

Time Alignment of Engine Emission Measurements

Master's Thesis in the Automotive Engineering

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CHALMERS UNIVERSITY OF TECHNOLOGY Göteborg, Sweden 2015 Master's thesis 2015:15

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ABSTRACT

Generally, steady state engine emission measurements are used to evaluate exhaust, which is far from the real condition. When driven in urban environments, the vehicle is affected by traffic lights, bus stations, pedestrians, etc. Therefore, transient driving states account for 50% - 90% of all working conditions, in which 30% - 50% are acceleration. Vehicles are, most of the time, working at starting, accelerating, decelerating and such transient states, and the air flow as well as the fuel flow rates are changing simultaneously. This will cause obvious change of the exhaust emissions such like hydrocarbon (HC) and nitrogen oxide (NO_X). Thus the emission parameters measured under steady state cannot represent the real exhaust level, which makes it necessary to study the transient emissions and measuring technique.

One cornerstone of dynamic emissions testing is the time alignment between measured emissions and exhaust flow from the engine. In the future, emissions development on engines will increase and thus increases the importance of time alignment. This means that new more accurate methods needs to be developed to increase accuracy.

In this study, 15.77% v/v CO_2 is dosed at the empty reactor setup, thus to observe the convolution effect of the exhaust system. In the absence of any catalytic or reactive material that interact with the gas phase species, the same step function is expected to be observed as the measured signal in the outlet. Nevertheless, the result is often a monotonically increasing function of time which slowly reaches the inlet concentration after certain elapsed time. Hydrodynamic dispersion is the reason of this phenomenon, because it will increase the residence time of fluid elements. Consequently, signal distortion will also affect the results of reactive experiments, and therefore it is necessary to characterize the reactor setup with respect to these perturbing effects in order to properly interpret the results of transient experiments. Having this fulfilled, the next step would be to propose a method to retrieve the true response of the reactor system for quantitative data analysis.

The residence time distribution of the detector and exhaust system was analyzed, thus to get the transfer function of the instruments. Through deconvolution of the step response and sinus response under certain conditions, we got the transfer function under various exhaust flow. Thus we can get the actual output emissions from the measured data. In this project we did various experiments under certain conditions, and in the next step we need to verify the usage of deconvolution under random conditions. So the methodology we developed in this thesis could not be used in reality for now, as more works need to be done in the future.

Key words: Time alignment, concentration measurements, transient experiments, deconvolution, engine, emissions, exhaust

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Notations

Abbreviation

BMD	Bag mini diluter
CFD	Computational fluid dynamics
CVS	Constant volume sampler
C _{pulse}	Response curve of pulse input signal
E	Normalized curve of C _{pulse}
EGR	Exhaust gas recirculation
LPM	Litre per minute
LTI	Linear, time-invariant

Nomenclature

А	Pipe cross-sectional area		
\mathbf{D}_{H}	Hydraulic diameter of the pipe		
L	Characteristic linear dimension		
М	Mole units of tracer into the fluid entering the vessel		
Q	Volumetric flow rate		
r	Fraction of recirculation mixing		
v	Velocity of the object relative to the fluid		
Re	Reynolds number		
ρ	Density of the fluid		
μ	Dynamic viscosity of the fluid		
υ	Kinematic viscosity		
g(t)	Transfer function		
g(s)	Transfer function in frequency domain		
h(t)	Combined transfer function		
h _L (t)	Gaussian function modeling of the transport delay and diffusion		
h _s (t)	Impulse response function modeling recirculation mixing		
h _f (t)	Impulse response function modeling bulk mixing		
x(t)	True (unperturbed) signal		
x(s)	True (unperturbed) signal in frequency domain		
y(t)	Convoluted signal		

y(s)	Convoluted signal in frequency domain
Δt	Total time delay
Δt_1	Time delay derived from engine exhaust manifold to tail pipe
Δt_2	Time delay derived from exhaust emissions flowing in the sampling pipe
	of the analyzer
Δt_3	Time delay of the emission analyzer
*	Convolution
*-1	Deconvolution

1 Introduction

With the swift growth of vehicle population in cities, exhaust pollution is coming to be one significant source of air pollution. Atmosphere pollution has transferred from industrial to urban, multiple and regional. This new type air pollution mainly from vehicles is becoming prominent day by day, and thus vehicle emission measurement becomes more important to do exhaust gas cleaning. In order to study the production of emissions and the controlling technology, it is important to correctly measure the poisonous emission contents. The development of engine technology is largely depending on the development of test and measurement technique. With the increasingly stringent emission regulations in each country, the emission testing technology is also improving continuously.

Generally, components of petrol engine emissions can be divided into two different kinds: One is the production resulting from completed burned fuel, the basic components consist of carbon dioxide, water vapor, excess air, residual nitrogen and so on, such gases won't cause any direct harm to human beings or other creatures (But carbon dioxide is a green house gas). The other one is uncompleted burned fuel and harmful oxides, such like carbon monoxide, unburned hydrocarbon, nitrogen oxides, sulfur dioxide, particles, etc. The second kind of emissions will cause harms to humans and other creatures in different degrees. Vehicle exhaust testing can provide exact emission contents, which is significant for environmental protection authorities and vehicle companies to grasp the actual emission levels as well as control pollution.

To resolve the core issues of sustainable development, we have to consider the problems of population, resources, environment and development. The main concept of sustainable development is that humans should coordinate the relationship among population, resources, environment and development, and to pursue development without prejudice to others and future generations. This requires us to develop energy saving and emission reduction technology, hence we need to develop advanced exhaust testing technique.

