THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING
IN MACHINE AND VEHICLE SYSTEMS

Online State Estimation in Electrified Vehicles Linked to Vehicle Dynamics

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Cover:
Example of functional architecture for state and parameter estimation

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ABSTRACT

Electric vehicles have the potential to significantly reduce energy consumption and emissions for personal and commercial road transport and number of electric vehicles is likely to increase in the future due to stricter emission legislations. In order to accelerate market penetration, the competiveness of electric vehicles should increase in comparison to conventional vehicles. Active safety is an area where electric vehicle have a possible advantage over conventional vehicle and that could reduce the number of fatalities and injuries in road traffic accidents.

However, the performance of active safety systems today is limited by the knowledge of vehicle state estimates and vehicle parameters, e.g. vehicle speed and the tyre-road friction coefficient. This thesis investigates the potential benefits of using the electric motor as a sensing element to improve state and parameter estimations and thereby also active safety systems. In particular, accurate torque estimation provided by electric propulsion is utilized as an additional source of information. The possibility of using active tyre force excitation for the estimation of the tyre-road friction coefficient is also investigated.

The results show that there is a potential to improve the longitudinal and, in some situations, the lateral tyre force estimation using electric motors. However, the estimates are sensitive to errors in the inertial parameters. A method for estimating the vehicle inertial parameters was therefore proposed. The estimate of the vehicle mass converged to within 3% of the measured value for the evaluated test cases. However, the estimation of the longitudinal centre of gravity position and the yaw inertia of the vehicle is sensitive to measurement errors and disturbances. This is mainly due to the weak link between lateral and longitudinal dynamics in normal driving conditions. An alternative method using the seat belt indicators was therefore proposed. This method improves the estimates of these parameters on average when compared to assuming default values.

A method to estimate the tyre-road friction coefficient with active tyre force excitation was also proposed. This method enables the estimation of the tyre-road friction coefficient when demanded from an active safety system. Electric motors offer several advantages for active tyre force excitation. The fast response and the ability to apply both positive and negative torque can improve the slip control of the wheels, which is crucial for vehicle stability during the intervention.

In summary, the improved wheel torque estimation has the potential to improve the tyre force estimation in both the longitudinal direction directly and in the lateral direction through improved inertial parameter estimation. Furthermore, the electric motor as an actuator provides further opportunities during active tyre force excitation.

Keywords: state estimation, vehicle dynamics, tyre-road friction estimation, electric vehicles, active safety, parameter estimation

“The most exciting phrase to hear in science, the one that heralds the most discoveries, is not "Eureka!" (I found it!) but That's funny...”

– Isaac Asimov

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THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A

Albinsson, A., Bruzelius, F., Jacobson, B., & Jonasson, M. (2014). Tire Force Estimation Utilizing Wheel Torque Measurements and Validation in Simulations and Experiments. In *The 12th International Symposium on Advanced Vehicle Control,(AVEC'14), Tokyo, Japan*.

Paper B

Albinsson, A., Bruzelius, F., Pettersson, P., Jonasson, M. & Jacobson, B. (2015). Estimation of the inertial parameters of vehicles with electric propulsion, Accepted for publication in *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*

Paper C

Albinsson, A., Bruzelius, F., Gustafsson T., Jonasson, M. & Jacobson, B (2015). Identification of tyre characteristics using active force excitation. In *The 24th International Symposium on Dynamics of Vehicles on Roads and Tracks (IAVSD'15), Graz, Austria*.

The author of this thesis had the main responsibility for designing and writing the estimator algorithms, performing simulations, analysing the results and writing the papers. The author did not conduct the experiments in Paper A. The experiments in Paper B and C were conducted by the author together with some of the co-authors.

NOMENCLATURE

Roman Symbols

a_x	$[m/s^2]$	*Longitudinal Acceleration*
a_y	$[m/s^2]$	*Lateral Acceleration*
a_z	$[m/s^2]$	*Vertical Acceleration*
B_{ratioF}	–	*Brake torque distribution ratio front/rear*
C_f	$[N/rad]$	*Cornering stiffness of front axle*
$C_{F\alpha}$	$[N/rad]$	*Cornering stiffness of tyre/axle*
$C_{F\sigma_x}$	$[N/-]$	*Slip stiffness of tyre/axle*
C_r	$[N/rad]$	*Cornering stiffness of rear axle*
e	$[m]$	*Rolling resistance coefficient*
F_{drag}	$[N]$	*Aerodynamic drag force*
F_{xf}	$[N]$	*Longitudinal force on the front axle*
F_{xi}	$[N]$	*Longitudinal force on tyre i=1,2,3,4*
F_{xr}	$[N]$	*Longitudinal force on the rear axle*
F_{yf}	$[N]$	*Lateral force on the front axle*
F_{yi}	$[N]$	*Lateral force on tyre i=1,2,3,4*
F_{yr}	$[N]$	*Lateral force on the rear axle*
F_{zi}	$[N]$	*Vertical force on tyre i=1,2,3,4*
I_z	$[kgm^2]$	*Yaw inertia of the vehicle*
I_{wi}	$[kgm^2]$	*Rotational inertia of wheel i=1,2,3,4*
l_f	$[m]$	*Distance from front axle to Centre of Gravity*
l_r	$[m]$	*Distance from rear axle to Centre of Gravity*
L	$[m]$	*Wheelbase*
m	$[kg]$	*Mass of the vehicle*

R_{ei}	$[m]$	*Rolling radius of wheel i=1,2,3,4*
v_x	$[m/s]$	*Local longitudinal velocity in vehicle reference frame*
v_y	$[m/s]$	*Local lateral velocity in vehicle reference frame*
V	$[m/s]$	*Vehicle speed*
T_B	[*Nm*]	*Total brake torque*
T_{EM}	[*Nm*]	*Propulsion Torque from Electric Motor*
T_{ICE}	[*Nm*]	*Propulsion Torque from Internal Combustion Engine*
X	$[m]$	*Global Position X*
Y	$[m]$	*Global Position Y*

Greek Symbols

α_f	[*rad*]	*Slip angle of front axle*
α_r	[*rad*]	*Slip angle of rear axle*
δ	$[rad]$	*Steering angle of the front wheels*
δ_{SWA}	$[rad]$	*Steering wheel angle*
μ	[-]	*Tyre-road friction coefficient*
ρ	$[kg/m^3]$	*Air Density*
σ_x	[-]	*Longitudinal slip ratio*
ϕ_x	$[rad]$	*Roll angle of the vehicle*
ϕ_y	$[rad]$	*Pitch angle of the vehicle*
ϕ_z	$[rad]$	*Heading Angle of the vehicle*
ω_{EM}	$[rad/s]$	*Electric Motor Speed*
ω_{ICE}	$[rad/s]$	*Engine Speed*
ω_{wi}	$[rad/s]$	*Wheel speed on each wheel*
$\dot{\omega}_{w3}$	$[rad/s^2]$	*Wheel angular acceleration, wheel i*=1,2,3,4
ω_x	$[rad/s]$	*Roll angular velocity*

ω_y	$[rad/s]$	*Pitch angular velocity*
ω_z	$[rad/s]$	*Yaw angular velocity*
$\dot{\omega}_z$	$[rad/s^2]$	*Yaw angular acceleration*

CONTENTS

1 Introduction

1.1 Background

Vehicles with electric propulsion have several advantages over vehicles with internal combustion engines. The main advantage of electric vehicles is their potential to reduce energy consumption and emissions, both locally and globally, for personal and commercial transports. Environmental issues and their link to transportation systems motivate a shift towards the electrification of vehicles. It is assumed that the number of electric vehicles will increase in the future due to the negative impact that combustion vehicles have on the environment.

In order to decrease the environmental impact of transportation systems, the transition to carbon-dioxide neutral energy sources should be accelerated. This can be partly be achieved by increasing the competiveness and consequently the sales of electric vehicles through the further investigation of their benefits over combustion engines. The transition from conventional vehicles to electric vehicles can thus be accelerated.

One of the main areas where electric motors offer new possibilities is in active safety systems. Improvements in active safety increase the customer value and consequently the competiveness of the vehicles. Investigating these new opportunities further could also benefit individuals and society as a whole due to the potential for reducing the number of traffic related injuries.

The focus of this thesis is to investigate how active safety systems can be improved for vehicles with electric propulsion. In particular the advantages with electric motors as a sensing element to estimate vital vehicle states and parameters are explored. With better active safety systems the competiveness of electric vehicles can be improved and the market penetration for electric vehicles can thus be accelerated.

1.1.1 Environmental aspect

The environmental issues that motivate a shift towards electric vehicles can be split into two different problems, local emissions and global emissions. Local emissions consist of several pollutants including carbon monoxide, soot, hydrocarbon and particulates [1]. In [2] a review of the research regarding the effect of particulate matter (PM) on human health is presented. Previous research has strengthened the evidence that the health effects from particulate matter emissions are biologically plausible. However, the exact mechanisms behind the health effects have to be further investigated. In [3], the economic cost of air pollution in the World Health Organizations (WHO) European region in 2010 was estimated to be US$1.575 trillion. This can be compared to the Swedish GDP of 2010 which was US$ 0.49 trillion [4]. Road transports are estimated to account for around 40% of the total economic costs for air pollution and 50% of the economic cost for ambient air pollutions [3]. This illustrates that local emissions from the transportation industry are not only a problem in developing countries but also a real problem in the European region.

The health issues related to local emission have led to stricter emission legislation for both commercial and passenger vehicles. Furthermore, new legislations aimed at improving the fuel efficiency of new vehicles, limit the fleet average CO_2 emission of passenger cars [5]. The stricter emission legislations forces car manufactures to investigate new solutions for decreasing the fuel consumption and local emissions of their vehicles. Electrification is one of the most promising solutions to this problem as its potential to reduce greenhouse gas emission has been shown in numerous previous studies, especially in combination with future renewable energy sources [6-8].

Despite the fact that electric vehicles are considered by most to be a more environmentally friendly option than conventional vehicles with internal combustion engines, the market penetration is progressing slowly. In 2013, electric vehicles had a new vehicle sales market share for passenger cars of only 1.8% in the European Union [9]. The reasons for not choosing an electric vehicle vary, as shown in a European interview study where the participants were asked to list the most important improvement feature for electric vehicles. The improvements that scored the highest were, range (32%), purchase price (32%), possibility to recharge at home without private garage (25%), and recharge time (9%) [10]. However, with the increasing interest for electric vehicles in the past years the battery price has fallen by around 14% annually from 2007 to 2014 and further reductions are predicted for the future [11]. Increasing the competiveness of electric vehicles would reasonably lead to a larger market penetration and motivating further technological development.

1.1.2 Road traffic safety aspect

Road traffic injuries are the leading cause of death globally for people between 15-29 years old. Furthermore, 59% of global road traffic deaths are accounted for by young adults between 15-44 years. On a global level approximately 1.24 million people die every year on the roads [12]. As a response to the traffic accidents some countries and cities have adopted the Vision Zero Initiative which states that no loss of life due to traffic accidents are acceptable [13]. Reaching this ambitious target requires further research and improvements in infrastructure and in the active and passive safety of the vehicles. Even though the number of fatalities and seriously injured people in traffic accidents in Sweden is decreasing, the total number of injured people has been fairly constant since 1960, see Figures 1.1 and 1.2 [14]. Passive safety is effective in mitigating the severity of the injuries sustained in a crash. However, active safety has the potential to also reduce the overall number of accidents and therefore also the number of injured people. Further improvements in active safety are thus vital to reach the Vision Zero goals. The statistics presented in this section emphasize the amount of work that remains within road traffic safety and consequently the opportunity to increase the competiveness of electric vehicles through improved active safety functions.

