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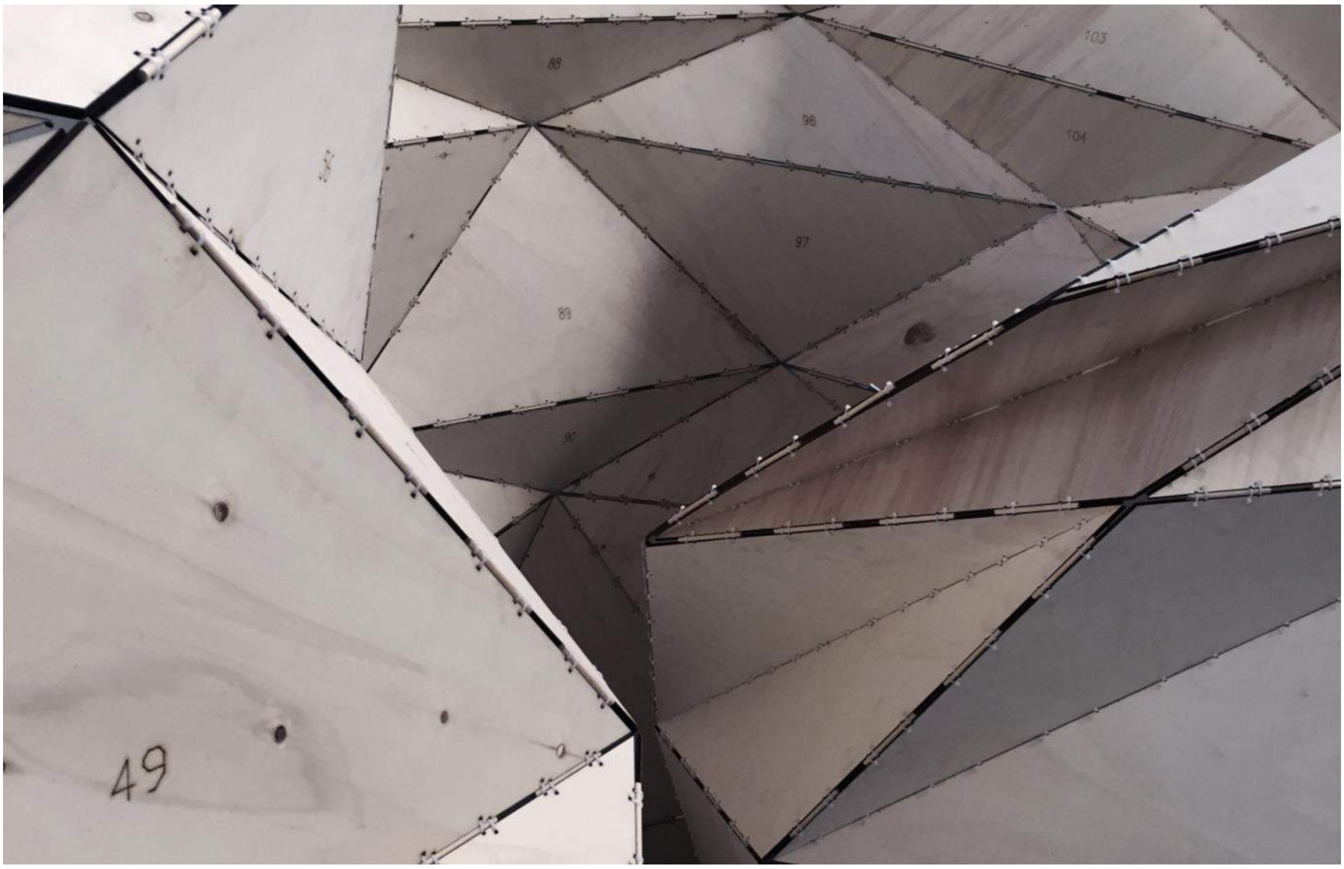

Structural Folding
A parametric design method for origami architecture

Master's thesis in Structural Engineering and Building Technology

Camilla Samuelsson and Björn Vestlund

Department of Applied Mechanics
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Cover:
Application of the design method on a full scale folded structure.

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Abstract

Folding of a plate will affect its stiffness, shape and expression. This principle can be used in architecture to create material efficient structures that are load bearing, space enclosing and expressive at the same time.

Folded plate structures are not very developed or commonly used in the building industry, apart from in the very basic form of simply corrugated sheets. To create more complex folded plate structures, the designer need to have an understanding of the mechanical and kinematical behavior associated with different fold patterns. Further, a method for parametrical modelling and analysis will give more possibilities and facilitate the design process.

This report results in a design method for folded plate structures. The proposed method includes: Investigation of different origami tessellations and their properties in simple paper models. Generation of two-dimensional crease patterns in *Rhinoceros* parametrical plug-in *Grasshopper*. Folding and shaping of the structure in *Grasshopper* physics simulation add-on *Kangaroo*. Structural analysis using *Grasshopper* FE add-on *Karamba*. Building of scale models for an intuitive understanding of the structure.

The final method is applied in the design and construction of a full-scale plywood pavilion. To be able to realize this pavilion, some more practical issues were investigated as well, including design of connections and internal constraints needed to stabilize the structure.

Key words: origami structure, folded plate structure, architecture and engineering, FE-modelling, parametric modelling, design methods, design process

Bärande veck
En parametrisk designmetod för origamiarkitektur
Examensarbete inom Konstruktionsteknik och byggnadsteknologi
Camilla Samuelsson and Björn Vestlund
Institutionen för tillämpad mekanik
Avdelningen för material och beräkningsmekanik
Chalmers tekniska högskola

Sammanfattning

Veckning av en skiva påverkar både dess globala och lokala form samt dess styvhet. Detta fenomen kan användas inom arkitektur för att skapa materialeffektiva strukturer som är bärande, rumsskapande och uttrycksfulla på samma gång.

Denna typ av strukturer är varken särskilt utvecklade eller vanliga i byggindustrin, utom i väldigt enkla former som till exempel korrugerad plåt. För att skapa mer komplexa veckade skivstrukturer, behöver en designer ha en förståelse för de mekaniska och kinematiska egenskaper som är kopplade till olika vikmönster. En metod för parametrisk modellering och analys kan ge större möjligheter och underlätta designprocessen.

Detta examensarbete resulterar i en designmetod för veckade skivstrukturer. Metoden innehåller; Undersökning av olika origamitessealtioner och deras egenskaper i pappersmodeller. Generering av tvådimensionella vikmönster i Rhinoceros parametriska plug-in Grasshopper. Vikning och formning i Grasshoppers fysiksimuleringstillägg Kangaroo. Spänningsanalys i Grasshoppers FE-tillägg Karamba. Byggande av fysiska modeller för en intuitiv förståelse för olika veckade strukturer.

Den slutliga designmetoden tillämpas i design och konstruktion av en fullskalig paviljong. För att kunna realisera detta undersöks också mer praktiska problem som till exempel utformning av leder och låsningsdetaljer.

Nyckelord: Origamistruktur, veckade skivstrukturer, arkitektur och teknik, FE-modellering, parametrisk modellering, designmetoder, designprocess

Contents

Preface

In this master's thesis a design method for folded structures has been developed and applied on a full scale structure. Design and modelling of folded plate structures has been explored in digital and in physical models by combining an engineering and an architectural point of view. The thesis is made by students from the Architectural Engineering programme (Arkitektur och teknik) in the master programme Structural Engineering and Building Technology. However, it has been carried out at the Department of Applied Mechanics with supervision from Dr Mats Ander.

The parts for the physical models we produced in the workshop at the department of architecture at Chalmers and in KKV (Konstnärernas Kollektivverkstad) where a laser cutter of proper size was found. A full-scale structure was built in Betonghallen at Chalmers.

The project was partly financed by ARQ foundation and Chalmers MasterCard.

The thesis could not have been done without some help. We would like to thank Mohsen Heshmati for advice about FE-modelling, Aaro Pirhonen for help in the metal workshop and all friends helping us folding the large scale structure into shape. Finally we would like to thank our supervisor Mats Ander for all the support, dedication and enthusiasm.

Göteborg May 2015-05-04

Camilla Samuelsson and Björn Vestlund

1 Introduction

The ancient art of origami originates from Japan. It comprises the art of folding a single paper into shapes without stretching, gluing or cutting. Origami techniques of folding are often referred to when talking about more complex folded structures. In structures it is usually not practical, as for paper origami, to fold the structure from one single sheet. Instead they are often made of several plates that are assembled into one structure.

Folded plate structures are thin rigid structural surfaces that are active in beam or plate action and which, due to their efficient use of material, can span remarkable lengths. They are structurally efficient in terms of material use. Further they can be very compact and stored in small spaces when flat folded. Folds can also be used to prevent buckling for thin compressive structures.

Constituting both enclosing envelope and structure, folded plate structures are good examples of when architecture and engineering are closely intertwined.

Still, they are not commonly used in building and design. Architectural design based on the concept of folding can be both architecturally expressive and material efficient. Folding can also be an interesting principle for deployable structures.

1.1 Aim

The purpose of this master thesis work is to find a useful design method for folded plate structures. This method should integrate parametrical modelling, FE-evaluation and structural optimization in a convenient way. This integration will be achieved by creating a design tool that combines existing software.

The method will finally be applied in the design and building of a full-scale pavilion.

A large part of the master thesis work will consist of investigations of mechanical behaviour, kinematics and geometry of folded plate structures in general.

1.2 Limitations

Due to the limit of time, some patterns of origami tessellations that we consider to be more interesting for structural purposes will be investigated thoroughly while others will only be treated briefly.

Software that already exists will be combined instead of new ones being created. Scripts and programs will be written only if needed to transfer data between programs. The method will be general and useful in a variety of folded plate applications. The specific software tool or model however, will be specific for our design and will not be made general for all kinds of folded structures.

No controlled loading experiments will be conducted due to cost and time limits.

Physical models in various sizes will be used to make qualitative mechanical and kinematical evaluations. The more accurate structural properties of the design will only be evaluated through computer analysis.

Since a large part of the master thesis consist of applying the design method to scale models and finally build a large scale structure a limitation during the design is that the structure should be flat foldable. This limitation was chosen to enable mounting of the parts when the structure is lying down, making a flat foldable structure is also beneficial to the production process limiting the amount of waste during fabrication.

1.3 Research question

How can both parametric software and physical models be used in the design, analysis and optimization of folded plate structures?

Which fold patterns have structural qualities? What are their mechanical characteristics and their architectural potential?

1.4 Method

To understand the subject and grasp which focus points that might be interesting to investigate, literature studies will be conducted, including reading of books, reports, articles, various internet sources and previous research.

Since this master thesis work contains moments of design the method partly has characteristics of a design method. This means for example sketching in physical and digital models as a way of open-minded exploration of a subject or field in search of possible applications and qualitative improvements. A design method often also include an iterative process rather than a linear one, meaning that it is allowed and even encouraged to work something through, draw conclusions and then start over from the beginning.

The iterative design process will be applied to find conceptual designs of a wall-like pavilion. This iterative process will include building of physical sketch models for structural, kinematic, functional and aesthetical examination. To make quantitative evaluations of the promising designs the object will be parametrically described and modelled in CAD software such as *Rhinoceros* together with the parametric control plug-in *Grasshopper*. Parameters that can be allowed to change will be identified including their range. A set of measurable quantities will then be chosen. The model will be analysed using finite element-software such as *Grasshopper* add-on *Karamba* or standard FE software such as *Abaqus*. One benefit of having the model-generating tool and model-evaluating tool in the same environment (or at least connected to each other) is that a feedback loop can be established. The parameterized model can in this way be altered according to information received from the evaluation in the previous steps and optimized with optimization software such as Grasshopper add-on *Galapagos*. This way of creating informed design can be very fast when rules for generation and evaluation of the digital model are in place.

2 Background

2.1 History

2.1.1 Origami

The word origami comes from the ancient art of folding a single paper into shapes without stretching, gluing or cutting. An advantage with the origami tessellations is that the 3D shape is specified by its 2D folded pattern due to the geometric constraints (Tachi, 2013).

Paper was invented in China in the 2:nd century AD, in the 12:th century it came to Japan where the work of paper art begun. Originally origami was about finding the character of a figure by using the least resources possible, to sketch the essence of the motive, and not about making a realistic replica with many folds. The oldest known publication of origami, Senbazura Orikata, was published in 1797 but the art of origami has been practiced long before that (Buri, 2010).

In the late 50's origami got popular worldwide due to exhibitions, in New York and Amsterdam, by the Japanese artist Aikira Yoshiwazawa. Mathematicians, engineers and architects, among others, got interested in the techniques and started to find applications for origami in a diversity of disciplines, like solar sails for satellites, aluminium cans and airbags (Buri, 2010). Mathematicians applied mathematical principles to the art and found the relations between three-dimensional geometry and fold patterns. Later on, because of this, origami folding became more popular and more advanced. It now became possible to make realistic replicas, of for example animals and insects (Lang, 2008).

The traditional art of origami can be divided into three branched; Classic origami, Modular origami, and Origami tessellations. Classic origami consists of strongly simplified, often two dimensional, images of animals, plants and objects, that are made from one square single piece of paper without cutting or gluing. Modular origami deals only with geometrical solids, like polyhedrons and spatial lattice structures, composed of individual blocks that are put together. Origami tesselations are also purely geometric, starting from a single sheet of paper with a geometric fold pattern and the result is often flat but it can also be three dimensional (Buri, 2010).

In nature the principles of origami can be found in leaves, insect wings and other thin structures where folding is used for stiffening (Buri, 2010). Recently, much of the progress made in the field of origami, has been drawn from nature and biomimicry. At the university of Tokyo Dr Taketoshi Nojima are doing research on the fold patterns found in nature in particular the growth process of plants, Jean Jacques Dupa, researcher at the Commissariat à l'Energie Atomique in France, has invented the area BiOrigami (Trebbi, 2014).

2.1.2 Folded Plate Structures

Folded structures started to appear in the beginning of the 20th century due to the search for efficient and light structures. Modern materials like reinforced concrete gave new possibilities. Eugene Freyssinet created the Orly airport in 1923, the first building with a folded roof structure (Šekularac,2012).

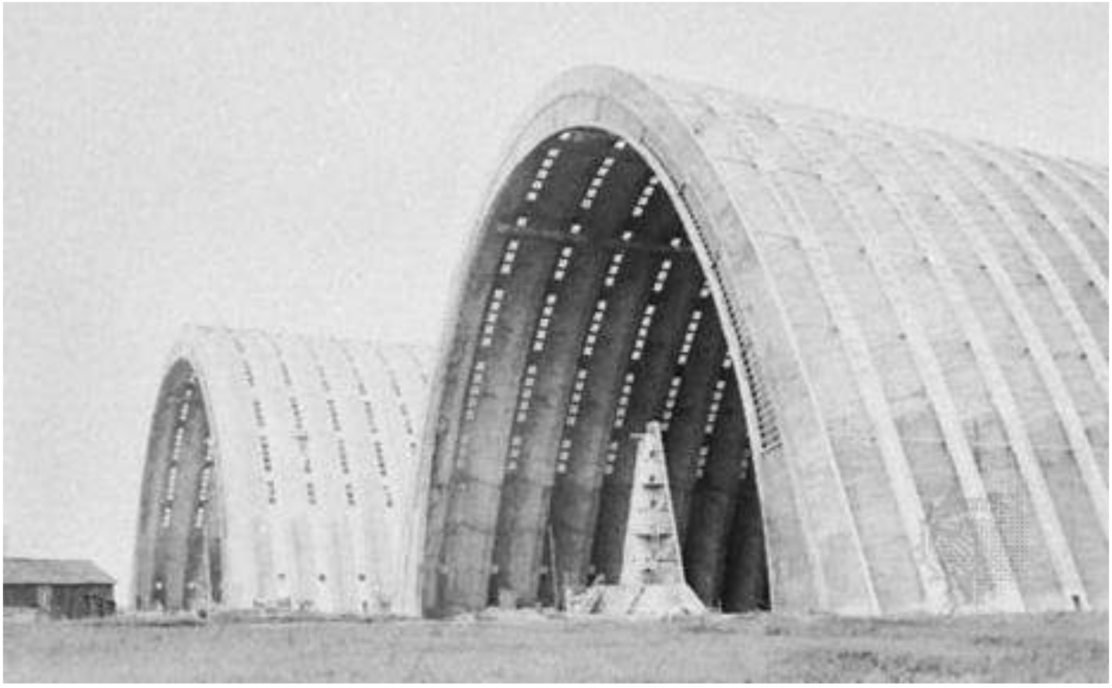


Figure 2.1 Hangar at Orly Airport (Encyclopedia britannica).

The first folded structures were shaped as vaults or domes, as can be seen in the Orly Airport, rather than as plate structures since the engineers in this time were more familiar with how domes and vaults behaved under load. In the 1940's the technical development had gone further and some of the initial challenges were mastered, like for example the problems with rigid joints. Experimentation with new kinds of folded plate structures began and other materials like steel, timber, composites and glass were investigated. Folded plates were initially mostly used in structures with large spans, like industrial buildings, since they offered the most efficient and cost effective option without any need of columns. An early example of this is the Atwater Kent Factory in Philadelphia that was built in 1923, since it is such an early example the spans are a bit shorter and the folded plate technique is not used to its full potential. Pre-stressing made thin plates possible and architects got interested in the folded plate structures. Folded plates were used in churches, exhibition halls, sport facilities and for many other purposes (Bechthold, 2008).

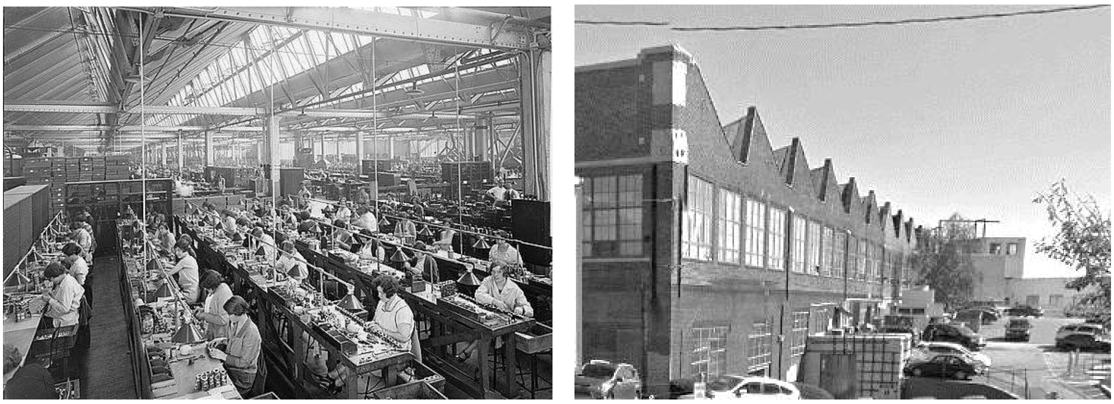


Figure 2.2 Example of a folded roof. Atwater Kent Factory in Philadelphia. Left (The encyclopedia of great Philadelphia) Right (Google street view).

2.1.2.1 Concrete

The early development of folded plate structures was totally dependent on the development of concrete and fibre reinforced composites. The first folded concrete structures had a simple one directional corrugation composed of repetitive elements. Pier Luigi Nervi was one of the engineers that contributed to the development of folded plate structures. As a part of his research for material and structural efficiency in the 60's, he refined the principle of vaults corrugated as folded surfaces. Most of the folded plate concrete structures he made was based on a simple geometry of parallel or inversely oblique folds, like the UNESCO Congress Hall built in the early 60's (Buri, 2010). There are also examples of plate structures of reinforced concrete from the late 50's and early 60's in USA. Those had a more advanced fold pattern like the Miami Marine Stadium and American Concrete Institute Building.

Figure 2.3 UNESCO conference hall in Paris, by Architect M. Breuer und Zehrfuss and Engineer. P. L. Nervi (UNESCO).

Figure 2.4 Miami Marine Stadium by Hiliaro Candela, built 1963 (Florida Memory).

Figure 2.5 The American Concrete Institute building in Detroit by Minoru Yamasaki built in 1958 (Michigan Modern).

2.1.2.2 Timber

In the beginning of the 60's there were a series of experiments going on with folded timber structures that were built from glue-laminated timber, plywood or layers of glued and nailed diagonal planks. Folded timber structures made after the 60's have been rare until the late 90's when large size timber products, like cross laminated timber panels, were developed resulting in a new interest in these structures. A recent example is the rehearsal room of Tannhausen designed by Regina Schineis in 2002, where both the roof and walls are folded with corrugations that follows the shape of the building. Structurally it is composed of two-hinged V-shaped frames and the structure, shape and interior cladding are formed by glue-laminated timber panels (Buri, 2010). The shape of the building is obtained from the acoustical requirements of the rehearsal room, where the folded shape prevents flatter echoes diffuses the sound (Scheis, 2004).

Figure 2.6 Exterior of rehearsal room of Tannhausen by Regina Schineis, built in 2002 (Hiendl & Schineis Architectenpartnershaft).

Figure 2.7 Interior of rehearsal room of Tannhausen by Regina Schineis, built in 2002 (Hiendl & Schineis Architectenpartnershaft).

2.1.2.3 Fibre-reinforced composites

New composite materials, like reinforced plastic, triggered the systematic research on folded plate geometries in the 60's. Renzo Piano, Z.S. Makowski and Peter Huybers all investigated folded plate structures made from Fibre reinforced polymers (FRP). They focused on structures based on anti-prisms which can be folded from a plane surface. In 1966 Piano realised a mobile shelter for a sulphur factory made from a diamond fold-like structure (Buri, 2010).

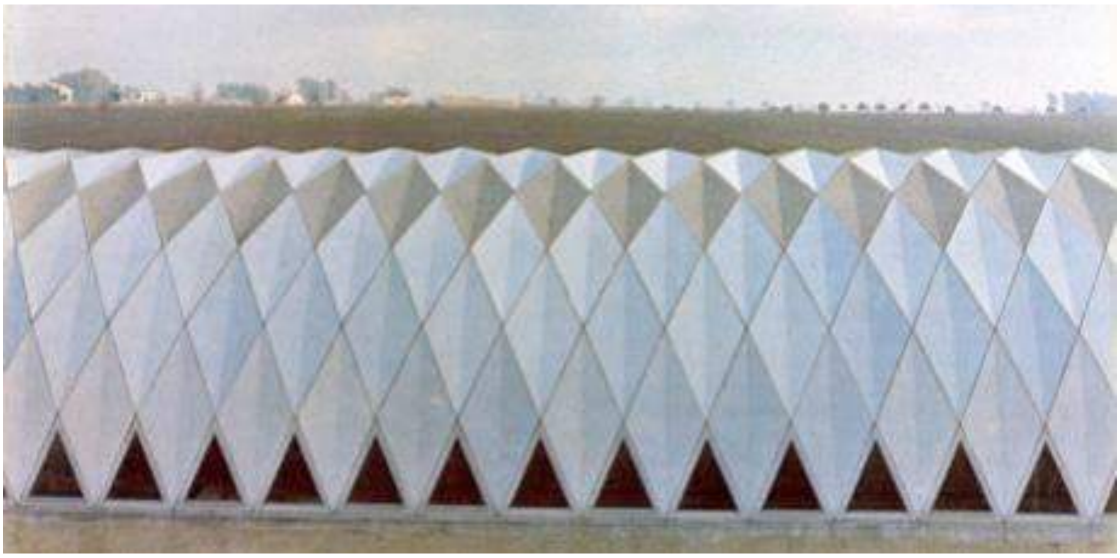


Figure 2.8 Shelter for sulphur factory by Renzo Piano, built in 1966 (Composites and Architecture).

2.1.2.4 Steel

Steel or iron has been used in folded structures since the 1820s in the form of corrugated iron sheets for roofing (Wikipedia, 2014). Steel folded plate structures appeared in USA in the 1960s and the use of these systems has been increasing rapidly in recent years. Folded plate structures made of steel are generally constructed from plates working as membranes taking both shear and normal forces when the spans are small. For larger spans the plate structures are usually made up from an inclined simple truss with diagonal truss element taking shear, the structures are covered with steel plates taking only normal force (Wu, 2010). Two examples of this technique are the Yokohama ferry port building and the Riverside museum.

Figure 2.9 Folded steel roof in Palm Springs by Donald Wexler, built in 1962 (Ultra Modern Style).

Figure 2.10 Ferry terminal in Yokohama International Port by FOA Architects, built in 2002 (Eikongraphia).

Figure 2.11 Riverside museum by Zaha Hadid, built in 2014 (Spotted by Locals).

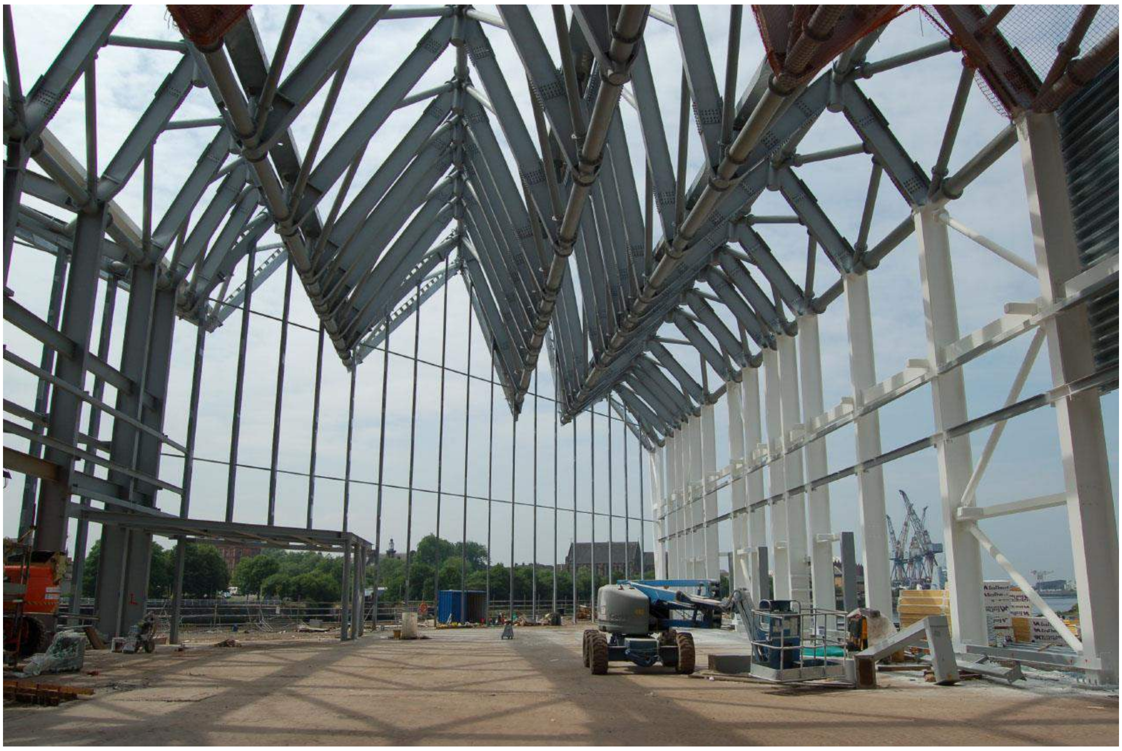


Figure 2.12 Riverside museum by Zaha Hadid during construcion, built in 2014 (Buildipedia).

2.1.2.5 Re-invention

As have been stated above there was a lot of work on folded plate structures going on in the 1960's but later the interest declined. The main cause was the possibility of electric-arch-welding, which made truss systems more interesting and efficient. Plate structures were almost forgotten. It is not until recently that these structures are being investigated again due to new possibilities of computer-aided production and new materials (Bechthold, 2008).

2.2 State of the Art

Existing applications of origami structures in structural engineering can be categorized into three areas; deployable structures, like solar sails or emergency shelters. Secondly folded shapes could be used for their stiffness and mechanical properties. The third area of use is as shock absorbing advises, for example car crash boxes with origami patterns that induce higher local buckling modes, or for packaging material (Schenk, Guest, 2010).

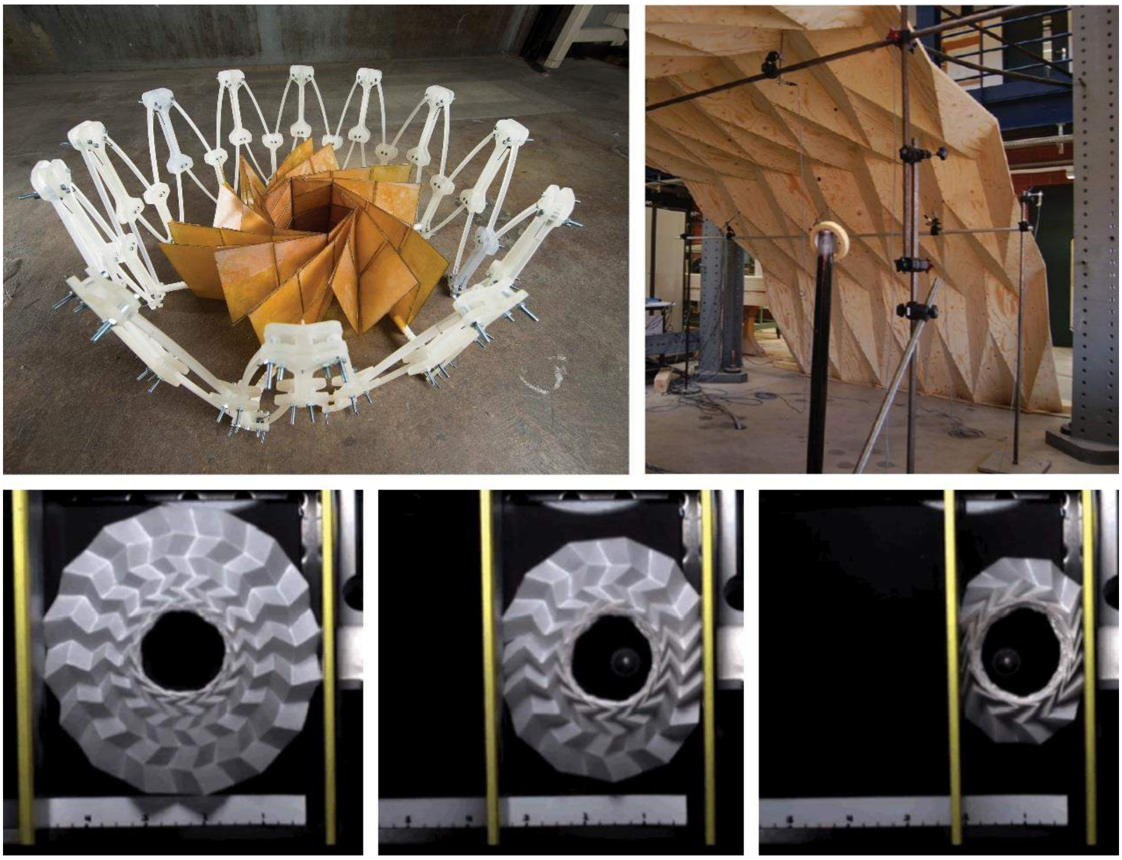


Figure 2.13 Top left: Solar panel project for NASA, working with the deployability of Origami structures, Top right: Research project at IBOIS, investigating the structural properties of origami. Bottom: Project at university of California, using the possibility of the origami structure to absorb energy.

At the university of Cambridge Mark Schenk and Simon D. Guest has until recently been researching on folded structures. In his PhD thesis from 2011 Schenk introduces a new type of shell structure, the folded shell structure, which globally can be regarded as a thin shell structure, on a mezzo-scale they consist of tessellation units, and on a local scale consist of thin walled shells joined at the fold lines. The mezzo-scale has been discovered especially interesting, due to the ability of changing the global Gaussian curvature (See chapter 3.1.1). Much of the work that has been done is about multi scale modelling of the sheets, making the possible to prescribe the global mechanics from deformations of the unit cells. It was discovered that the dominant mechanical behaviour of the whole structure is depending on the geometry rather than the material and that it is insensitive to imperfections in the fold pattern. He has also been looking into manufacturing methods and created a new manufacturing method suitable for these structures, based on cold gas pressure folding.

Some of the future work that needs to be done in the area according to Schenk is combining the kinematic analysis with material models and taking the large out of plane deformation into account in the folds including fatigue analysis. An application, among others, that he finds interesting for further work is the impact absorbing qualities of origami structures.

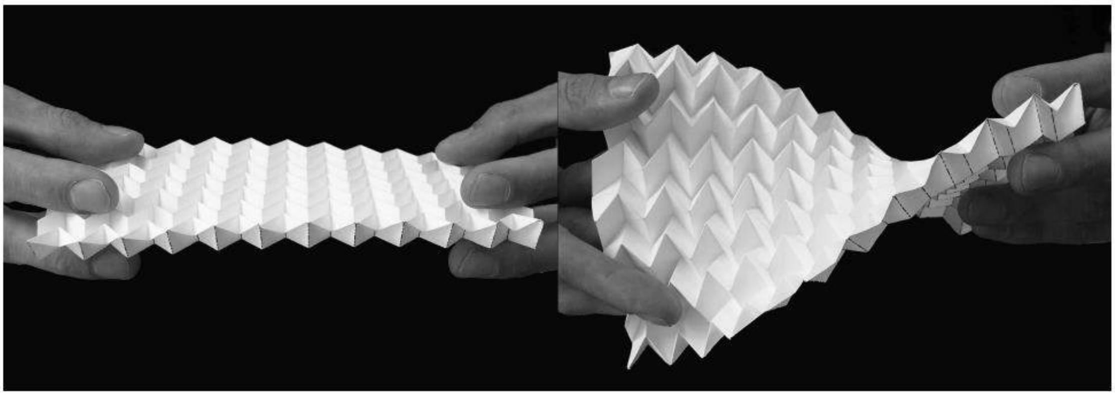


Figure 2.14 The possibility to twist the Miura-ori pattern into a saddle shape with negative Gaussian curvature (Schenk, 2011).

At the University of Tokyo research is being conducted by Tomohiro Tachi on rigid deployable origami, origami structures that consists of stiff plates and that is foldable. The research focus on making 3D shapes from flat objects and making them into units that can build various structures. He has developed rigid-flat-foldable discs and cylinders by introducing bi-directionally flat-foldable planar quadrilateral meshes resulting in 3D structures that can be flat folded in two possible ways.

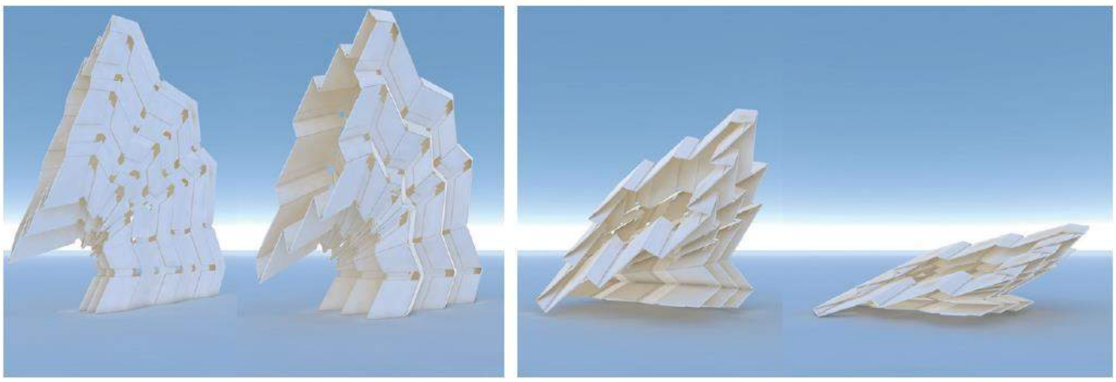


Figure 2.15 A cylinder of bi-directionally flat-foldable planar quadrilateral mesh (Tachi, 2010).

Tachi is also investigating thick origami and how to deal with the thickness of the plates in deployable structures. In his paper on thick origami he is presenting a new method for enabling rigid foldable origami structures that has thick panels to still have the kinematical behaviour of ideal origami surfaces. The method consists of trimming the intersecting parts of meeting panels and works for full-scale structures.

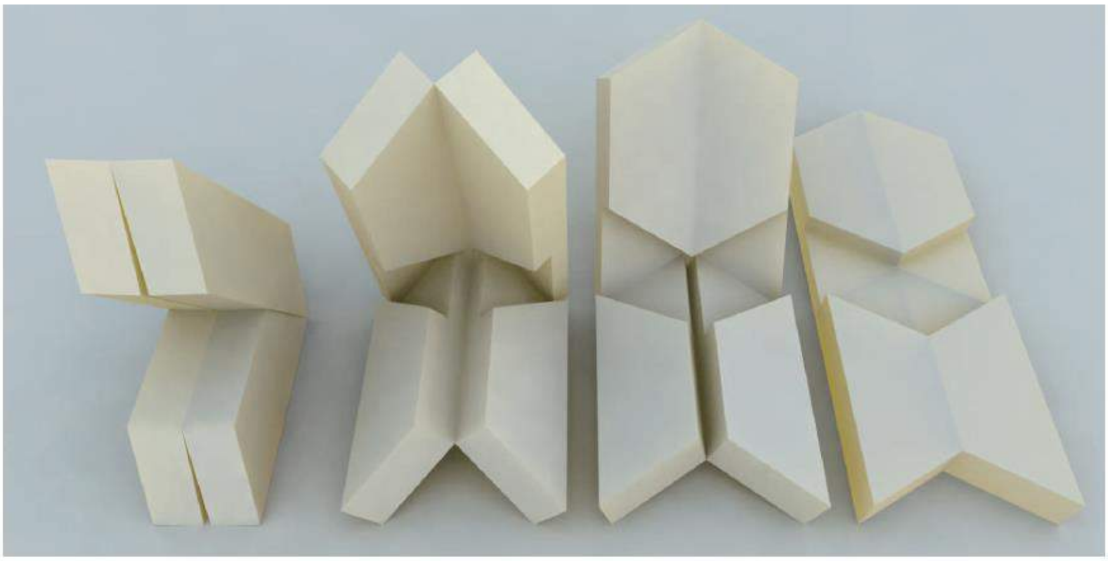


Figure 2.16 Thick rigid foldable origami structure (Tachi, 2011).

Tashi has also invented a computer based design method to obtain a folding pattern for a 3D object from a plane paper, called Origamizer, and another software, called Free Form Origami, that calculates the 3D form such that it can be created by folding a single piece of paper. In an interview in the spring edition of Approach magazine 2013 Tachi explains that computer based design methods today enables a rational way of producing less restricted free form designs, which has led to more interest among architects and engineers in the field of origami. He thinks that the next step in the future would be to apply the principles and theories of origami on architecture to create practical deployable structures that change the quality of a space. This could be possible due to upcoming computer design methods.

At the Laboratory for timber constructions IBOIS, at the École Polytechnique Fédérale de Lausanne, Hans Ulrich Buri and Yves Weinand has been doing research on foldable plate structures from cross laminated timber. In their research they have developed a form finding process based on origami that generates doubly-corrugated surfaces from two polygonal lines, the cross-section profile and the corrugation profile. This allows a fast representation of complex folded structures both in space and unfolded and a diversity of forms can be generated. This approach combines intuitive experiments and rational analysis to provide a design method which creates folded plate models in a 3D software. They have also studied the structural properties and connections of these structures in full-scale experiments.

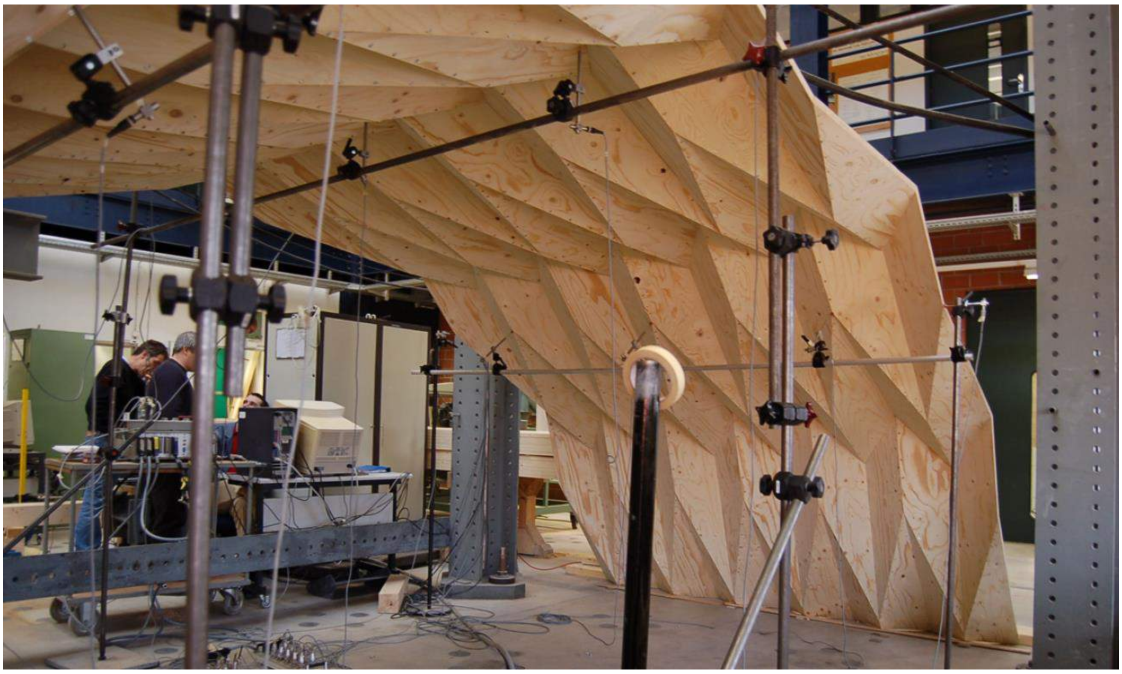


Figure 2.17 A full-scale test of folded plate structure (Buri, 2010).

Hans Ulrich Buri states some areas in his paper, Origami – Folded Plate Structures, that would be interesting for further research. One of the future focus points he mentions is implementation of the design method of corrugation profile and cross-section profile into a parametric software. This would accelerate the design process and make it possible to connect to calculation software for optimization of the structures. A parametric representation would be especially interesting for structures with triangular faces since it would the general shape to be deformed. Buri also writes that another interesting field to investigate is doubly curved structures, which are not possible with single patch geometries but could be created using multiple patches.

For further development of these structures calculations on their behaviour has to be improved. Connections have to be further developed to be efficient and durable. To be able to use these structures in buildings, integration of thermal insulation and exterior cladding has to be developed in regard to each specific project.

2.3 Rules and symbols of fold patterns

In the 1950 Akira Yoshizawa structured the art of origami folding by developing a language of symbols like arrows and lines representing the different folds. This made it possible to communicate and transmit information about origami folding. This language was later revised by Samuel Randlett and Robert Harbin, to the system that is used today, the Yoshizawa-Randlett system (Wikipedia 2014).

The symbols used in this report are based on the Yoshizawa - Randlett system:

Dashed line		Valley fold
Continuous line		Mountain fold
Thick Continuous line		Paper edge
Dotted line		Support fold that in the final model is left unfolded

2.3.1 Basic rules of crease patterns

When creating a crease pattern there are certain rules that have to be fulfilled to make it foldable. The first rule is that the faces have to be able to be coloured with two colours without two same-coloured faces being next to each other (Lang, 2008).

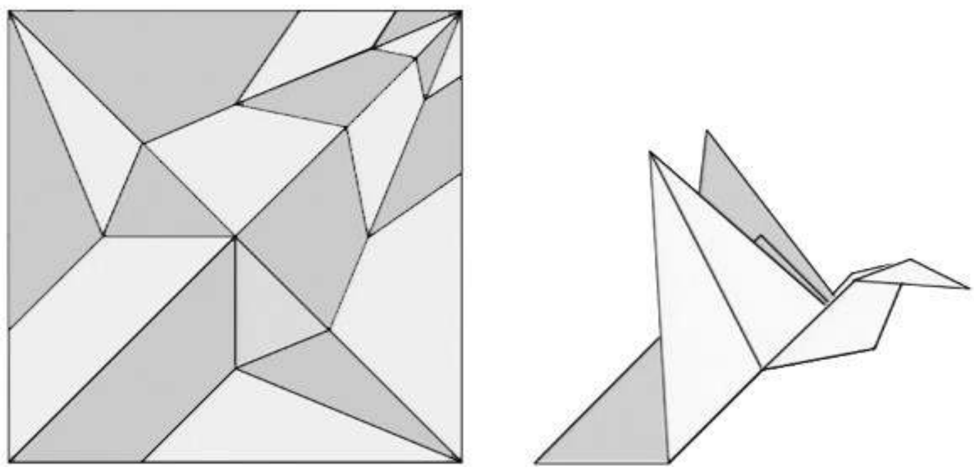


Figure 2.18 First rule for crease patterns, two-colourability (Lang, 2008).

The second rule is that in every internal vertex, the number of valley folds and mountain folds always differs by two. Two less or two more (Lang, 2008).

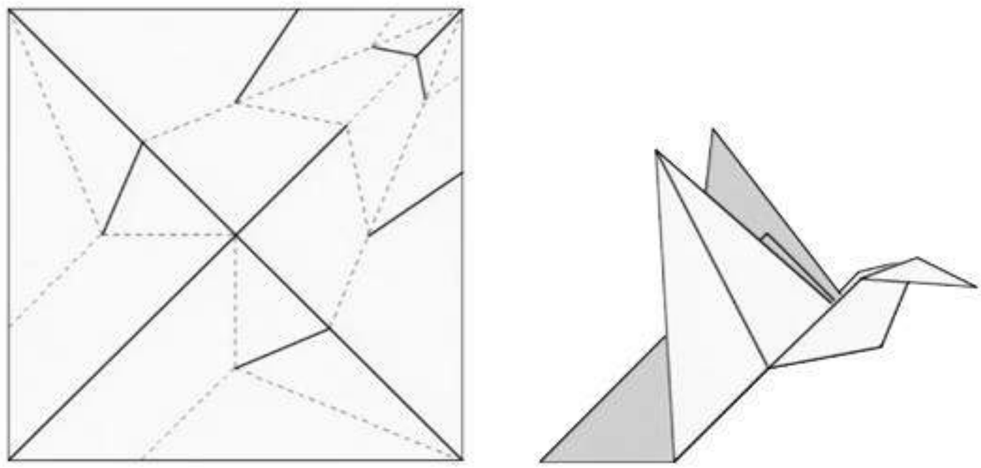


Figure 2.19 Second rule for crease patterns, valley-mountain fold relation ± 2 (Lang, 2008).

The third rule is that if all angles meeting in a point are numbered, all even angles will add up to 180 degrees and so will the odd numbered angles. The fourth rule is that a sheet can never penetrate a fold (Lang, 2008).

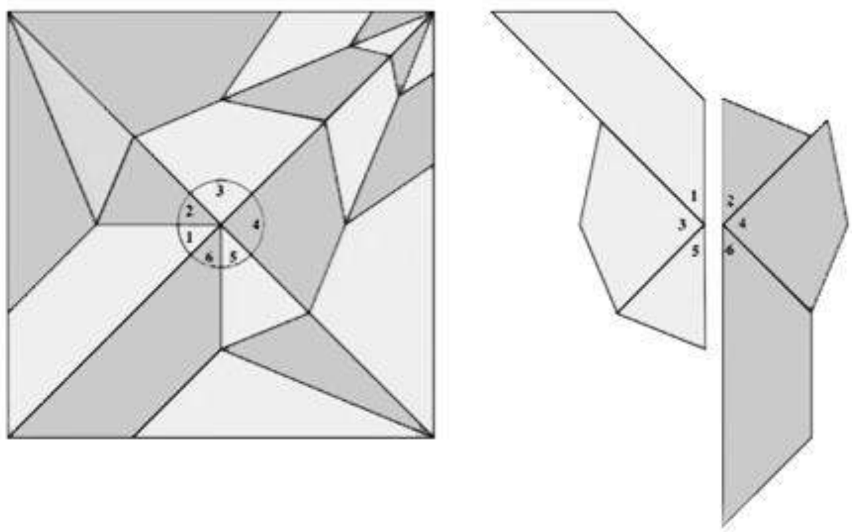


Figure 2.20 Third rule for crease patterns, sum of odd and even angles equals 180 degrees (Lang, 2008)

3 Theory

The word *plate* can refer both to out-of-plane-action plates, that can take bending, and to in-plane-action plates that are pure membrane members. In this report when the word plate is used without further description, it is referring to in-plane-action.

3.1 Origami engineering

3.1.1 Gaussian curvature

The principal curvatures of a given point on a surface are perpendicular and expressed as $k_1=1/R_1$ and $k_2=1/R_2$. The Gaussian curvature is obtained by multiplication of the two principal curvatures as $K=k_1k_2$. A point on a surface can either have a negative, zero or a positive Gaussian curvature. Those correspond to three different kinds of surfaces. In saddle surfaces or hyperboloids the principal curvatures have different signs, which means that the Gaussian curvature is negative. In single curved surfaces one k is zero and therefore K is also zero. For spheres k_1 and k_2 are both positive or negative which gives a positive Gaussian curvature.

Typically out of plane bending of a plate can only, without a lot of axial elongation, be singly curved. As is written above this corresponds to a Gaussian curvature of zero. Some folded geometries are special in the sense that they can change their Gaussian curvature when being bent. For instance, the Miura-ori pattern, when folded into a flat shape from paper, can easily be bent into a saddle surface as shown in chapter 3.6.1.

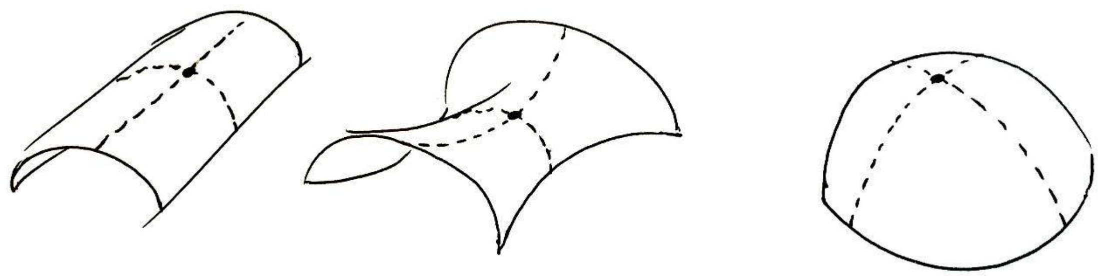


Figure 3.1 Points on surfaces with different gaussian curvature.

3.1.2 Poisson's ratio

Poisson's ratio for a material tells about how the material reacts on compression and tension. If a material is compressed in one direction it is either decreasing or expanding in the transversal direction. The Poisson's ratio is defined as the relative change in length in the transversal direction divided with the relative change in length in the main direction.

$$\nu = \frac{\frac{l_x - l_{x0}}{l_{x0}}}{\frac{l_y - l_{y0}}{l_{y0}}}$$

Almost all materials has a positive poisons ratio, meaning that it expands in the transversal direction when it is compressed in one direction. For many origami tessellations, however, the poisons ratio is negative. When the pattern is compressed in one direction it is also decreasing in the transversal direction, and if it is tensioned in one direction it is also expanding in the transversal direction.

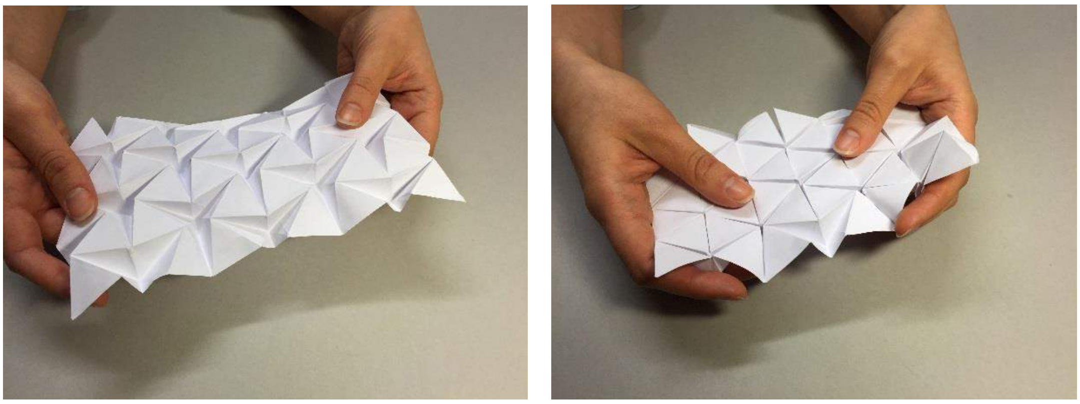


Figure 3.2 Negative Poisson's ratio observed in an origami tesselation.

3.2 Folded plate structures

Rigid systems are structures made from rigid materials, which means that they can take compression, tension, bending and shearing. Folded plate structures are rigid systems that consist of thin planar structural surfaces. Those can span over large distances without the need of a global vault or dome shape. The surfaces are active and works both as an enclosing envelope and as a structure, folded plate structures are therefore good examples of when architecture and engineering are closely intertwined. Another use of the folded structures can also be to prevent buckling of thin compressive structures (Bechthold, 2008).

3.2.1 Beam and plate action

The main folds represent lines of stiffness in the system, to create a stiff structure the folds must run approximately in the span direction. Depending on the height-to-depth ratio, folded plate structures are either governed by beam action or plate action. In a plate the loads paths do not follow cable or arch-like lines like in a beam. Instead loads from the upper part of the system begin to flow directly down to the supports following inclined stress paths. Plate action requires a significant structural depth which is about 50 to 100 percent higher than the depth required by a beam. The pure plate action occurs when the height of the plates are around half the span (Bechthold, 2008).

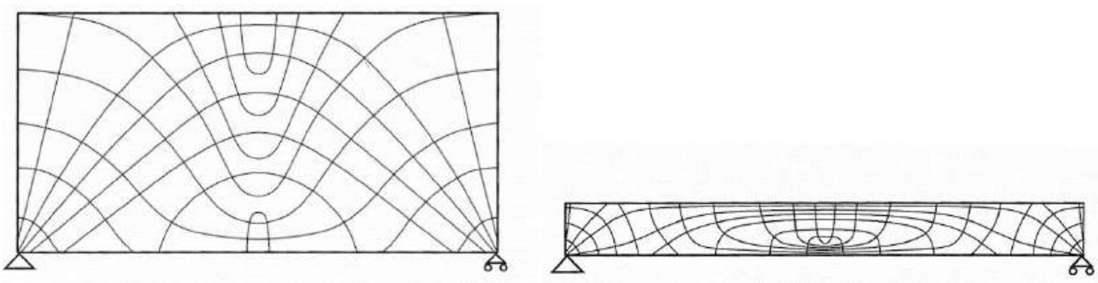


Figure 3.3 Comparison of stress patters between plate action (left) and beam action (right) (Bechthold, 2008).

3.2.2 Boundary supports

The support conditions for plates are important for stiffness and deformations. Since the inner plates are supporting each other, the largest deformation occur at the free edges at the boundaries. Free edges are less stiff, therefore it is not optimal to have a free edge in the compression zone due to the large risk of lateral buckling (Schodek, Bechthold, 2008).

If we look at a simple folded systems with two planar plates joined at one single fold in the span direction (*figure 3.4* (1)). The system shows large end deflection and instability when loaded, and the deformation in the middle is extensive. By securing the edges by a narrow plate, the instabilities are reduced (*figure 3.4* (2)). Best results are obtained if the added plate continues to the neutral axis of the system but not further since the cantilever then gets too large. We can see that the system is quite weak if it is not transversely stiffened to preserve the geometry during loading. A stiff diaphragm at the supporting ends makes the plate system much stiffer and the system now has the possibility to take significant loads (*figure 3.4* (3)) (Bechthold, 2008).

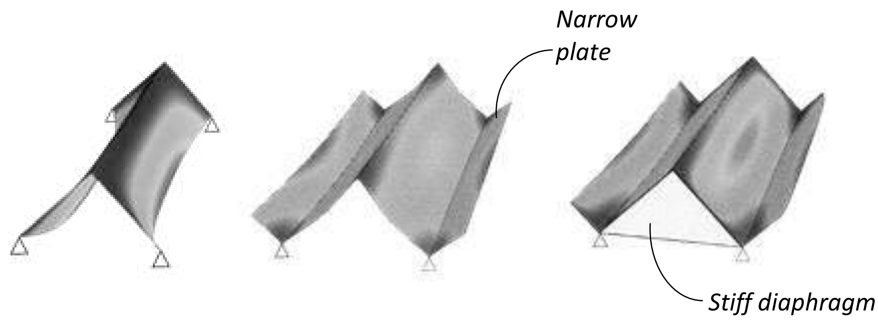


Figure 3.4 Effect of proper boundary supports (Bechthold,2008).