1.1 Background

For long period of time, engine exhaust emissions are measured through steady state running cycles to obtain emissions data to do exhaust analysis. In the real working procedure, however, conventional cars are often running into different situations such like stop, start-up, acceleration and deceleration. The engine, at the same time, will also work on idling, start-up, acceleration and deceleration conditions. These transient state working conditions with sharp changes of rotating speeds and loads account for a large proportion in the engine working cycles, which cause more severe pollution to environment. In the regularized driving cycles worldwide, 40% - 70% are transient working conditions, in which 25% - 40% are acceleration state. Especially when the

vehicle drives on urban path, unsteady state conditions account for more proportions which is around 50% - 80%, and 30% - 50% of which are acceleration state. Thus we can see that vehicle engines are frequently working under transient states, during which rotating speed and loads change sharply, and this will in turn cause changes of mixture formation as well as combustion continuously. The emission characteristics are also different from steady state conditions, so it has great theoretical and practical values to do some research about transient emission properties.

Most measured values reported by a measurement device have associated systematic and methodical errors. For steady state measurements the cumulative effect of these errors is expressed by the measurement uncertainty, which depends on the device capabilities, the calibration process, and the accuracy of the reference devices used for calibration.

While the measurement uncertainty is well suited to describe the accuracy of a steady state measurement, it most often does not cover any dynamic effects occurring during a transient measurement. Equipment manufactures declare response times for their devices; however effects of signal propagation, sample transportation, and signal processing are usually excluded and often poorly understood. One has to consider that any measurement of a transient signal is subject to these dynamic measurement errors.

The effect that a step change of the concentration signal is actually detected as a spread out signal is called dispersion. Hydrodynamic dispersion increases the delay time of exhaust elements, and the signal perturbation will also inevitably affect the transient exhaust measurement. Therefore, in order to test transient emissions, we need to determine the signal distortion effect of the instruments. Moreover, the math B.Ganesan[1] used to describe dispersion is equal to the mathematical function of convolution as described by S.Smith[2]. Then we are going to propose an appropriate way to get the true transient exhaust emissions.

1.2 Objective

Analysis of transient exhaust emissions requires high temporal resolution. And thus the response time of the analyzers is generally desired to be lower than the kinetic time scale to obtain the time-resolved information of the system. In order to calculate raw emissions, the exhaust flow and the concentration signals must all be related to a common location in the system(usually the tailpipe sample location) and the recorded signals shall not be altered by measurement and sampling system.

With the current methods, however, the transient exhaust measurement is still far from perfect. Some dynamic information of the exhaust concentration is lost because of the effect of convolution. We tried to relate everything to the state of the engine. What's more, the fuel signal has a time delay that results from the emissions dynamics. And when the working states are changing, the air flow and exhaust emissions stored in the system change, which could in turn change the relationships between air flow and the exhaust flow in the system.

A signal restoration method is therefore required in order to retrieve the true response of the system as the solution to the dispersion and filter function of the sample system and the exhaust system. It is still difficult to get the true exhaust, however, because any perturbation during the measurement will cause great fluctuation to the test result. Therefore, a regularization method is required to filter out these magnified perturbations. The main objective of this thesis aims at delivering a new method for time alignment between emissions measurement and engine exhaust flow giving more accurate mass calculations.

2 Literature Studies

2.1 Gaseous emissions measurement

There are generally two ways to measure engine exhaust gas components: one method is placing an in-situ detector inside the exhaust pipe, and the other method is extractive sampling of the gas into the analyzer. The former only requires attaching sensors directly to the exhaust pipe; the later requires sampling the gas from the exhaust pipe and appropriately conditioning the sample before introducing the gas into the analyzer. The components applied to the conditioning and pretreatment of the exhaust gas are called the sampling system. The sampling system plays an important role in the measuring system, next to the analyzers. The extractive sampling methods for the exhaust gas are classified broadly into "dilution sampling method", in which the exhaust gas is diluted by air or by synthetic air, and "direct sampling method" that requires no dilution. These methods are used selectively depending on the objectives or the condition of the exhaust gas.

The engine exhaust gas flow rate, or the total volume of the engine exhaust, is measured, and the concentration value measured by the analyzer can be converted to mass of emission. The flow rate of the exhaust gas can be measured either directly or indirectly. In this study, we use direct sampling method to measure the exhaust emissions.

2.1.1 Direct sampling method

The direct sampling method measures the concentration of sample gas without dilution. To calculate mass emissions with direct sampling requires simultaneous measurement of the exhaust flow rate. However, there are response time delays for the exhaust flow rate meter and each gas concentration analyzer. When the engine operating conditions change dynamically, the response time differences between instruments because of errors in the calculated results without time alignment. Historically, the direct sampling method has been used for emission measurement during steady-state conditions. The direct sampling method is frequently used for determining the Exhaust Gas Recirculation (EGR) ratio and/or the air-to-fuel ratio from exhaust gas concentration measurements. In current transient testing applications, the direct sampling method may be used for mass emissions calculations when the individual response times of the exhaust flow meter and gas analyzers are determined and the data timing are aligned before calculating results.

Figure 2.1 shows an example of the equipment configuration for the direct sampling method. A portion of exhaust gas is extracted from the tailpipe and filtered to remove particulate matter before the sample is delivered to the gas analyzers. Because the

exhaust gas generally contains high concentrations of water vapor, the sample gas is often conditioned with a dehumidifier (a cooler) and considered as a dry sample gas. When a dehumidifier is employed, almost all of the water vapor is condensed and removed from the sample. This reduces the exhaust volume and rises the concentrations of each remaining exhaust constituent. However, there are cases in which water vapor is not removed from the exhaust sample by dehumidification, when the exhaust contains components of interest that are water soluble or when water vapor does not interfere with the analysis. In such wet-based cases, the sampling line from the sample point all the way to the analyzer is heated to prevent water condensation.



Figure 2.1 Example of gas measurement configuration using direct sampling method[13]

2.2 Retrieval Methods

2.2.1 Analysis of measuring time delay

During measurement of transient signals, the changing pattern of the measured signals are different from real emissions. Because of the limited measuring method, the measured signals have certain time delay and amplitude distortion. Thus the measured concentration of each components do not correspond with the engine operating point at the same time, they differing from certain time delay. For continuous sampling test, the total time delay Δt consist of the following three parts:

1) Time delay Δt_1 deriving from engine exhaust manifold to the end of the tail pipe. Flowing dynamics of exhaust emissions in this section of pipeline is complex, especially when crossing through catalysts, the flowing state as well as the physical and chemical conditions will change. However, on the whole we can think that the time delay Δt_1 is varying with the change of exhaust flow. Time delay Δt_1 can be neglected when only considering to get the transient emission parameters at the vehicle tail pipe. 2) Time delay Δt_2 deriving from exhaust emissions flowing in the sampling pipe of the analyzer (the gas flow time delay from the analyzer sampling point to the analyzer sensor). This section of time is basically constant or changing little, and it does not change with the exhaust flow.