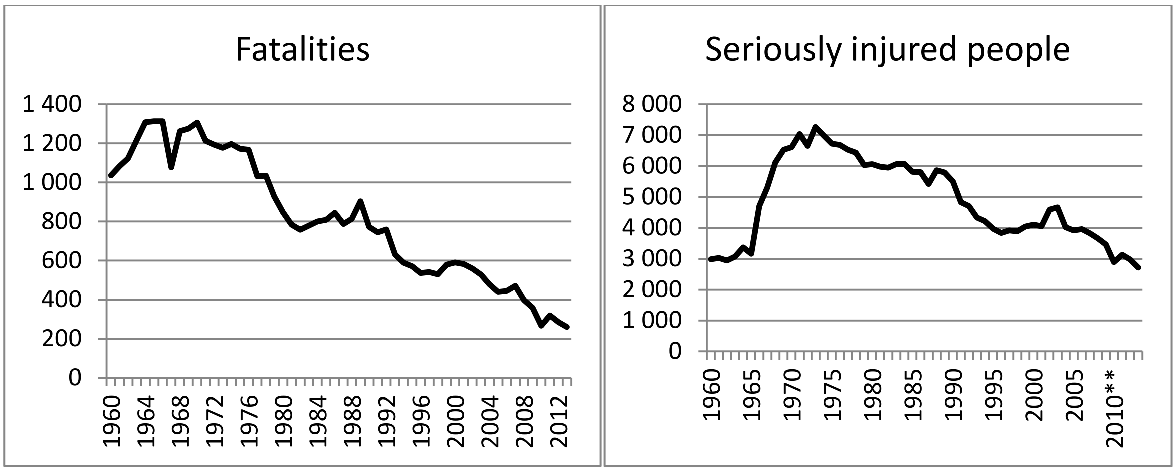

Figure 1.1, Number of fatalities and seriously injured people in Sweden annually due to road traffic accidents from police reports 1960-2013, data taken from [14]

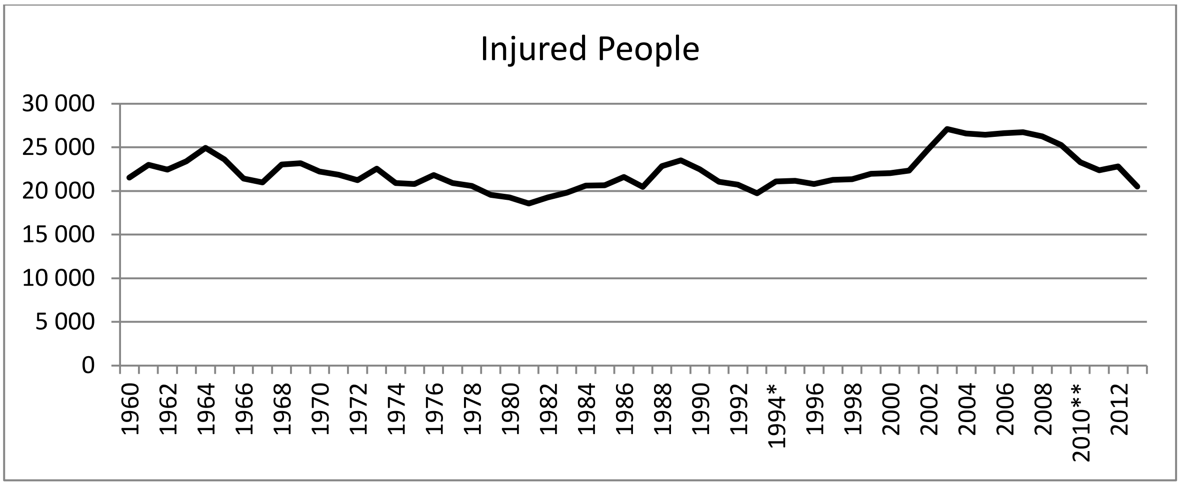

Figure 1.2, Number of injured people in Sweden annually due to road traffic accidents from police reports 1960-2013, data taken from [14]

1.2 Benefits of electric propulsion on a vehicle level

The major benefits of electric motors from a societal perspective are the prospect of reducing emissions in the transportation sector and the possibility of improving vehicle safety. On the vehicle level there are a number of benefits within electric propulsion. Connected to the environmental aspect are the fuel efficiency and local emissions. As a wheel torque actuator, the electric motor offers several benefits that can be utilized to improve active safety functions.

Electric vehicles are more efficient than conventional vehicles with internal combustion engines. According to [15] around 75-80% electric driveline efficiency can be achieved (including inverter, motor and reduction gear) for rural and highway driving cycles with regenerative braking. This does not include losses in battery and battery charger. With a battery efficiency and battery charger efficiency of 90% each [16], the total efficiency from plug-to-wheels is around 60-65%. In comparison, US light-duty vehicle powertrains had an estimated efficiency of 14-20% for a city driving cycle and 21-30% for a highway driving cycle [17].

Fully electric vehicles do not produce any local emissions from the propulsion system. Even plug-in hybrids reduce local emissions since the internal combustion engine is used less compared to a conventional vehicle. However, it should be mentioned that local emissions may be emitted close to the power plants depending on how the electricity is produced. The electricity production should therefore move towards renewable or carbon-dioxide neutral methods in order to reduce the overall emissions caused by electric vehicles. The noise emissions from electric motors are also lower compared to conventional vehicles with combustion engines, especially at low velocities where the engine is the main source of noise.

The response time of an electric motor is several milliseconds or 10-100 times faster than internal combustion engines and hydraulic brakes. The torque from electric motors is also more straightforward to estimate accurately compared the torque from internal combustion engines [18]. Due to the fast response and the accurate torque estimation, electric motors have good controllability properties which can be utilized to improve active safety systems.

Electric motors can generate both positive and negative torque. This enables efficient torque vectoring systems that can apply opposite torque on the two sides of the vehicle. Torque vectoring can be used not only for vehicle stability systems but also to improve vehicle performance. The possibility to apply a negative wheel torque enables the use of regenerative braking which increases the energy efficiency of the vehicle. Furthermore, electric motors can generate torque at zero motor speed. This means that electric vehicles do not need starting devices (clutches) and do not require multiple gear ratios to operate in a wide velocity range (gearboxes), thus reducing the system complexity. Electric motors have a high specific power that, in combination with the absence of gearboxes and clutches, allows for wheel or axle individual propulsion without any mechanical connections between the different motors.

1.2.1 The electric motor as an actuator to improve active safety

One way to improve the active safety functions utilizing electric motors is to study the benefits of the electric motor as an actuator. The main benefits from this perspective are the improved response, controllability, and the possibility to add both positive and negative wheel torque. Electric motors allows for faster, more accurate interventions without large actuator delays. These benefits have been previously investigated in [19].

1.2.2 The electric motor as a sensing element to improve active safety

In an estimation context, the accurate torque estimation from electric motors provides one of the largest benefits. Accurate information about the current propulsion torque on the wheels provides information about the current longitudinal tyre forces. This fact has been used in [20] to detect excessive wheel slip and improve longitudinal velocity and road slope estimation. With accurate wheel torque estimation, vehicle and tyre parameters that are, or could be, used in active safety systems can be estimated with higher accuracy online and the performance of active safety systems can therefore be improved. The estimate of the applied wheel torque and the benefits of the electric motor as an actuator also enable vehicle force neutral active tyre excitation that can be used to identify the tyre characteristics.

1.3 Motivation and purpose

The competiveness of electric vehicles should be increased in order to accelerate their market penetration and the shift towards a carbon-dioxide neutral transportation system. An area where electric vehicles have possible advantages over vehicles with combustion engines is active safety. The active safety and vehicle dynamics control functions of today are limited by the quality of the vehicle state and parameter estimates. States are defined here as time varying quantities that are affected by the time history of the system. Parameters are defined here as the quantities needed to describe how different states of interest affect each other. Parameters can be viewed as constants or slowly varying in time. However, depending on the application and the point of view, certain quantities can sometimes be regarded as states and sometimes as parameters. This thesis investigates the possibilities of utilizing the electric motor as a sensing element to improve current and future active safety and vehicle dynamics control functions. The prospect of estimating vehicle states and parameters that are vital to these control functions with information from the electric motor is also explored. More specifically, the possible benefits of using the accurate wheel torque estimation from the electric motor are examined.

The main focus of the thesis explores which vehicle states and parameter estimates can be improved by utilizing the electric propulsion as a sensing element and how the corresponding estimators should be designed to use the information from the electric motors. The vehicle functions that can be enhanced with improved vehicle state and parameter estimation and the requirements these functions put on the estimate in terms of accuracy are also discussed. Furthermore, the thesis investigates the possibility of using active tyre force excitation to identify the tyre characteristics online during normal driving.

1.4 Research Limitations

The thesis only considers on-road driving where the vehicle instrumentation is limited and the manoeuvres cannot be selected freely in time and space. The scope is thus limited to using electric propulsion as an addition to standard vehicle sensors and actuators that can be expected in a premium production vehicle of the year 2020. See Table 1.1 for signal sources that were used in the estimators in this thesis. It is reasonable to assume that vehicle state information through image processing will be available in the year 2020. However, this additional information is not considered in the present work since it is considered as a parallel research and development track. Although not used in the estimators presented in this thesis, Electric Power Assisted Steering (EPAS) has sensors that enable the estimation of the aligning torque of the tyres. This information can be used for friction estimation [21]. EPAS is common in production vehicles today but has not been utilized in the research so far.

It is assumed that the estimate of the electric motor torque is available to the estimator. Hence, no investigation of how the torque estimation from the electric motors can be improved has been done. The proposed methods should not depend on vehicle or tyre parameters that are not feasible to know for a production vehicle. The thesis does not consider the implementation of cooperative systems such as car-to-car or car-to-infrastructure communication. However, some of the results can be used to contribute to the information in such cooperative systems.

Table 1.1, Signal sources used for estimation

Sensor/Signal Source	Signals	Notation
Inertial Measurement Unit	Longitudinal Acceleration	a_x
	Lateral Acceleration	a_y
	Vertical Acceleration	a_z
	Roll rate	ω_x
	Pitch rate	ω_y
	Yaw Rate	ω_z
Wheel speed sensors	Wheel speed on each wheel	ω_{wi}
Steering wheel angle sensor	Steering wheel angle	δ_{SWA}
GPS (1Hz)	Position	X, Y
	Velocity	V
	Heading Angle	ϕ_z
Electric Motor	Estimated Torque	T_{EM}
	Motor Speed	ω_{EM}
Internal Combustion Engine & Powertrain	Estimated Propulsion Torque	T_{ICE}
	Engine Speed	ω_{ICE}

1.5 Scientific Contribution of this thesis

The thesis investigates how the estimate of the electric motor torque can be used to improve vehicle state and parameter estimation, especially how the estimation of inertial parameters and tyre forces can be estimated.

The thesis highlights general problems associated with tyre force estimation which needs to be resolved in order to achieve good estimator performance using standard vehicle sensors. The thesis also investigates the feasibility of using active force excitation to estimate the current tyre-road friction coefficient.

1.6 Thesis Outline

Chapter 1 presents the background and motivation behind the thesis. Chapter 2 discusses the importance of vehicle state estimation for active safety systems and vehicle dynamics control functions. In Chapter 3, an overview of online state and parameter estimation, and discussions regarding possible methods to estimate important states and parameters are presented. Chapter 4 summarizes the most important findings from the appended paper. In Chapter 5, discussions and conclusions regarding the results and the assumptions made in the thesis are provided. Chapter 6 outlines the future work within the research area.

2 Relevance of vehicle parameter and state estimation

The torque estimate from the electric motor can be considered as an additional source of information that can be used to improve the estimation of vehicle parameters or states. These estimates can be used in active safety systems to improve their performance which not only can reduce the number of injuries in road traffic accidents but increase the competiveness of electric vehicles. Various active safety systems require different states or parameters for their decision making. This chapter presents a number of estimates that have the possibility to improve active safety functions. The tyre-road friction coefficient is one of the states that has the potential to improve several different active safety systems.

An example of a functional architecture for an estimator that utilizes the electric motor torque as an input to identify the tyre characteristics is presented in Figure 2.1. The sub functions in the functional architecture correspond to the appended papers in this thesis. Paper A investigates tyre force estimation based on known wheel torque and highlights the sensitivity of the estimator to the inertial parameters. Paper B investigates how the electric motor can be used to estimate the inertial parameters of the vehicle. Finally, Paper C investigates how the tyre characteristics can be estimated with longitudinal tyre force excitation.

The published papers are related to each other as shown in the function architecture, Figure 2.1. The inertial parameter estimator and the tyre force estimator can provide useful information to other systems in the vehicle without requiring the tyre parameter estimator as is explained in the following sections. For estimating the longitudinal and lateral tyre slip, the corresponding vehicle velocities are required. The longitudinal velocity can be estimated from the wheel speed signals if at least one tyre is free rolling. Paper C shows how the longitudinal velocity can be estimated during active tyre force excitation when all tyres have large longitudinal slip ratios. Estimation of the lateral tyre slip has not been investigated in this thesis.

