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# NONLINEAR DYNAMIC ABSORBER TO REDUCE VIBRATION IN HAND-HELD IMPACT MACHINES

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**Keywords:** Nonlinear tuned vibration absorber, Tuned vibration damper, Impact machine, Optimisation.

**Abstract.** Hand-held impact machines have been used since the early 20th century. Up to day still a little has been changed in their fundamental design. Despite them being robust and efficient, vibration, noise, dust and poor ergonomics cause a large number of injuries to the operators, especially in the stone industry. In this paper, vibration dynamics of a pneumatic hand-held impact machine (HHIM) equipped with a nonlinear tuned vibration absorber (NLTVA) is in focus. The considered HHIM has the same parameters as the existing conventional one with respect to piston weight, impact energy and operating frequency. The paper presents the mathematical and computational models of vibration dynamics of the HHIM with NLTVA as well as the experimental set-up developed for model validation. The advantage of using a nonlinear spring is shown and how the effective vibration reduction frequency of the NLTVA can be made to follow the excitation frequency and thereby in principal achieving the same conditions as is the case for tuned pendulum absorbers for torsional vibrations where the resonance frequency follows the engine order. Using the verified and validated computational model, the sensitivity analysis of vibration dynamics has been performed in a broad set of feasible operational scenarios with respect to structural parameters of the HHIM equipped with NLTVA. Sensitivity analysis results have formed the base for optimisation of the parameters of NLTVA. It was shown by simulations as well as it was proved by experiment that the vibration of the HHIM equipped with NLTVA can be reduced significantly in a broader range of operating frequencies compared to the vibration of the same machine equipped with linear tuned vibration absorber. The results obtained confirm the possibility to design a user friendly low vibration impact machine efficiently operating in a broad frequency range. Optimised NLTVA combined with vibration isolation has shown to significantly reduce the vibration on the operator.

# **1 INTRODUCTION**

Vibration exposure from hand-held impact machines (HHIM) such as rock drills, rammers and breakers with a reciprocating action is a major cause for injuries to the workers in the industry. In order to improve the work environment in the stone industry, a project was started with the objectives to redesign the tools to achieve low vibration as well as improved ergonomics, dust removal and reduced noise while maintaining productivity [1]. Redesign of current hand-held pneumatic impact machines can reduce the vibration level and thereby reduce injuries to workers. Hand-arm vibration injury, often called Hand-Arm Vibration Syndrome (HAVS), is one of the most common reasons for work related injuries among this group of workers in the industry.

Although impact machines have been used since the early 20th century, little has been changed in their fundamental design to date. Despite them being robust and efficient, vibration, noise, dust and poor ergonomics cause a large number of injuries to the operators. Previous work has been done to reduce vibrations from these machines, some of which have been patented [2, 3]. One approach is to use traditional linear tuned vibration absorbers (TVA) invented in 1909 by Frahm and described by Den Hartog [4]. However, this technology is to a large extent limited in practical use on this application since it is only effective in a narrow frequency range. At higher frequencies, the TVA will instead increase the vibration and at lower frequencies will the effect rapidly decrease.

A thorough analysis of the dynamics of the HHIM together with an implementation of a TVA in the handle of the machine in combination with a vibration isolated handle is described in [5]. More general aspects of the NLTVA are analysed in [6] which show how different parameters influence the stability of the overall system.

However, by introducing nonlinear spring characteristic of the auxiliary mass, the effective frequency range can be greatly increased and thereby can the technology be effectively implemented to this kind of machines [7]. As a result, a new generation of impact machines is developed by approaching the redesign from a user perspective, and starting adhering to strict conditions of low vibration, noise and dust as well as sound ergonomics.

The objective of this study has therefore been to develop a user friendly low vibration impact machine using NLTVA together with integrated vibration isolation. A HHIM with a NLTVA combined with vibration isolation has shown to significantly reduce the vibration on the operator from 20 m/s<sup>2</sup><sub>haw</sub> to 2.7 m/s<sup>2</sup><sub>haw</sub> [1].