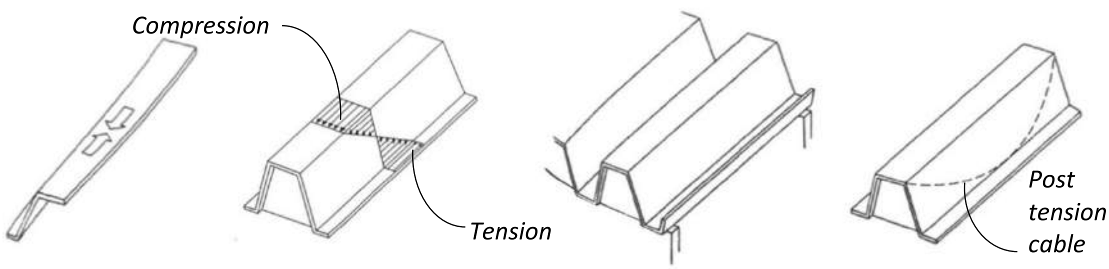


Figure 3.5 Other ways of stabilizing the folded structure (Schodek, Bechthold, 2008).

Plates in compression are easily subjected to lateral buckling if they are not laterally braced (*figure 3.5* (1)). To minimize this problem in folded plate structures free edges can be placed in the tension zone (*figure 3.5* (2)). Even if the free edges are in the tension zone they are not very stiff and can be stiffened by being bent or locally braced (*figure 3.5* (3)). Also post tensioning can be used to stiffen the plates in an efficient way even if it is mostly used in reinforced concrete plates (*figure 3.5* (4)).

3.2.3 Longitudinal and transversal behaviour

The way a simple folded plate system transfers surface loads to the supports can be described as two types of beam action. In the main direction of the folded structure, each inclined plate acts as a beam and is horizontally supported by the neighbouring plate. In the transverse direction, each plate element can be seen as a one way slab in bending supported by the folds. If a rigid connection between plate elements is used, a transverse strip behaves like a continuous beam with negative moments in the folds, which acts as supports, and positive moments in the spans. The loads that are transferred to the folds, act as line loads carried to the supports by the longitudinal beam action. This is however a simplified way of describing the behaviour of a folded plate structure, in reality the structural behaviour can be more complex (Schodek, Bechthold, 2008).

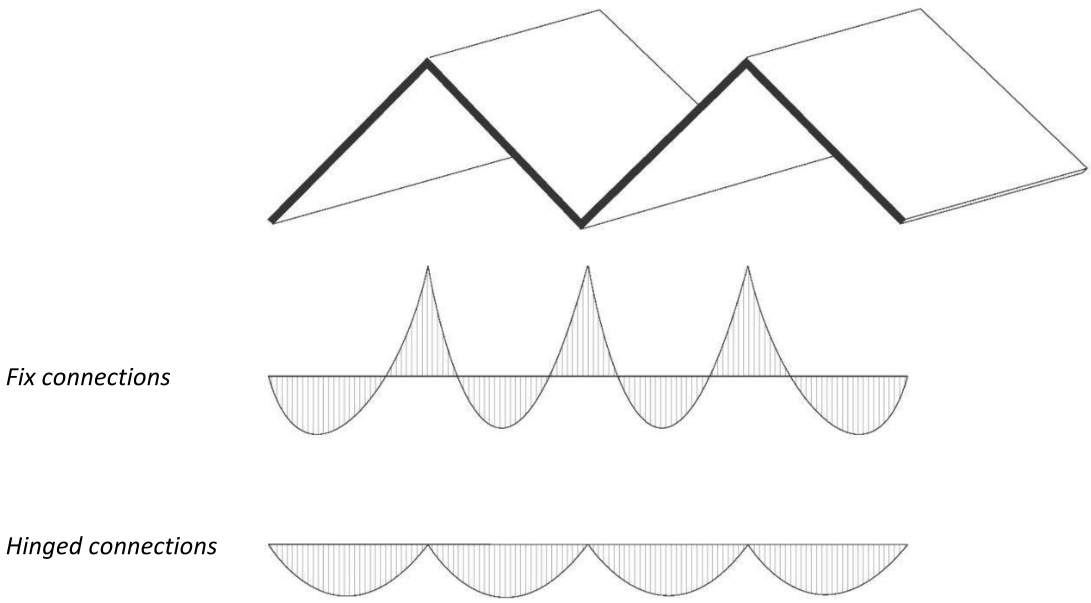


Figure 3.6 Moment diagram in transversal direction for a folded plate structure with fixed connections and hinged connections loaded with self-weight.

3.2.4 Angles

One important design parameter in folded plate structures is the angle between neighbouring plates since the angle together with the plate height determines the structural depth of the system. If the angles are too large the system loose it's stiffness since the plates do not get much edge support from the adjoining plate and too small angles makes the system inefficient since more folds and more material is needed to cover the same surface. It has been shown that angles close to 90 degrees (ca 70-110 degrees) are generally most efficient (Bechthold, 2008).

3.2.5 Stability

In complex folded plate structures it is not easy to determine stability, the simplest way is usually to consider the elevation view parallel to the main span and check for locations where the overall depth is reduced to only the plate thickness. These areas will behave as hinges since the out of plane bending strength of the plate is very small compared to the bending strength of the rest of the system. Hinges can be avoided by having adjacent plates that provide the required structural depth to give an overall stiff structure. Hinges in some places might be advantageous, making the structure work as a hinged frame, although more than three hinges in each section will result in a mechanism (Bechthold, 2008).

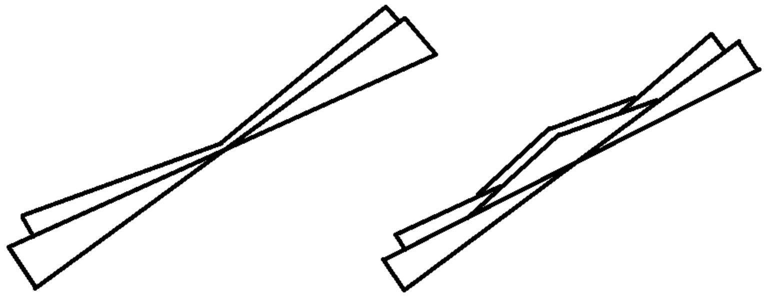


Figure 3.7 Hinges (left) and how they can be avoided globally by neighbouring folds that have a construction height (right).

3.2.6 Plate or truss

Folded plate structures in bending allow for openings in zones of low bending moments and shear, for example near the neutral axis. If too large and closely spaced openings are created the system might act as a *Vierendeel* truss or frame instead of as a folded plate structure. Triangular bar networks can also replace plates with a sheet material providing shear capacity, this structure will work equivalent to plate structures (Bechthold, 2008).

3.3 Reverse fold

Most fold tessellations are based on the reverse fold in one-way or another. The reverse fold is used to change the direction of straight folds and is therefore very useful when creating folded structures and origami tessellations.

3.3.1 Geometric relations

To understand the geometrical relations it is important to understand the two terms mathematical and physical reflection. Mathematical reflection gives the mirrored picture of the object, a reversed copy, it is reflection of an object **about** a plane or line. Physical reflection, gives the reflection in the way a light ray bounces off a surface, the object never crosses the reflection plane, it is reflection of an object **off** a plane or line (Buri, 2010).

The relation between the different angles and shape can be described by first looking at the flat reverse fold in two dimensions. A strip is folded into a single reverse fold by using β as the fold angle. When folded flat we have the angle alpha determining the fold angle compared to the strip.

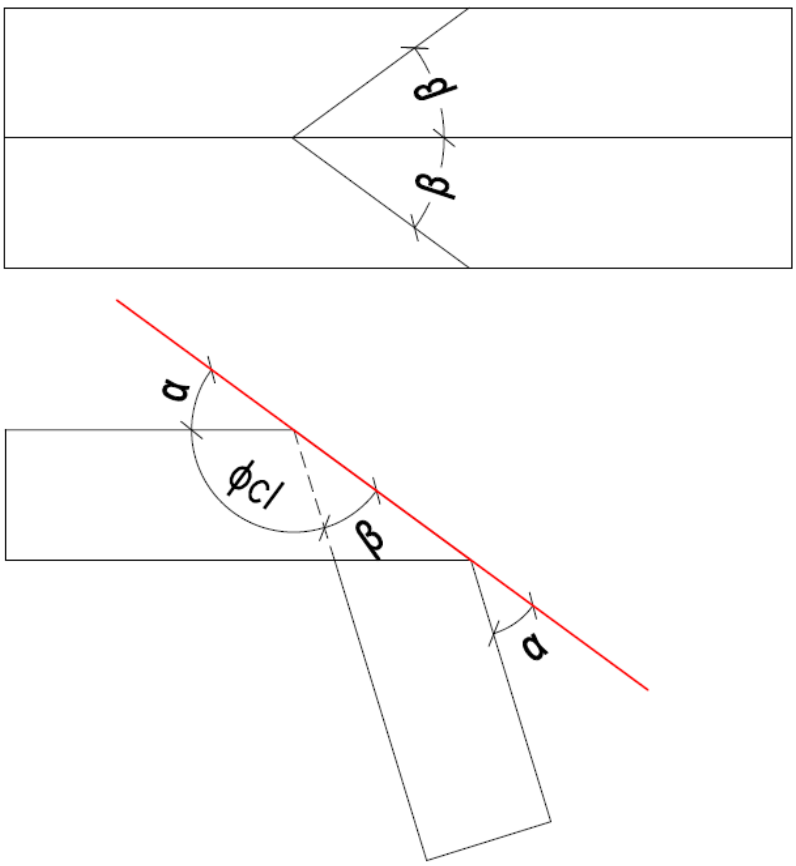


Figure 3.8 Mirror line and relations between angles.

Parallel lines and physical reflection off the red line gives that α must be equal to β and thus:

$$\varphi_{cl} = \pi - 2\beta \qquad \text{(Eq. 3.1)}$$

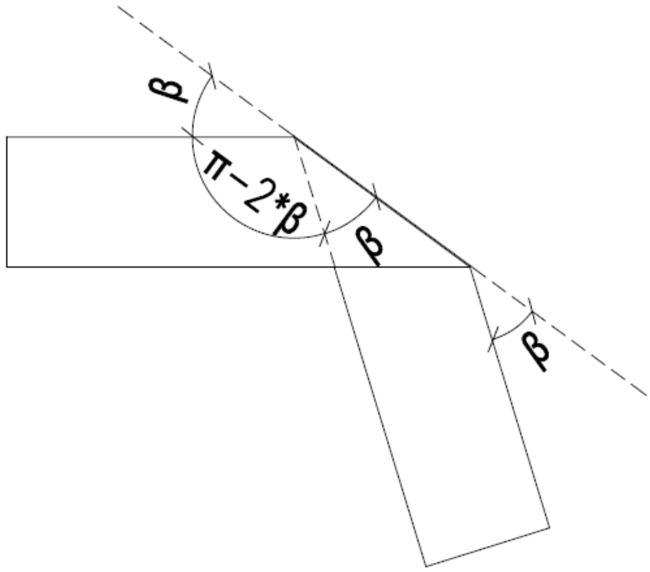


Figure 3.9 Relation between reverse fold angle and shape.

To determine the angles in three dimensions, when the fold is semi-closed, the angles can be determined by physical reflection off a plane instead of reflection off a line. A reversed fold in three dimensions has the angles as seen in the picture below, if we look at a section cut of the reversed fold we have similar relations between the angles as for the reversed fold in two dimensions, but the angle φ depend on gamma instead of on β.

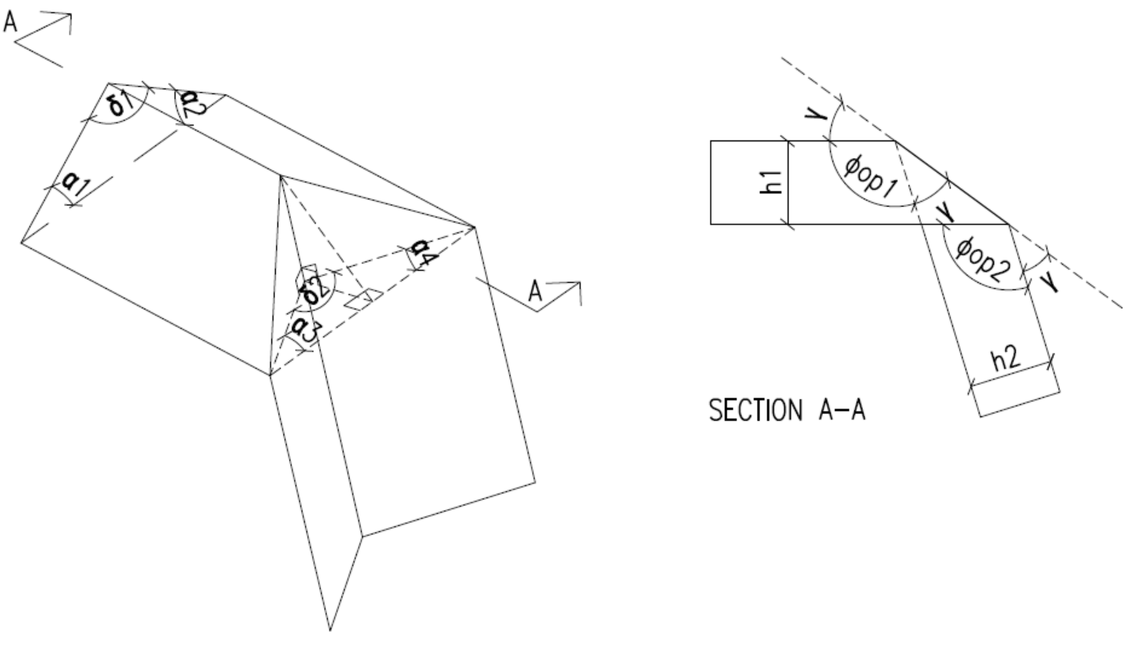


Figure 3.10 Relation between reverse fold angle and shape.

Physical reflection of the red plane gives that the parallel lines will remain parallel and the fold height *h*, will remain the same on both sides of the fold, this also means that the angles will be the same on both sides of the fold, $\delta_1 = \delta_2$, $\alpha_1 = \alpha_3$ and $\alpha_2 = \alpha_4$.

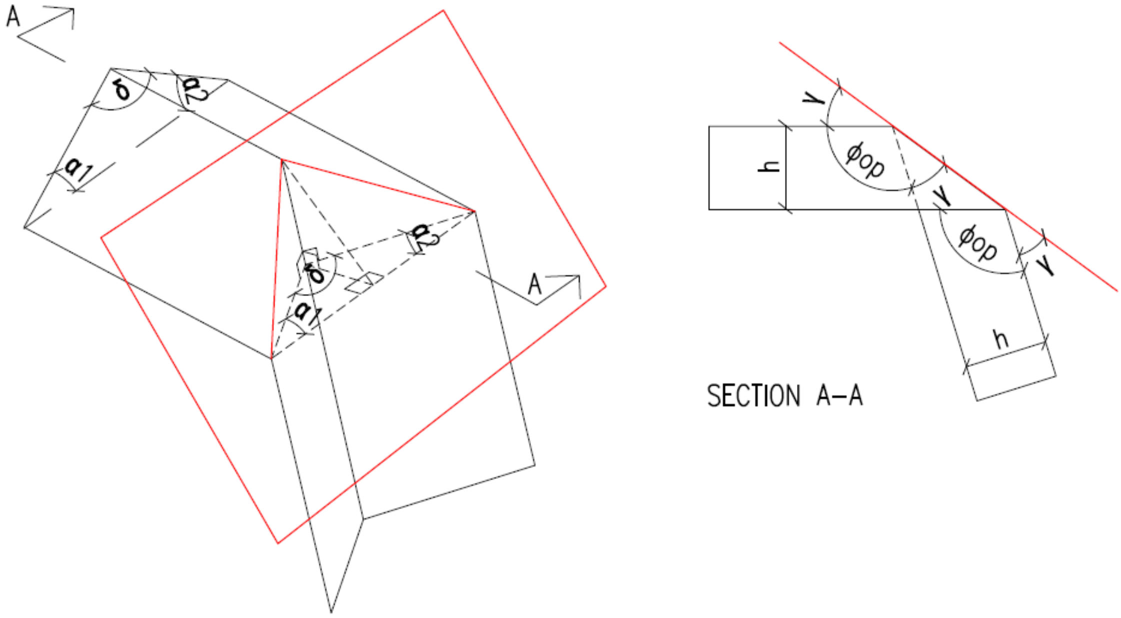


Figure 3.11 Relation between reverse fold angle and shape.

To find the relations between the angles α, beta and φ, we need to determine the relation between γ, α and β.

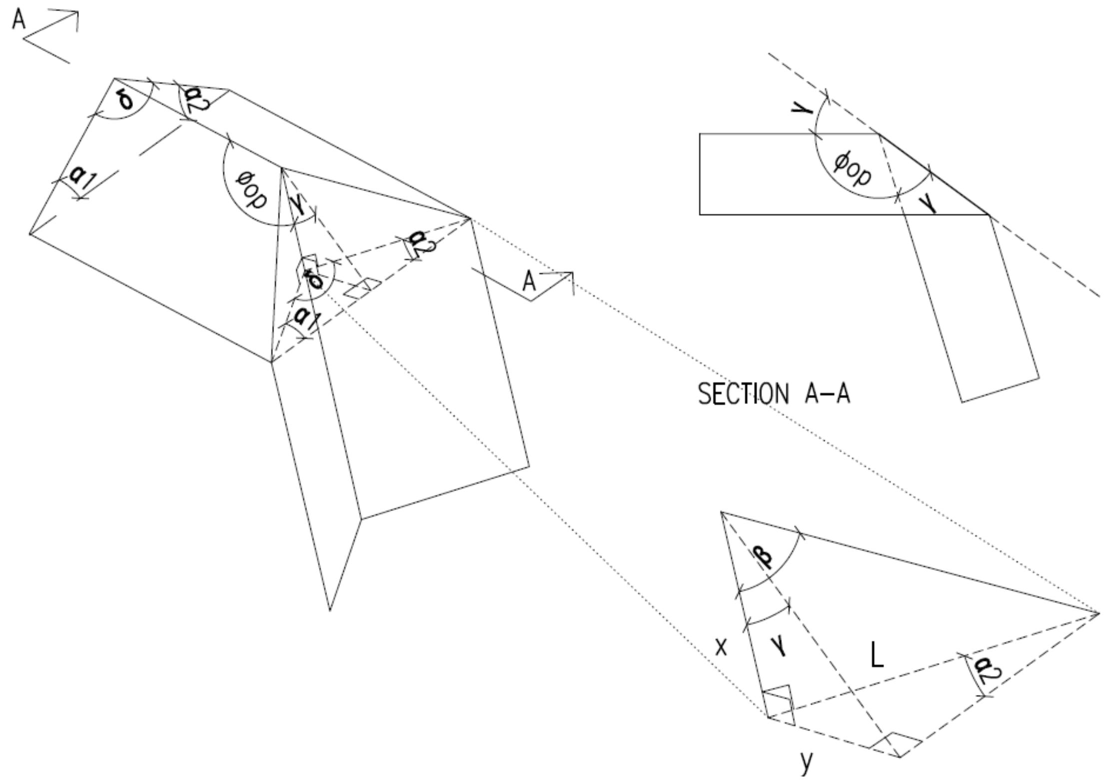


Figure 3.12 Relation between reverse fold angle and shape.

It can be seen from the figure that:

$$x = L/\tan(\beta) \quad \text{(Eq. 3.2)}$$
$$y = L/\sin(\alpha_2) \quad \text{(Eq. 3.3)}$$

$$\gamma = \alpha rctan\left(\frac{y}{x}\right) = \arctan(\sin(\alpha_2)\tan(\beta)) \qquad \text{(Eq. 3.4)}$$

Since the same relation hold between φ and γ in the semi-closed fold as between φ and β for the closed fold, φ relates to α and β as:

$$\varphi_{op} = \pi - 2\gamma = \pi - 2\alpha rctan\left(\frac{y}{x}\right) = \arctan(\sin(\alpha_2)\tan(\beta)) \qquad \text{(Eq. 3.5)}$$

3.3.2 Foldability

In many applications it is important that structures are completely foldable. For example if they need to be stored in an efficient way. Symmetric reverse folds are always completely foldable while asymmetric folds, with different beta angles, generally are not. For a reverse fold structure to be foldable certain requirements need to be fulfilled. Through experiments it has been discovered that if the longitudinal folds have the same angle from the bisectrix and if the valley and mountain folds are placed on different sides of the bisectrix the structure is completely foldable. Symmetric reverse folds always follow this criterion since the folds are going in the line of the bisectrix, but by using this criterion also asymmetric folds can be made completely foldable. In the picture below it can be seen that asymmetric patterns can be both foldable and non-foldable, and how the angles affect the foldability. It has been shown by Murata that the pattern is foldable if the relation between the angles in the vertex is as follows (Murata, 1966):

$$\alpha + \delta = \pi$$
$$\beta + \gamma = \pi$$

It can be shown that the requirement of the angles in relations to the bisectrix that was found can be translated into Murata's rule, so we thus know that this relation governs the foldability for all patterns:

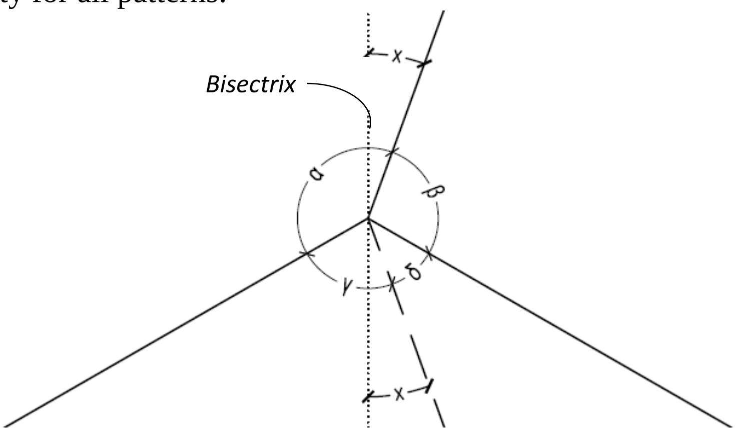


Figure 3.13 Relations between the folds and the bisectrix.

From *Figure 3.8* we see that:

$$\alpha - x + \gamma - x = \pi \quad \rightarrow \quad \alpha + \gamma - 2x \qquad \text{(Eq. 3.6)}$$

$$\delta + x + \beta + x = \pi \quad \rightarrow \quad \beta + \delta + 2x \qquad \text{(Eq. 3.7)}$$

$$\delta + x = \gamma - x \quad \rightarrow \quad \gamma = \delta + 2x \qquad \text{(Eq. 3.8)}$$
$$\delta = \gamma - 2x \qquad \text{(Eq. 3.9)}$$

Eq. 3.8 into Eq 3.6 and Eq 3.9 into 3.7 gives Miura's rule:

$$\alpha + \delta = \pi \qquad \text{(Eq. 3.10)}$$
$$\beta + \gamma = \pi \qquad \text{(Eq. 3.11)}$$

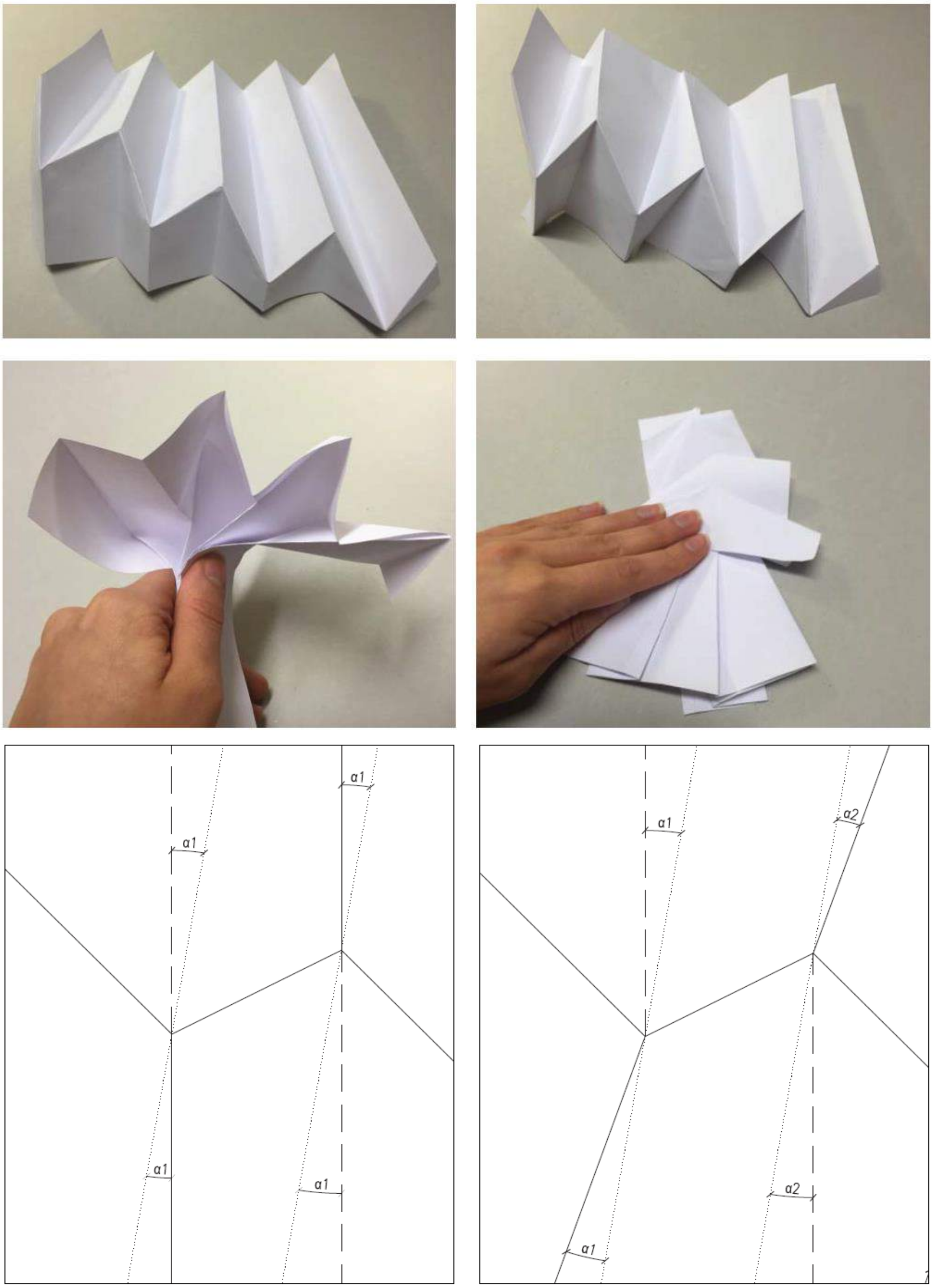


Figure 3.14 Relations determining foldability. To the left an asymmetric fold pattern that is not completely foldable and to the right a fold pattern that is. The dotted line represents the bisectrix.

3.4 Rigid-Foldable Thick Origami

In the previous chapter, rigid foldability was discussed assuming ideal zero-thickness surfaces. In a mathematical context, this is how origami is commonly regarded. In physical applications, the surface often has a non-negligible thickness that need to be taken into account. There are a few ways of dealing with problems related to thickness in rigid origami.

3.4.1 Existing solutions

A common door hinge can be used as an example of a rigid-foldable thick origami application with a single fold line. The structure can be fully developed (door is closed) and flat folded (door is open and parallel to the wall). This is possible since the position of the rotational axis is shifted from the ideal plane (middle of door) to the valley side of the fold line. This method is called axis-shift and works for doors and other simple fold applications.

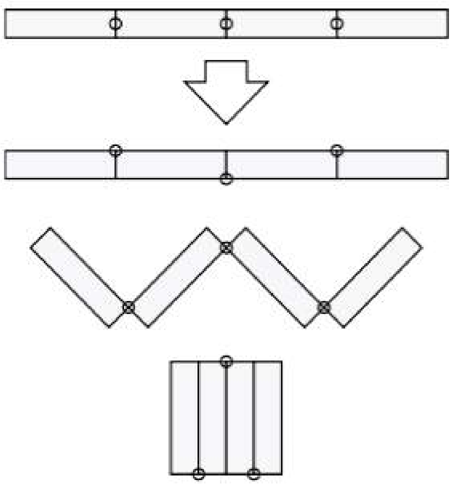


Figure 3.15 Simple fold thick-origami with hinges shifted to the corresponding valley sides of the mechanism. This structure can be folded from 0 to π (Tachi, 2010).

In the case of rigid origami mechanisms with interior vertices, the axis-shift method is still associated with some problems. The state of a rigid origami mechanism is described by the folding angles of its fold lines, which meet in interior and boundary vertices. For interior vertices, each facet has 3 degrees of constraints that correspond to x, y and z rotation around the point of fold line intersection. Due to this, fold lines must fold simultaneously around interior vertices in the kinetic motion of a rigid origami mechanism. When using the axis-shift method on crease patterns with interior vertices, since both valley and mountain folds meet in a vertex, the fold lines are not concurrent in the same plane. This generally produces six constraints throughout the transformation, three rotational and three translational. This usually results in an over constrained system, that is, a system where a continuous motion is not possible (Tachi, 2011).

For a Symmetric Miura-ori vertex, it is possible to connect shifted axes of rotation around a degree-4 vertex using plates with two thicknesses. This gives the mechanism one degree of freedom as a rotation, which corresponds to folding between 0 and π.

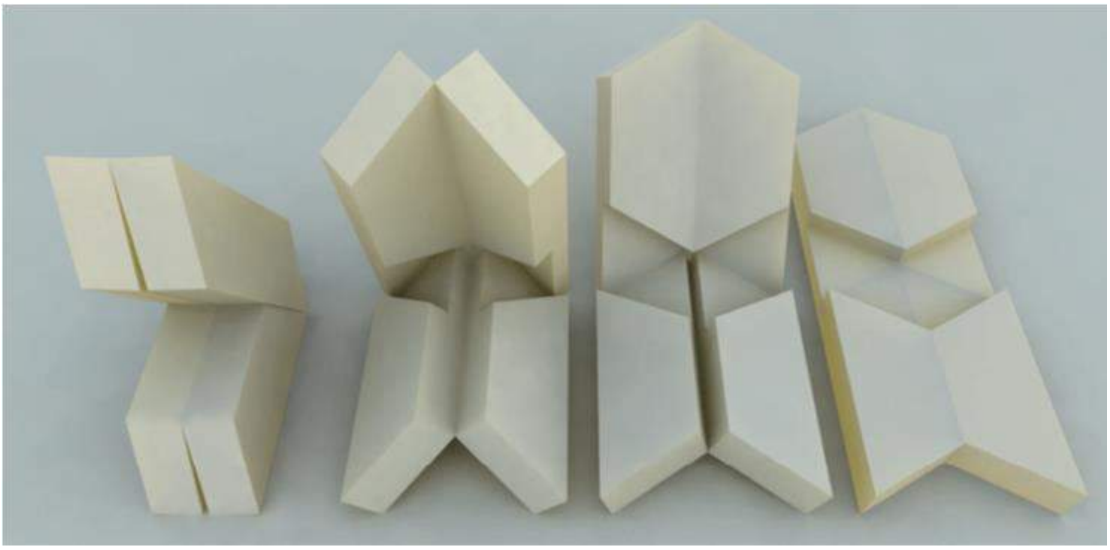


Figure 3.16 Symmetric Miura-ori vertex with shifted axes of rotation. Flat-foldable and fully developable due to two plate thicknesses (Tachi, 2010).

Another method is to give an extra degree of freedom to the facets by implementing sliding hinges that can move along their rotational axis. It is a bit unclear how this method affects the behaviour of a general origami structure. The figure below shows an example of where the method does not work very well. The displacement from the sliding accumulates at one of the edges of the structure.

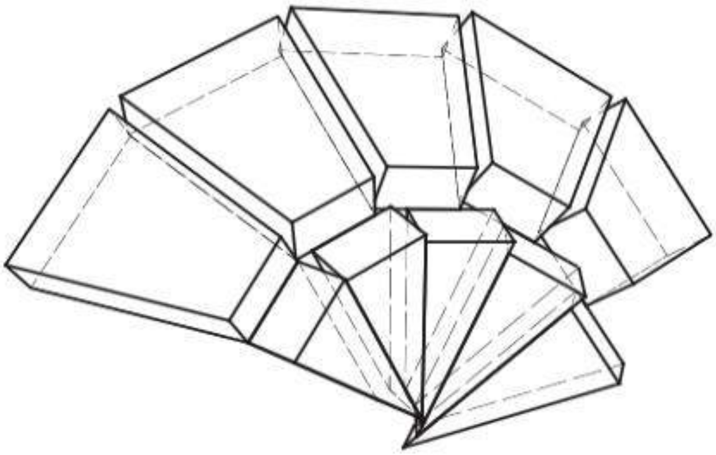


Figure 3.17 Folding mechanism with shifted axes of rotation. Rigid-foldable but problems occur due to accumulation of sliding displacements (Tachi, 2010).

One promising solution is based on the idea of keeping the hinges in the ideal plane, which allows the structure to follow the motion of an ideal zero-thickness origami model. To do this, a zero thickness surface is offset an equal distance in both perpendicular directions so that it gets a thickness. Folding of the model (in theory) to a certain fold angle $\pi - \delta$ results in intersecting parts. By trimming those intersecting parts gives a mechanism that is fully developable and foldable to the angle $\pi - \delta$.

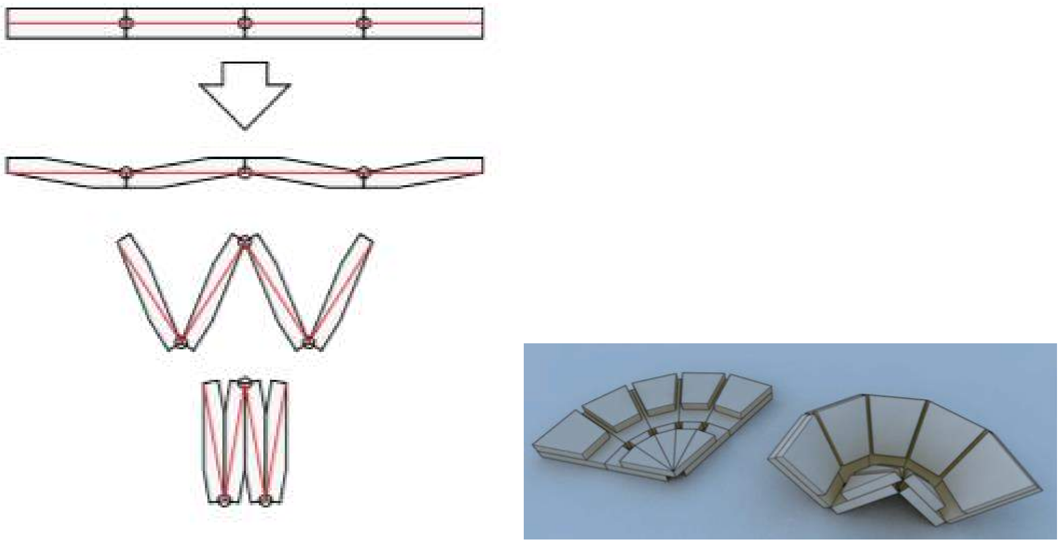


Figure 3.18 Mechanisms that can follow the motion of ideal zero-thickness origami by trimming of bisecting plates. Left: Tapered panels. Right: Constant thickness plates of different sizes that are sandwiching a hinge layer.

The described methods are only responses to panel collisions that occur at the fold lines. The problems with global collisions of the model are more or less the same for both ideal zero-thickness origami and thick origami.

3.5 Evaluated patterns

By reading reports, books and internet sources a collection of different origami tessellations have been found. Those are investigated to sort out the most structurally interesting. The patterns have been folded as paper models for rough examination of their behaviour. The basic patterns have been developed and varied to create other shapes and properties. They have also been evaluated and compared according to their stiffness/flexibility and deformation behaviour in some basic modes, and to their possibility to be folded from a flat sheet.

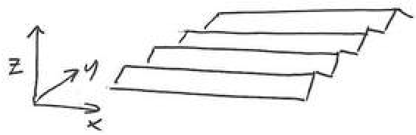


Figure 3.19 Definition of coordinate system.

To compare the different patterns and modes the directions in the form of x, y and z coordinates has to be set. The x-direction is in this comparison defined as the main direction of the folds while the y-direction is the transversal direction to the main folds.

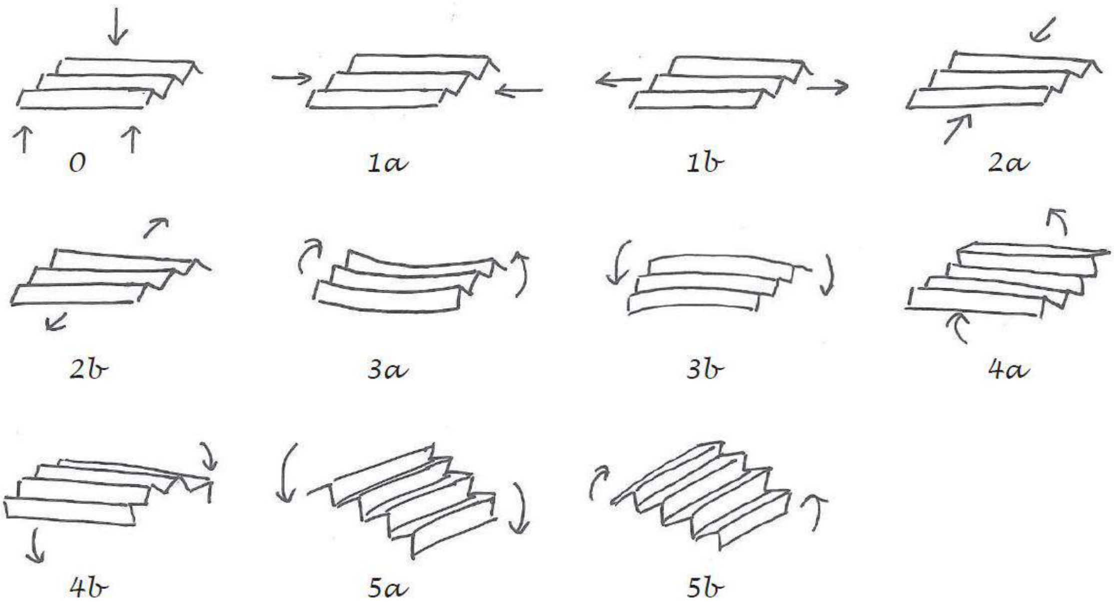


Figure 3.20 Modes for plane patterns.

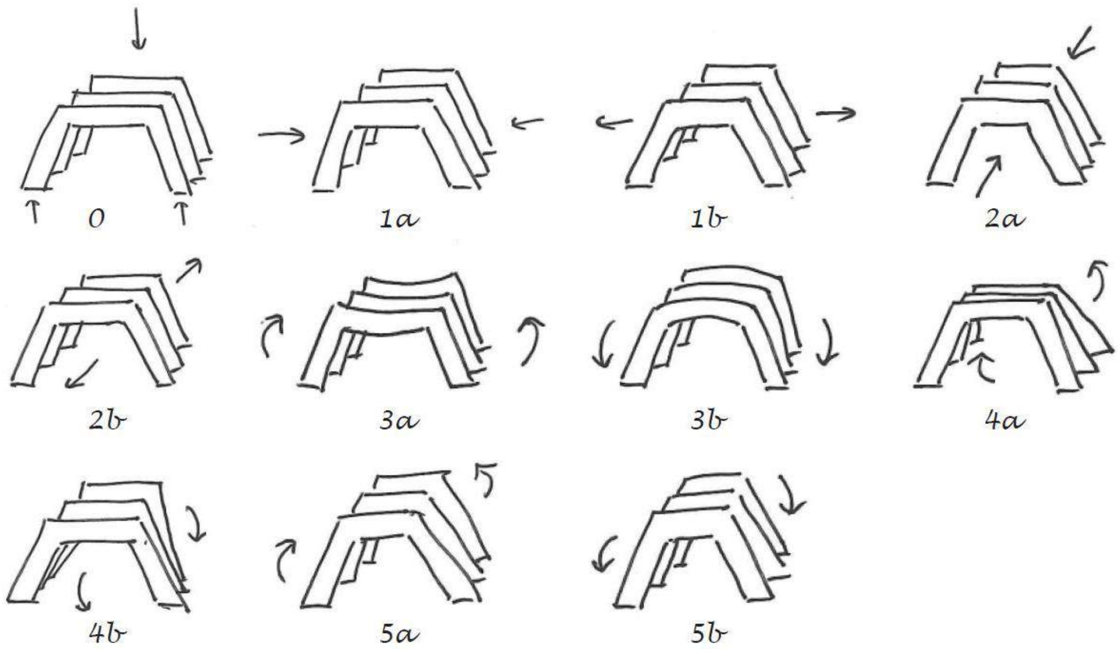


Figure 3.21 Modes for curved patterns.

Mode 0. *Vertical loading in z-direction.*
Mode 1a. *Compression mode in x-direction.*
Mode 1b. *Tension mode in x-direction.*
Mode 2a. *Compression mode in y-direction.*
Mode 2b. *Tension mode in y-direction.*
Mode 3a. *Bending mode in x-direction, top in compression, (bending around y-axis).*
Mode 3b. *Bending mode in x-direction, top in tension, (bending around y-axis).*
Mode 4a. *Bending mode in y-direction, top in compression, (bending around x-axis).*
Mode 4b. *Bending mode in y-direction, top in tension, (bending around x-axis).*
Mode 5a. *Diagonal bending mode in x-y-direction, top in compression.*
Mode 5b. *Diagonal bending mode in x-y-direction, top in compression*

3.5.1 The cut eggbox

The cut eggbox pattern is the only pattern considered that is not folded from a flat sheet. As can be seen in figure 3.22 it is made up from a sheet of the miura-ori pattern that is cut in the transversal folds, mirrored and glued together. The fact that it is made from a planar sheet with pieces fitting together, makes it possible to be produced in a material efficient way with minimized waste material, and this is the reason for this pattern being included in the investigation. The fact that it is cut in pieces and glued means that it is not completely unfoldable to a flat sheet. It is though completely foldable in both mode compression modes 1a and 2a and can thus be converted into a very space efficient form. Deformation of compression mode 2a results in deformation also in tension mode 1b. The same happens in tension mode 2b when compression mode 1a is deformed, to be able to deform in direction of 2a it thus also has to deform in mode 1b. The movement in the compression modes can thus be prevented by locking the translational displacement. The pattern deforms more easily in direction 2b than in direction 1b. It is stiff in bending mode 3a and 3b, while it is deformable in mode 4a, 4b and forms a saddle shape in mode 5. The fact that the cut eggbox pattern is not stiff in any compression direction means that it might be hard to use for structural purposes.

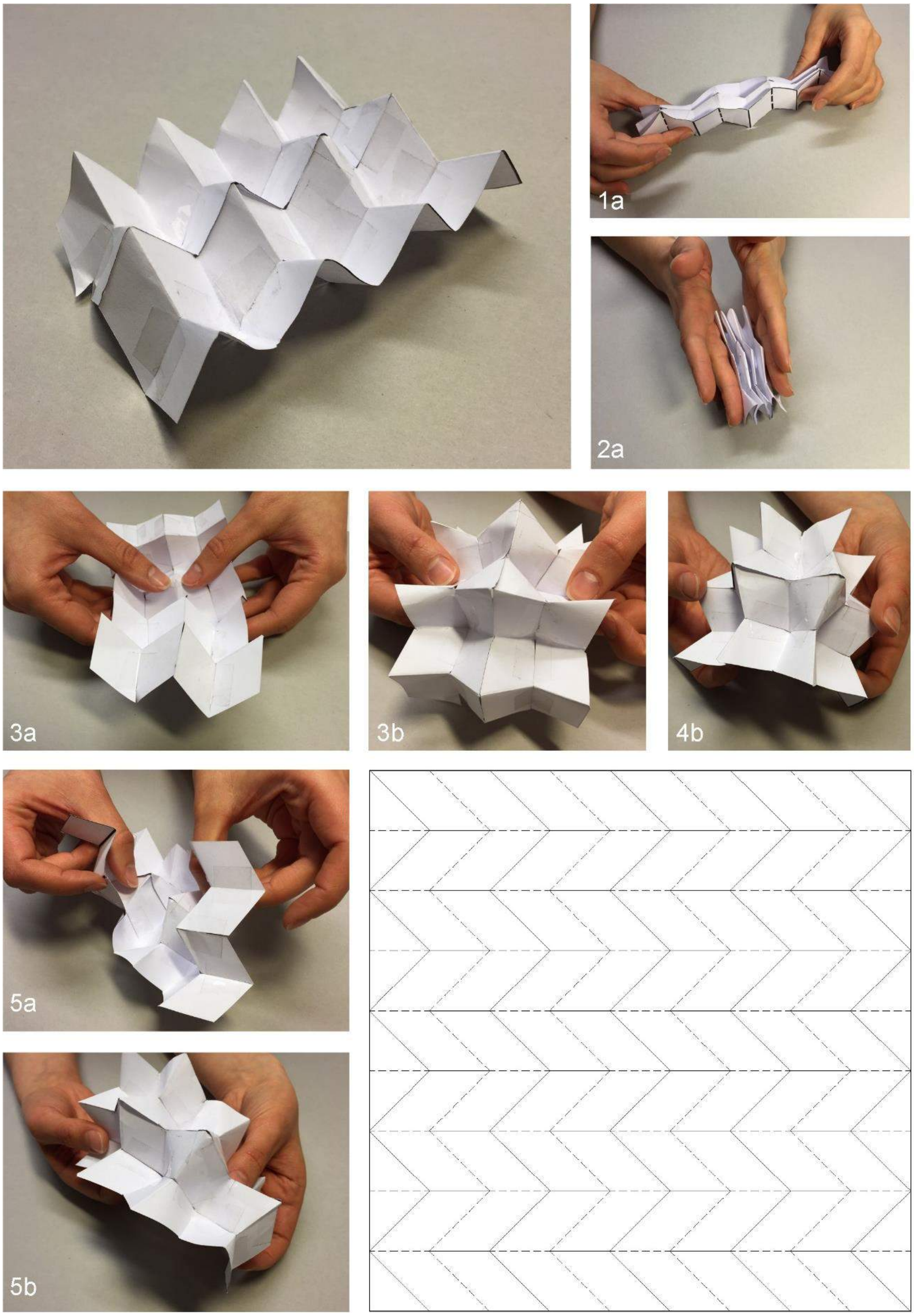


Figure 3.22 The cut eggbox pattern.

3.5.2 The folded eggbox

The folded eggbox can be seen in figure 3.23 and is completely compressible both in compression mode 1a and 2a and also flexible in tension modes 1b and 2b, but do not deform completely. When subjecting the structure to forces in tension mode 1b, it simultaneously deforms in compression mode 2a and the same relation can be found between tension mode 2b and compression mode 1a. This means, as for the cut eggbox, that deformation in these compression modes can be prevented by stiffening the transversal direction. The pattern is flexible in all the bending modes, when deformed into mode 3a and 3b respectively it also bends into mode 4a and 4b, it can be bent diagonally in mode 5a and 5b but do not form a saddle shape. The structure do not fold out to a flat sheet in neither of the tension modes due to the double surfaces that fix the structure, but the double surfaces also means that this pattern is not very usable when building in materials that have a thickness since the structure is based on two surfaces being folded flat against each other with a 360 degree joint, which in reality is hard to obtain in a good way. The folded eggbox has the advantage of being completely foldable in both compression directions and can thus be stored efficiently but is not very usable in this project since stiffness in at least one of the compression directions is required.

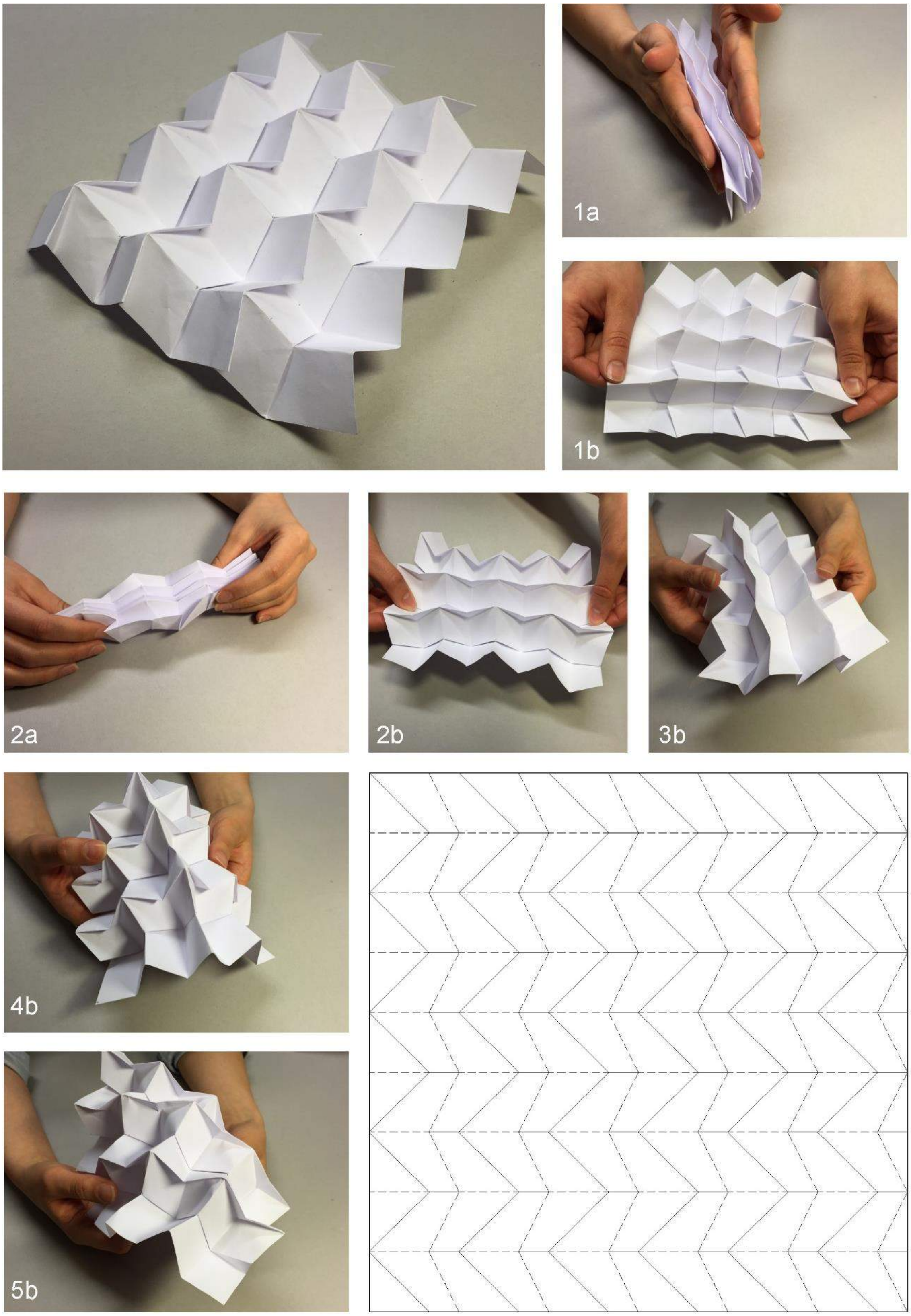


Figure 3.23 The folded eggbox pattern.

3.5.3 The diamond box

The diamond box pattern (fig. 3.24) is similar to the folded eggbox but has slightly different proportions and all the double surfaces is folded on the same side instead of on varying sides as for the eggbox. The diamond pattern is stiff in the main compression direction 1a and tension direction 1b, while it is completely foldable in compression mode 2a and is also flexible in tension mode 2b even though it cannot be folded out completely due to the double surfaces keeping the structure together. The pattern is deformable in all bending modes except 3a, in which the pattern is stiff against bending. It forms a saddle curve when subjected to mode 5a and 5b. The fact that this pattern has a stiff and flexible direction makes it interesting. So does the possibility to divide the pattern into parts that can be produced without too much waste. But the double surfaces make it hard though to use the pattern in reality since the elements have a thickness.

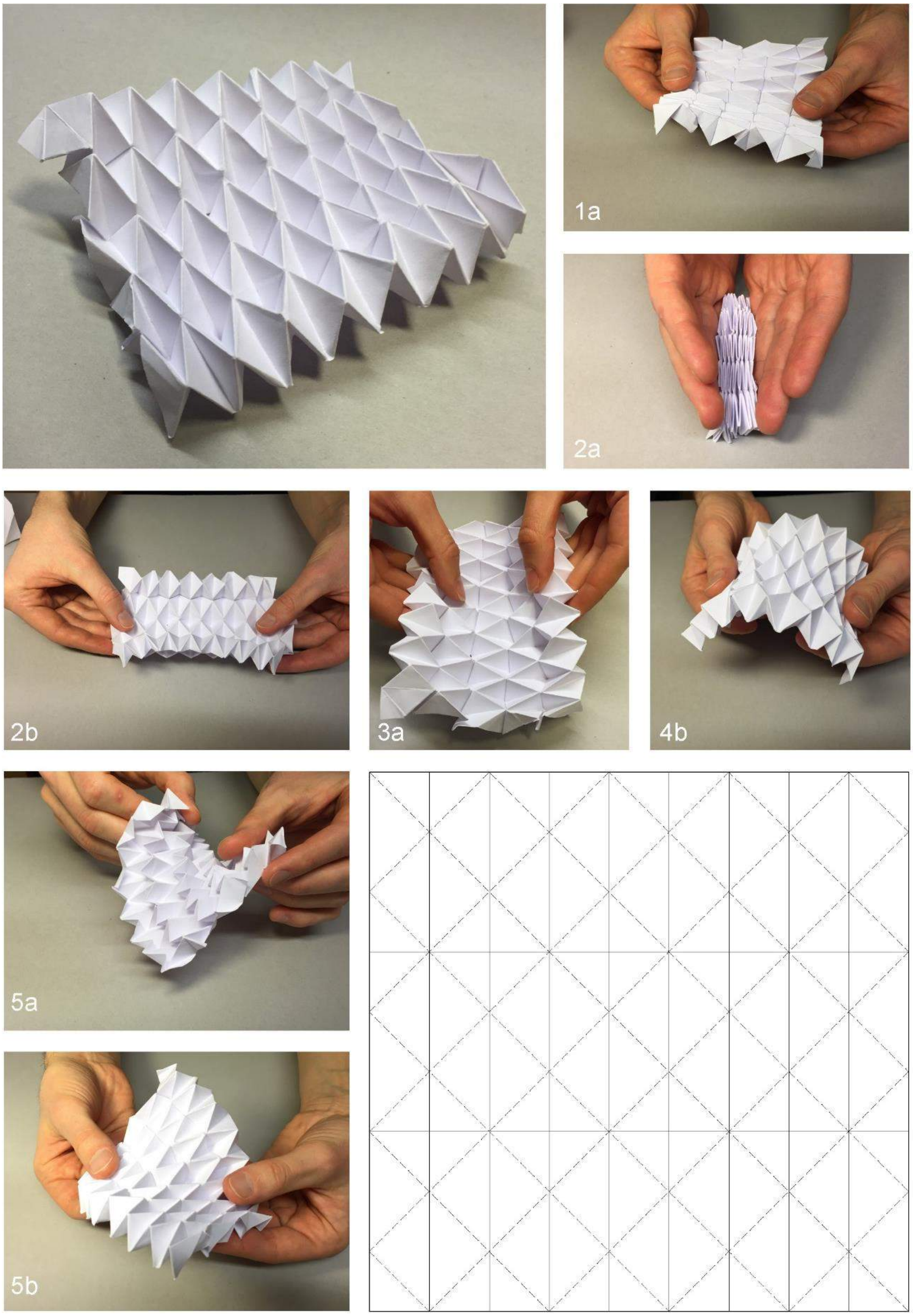


Figure 3.24 The diamond box pattern.