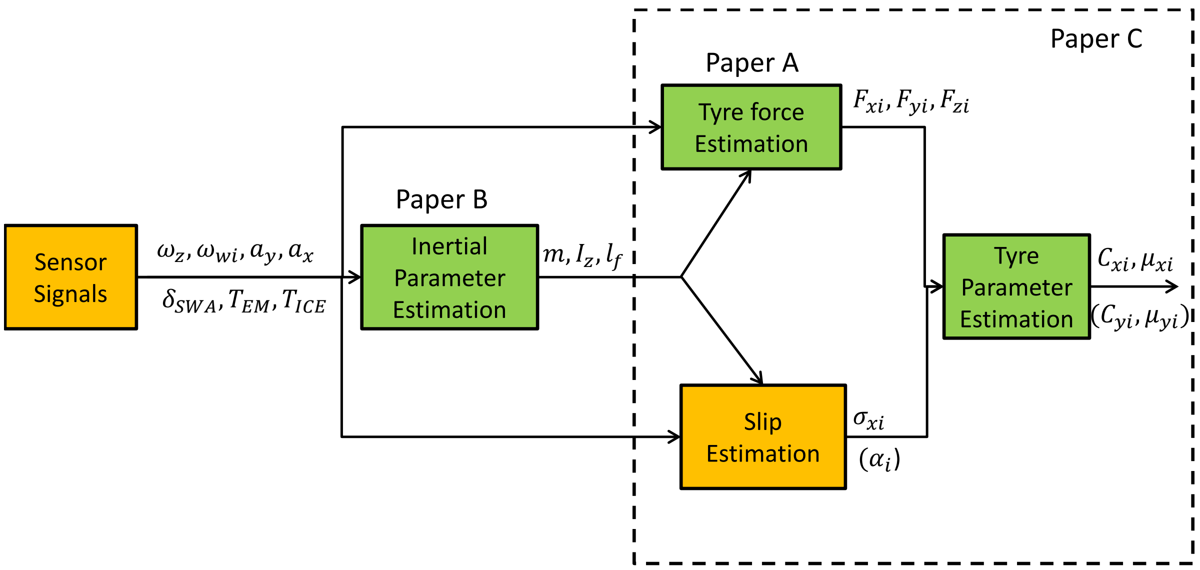

Figure 2.1, Example of functional architecture for state and parameter estimation

2.1 Inertial Parameters

The inertial parameters can vary significantly for a normal passenger car depending on the number of passengers and payload. For larger vehicles, a mass variation of around 20% is within the legal limits [22]. The added mass is not evenly distributed between all four tyres and changes to the position of the centre of gravity and the inertia tensor are likely as well. The handling and stability of the vehicle is affected by these changes, see [23]. Active safety systems that correct the vehicle motion often use reference models that represent the desired or nominal vehicle behaviour. When the vehicle motion deviates too much from the desired motion, based on the current driver inputs, the system intervenes. However, since no online estimation of the inertial parameters is traditionally available to active safety systems, the inertial parameters used in the systems must be set to reasonable values which represent a conservative or most probable load case. The system must therefore be robust to variations in these parameters and the threshold for system intervention must be set higher in comparison to the case with known parameters. If these systems are provided with accurate information about the current vehicle and tyre parameters that are used in the reference model, the intervention threshold could be set lower and the controller could be optimized for the specific load case. This should enable the system to have a faster and smoother intervention.

2.2 Tyre Parameters

The importance of tyres as vehicle components are indisputable as tyres constitute the only contact between the road and the car and the majority of forces used to control the vehicle are generated in these four contact patches. They have a large influence on the vehicle behaviour and stability, affect the over/under steer behaviour of the vehicle and limit the maximum forces that can be generated through steering and braking. Any tyres of the correct dimensions for the vehicle can be used in Sweden, provided that they fulfil other legal requirements. The information regarding the current tyres that are used is therefore limited. Furthermore, the road surface has a large influence on the slip-force characteristics of the tyre, which further increases the uncertainty of the tyre-road interaction. However, better information regarding the tyres that are currently fitted to the vehicle and knowledge about the current road surface, can for instance, be used to improve vehicle dynamics reference models used in the active safety system. The tyre-road friction coefficient limits the maximum horizontal forces that the tyres can generate and is therefore the one of the most important tyre parameters. Other parameters include the longitudinal slip stiffness and the cornering stiffness. These parameters describe the tyre behaviour when the tyres are operating in the linear region far from the friction limit.

For vehicles equipped with autonomous functions, knowing the current tyre-road friction limit is even more important than for vehicles without these functions. Autonomous emergency braking systems normally assume a high-friction surface. If the system was aware that the vehicle is driving on a low-friction surface the intervention can be executed earlier and the crash is then mitigated or completely avoided. The importance of the current friction information increases in conjunction with the level of automation. An autonomous vehicle must for instance, be able to adapt the vehicle speed before entering a corner. Without an estimation of the road surface in the corner, the worst case scenario must be assumed to guarantee vehicle stability in all conditions. An autonomously controlled vehicle would therefore have to assume that it is driving on a low-friction surface at any instance where such conditions are plausible. The customer acceptance of such as system would most likely be poor. The estimate of the tyre-road friction coefficient can also be shared with other vehicles to warn drivers or vehicle functions that they are approaching an area with low friction. However, the tyre-road friction coefficient is not only dependent on the road surface but on the tyres as well. The shared friction information can thus be used to give an approximate value of the current friction coefficient.

2.3 Vehicle States

The information that can be used by the on-board systems in the vehicle is limited by the available sensor information. Due to cost, the number of sensors in production vehicles is restricted. Hence, not all vehicle states that are beneficial for the active safety functions are directly measured. In order to obtain these missing states, sensor information from several sensors can be combined to provide an estimate. This is normally referred to as sensor fusion and allows the on-board systems to include states which cannot be directly measured by the available sensors. Two states which are commonly used by active safety systems are the longitudinal and lateral velocities of the vehicle. These states are important in detecting excessive lateral and longitudinal wheel slip and thereby maintaining vehicle stability in critical manoeuvres. Previous research has investigated how these states can be estimated; see [20, 24-26]. Volvo Cars has a lateral velocity estimator in production vehicles that is used to improve the vehicle stability functions.

2.4 Systems that can benefit from improved vehicle state and parameter estimation.

In this section a number of active safety systems are presented together with vehicle states that are or can be used by these systems, see Table 2.1. Accuracy requirements of the estimates can be investigated by studying the users of the information. The friction estimate used in the autonomous emergency braking systems has for example different requirements than the friction estimate used in traction control. The autonomous emergency braking systems require a friction estimate for the whole vehicle before the intervention to approximate the braking distance of the vehicle. The traction control on the other hand requires a friction estimate for each of the propelled wheels individually during the intervention. The requirement for the state or parameter estimation accuracy will thus be determined by the systems that use the estimate.

Table 2.1, Examples of functions which could benefit from improved state estimation

Function	Estimates that can improve the function
Electronic Stability Control	v_y, v_x, μ
Regenerative Braking [27]	v_y, v_x, μ, F_{zi}
Autonomous Emergency Braking	μ
Torque Vectoring	$\mu, C_{F\alpha}, F_{zi}, v_x, \phi_x, v_y$
Traction Control & ABS	$\mu, F_{zi}, C_{F\sigma_x}, v_x$
Rollover Stability Control	v_y, v_x, F_{zi}, ϕ_x
Evasive Manoeuvre Assist [28]	X, Y, ϕ_z, μ

3 Methods for online state and parameter estimation

Online estimation of dynamics states or parameters imposes different demands on the modelling of the vehicle when compared to offline simulation models. In simulation, the complexity of the vehicle model is generally not a problem as long as the computation time is reasonable. The parameters of the system can be determined prior to initialization and measurements are not required to run the simulation. For online estimation though, the number of parameters and the complexity of the model can be of large concern. The parameters can be unknown and using nominal parameter values can cause large estimation errors. Over time individual components will age and their characteristics can change. The owner will most likely also change the tyres as the vehicle gets older and the tyre characteristics will therefore be unknown to the online estimators. Furthermore, the number of sensors, and thereby the available sensor information, are limited. This means that some parameters or vehicle states will be difficult or impossible to estimate for a given sensor configuration. The vehicle model used in an online estimator must therefore be adapted to the available sensor information. Another issue with online estimation is the need for sufficient excitation in order to estimate specific parameters or states. Mass estimation can be used to illustrate the problem. Assuming that the estimator utilizes the longitudinal accelerometer and information about the longitudinal tyre forces to estimate the mass, the problem of sufficient excitation can then easily be understood by looking at the longitudinal equation of motion

$$ma_x = F_{x1} + F_{x2} + F_{x3} + F_{x4} - F_{drag} \tag{3.1}$$

Even if the aerodynamic drag force F_{drag} and the longitudinal tyre forces are known, the measurement of the longitudinal acceleration does not provide information about the vehicle mass without any acceleration. Hence, in order to estimate a parameter or state accurately the sensor signals must provide the necessary information, i.e. the excitation must be sufficient. Another example of the need for sufficient excitation is related to tyre-road friction estimation which is discussed more in detail in Section 3.6.2.

Furthermore, for online estimation the computational burden should be kept to a minimum. The estimator should therefore preferably use as simple of computations as possible and avoid using excessive memory since the algorithms should be implementable in production ECUs. Recursive methods that only require that the estimator remember a few states from the previous sample are therefore commonly used in online estimators.

3.1 Vehicle models for online estimation with a limited sensor set

Considering the limited information from the sensor set, the complexity of vehicle models used in online estimators has to be restricted. Furthermore, implementing vehicle models that require a heavy computational burden is not advantageous when the estimator should run in real-time on hardware with limited resources. These considerations favour simple vehicle models for online estimators. One way to resolve the lack of measurements is to have a more accurate vehicle model coupled with few sensor measurements. However, this requires that the parameters used in the model are well known and that the model used represents the real vehicle behaviour. As an example, the estimation of the longitudinal centre of gravity positions using the yaw rate and lateral acceleration signals together with the single-track model, Figure 3.1, can be considered. Assuming small steering angles, the lateral and yaw equations of motion for the bicycle model are,

$$ma_y = F_{yf} + F_{yr} + F_{xf}\sin\delta \tag{3.2}$$

$$I_z\dot{\omega}_z = F_{yf}l_f - F_{yr}l_r + F_{xf}l_f\sin\delta \tag{3.3}$$

In a steady state manoeuvre where the yaw acceleration $\dot{\omega}_z$ is zero and the longitudinal tyre forces are assumed to be negligible, the equation for the longitudinal centre of gravity position becomes, with $l_r = L - l_f$,

$$l_f = L - \frac{F_{yf}}{ma_y} L \tag{3.4}$$

Hence, if the front axle lateral force F_{yf} is measured together with the lateral acceleration, the longitudinal centre of gravity position can be determined using Equation 3.4. However, the sensors required to directly measure the lateral front axle force are expensive and normally not found in production vehicles. One solution to this is to introduce a linear tyre model. Assuming that the tyre is operating in the linear region, the front axle and rear axle lateral forces can be expressed as

$$F_{yf} = C_f \alpha_f \tag{3.5}$$

$$F_{yf} = C_r \alpha_r \tag{3.6}$$

The simultaneous estimation of the front and rear cornering stiffness, C_f, C_r, and the longitudinal centre of gravity position has been investigated previously [29, 30]. It was shown that it is not possible to estimate these parameters simultaneously for low frequency steering inputs [30]. To resolve this issue it is possible to assume that the tyre characteristics are known. However, any error in the assumed tyre characteristics will directly affect the estimation of the longitudinal centre of gravity position. Considering that the cornering stiffness can vary depending on the road surface roughness, this approach cannot be considered as robust.

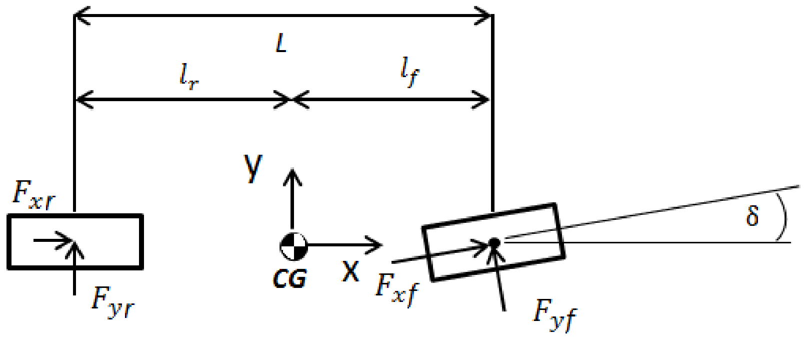

Figure 3.1, Bicycle model used to illustrate the implications of different estimation approaches

The example above illustrates a problem with using constant unknown parameters for the estimation of other parameters. When choosing a vehicle model, it is important to consider which parameters are required to describe the model. The parameters of the chosen vehicle model should in the ideal case be known and well defined and not vary during the lifetime of the vehicle. If the parameter varies within a limited range and a nominal value can be measured, they can be used in the estimator as long as the estimator accuracy is improved. The same conditions apply to unknown parameters where a reasonable value can be assumed. As long as including these parameters improves the estimator performance they can be used. Unknown parameters that are hard to approximate and that could potentially vary during the lifetime of the vehicle could still be included in the estimator if they are estimated online. This requires that the parameter estimator is independent of unknown parameters or states from estimators that use its estimated parameter value, i.e. avoid a loop of interdependent estimators. Using information that is not expected to be available to the estimator online is not recommended. A simpler vehicle model that removes the need to incorporate unknown parameters in the estimator should then be used instead.