3) The response time of the emission analyzer Δt_3 . It is the intrinsic property of the instrument. When applied to steady-state combustion engine emission measurement, the effects of signal distortion and time delay are low, thus the measurement results can be used directly. However, when it is applied to transient state engine emission measurement, there could come up with amplitude distortion and time delay Δt because of the rapidity and instability of combustion.

In this report, the TranKin[4] algorithm took all these three parts of time delay into account. TranKin was used to deconvolute the signal to be able to calculated the true mass signal. Thus what we got using deconvolution was the real engine exhaust emissions.

2.2.2 Dispersion and convolution

The effect of a step change of the concentration signal which is detected as a spread out signal is called dispersion, in that it gives rise to a distribution of the residence time of fluid elements. Brenner ans Edwards[3] defined dispersion as a global spatial spread of an initially inhomogeneously distributed packet of some conserved entity owing to a stochastic transport mechanism superposed upon a deterministic inhomogeneous, convective transport mechanism. Generally, the term convolution is used to describe this phenomenon, and deconvolution is used to describe signal processing done to correct for the effects of convolution. Signal distortion will affect the results of reactive experiments, and therefore it is necessary to characterize the measuring instruments with respect to these smearing effects in order to properly interpret the results of transient experiments. A method is needed to retrieve the true response of the reactor system for quantitative data analysis. In appendix A, TranKin is developed to process transient kinetic data with deconvolution algorithm[4].

2.2.2.1 Convolution

Smith described convolution as "a mathematical way of combining two signals to form a third signal[2]. It is the single most important technique in Digital Signal Processing. Using the strategy of impulse decomposition, systems are described by a signal called the impulse response. Convolution is important because it relates the three signals of interest: the input signal, the output signal, and the impulse response."

The mathematical operator used to denote convolution is "*". The inverse operation, deconvolution, is denoted by the operator "*-1". The mathematical details and code examples for convolution calculation can be found in [S.Smith].

The equation y(t) = x(t) * g(t) describes the convolution problem in the time domain,

Where

- y(t) convoluted signal
- x(t) true (unconvoluted) signal
- g(t) transfer function

$$y[i] = \sum_{j=0}^{M-1} g[j]x[i-j]$$
(2.1)

The formal definition of convolution, written in the shorthand is:

$$y[n] = x[n] * g[n]$$
 (2.2)

In this equation, g[n] is an M point signal with index indexes running from 0 to M-1.

2.2.2.2 Gas dispersion in tube

The decay of exhaust concentration signal results from the parabolic profile of the laminar flow velocity when it travels through the exhaust pipe. What's more, molecular diffusion will also cause effects in a way. The diffusion in flow direction causes axial dispersion, and the diffusion towards wall keeps the concentration profile uniform[4].

2.2.2.3 Gas dispersion in cell

When the geometry difference of cylindrical tube is taken into consideration, it comes to be more complicate to analyze the response time delay. The simulated output signal changes with time and thus to acquire the kernel that represents the influence of convolution.

2.2.3 Restoration Methods

The effect of convolution has been recognized for some years and there have been multiple attempts to restore the original signal by undoing the effects of convolution.

2.2.3.1 Calculation from Engine Power

S.Smith[2] tries to predict the signal prior to convolution of CO_2 and NO_X mass flows from instantaneous engine power. To do this, he first establishes a correlation between the convoluted concentration traces and an engine power signal with the same convolution applied. In a second step he then applies this correlation to the un-convoluted power signal, calculating a signal that is close to the true concentration traces. While this method may deliver acceptable results for emissions that are proportional to engine power like CO_2 , for NO_X the result was not as good, as NO_X is almost never proportional to power.

The method is also not usable for emissions from an engine with electric power assist system(EAS), as the EAS system significantly changes or even eliminates the correlation between emission and engine power.

2.2.3.2 Reconstruction by Deconvolution

James D.Pakko[6] and T.Summers[7] use deconvolution to reconstruct the true signal from the signal recorded by the analyzer. Deconvolution is the inverse operation of convolution. The use of deconvolution implies the assumption the the changes caused by the sampling system follow the mathematical process of convolution.

Consider a linear, time-invariant system that can represent the instruments response, as shown in Figure 2.2. The block system is supposed to reflect the input signal at the output.



Figure 2.2 Block diagram of the reactor setup[6]

In case of deconvolution, the input tracer signal x of the injection-sampling system and the response signal y are supposed to be available. If a certain linear flow model which used to describe the transport process is determined, the impulse response is also thought to be available. Deconvolution can be used to reconstruct any type of concentration signal, because it has nothing to do with engine types.

2.2.3.3 Deconvolution

The equation $x(t) = y(t) *^{-1}g(t)$ describes deconvolution. In order to solve the equation with simple algorithm, one can use the fact that a convolution operation in the time domain can be expressed as a multiplication operation in the frequency domain. Conversely, a deconvolution operation in the time domain can be expressed as a division operation in the frequency domain.

Time domain	Frequency domain		
y(t) = x(t) * g(t)	\Rightarrow	y(s) = x(s)g(s)	(2.3)
1		v(s)	

$$x(t) = y(t) *^{-1} g(t)$$
 \implies $x(s) = \frac{y(s)}{g(s)}$ (2.4)

Where

- y(t) convoluted signal in time domain
- x(t) original (unperturbed) signal in time domain
- g(t) transfer function in time domain
- y(s) convoluted signal in frequency domain
- x(s) original (unperturbed) signal in frequency domain
- g(s) transfer function in frequency domain

While the signals in the frequency represent the Fourier transform of the signals in the time domain.

y(s) = F y(t)	(2.5)
x(s) = F x(t)	(2.6)

$$g(s) = Fg(t) \tag{2.7}$$

For all calculations in the frequency domain it must be considered, the y(s), x(s), and g(s) are all signals where each value represents a complex number having a real and an imaginary part. All mathematical operations in the frequency domain must therefore follow the rules for complex numbers.