3.5.4 The magic ball

The magic ball pattern can be folded from a flat sheet, it is very flexible and can thus be formed into a variety of shapes. As can be seen in figure 3.25, on the next page, the magic ball pattern is not very stiff but is flexible in all modes except in compression mode 1a. The pattern have some interesting properties when subjected to certain modes, when exposed to mode 2a, the second compression mode, the structure bends upwards, as if been subjected to bending mode 3a, when subjected to compression modes 1b and 2b, the structure in the same time curls as if it was subjected to bending modes 3b and 4b respectively. Since the magic ball pattern is very flexible it is hard to use for loadbearing purposes.

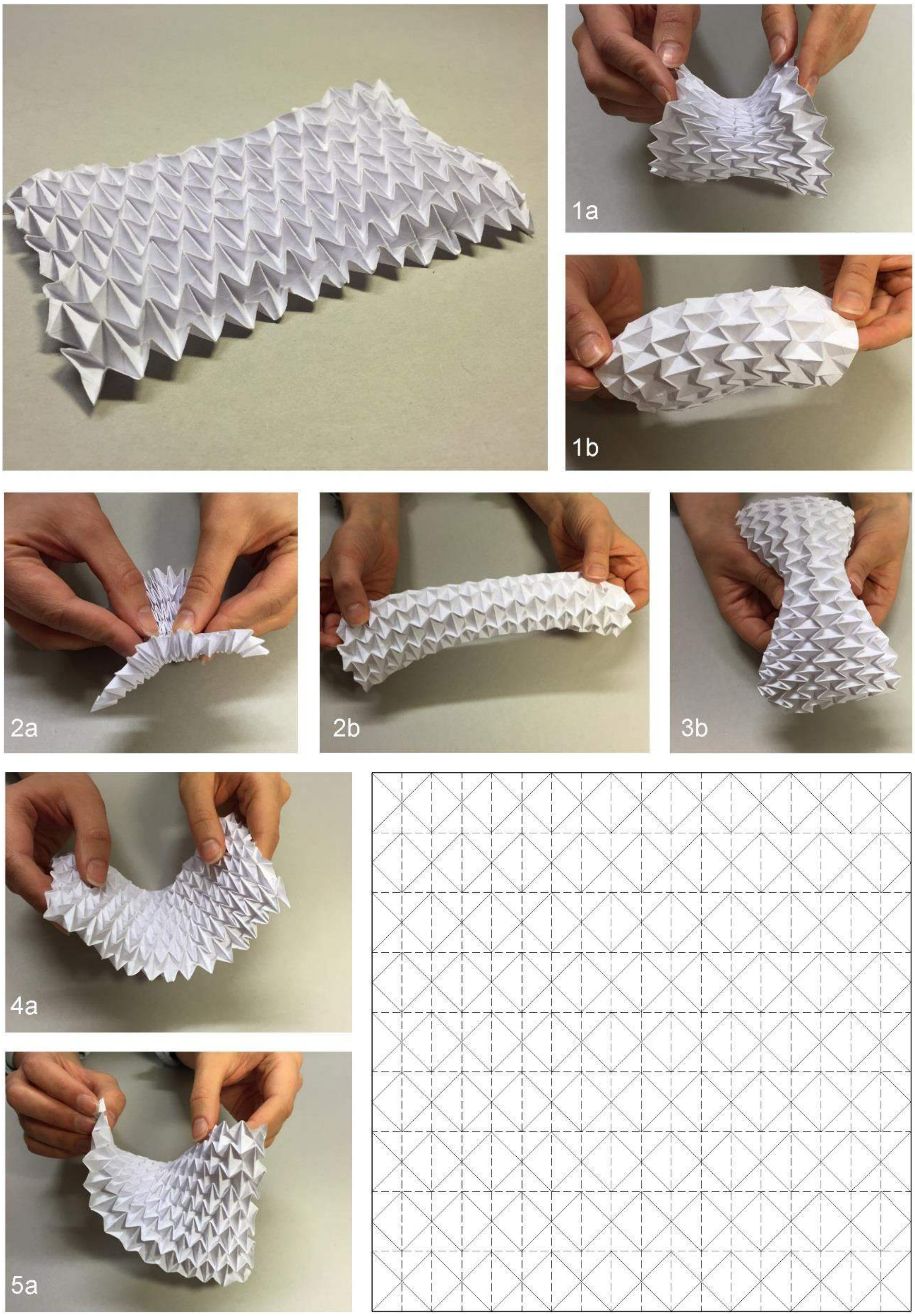


Figure 3.25 The magic ball pattern.

3.5.5 The fix magic ball

The fix magic ball pattern is based on the original magic ball pattern but with slightly different proportions to be able to lock the pattern in the meeting points. As can be seen in figure 3.26 the fix magic ball pattern is stiff in compression modes 1a and 2a, in tension modes 1b and 2b and in bending modes 3a, 5a and 5b, while it is flexible in the other bending modes. The behaviour when subjected to mode 3 and 4 is similar to the pattern for the original magic ball. The fixed magic ball could be structurally useable but it depends on if the fixed points can be locked in a good way, if not the pattern is completely flexible. The pattern is easily divided into strips and the panels can be produced from rectangular boards without too much waste material.

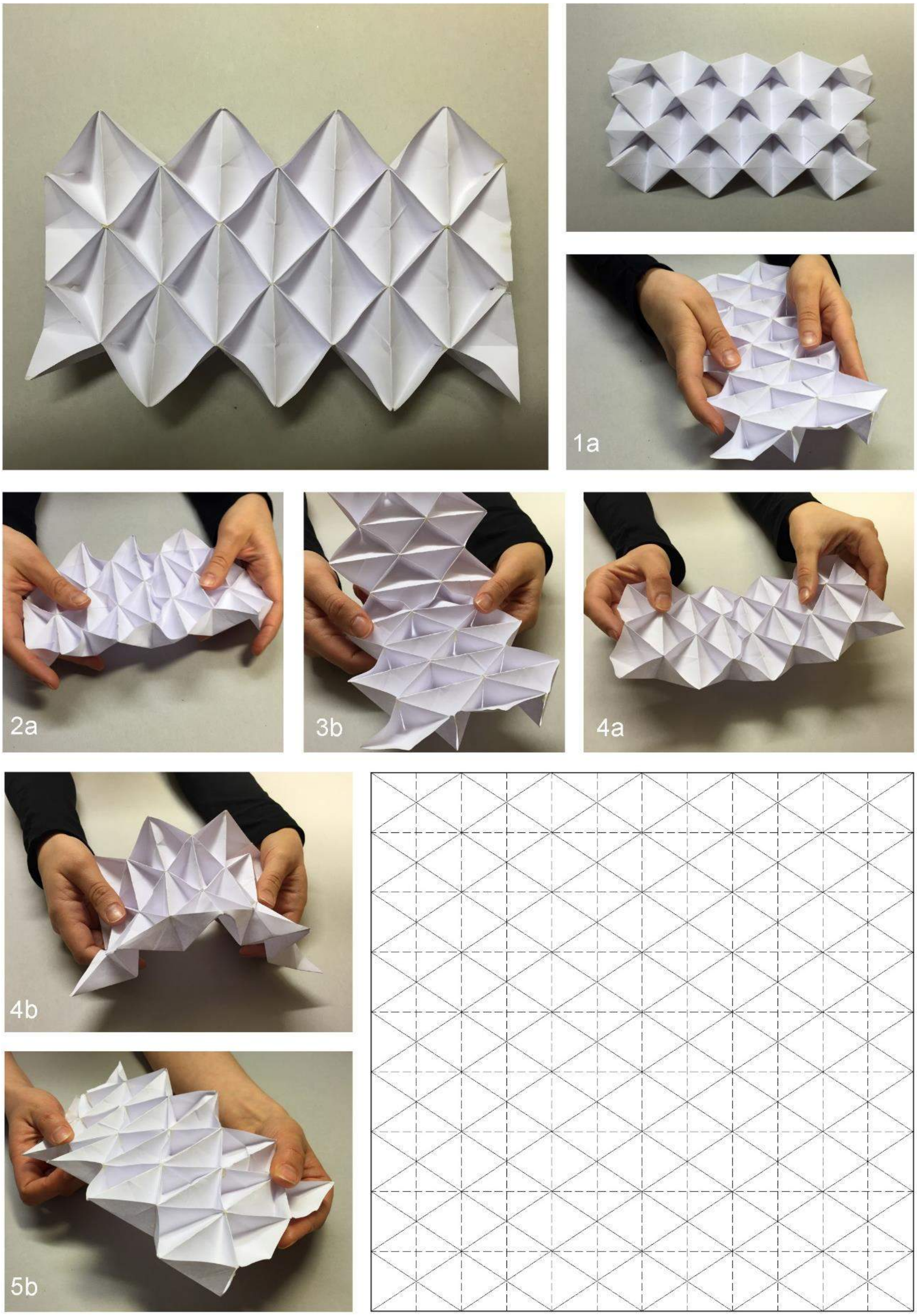


Figure 3.26 The fix magic ball pattern.

3.5.6 The reverse magic ball

The reverse magic ball pattern is made in the same way as the fix magic ball, to be able to lock the pattern in the meeting points. The difference from the fix magic ball pattern is that every second row is folded in the opposite direction. As can be seen in figure 3.27 the reversed magic ball pattern is stiff in compression modes 1a and 2a and tension modes 1b and 2b. While it is flexible in the bending modes except for the diagonal banding modes 5a and 5b. The behaviour when subjected to bending modes 3 and 4 is similar to the pattern for the original magic ball. Also here the pattern is easily divided into parts that can be produced from boards without waste.

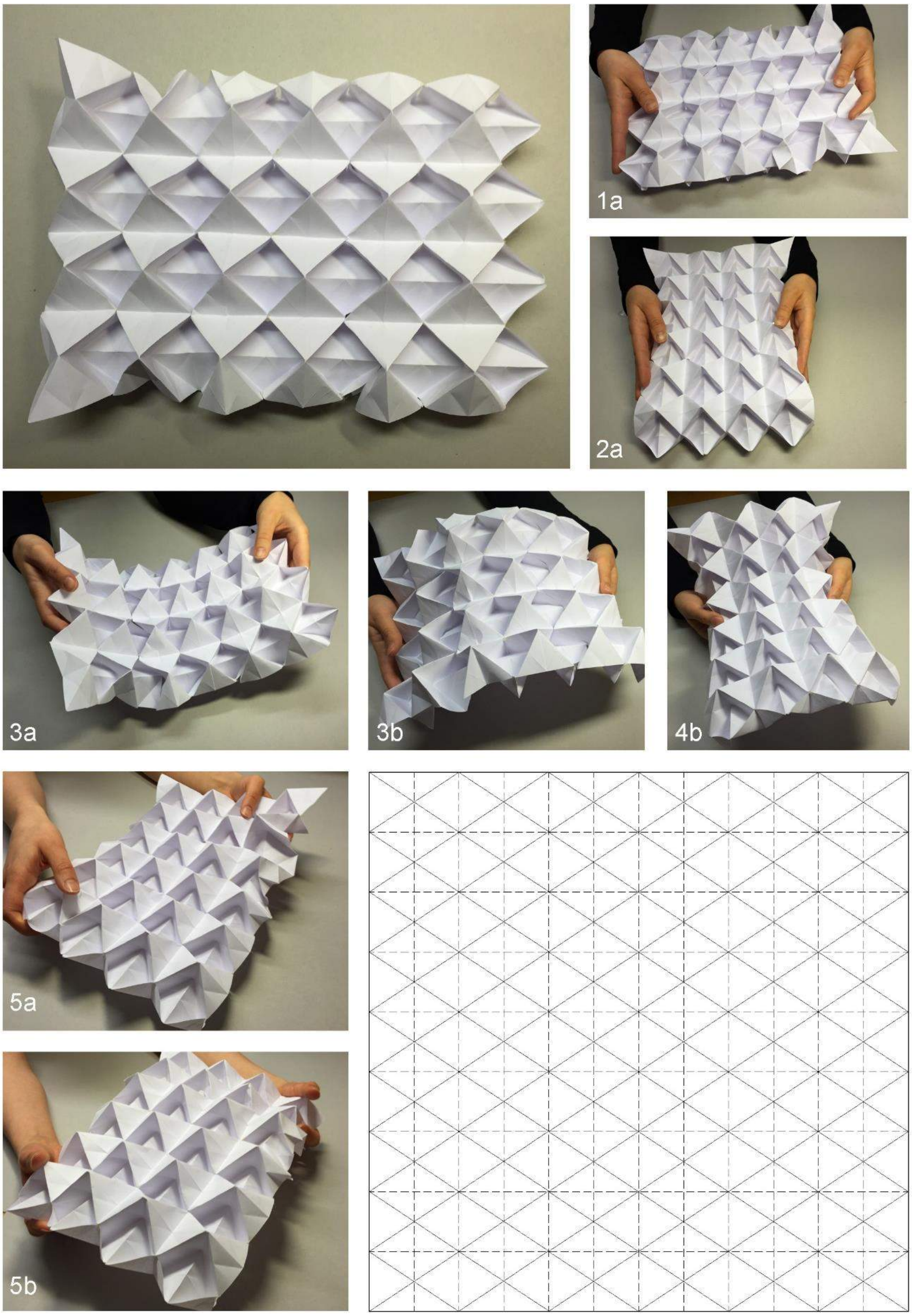


Figure 3.27 The reverse magic ball pattern.

3.5.7 Resch trigonal pattern

The trigonal Ron-Resch-pattern (fig. 3.28) behaves similarly in x, y and diagonal direction. As can be seen on the next page it is flexible in all modes although it is not completely foldable in neither compression mode 1a nor 2a. When subjected to bending in modes 3a, 4a and 5a the pattern deforms transversally in the opposite bending direction, deformation in mode 3a gives deformation in mode 4b and the same pattern occurs when exposed to mode 3b which also gives 4a and mode 5a and that gives the mirrored mode of 5b. The trigonal Resch pattern is not very stiff and is not suitable for structural purposes, but has other qualities for example that the surface can change in size by expanding or contracting. The pattern is not so easily divided into strips and can therefore be hard to produce from boards without a lot of waste.

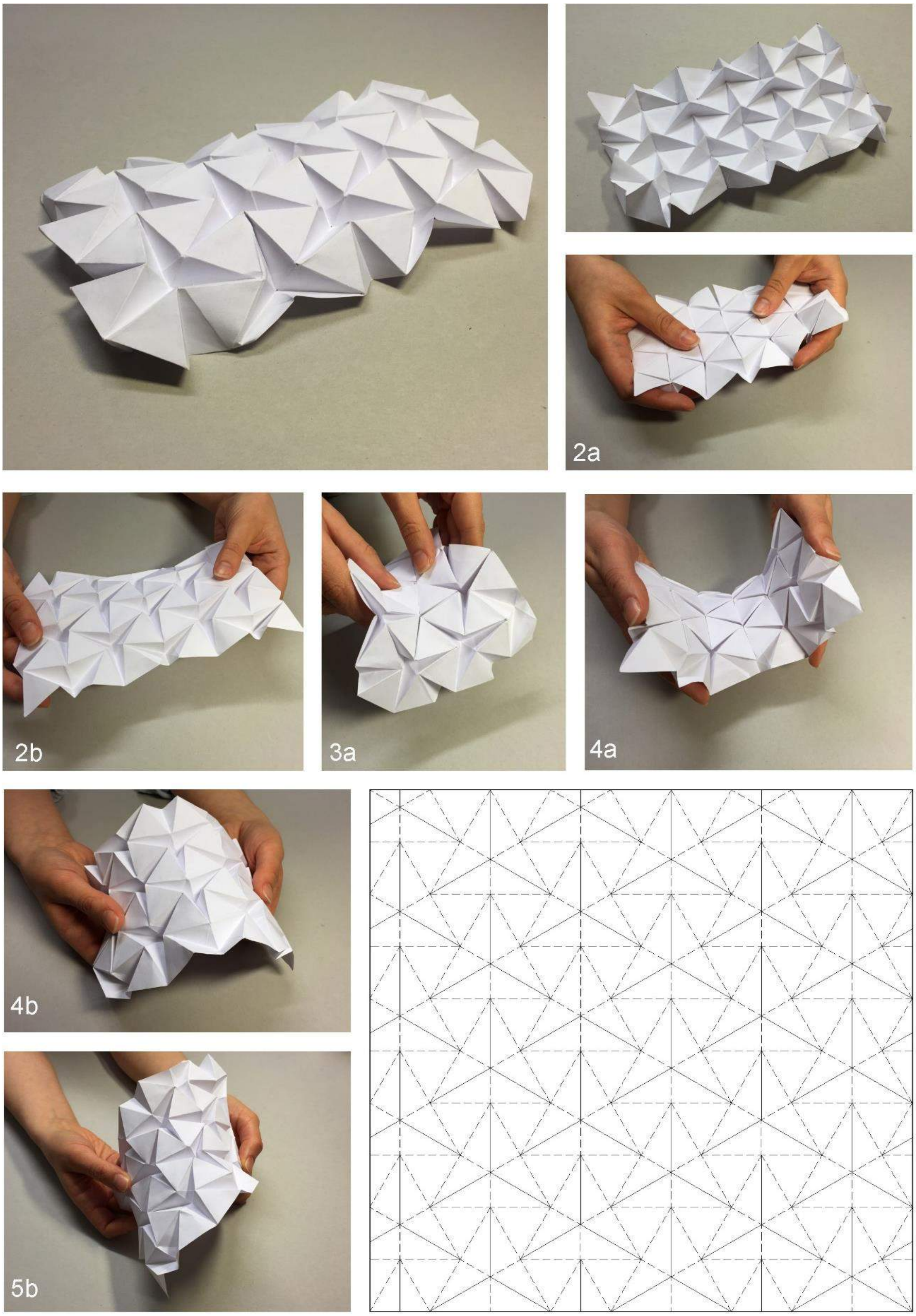


Figure 3.28 The Resch trigonal pattern.

3.5.8 Resch quadratic pattern

The quadratic Ron-Resch-pattern behaves very similarly to the triangular Resch pattern and has the same properties in x, y and diagonal direction as can be seen in figure 3.29. As for the triangular pattern the quadratic Resch pattern is flexible in all modes although it is not completely foldable in neither compression mode 1a nor 2a. When subjected to bending in modes 3a, 4a and 5a the pattern deforms transversally in the opposite bending direction, meaning that the deformations in these directions are connected. The quadratic Resch pattern is very flexible and is not suitable for structural purposes.

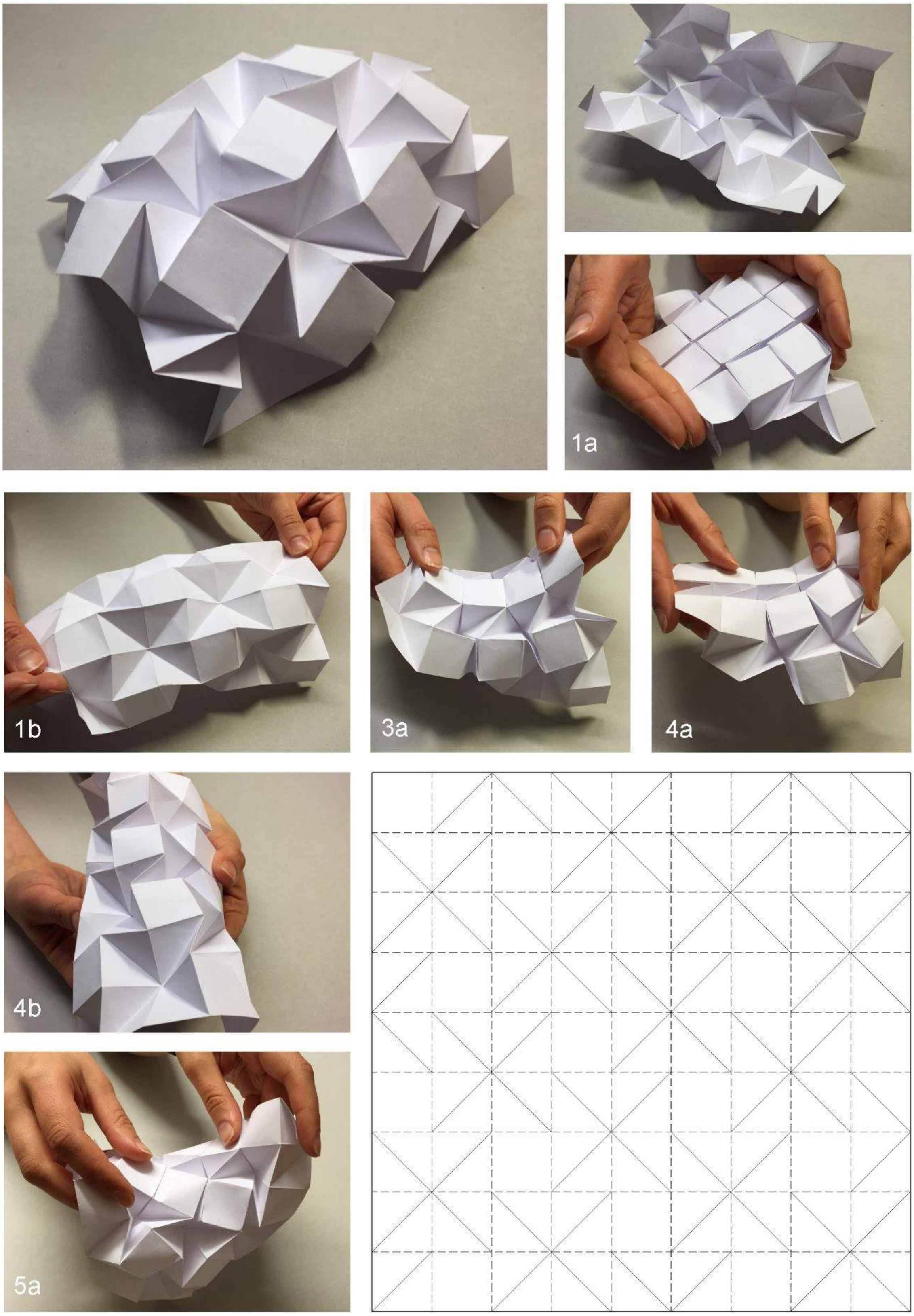


Figure 3.29 The Resch quadratic pattern.

3.5.9 Equidistant weave

The equidistant weave pattern (fig. 3.30) is built up from parallelograms and trapezoids. If the edges are locked the pattern is stiff in compression mode 1a and tension mode 1b. If the edges are not fixed, deformation in compression mode 1a also results in deformation in compression mode 2a, and deformation in tension mode 1b results in deformation in tension mode 2b. The pattern is completely foldable in compression mode 2a. It is stiff in bending modes 3a, 3b, 5a and 5b while it is flexible in bending modes 4a and 4b. It can be seen in the paper model that the pattern depends on that the elements can twist out of its plane and the pattern is therefore not buildable in stiffer materials.

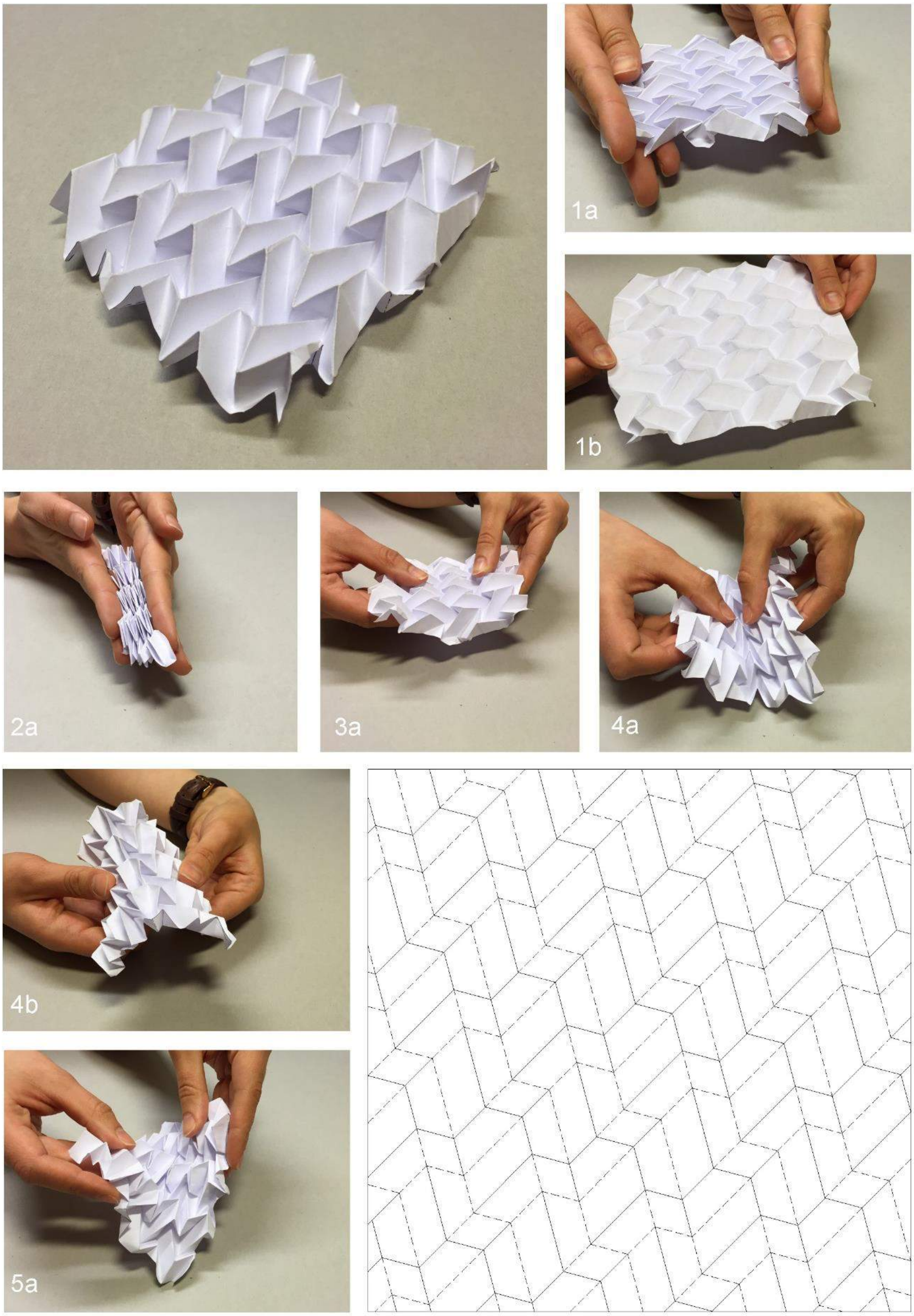


Figure 3.30 The equidistant weave pattern.

3.5.10 Simple folds, parallel

The simple fold pattern consists of a simply corrugated surfaces with parallel folds as can be seen in figure 3.31. The parallel simple fold pattern is stiff in the main direction, consisting of compression mode 1a and tension mode1b, while it is completely foldable/deployable in mode compression mode 2a and tension mode 2b. The pattern is also stiff in the first bending modes 3a and 3b but flexible in bending modes 4a, 4b, 5a and 5b. If subjected to deformation in bending mode 5a and 5b the pattern shapes into a saddle form. The parallel simple fold pattern is suitable for structural purposes but it is hard to vary the global shape of the pattern by changing the fold pattern and the shape.

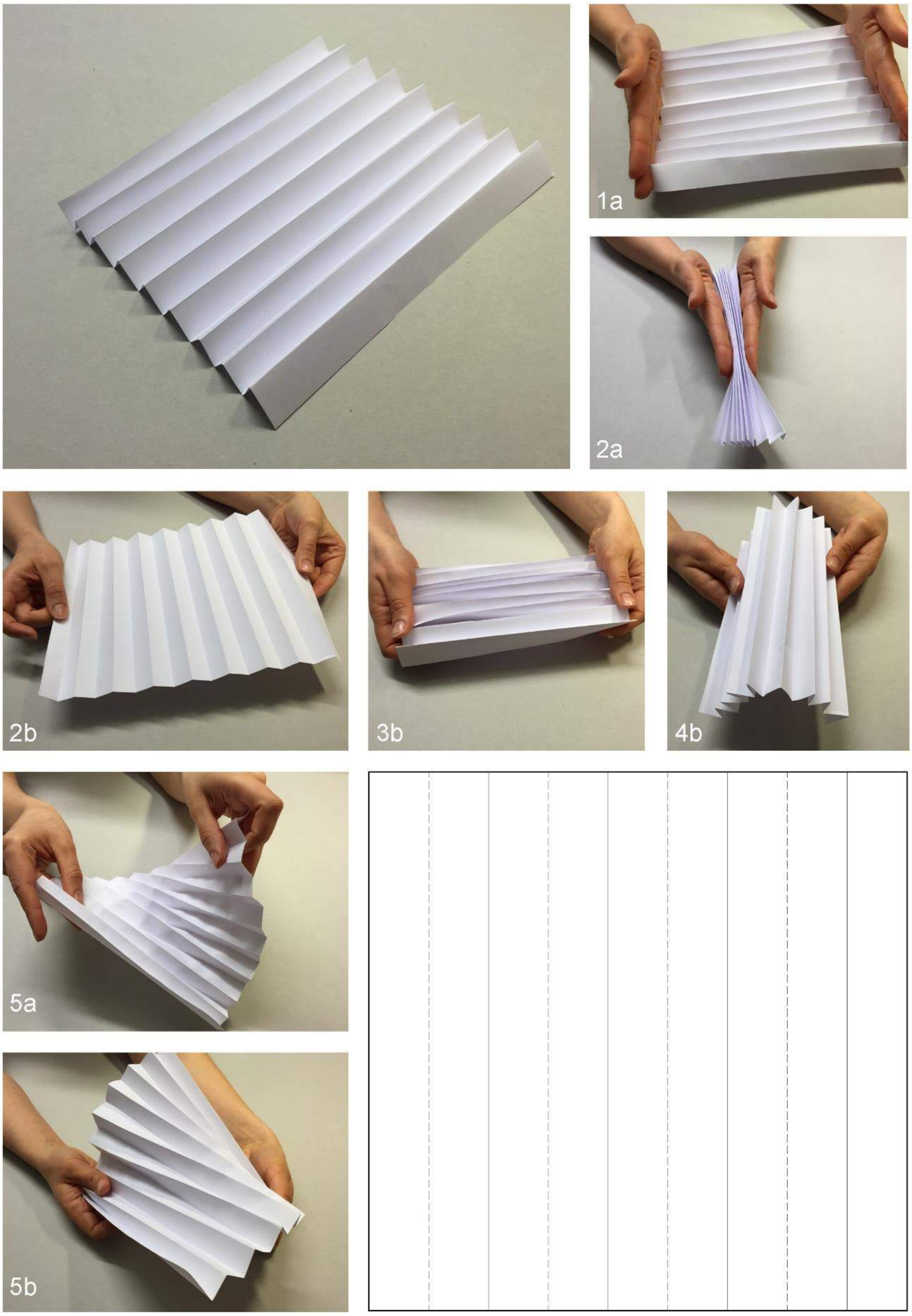


Figure 3.31 The parallel simple fold pattern.

3.5.11 Simple folds, oblique

The oblique simple fold pattern is similar to the parallel simple fold pattern but the folds are not parallel but in an angle to each other as can be seen in figure 3.32. It has the same properties as the parallel simple fold being stiff in compression mode 1a, tension mode 1b and bending modes 3a and 3b. While it is flexible in compression mode 2a, tension mode 2b and bending modes 4a, 4b, 5a and 5b. The oblique simple folds is also suitable for structural purposes but has the same problems as the parallel folds that the global shape is hard to change by altering the folds.

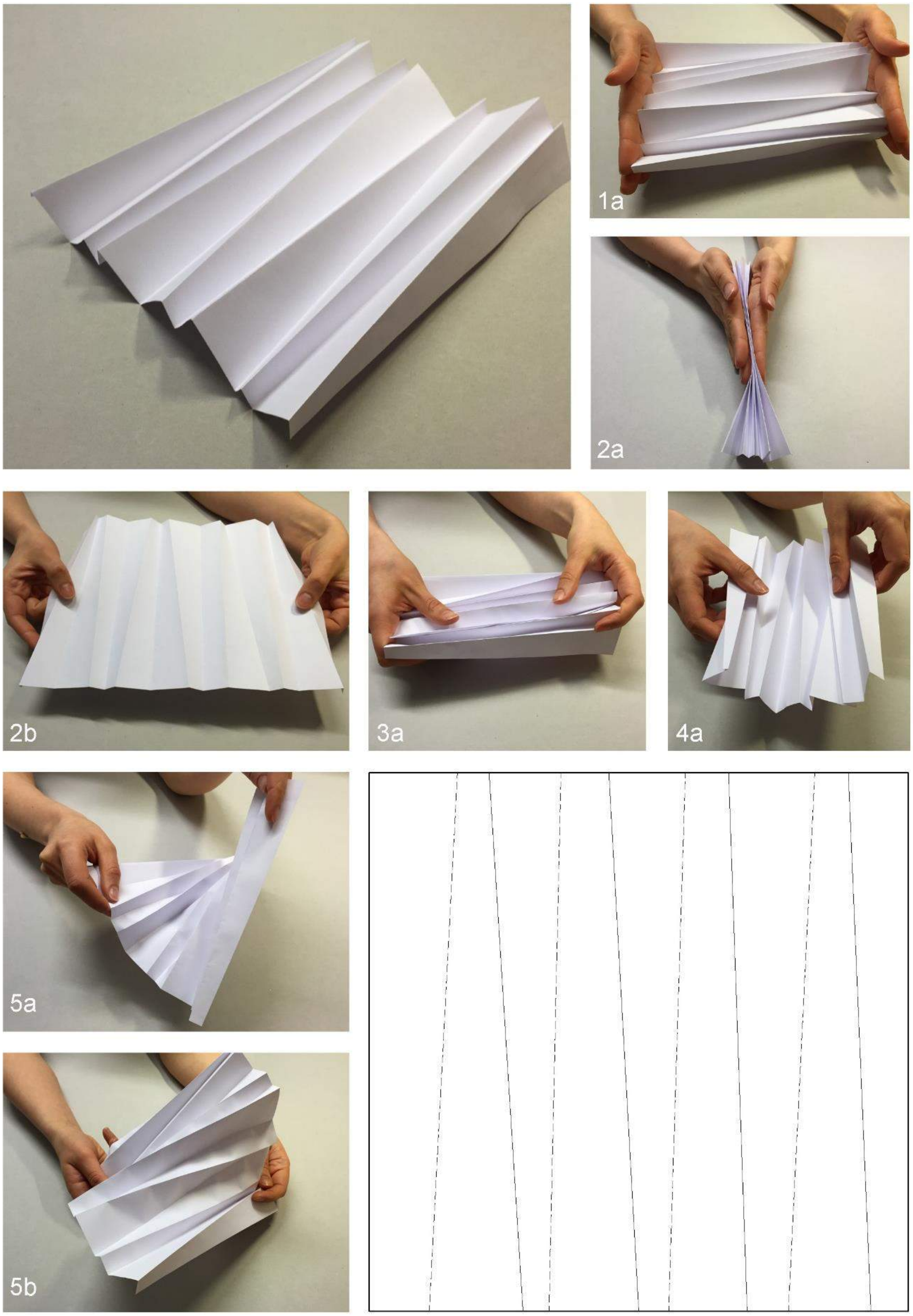


Figure 3.32 The oblique simple fold.

3.6 Chosen patterns

Three patterns that were found to be more interesting for this project were chosen for further investigation, the Miura-ori (Herringbone) pattern, the diagonal pattern and the diamond pattern. These patterns where chosen due to their ability to be shaped into different global forms by changes in the crease pattern. They were also chosen due their structural properties since they have a certain stiffness and are able to take bending and compression loads in some modes if other directions are locked. They are also able to be folded from a flat sheet which makes it possible to assemble the structure when lying flat and then shape it into its final form which makes them interesting for further investigation. These three patterns can also be divided into strips that make an efficient production and neither of the patterns has double surfaces which makes them constructible and therefore interesting for this project. Another factor that influenced our choice of patterns was that these also had been found most interesting for structural purposes by Hani Buri in his PhD thesis.

3.6.1 Miura-ori (Herringbone)

The Miura-ori pattern (fig. 3.33) is based on the reverse fold, consists of mirrored parallelograms and the main folds and the side folds are altered valley and mountain folds. The extension of the pattern is straight, as long as the pattern stays symmetrical with panels that are parallelograms. The fact that the panels are mirrored parallelograms makes it possible to decompose it into long strips with parallel edges which is interesting for production (Buri, 2010).The Miura ori pattern can undergo large deformations by the folds opening and closing. The fold patterns also enable the structure to locally expand and contract, and thereby changing their Gaussian curvature without stretching on material level. Gaussian curvature is a measure of the curvature at a point on a surface (see chapter 3.1.1) which remains unchanged when exposed to bending but not when the surface is stretched. By looking at the global Gaussian curve for the Miura sheet, it is initially flat and therefore have 0 curvature but can easily be formed into saddle shapes and then has a negative Gaussian curvature (Schenk, Guest, 2010).

The Miura-ori pattern is folded from a flat sheet and is stiff in the compression mode 1a, the tension mode 1b and in bending modes 3a and 3b if the edges are prevented from deforming. If not, bending mode 3a also results in deformation in bending mode 4b, and bending mode 4a results in deformation in bending mode 3b and 5a gives a reversed 5b resulting in a saddle curve. The Miura-ori pattern is completely foldable in the y-direction, compression mode 2a. The result from the mode tests above show that the Miura pattern have a negative Poisson’s ratio in its planar deformation mode but show the reverse behaviour in bending. It is very unusual for a material to have a negative Poisson’s ratio. Materials with positive poisons ratio usually deforms into a saddle curve in bending while materials with a negative poisons ratio deforms into a spherical shape but as can be seen the behaviour is the opposite for the Miura-ori, having poisons ratios with opposite signs in planar deformation and bending (Schenk, Guest, 2010).

The Miura-ori pattern has only one internal degree of freedom, and the only way to change the global shape of the structure is to vary the fold pattern. This is further described in chapter 3.7 Structural properties.

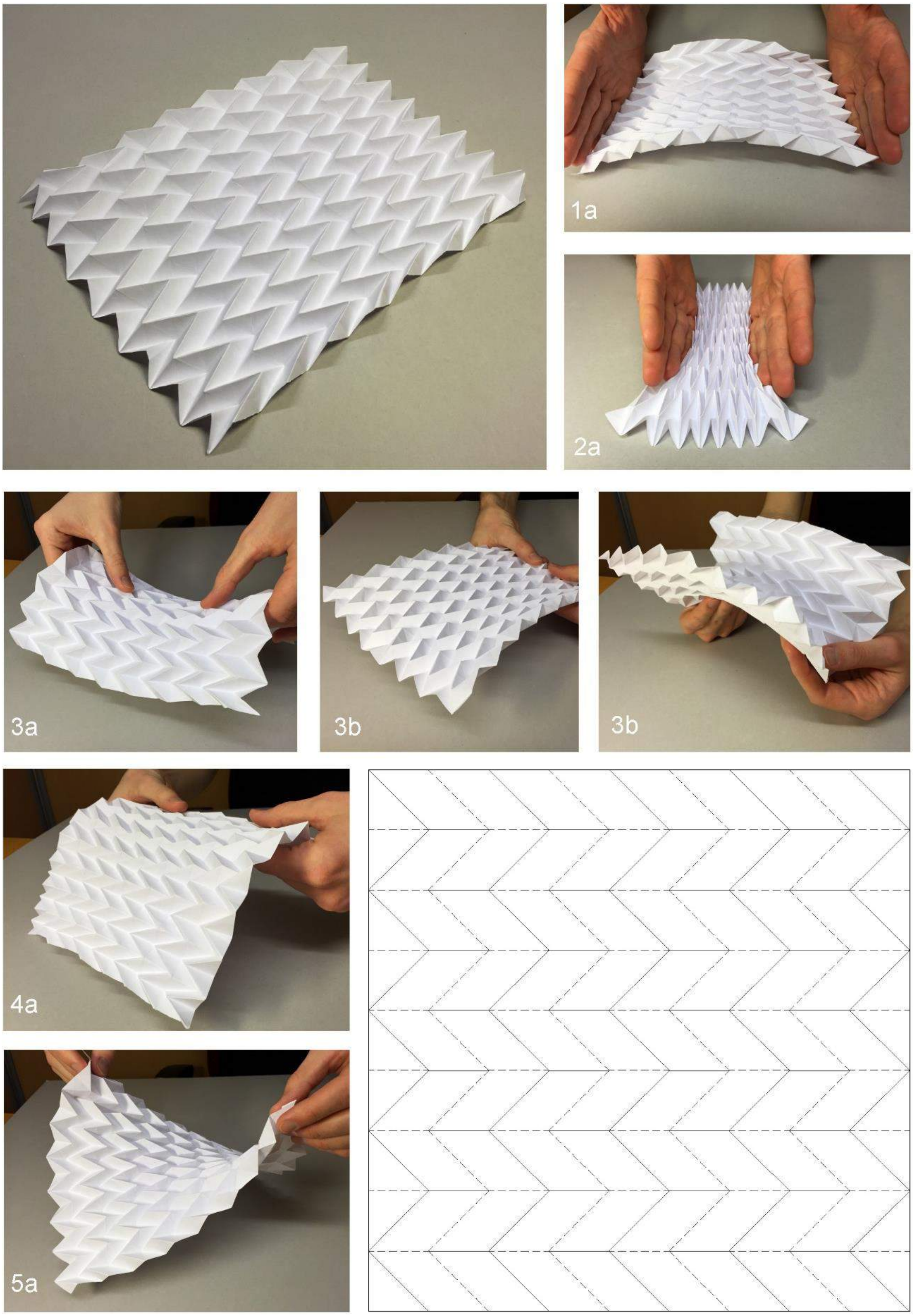


Figure 3.33 The Miura-ori pattern.

3.6.1.1 Crease pattern and global shape

The global shape of a model folded from the Miura-ori pattern will change when the crease pattern change. Changes of fold lengths and angles within a given topology will affect the folded shape in different ways.

In this pattern the zigzag amplitude increases gradually from left to right with each new vertical herringbone stripe. The general shape is still a flat surface.

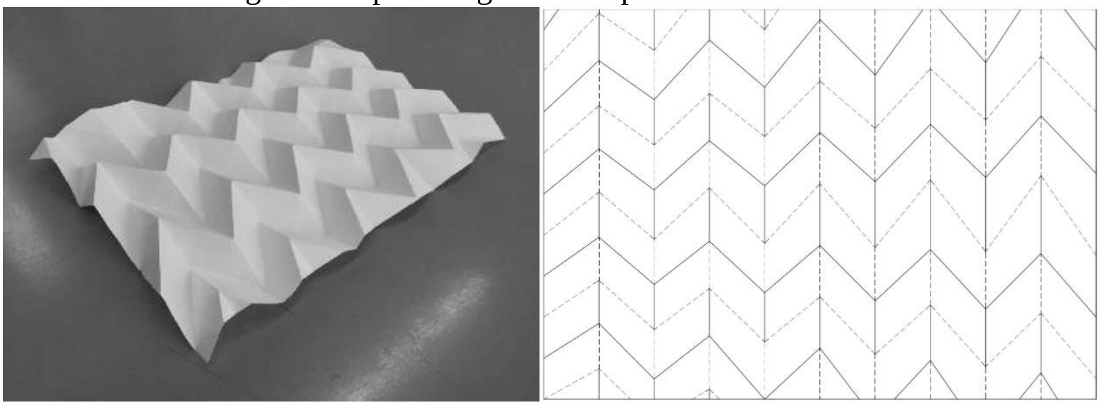


Figure 3.34 Left-to-right increasing amplitude.

Also different kinds of inclination of the vertical crease lines do not change the shape, which is still flat.

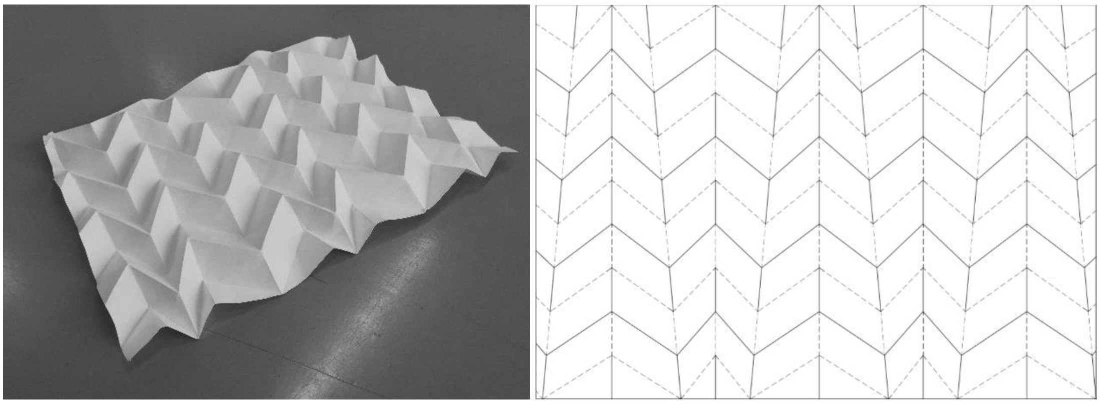


Figure 3.35 Inclined vertical crease lines.

When every second zigzag crease line has lower amplitude than the others, the folded shape will curve globally in the longitudinal direction becoming the Sogame-ori pattern. The larger the amplitude difference the larger the curvature.

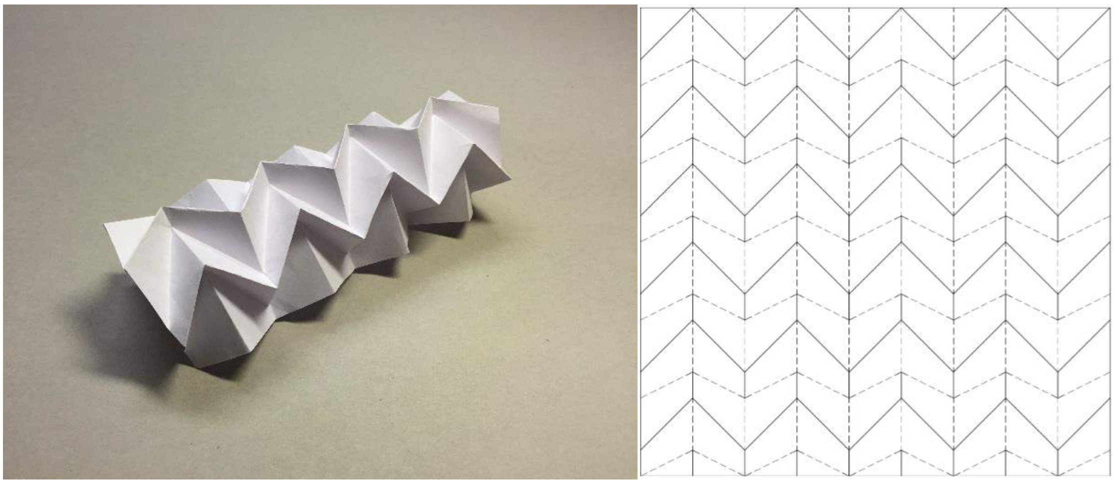


Figure 3.36 Top-down altering zigzag amplitude.

Gradually changing the amplitude difference from left to right also changes the longitudinal curvature, resulting in a doubly curved general shape.

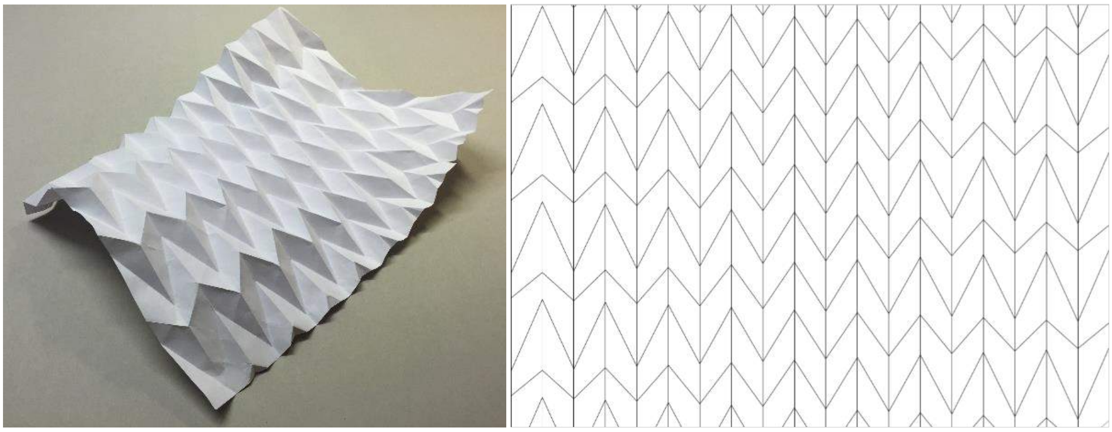


Figure 3.37 Top-down altering zigzag amplitude with left-to-right gradual shift.

3.6.2 The diagonal pattern

The diagonal pattern (fig. 3.38) is similar to the diamond pattern and is constructed from parallelograms that are folded along their diagonals, it is not symmetrical about a line but about a point and the parallelograms are not symmetrical about one of the bases of the trapezoid, as for the herringbone pattern. All diagonals are folded as valley folds and all edges as mountain folds as in the diamond pattern but the diagonal folds in one row are not in the same plane but describe a helix. The diagonal pattern occurs in thin cylinders which are subjected to compression and rotation (Buri, 2010).

The diagonal pattern as well as the diamond pattern forms into a curved shape when folded, it is stiff in the vertical mode 0, in the compression modes 1a and 2a, and in the tension modes 1b and 2b if the edges are fixed. If the edges are free it is completely foldable in the transversal compression mode, 2a which also leads to deformation in compression mode 1a, it is also deployable in tension mode 2b but results in deformation in tension mode 1b. The pattern is also stiff in bending mode 3a but not in bending mode 3b, 4a, 4b, 5a and 5b.

The diagonal pattern has a negative Poisson's ratio in compression/tension. The diagonal pattern has several degrees of freedom caused by the the folding/unfolding and bending degree of freedom in mode 4a/4b, which is further described in chapter 3.7 Structural properties.

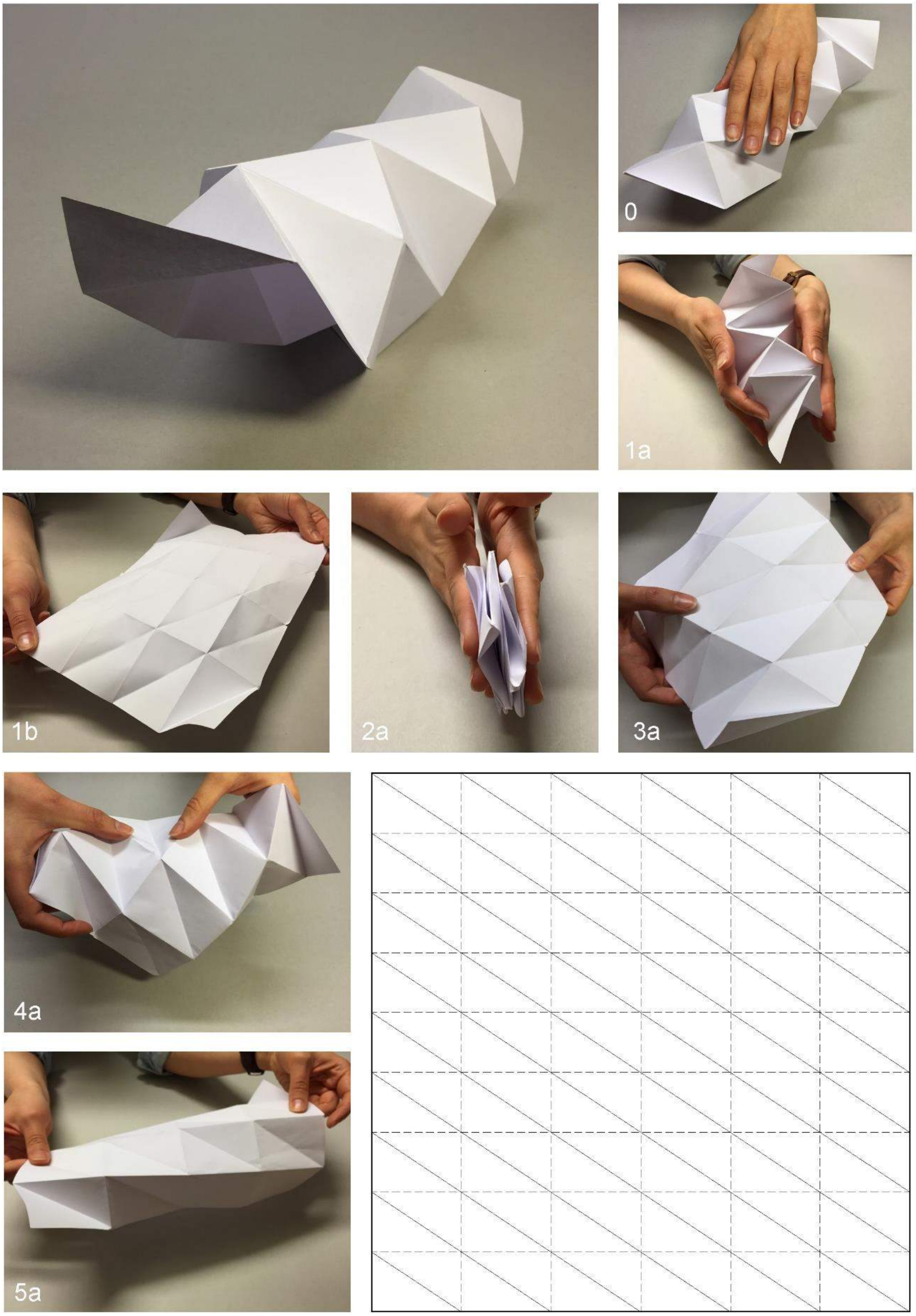
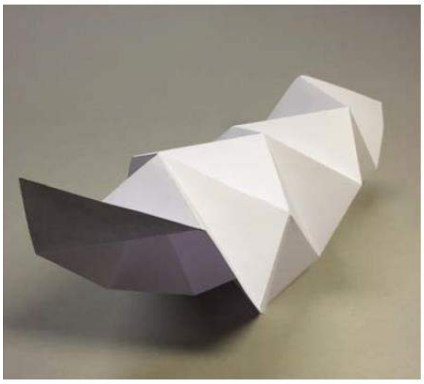


Figure 3.38 The diagonal pattern.

3.6.2.1 Crease pattern and global shape

The general shape of a model folded from diagonal the pattern will change when the crease pattern change. Changes of fold lengths and angles within a given topology will affect the folded shape in different ways.

The basic diagonal crease pattern corresponds to a cylindrical folded shape that curves in the direction of the diagonals.



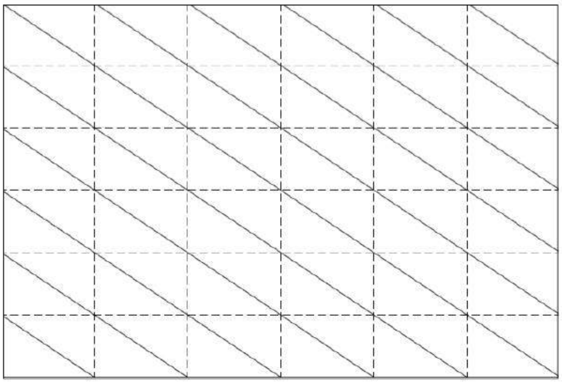
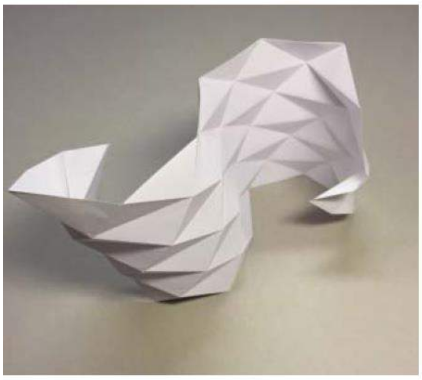


Figure 3.39 Basic crease pattern.

Since the pattern curves along the diagonals the diagonals have to be placed parallel to an edge to get a flat base. If the folds are changed from mountain to valley and vice versa the structure starts to curve in the other way and a global S-curved shape can be created.