3.2 Simulation versus experiments

Simulation tools are valuable in the development process of online estimators. They provide an ideal environment that can be used to verify the basic concept of the estimator. Furthermore, the influence of different vehicle parameters and errors in sensor signals on the performance of the estimator can be easily investigated in simulations. For the papers included in this thesis IPG CarMaker® was used as a simulation tool. The vehicle model used in IPG CarMaker® is significantly more advanced than the models used in the evaluated estimators. The assessment of the estimator performance in simulations is more realistic with high-fidelity simulation tools compared to using simple vehicle models since the effect of some of the modelling assumptions are captured in the evaluation.

Although simulations provide a good environment to test the initial feasibility of the estimator design, the real world performance assessment should be conducted with experimental test data. In a real vehicle, the sensor signals include disturbances and offsets. Furthermore, modelling errors which may not be evident in the ideal simulation environment may decrease the performance of the estimator. However, in experimental tests it is hard to isolate different error sources. Simulations and experiments should therefore be regarded as a complement to each other. Both are important parts of the estimator development process. The evaluation of the estimator should therefore be done both in simulations and experiments.

3.3 Recursive estimation methods

Two recursive estimation methods are presented in this chapter. Common for all curve fitting algorithms is the minimization of a cost function which describes the difference between the signal model, the model used in the estimator, and the measurements. Two common approaches for online estimation are briefly presented in this chapter, the least squares methods and the Bayesian approach.

3.3.1 Least Squares

Least square methods use a straightforward quantification of the error between the measurements and the signal model. Least squares methods minimize the square error between the measurement and the model. In the general case with a signal model $\boldsymbol{s}[k]$ [31]

$$\boldsymbol{s}[k] = \boldsymbol{h}(k, \boldsymbol{\theta}) \tag{3.7}$$

$$\boldsymbol{y}[k] = \boldsymbol{s}[k] + \boldsymbol{w}[k] \tag{3.8}$$

where $\boldsymbol{s}[k]$ is the signal model at time instants k with dimension $m \times 1$, where m is the number of measurements, $\boldsymbol{h}(k, \boldsymbol{\theta})$ is a known function relating the model parameters to the signal model, $\boldsymbol{y}[k]$ is the measurement vector, $\boldsymbol{\theta}$ are the unknown parameters and $\boldsymbol{w}[k]$ is the measurement noise**.** The cost function J that should be minimized to find the optimal solution in the least squares sense is then [31]

$$J = \sum_{k=1}^{k=N} (\boldsymbol{y}[k] - \boldsymbol{s}[k])^T (\boldsymbol{y}[k] - \boldsymbol{s}[k]) \tag{3.9}$$

For a linear model with a scalar measurement the signal model can be written as

$$s[k] = \boldsymbol{h}^T[k] \cdot \boldsymbol{\theta} \tag{3.10}$$

and

$$y[k] = s[k] + w[k] \tag{3.11}$$

The least square estimate of the unknown parameters in the signal model can then be updated recursively as [32]

$$\widehat{\boldsymbol{\theta}}[k] = \widehat{\boldsymbol{\theta}}[k-1] + \boldsymbol{K}[k]\big(y[k] - \boldsymbol{h}^T(k)\widehat{\boldsymbol{\theta}}[k-1]\big) \tag{3.12}$$

where

$$\boldsymbol{K}[n] = \boldsymbol{P}[k-1]\boldsymbol{h}(k) * \big(1 + \boldsymbol{h}^T(k)\boldsymbol{P}[k-1]\boldsymbol{h}(k)\big)^{-1} \tag{3.13}$$

Finally the covariance matrix can be recursively updated as:

$$\boldsymbol{P}[k] = \boldsymbol{P}[k-1] - \boldsymbol{K}[n]\boldsymbol{h}^T(k)\boldsymbol{P}[k-1] \tag{3.14}$$

In order to initialize the recursive least square algorithm values for $\widehat{\boldsymbol{\theta}}[0]$, $\boldsymbol{P}[0]$ has to be assigned. By choosing $\boldsymbol{P}[0]$ to have large diagonal elements, an arbitrary finite $\widehat{\boldsymbol{\theta}}[0]$ can be chosen. The convergence and performance will then be similar to the stage-wise (non-recursive) solution. [32]

3.3.2 The Kalman filter & Bayesian estimators

In general the Kalman filter and Bayesian estimators differs from the classical statistical estimation method used in the least square method. In the classical approach the unknown parameters of interest are assumed to be deterministic but unknown constants. In the Bayesian philosophy the unknown parameters are instead treated as random variables. Instead of estimating deterministic parameters, the realization of random variables is estimated and appear as internal states in the estimator [31]. This means that it is possible to include prior knowledge about the unknown parameters or states in the estimator. The Kalman filter, a recursive estimation algorithm, is one of the most commonly used Bayesian estimators which minimize the Bayesian mean square error defined as

$$BMSE(\hat{\theta}) = E[(\theta - \hat{\theta})^2] \tag{3.15}$$

This corresponds to choosing the mean value of the posterior Probability Density Function (PDF), the PDF of the unknown parameter θ when the data $\boldsymbol{x}$ has been observed [31]

$$\hat{\theta} = \int \theta p(\theta|\boldsymbol{x})d\theta \tag{3.16}$$

This is illustrated in Figure 3.2 where the mean, mode and median of a general PDF are shown. The mode is also called the maximum a posteriori (MAP) estimator. If the posterior PDF is Gaussian distributed, the mean, mode and median estimators are identical. Note that the Kalman filter does not provide a general solution to Equation 3.16, see [31].

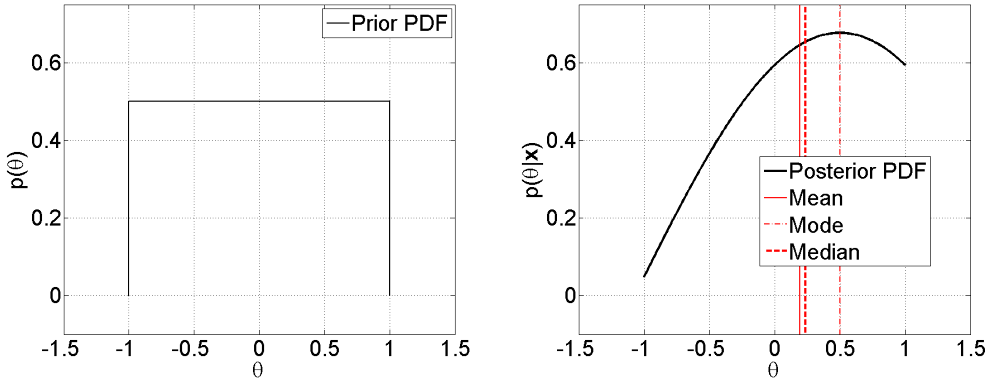

Figure 3.2, General prior and posterior PDF of a random variable

3.4 Tyre force estimation

Tyre force estimation is required for the online estimation of tyre parameters. Furthermore, to be able to use the estimated friction coefficient in vehicle dynamics control functions, e.g. torque vectoring or traction control, an estimation of the current tyre forces in some form is necessary. Knowing the current tyre forces aids these systems in determining how the applied wheel torque should be split between the different tyres.

Tyre force estimation has been investigated previously using a number of different proposed methods [33-37]. The methods proposed in these papers provide an accurate estimation of the tyre forces given that the conditions and assumptions made in the estimator are met. However, the methods differ due to the different prerequisites, available sensors and modelling assumptions. Without a reliable wheel torque estimation, for instance, the longitudinal tyre forces can only be estimated on one axle [37] as oppose to both axles with available wheel torque signals [33-36].

While the previously proposed approaches describe some useful methods for tyre force estimation, the focus tends to be on the manoeuvres or situations where the tyre force estimator performs well. The main issues that remain in order to have reliable tyre force estimation have not been discussed thoroughly in the previous research. Paper A highlights issues associated with tyre force estimation using a limited sensor set [38]. The main issues that should be solved are the estimation of the inertial parameters and the individual lateral tyre force estimation. The effect of having an erroneous mass parameter can be easily seen by studying the equations of motion for the bicycle model. Assuming that the nominal mass is 1500 kg and the vehicle has a 60/40% weight distribution front/rear, the distance from the centre of gravity to the front and rear axle can be expressed as,

$$l_f = 0.4L \tag{3.17}$$

$$l_r = 0.6L \tag{3.18}$$

Considering the vehicle at steady-state cornering at a lateral acceleration of 3 m/s^2, it can be assumed that the steering angle is small and that the longitudinal tyre forces are negligible. The front and the rear lateral axle forces can then be derived from the bicycle model in Figure 3.1 as,

$$F_{yF} = \frac{ma_y l_r}{L} \tag{3.19}$$

$$F_{yR} = \frac{ma_y l_f}{L} \tag{3.20}$$

Further assuming that the vehicle is carrying 300kg extra payload, corresponding to 20% of the total mass, which is represented by a point mass in the centre of gravity. Looking at the rear axle lateral force with the nominal mass parameter gives the error in the rear axle lateral force:

$$e_{FyR} = \frac{\left(m(1+0.2)a_y l_f - ma_y l_f\right)}{L} = \frac{0.2ma_y l_f}{L} \tag{3.21}$$

The error in the rear axle lateral force e_{FyR} can then be expressed as a fraction of the real rear axle lateral force

$$\frac{e_{FyR}}{F_{yR}} = \left(\frac{0.2ma_y l_f}{1.2ma_y l_f}\right) = \frac{0.2}{1.2} = 0.166666 \cong 16.7\% \tag{3.22}$$

Without any online estimation of the vehicle mass, the axle lateral force estimation error is 16.7%. The error can be even larger if the added payload is not evenly distributed between the front and rear axles. Hence, both the mass and the longitudinal centre of gravity position should be estimated online in order to achieve accurate tyre force estimation.

3.4.1 Individual lateral tyre force estimation

The difficulty of individual lateral tyre force estimation can be observed from the longitudinal, lateral and yaw equations of motion for the two track model

$$ma_x = F_{x1}\cos(\delta) + F_{x2}\cos(\delta) + F_{x3} + F_{x4} - F_{y1}\sin(\delta) - F_{y2}\sin(\delta) - \mathrm{F_{drag}} \tag{3.23}$$

$$ma_y = \left(F_{y1} + F_{y2}\right)\cos(\delta) + F_{y3} + F_{y4} + (F_{x1} + F_{x2})\sin(\delta) \tag{3.24}$$

$$\begin{aligned} I_z\dot{\omega}_z &= \left(F_{y1} + F_{y2}\right)\cos(\delta)\, l_f + \left(F_{y1} - F_{y2}\right)\sin(\delta)\frac{S}{2} - \left(F_{y3} + F_{y4}\right)\left(L - l_f\right) \\ &+ (F_{x4} - F_{x3})\left(\frac{S}{2}\right) + (F_{x2} - F_{x1})\left(\frac{S}{2}\right)\cos(\delta) + (F_{x1} + F_{x2})\sin(\delta)\, l_f \end{aligned} \tag{3.25}$$

The sums of the lateral forces on the axles are the dominating terms in the equations of motion. Therefore, trying to differentiate between the left and right lateral forces on one axle is difficult. The above model assumes the same steering angle on the left and right sides of the vehicle. It has been shown that for electric vehicles with 4WID (four-wheel independent drive) the individual lateral tyre forces can be estimated on the front axle when the difference between the left and right steer angles is large enough [39]. However, this requires that the individual steer angles are measured, which is not common for production vehicles.

For lower lateral acceleration a common approach is to distribute the lateral forces according to the vertical force distribution on the axle. Figure 3.3, shows the simulated friction utilization of the rear tyres versus lateral acceleration during a steady state constant radius manoeuvre for two different static toe angles. The simulation is done in IPG CarMaker® using a simple representation of the rear suspension. This figure illustrates the problem in assuming a lateral force distribution proportional to the vertical force of the tyre. For a more realistic vehicle setup with static toe angles, the left and right tyre utilization are not equal except at the lateral acceleration where the two lines intersect. If the rear suspension is tuned to have zero toe angle on the rear tyres, the assumption is accurate at low levels of lateral acceleration. Production vehicles have a tolerance on the accepted toe angles and the toe angle might change during the lifetime of the vehicle. Thus, it should not be expected that the vehicle has zero toe angle or even that the toe angle is known.