## 2 ENGINEERING MODEL OF HHIM

The pneumatic HHIM in question consists of the following machine functional components (MFCs): the housing of mass  $m_h$ , the main mass  $m_m$ , the piston of mass  $m_{pist}$  moving in the cylinder, the NLTVA with auxiliary mass  $m_a$ , the chisel, and the handle. The detailed engineering model of the HHIM is depicted in Figure 1. It is assumed that the MFCs are under the action of the external loads modelled by the exciting force  $F_e$  applied to the main mass and the internal loads modelled by the forces exerted by springs and dampers. The effective mass of the hand-arm system is quite small and is neglected in this study. The following additional notations are in use:  $k_m$ ,  $k_a$ ,  $k_h$ ,  $k_p$  are the stiffness between main mass and ground, main mass and auxiliary mass, main mass and housing, and the hand-arm stiffness negresenting the operator, respectively;  $c_m$ ,  $c_a$ ,  $c_h$ ,  $c_p$  are the damping between main mass and ground, main mass and auxiliary mass, main mass and housing, and the hand-arm damping representing the operator, respectively; a and  $F_0$  are the half gap length and the spring preload, respectively, characterising the TVA nonlinearity.



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Figure 1: Engineering model of the HHIM.

Figure 2: Sketch of the HHIM as 3-DoF vibrating system.

Parameter values used in the modelling are introduced in Table 1, except for  $k_a$ , a,  $F_0$  which are subject to optimisation. All values, except the three masses and  $k_h$ , have been estimated. For example, hand-arm stiffness  $k_p$  was taken from [8, 9] and hand-arm damping  $c_p$  was taken from [9]. Auxiliary damping  $c_a$  is kept as low as possible in the HHIM for maximum performance of the NLTVA. It is therefore given a low value in the model. It has been noted that all parameters pertinent to the coupling between main mass and housing as well as the couplings to ground have a minor influence on system response for excitations near nominal operating frequency since that is well above the first two resonance frequencies of the system [10]. Amplitude of the exciting force  $F_e$  in a HHIM prototype has previously been measured to be 351 N at an operating frequency of 26.1 Hz [10], which is utilised in this model. It was also concluded that a sinusoidal force is a good approximation, and its amplitude is assumed to increase quadratic with operating frequency since the displacement of the piston is constant.

	Masses [kg]			Stiffness coeffi- cients [kN/m]			Viscous damping coefficients [Ns/m]			
Parameter	m <sub>m</sub>	m <sub>a</sub>	$m_{ m h}$	k <sub>m</sub>	$k_{ m h}$	$k_{\rm p}$	c <sub>m</sub>	C <sub>a</sub>	$C_{\rm h}$	C <sub>p</sub>
Nominal value	2.7	1	3.1	500	14000	1000	100	1	20	60

Table	1:	Model	parameters.
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#### **3** MATHEMATICAL AND COMPUTATIONAL MODELS

With described engineering model, the HHIM is considered as a vibrating system with three degrees of freedom, the sketch of which is depicted in Figure 2. By introducing generalized coordinates  $x_1$ ,  $x_2$ ,  $x_3$  the equations of motion of the system can be written as follows

$$m_{\rm m} \ddot{x}_{\rm l} + (c_{\rm m} + c_{\rm h}) \dot{x}_{\rm l} - c_{\rm h} \dot{x}_{\rm s} + (k_{\rm m} + k_{\rm h}) x_{\rm l} - k_{\rm h} x_{\rm s} = F_{\rm e}(t) + F_{k}(\mathbf{x}) + F_{c}(\mathbf{x}, \dot{\mathbf{x}}) - m_{\rm m} g$$
(1)

$$m_{\rm a}\ddot{x}_2 = -F_k(\mathbf{x}) - F_c(\mathbf{x}, \dot{\mathbf{x}}) - m_{\rm a}g$$
<sup>(2)</sup>

$$m_{\rm h}\ddot{x}_3 - c_{\rm h}\dot{x}_1 + (c_{\rm h} + c_{\rm p})\dot{x}_3 - k_{\rm h}x_1 + (k_{\rm h} + k_{\rm p})x_3 = -m_{\rm h}g$$
(3)

Here in Eqs. (1-3),  $\mathbf{x} = [x_1, x_2, x_3]^T$ ,  $\dot{\mathbf{x}} = [\dot{x}_1, \dot{x}_2, \dot{x}_3]^T$  are the vectors of generalized coordinates and velocities, respectively.  $F_k(\mathbf{x})$  and  $F_c(\mathbf{x}, \dot{\mathbf{x}})$  are nonlinear spring force and damping force exerted by the NLTVA and applied to the main mass. g is acceleration due to gravity.