2.3 Transfer function

James D.Pakko[6], T.Summers[7] and B.Ganesan[1] approximate the transfer function by mathematical equations consisting of combinations of Gaussian functions, impulse response functions, or Gamma functions[6]. T.Summers noted that the first half of the response function is similar to the Gaussian function, but shows a longer decay at the end of the trace.



Figure 2.3 Typical transfer function[7]

The reason for this shape is thought to be that in the sample system convolution is caused by the combination of transport time through the system, diffusion, and mixing in the plenum. The combination of the effects is mathematically described by the following model:

$$h(t) = h_L(t) * (r h_s(t) + (1 - r) h_f(t))$$
(2.8)

Where

h(t) is the combined transfer function

 $h_L(t)$ is the Gaussian function modeling the transport delay and diffusion

- h_s(t) is the impulse response function modeling recirculation mixing
- $h_{f}(t)$ is the impulse response function modeling bulk mixing

r is the fraction of recirculation mixing



Figure 2.4 Calculating model including diffusion and mixing[6]

Using the first derivative of the step response is an alternative to approximating the transfer function with a mathematical model. When calculating the first derivative of system response data, any noise in the signals is amplified and the calculated first derivative shows some high frequency oscillations. While the Fourier transform of the modeled transfer function would give a nice smooth function in the frequency domain, the Fourier transform of the noisy first derivative would look similar, except the fact that it would have some high frequency noise.

In the studying of exhaust system's transient response time, the forward process is determined by the convolution of the true signal, combining with the intrinsic broadening effects in the transfer function of the system. Deconvolution is developed for enhancement of time resolution in transient experiments. The analysis is based on the discrepancy principle for calculation of the regularized parameter, and this provides satisfactory convergence and can handle the noise in measurement in a way.

2.4 Normalization

Normalization refers to the creation of shifted and scaled versions of statistics, where the intention is that these normalized values allow the comparison of corresponding normalized values for different data-sets in a way that eliminates the effects of certain gross influences, as in an anomaly time series.

Let us find the **E** curve for a vessel of volume V m³ through which flows v m³/s of fluid. For this instantaneously introduce M units of tracer into the fluid entering the vessel, and record the concentration-time of tracer leaving the vessel. This is the C_{pulse} curve. From the material balance for the vessel we find

(Area under the
$$C_{pulse}$$
 curve): $A = \int_{0}^{\infty} Cdt \cong \sum_{i} C_{i} \Delta t_{i} = \frac{M}{v} \qquad \left[\frac{kg \cdot s}{m^{3}}\right]$ (2.9)

(Mean of the
$$C_{pulse}$$
 curve): $\bar{t} = \frac{\int_{0}^{\infty} tCdt}{\int_{0}^{\infty} Cdt} \cong \frac{\sum_{i} t_{i}C_{i}\Delta t_{i}}{\sum_{i} C_{i}\Delta t_{i}} = \frac{V}{v}$ [s] (2.10)

All this is shown in Figure 2.5.



Figure 2.5 The useful information obtainable from the pulse trace experiment[8]

To find the **E** curve from the C_{pulse} curve simply change the concentration scale such that the area under the curve is unity. Thus, simply divide the concentration readings by M/v, as shown in Figure 2.6.

$$\mathbf{E} = \frac{C_{pulse}}{M/\nu} \tag{2.11}$$

Using this equation, the E curve would be the normalized transfer function.

We have another residence time distribution (RTD) function \mathbf{E}_{θ} . Here time is measured in terms of mean residence time $\theta = t/\bar{t}$. Thus

$$\mathbf{E}_{\theta} = \bar{t}\mathbf{E} = \frac{V}{v} \cdot \frac{C_{pulse}}{M/v} = \frac{V}{M}C_{pulse}$$
(2.12)



Figure 2.6 Transforming an experimental C_{pulse} curve into an *E* curve[8]

2.5 Conclusion from literature study

In this chapter we learned means of measuring gaseous emissions, here we used direct sampling method. Besides, we studied the reason for the time-delay during measurements. There were generally two ways to retrieval the original signal: calculation from engine power and reconstruction by deconvolution. Deconvolution was preferred as it could restore most of the original signal and had nothing to do with engine types.

3 Emission measurement and analysis

Due to the time required to transport the gas through the sample system the analyzer response is delayed. This effect is well know and all international regulations require correcting the dynamic analyzer response for this delay. The regulations require shifting the measured signals by the time from the step input to the T50 time - usually call "transformation time".



Figure 3.1 Analyzer response to a step input [5]

3.1 Characterization of the instrument

In order to evaluate the property of the instrument, we firstly run a calibration-like experiment in the measurement device. Here 15.77% v/v CO₂ is dosed at the inlet, without any catalytic or reactive material that can interact with the gas phase spices, the spread out step function is observed as the measured signal in the outlet. Figure 3.2 show the equipment configuration for the primary evaluation.



15.77% CO2

Figure 3.2 Equipment configuration for the primary configuration(OVN stands for a pump used for sampling)

As we can see in the figure above, the sampling point starts from the tube connected after the adjusting valve. Here the OVN is actually a pump that used for exhaust gas sampling from the tailpipe.

Due to changes that the gas exposed during transport from the probe to the analyzer, the dynamic change of the recorded signal is slower than the true signal. As shown in Figure 3.3, the response signal is rising with time until it reaches to the same value with the input concentration.



Figure 3.3 Instrument response to a step change in CO2 concentration

Here we use voltage to represent CO_2 concentration, 10 volt denotes 16% volume percent. In order to determine the repeatability and reproducibility of the system, a number of experiments were done in different days.