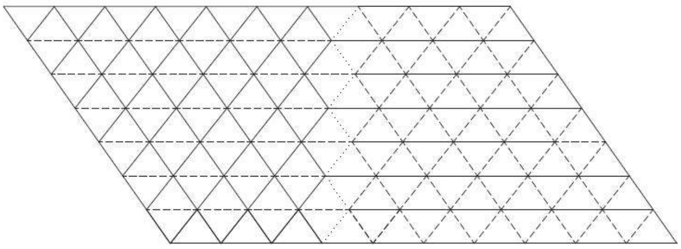


Figure 3.40 Rotated basic pattern in S-shape.

To create a curved structure with a conical shape with a changing radius more material in one side is required, resulting in an S-shaped crease pattern. It is important though that there are no folds perpendicular to the curve direction, if there is, the pattern will just fold in these lines and the structure will not be stable as can be seen in the picture below (Fig.3.17).

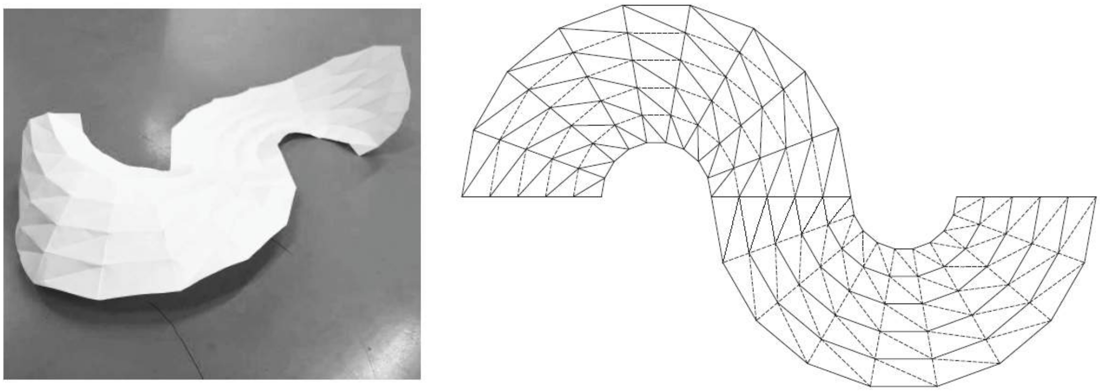


Figure 3.41 Radial basic pattern in S-shape.

The problem described above can be solved by making the radial folds angled so that they are not perpendicular to the diagonals when folded.

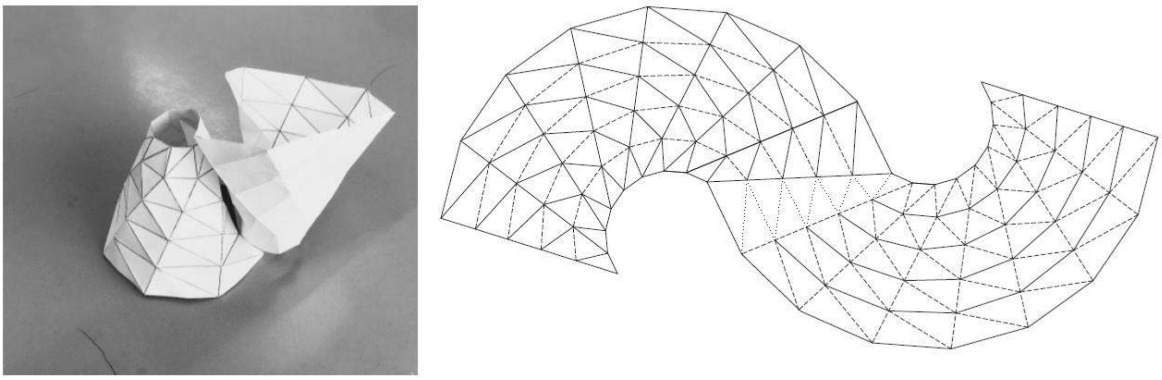


Figure 3.42 Shifted Radial S-shaped pattern.

The diagonal pattern can also be made by lines going diagonal to the global orthogonal coordinate system rather than diagonal to a polar coordinate system as above. The structure will then not be conical just longer in the part consisting of more material and larger elements.

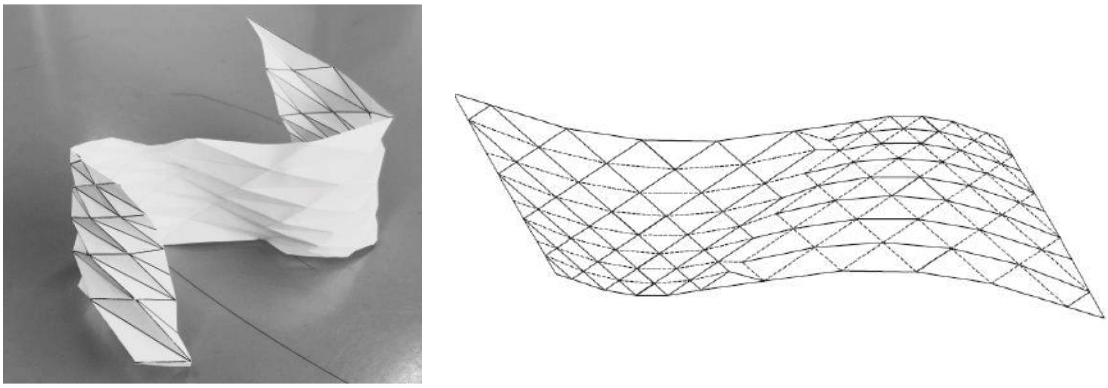


Figure 3.43 Straight S-shaped diamond pattern.

3.6.3 The diamond pattern

The diamond pattern (fig. 3.44) is also called Yoshimura pattern after the Japanese scientist who discoveded it. It is based on the reverse fold and it is built from diamonds folded along their diagonals. The diamond pattern forms a cylinder when folded. It is stiff in vertical mode 0, compression mode 1a and in tension mode 1b if the edges are prevented from moving. It is completely foldable in compression mode 2a and folds out into a flat sheet in tension mode 2b but it requires that the structure is free to move in compression mode 1b and tension mode 1a respectively. In bending the diamond pattern is stiff in mode 3a, 3b, 5a and 5b, while it is flexible in mode 4a and 4b. The diamond pattern has the advantage of being completely foldable in one direction while it is stiff in other compression directions and in bending.

From the investigations described above we can see that also the diamond pattern has a negative Poisson's ratio in compression/tension. The diamond pattern has several degrees of freedom caused by the folding/unfolding and bending degree of freedom in mode 4a/4b, which is further described in chapter 3.7 Structural properties.

If the pattern is symmetric, having congruent diamonds, the curvature of the structure approaches a circle segment and the diamond fold pattern can form a double curved surface if bent around an axis perpendicular to the cylinder. It is the generalised buckling pattern of a thin walled cylinder under axial compression (Buri, 2010).

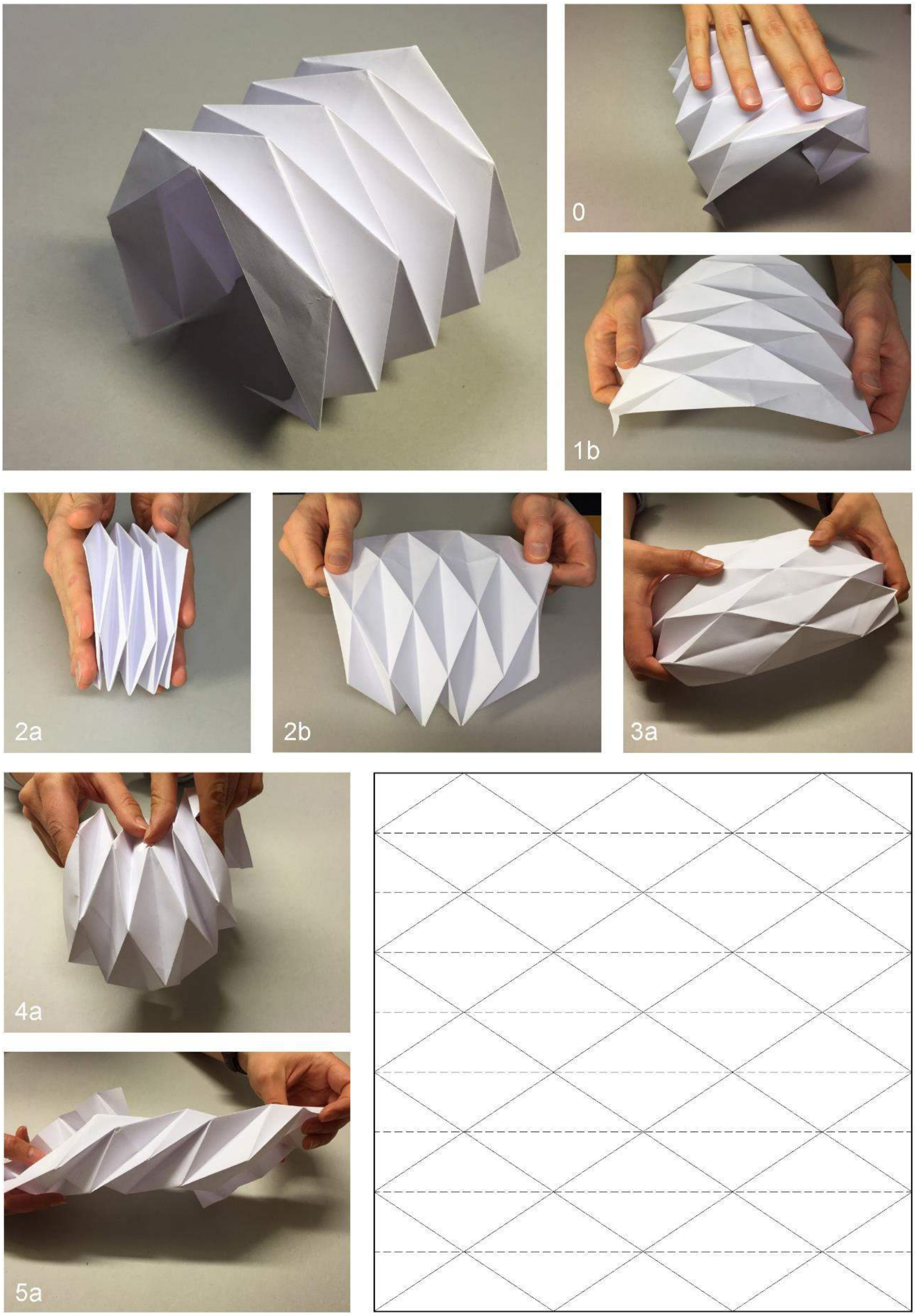


Figure 3.44 The diamond pattern.

3.6.3.1 Crease pattern and global shape

Another way to create a conical S-shape is to base the crease pattern on a radial diamond pattern, how evident the funnel shape will be is determined by the S-curve of the crease pattern.

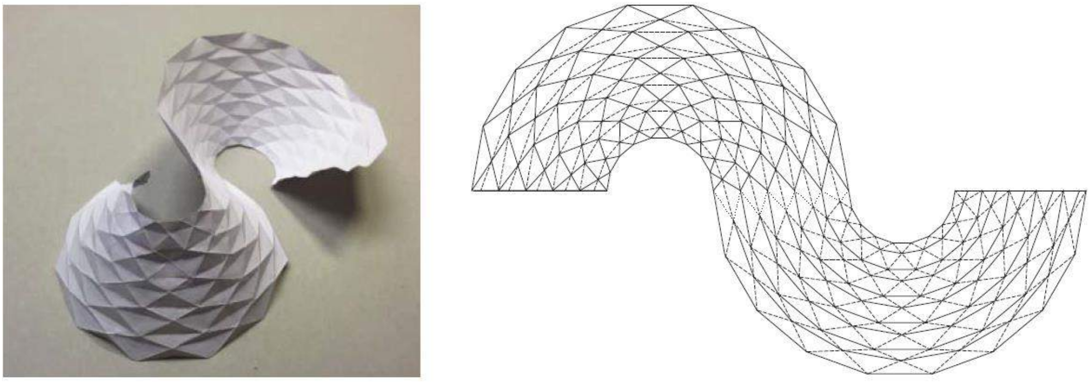


Figure 3.45 Radial S-shaped diamond pattern.

A prominent S-curvature results in some elements becoming very low and wide, a smaller S-curvature gives less funnel shape but the size of elements is more even.

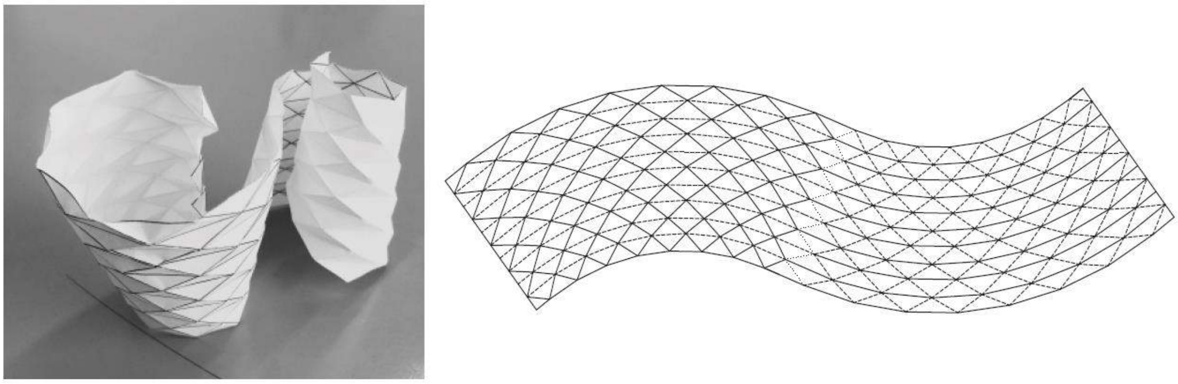


Figure 3.46 Radial diamond S-shaped pattern.

Fold pattern can be changed to create another effect locally but the global structure will still look the same, in the picture below the diamond pattern is altered so that the element size is increasing in one row and decreasing in the next.

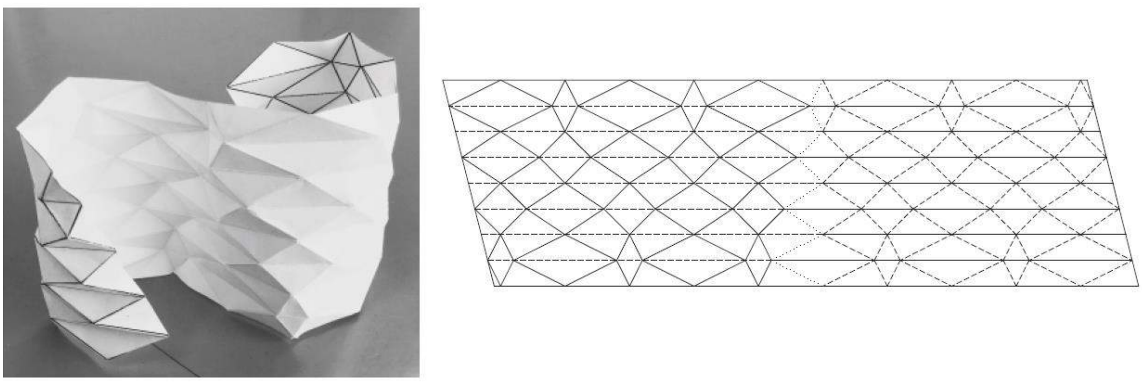


Figure 3.47 Increasing/Decreasing S-shaped diamond pattern.

3.7 Structural properties

3.7.1 Miura-ori (Herringbone)

3.7.1.1 Stability

The herringbone tessellation consists of quadrilateral faces, in each non-boundary node four faces and four edges meet. As a bar structure, due to the quadrilateral faces, a herringbone structure is not stable. However, as a plate structure and given that enough boundary constraints are provided, it is stable. When truncated, the structure is still stable but the larger the cut of piece is the easier it becomes for deformation to occur due to local one-way plate bending. By keeping the whole quadrilateral face but removing a small piece of the hinge near every node, so that the hinges do not meet, the possibility for one way bending of the plates is limited but the structure still loses a little stiffness.

Since the faces are not triangular it is expected that the structure is not stable when made as a lattice structure. According to Ture Wester (1984) a polyhedron is unstable if it is constructed by more than three edges meeting in each node. This doesn't seem to be the case for the herringbone pattern and is probably due to the fact that the folded structures have both positive and negative angles while the polyhedrons have only positive angles.

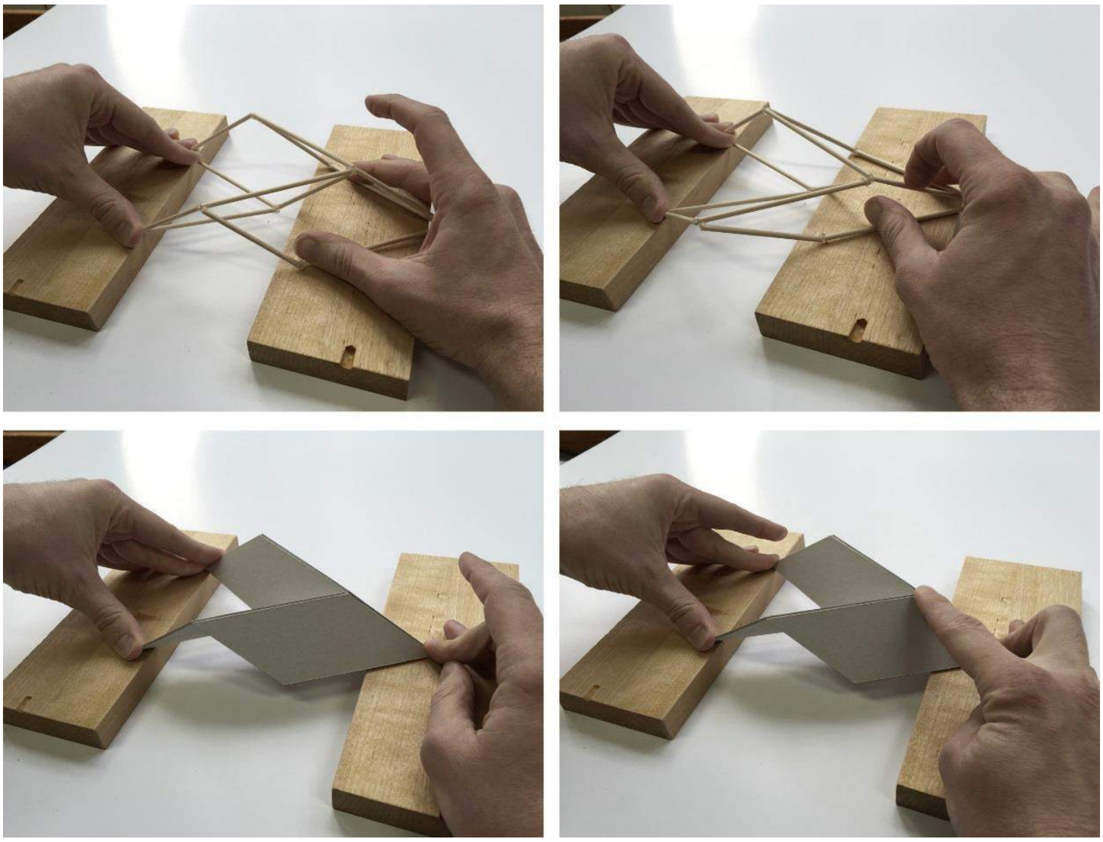


Figure 3.48 Herringbone unit as bar structure (1,2) and as hinged plate structure (3,4).

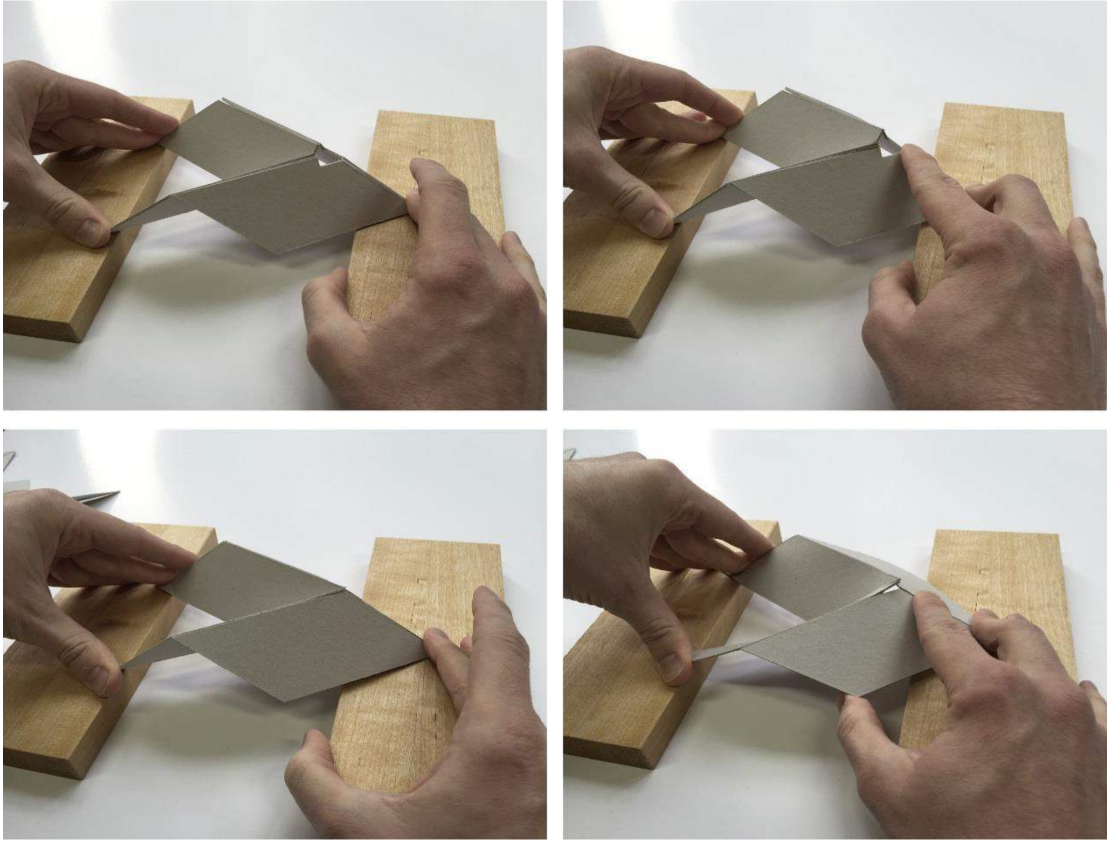


Figure 3.49 Truncated herringbone unit as hinged plate structure (1,2) and herringbone unit with shortened hinges (3,4)

3.7.1.2 Degrees of freedom

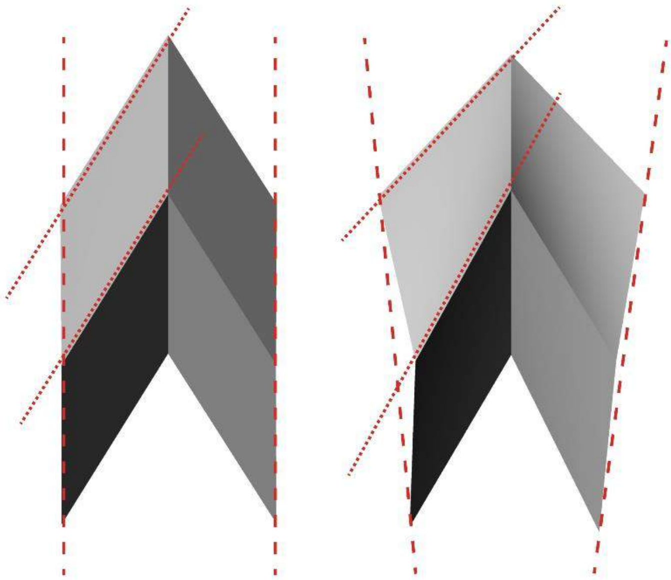


Figure 3.50 Folded herringbone unit with hinged connections. (Left) Both vertical creases have the same fold angle and the plates are rigid parallelograms. (Right) The lower vertical crease has a larger fold angle, which causes the non-rigid plates to twist.

The herringbone pattern, when built with hinged connections and rigid plates, has only one internal degree of freedom. This means that all fold angles are in a one-to-one relation, given that all valley and mountain-folds in the crease pattern are positive and negative correspondingly. If the quads are allowed to twist and become non-planar, the fold angles are no longer in a one-to-one relation. The implication of this characteristic, on a structure with rigid plate elements, is that the structure cannot be shaped other than being more or less folded according to one parameter. Non-rigid plates allow for much more freedom to shape a structure, this is the reason a herringbone paper model easily can take multiple different shapes (Figure 3.2). Obviously, the flexibility gained from non-rigid plates also increases the number of boundary constraints needed for the structure to remain in shape.

3.7.1.3 Stress patterns/Stress concentrations

An analysis of the herringbone pattern was made in Abaqus to investigate where there are stress concentrations. Also the differences in stresses between models with stiff and hinged connections were investigated. The structure was fixed for translational movement in all directions along the whole boundary and loaded by gravity in the z-direction, which induced global bending.

In the following pictures we can see that there are large stress concentrations of both tension and compression close to the fold pattern vertices. These stress concentrations are the same regardless of if the connections are hinged or fixed. There are high negative stresses in the fixed connections, due to their possibility to transfer moment. When the connections are hinged the stress concentrations along the folds do not occur in the same way but instead there are larger stresses in the panels due to bending. This occurs since every panel work as a simply supported member when the connections are hinged, in difference to a structure with fixed connections where the plates act as a continuous structure. This can also be seen in picture 3.55 and 3.56 showing the principal stresses. In the structure with hinges the principal stress direction is mainly in the direction across the plate while in the structure with fixed connections the principal stress direction is mainly along the folds.

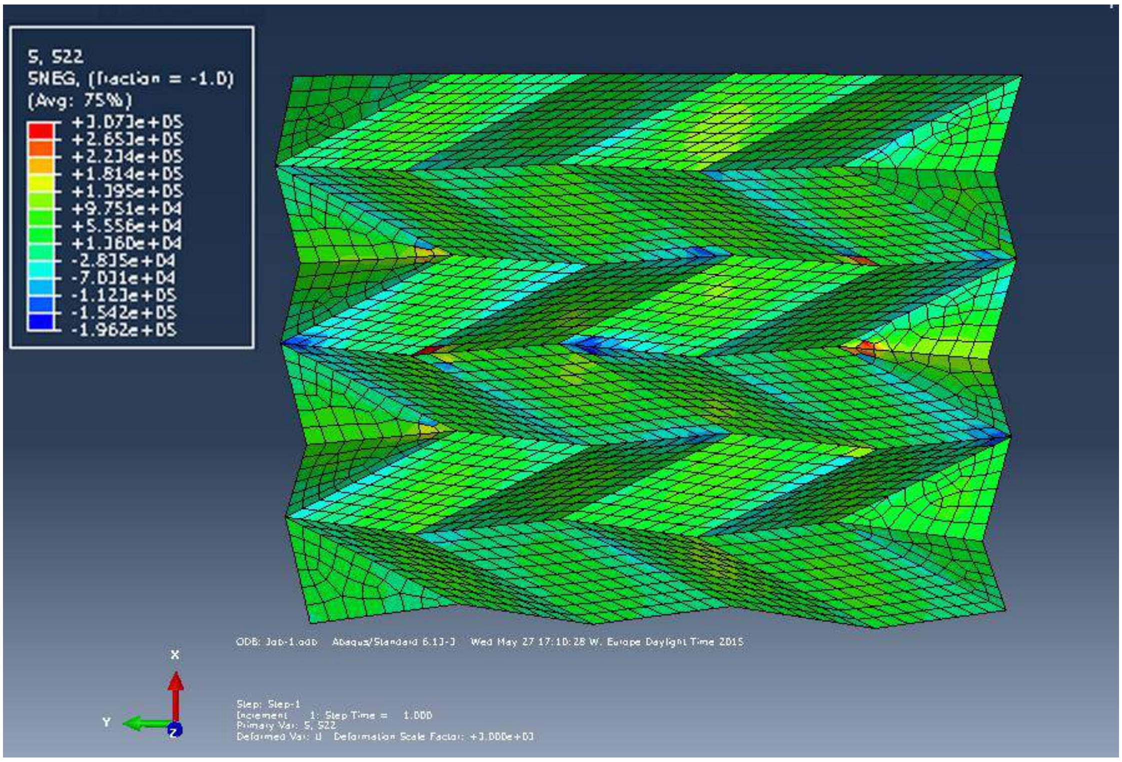


Figure 3.51 Principal stress, S22, Herringbone pattern with hinged connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

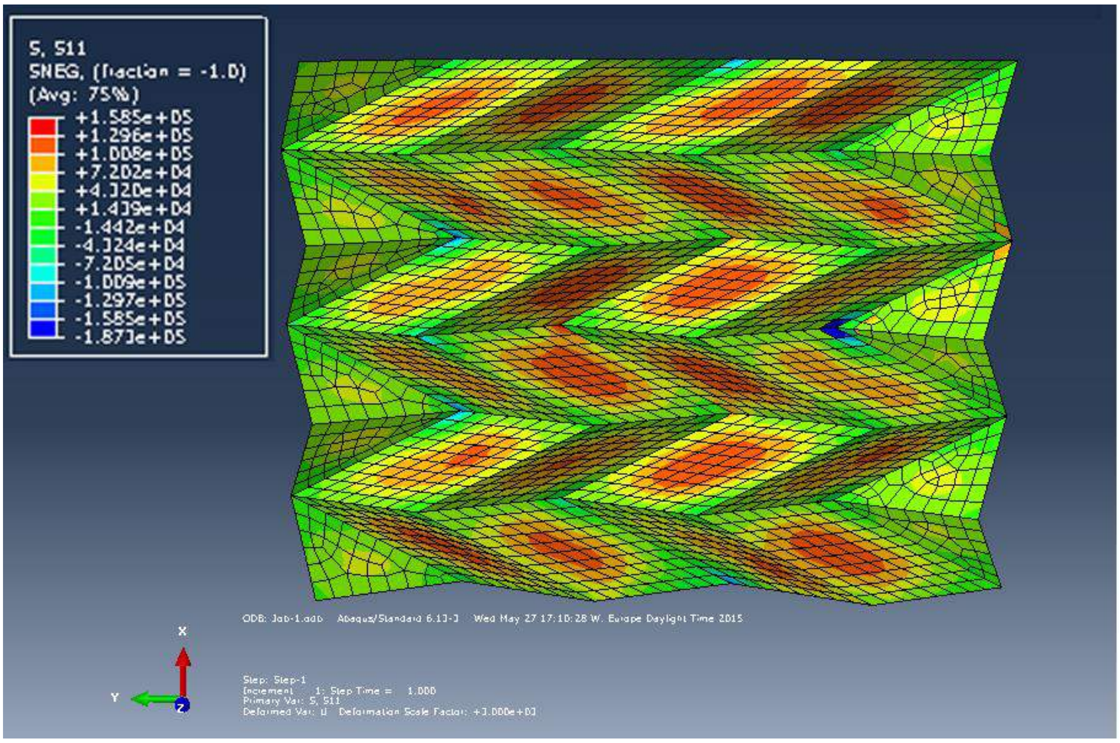


Figure 3.52 Principal stress, S11, Herringbone pattern with hinged connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

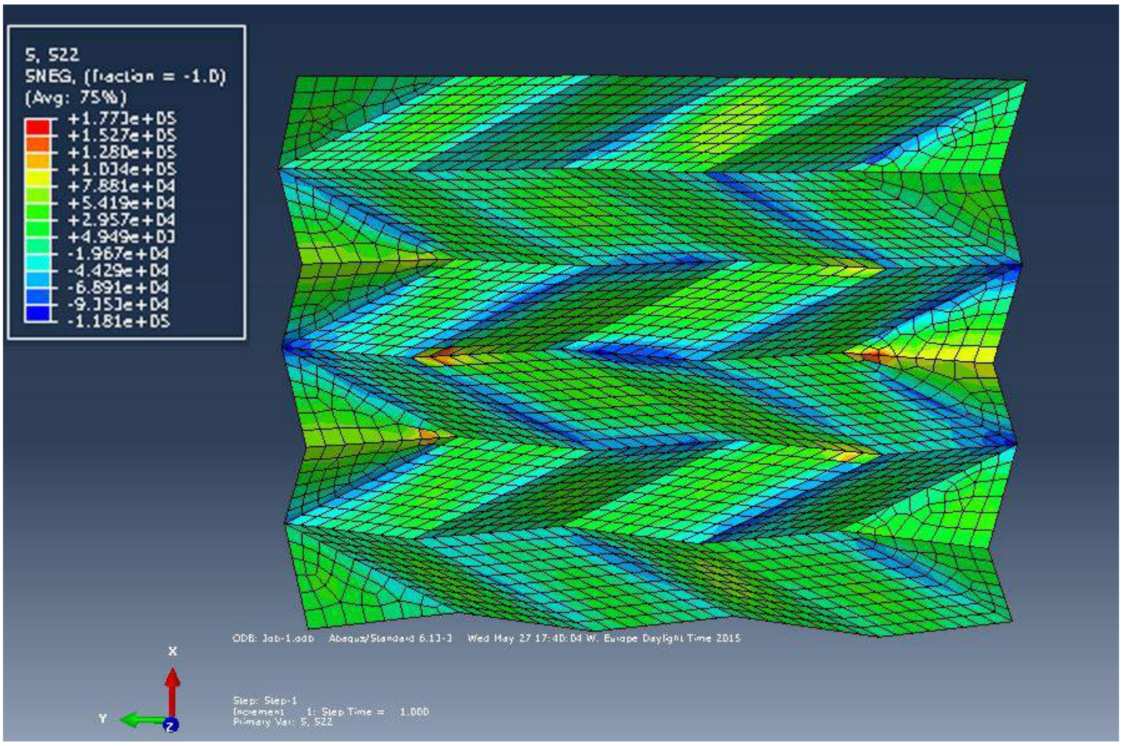


Figure 3.53 Principal stress, S22, Herringbone pattern with stiff connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

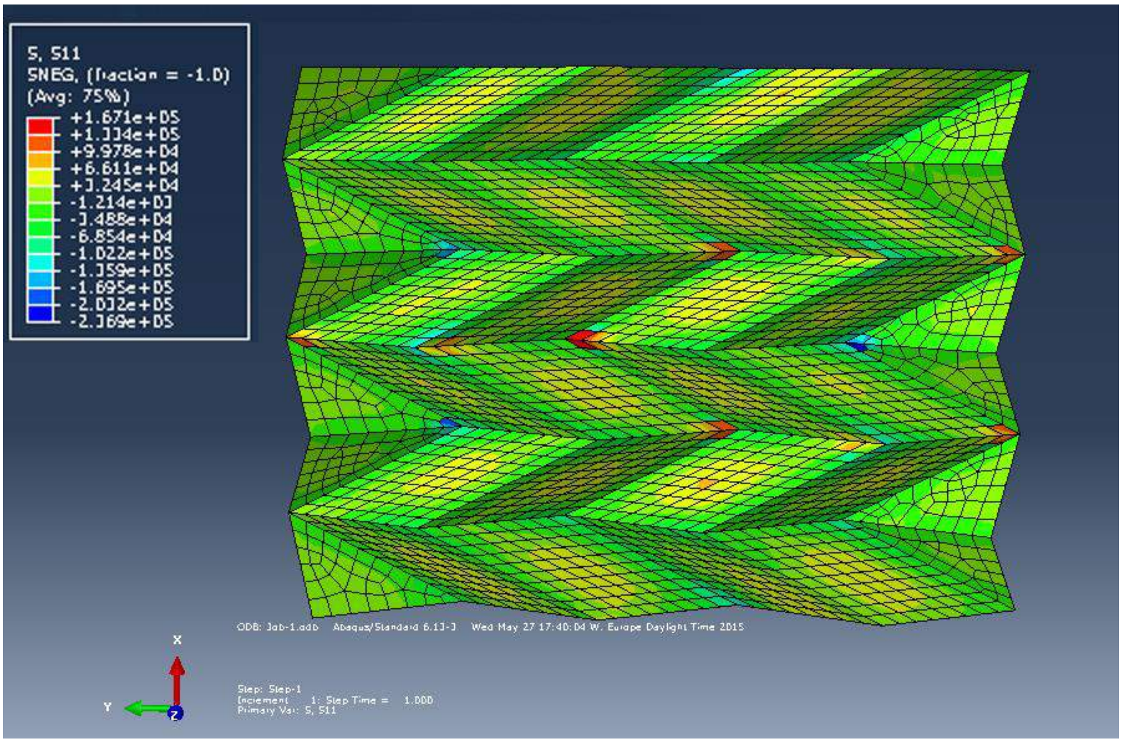


Figure 3.54 Principal stress, S11, Herringbone pattern with stiff connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

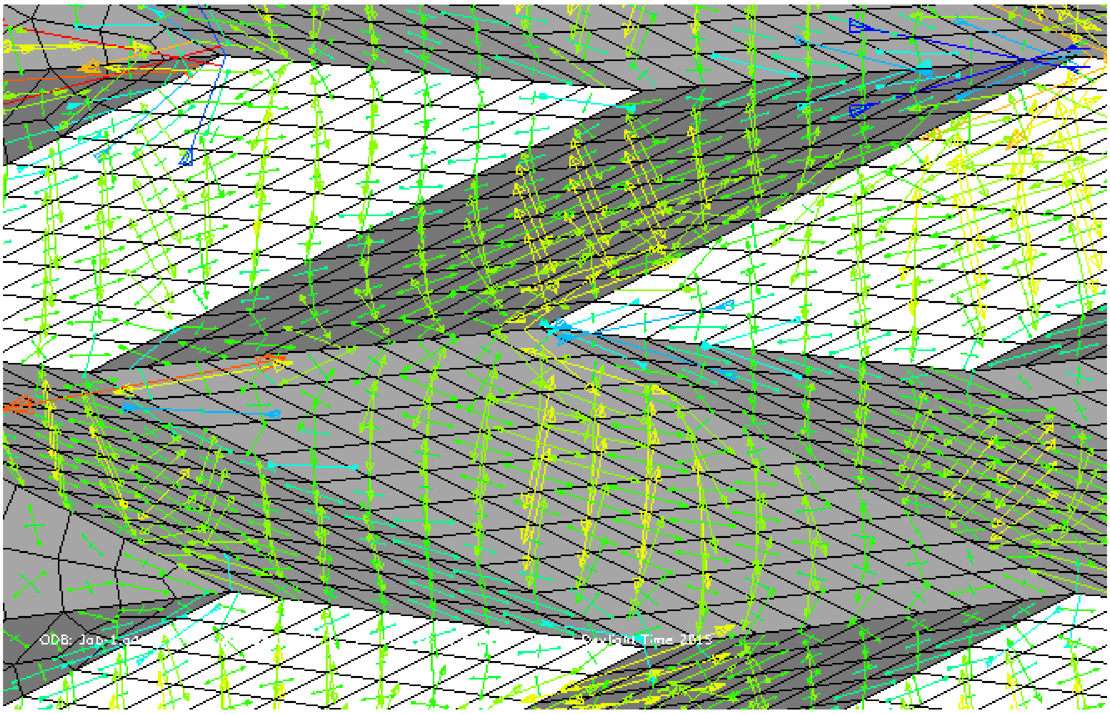


Figure 3.55 Herringbone pattern with hinged connections loaded in z-direction (perpendicular to the plane) with principal stresses and their directions, red arrows shows tension while the blue represents compression.

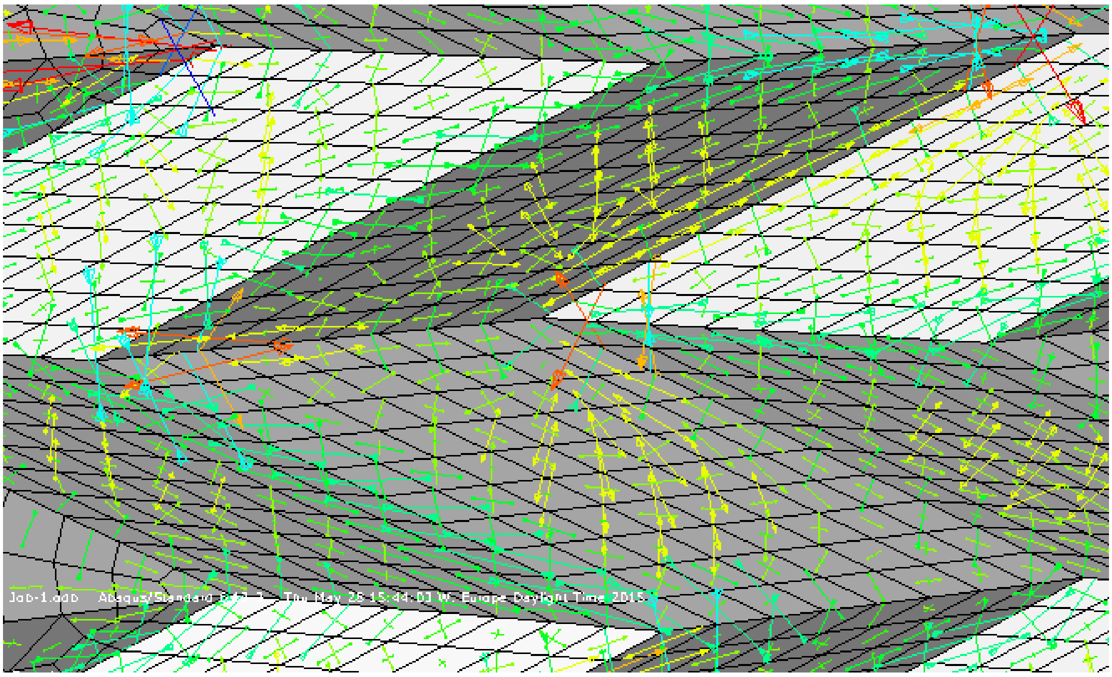


Figure 3.56 Herringbone pattern with fix connections loaded in z-direction (perpendicular to the plane) with principal stresses and their directions, red arrows shows tension while the blue represents compression.

The pattern was also loaded in x-direction and fixed against all translational movement at the lower border while the other nodes at the border where fixed against translational movement in z- and y-direction.

It can be seen that the stress concentrations close to the fold pattern vertices remains and that the stresses vary in the structure and the elements in the bottom of the structure are subjected to more stresses. Since the stress concentrations close to the vertices are so prominent it is probably here the structure will break and this part thus has to be designed carefully.

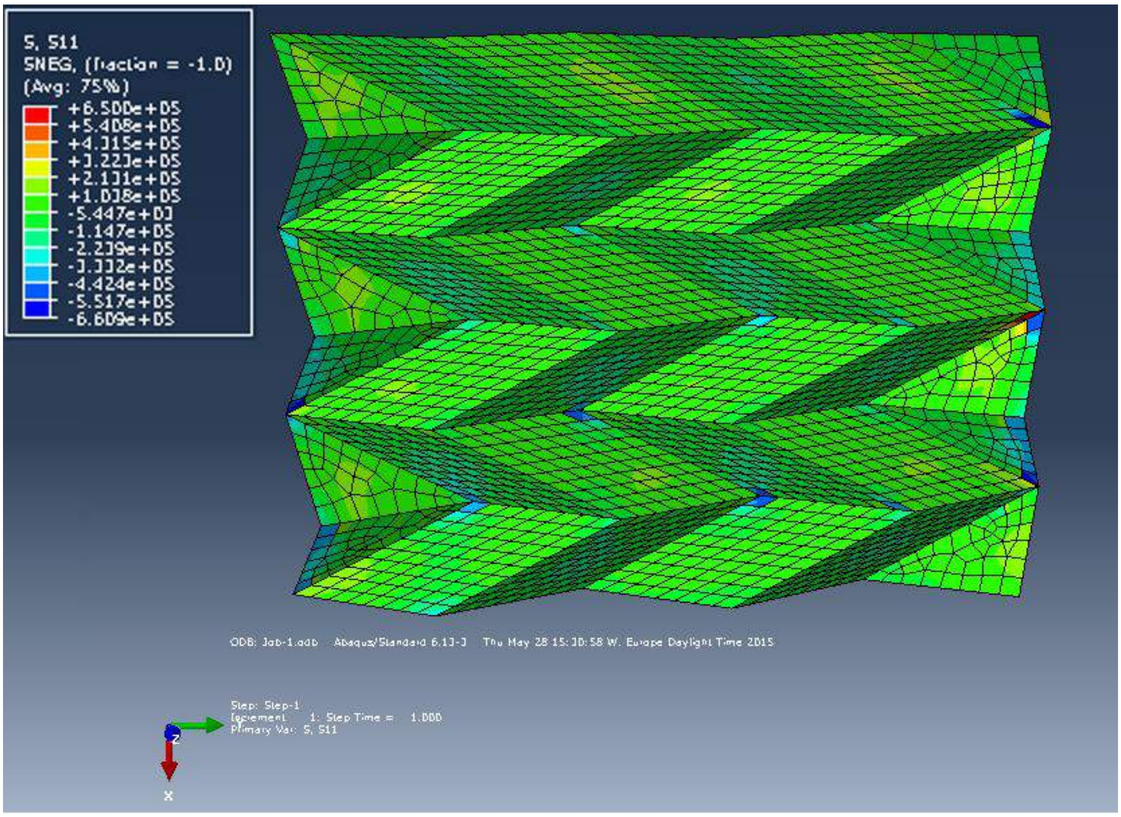


Figure 3.57 Principal stress, S11, Herringbone pattern with hinged connections subjected to gravitational load in the x-direction. Blue represents compression and red represents tension.

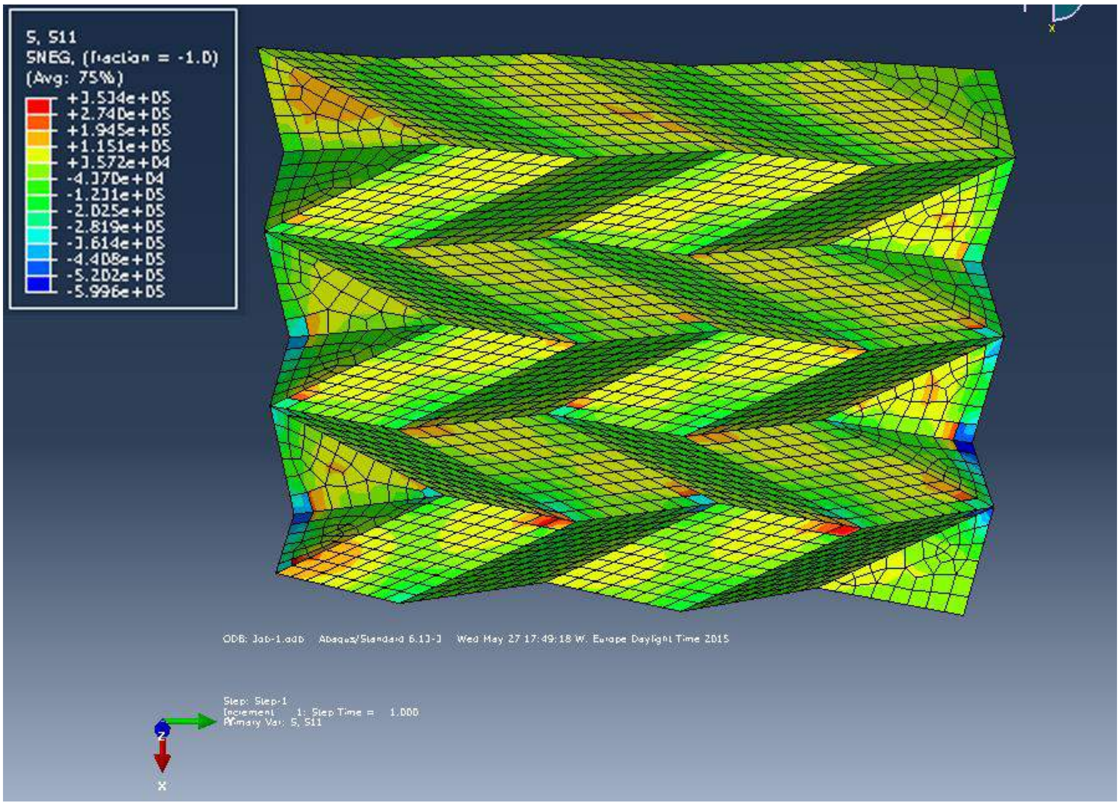


Figure 3.58 Principal stress, S11, Herringbone pattern with stiff connections subjected to gravitational load in the x-direction. Blue represents compression and red represents tension.

3.7.2 Diamond pattern and Diagonal pattern

3.7.2.1 Stability

To determine whether the diamond pattern works as a bar structure or as a plate structure or as both, three models were built; one with full plates, one with cropped plate corners, and one model with only bars at the plate edges. The first two models were made from double cardboard plates with hinged connections of double sided sticky tape placed in between two plates. The third model was made from round wooden rods mounted together with a glue gun.

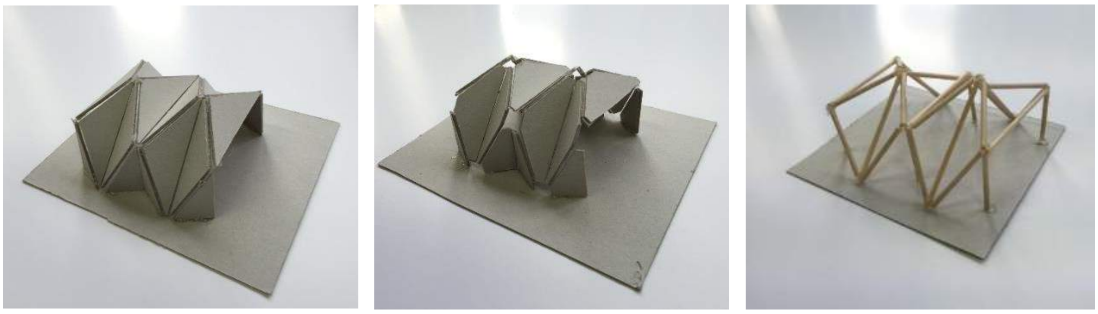


Figure 3.59 Diamond pattern built as plate and lattice structure.

When loading the models it can be seen that all the three models are stable even if the cropped plate model is less stiff and deforms a bit more than the full plate model. The bar structure can also be verified by checking if every node is connected to three other already stable points that are not in the same plane, which it is, meaning that the model is stable.

This is expected since the faces of the diamond pattern consists of triangular surfaces, for polyhedrons this means that the structure is stable and it is therefore also reasonable to think the same rules would apply to folded structures. It would also be reasonable to think that the cropped plate model should be unstable since it is built up by more than three edges meeting in each node, applied to polyhedrons meaning that the structure is not stable (Wester, 1984). This doesn't seem to be the case for the diamond pattern or for the herringbone pattern, as previously described, the fact that the folded structures have both negative and positive angles in difference to the polyhedrons seem to result in other rules for the stability.It can thus be concluded that the diamond pattern is stable both as a lattice structure and as a plate structure but that it benefits from having connections along the whole plate edge and from the plates going whole the way to the vertices.

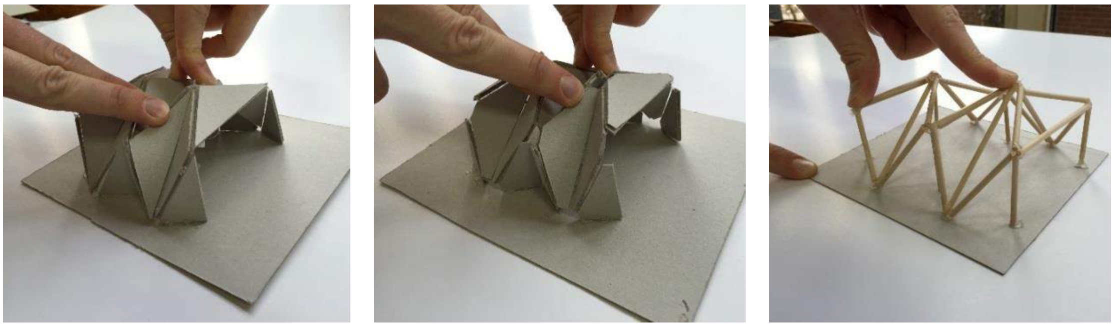


Figure 3.60   Stability as plate and lattice structure.

3.7.2.2 Degrees of freedom

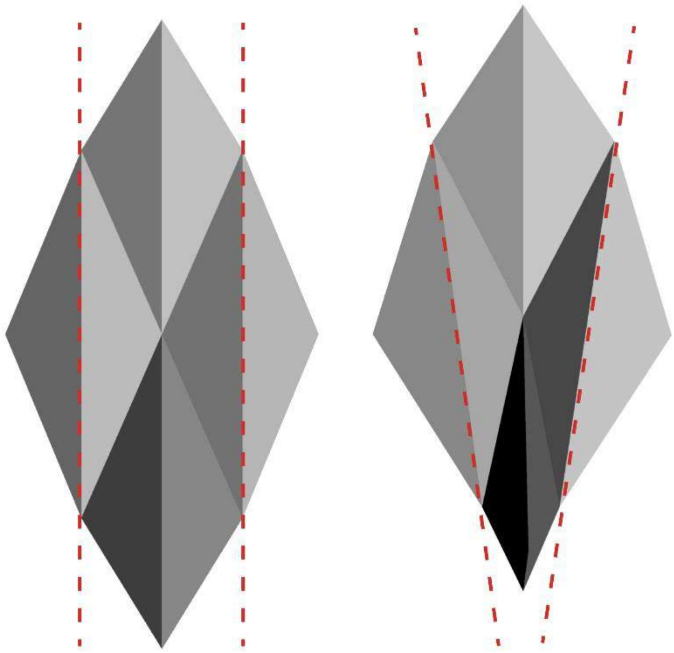


Figure 3.61 Folded diamond unit with hinged connections. Both middle vertical creases have the same fold angle and the plates are rigid triangles (Left). The lower middle vertical crease has a larger fold angle, which is possible without any deformation of the rigid triangular plates (Right).

The diamond pattern has, when constructed with hinged connections and rigid plates, multiple internal degrees of freedom. The fold angles are not in a one-to-one relation, which allows even a rigid plate diamond structure to be shaped with more than one folding parameter. In fact, since the corner points of a triangle face always are in the same plane, in plane deformations or buckling are the only deformation modes of the plates in this structure. This means that the behaviour of a folded paper model in a good way resembles a rigid plate structure when it comes to this pattern (Figure 3.16). In fact this goes for all patterns with triangular faces (Buri, 2010).

3.7.2.3 Stress patterns/Stress concentrations

An analysis of the diamond pattern was made in Abaqus in the same way as for the herringbone pattern. Stress concentrations in pattern and the differences in stresses between stiff and hinged connections were investigated. The structure was fixed for translational movement in all direction along the whole boundary and loaded by gravity in the z-direction, which induced global bending.

In the pattern with hinged connections there are large negative stresses in the corners of the elements while for the pattern with stiff connections the stress concentrations occur in the middle of the connection. There are also stresses in the middle of the elements due to bending and these stresses are larger for the pattern with hinged connection due to the same reason as was described for the herringbone pattern. The stresses in the middle of the plate is only affecting the plate itself and can be taken care of by choosing a proper material while the stress concentrations at the connections and corners requires a careful joint design. In figure 3.62 and 3.63 it can be seen that for the structure with hinged connections there is a lot of tension across the plate while for the structure with stiff connections the compression over the connection is large.