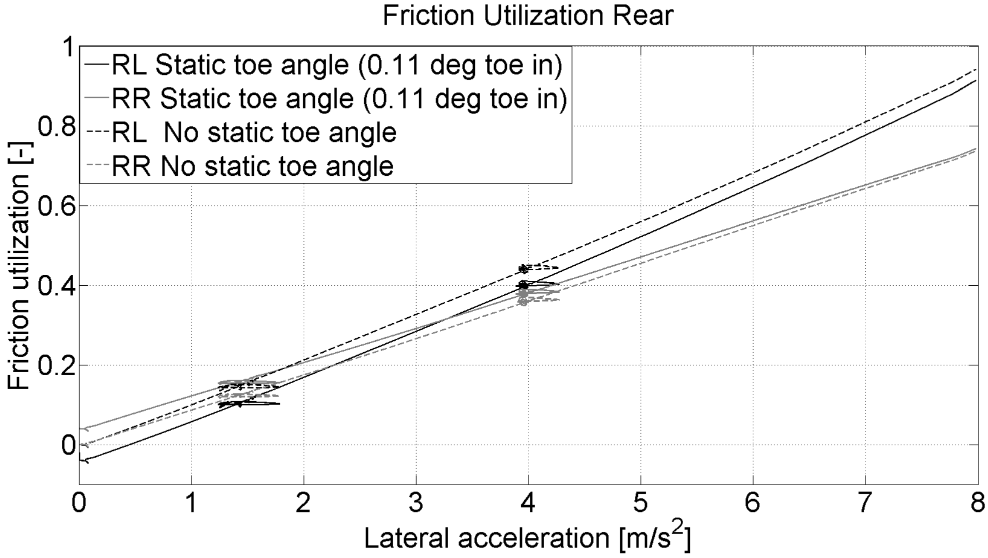

Figure 3.3, Friction utilization, constant radius manoeuvre, radius=20m

3.4.2 Tyre force estimation during braking

Wheel torque estimation based on the brake pressure in a friction brake system can be challenging due to the varying friction coefficient between the brake pad and brake disc. The friction between the brake pad and brake disc is dependent on many factors such as the sliding velocity, contact pressure, presence of water and temperature [40]. However, with a known brake torque distribution between the front and rear axles, the longitudinal tyre forces can be estimated from the accelerometers. The assumption that the brake torque distribution is known, is only valid if the friction coefficient between the brake pad and the brake disc is equal on all four wheels and if the brake pressure is known on at each wheel. However, an error in the brake force distribution does not affect the total longitudinal force estimation but only the distribution between the wheels. The front and rear axle longitudinal forces are estimated using a RLS method with forgetting factors [32], see Equations 3.12-3.14 where Equations 3.13 and 3.14 are modified to incorporate the forgetting factor. For driving straight ahead, the components of the RLS algorithm for the estimation of the total brake torque, from the longitudinal equation of motion and the wheel dynamics equations for each wheel, can be formulated as,

$$y(k) = ma_x + \left(\frac{-T_{EM1} + I_{w1}\dot{\omega}_{w1} + F_{z1}e}{R_{e1}} + \frac{-T_{EM2} + I_{w2}\dot{\omega}_{w2} + F_{z2}e}{R_{e2}}\right) + \left(\frac{F_{z3}e+I_{w3}\dot{\omega}_{w3}}{R_{e3}} + \frac{F_{z4}e+I_{w4}\dot{\omega}_{w4}}{R_{e4}}\right) + \frac{1}{2}\rho C_{DA} v_x^2 \quad (3.26)$$

$$\widehat{\boldsymbol{\theta}}(k) = T_B \quad (3.27)$$

$$\boldsymbol{h}(k) = -B_{ratioF}\left(\frac{1}{2R_{e1}} + \frac{1}{2R_{e2}}\right)\cos(\delta) - (1 - B_{ratioF})\left(\frac{1}{2R_{e3}} + \frac{1}{2R_{e4}}\right) \quad (3.28)$$

$$\boldsymbol{\alpha} = 0.5 \tag{3.29}$$

where e is the rolling resistance coefficient, I_{wi} is the wheel inertia of wheel i, F_{zi} is the vertical force on wheel i, R_{ei} is the rolling radius of wheel i, B_{ratioF} is the brake torque distribution between the front and rear axles, δ is the steering angle of the wheels, C_{DA} is the aerodynamic drag coefficient, m is the vehicle mass, a_x is the measured longitudinal acceleration, T_B is the total brake torque on all four wheels and $\boldsymbol{\alpha}$ is the forgetting factor. The front and rear axle longitudinal forces, based on the total brake torque estimation, can hence be calculated as

$$F_{xf} = -\frac{T_b(B_{ratioF})}{R_{eF}} - \frac{F_{zF}e}{R_{eF}} - \frac{I_{w1}\dot{\omega}_{w1}}{R_{e1}} - \frac{I_{w2}\dot{\omega}_{w2}}{R_{e2}} \tag{3.30}$$

$$F_{xr} = -\frac{T_b(1 - B_{ratioF})}{R_{eR}} - \frac{F_{zR}e}{R_{eR}} - \frac{I_{w3}\dot{\omega}_{w3}}{R_{e3}} - \frac{I_{w4}\dot{\omega}_{w4}}{R_{e4}} \tag{3.31}$$

The estimator performance during a braking manoeuvre is shown in Figure 3.4. With a known brake force distribution, the proposed method can estimate the longitudinal axle forces for braking manoeuvres with moderate accelerations. If the ABS is active the simple brake force distribution is no longer valid and the estimation accuracy will decrease. When these systems are active the estimator should be turned off. In order to estimate the longitudinal tyre forces during heavy braking, the brake pressure at each wheel should be measured. The brake torque can then be estimated from the brake pressure instead of the longitudinal acceleration. However, this requires a good knowledge of both the brake pressure at each wheel and the relation between the brake pressure and brake torque. This brake force estimator can be extended to include lateral manoeuvres as well.

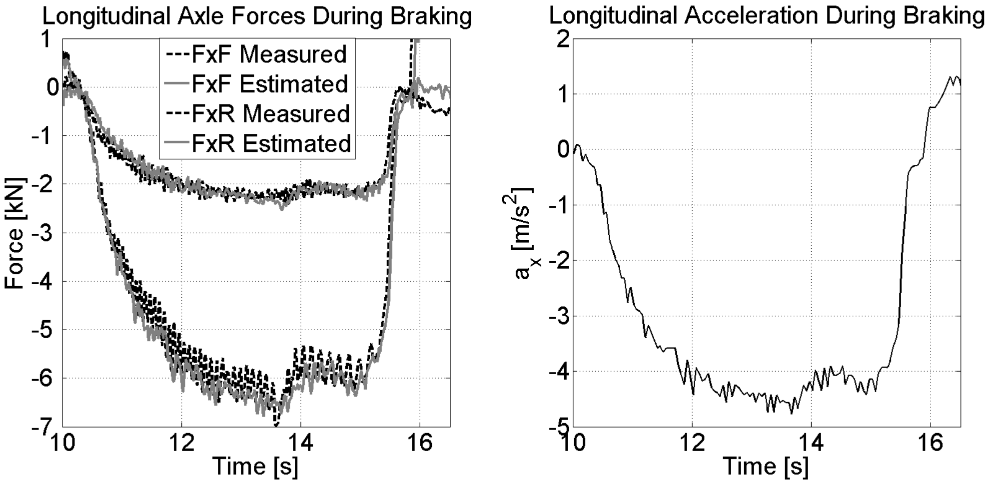

Figure 3.4, Longitudinal tyre force estimation and longitudinal acceleration during braking manoeuvre, initial velocity 28 m/s. The two black lines represent the measured forces and the two grey lines the estimated forces. The front axle force has a larger magnitude than the rear axle forces.

3.5 Vehicle inertial parameter estimation

Vehicle inertial parameter estimation is a well researched subject and a large number of articles investigate vehicle mass estimation since it can be considered one of the most important inertial parameters. Most methods are based on an estimation of the applied wheel torque and the longitudinal equation of motion [41-44], see Equation 3.32. Although the basic idea is the same, the estimation methods differ depending on the available sensor information and assumptions made in the estimators. Some authors use predefined values of the rolling resistance or the aerodynamic drag coefficient in order to estimate the mass [41, 42, 45]. This means that the error in the longitudinal equation of motion due to small errors in these parameters affects the mass estimation. An approach where these parameters are estimated simultaneously as the mass, utilizing a dual antenna GPS for mass and slope estimation, is presented in [43].

$$\begin{aligned} ma_x &= \frac{T_1}{R_{e1}} - \frac{F_{z1}e}{R_{e1}} - \frac{I_{w1}\dot{\omega}_1}{R_{e1}} + \frac{T_2}{R_{e2}} - \frac{F_{z2}e}{R_{e2}} - \frac{I_{w2}\dot{\omega}_2}{R_{e2}} \\ &+ \frac{T_3}{R_{e3}} - \frac{F_{z3}e}{R_{e3}} - \frac{I_{w3}\dot{\omega}_3}{R_{e3}} + \frac{T_4}{R_{e4}} - \frac{F_{z4}e}{R_{e4}} - \frac{I_{w4}\dot{\omega}_4}{R_{e4}} - \frac{1}{2}\rho C_{DA} v_x^2 \end{aligned} \tag{3.32}$$

Even though the mass can be estimated using standard vehicle sensors, a straightforward estimation of the longitudinal centre of gravity position and the yaw inertia is difficult without adding extra vehicle sensors or information about the tyre characteristics. It has been shown in previous research that the simultaneous estimation of the tyre characteristics (front and rear axle cornering stiffness) and the longitudinal centre of gravity position using the yaw and lateral dynamics of the vehicle, is difficult without high frequency steering inputs [30]. This is due to the insufficient information in the data at low frequency manoeuvres. The contribution from the centre of gravity position and the cornering stiffness can therefore not be separated. High frequency steering inputs are normally not found in everyday driving. Previous articles have solved this issue by assuming that the tyre characteristics are known [46-48] or by specifying the driving manoeuvres [49]. However, the tyre characteristics for a production vehicle are unknown and the manoeuvres cannot be selected freely in space and time, making these methods unsuitable for online estimation.

Another approach to estimating these parameters is to utilize the sprung mass response to road inputs by adding extra sensors to the vehicle or utilizing suspension deflection measurements to estimate the vertical force on each tyre [50, 51]. The main drawback with adding extra sensors is the added cost and complexity.

Paper B proposes a mass estimation method which includes estimations of the rolling resistance coefficient and the aerodynamic drag coefficient. Two estimators for the estimation of the longitudinal centre of gravity position and yaw inertia are also proposed. The estimators are based on a standard vehicle sensor set and the electric motor torque estimate. The mass estimate consistently converged to values close to the measured vehicle mass. Two methods were proposed in Paper B for the estimation of the longitudinal centre of gravity position and the yaw inertia using the standard vehicle sensor set. The first approach was not robust to small measurement errors in the experimental evaluation. The second approach was shown to improve the estimation of these parameters on average and reduce the variance of the error. However, the actual error for a given load case remains unknown with this method. The main advantage with using electric propulsion for inertial parameter estimation is thus the small mass estimate error due to accurate wheel torque information.

3.6 Friction Estimation

Tyre-road friction estimation is an intense research topic in both academia and industry as there are a large number of papers and reports dealing with this subject. Friction estimation can be divided into a large number of categories. One of the more common classifications is cause- and effect-based friction estimation methods, see [52]. Cause-based approaches detect the causes of varying road-friction such as temperature, road surface, presence of water, etc. Effect-based approaches, on the other hand, detect the effect of different road surfaces by observing the vehicle response. A popular cause-based approach is to use some form of vision system to identify the road surface. The advantages of cause-based approaches are the possibility to detect the road surface friction ahead of the vehicle and the fact that it does not require any tyre force excitation [52, 53]. One of the main drawbacks with cause-based approaches is that they neglect the influence of the tyre on the tyre-road friction coefficient. Different tyres can have significantly different friction coefficients on the same road surface.

Effect-based approaches measure the effect of the road conditions on the tyre or vehicle response. A large number of effect-based approaches exist and the different methods require different sensors sets, see [52]. One of the most common approaches is to measure or estimate the tyre force, or aligning moment, and the slip of the tyre. By identifying a varying number of tyre model parameters, the friction can be estimated. Paper [54] proposes a method where the slip stiffness is identified and the maximum tyre-road friction coefficient is assumed to be a function of the slip stiffness. Paper [55] further investigates the possibility of separating different roads surfaces based on the slip stiffness of the tyre. However, the slip stiffness is highly dependent on many different factors and therefore, a generic correlation between the slip stiffness and the friction coefficient is not found in academic research to the knowledge of the author. Furthermore, these methods have to be calibrated for different tyres and the slip stiffness of the tyre must be known for each surface that should be identified online.