The reason for this shape is thought to be that in the sample system convolution is caused by the combination of transport time through the system, diffusion, and mixing in tubes. When the gas flows through filters and manifolds, then bulk mixing and recirculation mixing occur, as shown in Figure 3.4.



Figure 3.4 Gas mixing in a tube[6]

3.2 Repeatability and reproducibility

Repeatability or test-retest reliability is the variation in measurements taken by a single person or instrument on the same item and under the same conditions. An imperfect test-retest reliability would cause certain variability. Such variability can be caused by, for example, intra-individual variability and intra-observer variability. A measurement may be said to be repeatable when this variation is smaller than some agreed limit.

Reproducibility is the ability of an entire experiment or study to be reproduced, either by the researcher or by someone else working independently. It is one of the main principles of the scientific method and relies on other things being equal. The result values of distinct experimental trials are said to be commensurate if they are obtained according to the same reproducible experimental description and procedure. The closeness of agreement between independent results obtained with the same method on identical test material but under different conditions (different operators, different apparatus, different laboratories and/or after different intervals of time). The standard deviation and average values of various experiments done on two different days are as shown in Table C-1 and Table C-2, respectively, which are attached in Appendix C.

Here the 'deviation' is defined as the sum of squares of deviations from the average value. According to the experiments, from which we can derive the repeatability and reproducibility of this system, comparison of the deviation and average values are made as shown in Figure 3.5.



Figure 3.5 (a) Repeatability and (b) reproducibility of the instrument

The next step is calculating the confidence interval of both days to see if they are overlapped. For the first day the 95% confidence interval is [9.79, 9.86], and it is

[9.78, 9.85] for the second day, the difference between which is just 0.01. We can see that the standard deviation for different experiments are limited to certain interval which is smaller than 0.02. Nevertheless, the discrepancy of average values is also under 0.1, from which we can conclude that the repeatability as well as the reproducibility of the instrument is fine.

3.3 Time delay of the response signal

In the time domain the transfer function of is equal to the first derivative of the response to a step input signal. For practical use in engine tests, this means that this impulse response is equal to the first derivative of the signal recorded during a system response. Therefore system response data is used to determine the transfer function. Convolution of the input signal with the transfer function of the system can be represented as:

$$\int_{a}^{b} g(t-t')x(t')dt' = y(t)$$
(3.1)

where g is the transfer function of the system, x is the unknown input and y is the measured output concentration.

Generally speaking, all the possible sources of signal distortion which have a linear effect on the output signal can be lumped into a transfer function which is also referred to as the kernel of the convolution equation. Therefore, determination of the kernel is an essential part of the characterization study.



Figure 3.6 Example of simple system response to a rectangular impulse[1]

Diffusion due to concentration gradient within the sample stream has caused an analyzer to respond to a short rectangular input impulse with an asymmetrical response, which is shown in Figure 3.6.

3.4 Experiments of the exhaust system

As shown in Figure 3.7, in this experiment we use an air injector to provide different flowing rate air mixing with certain pressure 100 percent CO_2 to simulate the engine emissions under various driving conditions. The air flow rate are set to be 250 slpm, 500 slpm, 750 slpm, 1000 slpm respectively, with the CO_2 pressure set as 2.5bar and 5bar separately.



Figure 3.7 Experiment instruments: 1-CO₂ *bottle 2-Mixer 3-Catalyst 4-Diesel particulate filter 5-Selective catalytic reduction 6-Silencer 7- Air Injector*

3.4.1 Response of the exhaust system to impulse

Firstly, we have to consider the number of tail pipes, as we can see in Figure 3.9, the exhaust coming out either from one or two of the pipes. The differences for each condition are compared in the following figure.



Figure 3.8 Comparison of single and dual tail pipes

The delay time for both conditions are same, but the response time for single tube is more rapid. Considering the experiments and reality, here we use one tail pipe to sample and measure the exhaust emissions, which is shown in Figure 3.9.



Figure 3.9 Using single pipe to measure emissions

We set a 500ms rectangular impulse and measure the response signal at sample point, thus can estimate the time delay caused by hydrodynamic dispersion in laboratory reactor systems. From the experiments we did under various air flow rates and pressures, we could acquire the following figures.



Figure 3.10 Sample system response to a rectangular impulse at 1000 slpm,5 bar



Figure 3.11 Sample system response to a rectangular impulse at 250 slpm,5 bar



Figure 3.12 Sample system response to a rectangular impulse at 250 slpm, 2.5 bar(10V = 16% CO2)

From the figures above we can see that the response asymmetrical curves resemble Gamma distribution[7]. The delay time is affected by the air flow rates, which means that the faster of the flow rate, the shorter of the delay time. That is because if the gas flow transports faster in the exhaust system, the dispersion time tends to be shorter. And the CO_2 pressure will affect the exhaust emissions concentration at the sample point. If the injected CO_2 pressure comes higher under the same flow rate, the measured concentration will also be higher.

3.4.2 Reynolds number

The Reynolds number (Re) is a dimensionless number that used to characterize the fluid flow. It is defined as the ratio of fluid inertial force and viscous force.

When the Reynolds number is small, the effect of viscous force to the fluid is greater than inertial force. Thus flow velocity perturbations to the flow field will decay due to viscous forces, and the fluid is stable laminar flow. On the contrary, turbulent flow occurs at high Reynolds number and is dominated by inertial force, which tends to produce chaotic vortices, eddies and other flow instabilities.

The characteristic length of the measuring exhaust system is 63.2mm, and the kinematic viscosity changes little under different pressure. Thus the Reynolds numbers under various flow rates are as shown below.

$$R_e = \frac{Q D_H}{\nu A} \tag{3.2}$$

Where

Q	is the flow rate
D_{H}	is 63.2mm

υ	is 14.8 mm ² /s
А	is pipe area
Temperature	is room temperature.

• • • • • • • • • • • • • • • • • • • •	<i>v</i>
Flow Rate [slpm]	Reynolds Number
250	5680
500	11300
750	17000
1000	22700

Table 3.3 Reynolds numbers under different flow rates

According to the calculations above, we can see that all the Reynolds numbers are larger than 4000, thus they are all turbulent flows. The flow conditions (turbulent flow) are same for all experiments.