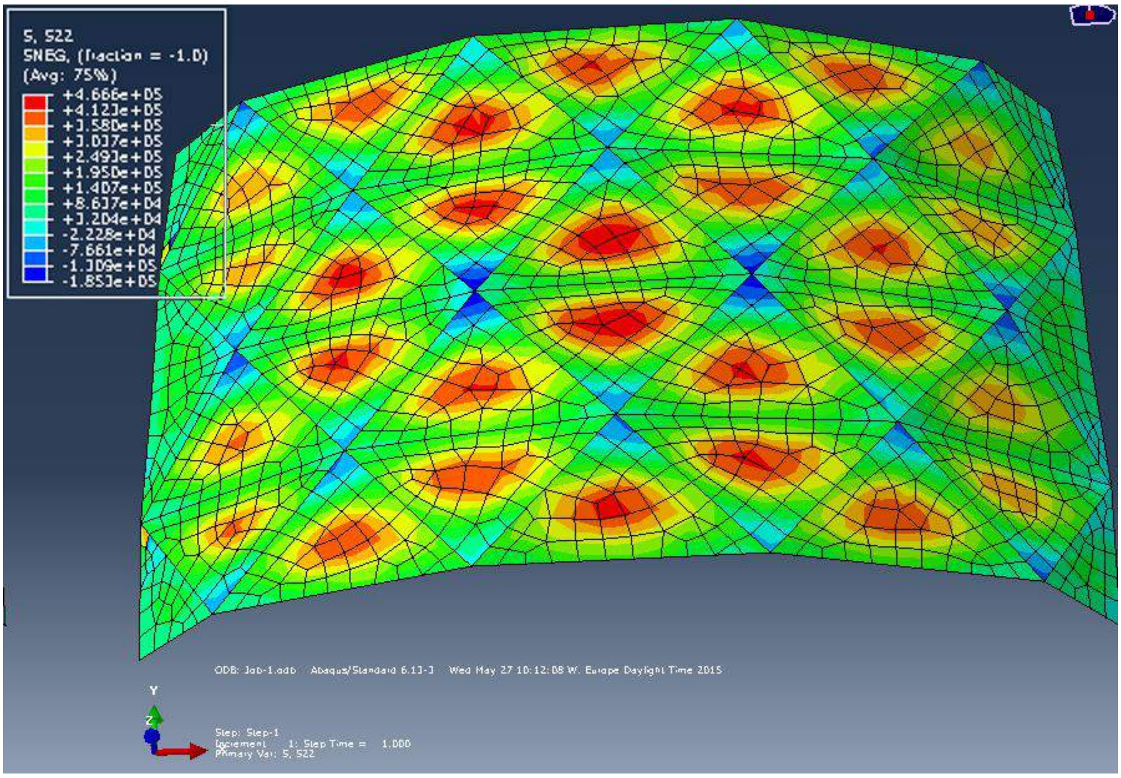


Figure 3.62 Principal stress, S22, Diamond pattern with hinged connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

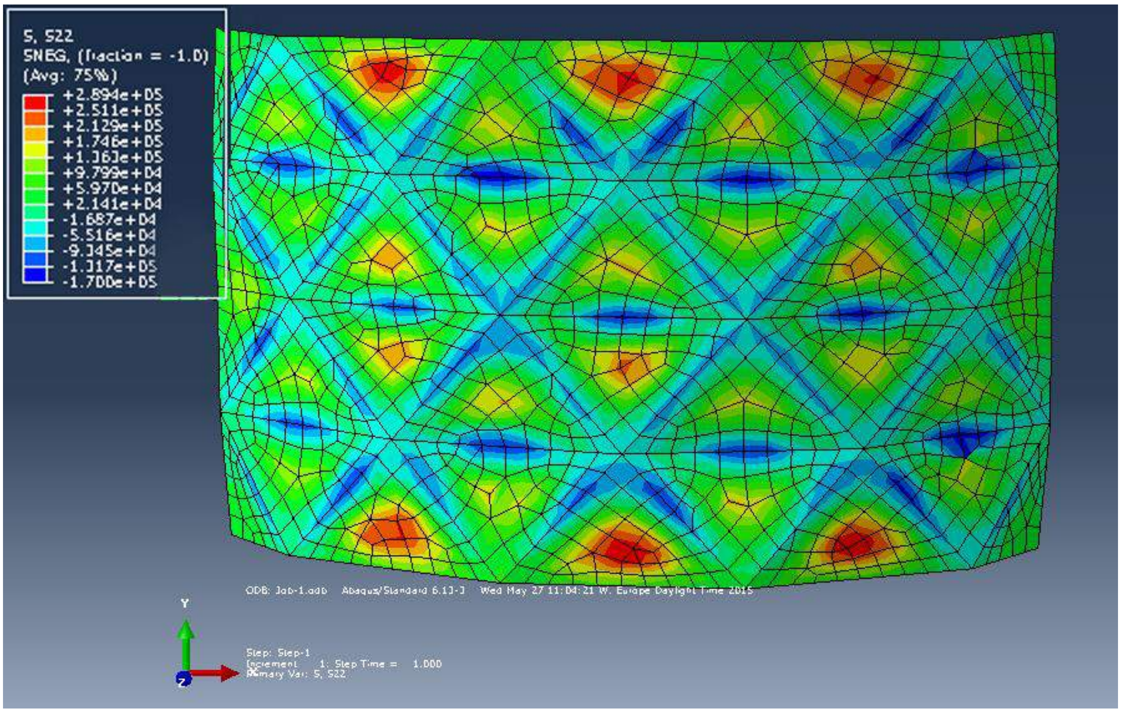


Figure 3.63 Principal stress, S22, Diamond pattern with fixed connections subjected to gravitational load in the z-direction (perpendicular to the plane). Blue represents compression and red represents tension.

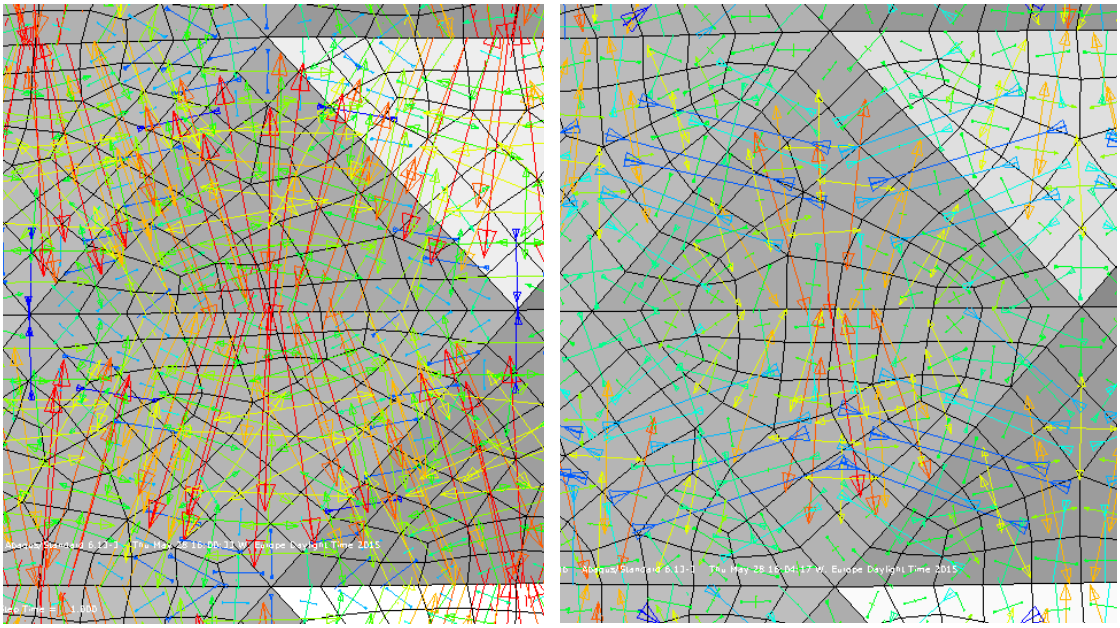


Figure 3.64 Diamond pattern loaded in z-direction (perpendicular to the plane) with principal stresses and their directions, red arrows shows tension while the blue represents compression (hinged connection to the left, fixed connection to the right).

The pattern were also loaded in y-direction and fixed against all translational movement at the lower boarder while the other nodes along the border where locked against translational movement in z- and x-direction.

It can be seen that the stress concentrations close to the nodes remains even for this load case and the elements in the bottom of the structure is subjected to more stresses. In the structure with fixed connections positive stresses occurs in the longitudinal connections and negative stresses mostly in the top elements. The stress concentrations at the corners are large and occur both for loading in y- and z- direction, these are the weak points of the structure and has to be carefully considered in the design.

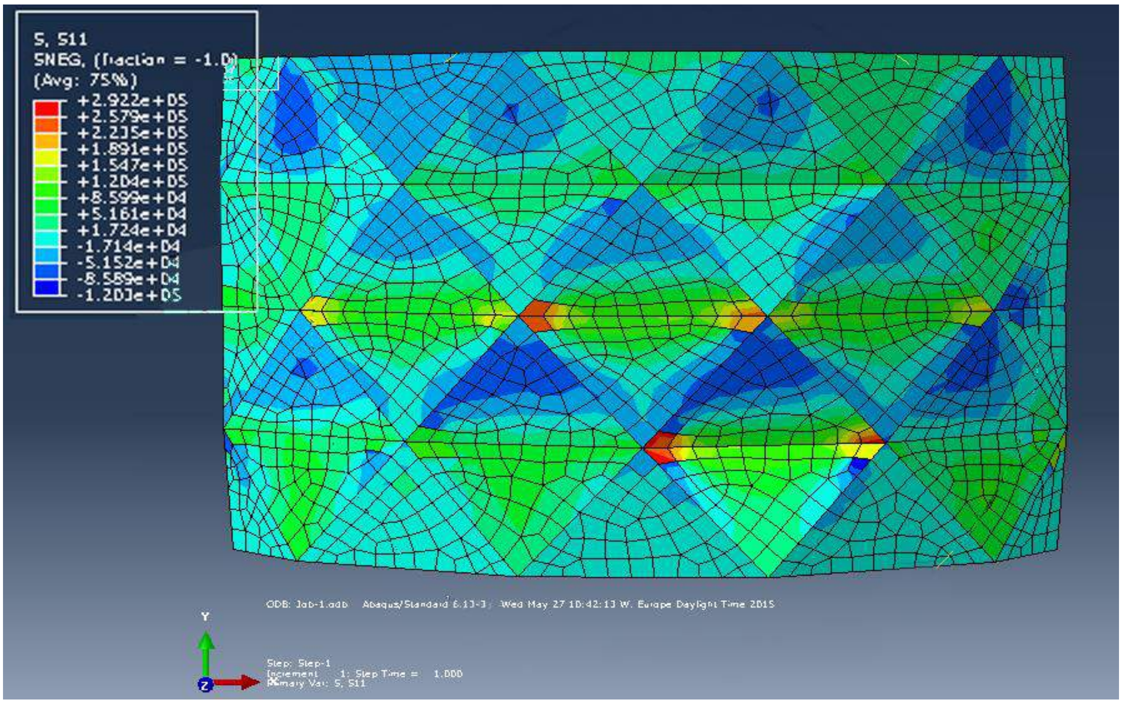


Figure 3.65 Principal stress, S22, Diamond pattern with hinged connections subjected to gravitational load in the y-direction. Blue represents compression and red represents tension.

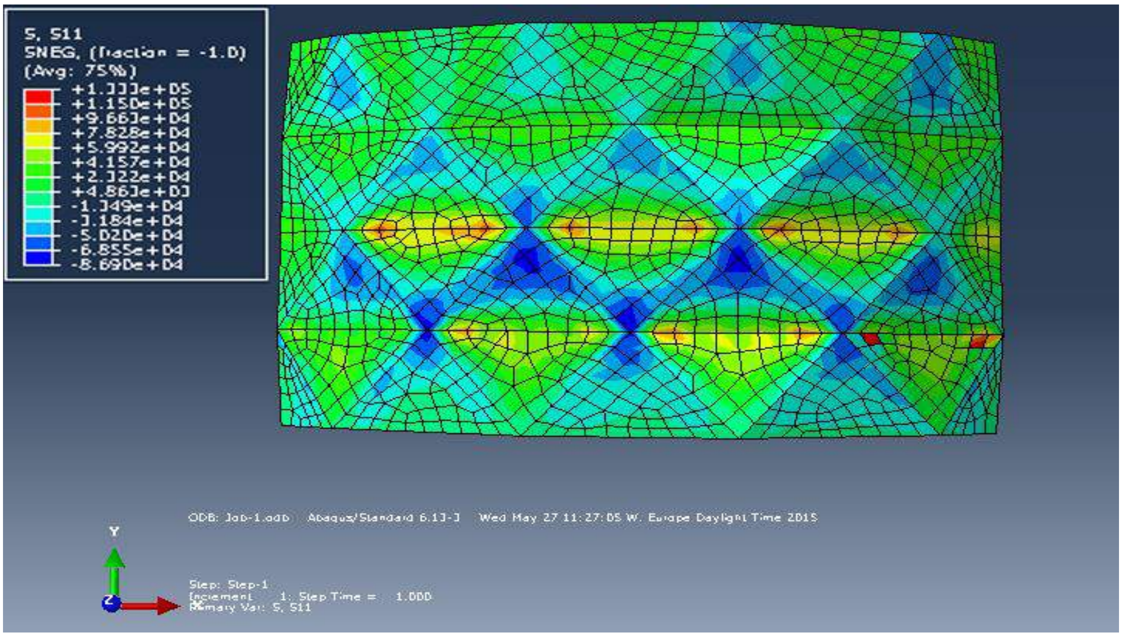


Figure 3.66 Principal stress, S22, Diamond pattern with fixed connections subjected to gravitational load in the y-direction. Blue represents compression and red represents tension.

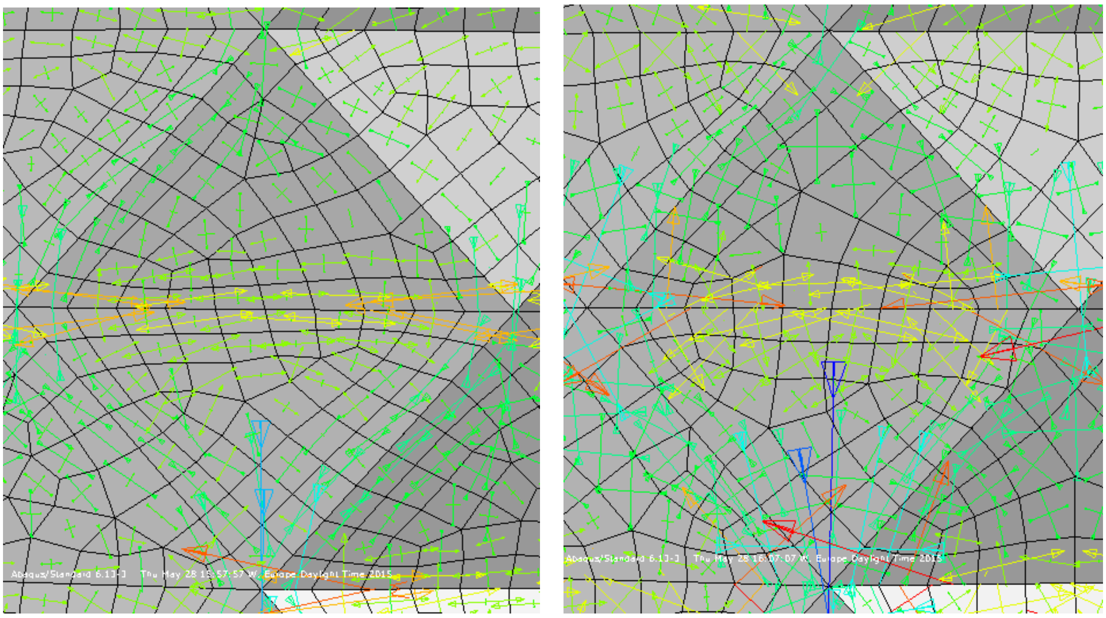


Figure 3.67 Diamond pattern loaded in y-direction with principal stresses and their directions, orange arrows shows tension while the blue represents compression (hinged connection to the left, fixed connection to the right).

3.8 Canonical stiffness

The concept of canonical stiffness is an alternative way of describing the stiffness of a structure. From the stiffness matrix, deformation patterns and corresponding loads can be obtained. The canonical stiffness is based on the static eigenvalues of a finite element model of linear elastic materials. In the static eigenvalue analysis the information about the stiffness of the structure is transformed from the *n* x *n* stiffness matrix, *K*, to *n* inherent stiffnesses, λ_i, called canonical stiffnesses (Olsson, K-G., 2005).

Designing based on the static Eigenvalues is a way to evaluate the structural properties of a structure in an early conceptual face. The method consist of identifying the deformation modes for which the structure is very weak and finding which load combinations this corresponds to. By finding the force directions in which the structure is vulnerable, the design can be modified so that the weakest Eigen modes and the Eigen modes which corresponds to probable force actions are eliminated. The factors affecting the deformation patterns are the material stiffness, local and global shape, external supports and internal joints, and these are thus determining the Eigen modes and the stiffness (Olsson, P., 2003).

The canonical stiffness is important to include in the design process since it evaluates the global qualities, while other analyse methods are generally focused on the local behaviour which results in decisions only affecting the local solutions and not the global force path and stiffness (Olsson, P., 2003). The canonical stiffness is also a way to digitally evaluate the behaviour of the structure and to get an understanding of the structural and mechanical properties early in the design process. This saves time and resources compared to a prototype based trial and error process (Olsson, K-G., 2003). It is often hard to anticipate which loads the structure will be exposed to apart from the main load case, by this method the load cases that are especially dangerous to the structure can be found, something that is very interesting for complex and previously unknown structures that are to be construct in new settings (Olsson, P., 2003).

The static Eigenvalue problem for the whole structure is solved by:

$\boldsymbol{K}\boldsymbol{a}_{\boldsymbol{i}} = \boldsymbol{f}_{\boldsymbol{i}} = \lambda_i \boldsymbol{a}_{\boldsymbol{i}}$ which also can be written:

$(\boldsymbol{K} - \lambda_i \boldsymbol{I})\boldsymbol{a}_{\boldsymbol{i}} = 0$

Where $\boldsymbol{K}$ is the structural stiffness matrix, $\boldsymbol{I}$ is the identity matrix, f_i is the force vector, λ_i is the Eigenvalue *i* and $\boldsymbol{a}_i$ is the corresponding Eigenvector. The Eigenvalue, λ_i, gives a value for the stiffness of the structure and the Eigen mode, $\boldsymbol{a}_i$, gives the corresponding deformation pattern, which is sorted from weakest to stiffest by the size of the Eigenvalues. Since the displacement is proportional to the load, the Eigen modes can be seen as load cases on the structure. The lowest modes usually shows deformation of the whole structure and can therefore be used to find the load directions for which the global structure is weak. During a design process also other values like the strain energy can be interesting. It can be obtained by normalizing the Eigen vectors (Olsson,P.,2003).

$\boldsymbol{a}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{a}_{\boldsymbol{i}} = \boldsymbol{1}$ this gives:

$\boldsymbol{a}_i^{\boldsymbol{T}} \boldsymbol{K} \boldsymbol{a}_i = \lambda_i$ or, if the eigenvectors are normed as above:

$$U_i = \frac{\lambda_i}{2}$$

Where U_i is the strain energy stored in the structure when exposed to the normalized displacements $\boldsymbol{a_i}$. This means that the Eigenvectors can be seen as the deformations that corresponds to different strain energies, and that the Eigenvectors corresponding to the lowest Eigenvalues shows the deformation modes that needs the least strain energy to occur (Olsson,P., 2003).

The formulas above is valid as long as the structure consists of parts working as rods, membrane plates or solids but not for parts working as beams or slabs since the degrees of freedom then will consist of both translations and rotations meaning that the Eigenvector will contain terms with different units. If this is the case the constitutive relations of the structure can then be written (Olsson, K-G., 2005):

$$\boldsymbol{K}\boldsymbol{a}_i = \boldsymbol{f}_i = \lambda_i \boldsymbol{C}\boldsymbol{a}_i \quad \text{or:}$$

$$(\boldsymbol{K} - \lambda_i \boldsymbol{C})\boldsymbol{a}_i = 0$$

where $\boldsymbol{C}$ is:

$$\boldsymbol{C} = \begin{bmatrix} c & 0 & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 \\ 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & d \end{bmatrix}$$

If c is chosen to 1 [-] and d to 1 [m^2/rad] $\boldsymbol{C}$ is equal to $\boldsymbol{I}$ and the eigenvalues will have the unit [N/m], which is equal to stiffness. Simplification can also be made by eliminating stiffness related to rotations from the analysis by setting d to 0. The corresponding normalization and strain energy are (Olsson, K-G.,2005):

$$\boldsymbol{a}_i^{\boldsymbol{T}} \boldsymbol{C} \boldsymbol{a}_i = \boldsymbol{1}$$

$$U_i = \frac{\lambda_i}{2}$$

The stiffness components are absolute values and can therefore be compared between different calculations. Strain Energies are different, they work well when comparing different modes in the same model, with same supports and same element division, but cannot really be compared between different models and especially not if the degrees of freedom is changed. With more degrees of freedom the translation of each node is smaller and therefore the strain energy also is lower. To be able to compare different structures and models, a scale factor must be determined for the lowest Eigen mode in each model and then this scale factor must be used for scaling all the modes. The scale factor, k, can be obtained by setting $\boldsymbol{a_{ref}}$, U_i or ε_{ref} to1 and using the expression (Olsson, K-G.,2005):

$$\boldsymbol{a}_i^{\boldsymbol{T}} \boldsymbol{C} \boldsymbol{a}_i = \boldsymbol{k}$$

Two of the most important parameters affecting the canonical stiffness is the stiffness of joints and material, which can be explained as simple springs:

Translational stiffness: k=F/u
Rotational stiffness: k=M/q

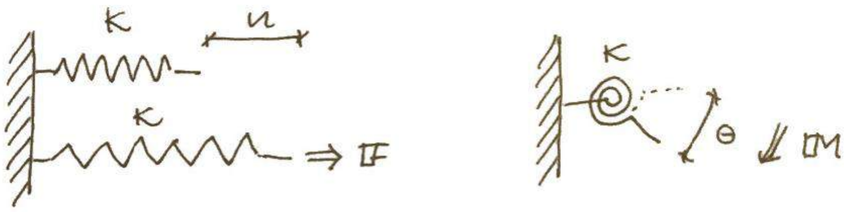


Figure 3.68 Definition of translational stiffness (left) and rotational stiffness (right).

Generally it is the stiffness that governs the inner load paths of a structure. If two springs, a weak and a stiff, are parallel connected a major part of the load will be transferred through the stiffer spring. The compound stiffness is obtained by adding the stiffness of the two springs.

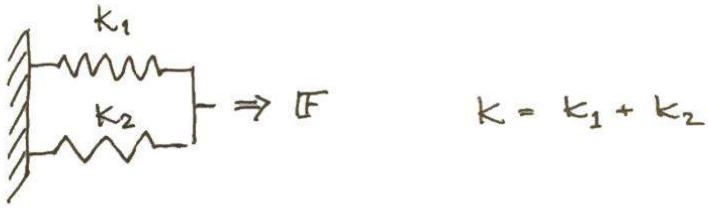


Figure 3.69 Parallel connected springs.

If the springs are attached in series, the effect of the weaker spring will dominate the behaviour since it is the weak link. As can be seen below the compound stiffness is close to the weaker spring stiffness.

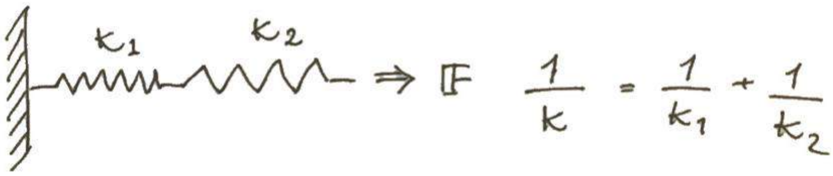


Figure 3.70 Serial connected springs.

The local stiffness of a connection depends primarily on the stiffness of the local material and the local geometry, if the connection consists of wood and glue it will probably act as two springs in series and the weaker spring, the glue, will determine the properties of the joint. The global stiffness is instead dependent on the distribution of the stiffness of the components, like the behaviour of parallel springs. Especially important is the distribution of stiffness of the joints, particularly the rotational stiffness since it has a larger impact on the global stiffness and is harder to control. Since the load takes the stiffest path, a joint which has been damaged, should not be strengthen and made stiffer but rather be made weaker while another path is made stiffer to transfer more load. (Olsson,P.,2003)

4. Design process

To find relevant and useful ways of modelling and optimizing folded plate structures is a central part of this master thesis. Form finding of a plate structure in bending differs and is not as straight forward as the form finding of a shell structure. Whereas in a shell or membrane structure, the optimized shape is when all stresses are pure compression or tension, the optimized shape of a folded plate structure under bending does not have such a clear definition. Further, the material cannot be arbitrarily distributed as in an optimization process of truss beams. Since the whole structure is folded from a large flat sheet with uniform thickness, the only way to vary the stiffness in a section is to make changes in topology, crease pattern and folding angles. When making those changes, generally, the shape of the whole structure is affected, which in turn alters the stress distribution. The goal of the optimization in a cantilevering partition wall/pavilion loaded from self-weight can be chosen as many different things. One way is to define some absolute quantifiable requirements in terms of shape etc. and then run a genetic algorithm like Galapagos (described in chapter 4.1.1), to find the solution, fulfilling the requirements and with say the least maximum deflection or the least internal elastic energy. Another way is to weight quantities related to shape and function together with results from structural evaluation and in this way make a trade-off between geometrical and structural properties of the wall. For the genetic optimization engine Galapagos to be able to compare variable combinations, all measured quantities has to be weighted and combined into one single value. The absolute conditions are in fact a smoothly varying measured quantity, translated into a non-continuous function that can either be say 0 or 1000 and added to the value to be minimized/maximized.

When making a parametric model to be optimized by a tool for genetic optimization, a reasonable amount of parameters has to control the model in a way relevant to the cause. A parametric model in which each node in the flat unfolded topology can move freely in its plane independent of the other nodes and where angles could be controlled independently in the folds could theoretically take any possible form within the folding pattern at hand. Such a model however would require a lot of computer power to run in a genetic optimization algorithm. Therefore it is necessary to parameterize the model in a way as simple as possible but that still allows for the promising alternatives to appear in the optimization process.

The steps that are conducted in the search of our design, and their inherent chronological order are presented in the following image.

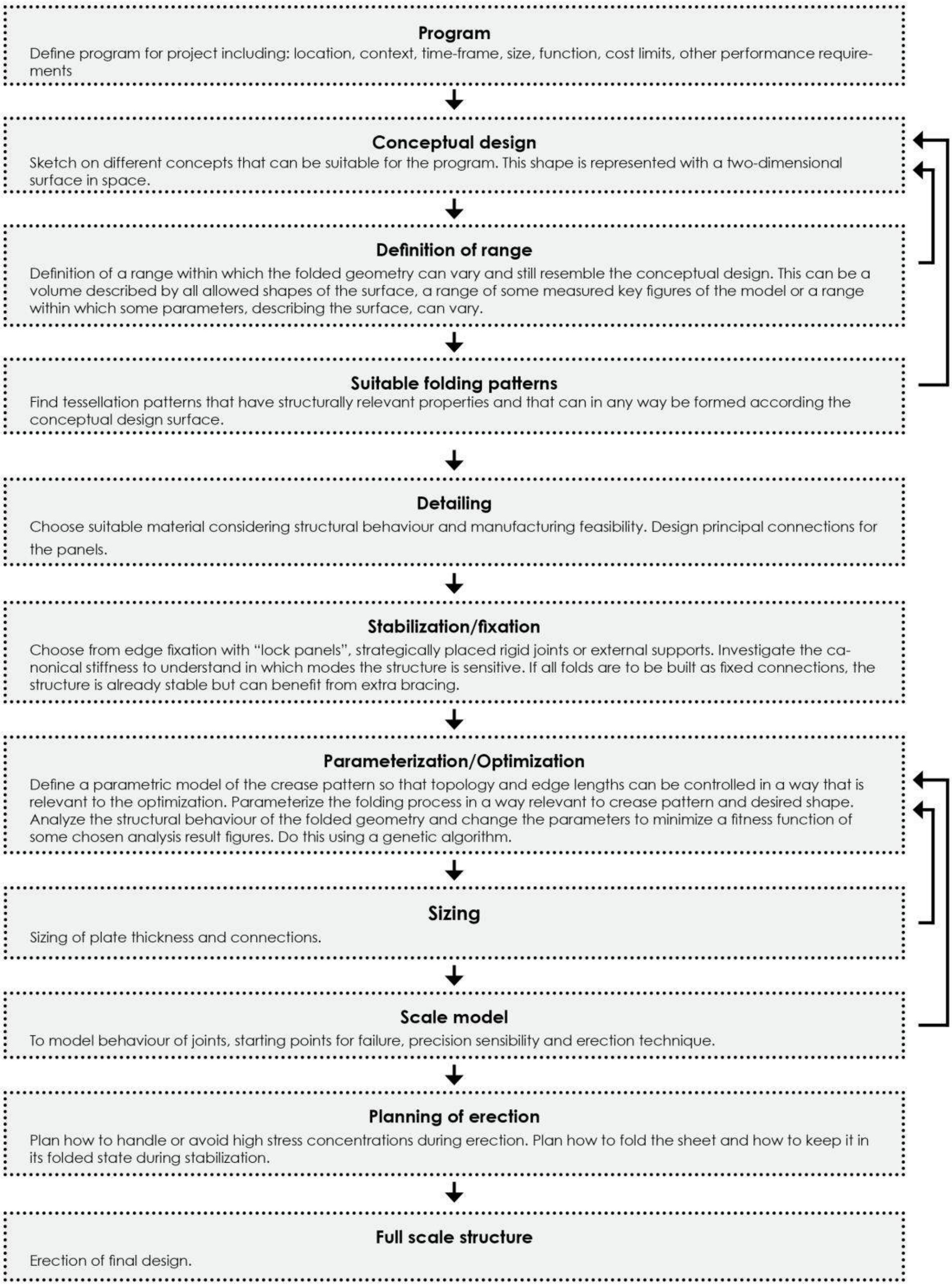


Figure 4.1 Process steps, first version.

The choice of design method is not self-evident. As always, several different more or less well defined design methods could be applied. For instance, the iterative approach could be replaced by a more linear one, or the chronological order of the steps could be turned upside down so that the function of the pavilion would be found after the structural optimization is conducted, which would probably result in a very different design. The analytical, and relatively well-structured and clearly defined method could as well be a more loosely described method based on intuition and craft. For the purpose of this master's thesis, it is facilitative to first decide a function and a shape which optimal crease pattern can be searched for. The analytical approach is also necessary for us to make as well as communicating qualified design decisions.

4.1 Digital Modelling

4.1.1 Software

Rhinoceros, developed by Robert McNeel & Associates, is a NURBS-based 3D CAD software. It contains a large array of tools for creating and editing surface structures. Rhinoceros in itself does not allow one to control the geometry parametrically, why editing complex structures can be very time consuming.

Grasshopper, developed by Robert McNeel & Associates, is a visual programing language. It consists of components with various functions that can be connected to each other to create parametric models in the same way as when programming but without the need of scripting knowledge. With the recent emergence of easy-to-use parametric modelling software, more and more architects and engineers can adapt new ways of thinking about computer-aided design. A parametric geometrical model can be described as a model where the relations between the nodes are logically described by different functions so that the complete model can be manipulated changing only one or more parameters. With such a model, one of many possibilities is for the designer to take the role of a coordinator of different logical systems and algorithms rather than a coordinator of geometry per se. Parametric models are beneficial when working with complex geometries but also when there is a need to model large number of variations of a model for evaluational purposes. Therefore, parametric modelling comes in handy for the optimization processes in this thesis work.

Galapagos is a built in Grasshopper component performing so called genetic algorithms (GA). GA is a search heuristic that uses a kind of trial and error method, inspired by the biological evolution, to generate solutions to optimization problems. The Galapagos component needs as input a genetic representation of the solution domain and a fitness function. The genetic representation can be a set of parameter sliders in grasshopper that controls a model. A fitness function maps values of one or more variables onto a real number, which the solver seeks to minimize or maximize.

Kangaroo, developed by Robert Cervellione, Giulio Piacentino, and Daniel Piker, is an add-on for Grasshopper that uses particle attraction to simulate a wide range of physical behaviour of a model. Particles connected with simple damped springs are combined to approximate non-rigid structures. In Kangaroo it is possible to interact with the model as the simulation is running. It is used to simulate the mechanism behaviour of a folded sheet, to manipulate the degree of folding in different parts of a model and to get a response realistic response from the rest of the model without having to describe relations between all folding angles mathematically. With this software a model can be shaped from a folded plate mechanism with flexible edge joints in a way similar to shaping a paper model by hand.

Karamba, developed by Clemens Preisinger, is another add-on for Grasshopper, which is used to make FE-simulations. Karamba is simpler than standard FE-modelling software such as Abaqus or Ansys, it can for example only simulate linear elastic behaviour of a model. However, since it is a part of Grasshopper, it allows an FE-analysis to be a part of an optimization algorithm conducted within this environment, which will be very useful in this master thesis.

Abaqus, by Dassault Systèmes, is a standard finite element simulation software from *Dassault Systems*. Geometries can be imported from other CAD software or created directly in Abacus. In this master thesis work, Abacus is used when a more precise visualization of FE-analysis results are needed and to verify results obtained with Karamba.

4.1.2 Origami modelling methods

There are several ways to establish parametric models of folded structures and there exists a few examples of origami modelling software. One example is *Origamizer* developed by *Tomohiro Tachi*. In Origamizer it is possible to chose and manipulate crease patterns, fold the patterns and play around with the folded geometry in different ways. To be able to combine the digital origami models with other software, this master thesis work will only focus on origami models created in more general CAD environment such as Rhinoceros and Grasshopper. Below are some principals for origami modeling in Grasshopper.

Method 1 - Explicit node trajectories

To make a model using this method, a topology of nodes and lines is established. Describing the movement in space of each node as explicit functions of one or more variables or parameters simulates “Folding” of the model. A model built with this method, when done correctly, will behave as a weightless rigid-body model and can be folded without any deformation of the surfaces. Even though the functions can be very complicated, the model definition can be relatively short in terms of steps or grasshopper components, and since the computations are linear and does not include any iterations the model is “lightweight” and can be folded smoothly without much computer power.

Method 2 - Relative surface translations

Instead of having explicit functions describing trajectories of each node, a simpler but still purely geometrical/mathematical way of modelling is to define rotation and movement of a predefined surface in relation to its neighbours. This can be done in Grasshopper by having the translations of a surface being dependent of geometrical measurements of translations of another surface, allowing a foldable plate structure to be described mathematically but implicitly using translation and measurement components in Grasshopper. This method might be more straightforward than the previous one but it can still be complicated to model irregular crease patterns.

Method 3 - Finite Element, Karamba

Folding of a plate structure can be simulated using a finite element model. Assembling quasi endlessly stiff plates with connections of moment free hinges can do this. Some of the boundary nodes are then assigned with prescribed displacements giving the deformed model its folded shape. This method has a large potential, but to use it in an optimization process for practical reasons the FE-model has to be established in the Grasshopper environment. The only existing FE-software available as a Grasshopper-add on is Karamba, which at the moment does not allow for shell elements to be connected with true moment free hinges. It is possible to approximate hinges by connecting two “thick” shell elements with stripes of very thin and very stiff shell elements; those elements are easily bent but axially very stiff.

Method 4 - Particle attraction, Kangaroo

The Grasshopper add-on Kangaroo is using particles and their mutual attraction to simulate physical behaviour of different kinds in a CAD model. Kangaroo has components that can be used to model hinges between triangles, with a “resting angle” and ”strength”. There is even a component called “Origami” that is a cluster of components such as “Hinge”, “Planarize quads” and “Spring Force” to imitate the behaviour of a folded sheet material. The Origami component needs as input: a mesh with all creases and the boundary of the whole surface, the valley and mountain creases, the rest angle of valley and mountain folds respectively and the hinge strength which could also be described as the degree of folding. This component in its default setup works fine for models consisting of a repetition of unaltered modules. But since it has only one input for rest angles and hinge strength, conflicts occur when trying to fold more asymmetric crease patterns resulting in large non-negligible deformations of the plates or, when increasing plate stiffness, some adjacent plates remain in the same plane i.e. some creases are not folded. To deal with this, the Origami component can be modified so that the hinges can be given unique strength and rest angles that are adjusted not to be contradictory. Kangaroo allows nodes of a model to be given different degrees of freedom; they can be free or fixed to a point, line or a plane of choice. However there is no such thing as prescribed displacement, the folding of a model can only be coordinated by controlling the spring forces, strengths or rest lengths.

4.1.3 Comparison of methods

Description
Methods 1 and 2 are suitable for simple repetitive patterns, folded in uniform way. As soon as the intricacy increases, a mathematical description of a folded model is no longer a feasible alternative. Methods 3 and 4 are easier to work with since an arbitrary CAD model can be converted into a structural model. Displacements or forces of choice can then be applied and the physics engine handles the simulation.

Formability
Since the mathematically derived models are defined, in method 1 with injective functions and in method 2 by a sequence of components where the data flow is linear and unidirectional (the position of the first surface(s) determines the position of the following surface(s) and so on), this means that the folding process is also injective and is induced by the variables of the functions or the translation of the first surface in the sequence. Shaping of the modelled structure with a given crease pattern by variations in degree of folding over the model has to be done, when it comes to method 1 and 2, it can be done by modifications in the functions themselves or in the algorithm describing the relative translations. This can be compared to a model simulating physical behaviour where displacements and forces can be introduced anywhere in the model to shape it. This resembles the way one would interact with and shape a real physical model such as a paper model.

Precision
The mathematically described models are very precise. Their surfaces do not change in size or shape, i.e. all surfaces in the unfolded state are congruent to their corresponding surfaces in a folded state. The FE-method is good to approximate the stress distribution and deformation of an already partially folded and braced structure under load. However, when it comes to modelling the folding itself, Karamba does not seem to handle the large deformations induced by the prescribed displacements associated with this method very well. Large element deformations occurs which makes the deformed structure useless as approximation of a rigid origami mechanism. Kangaroo is using dynamic relaxation, an iterative process computing forces between nodes followed by a pseudo-dynamic process in time resulting in a state of equilibrium. The plates described by those nodes will therefore have to deform elastically for the simulation to work. Now, at equilibrium the plates may be stress free again but due to a limited number of iterations or necessary built in stresses, this is not always the case. The deformations of plates can be limited to negligible proportions.

4.1.4 Chosen method

Due to the complexity of the mathematically derived models for non-symmetric patterns and the problem with Karamba not being able to handle large deformations, the conclusion is that Kangaroo will be the most useful tool for modelling and shaping folded structures.

4.2 First design iteration

The first design iteration was made mainly to verify the process steps to see if it was possible to work as planned and to find problems in the different parts of the design process that need to be solved. The design of the structure itself in this first iteration was thus not so important and the focus was rather on how to control the shape, how to model the structure in a way that corresponds to the real behaviour and to find relevant ways of how to optimize the structure based on shape, stability and stiffness. Therefore sizing, planning of erection and full scale structure was not conducted in this iteration. The first iteration was based on the Miura-ori pattern since it is one of the three patterns that has been found structurally relevant and is the pattern that sets the largest requirements on the joint solutions since it folds into itself making the impact from the material thickness very prominent.

4.2.1 Program

Where:
A-workshop.

What:
A balancing vertically curved wall with a certain minimum overhang.

Why:
The purpose is to test the design method.

Limitations:
The structure should be folded from a flat surface of connected panels to facilitate the assembly. The structure will not be built in large scale, only as cardboard model.

4.2.2 Conceptual design

We wanted to create a wall like structure that subdivides the room and creates a semi-enclosed space. For the first design iteration we also preferred a simple shape that was easy to investigate but still fulfilled these requirements. From this we came up with a conceptual design as single-curved wall with an overhang.

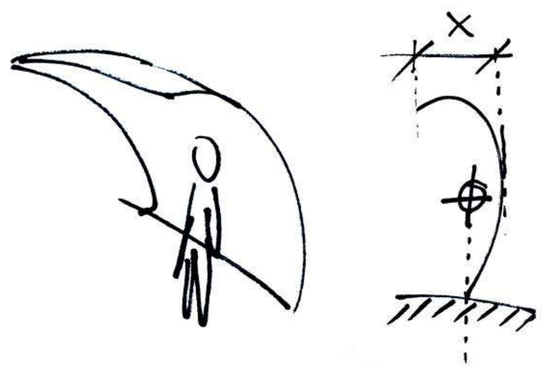


Figure 4.2 Conceptual design.

4.2.3 Definition of range

The range within which the structure was allowed to vary was determined by the parameters controlling the digital model combined with certain requirements expressed in the fitness function. The parameterization allowed the wall to bend out of its plane or to be flat. The functional requirements expressed in the fitness function were two, the mass centre should be above the support area and the total depth of the structure should be at least 1.5 m.

4.2.4 Suitable folding patterns

The Miura-ori or *Herringbone* pattern can easily be single curved in its longitudinal direction with the right adjustments in the crease pattern. It was therefore a suitable pattern for this application.

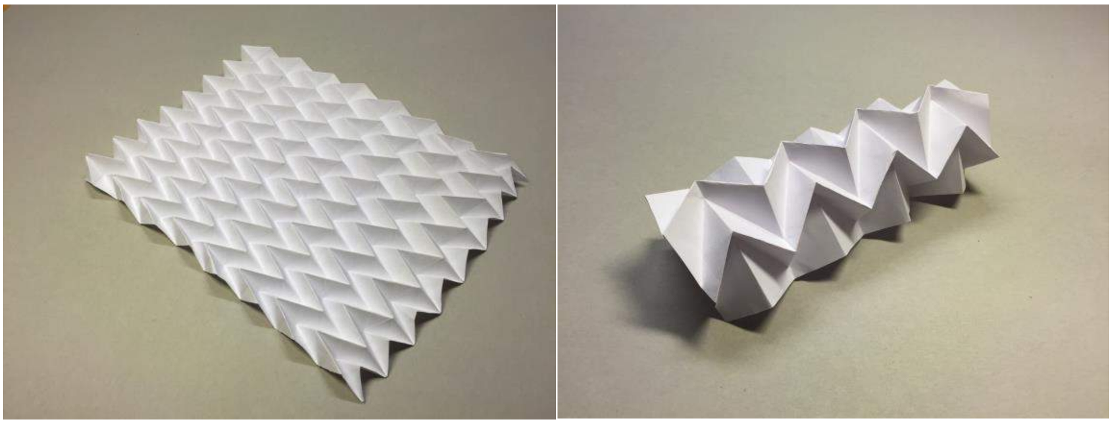


Figure 4.3 Miura-ori, flat and curved

4.2.5 Detailing

This first model was built in two versions, one with two layers of 1.5mm cardboard and one with one layer of 3mm cardboard. The cardboard was used because it fulfils the structural requirements in this scale and moreover it is cheap and easily cut in a CNC laser cutter. The reason for doing two versions was that two different connection principles were used. In the two-layer model, a fabric sheet was sandwiched in between the panels and worked as a hinge along the fold lines. In the one-layer model sticky tape was used in the connections in a way that vary between valley and mountain folds.

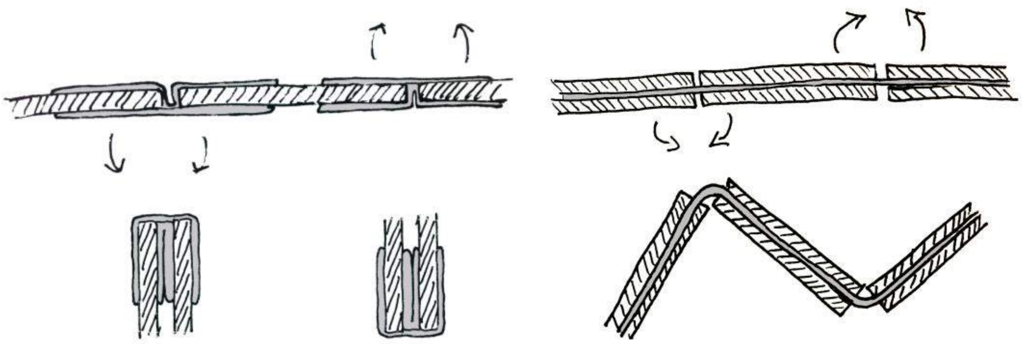


Figure 4.4 Tape connection with axis shift (left) and sandwiched fabric connection (right).

4.2.6 Stabilization/ fixation

The Miura-ori pattern has only one internal degree of freedom and is therefore theoretically stable (no-mechanism) if one fold is fixed. The structure of course needs a minimum amount of constraints to prevent rigid body movements.

In this model all nodes that were in contact with the ground were pinned to the foundation. Further bracing was also added to the structure in a second phase. This bracing consisted of extra panels, *locking panels*, around the free edges of the folded structure. Those panels had to be mounted on the already folded model and was made to increase stiffness but they also affected the aesthetic expression of the wall.

Even though the wall is stable in theory it has deformation modes for which it is weaker. More about this can be read under chapter 3.8 Canonical stiffness.

Karamba was used to find some eigenmodes for the wall geometry. In this analysis all folds where modeled as fixed and all nodes along the bottom boundary was pinned. The first eigenmode (left figure) shows a twisting movement that corresponds to a slight twist of the quad faces that could easily be re recreated in the physical model. The third eigenmode (right figure) which is a combination of twisting and longitudinal bending requires more energy and was more difficult to recreate in reality.

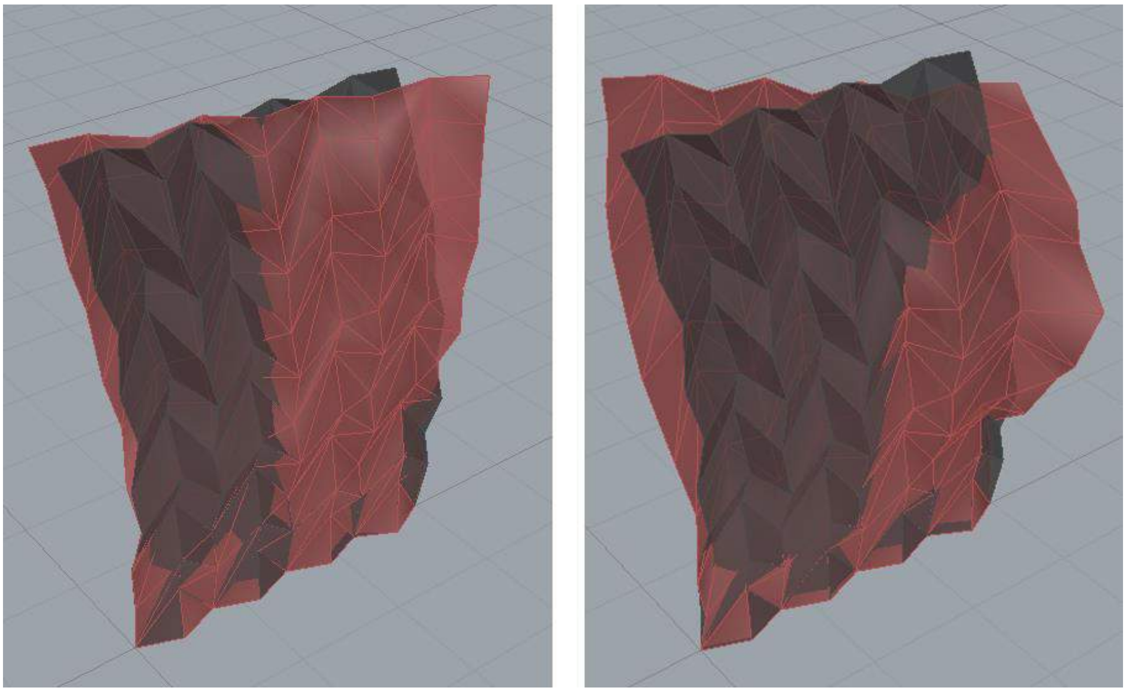


Figure 4.5 Eigen mode 1 (left) and 3 (right) in red. Original wall reference in grey.

4.2.7 Parametrization/Optimization

The model was constructed by modifying the Origami component in Kangaroo, as described in previous chapter. The base pattern used was the Herringbone pattern and the shape was varied based on the amplitude of the zigzag lines. The amplitude was constant through every zigzag line but could differ between rows.

The amplitude of the zigzag line was controlled by a curve based on three points which y-coordinates could vary. This curve was divided into segments for the number of rows that the pattern was going to have in the z-direction, and the shape of the curve controlled how much the base lines in z-direction (red marks) would be offset (blue marks).

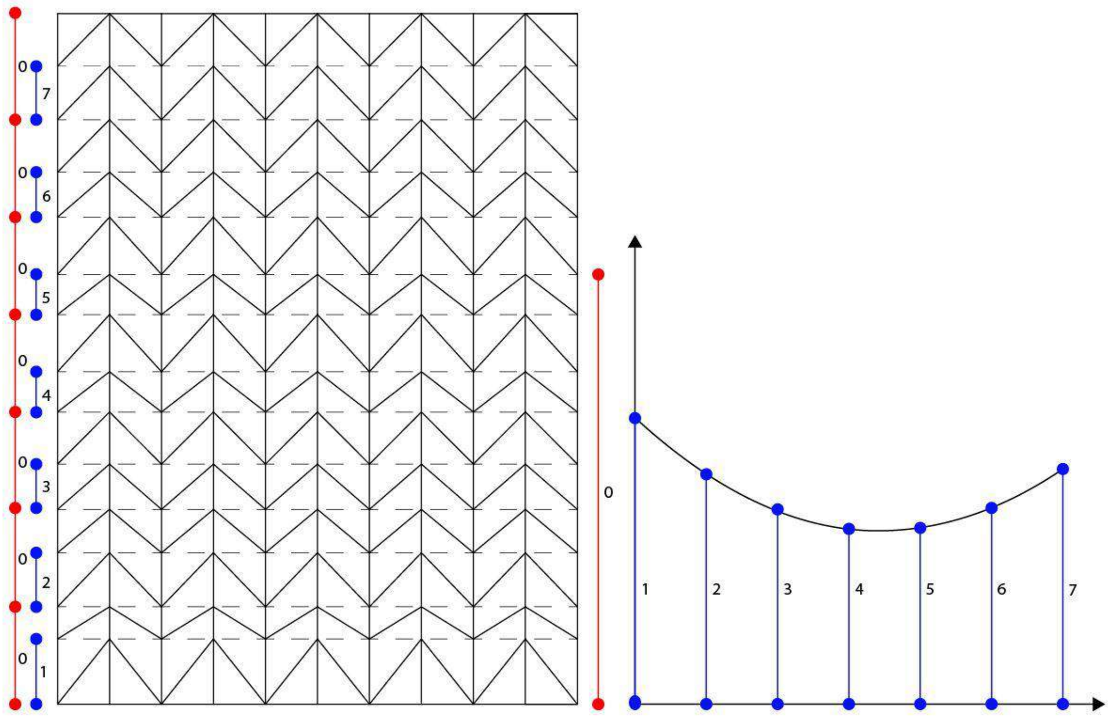


Figure 4.6 Parameterization of crease pattern.

To make the pattern curve, every other zigzag line must have lower amplitude, as seen in the following picture.

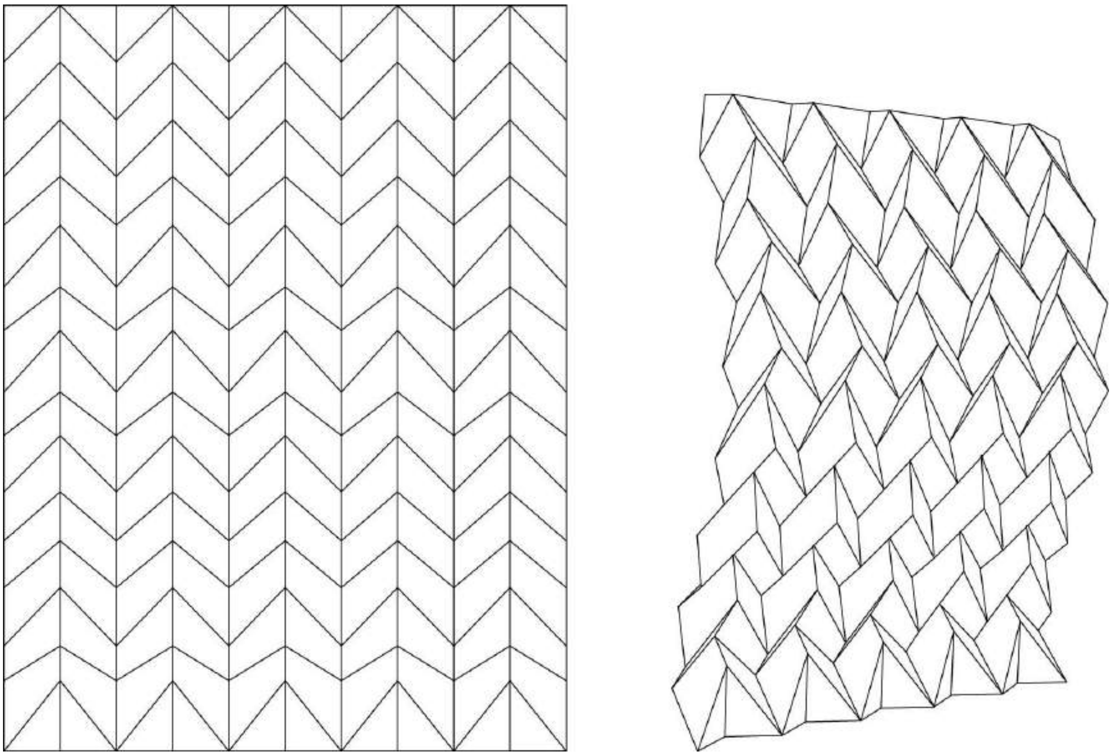


Figure 4.7 Crease pattern and corresponding shape when folded.

The optimization was based on certain shape requirements; how much the structure should cantilever, that the mass centre was positioned over the supports and minimum internal elastic energy in the structure. It was only three parameters that determined the shape, the three y-values for the curve points. The amount of panels and the width of the panels were pre-set and were not evaluated during the optimization.

The genetic evaluation algorithm can only optimize one quantity and if there is more than one value that should be minimized those have to be weighted and added together according to a fitness function.

In this model the internal energy should be minimized, at the same time the mass centre should be above the support area and the total depth of the structure should be at least 1.5 m. These last two requirements are binary but had to be included in the value that should be minimized. This was done by adding a large value to the internal energy in cases when the mass centre was outside the surface area or if the total depth was smaller than 1.5 m. The addition of this extra value blowed the differences in internal energy out of proportion and thus made the corresponding combination of in-data to end up in the bottom of the list of evaluated combinations. The top combinations was all balanced and had a depth of at least 1.5m. To evaluate the internal energy of the structure, the FEM plug-in Karamba was used and the hinges were modelled as thin stripes with a high Young's modulus. The material used in this model was steel since it was the default material and the material did not matter in the scale models. The connections were modelled as hinges since it would be easier to construct the model if the panels could be joined when lying flat and the structure then could be folded into its final shape and locked.

4.2.8 Sizing

No quantitative sizing was made this iteration since this would be redundant in for a model this size only loaded by self-weight.

4.2.9 Scale model

The crease pattern was numbered and exported from Rhinoceros as a dwg-file to be compatible with the laser cutter. The panels were then cut from cardboard in the laser cutter and the pieces were put together in the right place according to the drawing. Two different solutions were tried for the connections, first we used a double set of panels that we glued to both sides of a fabric, the fabric then worked as hinges and the model could be glued together lying flat and then folded into its final shape. The panels on one side were glued to the fabric with a small distance of ca 1 mm between each other and the setup were put under pressure while drying. After drying panels were glued to the backside of the fabric in the same position as the panels at the front side. When the glue was dry the extending fabric were cut and the structure where folded into its final shape and glued to a foundation foam board plate.