Different expressions for  $F_k(\mathbf{x})$  and  $F_c(\mathbf{x}, \dot{\mathbf{x}})$  can be chosen and thus different concepts of the TVA can be studied. The following representations for these forces are proposed

$$F_{k}(\mathbf{x}) = \begin{cases} F_{0} + k_{a}(x_{2} - x_{1} - a) & \text{if } x_{2} - x_{1} > a \\ -F_{0} - k_{a}(-x_{2} + x_{1} - a) & \text{if } x_{2} - x_{1} < -a \\ 0 & \text{else} \end{cases}$$
(4)

$$F_c(\mathbf{x}, \dot{\mathbf{x}}) = c_a(\dot{x}_2 - \dot{x}_1) \tag{5}$$

Here in the expressions (4) and (5), the parameters a,  $F_0$ ,  $k_a$ , and  $c_a$  are the half gap length, the spring preload, the stiffness between main mass and auxiliary mass and the damping between main mass and auxiliary mass of the NLTVA.

The following direct dynamics problem for the HHIM is formulated.

Problem A. Let the engineering model of the HHIM, the exciting force applied to the main mass  $F_e(t)$ ,  $t \in [0, t_f]$  and the internal loads modelled by the forces exerted by springs and dampers of the NLTVA  $F_k(\mathbf{x})$  and  $F_c(\mathbf{x}, \dot{\mathbf{x}})$  be given. It is required to determine the vectors of generalized coordinates  $\mathbf{x} = [x_1, x_2, x_3]^T$  and generalized velocities  $\dot{\mathbf{x}} = [\dot{x}_1, \dot{x}_2, \dot{x}_3]^T$  of the steady-state performance of the HHIM that satisfy the equations of motion Eqs. (1-3) and arbitrary prescribed initial conditions  $\mathbf{x}(0) = \mathbf{x}_0$ ,  $\dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0$ . Note that the final state of the machine as well as the final time instant are not prescribed, i.e. the components of the vectors  $\mathbf{x}(t_f), \dot{\mathbf{x}}(t_f)$  and the final time instant  $t_f$  are free. All structural parameters of the machine including parameters of the NLTVA are assumed to be given.

In order to solve *Problem A*, a computational model of the HHIM in question has been developed and implemented in MATLAB<sup>©</sup>. The core of the computational model is the numerical algorithm of the solution of the initial value problem for Eqs. (1-3) based on MATLAB function *ode45* with the value of both the relative and absolute error tolerances equal to  $10^{-8}$ .

# 4 MODEL VERIFICATION AND VALIDATION

#### 4.1 Model verification

To ensure correctness and accuracy of numerical solutions obtained from the computational model, a verification of the model is done by comparing its results with reference results.

Firstly, the modelled system was configured as a linear system by setting both the gap a and preload  $F_0$  in Eq. (4) to zero. Perfect correlation was seen between the numerical solution and the analytical solution of the same system, with respect to displacements, velocities, accelerations and phase angles. Secondly, simulations were run for the unforced undamped auxiliary system, i.e. the auxiliary mass restricted by the nonlinear spring described by Eq. (4), initiated from a spring compression b. Since that system is not dissipative, the auxiliary mass should oscillate with the same amplitude indefinitely. The said system was simulation for 20 seconds for several different configurations of the parameters  $k_a$ , a,  $F_0$  and b. The undamped resonance frequency has been derived analytically in [10] and is given by

$$f_{\rm res} = \sqrt{\frac{k_{\rm a}}{m_{\rm a}}} \left[ 2\pi - 4 \arcsin\left(\frac{F_0}{F_0 + k_{\rm a}b}\right) + 4a \sqrt{\frac{k_{\rm a}}{k_{\rm a}b^2 + 2F_0b}} \right]^{-1}$$
(6)

#### 4.2 Model validation

Validations of the computational models have been made in two experimental set-ups, a dedicated test rig [11] and in a pneumatic HHIM prototype. In the test rig, all relevant parameters could be controlled and measured and most important, the excitation frequency could be

varied from stand still up to 18 Hz. The test rig employed the TVA nonlinearity created by a gap but no preload in this test. In the HHIM prototype on the other hand, all parameter were close to a real HHIM although the exciting force frequency could not be varied easily and parameters as the stiffness to ground, damping and exciting force needed to be estimated or measured indirectly.

## Validation in test rig

In the test rig, the force is generated from an electric motor to a pneumatic piston connected to a force transducer that is attached to the main mass, Figure 3. The force can be varied to frequency by the motor speed and to amplitude by restricting the air openings in the pneumatic cylinder or changing the stroke length. Displacement of the main and auxiliary mass is measured by laser displacement sensors. In the experiments, the force is increasing linearly with a peak of 45 N at the frequency 18 Hz. The set-up of the system is chosen so that the main mass has a resonance frequency at 2 Hz and the auxiliary mass is tuned to give the lowest vibration of the main mass at 9 Hz.



Figure 3: Test rig.