4 Deconvolution of experiment data

In order to measure dynamic exhaust emissions, we need to know the measured results and transfer function of the exhaust system under different conditions. Here we measured the exhaust CO_2 concentration under various exhaust flow rate. Firstly, pulse input CO_2 concentration was injected into the system, thus we could get the transfer function curves for each experiment. Then we injected step input signals under the same experiment conditions to get the response curves. After deconvolution of the step response curves, we got the transfer function of the exhaust system under certain conditions. Then we can evaluate the usability of TranKin when comparing the transfer functions got from two different tests. If the deconvoluted results fit well with the pulse response, then we can continue the sinus input experiment[4.3], which means that the input CO_2 concentration is changing as sinus pattern, and try to find the transfer function under transiently changing exhaust flow rates.

4.1 Pulse response

In the first step, we inject 100 percent CO_2 into exhaust emissions under different flow rates and injection pressure. Here we set the conditions to be 250 slpm, 500 slpm, 750 slpm and 1000 slpm under 5 bar and 2.5 bar. The CO_2 pulse time is 500ms in each experiment.

Take 250 slpm, 5 bar and 250 slpm, 2.5 bar for example, after instantaneously introducing certain units of tracer into the exhaust fluid, the response curve increases to the top and then begins to decay. The response figure is as shown in Figure 4.1.



(a) 250 slpm, 5 bar (10 Volt=16% CO₂)



(b) 250 slpm, 2.5 bar(10 Volt=16% CO₂) Figure 4.1 Pulse response under the same flow rate and different injection pressure

The response curve is actually the transfer function of the exhaust system under 250 slpm, 5 bar and 250 slpm, 2.5 bar. In order to be compared with the deconvoluted step response, the curves need to be normalized. Here we use the response data to divide the area under the C_{pulse} curve, and the **E** curve we get would be the normalized transfer function.

In order to find the effect of pressure, we compare the curves got from the same flow rate and different pressures, which is shown in Figure 4.2. From the figure we can see that the delay time is actually the same, and the increase and decay trend is also the same. Thus we can conclude that the delay time of the exhaust emissions is independent of CO_2 pressure. It should only depend on the flow rate as well as the property of the exhaust system.

Here we can see that there is a narrow space between these two curves. It may because that when the injected CO_2 pressure arises, the concentration gradient in the exhaust pipe comes to be small. Thus the response rate could be faster under higher pressure. Thus the true signal we got from deconvolution under high pressure could match better with the original input signal as it is more sensitive to the change of concentration.



Figure 4.2 Comparison of the delay time under different pressure after normalized



Figure 4.3 Comparison of the delay time under different flow rates

As shown in Figure 4.3, with increasing of exhaust flow rate, the delay time of the measured emissions comes to be shorter. What's more, the dispersion time would also

decrease at the same time. This is because the increased turbulence will increase mixing and thus the dispersion. But since the residence time is longer, the dispersion is greater.

However, we also need to check the accuracy of the data we have got. In order to do this, we should calculate the area under the deconvolution curve to see if they are the same. The results are as shown in Equation 4.1. We can see that they are almost the same.

$$A_{250slpm,2.5bar} = 0.1054 \qquad A_{500slpm,2.5bar} = 0.1074 \qquad A_{750slpm,2.5bar} = 0.1066$$
(4.1)

4.2 Step response

In the step input experiment, we set the instruments to test under the same conditions with pulse experiment. The maximum value of the injected CO_2 concentration is determined from the amplitude of pulse response. For experiment under 250 slpm, 5 bar, the maximum value of the step input is 2.72% volume percent. Here we use a needle valve to tune the CO_2 concentration. The step response curve is shown in Figure 4.4.



Figure 4.4 The step response under 250 slpm,5 bar

Here we can see that there are some spikes in the figure, that is because of the electronic noise of the instruments. In order to deconvolute the step response, we have to clean up the obvious spikes in the curve. The cleaned experiment data is shown in Figure 4.5.



Figure 4.5 Modified experiment data under 250 slpm,5 bar

Then we need to use TranKin to deconvolute the cleaned data, here the regularized parameter alfa is set to be 150[9]. The deconvoluted curve is shown in Figure 4.6.



Figure 4.6 Deconvoluted step response curve under 250 slpm,5 bar

In order to compare with the spike response we get before, the deconvoluted curve has to be normalized. The normalized curve is shown in Figure 4.7.



Figure 4.7 Normalized step response curve under 250 slpm,5 bar

At last, we have to compare the normalized pulse response and the step response to see if the deconvoluted curve is correct. Here we can see it in Figure 4.8.



Figure 4.8 Comparison of the pulse response and the deconvoluted step response

From the figure above we can tell that the changing trend of pulse response and deconvoluted response fit well with each other. Though deconvolution could recover most of information lost by convolution, it is not able to provide a complete recovery in most cases. The peak value of the deconvoluted curve is determined by the regularized number alfa, and the larger of the number, the lager of the difference between the pulse response curve and the deconvoluted step response curve. As we can see in Figure 4.9, when alfa was set to be 600, there was an obvious difference at the delay time. We have to find an acceptable compromise between the recovery of the data and the added errors. And this confirms that deconvolution can be used to measure the exhaust emissions as it provides a significant improvement.



Figure 4.9 Comparison of the pulse response and the deconvoluted step response

Here the difference between these two curves was derived from unmatched injected CO_2 . Because the pulse input experiments and the step input experiments were done in different time period, thus the CO_2 concentration and pressure in the gas tank could be different. If taken this condition into consideration, such error was acceptable.



Figure 4.10 The input signal got from the deconvoluted transfer function

We use the deconvoluted transfer function to get the step input signal to see if the result is true, which is as shown in Figure 4.9. The curve rises straightly to the maximum value and keep stable, this is pretty close to the real input. Therefore, we can confirm that deconvolution is a fine way to measure the engine exhaust emissions.