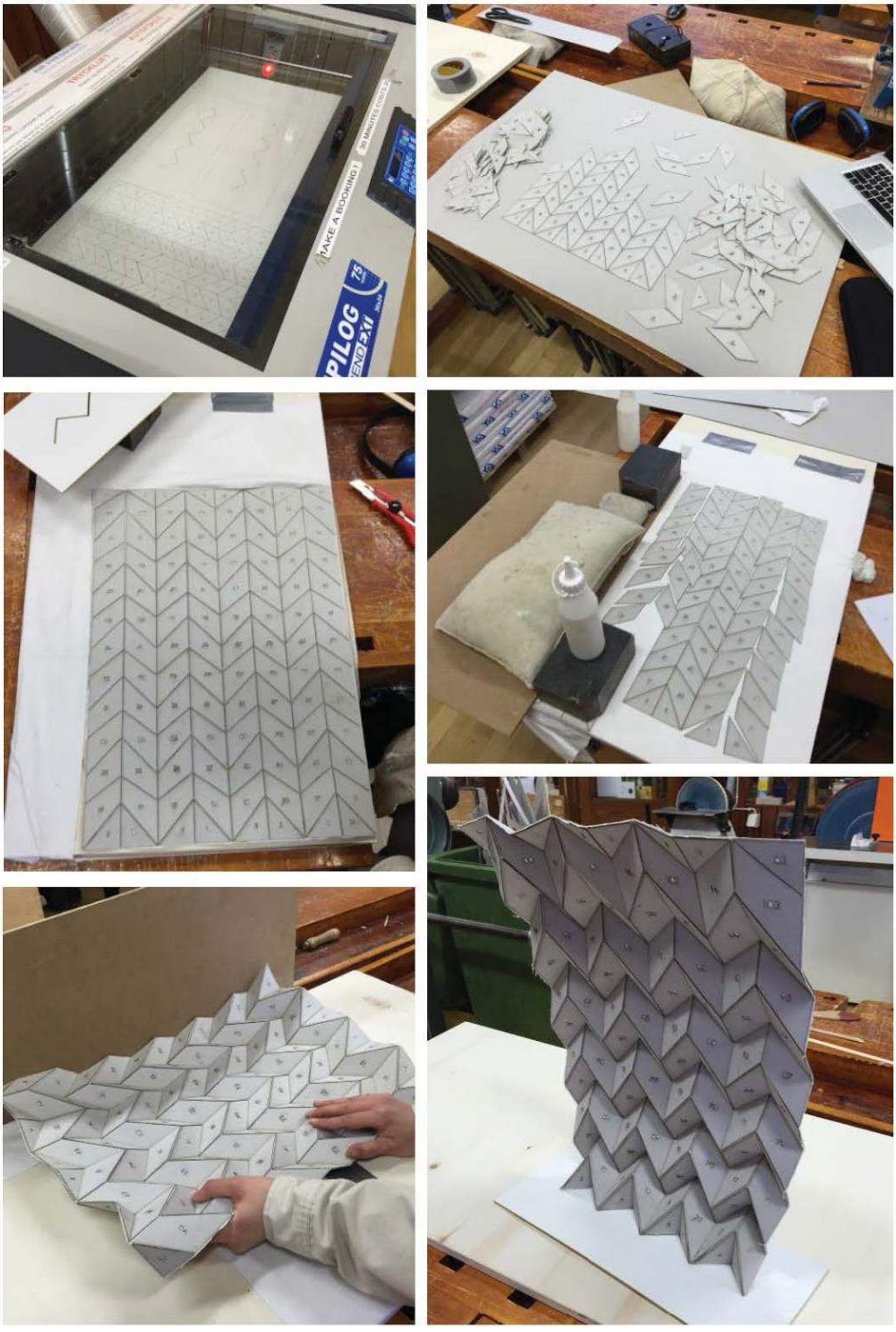


Figure 4.8 *Two-layered model with sandwiched fabric connections.*

The second model was joined by sticky tape from two sides in the same way as the "binder principle" meaning that the plates were joined by a sticky tape from one side lying flat and then joined from the other side by a sticky tape that was folded in between the edges of the plates. The structure was then mounted on the baseplates and fixed by the lock panels.

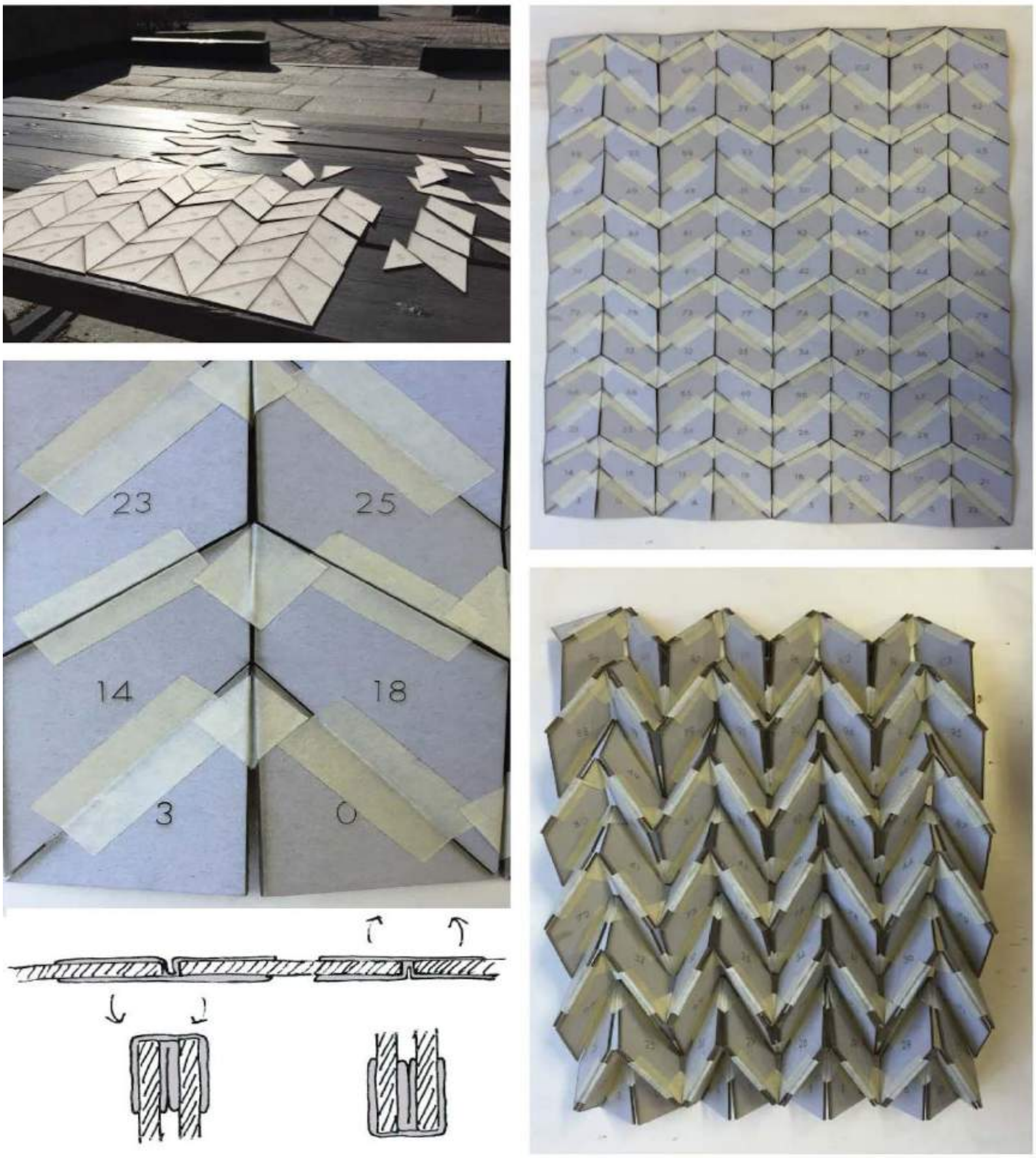


Figure 4.9 One-layer model with sticky tape joints.

4.2.10 Planning of erection

In the first iteration the full scale structure was not made so planning of erection was not conducted.

4.2.11 Full scale structure

The full scale structure was not made in this iteration.

4.2.12 Conclusion

The result did not turn out as planned due to the joint solutions, in the first model with textile connections the plates were mounted too close together which made the structure very hard to fold into shape and the final lock plates were impossible to mount since the stresses in the structure got too large and made the cardboard plates break. In the second model, there were problems since the joints were placed at the side of the plate, and on different sides depending on which direction they were going to be folded in, which made the impact from the thickness of the plates very large and it pulled the sticky tape away creating large gaps between the plates in the vertical connections resulting in a very different and a lot more flexible shape due to the lack of ability to transport shear forces between the plates.

In the first model with the textile connections it could be seen that the deformation modes from the canonical stiffness analysis coincided with the modes that we could find in the physical model, the first modes was the free top corners moving, the model was also relatively sensitive to tension/compression upwards, compression downwards on one side and torsion of the whole structure.

When fastening the lock plates the structure got stiffer and since the top corners were fixed the deformation modes regarding these disappeared. When mounting the lock plates in the second model, it got a bit stiffer and did not deform as much as before but the model was still very flexible and did not work as planned.

As can be seen in the pictures of the first model the shape work as planned creating a semi enclosed space even if the effects could be more prominent. The light and shadow also create an interesting effect that enhances the fold pattern.

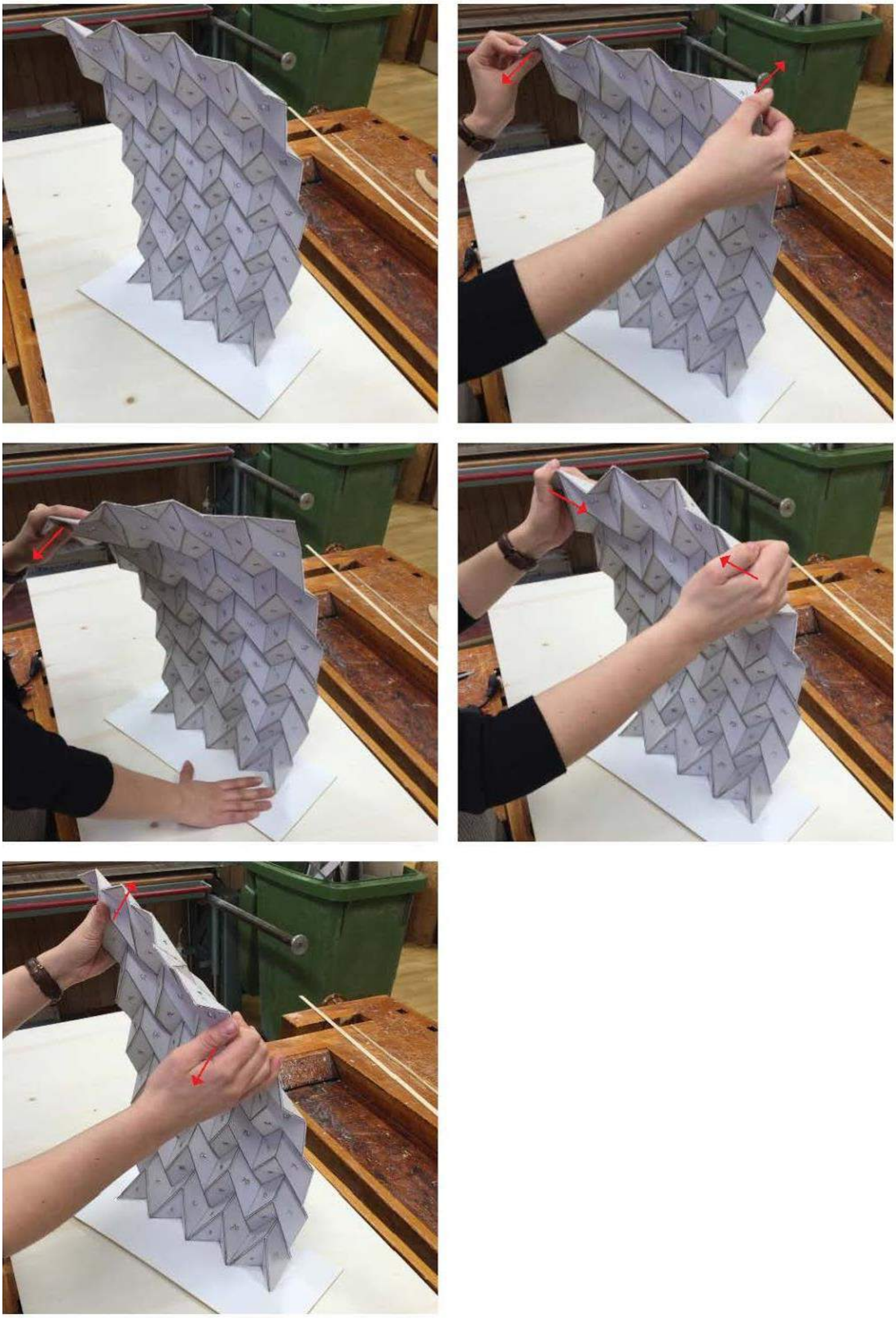


Figure 4.10 Qualitative evaluation of structural behaviour, no locking panels.

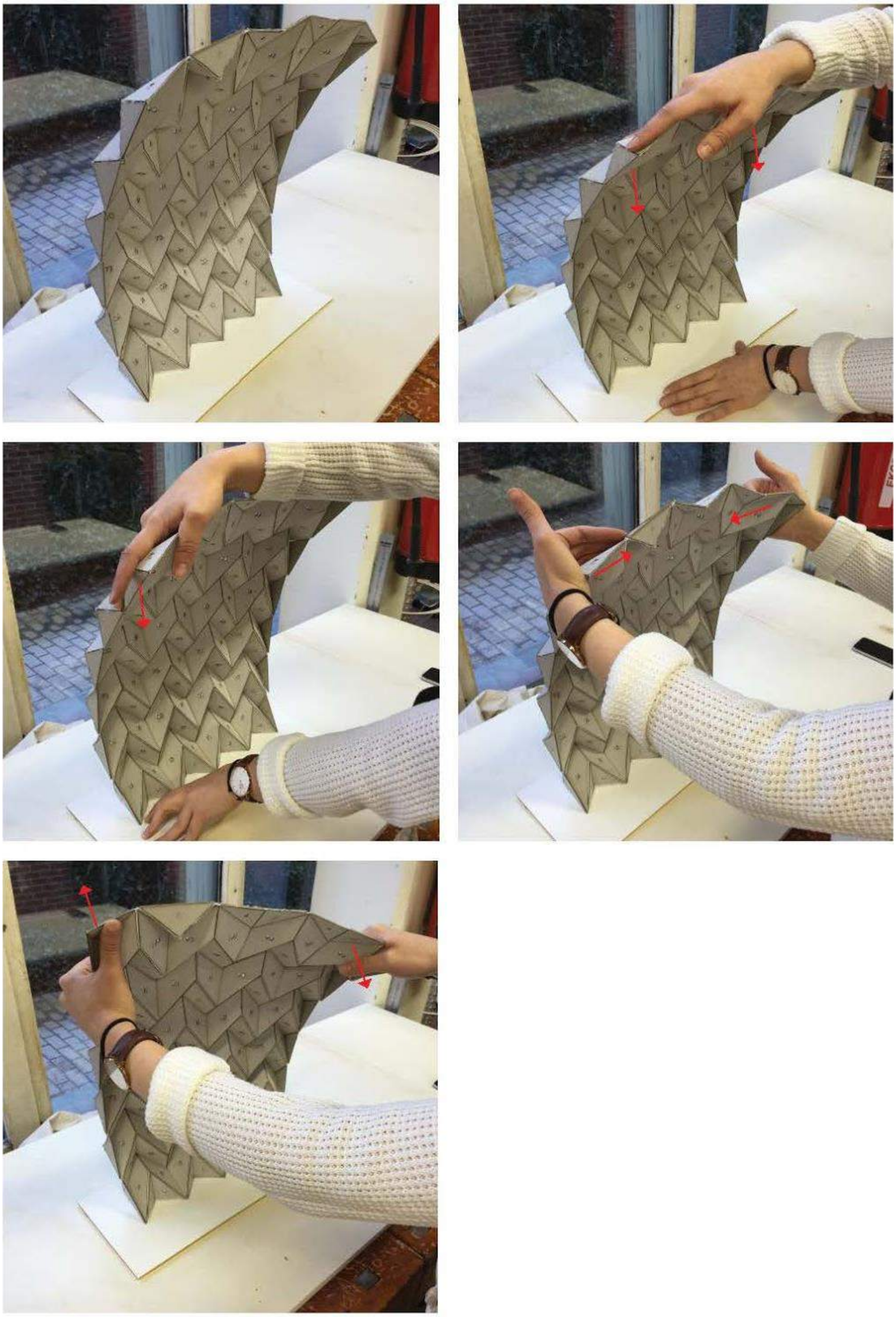


Figure 4.11 Qualitative evaluation of structural behaviour, with locking panels.

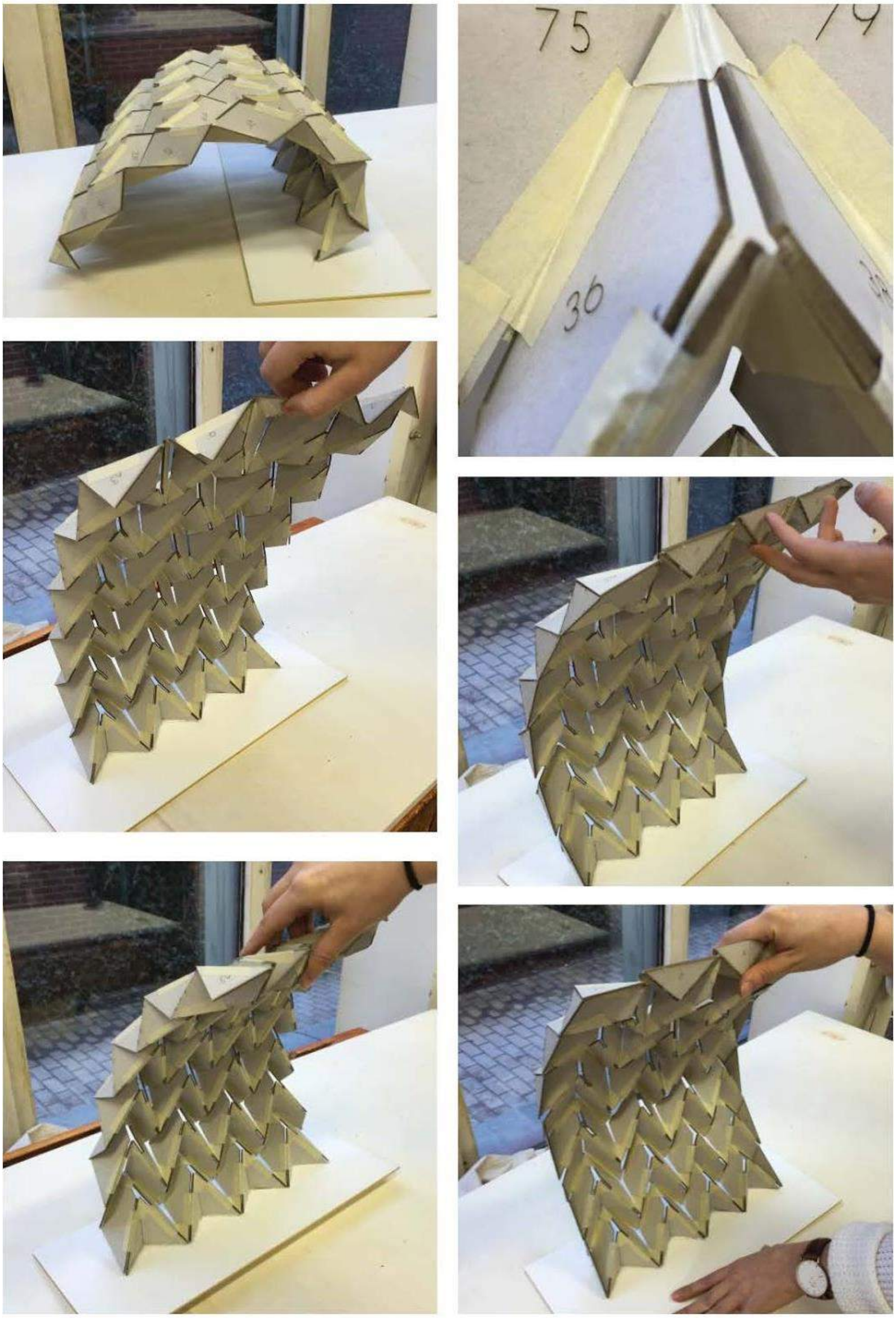


Figure 4.12 Qualitative evaluation of structural behaviour.

4.2.13 Developments

After the assembling and the intuitive structural evaluation of the two scale models, some conclusions could be drawn regarding the connections. It was clear that the imperfections caused by having the rotational axes in the level of the top or bottom surface of the plates had a large influence on the global curvature of the structure. It also showed that a fabric provided a good moment free hinge and structural capacity in tension, but of course no capacity in compression in itself. Since a fabric need to be stretched to have any capacity at all, and since a distance need to be kept between the plates when lying flat on the ground for the folding to be able to take place, some axial and transversal play can take place which, when added together, cause visible changes to the whole shape.

By tying a rod with a circular cross section and the same diameter as the plate thickness to both adjacent plates respectively, the two plates can rotate around an axis that is in the same level for both valley and mountain folds. A first attempt was made with a 0.35mm nylon line and pre drilled holes. The line was continuous through all the holes of one side and fastened at the plate corners, this made is difficult to control the pretension of the line which resulted in that the rod could, when forced to, jump out of the plates mid-plane.

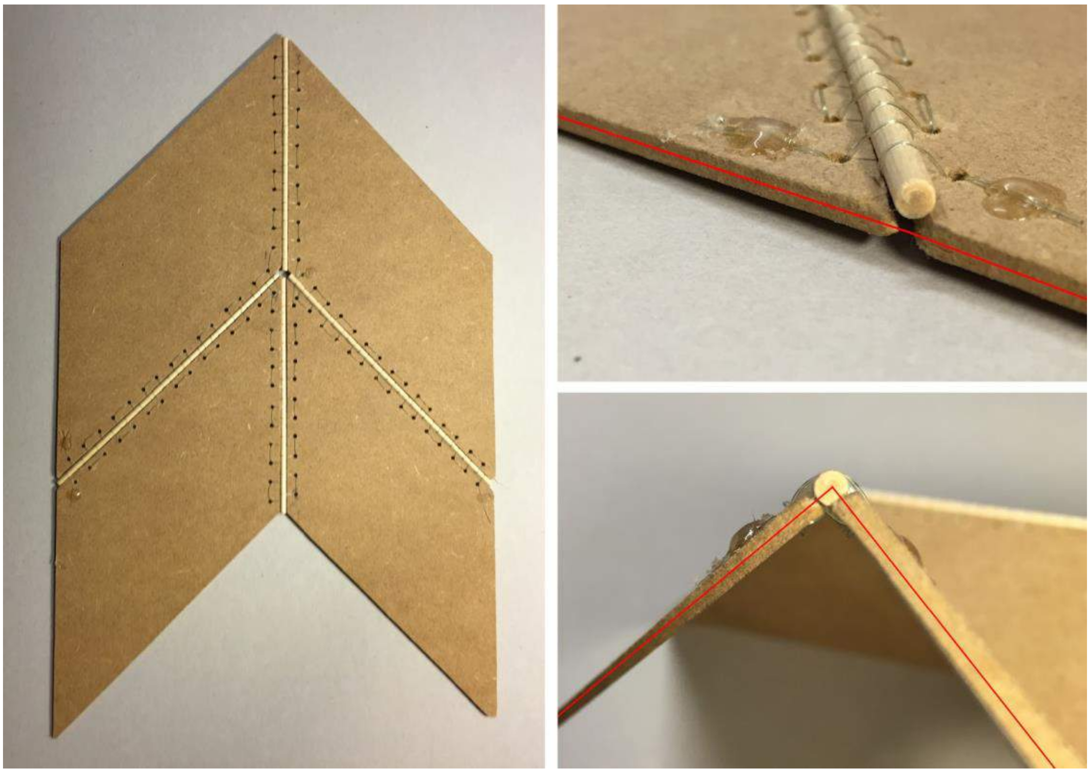


Figure 4.13 MDF plates connected with sticks and fishing line.

Screw eyes driven into the edges of the plates, penetrated with a wooden rod, can provide a hinge with less play and friction. Holes need to be predrilled in the plane of the plates. This cannot be done in the kind of CNC mill that we use, therefore this method would require a lot of manual work. Small diameter screw eyes also showed hard to find.

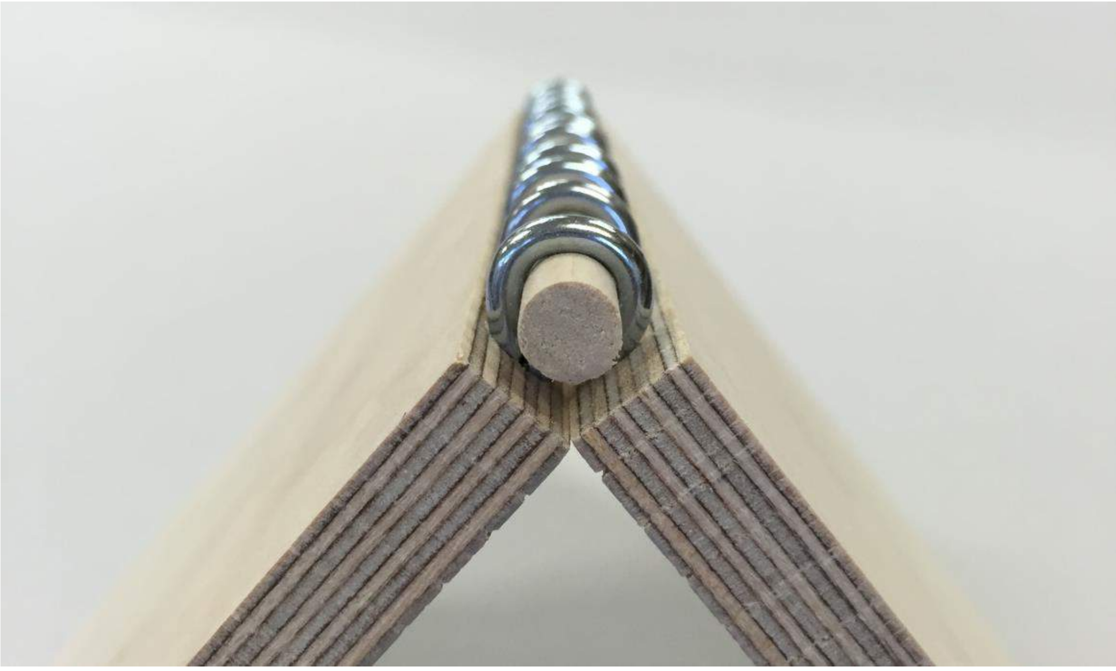


Figure 4.14   Screw eye connection.

Hinges submerged so that the axis of rotation ends up in the structures mid-plane also provides a connection with low play and friction. Fabrication however was labour intensive and hinges are expensive.

Figure 4.15   Metal hinge connection.

4.3 Second design iteration

To the second design iteration the process steps were modified. We realized in the first iteration that to design a shape and then find a folding pattern that suits it is not a very good way to go. Instead we discovered that these steps are intertwined and that the best result is obtained by finding a conceptual design and then find a fold pattern that has the properties to be shaped into the same kind of shape as desired. Then investigating the possibilities of this pattern, how it can be varied and what shapes it generates, and take the possibilities of the pattern back to the conceptual design.

In this iteration we used the scale model as a way to explore the stability of the pattern and there was thus no point in parametrization and sizing of the model. The stability was investigated in paper models before building the scale model but we did not know how prominent the stability issues found in the paper models would be in the scale model.

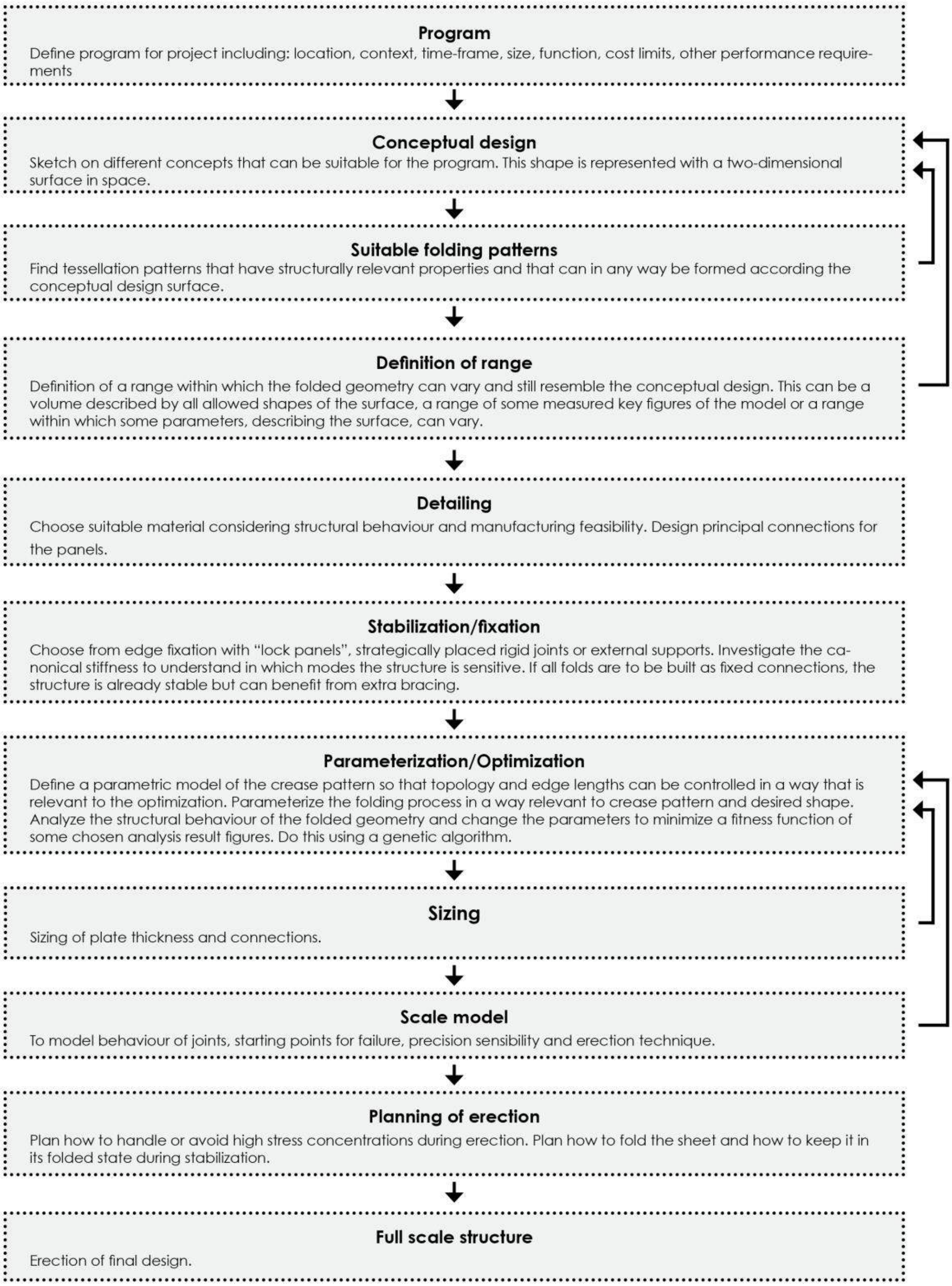


Figure 4.16 Process steps, second version.

4.3.1 Program

Where:
Betonghallen (The concrete hall), Chalmers

What:
A wall structure that creates a subspace in the large room.

Why:
Example of an application of the optimization and design process for folded plate structures proposed in this master thesis.

Limitations:
The structure need to be foldable from a flat sheet to make the production easier since the plates then can be mounted together when lying flat. Another reason for the flat foldability limitation is to minimize waste during production. The shape and size of the plates should be adapted to fit 1200mm x 2400mm plywood sheets in a good way so that as little material as possible is wasted during production.

4.3.2 Conceptual design

To come up with ideas for the wall, post-it notes were shaped. The semi-enclosing shapes and the non-symmetrical shapes had some interesting aspects that we wanted to go further with.

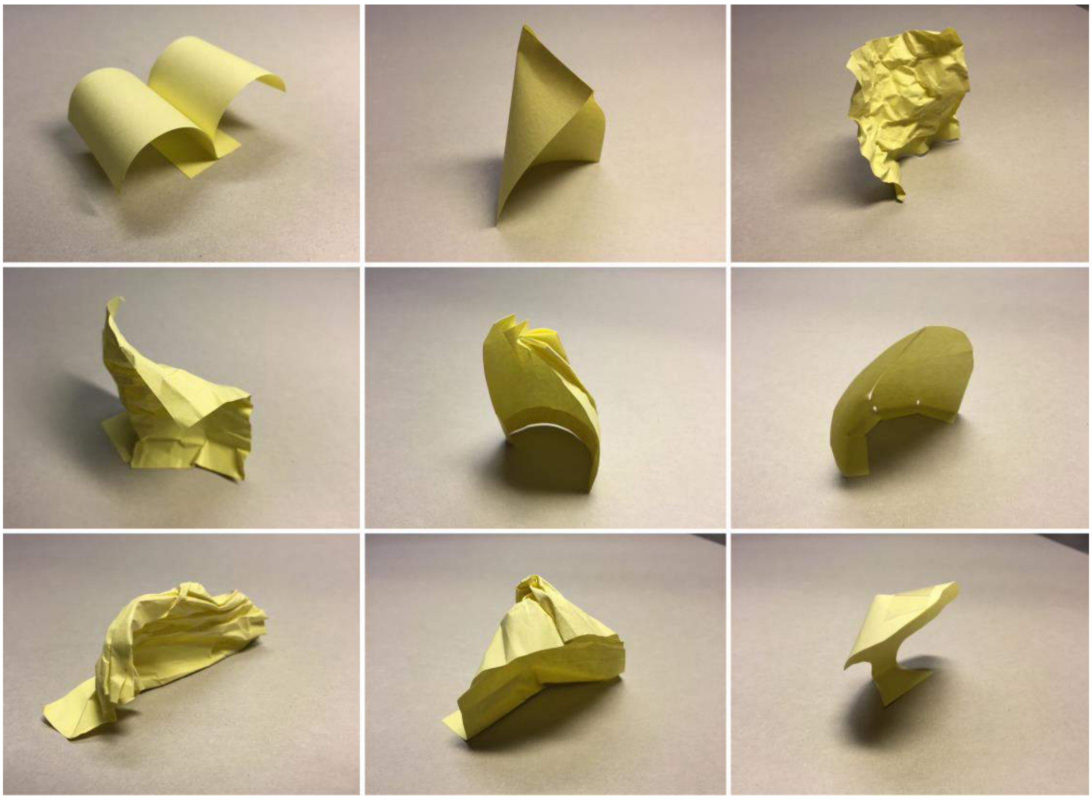


Figure 4.17 Post-it concept brainstorming.

After evaluating the different shapes we decided to continue with the S-shaped wall, since this design provides semi-enclosed spaces on both sides and does not have a front side and a back side in the same way as the other shapes.

We continued to work with the S-shaped curve and begun to investigate how the shape could be altered to create different impressions of the two spaces. This lead to varying section profiles with walls leaning inwards and outwards making an onion-shaped room and a bell-shaped room.

Figure 4.18 Conceptual design sketches.

When further investigating the properties of the diamond pattern we discovered that it is impossible to achieve a double curved wall with this pattern if it should be foldable from a flat sheet without cutting it into several branches. Therefore we instead decided to make the structure vary conically, still having one room with walls leaning outwards and one room with walls leaning inwards.

4.3.3 Suitable folding patterns

The diagonal pattern has shown to be suitable for creating walls that are s-shaped in plan (xy-section). As were found in the investigation of the diagonal pattern, deformation in x- and y direction is required for the structure to deform in z-direction. The structure is therefore stiff against vertical deformation if it is locked in these directions. With some manipulation in the crease pattern cone shapes can be achieved, something that could be spatially interesting in full scale.

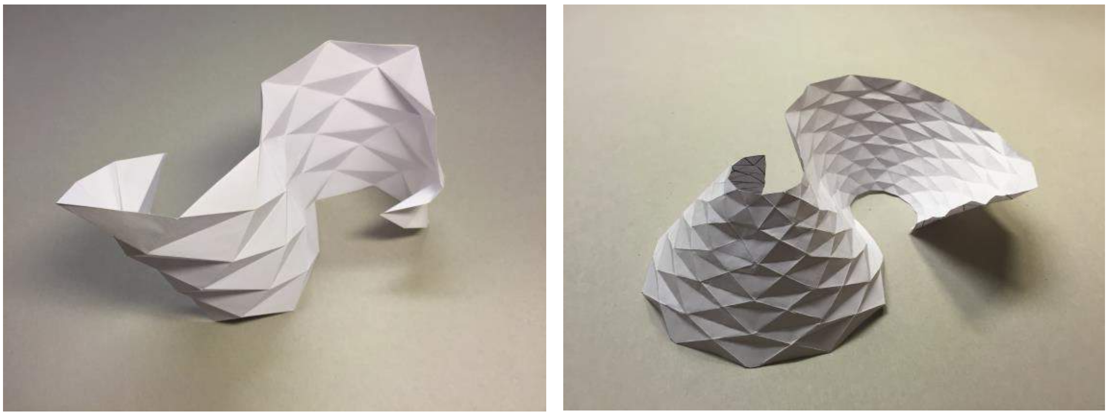


Figure 4.19 Enclosing wall from diagonal pattern.

4.3.4 Definition of range

This step was not fully conducted in this iteration since no parametrisation was done and was mainly included in connection to the conceptual design.

4.3.5 Detailing

The previous attempts to use a wooden rod in the connection between panels was kinematically successful but the nylon line method used to tie the parts together was time consuming and axial elongation (stretching) in the nylon fishing line resulted large global deformations. A development of the method was made including 2.5 mm cable ties, a thicker rod and 6 mm poplar plywood. The plates had pre-cut holes for the cable ties so that plate and rod could abut. The result is a connection with very little elongation under tensional forces, and which allow the plates to rotate between (a little more than) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ radians. With a larger diameter of the rod the possible rotation angle would increase and vice versa. When the plates are folded or rotated around the rod and simultaneously squeezing it with a certain force, it can happen that the rod, like in the previous model, moves a bit out of its mid plane as the cable ties stretch.

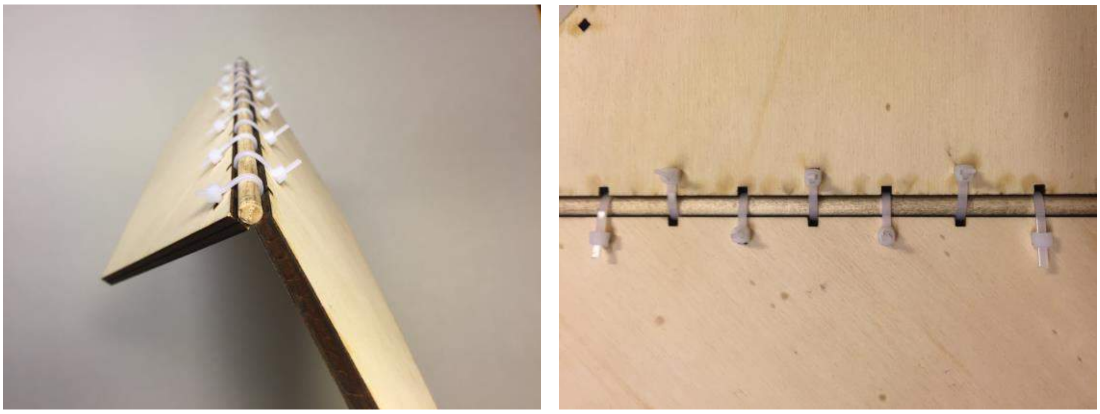


Figure 4.20 Poplar plywood (6mm) with cable tie and rod connection.

4.3.6 Stabilization/ fixation

The stabilization and fixation were partly studied in small paper models before building the scale model, but in this iteration the stabilisation and fixation were mainly investigated in the scale model. This was done as a step in exploring how the instability issues found in paper models correlate to the larger scale models and as a way to understand the properties of the diamond pattern.

4.3.7 Parametrization/Optimization

Since this iteration mostly was an investigation of stability no parametrization or optimisation was done in this step.

4.3.8 Sizing

No sizing was done in this iteration since the material used in the scale model was the same material and the same thickness as planned for a large scale structure and thus over dimensioned for this scale.

4.3.9 Scale model

The scale model was made from poplar plywood with connections of wooden rods and cable ties. From the Rhinoceros model the crease pattern were exported to AutoCad where numbers and holes for the cable ties were drawn onto the plates. The plates were then manually fit onto the 800 x 500 mm plywood plates. The parts where cut out in the laser cutter and sorted by number. The diamonds where then mounted together and assembled in to a structure while hanging in fishing lines since we needed to be able to reach both sides of the plates when fastening the cable ties.

First we tried to attach all the diamonds in each row into strips that then were assembled in to the full pattern. It was though hard to fasten the cable ties when assembling whole strips and therefore the diamonds where instead directly mounted onto the structure. We also discovered that it was easier to assemble the pattern when it was lying down on a table then when hanging in fishing lines.

When all the plates were assembled the structure was folded into its shape and lock plates were mounted on the short edges. This was however not enough to get the structure stable since the lock plates only stiffened every second angle in the short edges. A round wooden stick was attached to the edges to lock the rest of the angles, the model was then anchored to the bottom plate by cable ties at the bottom edge.

Figure 4.21 Production process, model without stiffening edge bars.

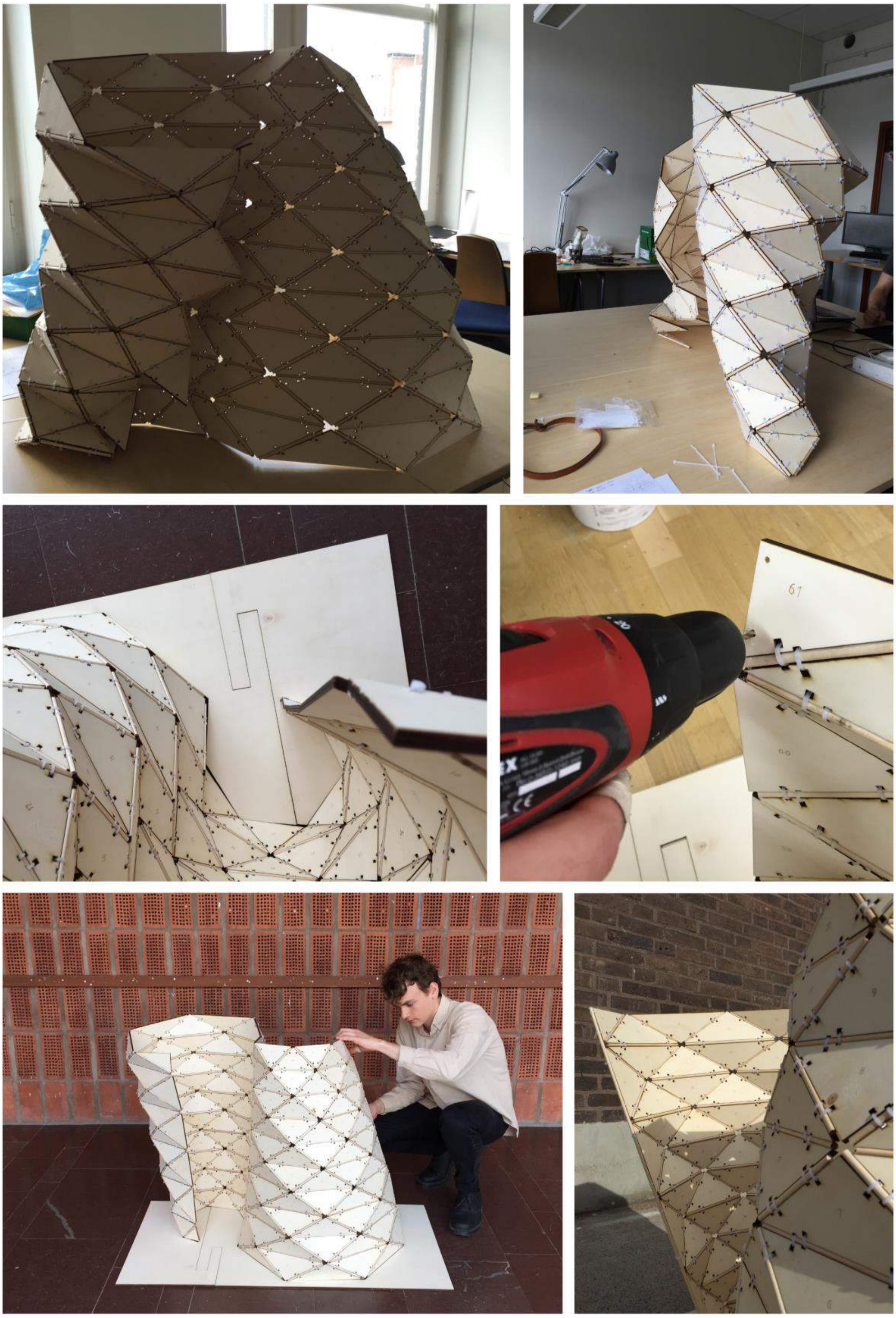


Figure 4.22 Production process, stabilizing model with edge bars.

Figure 4.23 Model pictures.

4.3.10 Planning of erection

Since no full scale structure was built in this iteration this step was not fully conducted. The planning of erection was though developed with respect to the next iteration and we experimented with different mounting techniques. We tried out both assembling the structure hanging and laying down and to assemble strips first and then put them together and to mount every diamond on to the structure separately. To achieve an efficient assembly process we discovered that the best way is to mount the structure when it lies down as long as it is possible to reach both sides of the panels. To be able to reach the panels in a good way it is also easiest to mount one diamond at a time.

4.3.11 Full scale structure

No full scale structure was built in this iteration.

4.3.12 Conclusion and developments

In the scale model it can be seen that the instability issues discovered in paper models are present also in the larger scale and thus have to do mainly with the structure and not the material. The diamond pattern has, as expected, many degrees of freedom and the structure can still move even though the short edge angles are fixed. As can be seen in the picture below the upper corners are moveable and affect the global shape and global stability. The upper edge is also flexible but this deformation does only affect the local stability. However, if holding the upper corner in locked position the whole structure becomes globally very stable. To solve the problem with the stability we came up with some possible solutions:

Lock the angles at the edges with brackets
Fixing all edges would be one way to stabilize the structure, however the plate material itself is a bit flexible and only locking the angles would not help against this deformation.

Stabilise the edges with a bent pipe or steel bar
By adding a steel bar along the whole edge the structure would be stable but it requires a lot of precision in the bending to make it fit all around the structure.

Shape the structure as an eight and lock the short edges together
Attaching the loose ends to each other by shaping the structure as an eight would remove some degrees of freedom, but the structure would still not be completely stable.

Change the crease pattern to Miura-ori
Miura-ori does just have one degree of freedom and would be more stable than the diamond pattern, but the Miura-ori pattern does not have the same abilities to form an S-shaped curve.

Bend the short boundaries down and lock them to the ground
By attaching the short edges to the ground the pattern becomes globally stable since three edges are restrained. This however brings some restraints to the shape.

Another part of the process that was difficult was the production of the wooden sticks. They had all different length and had to be right sized. If not the structure were either not foldable or there where uneven and unpleasant holes in the nodes.
To get rid of this problem we discussed some different solutions, either to cut the rods in a CNC-machine, then the different length would not be a problem, or to limit the rods to a few different lengths to make them easier to produce by hand.

Also the tensioning of the cable ties were a difficult step, it was hard to tension them equally to archive an even friction and it was also a slow process with first tensioning the cable ties by hand and then cut them. To solve this a cable tie tool should be used for next iteration.

The conical S-shape works well in creating a semi enclosed space, it results in two separate rooms with very different expressions. The folding also results in light effects that enhances the shape and the fold pattern and gives the space a spiritual sensation. The shape also results in light play on the ground caused by light falling through the gaps between the plates. These effects could be elaborated further by varying the size of the panels and work with the openings created by the connections to create more sophisticated light play.

Figure 4.24   Stablility analysis.

4.4 Third design iteration

In the third design iteration the program, conceptual design and pattern type where the same as in iteration 2. The process steps where updated though, since we realized that the stabilisation phase and the shape where tied to each other if the short edges should be fixed to the ground. No scale model where built except from folded paper models since roughly the same details and materials as in iteration 2 were used.

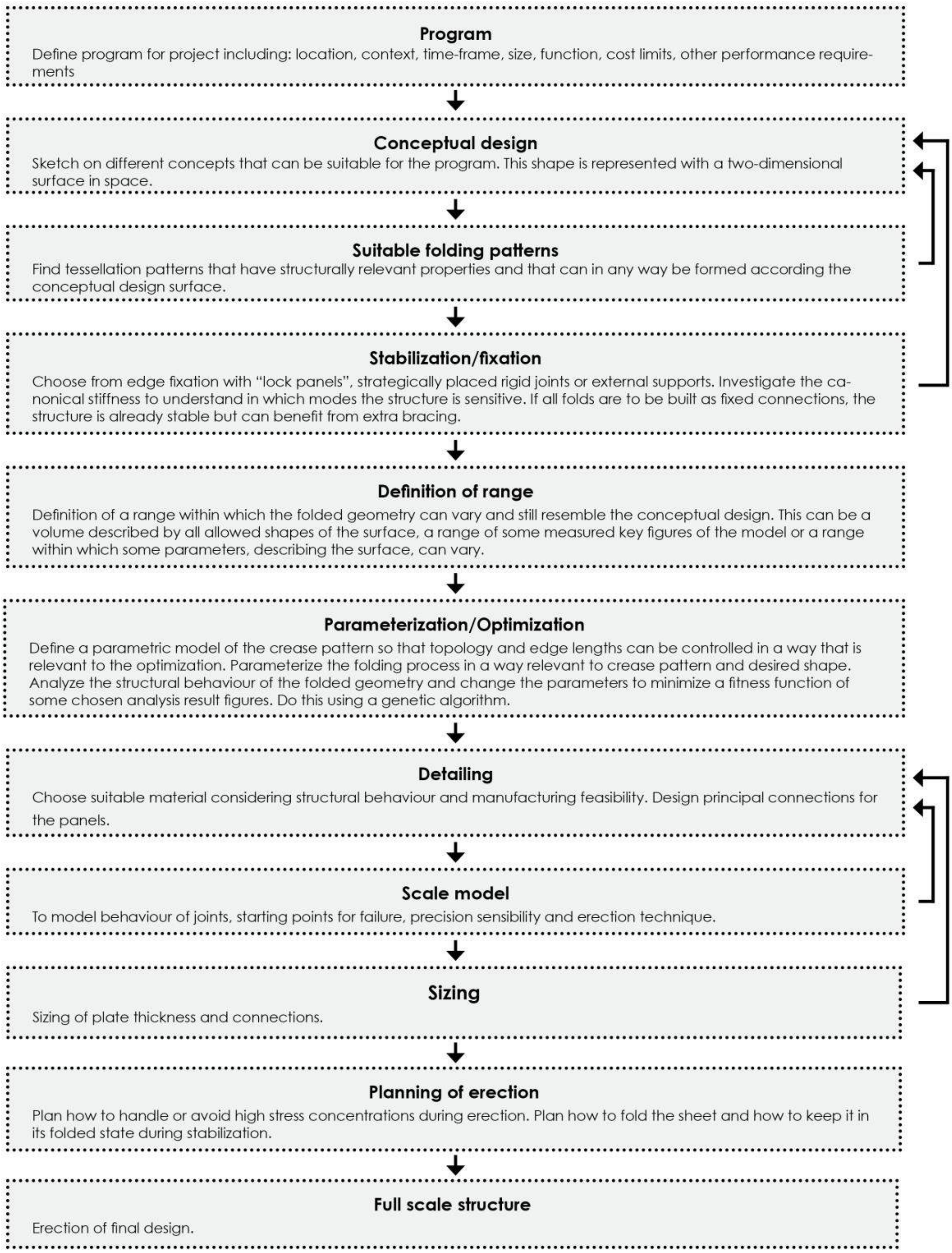


Figure 4.25 Process steps, third version.

4.4.1 Program

Where:
Betonghallen (The concrete hall), Chalmers

What:
A wall structure that creates a subspace in the large room.

Why:
Example of an application of the optimization and design process for folded plate structures proposed in this master thesis.

Limitations:
The structure need to be foldable from a flat sheet to make the production easier since the plates then can be mounted together when lying flat. Another reason for the flat foldability limitation is to minimize waste during production. The shape and size of the plates should be adapted to fit 1200 mm x 1200 mm plywood sheets in a good way so that as little material as possible is wasted during production.

4.4.2 Conceptual design

No new concept was developed, the same as in the previous iteration was used but developed regarding stability.

4.4.3 Suitable folding pattern

The diamond pattern was used also in this iteration since it can give the desired shape and since it has triangle facets, which cannot twist. This will make it easier to predict the behaviour of the full scale structure.

4.4.4 Stabilization fixation

From the previous iteration we discovered that the structure needed to be constrained in the edges to be stable, from the possible solutions that we came up with, we chose to work further with fixing the short edges to the ground and if needed lock the angles at the upper edge with steel bars. We chose this option since we found it being the option affecting the shape least. We made some sketch models investigating new conical S-shaped forms also making sure that fixing the structure to the ground would make it stable. We could see very clearly that the stability was improved when the pattern was designed so that both short edges was possible to fix to the ground, se picture nr 1, 2, 4, 5, 7, 8 and 9 in following figure. As can be seen, it can be done in several ways.

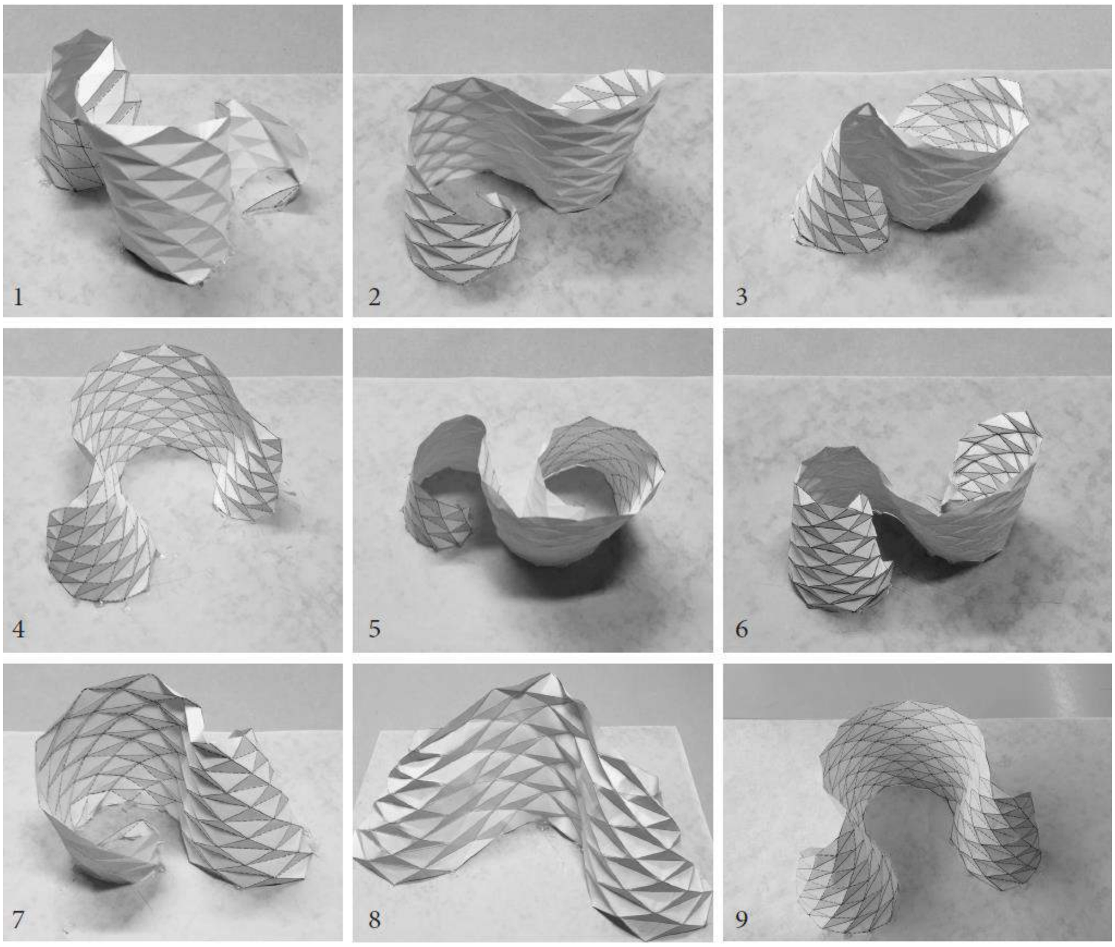


Figure 4.26 New models.

If the structure is designed as in iteration 2 the free corner can move, as can be seen in the picture below. The red line shows the part of the structure that is attached to the ground.