The results from the experiments compared to the simulations are shown in Figure 4. It is clearly seen that there is a substantial reduction in vibration amplitude of the main mass when the NLTVA is active in a broad frequency range. The deviations are mainly believed to be due to that the experimental exciting force is not purely sinusoidal but has a substantial degree of harmonic content. There are also uncertainties in the spring force for the auxiliary mass since it was compressed partly beyond its specifications. The exciting force in the simulation model was purely sinusoidal.



Figure 4: Experimental and simulated results of main mass vibration for active and inactive auxiliary mass.

## Validation using the prototype

Since the working frequency and the amplitude of the exciting force in the prototype cannot be varied, gap length a was varied to see if the model would predict the changes in reality. To be able to measure the motion of the auxiliary mass, the housing was removed during

measurements and simulations. Since the operating frequency varied slightly between measurements, the frequency from each measurement was used as the excitation frequency in respective simulation. The gap lengths used were 3.2, 6.1 and 9.7 mm and corresponding measured operating frequencies were 26.5, 25.8 and 25.4 Hz, respectively. The simulated and measured root mean square (RMS) displacements for different gap lengths are compared in Figure 5. RMS response of the experimental data was calculated after applying a 5th order high-pass Butterworth filter with a cutoff frequency of 6.5 Hz. The comparison shows very similar results, especially when taking into account that many of the parameters in the prototype cannot be measured very accurately. The model predicts the change in vibration level of both the auxiliary and main mass well.



Figure 5: Experimental and simulation results for the prototype when the gap length is varied.

# 5 SENSITIVITY ANALYSIS AND OPTIMISATION

To investigate how the three NLTVA parameters  $k_a$ , *a* and  $F_0$  that are subject to optimisation affects the vibration suppression, each parameter was varied around its nominal value. The results from varying single parameters separately can be seen in Figure 6.



Figure 6: Vibration of the handle for different values of the NLTVA parameters. Only one parameter is varied at a time while other parameters are kept at their nominal values;  $k_a = 40$  kN/m, a = 5 mm,  $F_0 = 190$  N.

Figure 6 clearly shows that any of the NLTVA parameters can be adjusted to change the frequency that has maximum suppression, i.e. the tuned frequency. When the stiffness  $k_a$  is increased, the tuned frequency moves upwards and vice versa if the stiffness is lowered. The unwanted resonance above the tuned frequency is also moved considerably upwards in frequency when the stiffness is increased, and vice versa when the stiffness is decreased. The resonance is however not affected significantly when the gap length a and preload  $F_0$  are changed. This suggests that an optimal result would be a high stiffness  $k_a$  to move the unwanted resonance to a higher frequency, combined with a lowered preload  $F_0$  and/or an increased gap length a to keep the minimum, i.e. the tuned frequency, around 28 Hz.

The following optimisation problem is formulated in order to optimise the proposed NLTVA in the considered HHIM:

$$\left(k_{\mathrm{a}}, a, F_{\mathrm{0}}\right)_{\mathrm{optim}} = \underset{k_{\mathrm{a}}, a, F_{\mathrm{0}}}{\operatorname{argmin}} L\left(k_{\mathrm{a}}, a, F_{\mathrm{0}}\right) \tag{7}$$

Here in Eq. (7), L is the objective function and  $k_a$ , a and  $F_0$  are the NLTVA parameters subject to optimisation. The objective function is defined as the area under a weighted response curve in a frequency range that spans  $\pm 15$  % of the nominal operating frequency, 28 Hz, to account for variations.

$$L(k_{\rm a}, a, F_{\rm 0}) = \int_{23.8\,\rm Hz}^{32.2\,\rm Hz} \dot{x}_{3,\rm RMS}(f, k_{\rm a}, a, F) \cdot W(f) \,\rm df \tag{8}$$

In Eq. (8),  $\dot{x}_{3,\text{RMS}}$  is the velocity RMS response at the handles, computed by the numerical model, and W(f) is a weighting function that provides priority of minimisation at the nominal operating frequency, 28 Hz, but still lets the response at the frequency bounds 23.8 Hz and 32.2 Hz be accounted for. The weighting function is a normal distribution centred at the nominal frequency with a standard deviation of 2.83 Hz. Choosing this standard deviation makes the weighting function being three times as large at the nominal frequency as at the frequency bounds. In the optimisation routine, the frequency range of interest,  $f \in [23.8, 32.2]$  Hz, is discretised into 15 points such that the density of points is proportional to the weighting function W(f). The objective function is computed by summing the velocity RMS responses evaluated at the discrete frequencies. The minimisation problem in Eq. (7) is solved in MATLAB with *lsqnonlin*, a nonlinear least squares solver implementing the trust-region-reflective method. During optimisation, all parameters are restricted by a lower bound of zero and upper bounds which are 500 kN/m for stiffness  $k_a$ , 40 mm for gap *a* and 500 N for preload  $F_0$ . To increase the chance of reaching the global optimum sought in Eq. (7), multiple starts are run, each with an initial guess randomised uniformly within the parameter bounds.