4.3 Sinus experiment

In order to test the response at transient exhaust emissions, we set the exhaust flow to change in sinus pattern. The maximum rate is 750 slpm, the minimum is 250 slpm, and the time period is 10s, which is shown in Figure 4.11. The CO₂ concentration is tuned to be 12% when the flow rate is 250 slpm at 2.5 bar, and the injection lasts 10s every other 20s. The injected CO₂ flow rate is shown in Equation 4.2. Here we set the CO₂ flow to be injected at the exhaust wave peak. The test response can be seen in Figure 4.13.

$$\frac{Q_{co_2}}{Q_{co_2} + 250 \, slpm} = 0.12 \qquad \Rightarrow \qquad Q_{co_2} = 34 \, slpm \tag{4.2}$$

Therefore, the CO₂ flow rate in the total process is

$$Q_{CO_2} = \begin{cases} 34, & 10 + 30n \le t \le 20 + 30n, n = 0, 1, 2, 3... \\ 0, & other \end{cases}$$
(4.3)



Figure 4.11 Sinus input exhaust flow



Figure 4.13 Sinus input response at 2.5bar

Here the injected signal is a theoretical calculation of the concentration at the injection point. As we can see that the response signal has time delay with input concentration, and the dispersion time is longer. We need to deconvolute the sinus response to get the transfer function. Firstly, we use the same transfer function obtained from 500 slpm steady flow experiments for the sinus experiment. This will assume that the dispersion and delay time is constant.

In order to check if the transfer function is correct or not, we need to calculate the total CO_2 mass during sinus exhaust flow. Here we will check the results for four different cases, which is show in Table 4.1.

*Table 4.1 Four different cases under which we calculate the total CO*₂ mass

Case 1	Calculate from experiment data
Case 2	Calculate with the time delay is shifted
Case 3	Calculate with the transfer function of step experiment
Case 4	Calculate from interpolation

Firstly, we get it from the experiment data, as shown in Equation 4.4. Here the y(t) refers to the data got directly from sample point, which is shown in Figure 4.13.

$$m = \sum_{t=1}^{end} y(t)_{\exp} * Q(t) * \rho = \sum_{t=1}^{end} y(t)_{\exp} * 10^{-6} * (Q_{air}(t) + Q_{CO2}) * \frac{10^{-3}}{60} * 1.83 * 1000 = 93g$$
(4.4)

In case 2, we calculate the total mass with the delay time from the measurement point to the MEXA system is removed, which is shown in Equation 4.5. Here the delay time is 4.5s.

$$m = \sum_{t=1}^{end} y(t-\tau) * Q(t) * \rho = \sum_{t=1}^{end} y(t-\tau) * 10^{-6} * (Q_{air}(t) + Q_{CO2}) * \frac{10^{-3}}{60} * 1.83 * 1000 = 96g$$
(4.5)

In case 3, we use the transfer function of step experiment under 500 slpm, 2.5bar, as shown in Figure 4.14, to get the input signal, and then calculate the total mass flow. Figure 4.15 is the deconvoluted curve.



Figure 4.14 Transfer function of step experiment under 500 slpm, 2.5 bar



Figure 4.15 Input signal got from deconvolution[Equation 2.4]

$$m = \sum_{t=1}^{end} y(t)_{stepdeconv} * Q(t) * \rho = \sum_{t=1}^{end} y(t)_{stepdeconv} * 10^{-6} * (Q_{air} + Q_{CO2}) * \frac{10^{-3}}{60} * 1.83 * 1000 = 106g$$

(4.6)

However, since the exhaust flow is changing with time, the delay time will also change. The delay time at different flow rate is shown in Table 4.2.

Tuble 4.2 Delay time at afferent flow rate				
Flow Rate[slpm]	250	500	750	1000
Delay Time[s]	14	9	7.5	6.5

Table 4.2 Delay time at different flow rate

From the data above we can get the Figure 4.16.



Figure 4.16 Delay time at different flow rate

Therefore, we can make an interpolation with the curve above. In the sinus response, the minima of the exhaust flow appears at the highest amplitude point, and the peak appears at the lowest point. We can see that Figure 4.17 shows the original y(t) and the new, corrected y(t). Thus we can calculate the CO₂ mass according to the equation above, which is shown in Equation 4.7.



Figure 4.17 Comparison between the original data and corrected data

$$m = \sum_{t=1}^{end} y(t)_{deconv} * Q(t) * \rho = \sum_{t=1}^{end} y(t)_{deconv} * 10^{-6} * (Q_{air} + Q_{CO2}) * \frac{10^{-3}}{60} * 1.83 * 1000 = 97g$$
(4.7)

From the equations above we can see that the result got from the transfer function of 500 slpm(in case 3) is not suitable for the calculation of CO_2 mass, but for case 4 the results get better. Because it overlooked the dynamic change of the flow rate as well as the delay time. Here the reason for why the result of each case is close to the reality or not is shown in Table 4.3.

Case 1	The result is far from reality. Because it was calculated directly from the				
	experiment data, which had time delay in measurement.				
Case 2	The result is close to reality. Because it took the time delay into				
	consideration and shifted it away.				
Case 3	The result is not close to reality. Because it overlooked the dynamic change				
	of the flow rate as well as the delay time.				
Case 4	The result is close to reality. Because it used interpolation to calculate the				
	delay time at each point and moved it away.				

Table 4.3 The reason for why the result of each case is close to the reality or not

5 Conclusion

Conventional vehicles are working at transient states in most of the time. In order to correctly measure the engine emissions, new technique needs to be developed. However, because of the transmission procedure and dispersion, the dynamic property of exhaust is complicated. And this result to the measured concentration does not correspond with operating points. Instead, there is certain time delay. In the thesis, we use deconvolution to do time alignment of engine emission measurements. We have done the following work.

(1) Firstly, we did calibration-like experiments to test the responding property of the instruments. Use 15.77% volume percent CO_2 to do the tests, and observe the pulse and step signal response. Then we made sure that the repeatability and reproducebility of the system met the testing requirement, here the relative standard deviation is lower than 0.2%.