A more robust approach is to identify the maximum friction coefficient based on the measured or estimated slip and tyre forces when the tyres are operating in the nonlinear region closer to the friction limit. However, these methods require large tyre utilizations, see [56]. By using information about the aligning moment as an extra signal, the required friction utilization can be somewhat reduced, see for instance [21, 57]. The estimation of the aligning moment of the tyre is normally based on the sensors from the Electronic Power-Assisted Steering (EPAS). The main drawback with this approach is the need for lateral excitation of the vehicle and the difficult task of estimating the aligning moment on a real vehicle. The main drawback with all of the effect-based approaches is the fact that they only measure the current friction coefficient. They have no possibility to measure the friction in front of the vehicle. This can be solved by sharing the friction information with other road users in a cloud-based service, see for instance the Road Status Information project [58]. However, the tyre's influence on the tyre-road friction coefficient for the vehicles which receives the information is then neglected. The friction estimation method investigated in this thesis is based on the estimated wheel slip ratios and longitudinal tyre forces. The tyre-road friction coefficient is estimated by fitting a nonlinear tyre model to the measurements. This approach does not require any additional sensors and can therefore be implemented in production vehicles at a low cost. The main drawback however is the need for sufficient friction utilization.

3.6.1 Tyre models for friction estimation

A large number of different tyre models have been used in previous research to estimate the maximum road friction coefficient. One of the commonly used models is the brush model with parabolic pressure distribution, see [21, 56, 57] for methods utilizing the brush model. The brush model is a physical model of the tyre-road interaction that describes the nonlinear tyre behaviour. Another common approach is to use a linear tyre model and correlate the slip stiffness with the friction coefficient, see [54, 55, 59]. It is also possible to generate tyre data offline, for instance using data which has been parametrized for the magic tyre formula [60]. This approach has been investigated previously [33, 61]. However, the robustness of the offline tyre data approaches to variations in tyre characteristics can be questioned. The owner is free to use any tyres that meet the legal requirements and the tyre characteristics can vary significantly during the lifetime of the vehicle. Furthermore, in order to cover all possible road surfaces, tyre data must be available for the road surfaces where the vehicle will be used. However, with the limited tyre excitation that can be expected during normal driving, it is difficult to identify parameters of complex tyre models. Prior information about the tyres that are fitted to the vehicle is therefore required in these situations. The slip based friction estimators have the same drawbacks as the offline tyre data approaches. Even though there is a correlation between the slip stiffness and the road surface, the actual slip stiffness values vary for different tyres and is sensitive to temperature, tyre wear and tyre pressure [62]. Due to the uncertain correlation between the slip stiffness and the friction coefficient, and because of the fact that the slip stiffness varies depending on the road surface, at least these two tyre parameters should be estimated online in order to have a reliable friction estimate.

The tyre models can be divided into different categories, longitudinal slip, lateral slip and combined slip, depending on for the situations they are valid. The models that only handle longitudinal slip are not valid when lateral tyre slip is also present and vice versa. The combined slip models include the effect of both longitudinal and lateral slip and their interaction. These models can thus handle driving manoeuvres with lateral and longitudinal slip. For longitudinal slip, different tyre characteristics can be used for propulsion and braking to account for the non-symmetric characteristics of the tyre. Depending on the chosen tyre model some driving manoeuvres have to be excluded. Models that account for the combined slip can handle more driving manoeuvres than models that only include longitudinal and lateral slip. Naturally, the complexity of the tyre model is increased when both the lateral and longitudinal slips is included. In some situations it can therefore be beneficial to use a simpler model.

3.6.2 Tyre excitation needed for friction estimation

The level of excitation is one of the main issues for effect-based friction estimation, particularly for methods which utilize nonlinear tyre models in the estimator. The minimum number of samples and the minimum friction utilization needed to accurately estimate the friction coefficient in an ideal case was investigated using the Cramer-Rao Lower Bound (CRLB). The CRLB shows the lower bound of the variance of the estimated parameters that any estimator can achieve. No estimator can be found with smaller variance. It might not be possible to find an estimator which achieves the CRLB but it gives an indication of the minimum number of samples and the friction utilization that is needed for accurate friction estimation. The CRLB is given by

$$var(\hat{\theta}_i) \geq [\boldsymbol{I}^{-1}(\boldsymbol{\theta})]_{ii} \tag{3.33}$$

where

$$[\boldsymbol{I}(\boldsymbol{\theta})_{ij}] = -E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial\theta_i \partial\theta_j}\right] \tag{3.34}$$

where the PDF, $p(\boldsymbol{x};\boldsymbol{\theta})$, for the data given the measurement points $\boldsymbol{x}$ as a function of the unknown parameters $\boldsymbol{\theta}$. Assuming that the measurement noise is White Gaussian Noise (WGN) implies that the PDF is given by

$$p(\boldsymbol{x};\boldsymbol{\theta}) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} e^{-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}(x[k]-s[k;\theta])^2} \tag{3.35}$$

where σ^2 is the variance of the measurement noise, N is the number of observed samples, $x[k]$ is the measurement at time instant k, and $s[k;\boldsymbol{\theta}]$ is the signal model value at time instant k given the true parameter values $\boldsymbol{\theta}$**.** The brush model with parabolic pressure distribution is chosen as the signal model. The brush models is given by

$$F_x = -C_{F\sigma_x}\sigma_x + \frac{{C_{F\sigma_x}}^2\sigma_x|\sigma_x|}{3\mu F_z} - \frac{{C_{F\sigma_x}}^3\sigma_x^3}{27\mu^2F_z^2} \tag{3.36}$$

with $C_{F\sigma_x}$ and μ, the slip stiffness and the maximum friction coefficient respectively chosen as the unknown parameters that should be estimated and where F_z is the vertical load on the tyre, assumed to be constant, σ_x is the slip ratio and F_x is the longitudinal tyre force. To generate the data points $\boldsymbol{x}$, white Gaussian noise is added to the signal model with a linear increase of the slip ratio σ_x. Expressions for the PDF, the derivatives of the log-likelihood function and hence the CRLB, can then be derived, see Appendix A. The CRLB can be calculated from the expressions in appendix A for different numbers of samples and different levels of friction utilization for a given noise level. The variance of the measurement noise σ^2 was set to be 8500 N^2 to correspond to the noise on the rear axle force estimation in Paper C. Tables 3.1 and 3.2, show the CRLB for different levels of friction utilization and different number of samples for this noise level, from [63]. These results are valid for a tyre that can be perfectly described by the brush model with a slip stiffness of 110 000 N/- and a vertical force of 7400N representing the characteristics for one axle. The columns in the tables represent the maximum friction utilization during the manoeuvre and the rows correspond to the number of samples (data points) up to the maximum friction utilization. For the maximum friction utilization of 0.6 and with 110 samples, an optimal estimator would be able to estimate the friction coefficient to within $1 \pm 0.0441 * 2 = 1 \pm 0.0882$ with 95% certainty given that the assumption of the noise level and the tyre characteristics are fulfilled. In production vehicles however, the tyres will not be perfectly described by the brush model and noise will be present on the wheel speed signals used to calculate the wheel slip ratio. Larger friction utilization will hence be required to estimate the maximum road friction coefficient with similar accuracy.

Even though the CRLB will not describe the actual required friction utilization it provides an idea of the required excitation levels and general trends. From the tables below a general trend can be observed. In order to improve the accuracy of the estimate, high tyre utilization and a large number of samples should be used. However, this contradicts most other requirements, e.g. safety, comfort, energy consumption, tyre wear etc. where a fast intervention with low friction utilization is preferable. The CRLB can be used to approximate the minimum utilization and number of samples required for a given tyre-road friction coefficient accuracy.

Table 3.1, Standard deviation of μ-estimate for μ=1 for a given measurement noise, [63]

$\mu_{util,max}$	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
N						
10	0.7787	0.4184	0.2203	0.1357	0.0784	0.0471
60	0.3383	0.1820	0.0961	0.0593	0.0344	0.0207
110	0.2514	0.1353	0.0714	0.0441	0.0255	0.0154
160	0.2089	0.1124	0.0593	0.0366	0.0212	0.0128
210	0.1825	0.0982	0.0519	0.0320	0.0186	0.0112
260	0.1642	0.0883	0.0466	0.0288	0.0167	0.0101
310	0.1504	0.0809	0.0427	0.0264	0.0153	0.0092
360	0.1396	0.0751	0.0397	0.0245	0.0142	0.0086
410	0.1309	0.0704	0.0372	0.0230	0.0133	0.0080
460	0.1236	0.0665	0.0351	0.0217	0.0126	0.0076

Table 3.2, Standard deviation of μ-estimate for μ =0.3 for a given measurement noise [63]

$\mu_{util,max}$	**0.1**	**0.2**	$\mu_{util,max}$	**0.1**	**0.2**
N			**N**		
10	0.6196	0.1001	**260**	0.1307	0.0213
60	0.2693	0.0438	**310**	0.1197	0.0195
110	0.2001	0.0325	**360**	0.1112	0.0181
160	0.1663	0.0271	**410**	0.1042	0.0170
210	0.1453	0.0236	**460**	0.0984	0.0160

These results can be compared to experimental results from the study in paper C, see Figure 3.5. For a friction utilization of around 0.8, with a maximum friction coefficient of around 1.15, and with 360 samples, the parameters of the brush model presented in Equation (3.36) was fitted to the measured data. The friction coefficient was estimated using an offline nonlinear least squares method with the force as the measurement. The estimate of the tyre-road friction coefficient that minimized the measurement was 1, which is 0.15 lower than the approximated value of 1.15 found in an ABS braking manoeuvre. A friction utilization of 0.8 corresponds to around 70% of the maximum friction coefficient. This can be compared to a friction utilization of 0.7 with 360 samples in Table 3.1. According to the CRLB the friction can be estimated to $1 \pm 0.0142 * 2 = 1 \pm 0.0284$ with 95% certainty. Scaling this to the friction coefficient of 1.15 the accuracy should be approximately 0.033 of the true friction value. However, as shown by the experimental results the actual error is much larger. This is due to a number of uncertainties which are not accounted for in the CRLB calculations. The brush model cannot describe the tyre behaviour perfectly and this will cause errors when fitting the tyre model to the data. Furthermore, in the real measurements, the wheel speed signals include disturbances and noise which are not accounted for in the CRLB.

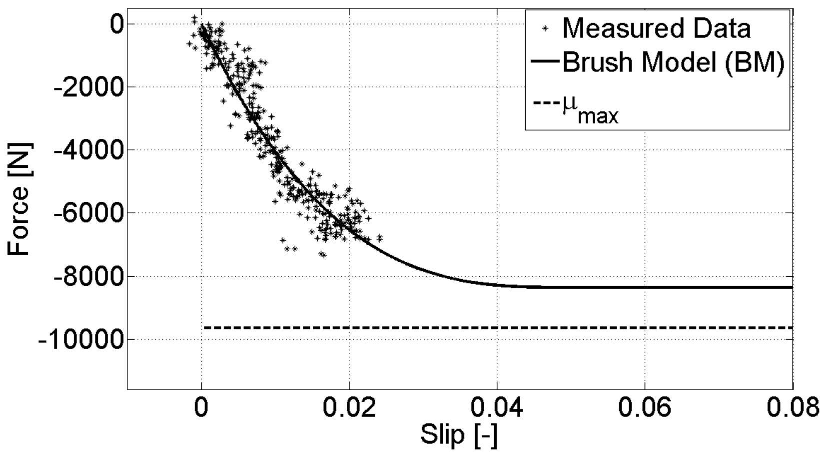

Figure 3.5, Brush model fitted to real measurement data with the reference friction value shown as a dotted line.

3.6.3 Active tyre excitation

The level of excitation required in order to accurately estimate the friction coefficient is large even in the ideal, as shown in Section 3.6.2. These higher levels of tyre utilization are not common during normal driving. Hence, only relying on the driver input to achieve the required tyre utilization will not provide many opportunities to estimate the friction coefficient, see Figure 3.6 for an illustration of the problem. In order to differentiate between the linear model and the nonlinear model, the tyre forces must be large enough. In Figure 3.6, normal driving is depicted as keeping the longitudinal acceleration below 2 m/s^2 for illustrative purposes. The limits for normal driving can vary significantly between different drivers. Furthermore, the availability of a reliable friction estimate cannot be controlled by the systems that will use it. Active tyre force excitation is one option for resolving these issues. Previous research has investigated the idea of active tyre excitation by longitudinal force generation in simulations and a few patents exist that propose similar solutions [64-66]. Another method has also been proposed where an additional yaw moment is added, together with corrective steering, to keep the vehicle on the intended path [67]. Before introducing active tyre excitation further, a classification of different approaches is in order. Due to the small number of research papers and patents investigating the idea of active tyre excitation, no classification exists so far. Significant for all active tyre force excitations systems is the fact that the systems change the actuation of the vehicle during normal operation with the purpose of estimating tyre parameters.