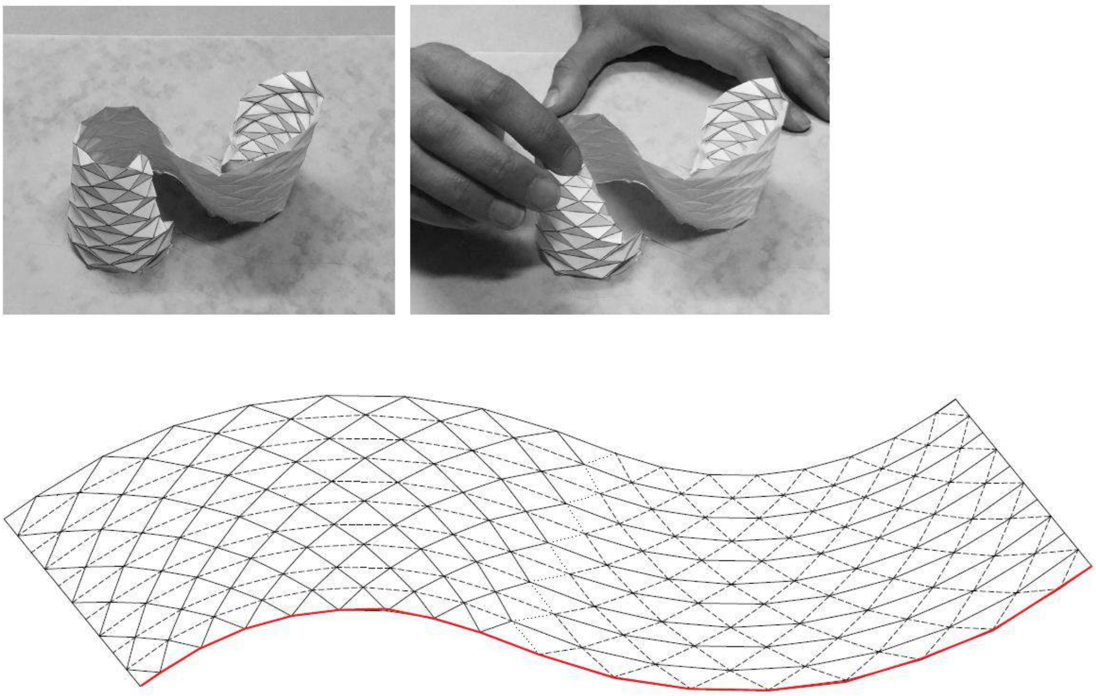


Figure 4.27 Stability and crease pattern of a structure with only the bottom edge attached to the ground.

To improve the stability the pattern can be reduced to try to eliminate the corners. In the pattern below one of the short edges is fixed to the ground and the free corner at the other short edge is less prominent but as can be seen below the structure is still not stable.

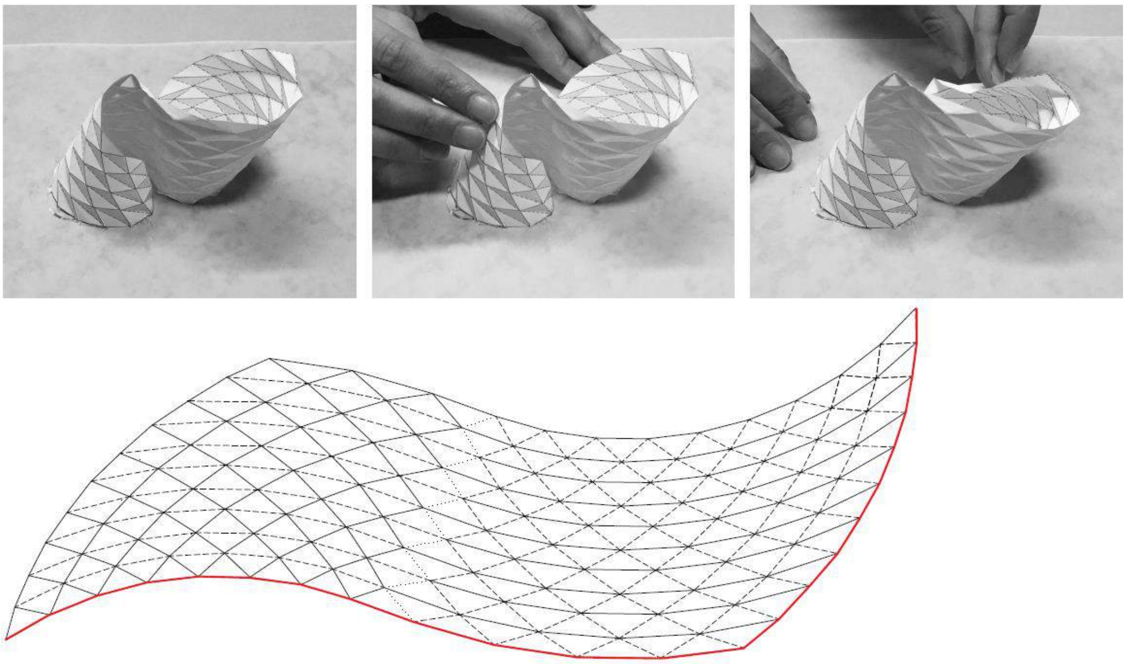


Figure 4.28 Stability and crease pattern of a structure with one short edge attached to the ground.

By cutting the pattern in another way both short edges can be fixed to the ground, it though results in a very noticeable corner in the structure. If no plates are to be cut the shape of the structure must follow the pattern and then gets a concave shape in one end, which results in this noticeable corner. As can be seen below the structure is stable except for local deformation at the free top edge.

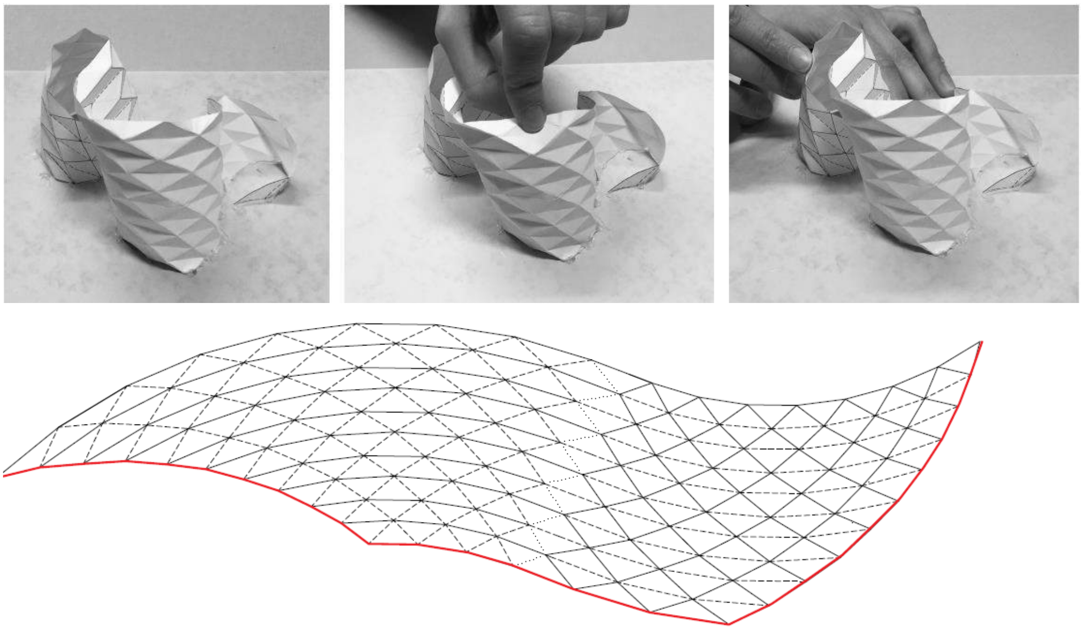


Figure 4.29 Stability and crease pattern of a structure with both short edges attached to the ground.

To make the corner less prominent the structure can be prolonged but the result is then that both ends inclines inwards, to get a conical incline at an end the pattern has to be cut in a concave shape, like above.

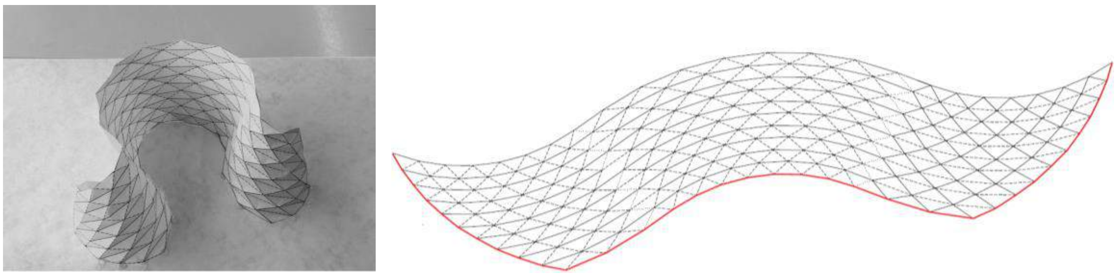


Figure 4.30 Alternative shape and crease pattern with both short edges attached to the ground.

Karamba was used to investigate the canonical stiffness of the structure and find some interesting eigenmodes for the wall geometry. In this analysis all folds were modelled as fixed and all nodes along the bottom boundary was pinned. The first eigenmode (left figure) has large displacements along the free edge of the wall. Even though this edge is stable theoretically since folds are modelled fixed, it is weak compared to parts fully surrounded by other panels. In the fourth eigenmode (right figure) large displacements can be seen not only in the top layer of facets but also in the wall structure as a whole.

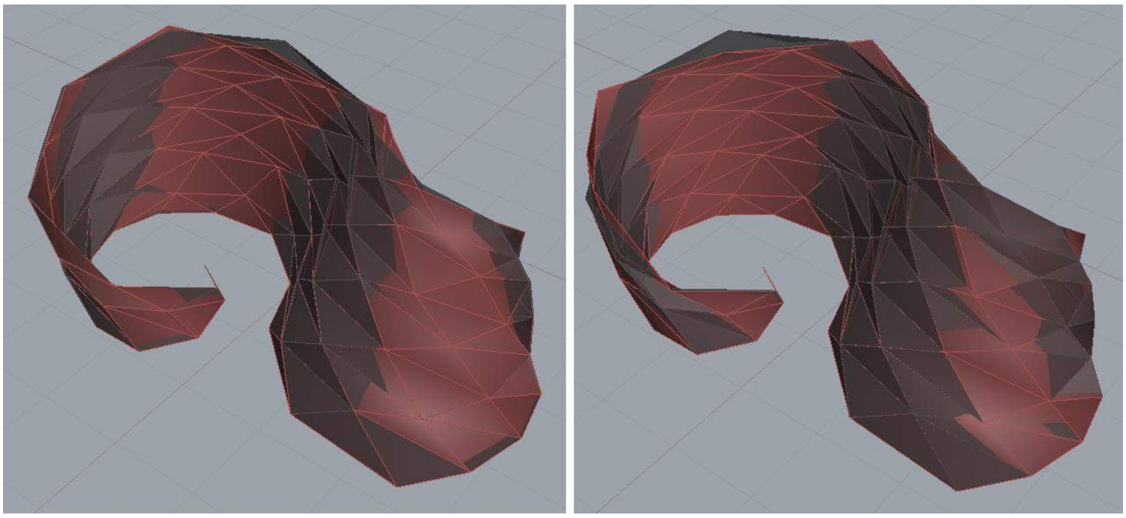


Figure 4.31 Eigen mode 1 (left) and 4 (right) in red. Original reference geometry in grey.

4.4.5 Definition of range

To be able to enter the structure, the openings has to be at least 60 cm wide and the rooms large enough for one person to stand in. For the structure to able to partition the space into separate rooms the wall has to be at least 2.5 m high.

4.4.6 Parameterization/Optimization

Since the time was limited we decided to focus the structural optimisation to adapt to the vertical stresses since this was the main load on the structure and we optimised only depending on the vertical principal stress components.

We started with a hypothesis that higher elements gives less total energy in the structure for a vertical load case, this was investigated in Karamba, and we saw that our hypothesis were correct. To achieve an even distribution of stresses in the structure we optimized the pattern so that the element height for the elements in each vertical strip was adjusted depending on stresses in the elements.

In the tests the structures where loaded with gravity in vertical direction. The patterns in the investigations were 4.5 m x 8 m and had a plate thickness of 0.01 m. The patterns were folded so that the span was 7.15 m and the structure was modelled with hinged connections. The bottom edge was locked in all translation and rotation directions while the upper edge was locked in all directions except in the direction of the load.

The first pattern with high diamonds had a maximum displacement of 0.099 m while it for the second pattern was 0.39 m. The same relation could be found in the total axial energy and total bending energy in the plates, for the first pattern the total axial energy was 0.0034 kNm and the total bending energy 0.039 kNm while for the second pattern it was 0.11 kNm and 0.077 kNm respectively. This means that higher elements are better for load in vertical direction.

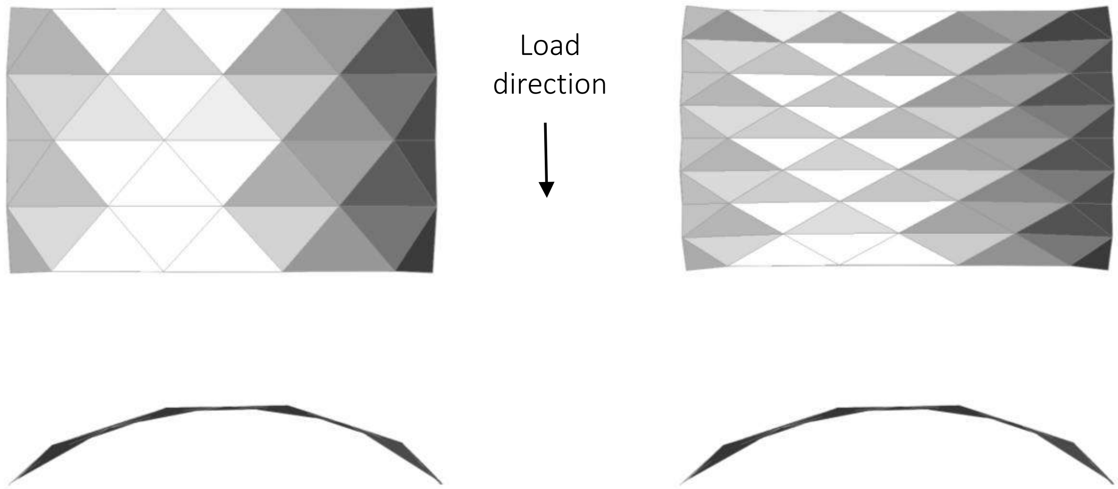


Figure 4.32 Test of total energy and deflection for panels with different height.

To verify that this relation has to do with the width/height ratio and not with the size of the elements, a control pattern was made with smaller elements having the same proportions as pattern 1.

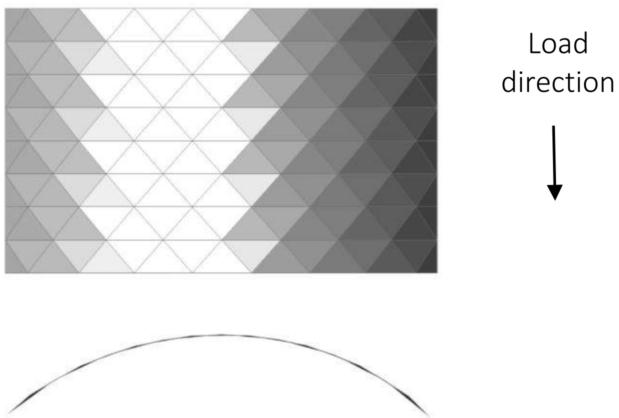


Figure 4.33 Test of total energy and deflection for panels with different height.

For the control pattern the deflection was 0.0057 m, and axial energy and the bending energy in the plates were 0.00032 kNm and 0.00043 kNm respectively. We can see that this is even lower than for pattern 1, and thus must the larger deflection and bending energies in pattern 2 depend on the height/width ratio. From this we can conclude that to make the vertical stresses more even in the structure the elements with high vertical components of the principal stresses should be made higher than the ones with low vertical stress components. It is also beneficial with small elements for the total energy in the structure. However this mostly depend on that the structure with smaller elements is less folded than the one with large elements. If they have the same global form and the plates in the structure with smaller elements is oriented more vertically and can take vertical load in a better way. For the stability of the structure it is better to have a more folded structure otherwise the elements will easily fold in the wrong direction. A compromise between these criteria thus has to be made.

The design method for an S-shaped wall in the third iteration, starts with finding the global form from an even fold pattern in Grasshopper, deciding the size of the two rooms, the height of the structure, the number of elements and the inclination of the cones. The pattern is built from a curve that is controlled by four points that can move in x- and y-direction and the curvature of the curve decides the degree of inclination of the cones. The curve is offset in two directions and the amount of curves offset on each side determines the number of elements in the vertical direction of the wall. The curve is divided into segments and in each node the normal to the curve is obtained, the intersection between the normal and the curves determines the nodes for the diamonds of the pattern. Lines are then drawn between the node points to create the diamonds and the diagonals making the valley and mountain folds. The origami component together with the Kangaroo Physics component is then used to fold and shape the structure. The Origami function is modified so that the diagonals have less fold strength than the diamonds, letting the diamonds govern the folding to reduce the risk of errors occurring in the folding process. This is done by comparing the midpoints of all the lines of the mesh with the midpoints of the diamond lines and if they correlate the fold strength value is replaced. The fold angle can also be changed along the length of the structure so that the global shape can be changed, this is controlled by a curve where the x-direction represent the position along the pattern and the y-direction determines the rest angle of the folds. This curve depends on four points which x- and y-coordinates can be changed. The rest angle determines the angle for which the particle attraction in Kangaroo is striving to achieve and all angles in the structure will thus not be exactly this. The structure is first folded and then the nodes at the lower borders are pulled down to the zero plane by particle attraction with the Kangaroo component *Pull to surface* to get a shape that has a flat base surface.

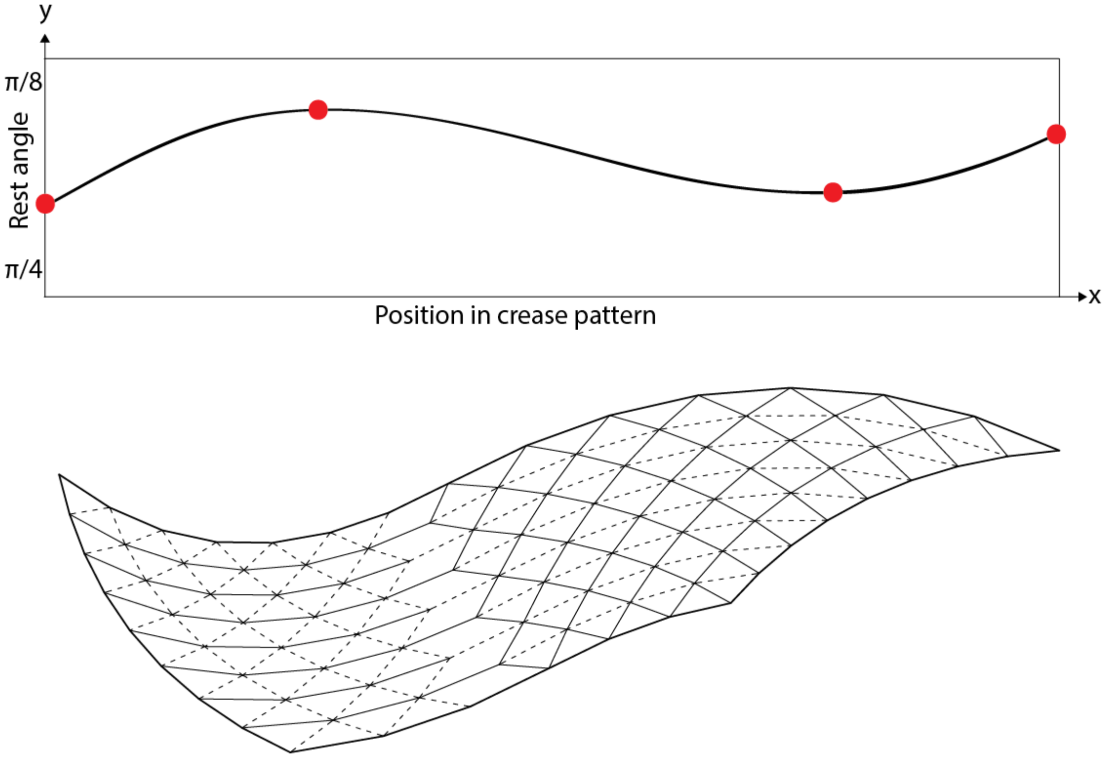


Figure 4.34   Base pattern and different rest angle along the pattern.

The global shape is then analysed with Karamba to receive the vertical component of the mean principal stresses in each facet. The only load causing those stresses is self-weight. A custom Grasshopper component called *Vertical Stress Distribution* does this. It requires the unfolded crease pattern and the same crease pattern folded to desired shape as input.

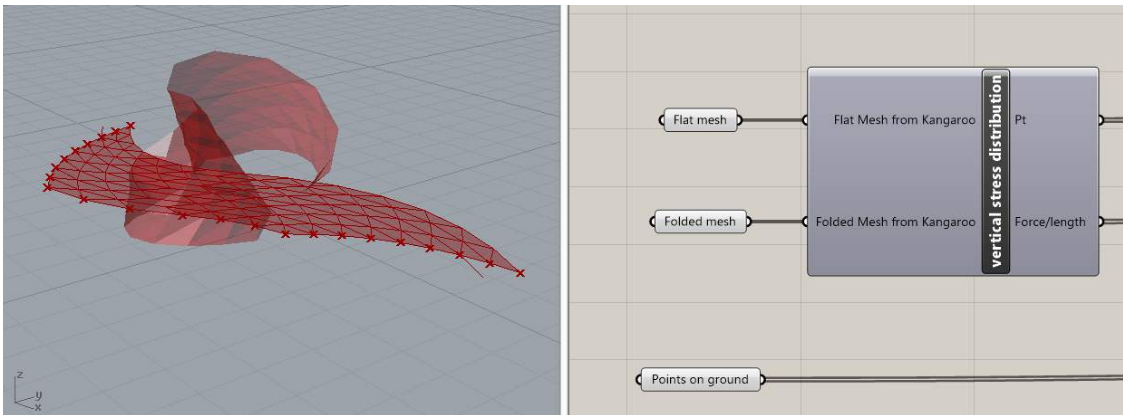


Figure 4.35   The custom made Grasshopper component that will give z-components of principal stresses in each panel.

Looking inside this component, it can be seen how the z-components of the two principal stresses are summed for each panel.

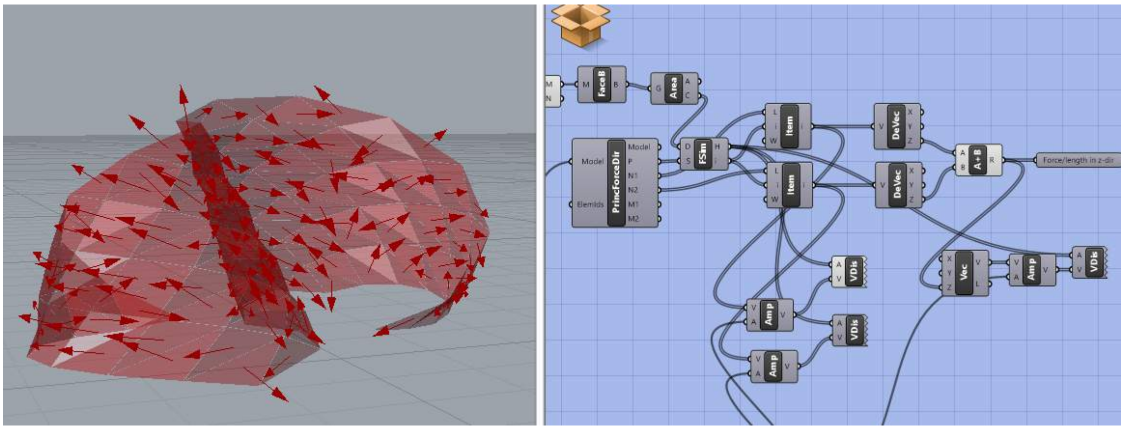


Figure 4.36 First principal stresses.

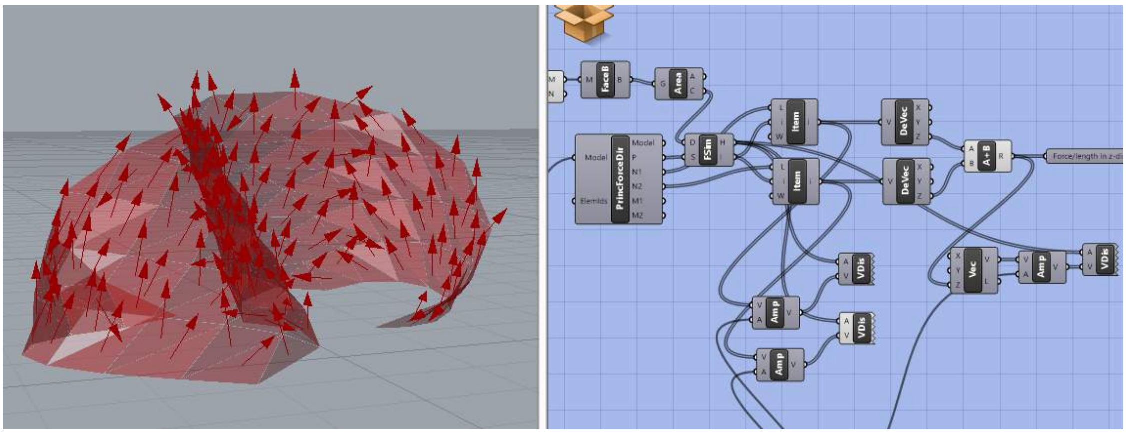


Figure 4.37 Second principal stresses.

The values of those z-components are put in a list with the same length as the number of panels in the model.

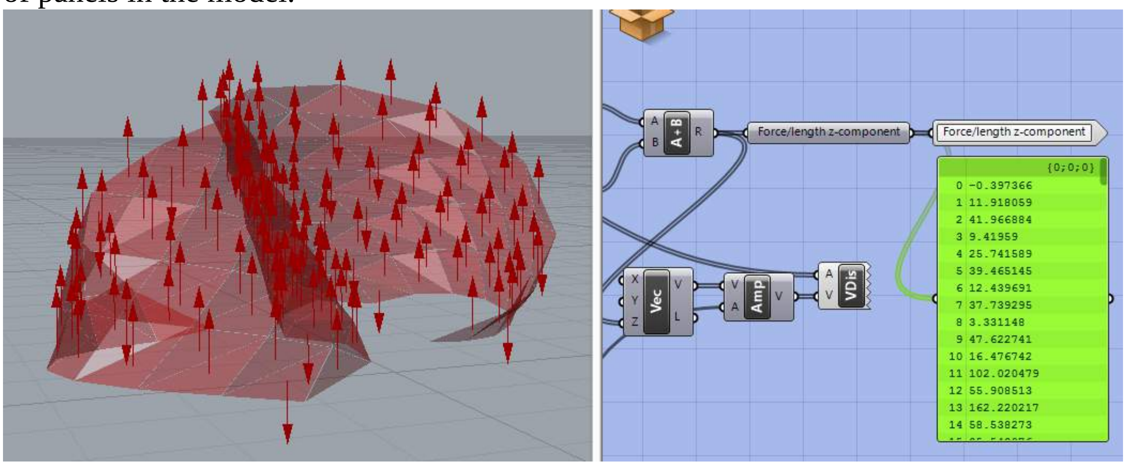


Figure 4.38 Sum of z-components of both principal stresses. The vector lengths are put in a list.

The fold pattern is then adjusted so that elements that has larger vertical principal stress components gets higher while the elements with low vertical principal stress gets lower. In detail, this is done like this: Each strip of facets is handled individually but in the same parametric model. Let’s look at one strip of facets as an example. Each diamond pair of facets (or single facets for those near the pattern boundary) is assigned a number. This number is the sum of the z-components of the mean principal stresses for the same pair of facets taken from the list in the figure above. The principal stresses of each strip of panels are summed up as $\sigma_{tot} = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} + \sigma_{z4} + \sigma_{z5}$. Which means that $\sigma_{mean} = \frac{\sigma_{tot}}{5}$.

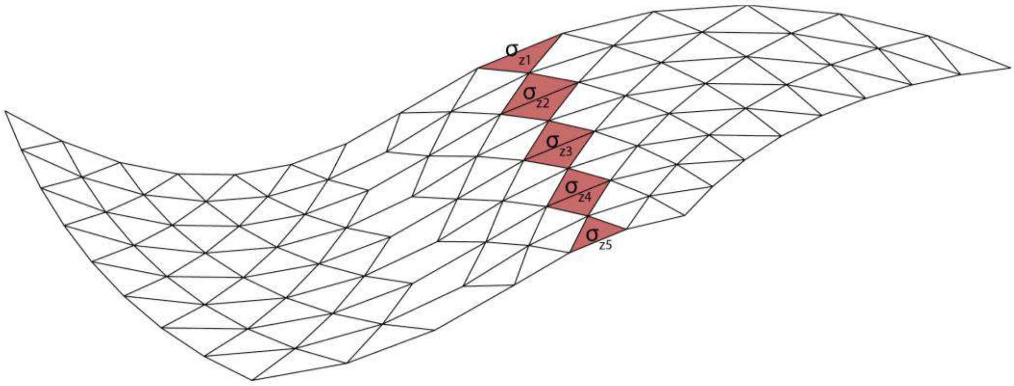


Figure 4.39 Vertical component of the principal stresses of one strip of facets.

The distances between pattern boundary and the crease line intersection points in each strip are l_1 ... l_5 in the original pattern. And of course $l_{tot} = l_1 + l_2 + l_3 + l_4 + l_5$

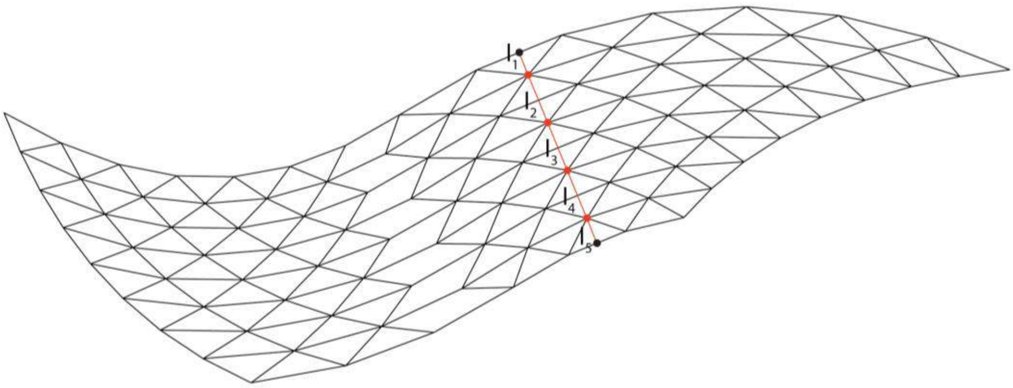


Figure 4.40 Distances between crease line intersection points in one strip of facets.

Each of the distances l_1 ... l_5 are adjusted so that their percentage of l_{tot} is the same as the corresponding principal stresses percentage of σ_{tot}. The new distances in the modified mesh are then $l_1 = \frac{\sigma_{z1}}{\sigma_{tot}} * l_{tot}$, and so on for the whole strip. The same thing is done for all strips in the crease pattern.

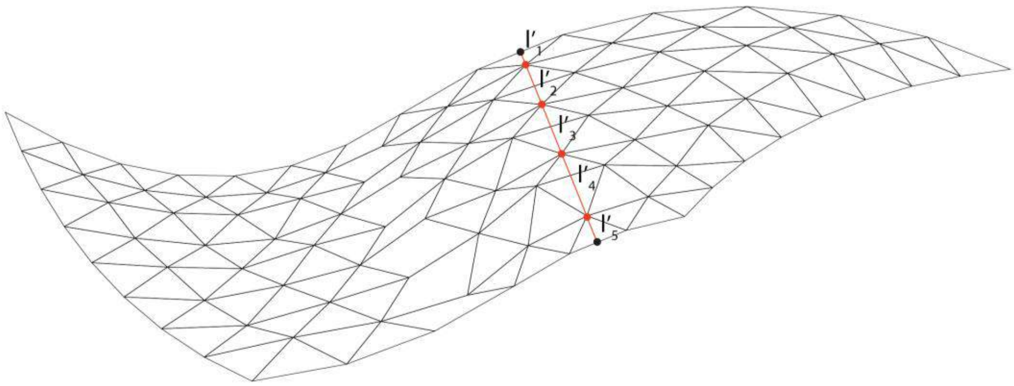


Figure 4.41 Adjusted crease pattern. Distances between crease line intersection points highlighted in one strip of facets.

Another Grasshopper component called *Crease Pattern Adjustment* does the crease pattern adjustments described above using output data from the *Vertical Stress distribution component.* The outputs are *points* which is a list panel centroids, and *Force/length* which is the list of principal stresses multiplied with panel thickness [kN/m]. Both lists have the same lengths and are sorted so that each centroid corresponds to the principal stress in that panel. Handle curves are added to the crease pattern to pull the internal nodes into the modified position. A number slider allows for control of how much the new crease pattern is modified. Zero means that nothing is changed compared to the base pattern and one means that the handle length of each pair of panels correspond exactly to the quotient (principal stress of that pair of panels)/ (the sum of principal stress in the strip of panels) multiplied with the total length.

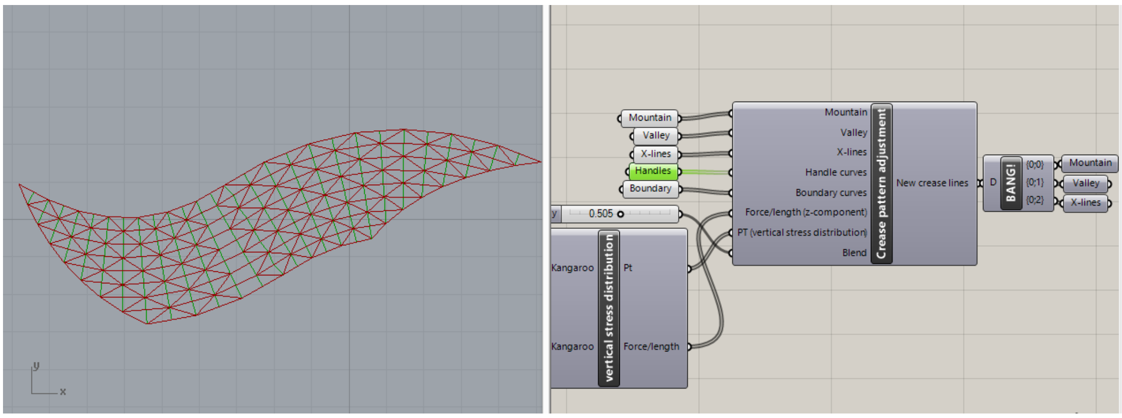


Figure 4.42 Crease pattern with extra "handle lines" and custom made Grasshopper component for crease pattern adjustment.

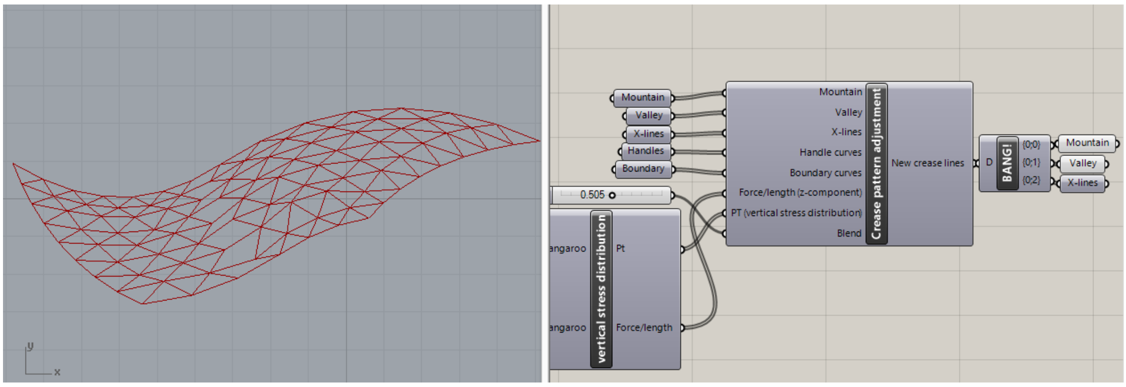


Figure 4.43 Modified crease pattern.

The new pattern is now modified once more so that the mid strip of Herringbone panels becomes a bit narrower. Further, a few nodes are moved slightly so that intersecting lines of the same kind (mountain or valley) are not parallel, which would cause problems later on. Those two last modifications of the crease pattern are done manually in Rhino.

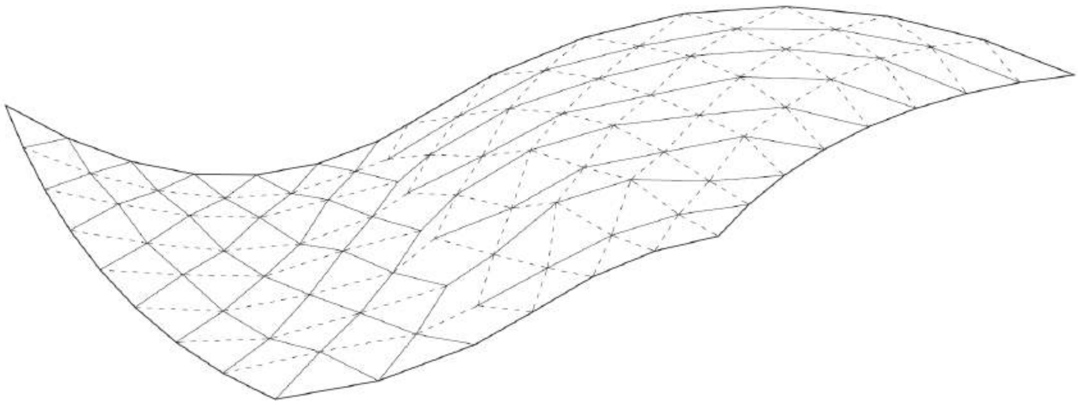


Figure 4.44 Final crease pattern.

The folded geometry from the base pattern and final pattern are compared using the *Utilization* component in Karamba. The smallest and the largest panel utilization ratio are extracted from both geometries. Settings for material and panel thickness are equal in the two versions. Also the hinges are modelled in the same way. The model folded from the original mesh has a higher maximum and a lower minimum utilization ratio than the model folded from the modified crease pattern. Since the panel with the highest utilization ratio will be decisive for the sizing of all the panels it is beneficial when this value is lower.

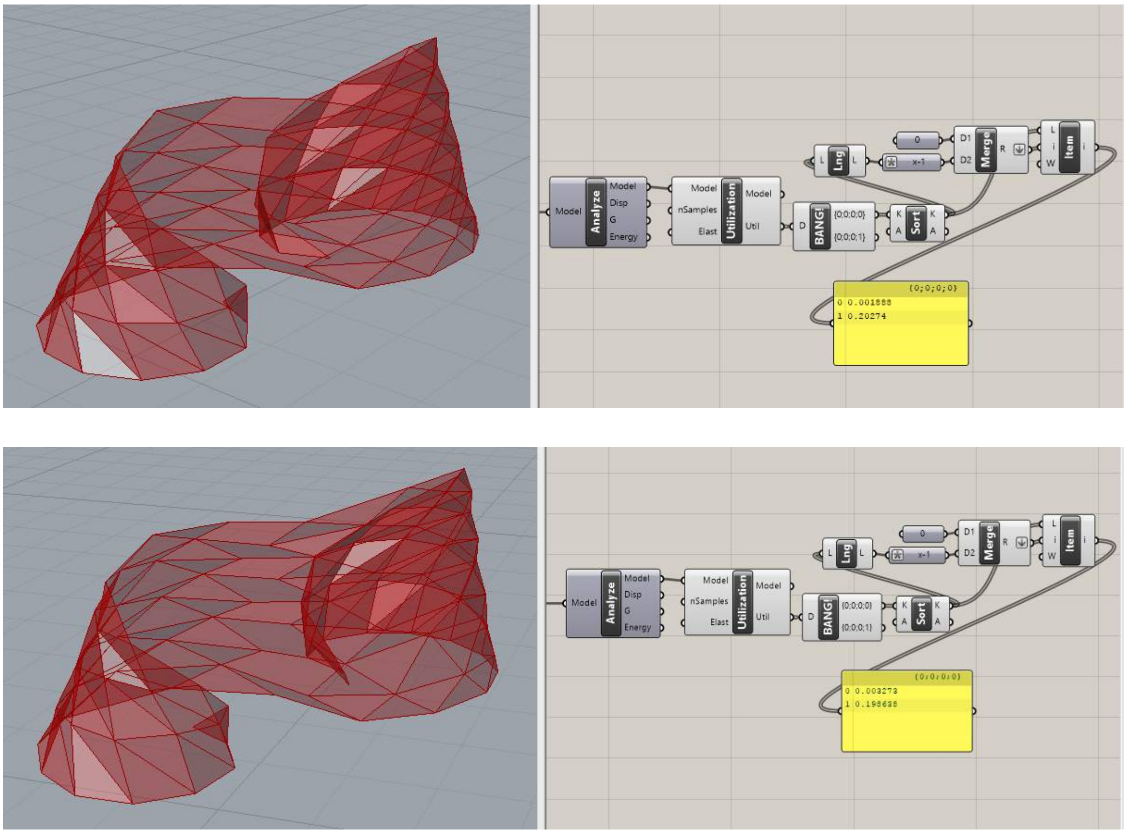


Figure 4.45 Utlilization ratio of the original and the crease patterns.

Since it was shown difficult to modify the crease pattern without large changes in the shape of the folded structure, we chose to adjust only the heights of the panels. This could be done relatively independent of the overall shape. We also chose to adjust the panel heights according only to measured z-components of principal stresses. There are of course other important stress patterns in the structure. Our choice of measured stresses and changed dimensions was more a theoretical exercise than a practical and plausible optimization. This might be why the difference in maximum utilization ratio of the two models is quite small.

4.4.7 Detailing

The birch rods that were used in the connections in iteration 2 all had unique lengths. The Lengths themselves are easy to extract from the parametric crease pattern, but the manufacturing of the rods had to be made manually. This was time consuming and it was hard to organize and label the manufactured rods. To make the connections easier to manufacture, many short rods of equal length was replacing the long rods of unique lengths. This means that one facet edge can have many short rods after each other but with a distance in between. This distance can vary since it is regulated by the laser cut holes rather than the rods themselves. This small example illustrates well the difference between digital production and manual production.

The lines that are to be used by the laser cutter are generated by another custom-made grasshopper component called *Lines for laser cutting.* It creates cut lines for holes and panels. The input parameters are some key figures for the hole distribution and size and for the rod thickness. The output data are cut lines, panel centroids, number of cable ties needed and total cut length. For the current design, hole widths are set based on width of the cable ties that will be used. The spacing in between each pair of holes, and in between each pair of holes are set so that each wooden rod will have a length of 100 mm. The minimum spacing between the rods are set to 60 mm which means that holes for as many rods as possible, without this limit to be overridden, will be drawn.. The number of cable ties is well enough in terms of capacity when the structure is standing in its intended position. The choice of cable tie density is based on intuition and experience from the built prototypes. Too few cable ties could result in buckling of the panels along their edges and could also result in a less accurate hinge mechanism.

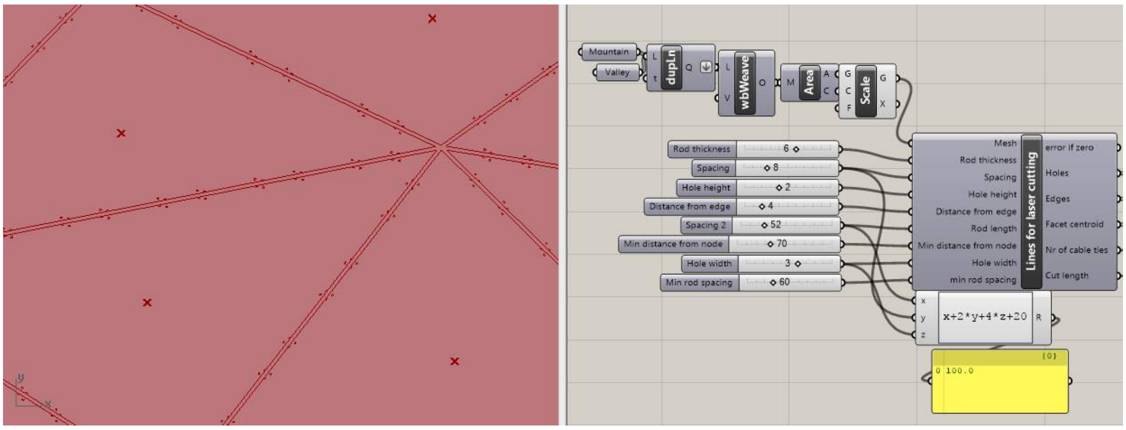


Figure 4.46 Custom made Grasshopper component generating lines for the laser cutter.

4.4.8 Scale model

No scale model was built in this iteration. It would have been useful, but it would also have required a certain amount of time and effort. The scale model from the previous iteration had given enough experience for this step to be left out.

4.4.9 Sizing

The structure that is about to be built in practice will be subjected to very unpredictable load combinations during erection. A sizing of the folded structure based on gravitational loads will not suffice for the erection phase. A lightweight 6 mm poplar plywood was chosen as plate material and 2.4 mm wide cable ties with a declared capacity of 80 N each was chosen for the connections. Rough estimations shows that the number of cable ties used and the panel material is well enough for the loads caused by gravity on the standing pavilion. To minimize strain elongation and play in the connections, and to minimize the risk of the wooden rods to “jump” out of place, it is desired to keep the effective length of the cable ties as small as possible. On the other hand placing the holes for the cable ties closer to the plate edge will reduce the capacity of the plate to hold the cable tie under tension. The distance from the panel edge to the hole edge was designed so that the capacity of the plate at location of the hole was equal to the capacity of the cable tie itself. A larger distance would not help the capacity but worsen the precision. A shorter distance would require more holes and cable ties which means less material efficiency. Calculations for estimation of the distance of the holes from the panel edges can be found in appendix A1, and it can be seen that from this estimation the needed distance is 4.5 mm.

4.4.10 Planning of erection

The erection was planned according to the following procedure. In the figure below the steps are illustrated as well.

First the panels are tied together in diamond shaped pairs. Wooden rods are mounted on all right hand side panel edges. The diamond shaped panel pairs are mounted into two sheets, one for each of the two enclosed spaces. The right sheet is flipped upside down and the two sheets are then folded inwards from their short edges. The two rolls are now tilted into their right position with help from wires and an overhead crane. The two parts are put together and fastened to each other using cable ties. Now, steel rods can be fastened to stabilize the structure so that the wires can be removed.

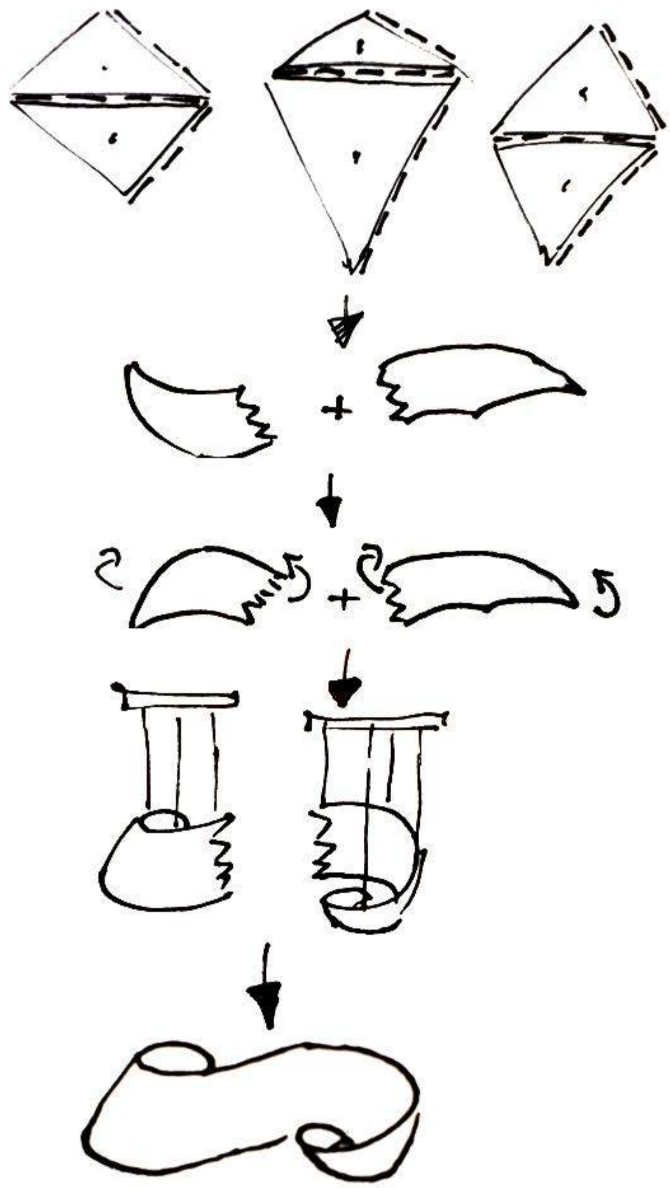


Figure 4.47 Illustration of the different steps in the erection process

To understand which the weak points of the structure are and how it has to be constrained, we evaluated the structure in Abaqus. The model was loaded with self-weight in the vertical direction and pinned along the bottom line and the angles along the top boundary were constrained. As can be seen in the picture below, showing Mises stresses, the main problem area is at the top in the middle of the structure. The two herringbone plates in the top are subjected to large stresses in the middle and the neighbouring plates at the top edge of the structure have large stresses in the corners. Also the corners in the bottom of the highest part, above the convex corner, show high stresses.

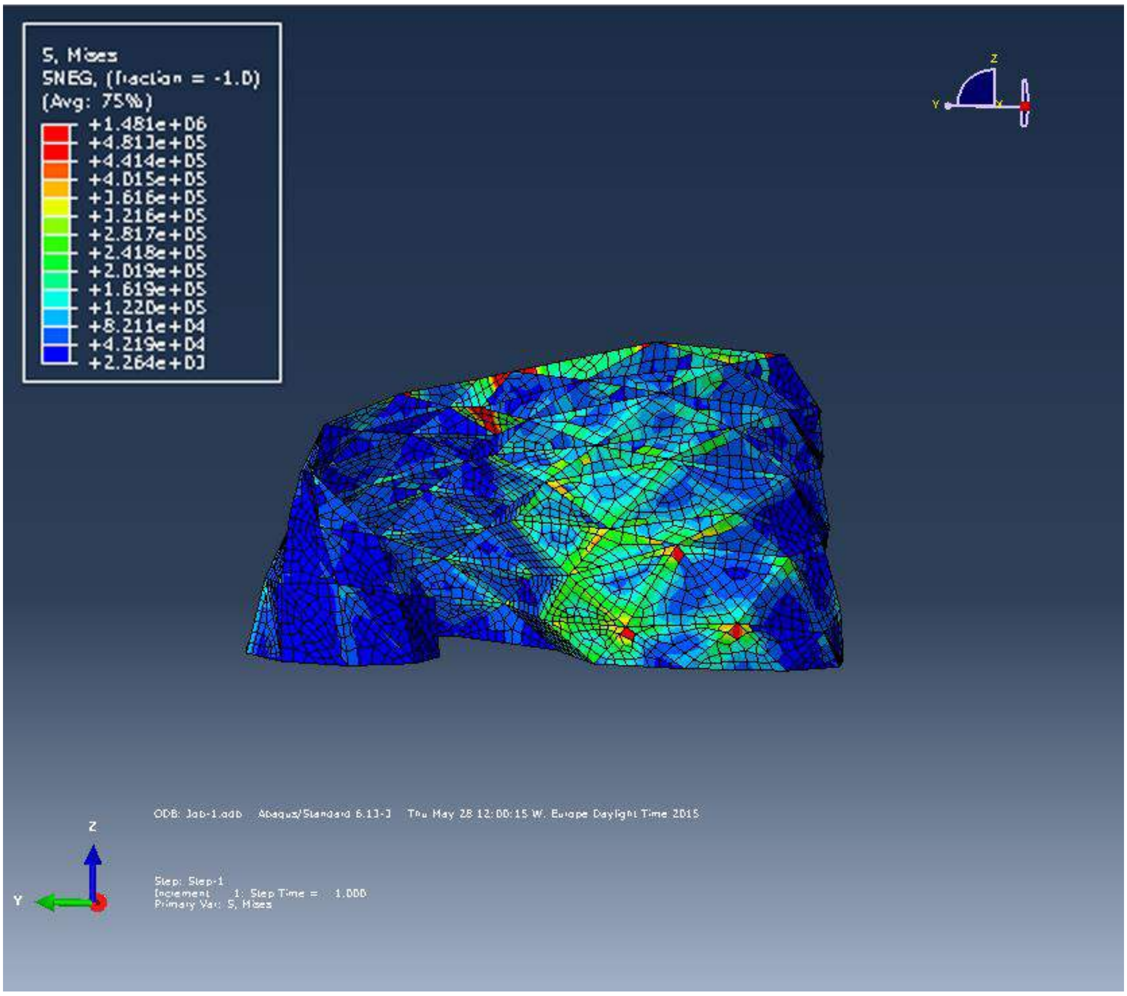


Figure 4.48 Mises stresses of the structure.

Looking at the principal stresses and their directions we can see that in the end of the out-leaning part of the structure there are high tensional stresses. This means that the corner might need to be anchored down to the ground in order for the structure to be completely stable. In the other corner it can be seen that the tensional stresses are much smaller and are thus not a problem.

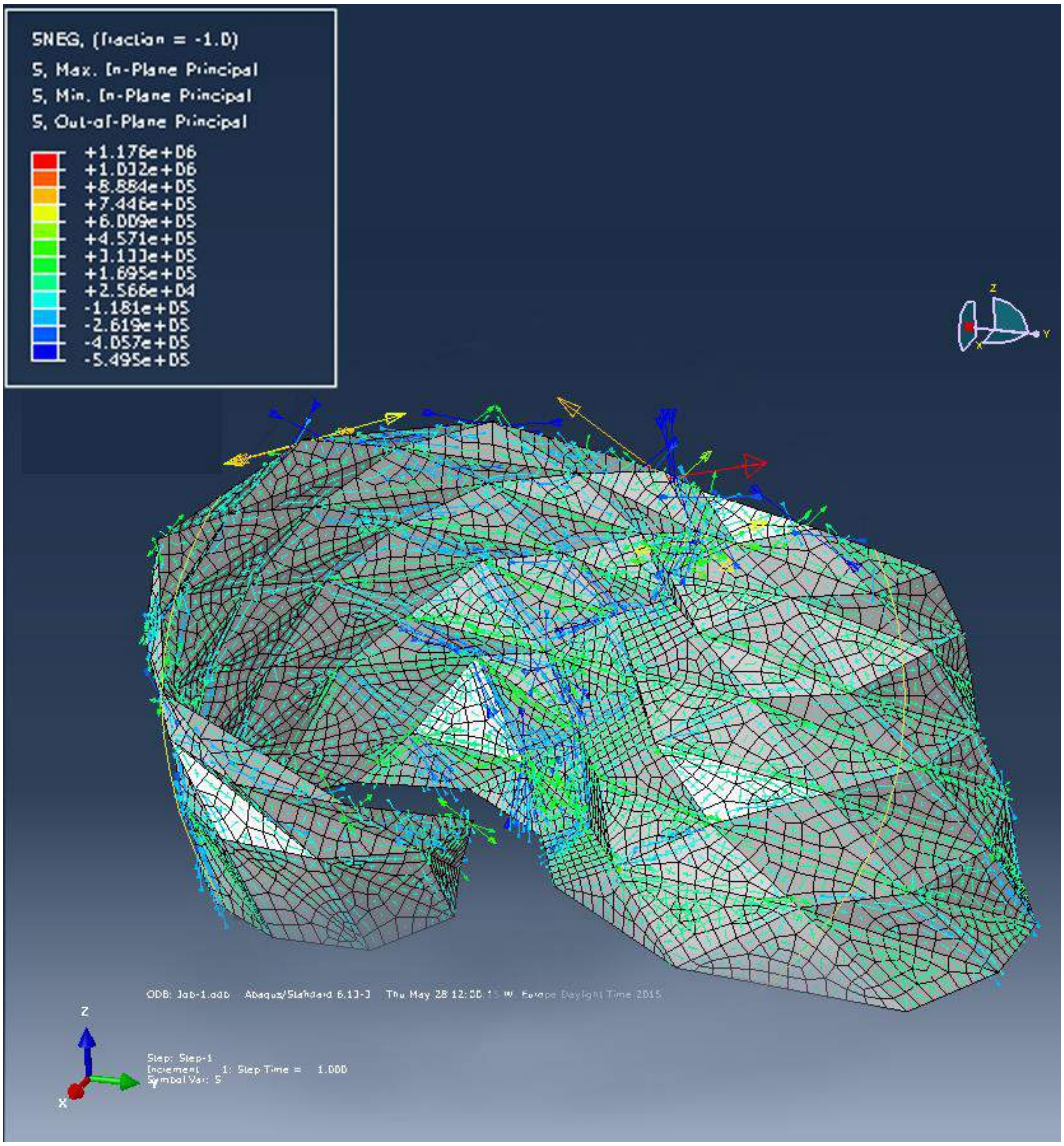


Figure 4.49 Principal stresses and their directions.