(2) Test the pulse response under 250, 500, 750, 1000 slpm at 5 bar and 2.5 bar. The response curves were transfer functions of the system under each exhaust condition. And we could derive that the delay time was only decided by flow rate. What's more, the CO_2 pressure will affect the exhaust emissions concentration at the sample point. If the injected CO_2 pressure comes higher under the same flow rate, the measured concentration will also be higher.

(3) Test the step response under different conditions, and then we have to modify the data with random spikes. Deconvoluting the response signal, thus we got the transfer function, and comparing it with pulse response. We could conclude that the true signal we got from deconvolution fitted well with the original transfer function. Therefore, deconvolution could be used to do time alignment.

(4) At last, we did sinus experiment in order to confirm that deconvolution was useful to do dynamic measurements. The total exhaust emissions we got from deconvolution were almost the same with the true injected signal, the difference was 4.12%. It looks promising and it is an important first step. However, there is still more work need to be done in the next step, in that we still cannot decide the usage of the methodology under random exhaust conditions. So this method could not be used in reality for now.

6 Future Work

In the report, we analysed the time delay of the measured engine emissions signal and tried to retrieve the original data. We did quite a lot of experiments under various conditions and used deconvolution to do the inverse problem, it turned to be useful to get the real exhaust emissions.

However, in order to do time alignment of engine emission measurements accurately in any type of driving conditions, more work needs to be done in the future. Here we already did impulse input experiments, step input experiments and sinus input experiments, and got the original engine emissions from the experiment data. In the next step, we are supposed to do arbitrary input experiments, in that most vehicles are driving in transient states in the public traffic conditions. Then from the measured transient state data, we can acquire the real exhaust emissions using deconvolution. Thus we can successfully do time alignment of engine emission measurements in any kind of driving conditions.

There is still one more thing need to be noticed that the regularized parameter could affect the result of TranKin. When the parameter is small, the amplitude of the deconvoluted step-response is even higher than the impulse response after normalization. But when the regularized parameter is higher, the deconvoluted step-response would become wider on the x axis than the impulse response though the amplitude comes smaller. That is because while TranKin is an algorithm that stabilize the solution to the inverse problem, that comes at the cost of losing some accuracy of course. Then we have to find an balanced regularized parameter that could be used under various conditions.

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Acknowledgments

In this study, pulse input and step input tests have been done with indented injected air flow rate and CO2 concentration. The tests have been carried out from January 2015 to May 2015. The work aims to develop a new method to measure transient exhaust emissions. The project is carried out at the Volvo Car Group, Sweden. All the tests are financed by Volvo Car Group.

This part of the project has been carried out with Jonas Sjoblom as supervisor. All tests have been carried out in the laboratory of the Department of Emission Engineering at Volvo Car Company. Mr. Olle Berg and Mr. Lasse Simonson are highly appreciated for their help with planning the tests. I would also like to thank Soheil Soltani for his kind help.

Finally, it should be noted that the tests could never have been conducted without the sense of high quality and professionalism of the laboratory staff.

Göteborg May 2015

Guo Dong

Appendix A - TranKin

Please refer to the web page attached below. <u>http://www.chalmers.se/en/staff/Pages/soheil-soltani.aspx</u>

Appendix B - Experiments Data



The documents attached above are the experiments data got under different conditions. The injected air flow rates ranked from 250 slpm to 1000 slpm under 2.5 bar and 5bar. What's more, the experiments were done under room temperature. For the impulse input experiments, the impulse time was set to be 500 ms with 100 percent carbon dioxide. As for the sinus experiments, the time period is 10 seconds, the minimum injected air flow is 250 slpm and the maximum value is 750 slpm. When the injected air flow is 250 slpm constantly, CO_2 volume concentration 12%. What's more, the injected CO_2 pressure is 2.5bar while the spike time last 10 seconds at wave peak every 20 seconds.

Appendix C- Repeatability and Reproducibility

Experiment	Deviation [Volt]	Standard Deviation [Volt]	Relative Standard Deviation[%]	Average Value [Volt]
1	0.131	0.0189	0.19	9.81
2	0.171	0.0186	0.19	9.81
3	0.146	0.0172	0.18	9.82
4	0.117	0.0172	0.17	9.81
5	0.163	0.0172	0.18	9.82
6	0.138	0.0170	0.17	9.83
7	0.175	0.0166	0.17	9.83
8	0.159	0.0165	0.17	9.82
9	0.192	0.0180	0.18	9.82
10	0.116	0.0175	0.18	9.82
11	0.167	0.0170	0.17	9.82
12	0.154	0.0185	0.19	9.82
13	0.132	0.0169	0.17	9.82
14	0.189	0.0182	0.18	9.83
15	0.159	0.0178	0.18	9.83
16	0.121	0.0171	0.17	9.83
17	0.133	0.0173	0.18	9.83
18	0.139	0.0157	0.16	9.83

Table C-1The standard deviation and average values on the first day

Experiment	Deviation [Volt]	Standard Deviation [Volt]	Relative Standard Deviation [%]	Average Value [Volt]
1	0.125	0.0180	0.18	9.81
2	0.162	0.0189	0.19	9.81
3	0.151	0.0183	0.19	9.81
4	0.133	0.0182	0.19	9.81
5	0.174	0.0180	0.18	9.81
6	0.110	0.0179	0.18	9.81
7	0.104	0.0180	0.18	9.81
8	0.123	0.0201	0.21	9.81
9	0.183	0.0194	0.20	9.82
10	0.181	0.0192	0.20	9.82
11	0.191	0.0190	0.19	9.81
12	0.173	0.0190	0.19	9.81
13	0.152	0.0179	0.18	9.81
14	0.189	0.0183	0.19	9.82
15	0.177	0.0180	0.18	9.82
16	0.192	0.0172	0.17	9.81
17	0.181	0.0187	0.19	9.82
18	0.176	0.0178	0.18	9.81

Table C-2The standard deviation and average values on the second day