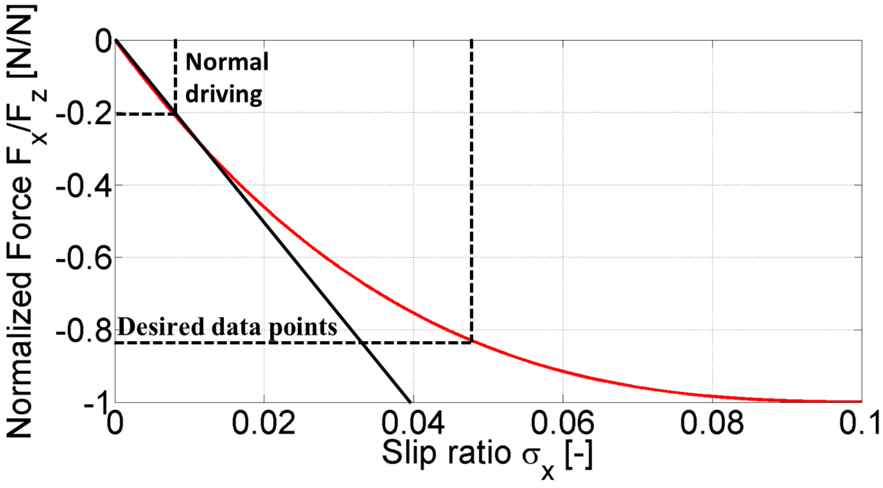

Figure 3.6, Example of tyre characteristics to illustrate the need for active tyre force excitation. The red line is the true tyre behaviour; the black line is a linear model which has been fitted to the tyre characteristics in the linear region.

In Figure 3.7, a classification is proposed which divides active tyre excitation into two main categories: driver induced and system induced excitation. In driver induced excitation the friction coefficient is estimated during manoeuvres that are initiated and controlled by the driver. One such example is active tyre excitation during braking. The brake controller can then decide to limit the number of braked wheels or distribute the brake torque in order to achieve a larger friction utilization on these tyres. The driver induced excitation can hence be viewed as, changing the control allocation during driver initiated manoeuvres in order to estimate the friction coefficient. The system induced category includes systems where the driver remains in control of the vehicle but the active tyre excitation may be done without the driver demanding any change in the vehicle states, for instance during cruising on a highway. The main difference is therefore that the driver induced category only includes manoeuvres where the driver actively demands tyre forces and controls the magnitude and direction of the tyre forces. The main benefit with driver induced excitation is that the driver expects some change in the vehicle behaviour and is thus less sensitive to disturbances in the vehicle motion. On the other hand, the friction coefficient can only be estimated during these manoeuvres and the availability of the estimate is therefore limited. In comparison to the current situation though, the availability of a friction estimate is increased. The system induced active tyre excitation has the benefit of choosing when the intervention should be executed, for instance when demanded from an active safety system. However, the driver does not expect any changes in the vehicle motion and might be more sensitive to disturbances in the vehicle motion.

These two main categories are further split into three different subcategories depending on if the active tyre force excitation is performed in such a way that the tyres generate longitudinal tyre forces only, lateral tyre forces only or a combination of lateral and longitudinal tyre forces. These three subcategories describes the manoeuvres and how the active excitation is performed, e.g. during straight driving, during cornering, through steering or braking wheels etc. Lateral excitation includes manoeuvres where the lateral tyre forces are dominating and the longitudinal tyre forces are small. The opposite is true for the longitudinal excitation category.

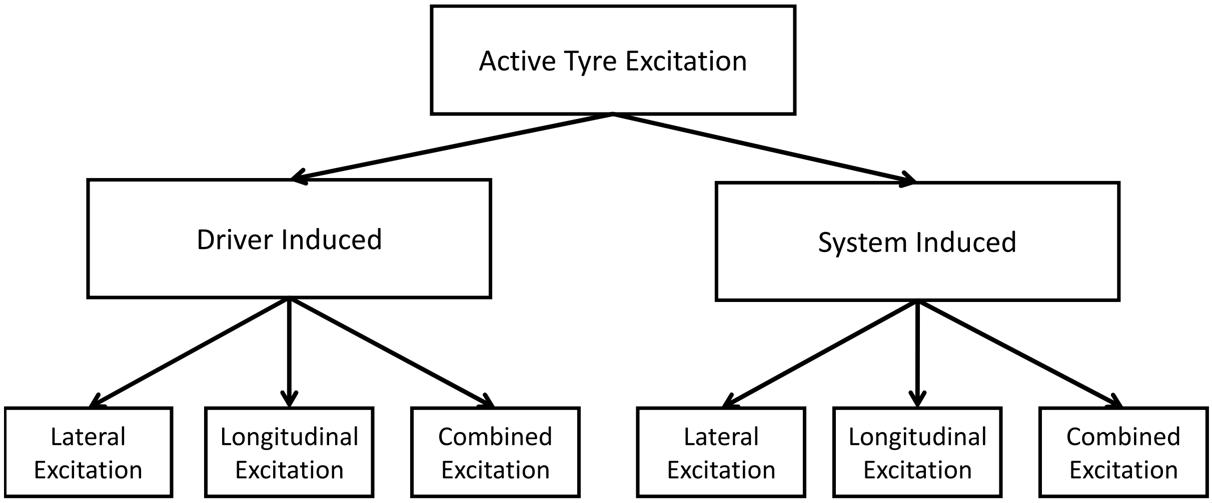

Figure 3.7, Classification of methods for active tyre force excitation

Two previously published papers investigate the basic idea of active tyre excitation [64, 67]. However, in order to implement these systems in production vehicles further investigations are required. On a vehicle level the main issues include, safety, comfort, component wear and energy consumption. The safety aspect is the most critical issue that needs further investigation. To implement these systems in a production vehicle, vehicle stability must be guaranteed during all interventions. Defining boundaries for when it is safe to initiate an active tyre force excitation and investigating how these systems should be designed to maintain vehicle stability during the intervention is therefore of great importance.

Another area which requires a lot of work is the estimation and excitation itself. Not only from a safety point of view but from an estimation perspective as well. Without active tyre excitation the tyre forces cannot be controlled and the estimator must use the data which is provided. With active tyre excitation though, the tyre forces can be controlled to improve the estimation accuracy. The tyre excitation can and should hence be designed with respect to the estimator. Investigating how the tyre excitation should be optimized for the estimator is therefore a key point for improving the friction estimation accuracy.

Connected to the estimator design is the limited number of sensors available. Since there are no direct measurements of the states which are needed for the friction estimation, tyre force, slip ratio etc., these states have to be estimated as well. Estimating these quantities with the accuracy required for friction estimation is one of the major challenges. Furthermore, the available actuators limit the number of different ways to perform a vehicle force-neutral intervention. Ideally, at least one wheel should free rolling so that the longitudinal velocity can be estimated. However, for a conventional vehicle without torque vectoring it is not straightforward to do this. It can be done by applying the friction brakes on one of the front wheels to remove the propulsion torque from the engine. It would then be sufficient to apply the brakes on one rear wheel to have a neutral intervention. However, estimating the longitudinal forces on the rear axle without any accurate estimation of the applied wheel torque is difficult in this situation and the tyre force estimation would hence suffer.

4 Summary of papers

4.1 Paper A, Tire Force Estimation Utilizing Wheel Torque Measurements and Validation in Simulations and Experiments

Paper A investigated how the tyre force estimation can be improved with accurate knowledge of the applied wheel torque from the propulsion system. This represents using an electric motor for propulsion with accurate torque estimation. The main benefits could be found in the estimation of the longitudinal tyre forces. However, knowledge about the wheel torque distribution between the left and the right wheels is required for an accurate estimation of the lateral axle forces when torque vectoring is used.

Furthermore, a number of issues with tyre force estimation using standard sensors were identified. The main issue highlighted was the need for an online estimation of the vehicle inertial parameters in order to have accurate tyre force estimation. It was also shown that individual lateral tyre force estimation is challenging without adding extra sensors. An approach which is commonly found in literature where the axle lateral force is distributed according to the vertical load was evaluated. This method did not perform as well as expected due to the many assumptions used to derive the method. The estimation of the axle lateral force was found to be robust against large errors in the vertical force estimation.

The results also showed that information about the road bank angles is redundant for the estimator since the accelerometer measurements capture the effect of the road banking.

4.2 Paper B, Inertial parameters estimation for vehicles with electric propulsion

The main benefit of electric motors for inertial parameter estimation is the accurate torque estimation. One result from this paper shows that it is possible to accurately estimate the mass using the torque estimation from the electric motor. This is done without making any assumption regarding the rolling resistance coefficient or the aerodynamic drag coefficient but requires a certain excitation.

A method to estimate the longitudinal centre of gravity position and the yaw inertia was proposed. The method is based on the planar equations of motions and the electric motor torque estimation. The results showed that this method was not accurate or robust enough to be used in a production vehicle. This is mainly due to uncertain wheel steering angles and the weak interaction between the front lateral tyre forces and the longitudinal acceleration.

A more straightforward approach was also proposed for the estimation of the longitudinal centre of gravity position and the yaw inertia. This method is based on the mass estimate and the seatbelt indicators. The method was shown to perform, on average, better than using default parameter values for an unladen vehicle, provided that the mean values of the passenger and driver weights are known.

This paper illustrates some of the difficulties with estimating the longitudinal centre of gravity position and the yaw inertia using a standard sensor set. In order to have a more accurate estimation of these parameters, extra sensors should be added to the vehicle.

4.3 Paper C, Identification of tyre characteristics using active force excitation

Paper C investigates the possibility of using active torque excitation to estimate the tyre-road friction coefficient and the slip stiffness. An intervention, where the propulsion torque on the front axle is increased linearly and the rear brake pressure is controlled to keep a constant velocity, was performed in a test vehicle. An estimation method was then proposed that removes the need to directly measure the longitudinal velocity of the vehicle. This method was compared to a reference model with measured longitudinal velocity.

The main results show that it is possible to estimate the tyre-road friction coefficient during this intervention if the tyre excitation is large enough. The proposed method to estimate the friction coefficient without measuring the longitudinal velocity was found to be more sensitive to measurement noise and disturbances on the wheel speed signal when compared to the reference model. As a result, this method tends to underestimate the friction when the tyre excitation is too low. The tendency to underestimate the friction coefficient was shown to be dependent on the noise level on the wheel speed signals. The estimator performance and the possibility to separate between low and high friction surfaces can therefore be improved by reducing the noise and disturbances on the wheel speed signals. The wheel speed signals were taken directly from the CAN-bus during the experiments.

These experiments were performed without any electric propulsion. However, adding electric propulsion to the rear axle would have several advantages. Firstly, the accurate torque estimation improves the longitudinal tyre force estimation since the disturbances from the accelerometer is avoided. Furthermore, the controllability of the rear axle torque gives new possibilities for a more advanced torque profile during the excitation and allows for better slip control. Furthermore, if electric propulsion is used instead of the friction brake to apply a negative torque to the rear axle, the energy consumption during the intervention can be reduced by regenerating some of the energy.

5 Discussion and Conclusion

Some benefits of using the electric motor as an additional sensing element have been shown in this thesis. These benefits are mainly based on the accurate torque estimation from the electric propulsion. Other signals from the electric motor have not been considered in this thesis, e.g. electric motor speed etc.

The main benefit of the accurate torque estimation from the electric motor is the improved knowledge about the current longitudinal tyre forces. This information can be utilized to improve parameter and state estimates. Estimates which are directly related to the applied longitudinal tyre forces will benefit from this information. However, as shown in [20] other estimates can also use this information for decision making or switching between different estimation strategies. This thesis provides a few examples of how the estimated electric motor torque can be used for the online estimation of vehicle and tyre parameters and states.

