbf{R}}]_{nn}}} \tag{8}$$

Fig. 1 shows a photo and drawings of the Bowtie antenna. It was designed to cover the frequency range of 1.5 ~ 3 GHz. The 4 ports of the Bowtie antenna are marked in Fig. 1. Due to the symmetry of the antenna, it is sufficient to present correlations between Ports 1 and 2, Ports 1 and 3, and Ports 1 and 4.

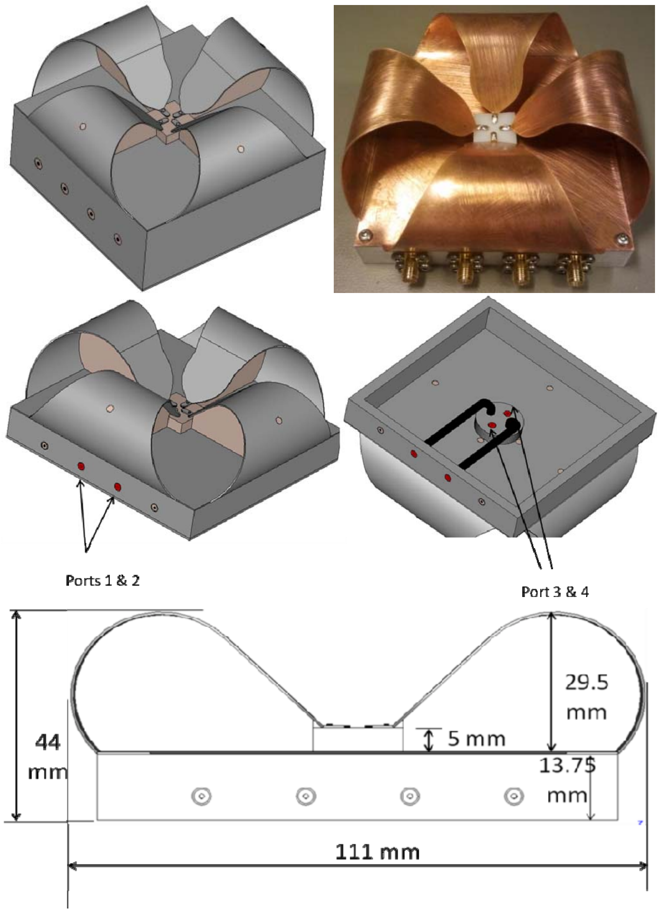
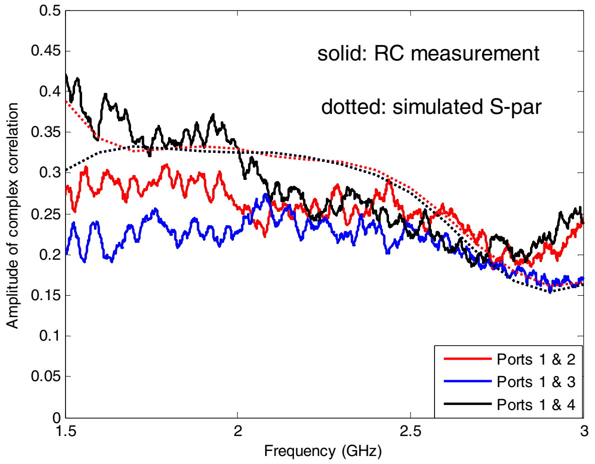


Fig. 1. Photo and drawings of the Bowtie antenna [10].

Fig. 2 compares the correlation calculated from S-par with that measured in the RC and with that calculated from antenna patterns, respectively. There are in general good agreements between the correlations obtained using different methods. Note that due to manufacture tolerance the curvatures of the petals of the actual Bowtie antenna are not identical, which probably causes the slight disagreement between the measured correlation and the correlation calculated from simulated S-par.

The S-parameter based formula for correlation is derived under the assumption of a lossless antenna. It will still be good to know how large the error will be if there are losses. In order to study this, we numerically add losses to the antenna ports of the Bowtie antenna. For simplicity, we assume identical loss on all the 4 ports. Fig. 3 shows the calculated correlation using S-parameters and embedded antenna patterns. As expected, the correlation obtained from the antenna patterns is not affected by these Ohmic losses added on the ports, which is correct and in agreement with the physics of correlation. The correlation calculated from the S-parameters appears, on the other hand, to be sensitive to the loss. It can be seen from Fig. 3 that the 0.1-dB cable insertion loss has little effect on the S-par based correlation. Yet there is noticeable degradation for 0.5-dB cable insertion loss and the degradation is severer for larger loss. Hence, extra care should be exerted when using the S-parameter based correlation. It will show wrong results if there are losses in the feed cables larger than 0.1 dB.



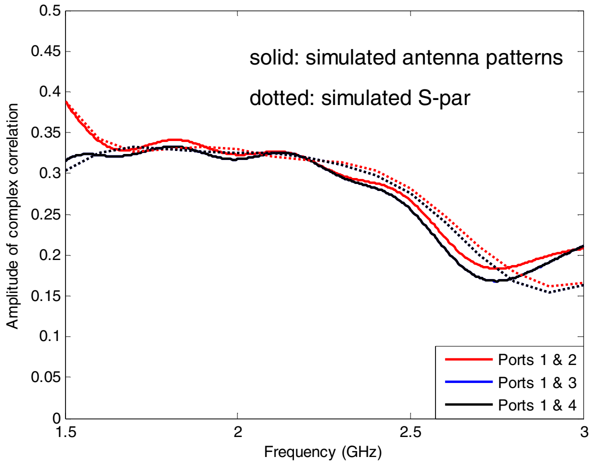


Fig. 2. Bowtie correlation. (upper) RC measured correlation vs correlation calculated from S-par; (lower) correlation calculated from S-par vs correlation calculated from antenna pattern.

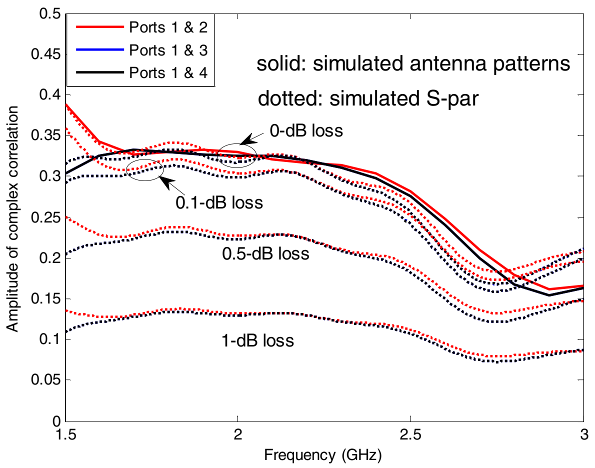


Fig. 3. Correlation from simulated S-parameters and antenna patterns with various cable insertion loss.

IV. CONCLUSIONS

In this paper, we present an S-parameter based expression for the complex-valued correlations between antenna ports. Using the 4-port wideband Bowtie antenna, the formula is verified by comparing the calculated correlation based on S-parameters with that based on embedded radiation patterns. There is good agreement between the correlation calculated from embedded antenna patterns and that calculated from S-parameters. In addition, RC measurement has been performed. Due to the manufacture tolerance of the Bowtie antenna, certain disagreement exists between the measured and simulated correlation from S-parameters. Finally, it should be noted that the S-parameter based correlation formula is valid only for lossless antennas. We have found by numerical studies that the losses in the built-in feed cables need to be smaller than typically 0.1 dB in order to give a good estimate of the actual correlation. When the losses in the feed cables are larger than 0.5 dB, the correlation results calculated from the S-parameter are not reliable anymore.

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