#### **Optimisation results**

The optimisation has been run using over 250 randomised starts. About half of the optimisation runs resulted in parameters that gave very low objective function. The parameters that gave the lowest found objective function value were  $k_a = 164.23$  kN, a = 10.21 mm and  $F_0$ = 98.2 N. The parameters are as expected considering the findings of the sensitivity study. The stiffness is very high, which places the unwanted resonance frequency far above the operating frequency, and the increased gap length a moves the tuned frequency down to 28 Hz. Figure 7 compares the response of the handle between using the optimised NLTVA and the traditional linear TVA. The suppression band with the optimised NLTVA is much wider than with the linear TVA, clearly demonstrating the advantage of using a NLTVA.

It is thus shown that there is a substantial advantage in using a nonlinear spring characteristic instead of the traditional linear spring for the auxiliary mass in order to achieve a broader useful frequency range and thereby a more robust system. The core reason for the improvement can be explained to a large extent by that the phase of the auxiliary mass remains close to  $180^{\circ}$  in relation to the exciting force over a very broad frequency range, Figure 8 and Figure 9. The counterphase of the auxiliary mass is maintained because the resonance frequency of the auxiliary system to a large extent follows the increasing excitation frequency, Figure 10. The reason for this in turn is that the displacement amplitude of the auxiliary mass increases with higher excitation frequencies, which is also seen in Figure 10, and, since the auxiliary spring characteristic from the optimised  $k_a$ , a and  $F_0$  is nonlinear stiffening, the resonance frequency of the auxiliary system will then increase.



Figure 7: Vibration of the handle. Comparison between using the optimised NLTVA, the LTVA tuned to 28 Hz, deactivated TVA (vibration isolation only) as well as no means of vibration reduction at all.

In Figure 8 and the close-up in Figure 9, the phase and amplitude are compared between using a linear and a nonlinear TVA that are both optimised to reduce vibrations in the area around 28 Hz of the system.



Figure 8: Phase and displacement for NLTVA and LTVA.



Figure 9: Phase and displacement for NLTVA and LTVA, close-up.

What can be seen from these figures is that the linear auxiliary mass is only in counterphase to the excitation up to about 30 Hz. Just above 30 Hz, there is a phase shift and a severe amplification of the vibration. In the nonlinear system on the other hand, the auxiliary mass maintains the counterphase and reduces vibration even up to about 60 Hz. The resonance introduced by adding the TVA to the system is considerably far above and on a safe distance from the tuned frequency in the nonlinear case, making the effectiveness of the NLTVA very insensitive to varying excitation frequency.

In Figure 10, it is shown how the resonance frequency of the auxiliary mass is increasing with increased excitation frequency. The analytical resonance frequency is given by Eq. (6) and is a system where the exciting force is increasing quadratic with increased frequency. This condition is valid for most mechanical systems which vibrations are mass controlled and where there is a constant displacement of moving components over the operating frequency.



Figure 10: Analytical resonance frequency and spring compression versus excitation frequency.

The relation that the resonance frequency is increasing with excitation frequency can also be found in the closely related "sister" technology with tuned pendulum torsion absorbers [4]. The tuned pendulums are used to create torque on a shaft to reduce torsional vibrations, e.g. in the shaft from a combustion engine. In the tuned pendulum system, the resonance frequency is proportional to the rotating speed which means that the resonance frequency will follow the engine order and thereby have an effective vibration reduction throughout the whole speed range.

## 6 CONCLUSIONS

A 3-DoF computational model for a pneumatic HHIM utilising a NLTVA in combination with vibration isolation was developed and implemented in MATLAB. The numerically modelled HHIM is excited by a harmonic force exerted by a piston reciprocating with constant amplitude. The model was verified against analytical results and validated against experimental data from a dedicated test rig as well as from a HHIM prototype.

Sensitivity analysis and optimisation of the three NLTVA parameters; stiffness, gap and spring preload, have performed based on the numerical model. The obtained results showed that, for the considered HHIM, the optimised NLTVA is superior to a linear TVA tuned to the same frequency, especially in terms of its insensitivity to variations in operating frequency. In combination with vibration isolation, the optimised NLTVA can significantly and reliably reduce vibration on the operator in a broad range of operating frequencies.

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