Although some comparisons have been made between the torque estimation accuracy based on an electric motor and an internal combustion engine, it is difficult to make any general comparisons. Depending on the method used to estimate the torque from an internal combustion engine, the estimation accuracy can vary. However, the details of these torque estimation algorithms have not been studied in this thesis. In Paper C, it is shown that the torque estimation from internal combustion engines can be accurate enough for online estimators. The internal combustion engines are complicated systems with many moving parts, especially modern engines that are supercharged to a larger extent. In contrast, the electric motor has few moving parts and the torque is straightforward to estimate. It is therefore viable to regard the electric motor torque estimate as a signal that can be a source of information for other estimators.

The accuracy of the estimates presented in the thesis is dependent on accurate wheel torque estimation. Thus, with improved wheel torque estimation these estimates can be improved as well. Since the potential to increase the performance of active safety systems are linked to the accuracy of the estimates, the accuracy of the wheel torque estimation determines how much the performance of these systems can be improved. If the wheel torque estimation from electric motors is generally more accurate than the torque estimate in vehicles with combustion engines, a potential to improve active safety systems for electric vehicles through better vehicle state and parameter estimation exists, and consequently is a possibility for improving the competiveness of electric vehicles. However, further studies are required to quantify the differences in torque estimation accuracy.

5.1 Assumptions

The flexibility of the powertrain is neglected in Paper B and C. The elastic deformation, in combination with the inertia of the individual powertrain components, filters the torque from the engine or motor to the wheels. In fast transient events the wheel torque might therefore differ from the steady-state wheel torque that would be achieved with the same motor/engine torque. In Paper C, the active tyre force excitation is performed slowly in order to minimize the influence of the tyre or powertrain dynamics. In Paper B, the estimation is based on the driver inputs. Hence, manoeuvres where the powertrain elasticity influences the estimation performance can occur since the conditions for activating the estimators do not consider the derivative of the applied wheel torque. For most practical cases though, the influence of the powertrain elasticity should be small, since drivers normally aim to have an acceptable comfort level while driving.

The sensor set that was available to the estimators was limited to a reasonable sensor set for a production vehicle in 2020. This means that the vehicle has limited sensor information that can be used in the estimator. With more sensors the amount of information that is available to the estimators is increased, which has the potential to improve the estimation accuracy. However, the main focus of the thesis was to investigate the potential benefits of using the electric motor torque as an input to the estimators. Regardless, there are a number of potential sensors that could be fitted to the vehicle to improve the performance of the estimators.

Wheel angle sensors can be used to enable individual lateral tyre force estimation on the front axle [39]. Accelerometers and suspension deflection sensors can be added to improve the estimation of the longitudinal centre of gravity position and the yaw inertia [50, 51]. These sensors are some of the more reasonable additional sensors that could be added to the vehicle. An external GPS can be added to the vehicle to improve the estimation of the longitudinal velocity and hence the slip ratio estimation, see Paper C.

Furthermore, the environmental sensors that are used for the autonomous emergency braking and the autonomous cruise control systems can be used for estimating the velocities of the vehicle. This would provide additional information which can be used for longitudinal and lateral velocity estimation. The vehicle state information from these sensors should therefore be included in the future development of other state or parameter estimators.

The parameters used in the proposed estimators were limited to parameters that can reasonably be known in production vehicles. By providing the estimators with additional information regarding other vehicle and tyre parameters, further improvements in estimation accuracy can be achieved. On the other hand, introducing new uncertain parameters can decrease the estimation accuracy if the model differs too much from the real measurements.

As shown in Paper B, the parameters required for longitudinal tyre force estimation can be estimated online in the vehicle. However, the longitudinal centre of gravity position and the yaw inertia are still challenging to estimate online with a production vehicle sensor set. Additional sensors might therefore be required in order to enable the estimation of these parameters. The longitudinal centre of gravity position determines the vertical force distribution between the front and rear tyres and will thus directly affect the friction estimation accuracy for the proposed method in Paper C. The effect of an erroneous vertical force distribution and its impact on the accuracy of the friction estimate should therefore be further investigated. As discussed in Section 2.4 a target system is required to specify the minimum estimation accuracy. The current knowledge on the vertical force distribution might hence be sufficient for some systems but not for others.

An estimation or measurement of the longitudinal velocity is required for the active tyre excitation intervention in Paper C. Because of the actuator limitations, all tyres have large slip ratios during the intervention. No wheel can therefore be used as a reference to estimate the vehicle velocity. This is the motivation behind the advanced algorithm in Paper C where the vehicle velocity is estimated simultaneously with the tyre parameters. The assumptions made in order to estimate the longitudinal velocity make the estimator more sensitive to noise and disturbances on the wheel speed signals. Furthermore, the accuracy of the velocity estimation is dependent on the accuracy of the tyre parameter estimation. An external estimation or measurement of the longitudinal velocity that is independent of the tyre parameter estimation would thus be preferable. However, the peak in the tyre force as a function of the slip ratio can occur for as low slip ratios as five percent or lower, depending on the tyre and road surface, see Figure 3.5. An external estimation or measurement of the longitudinal velocity must therefore have a high accuracy and low variance. More importantly, the change in longitudinal velocity during the manoeuvre must be accurately measured. A small constant error during the manoeuvre can be handled better by the friction estimator than an error that varies during the manoeuvre. The longitudinal velocity is normally not measured directly in production vehicles due to the expensive sensors required.

5.2 Applications and Implications

One of the main applications from the results in this thesis is the ability to estimate the friction coefficient during normal driving. This enables the possibility to estimate the friction freely in time and space. By sharing this information with other road users, a map of the current road surface conditions for different roads can be obtained. This information is valuable for other drivers, on-board active safety systems and the road authorities. Other drivers can use the information to adapt their driving when approaching a low friction area. The active safety systems can adapt the controllers or the decision making to the current road conditions. Road authorities can use the information to distribute their resources in order to keep the roads clear from snow and ice. Without communication between vehicles, the friction coefficient on the road ahead of the vehicle will be unknown for cause-based estimation approaches. For future autonomous vehicles, adaptation of the vehicle velocity before entering a corner is crucial to ensure vehicle stability. On low friction surfaces the maximum possible cornering speed can be greatly reduced in comparison to high friction surfaces. Reliable friction information is thus a crucial issue that needs to be resolved in order to have fully autonomous vehicles on the roads.

The inertial parameters estimation and the tyre force estimation enable the tyre parameter estimation. The inertial parameters are also useful for adapting the reference models in active safety and vehicle dynamics control functions. The tyre force estimator quantifies the tyre forces on the wheels which together with a friction estimator indicate the remaining force potential of each tyre. The estimators in the three papers are hence interconnected.

6 Future Work

The main focus of the future work will be connected to the active tyre excitation. Many fundamental questions remains before this type of system can be implemented in a production vehicle. One of the most important issues is the stability of the vehicle during the manoeuvre, which needs thorough investigation. The intervention must be performed in such a way that the vehicle is stable or can be stabilized during the whole intervention. Inhomogeneous surfaces and sudden steps in the tyre-road friction coefficient pose the largest challenges since the tyres may go from low to large friction utilization very quickly. The ability to quickly control the wheel slip is then important for the vehicle stability in these situations. Since electric motors can apply both positive and negative torque, the slip ratio can be controlled faster in comparison to using the friction brakes. Related to the vehicle stability during the manoeuvre, the system also has to decide when it is safe to initiate an intervention. It is for instance not suitable to initiate an intervention when the vehicle is cornering with large lateral accelerations or if the vehicle is approaching a sharp corner or crossroad. The ability to quickly and safely abort the intervention in case of any unexpected disturbances is thus crucial.

From the estimation perspective, investigations should be made regarding the tyre excitation itself. With system induced active tyre excitation, the excitation can be chosen by the system and adapted depending on the estimator design and noise levels. Deciding the wheel torque profile used during the excitation can therefore improve the performance of the estimator. The measurement noise on the wheel speed signals and the noise on the estimated tyre forces should also be reduced in order to improve the performance of the estimator. The sensitivity of the friction estimation to errors in vehicle parameters should also be investigated.

The friction estimation method presented in this thesis is only focused on excitation during straight driving. If active tyre excitation is used during cornering, lower wheel torques are required in order to reach large tyre excitations. Both the energy consumption during the intervention and the required power from the propulsion system can then be reduced. Possible vehicle configurations and the possibility to implement different excitations methods should therefore be investigated.

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Appendix A, PDF Derivatives

The PDF is given by:

$$p(\boldsymbol{x}; C, \mu) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} e^{-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}\left(F_x[k]-\left(-C\sigma_x[k]+\frac{C^2\sigma_x[k]|\sigma_x[k]|}{3\mu F_z}-\frac{C^3\sigma_x^3[k]}{27\mu^2F_z^2}\right)\right)^2} \tag{A.1}$$

The derivatives of the log-likelihood function can then be derived as:

$$\frac{\partial^2 \ln p(\boldsymbol{x}; C, \mu)}{\partial \mu^2} = -\frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\left(F_x[k] - \left(-C\sigma_x[k] + \frac{C^2\sigma_x[k]|\sigma_x[k]|}{3\mu F_z} - \frac{C^3\sigma_x^3[k]}{27\mu^2F_z^2}\right)\right)\left(-\frac{2C^2\sigma_x^2[k]}{3\mu^3F_z} + \frac{6C^3\sigma_x^3[k]}{27\mu^4F_z^2}\right) + \left(\frac{C^2\sigma_x^2[k]}{3\mu^2F_z} - \frac{2C^3\sigma_x^3[k]}{27\mu^3F_z^2}\right)^2\right) \tag{A.2}$$

$$\frac{\partial^2 \ln p(\boldsymbol{x}; C, \mu)}{\partial \mu \partial C} = \frac{\partial^2 \ln p(\boldsymbol{x}; C, \mu)}{\partial C \partial \mu} = -\frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\left(F_x[k] - \left(-C\sigma_x[k] + \frac{C^2\sigma_x[k]|\sigma_x[k]|}{3\mu F_z} - \frac{C^3\sigma_x^3[k]}{27\mu^2F_z^2}\right)\right)\left(\frac{2C\sigma_x^2[k]}{3\mu^2F_z} - \frac{6C^2\sigma_x^3[k]}{27\mu^3F_z^2}\right) + \left(\sigma - \frac{2C\sigma_x^2[k]}{3\mu F_z} + \frac{3C^2\sigma_x^3[k]}{27\mu^2F_z^2}\right)\left(\frac{C^2\sigma_x^2}{3\mu^2F_z} - \frac{2C^3\sigma_x^3[k]}{27\mu^3F_z^2}\right)\right) \tag{A.3}$$

$$\frac{\partial^2 \ln p(\boldsymbol{x}; C, \mu)}{\partial C^2} = -\frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\left(F_x[k] - \left(-C\sigma_x[k] + \frac{C^2\sigma_x[k]|\sigma_x[k]|}{3\mu F_z} - \frac{C^3\sigma_x^3[k]}{27\mu^2F_z^2}\right)\right)\left(-\frac{2\sigma_x^2[k]}{3\mu F_z} + \frac{6C\sigma_x^3[k]}{27\mu^2F_z^2}\right) + \left(\sigma_x[k] - \frac{2C\sigma_x^2[k]}{3\mu F_z} + \frac{3C^2\sigma_x^3[k]}{27\mu^2F_z^2}\right)^2\right) \tag{A.4}$$

Note that:

$$E\left[F_x[k] - \left(-C\sigma_x[k] + \frac{C^2\sigma_x[k]|\sigma_x[k]|}{3\mu F_z} - \frac{C^3\sigma_x^3[k]}{27\mu^2F_z^2}\right)\right] == 0 \tag{A.5}$$

The derivatives used in Equation 3.34 can hence be reduced to:

$$-E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial\mu^2}\right] = \frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\frac{C^2\sigma_x^2[k]}{3\mu^2 F_z} - \frac{2C^3\sigma_x^3[k]}{27\mu^3 F_z^2}\right)^2 \tag{A.6}$$

$$-E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial\mu\partial C}\right] = -E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial C\partial\mu}\right] =$$
$$= \frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\sigma_x[k] - \frac{2C\sigma_x^2[k]}{3\mu F_z} + \frac{3C^2\sigma_x^3[k]}{27\mu^2 F_z^2}\right)\left(\frac{C^2\sigma_x^2[k]}{3\mu^2 F_z} - \frac{2C^3\sigma_x^3[k]}{27\mu^3 F_z^2}\right) \tag{A.7}$$

$$-E\left[\frac{\partial^2 \ln p(\boldsymbol{x};\boldsymbol{\theta})}{\partial C^2}\right] = \frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\sigma_x - \frac{2C\sigma_x^2[k]}{3\mu F_z} + \frac{3C^2\sigma_x^3[k]}{27\mu^2 F_z^2}\right)^2 \tag{A.8}$$

The negative of the expected value for these derivatives is hence the negative value itself.