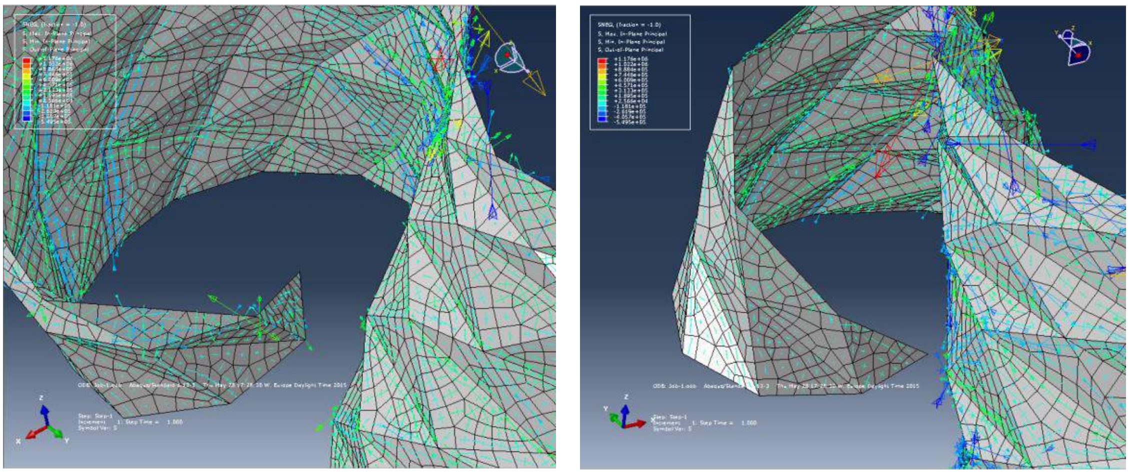


Figure 4.50 Principal stresses and their directions.

4.4.11 Full scale structure

The full-scale structure was built from the same material as the scale model in iteration 2, the only difference in material from the previous step was that the edge angles where locked with steel bars.

The poplar plywood sheets where split in two pieces of 1220 x 1250 mm, to fit the laser cutter at KKV, Konstnärernas Kollektivverkstad. From Rhinoceros, the final pattern was exported to AutoCAD where the panels where numbered and manually fit to the plywood sheets. The CAD file was then converted to eps format to be able to be opened in CorelDraw from which the laser cutter was run. The panels where cut out and the diamonds where mounted together with wooden sticks and cable ties. Wooden sticks were also attached to the right side of the panels to make the assembly easier. The cable ties where tensioned and cut with a cable tie tool. See picture 4.54.

The diamonds for the first half where mounted together when lying flat and then this part where folded and shaped. Several tables where used when assembling the structure to be able to reach both sides of the plates, which is required when mounting the cable ties. The second part where assembled, folded and shaped in the same way. It was not possible to use the overhead crane so the structure had to be folded together by hand. Before the iron bars where attached the structure where not stable and had to be secured by wooden studs and weights. See picture 4.55.

When shaping the structure we realised that it had to be more folded than we had planned. In Rhinoceros we shaped the model more opened and less folded, but in reality this was hard, since the fold angle where so low that the folds easily flipped over in the wrong direction, we thus had to find a new shape that was more curved to make the structure stable resulting in a spiral in the part leaning outwards, as can be seen in the picture below.

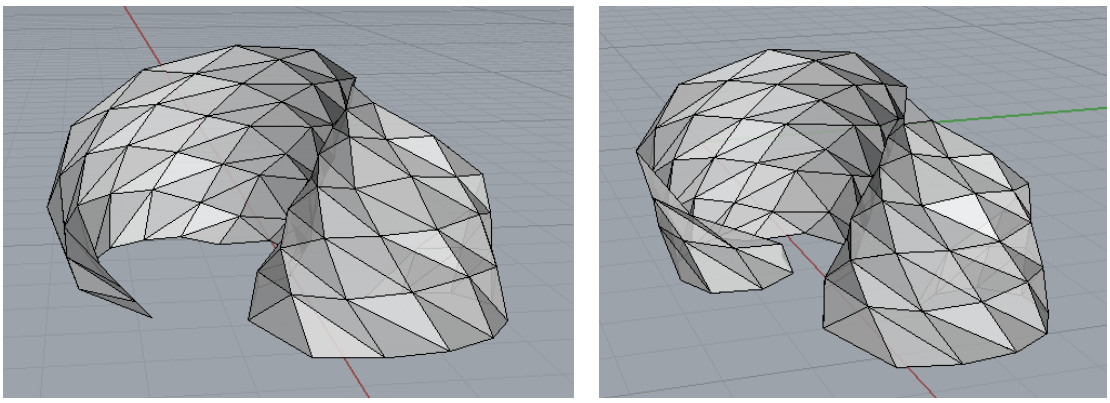


Figure 4.51 Original and modified shape.

There were also more problems with the concave corner then we thought it wold be; it made the adjacent plates fold in the wrong direction and the edges to lift from the ground. We went back to the computer model and saw that in order for the pattern to keep its shape and for all edges to be in the ground the corner had to be cut off. The edge between the plates had to be shortened by a third of its length. The corner plates where demounted, cut and reattached to the structure. This worked well and the structure could now be formed and folded into its right shape, see picture 4.56.

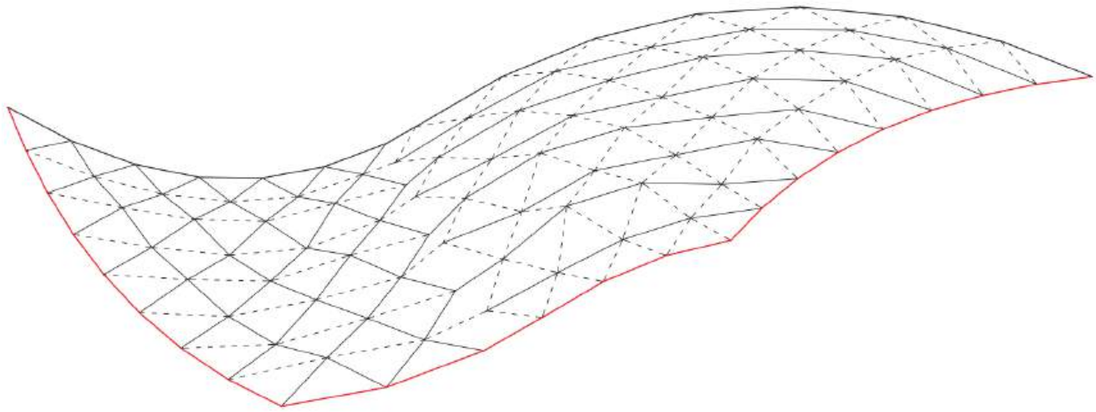


Figure 4.52 Original crease pattern.

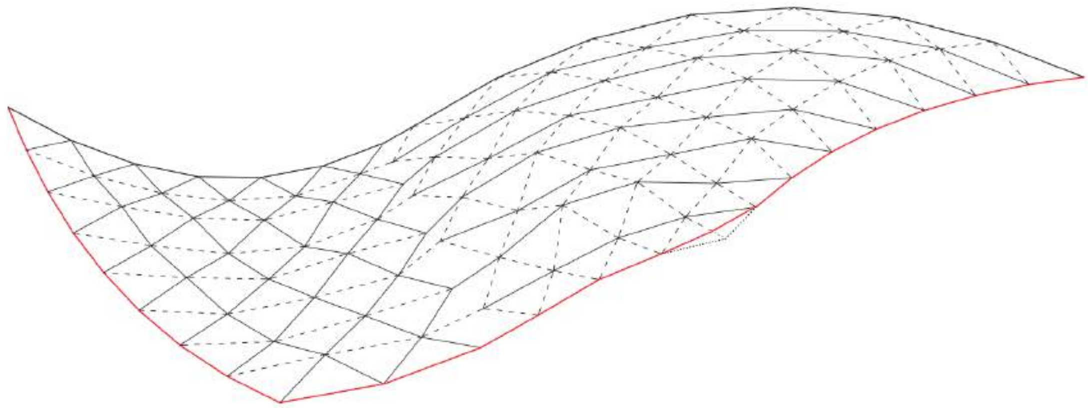


Figure 4.53 Modified crease pattern with cut corner.

Steel bars where bent in the correct form by measuring the angles at the structure. To stabilize the structure, the steel bars where then mounted with cable ties in all angles of the edge to fix the shape, se picture 4.57.

Figure 4.54 Material, laser cutting and mounting of the full scale structure.

Figure 4.55 Assembling and folding of full scale structure.

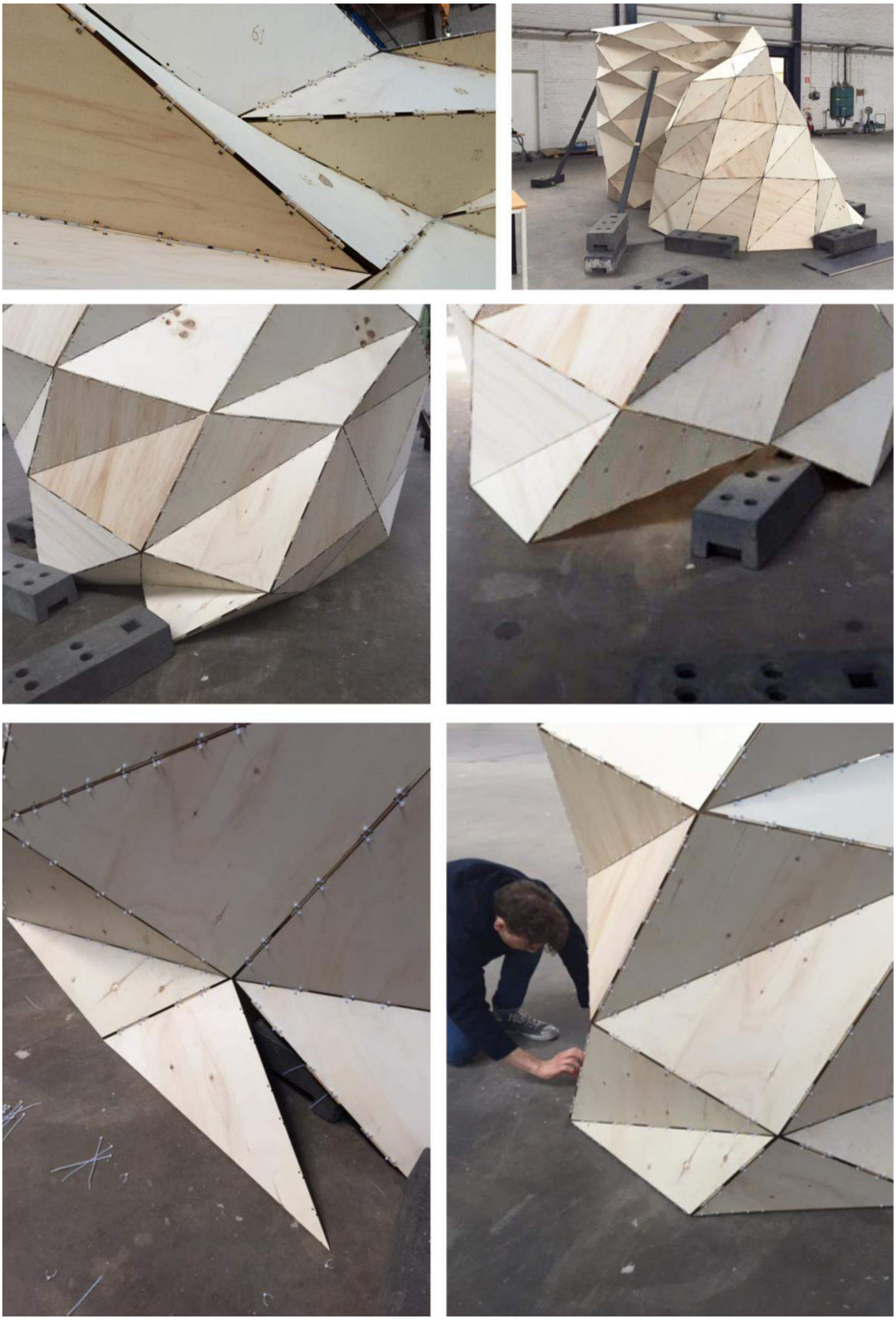


Figure 4.56 Modifying the structure by cutting of a corner.

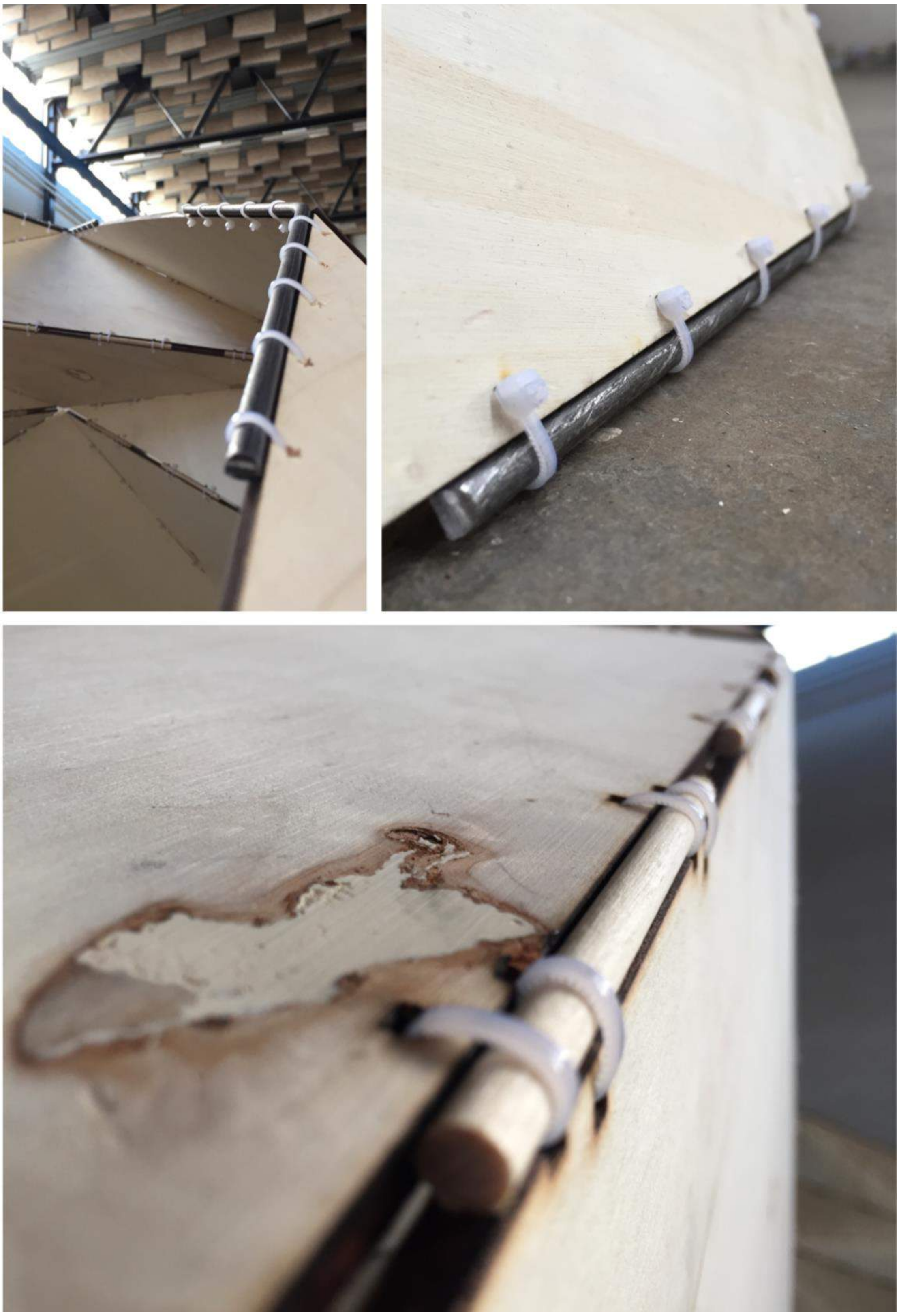


Figure 4.57 Stabilization and connections.

Figure 4.58 Full scale structure.

4.4.12 Conclusion and developments

The vertical strip of quad elements in the structure can be seen as a strip of the herringbone pattern. This pattern has fewer degrees of freedom than the diamond pattern and can therefore appear more stable. In the scale model from iteration 2, it felt like the herringbone strip stiffened the mid part of the wall. This effect however, depends on the quads to be stiff in terms of out of plane bending. For the full-scale structure in this iteration, the quads size made the elements weaker in bending. This combined with the stress concentrations (which can be seen in the FE-images) in the top mid part of the structure made the quad panels in this spot twist out of their original plane.

The traverse that we planned to use was not accessible to us at the time of erection. Instead we had to use studs, weight blocks and a few extra arms to keep the structure in place before stabilization. Logistics and equipment access should have been planned better.

We also realized that even though small fold angles work well in the computer modeling it is hard to make it work in reality. Small angles easily flips in the wrong direction making the whole structure collapse. Also the corner was a bigger problem than we thought it would be and is something that has to be carefully considered for future structures

To move the pavilion a couple of Masonite boards were placed beneath it so that it could be pulled. A more rigid bottom plate with wheels would have made transportation easier.

When the pavilion was pulled outside for a daylight photo-shoot, the wind loads made it collapse. The wind forces accumulated as bending forces in the edge panels and since they were fixed by steel rods some of them had to break for the structure to unfold. A continuous stiff steel rod or pinning to a ground plate would have prevented the edge plywood panels from being subjected to bending and the collapse may not have been initiated.

Over all the last iteration was successful, the structure resulted in two spaces with very different impressions enriched by the light. The shape and fold pattern were developed in this iteration resulting in a more playful effect due to the different sized panels and the different room shapes. The spiral shape resulted in a space totally embraced by the structure but opening up towards the sky resulting in a very peaceful and spiritual room, the other room was more covered and gave a darker more sheltered feeling where the light play from the light falling through the gaps between the wooden rods were very prominent and created a sensational effect.

Figure 4.59 Twisted quad element, traverse substitutes, transportation, wind failure and damaged elements near the fixed boundary.

5. Discussion

5.1 Sustainability

5.1.1 Material efficiency

To make early, qualified design decisions regarding the shape of a building and its structural system, can have huge impact on the amount of material that is needed in a structure.

Folding can make thin sheets or plates load carrying over long spans and with a high local stability. The potential of a folded sheet material to be both an efficient structure, envelope and to provide an architectural expression to a building makes it interesting from a material saving point of view.

This thesis is about folded plate structures, but the iterative design process and interdisciplinary approach is relevant and useful in many different design problems.

5.1.2 Social Sustainability

Parametrical tools and digital manufacturing methods open up possibilities to avoid standardized and mass produced architecture. This could be used to create more site specific buildings, which can strengthen a site and the identity of its inhabitants. An interdisciplinary way of working where architects and engineers collaborate throughout the whole design process reduces the risk of mistakes and that important aspects are forgotten. Buildings with a high quality regarding technical performance, durability and spatiality have positive consequences.

5.2 Conclusions

I this master thesis we have found a design method for folded plate structures, including relevant ways of optimisation and structural evaluation. This has included modifying and creating grasshopper components and developing a way to combine experimental folding and sketching by hand with computer based design. We have showed useful ways of dealing with design, parametric and physical modelling, analysis and optimization of complex folded plate structures.

We discovered that a unique property of the folded origami tessellations is the way the design of the semi scale and the crease pattern results in the global shape and structure. The way these structural levels are closely intertwined results in architecturally interesting limitations and possibilities. We investigated the mechanical properties and found three fold patterns with load bearing structural qualities that could be used for architectural purposes, the Herringbone pattern, the Diamond pattern and the Diagonal pattern.

We also can conclude that a useful tool for a parametric modelling of folded plate structures is the Grasshopper add-on Kangaroo. With a good principal understanding of a specific pattern, the patterns internal DOF:s and the relation between crease pattern and folded shape, it is by using Kangaroo possible to model folding of complex origami mechanisms without a mathematical description of all vertex trajectories.

We decided to work only with origami-like folded plate structures, which in their unconstrained state can be fully developed flat surfaces. This meant that all manipulations with the crease pattern of a structure were tied to changes in the general shape of that structure. We also limited the panels to be of constant thickness within each model, physical as well as digital. This limitations was chosen for reasons related to waste reduction, manufacturing and as a general aesthetic obstruction. With these limitations, we also limited the possibilities for structural optimization of the models. The structures are not optimized in the sense that the material utilization ratio is near one everywhere. But rather that the generated geometry is the most efficient out of a large array of alternative variations.

In theory the origami folded plate structures are ideal for production since all the panels fit together with no space in-between when lying flat. Some practical issues like the size of the laser cutter though resulted in some waste of material during production since the pattern had to be divided to fit onto several small sheets. Another pattern that can be divided into straight strips could maybe have been beneficial.

We have tested our design method by applying it to scale models and to a full scale structure. To be able to do this we also had to solve some practical issues like the connections and assembly process. We have developed a hinged connection using wooden rods and cable ties together with pre-cut holes in the panel. The connection gives a rotational axis that lies in the mid plane of the panels, allowing the mechanism to follow the kinematic motion of an ideal zero thickness origami model.

We have also showed that it is possible to implement and integrate structural optimization in a larger design process for folded plate structures.

5.3 Outlook

For further work there are many other possibilities with folded structures that need to be looked into. One of them, which we have just touched upon, is the architectonical qualities of light and shape and how the folding affects the impression of a room.
To continue the work of making it easier for designers to use folding and folded shapes in their practice it could also be interesting to look into the fields of room acoustics, micro climate and air streams around folded shapes. More complex parametric models could be established to find more shapes and better stress distributions. Details and connections for larger scale applications could be designed. It should be investigated if the production could be completely automatized in the future, starting with digital design, and laser cutting production and finishing in automatic robot assemblage.

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6.1 Figures

Figure 2.1	Encyclopedia Britannica http://media-1.web.britannica.com/eb-media/07/24607-004-DB0D5ED5.jpg/ [2015-01-27]
Figure 2.2(left)	The encyclopedia of great Philadelphia http://philadelphiaencyclopedia.org/archive/workshop-of-the-world/workshop-akent-loc/ [2015-03-27]
Figure 2.2(right)	Google street view https://www.google.se/maps/@40.016518,-75.174493,3a,75y,110.07h,72.49t/data=!3m4!1e1!3m2!1sP6MnhCrCUnyycbT8nuMN6Q!2e0?hl=sv [2015-03-27]
Figure 2.3	UNESCO http://www.unesco.org/new/fileadmin/MULTIMEDIA /HQ/BPI/ _images/hq/06_01.jpg [2015-01-27]
Figure 2.4	Florida Memory, State library and archives of Florida https://www.floridamemory.com/items/show/58828 [2015-03-27]
Figure 2.5	Michigan Modern, design that shaped America http://www.michiganmodern.org/buildings/american-concrete-institute-building [2015-03-27]
Figure 2.6	Hiendl & Schineis Architectenpartnershaft http://hiendlschineis.com/projekte/spielen_lernen/musiksaal-_thannhausen/ [2015-03-27]
Figure 2.7	Hiendl & Schineis Architectenpartnershaft http://hiendlschineis.com/projekte/spielen_lernen/musiksaal-_thannhausen/ [2015-03-27]
Figure 2.8	Composites and Architecture http://compositesandarchitecture.com/?p=100 [2015-03-27]
Figure 2.9	Ultra Modern Style http://ultramodernstyle.com/wexler-steel-folded-plate-roof-houses-for-the-future/ [2015-03-30]
Figure 2.10	Eikongraphia http://www.eikongraphia.com/wordpress/wp-content/FOA.jpg [2015-01-27]

Figure 2.11	Spotted by Locals http://www.spottedbylocals.com/glasgow/riverside-museum/ [2015-03-30]
Figure 2.12	Buildipedia http://buildipedia.com/aec-pros/engineering-news/case-study-zaha-hadid-architects-riverside-museum-of-transport-and-travel-part-5/ [2015-03-30]
Figure 2.13(U. left)	NASA http://www.nasa.gov/jpl/news/origami-style-solar-power-20140814/#.VRlBwvmsVWY/ [2015-03-30]
Figure 2.13(U. right)	Buri, H. (2010). *Origami - Folded Plate Structures.* Ph.D. thesis, École Polytechnique Fédérale de Lausanne. Lausanne: Univ.
Figure 2.13(Bottom)	New Scientist http://www.newscientist.com/article/dn27087-origami-d [2015-03-30]
Figure 2.14	Schenk, M. (2011). *Folded Shell Structures.* Ph.D. thesis University of Cambridge. Cambridge: Univ.
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Figure 2.16	Tachi, T. (2011). Rigid-foldable thick origami. I Patsy Wang-Iverson, Robert J Lang, Mark Yim (red.). *Origami^5*. Boca Raton: Taylor and Francis group, 253-263 pp.
Figure 2.17	Buri, H. (2010). *Origami - Folded Plate Structures.* Ph.D. thesis, École Polytechnique Fédérale de Lausanne. Lausanne: Univ.
Figure 2.18	Lang, R. (2008): *The math and magic of origami,* TED Talks, http://www.ted.com/talks/robert_lang_folds_way_new_origami [2015-02-26]
Figure 2.19	Lang, R. (2008): *The math and magic of origami,* TED Talks, http://www.ted.com/talks/robert_lang_folds_way_new_origami [2015-02-26]

Figure 2.20	Lang, R. (2008): *The math and magic of origami*, TED Talks, http://www.ted.com/talks/robert_lang_folds_way_new_origami [2015-02-26]
Figure 3.3	Bechthold, M. (2008). *Innovative Surface Structures_ Technologies and Applications.* Oxon: Taylor and Francis.
Figure 3.4	Bechthold, M. (2008). *Innovative Surface Structures_ Technologies and Applications.* Oxon: Taylor and Francis.
Figure 3.5	Bechthold, M. (2008). *Innovative Surface Structures_ Technologies and Applications.* Oxon: Taylor and Francis.
Figure 3.15	Tachi, T. (2011). Rigid-foldable thick origami. I Patsy Wang-Iverson, Robert J Lang, Mark Yim (red.). *Origami*[5]. Boca Raton: Taylor and Francis group, 253-263 pp.
Figure 3.16	Tachi, T. (2011). Rigid-foldable thick origami. I Patsy Wang-Iverson, Robert J Lang, Mark Yim (red.). *Origami*[5]. Boca Raton: Taylor and Francis group, 253-263 pp.
Figure 3.17	Tachi, T. (2011). Rigid-foldable thick origami. I Patsy Wang-Iverson, Robert J Lang, Mark Yim (red.). *Origami*[5]. Boca Raton: Taylor and Francis group, 253-263 pp.
Figure 3.18	Tachi, T. (2011). Rigid-foldable thick origami. I Patsy Wang-Iverson, Robert J Lang, Mark Yim (red.). *Origami*[5]. Boca Raton: Taylor and Francis group, 253-263 pp.

Appendix A1

-Estimation of edge distance.

Local shear failure

Estimation of distance needed between holes for the cable ties and the edge of the panel.

Loaded edge:

$alpha := 90 \cdot deg$ — Angle between load direction and loaded end

$d := 2.4$ — Nail (strip) diameter [mm]

$t := 6$ — With of panel [mm]

Minimum edge distance

$a_{3t} := (3 + 4 \cdot \sin(alpha)) \cdot d = 16.8$ — [mm]

Applied load:

$Force := 1.35 \cdot 80 = 108$ — Capacity of cable ties [N]

$p_k := 350$ — Characteristic timber density [kg/m^3]

$f_{hk1} := 0.082 \cdot (1 - 0.01 \cdot d) \cdot p_k = 28.011$ — [N/mm^2]

$f_{hk2} := 0.11 \cdot p_k \cdot d^{(-0.3)} = 29.607$

Load resistance:

$F_{vRk1} := f_{hk1} \cdot t \cdot d = 403.361$ — [N]

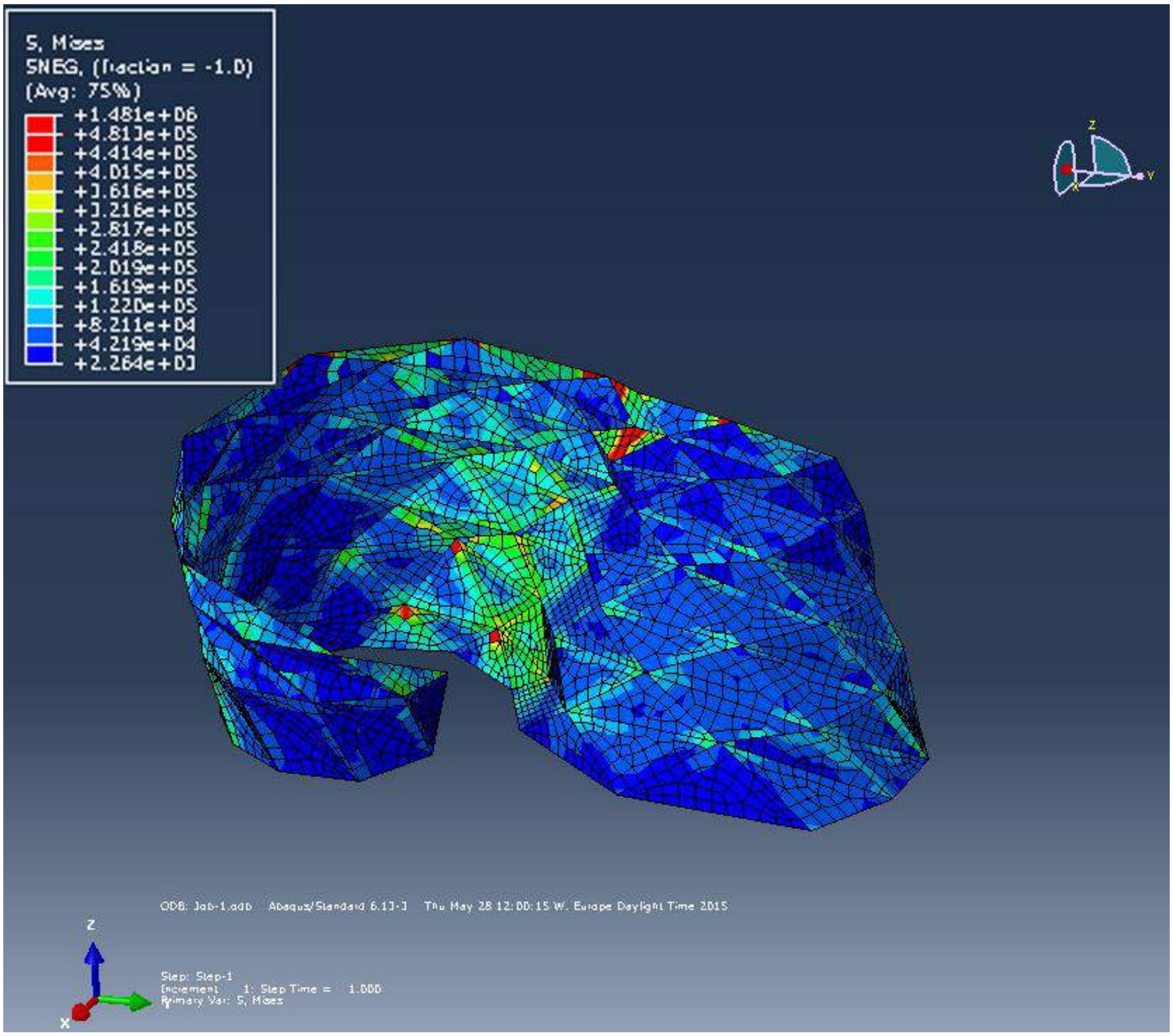
Estimation of needed edge distance

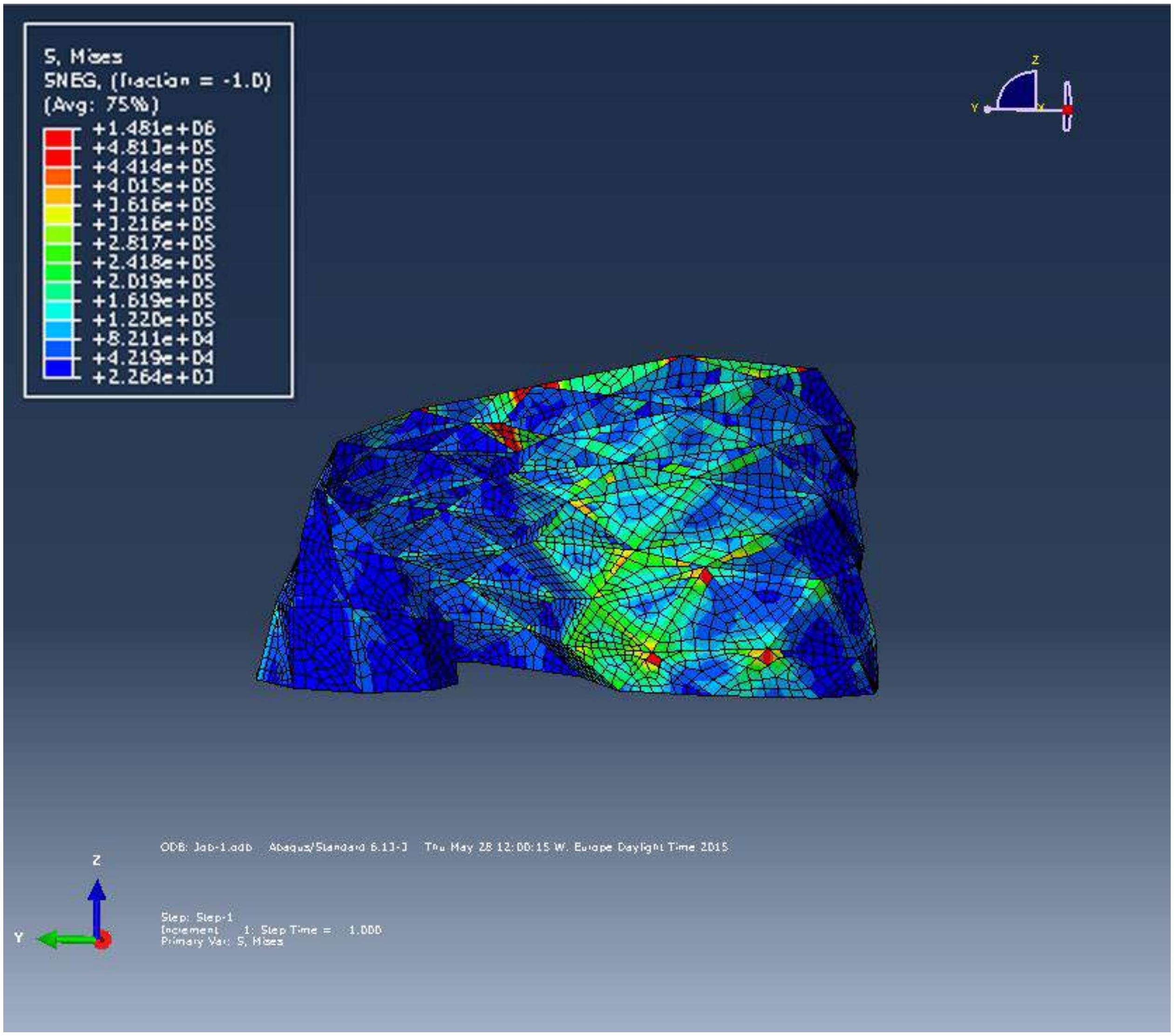
$$ratio := \frac{F_{vRk1}}{Force} = 3.735$$

$$\frac{a_{3t}}{ratio} = 4.498$$

Appendix A2

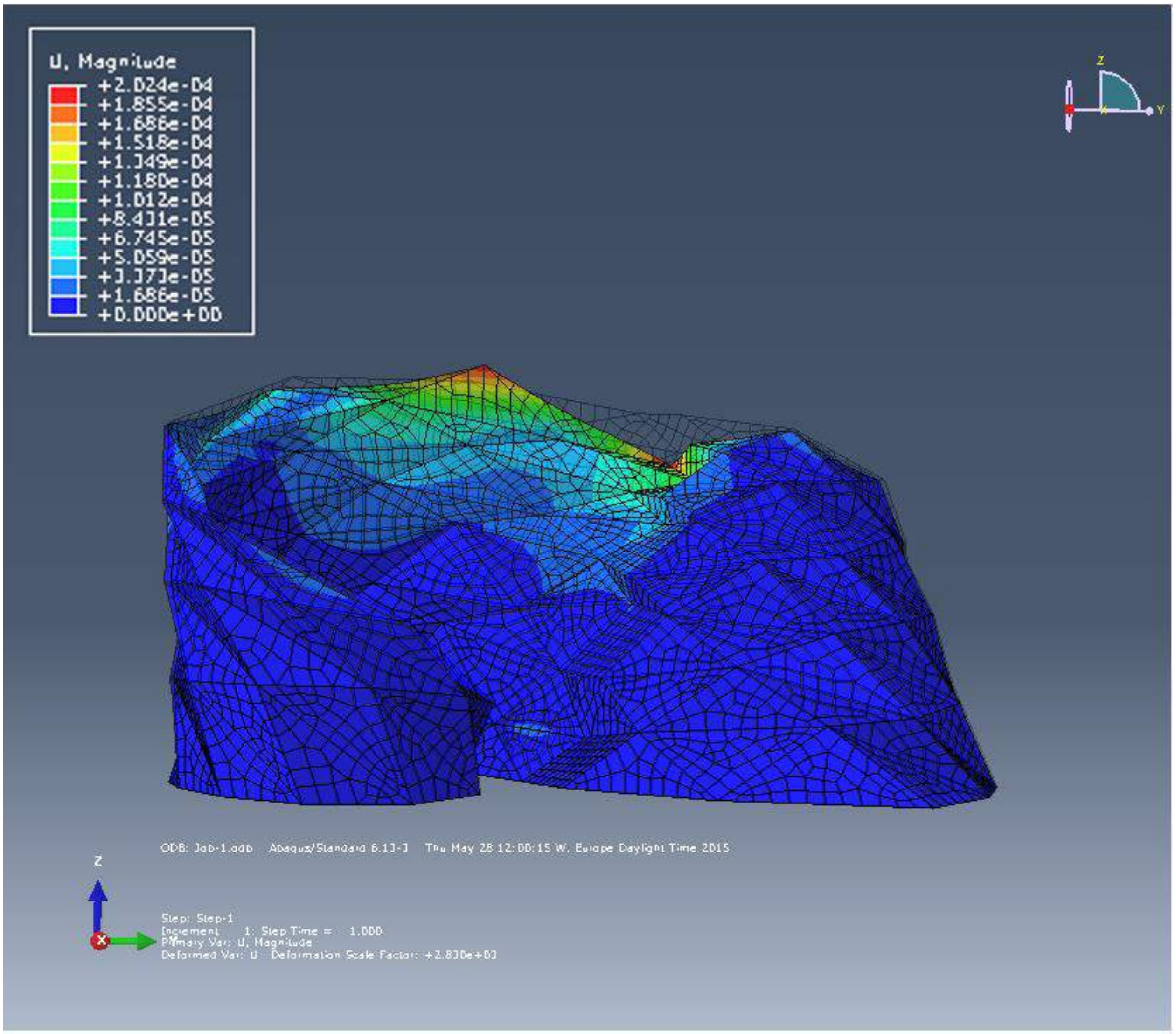
-FE analysis of the final structure showing Mises stress.




S, Mises
SNEG, (fraction = -1.0)
(Avg: 75%)
+1.481e+06
+4.813e+05
+4.414e+05
+4.015e+05
+3.616e+05
+3.216e+05
+2.817e+05
+2.418e+05
+2.019e+05
+1.619e+05
+1.220e+05
+8.211e+04
+4.219e+04
+2.264e+03
ODB: Job-1.odb Abaqus/Standard 6.13-3 Thu May 28 12:00:15 W. Europe Daylight Time 2015
Step: Step-1
Increment 1: Step Time = 1.000
Primary Var: S, Mises
Z
Y

Appendix A3

-FE analysis of the final structure showing deformation



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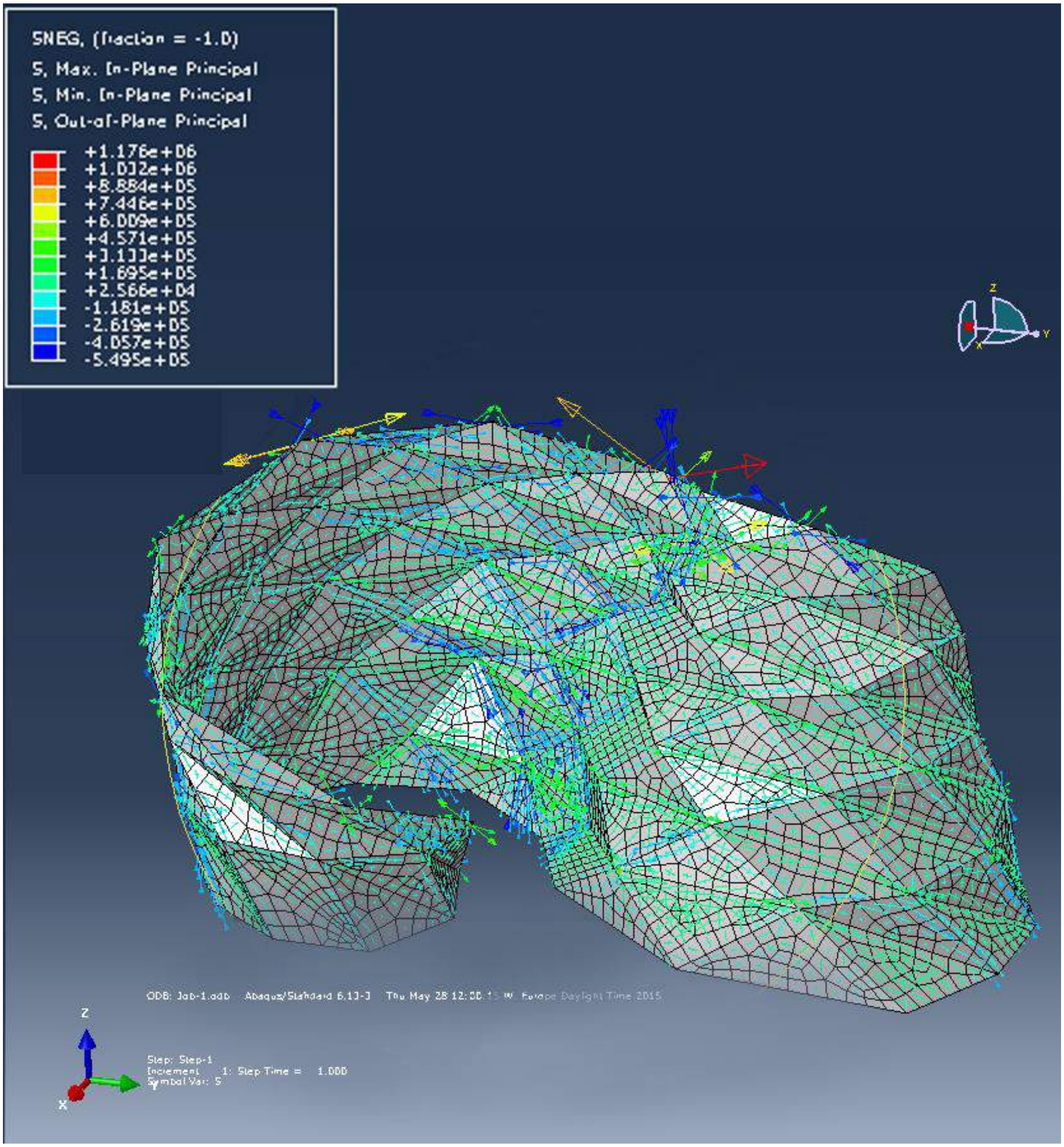
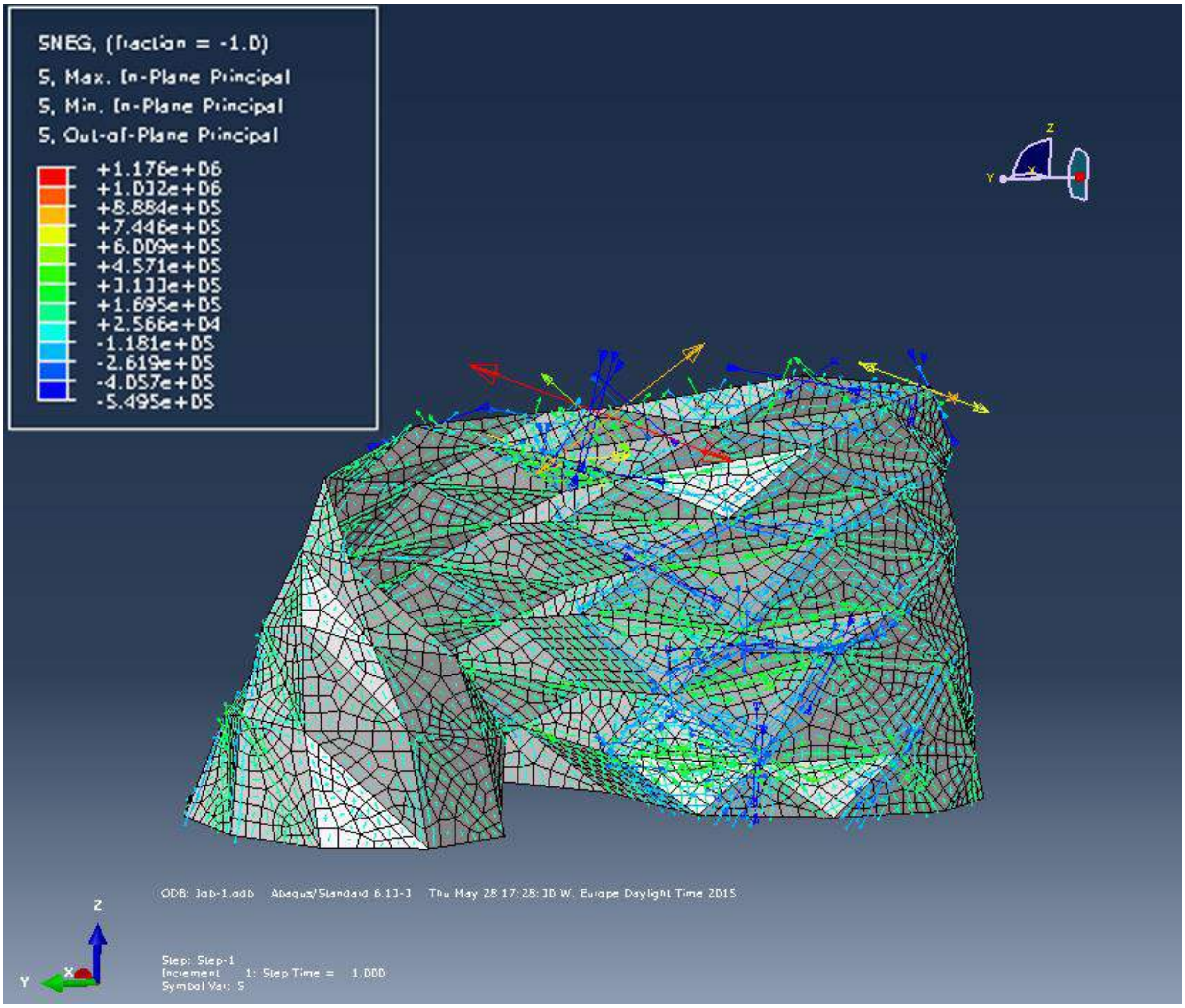
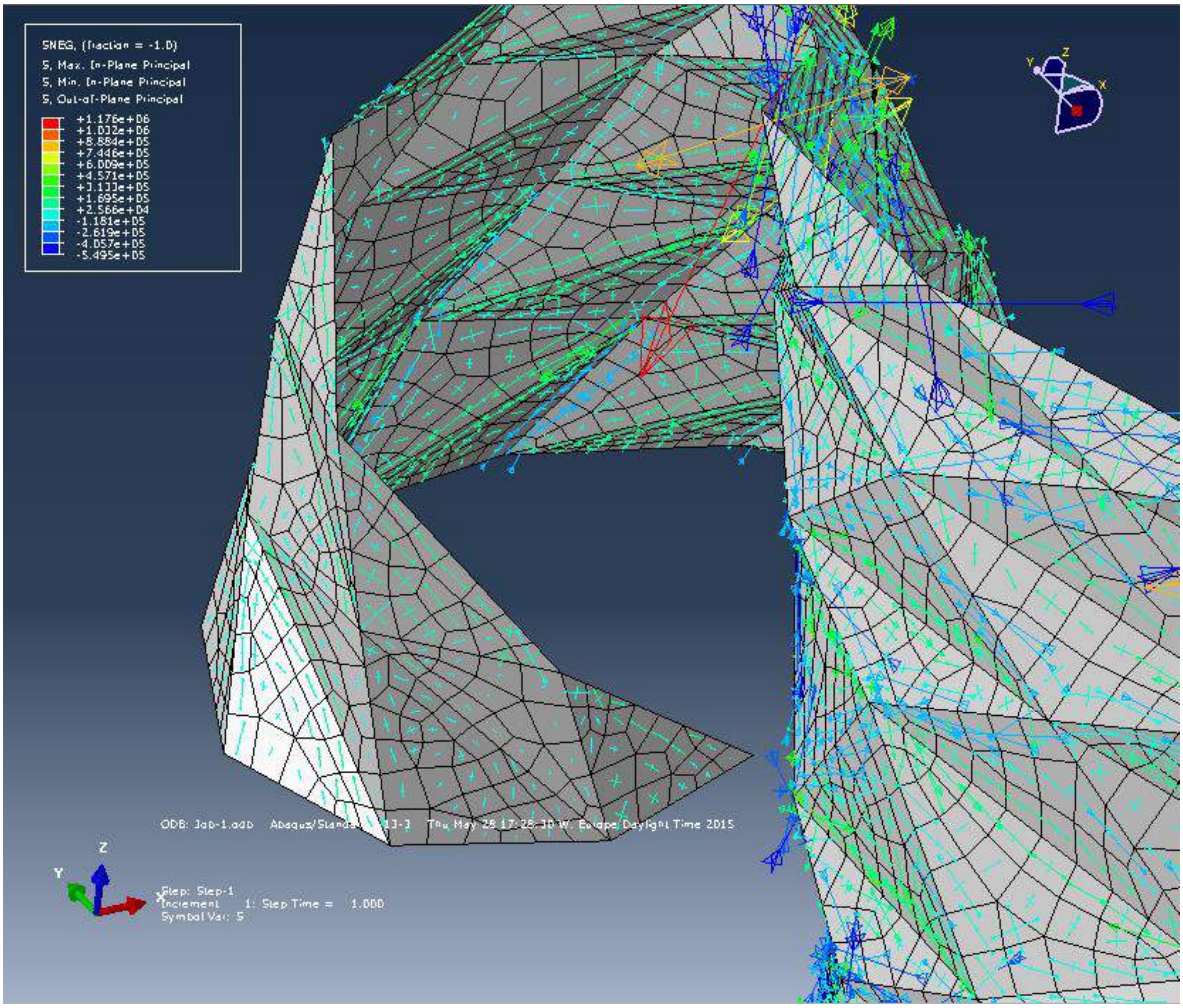
Appendix A4

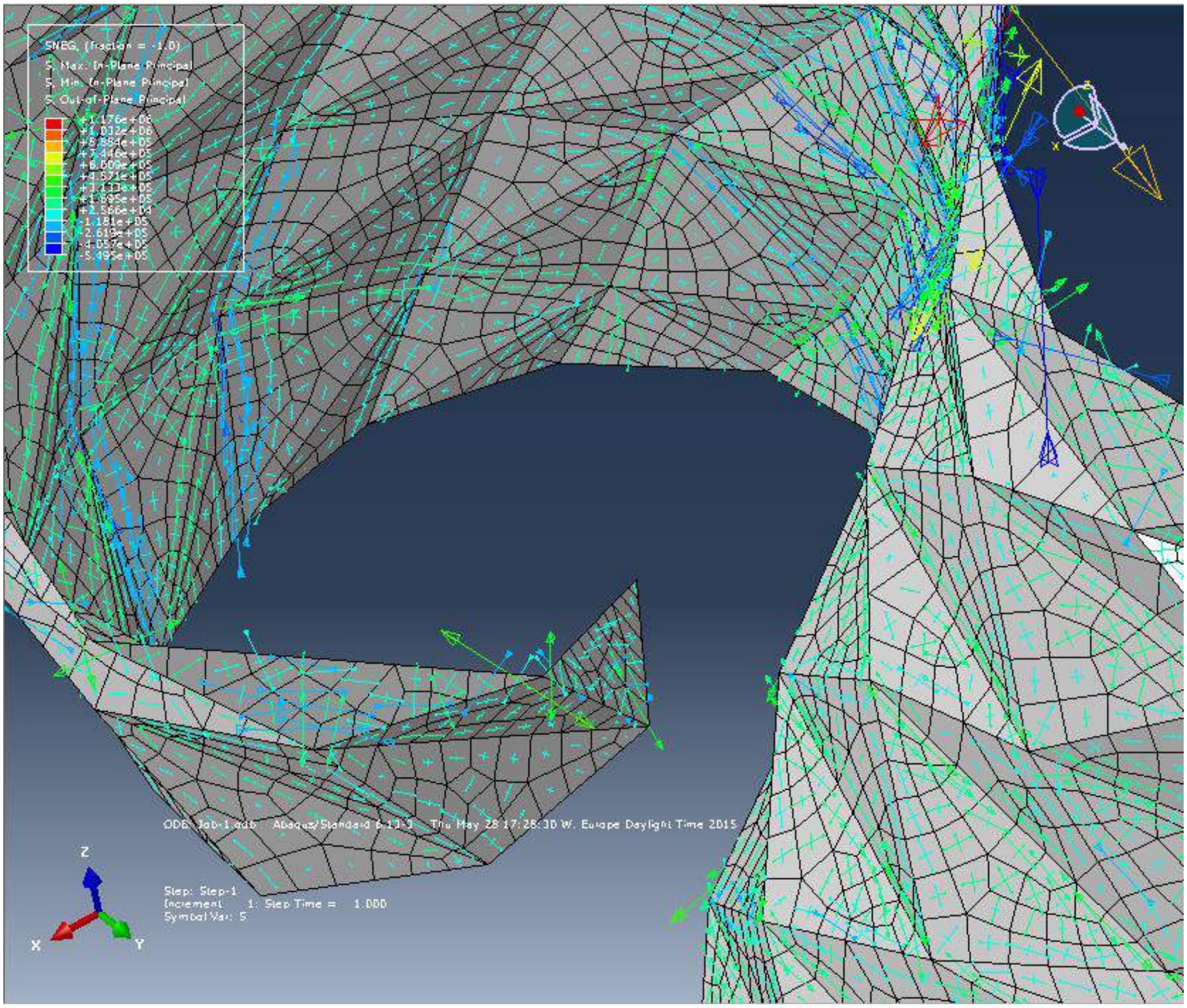
-FE analysis of the final structure showing principal stresses and their directions.





SNEG, (fraction = -1.0)
S, Max. In-Plane Principal
S, Min. In-Plane Principal
S, Out-of-Plane Principal
+1.176e+06
+1.032e+06
+8.884e+05
+7.446e+05
+6.009e+05
+4.571e+05
+3.133e+05
+1.695e+05
+2.566e+04
-1.181e+05
-2.619e+05
-4.057e+05
-5.495e+05
ODB: Job-1.odb Abaqus/Standard 6.13-3 Thu May 28 17:28:30 W. Europe Daylight Time 2015
Step: Step-1
Increment 1: Step Time = 1.000
Symbol Var: S

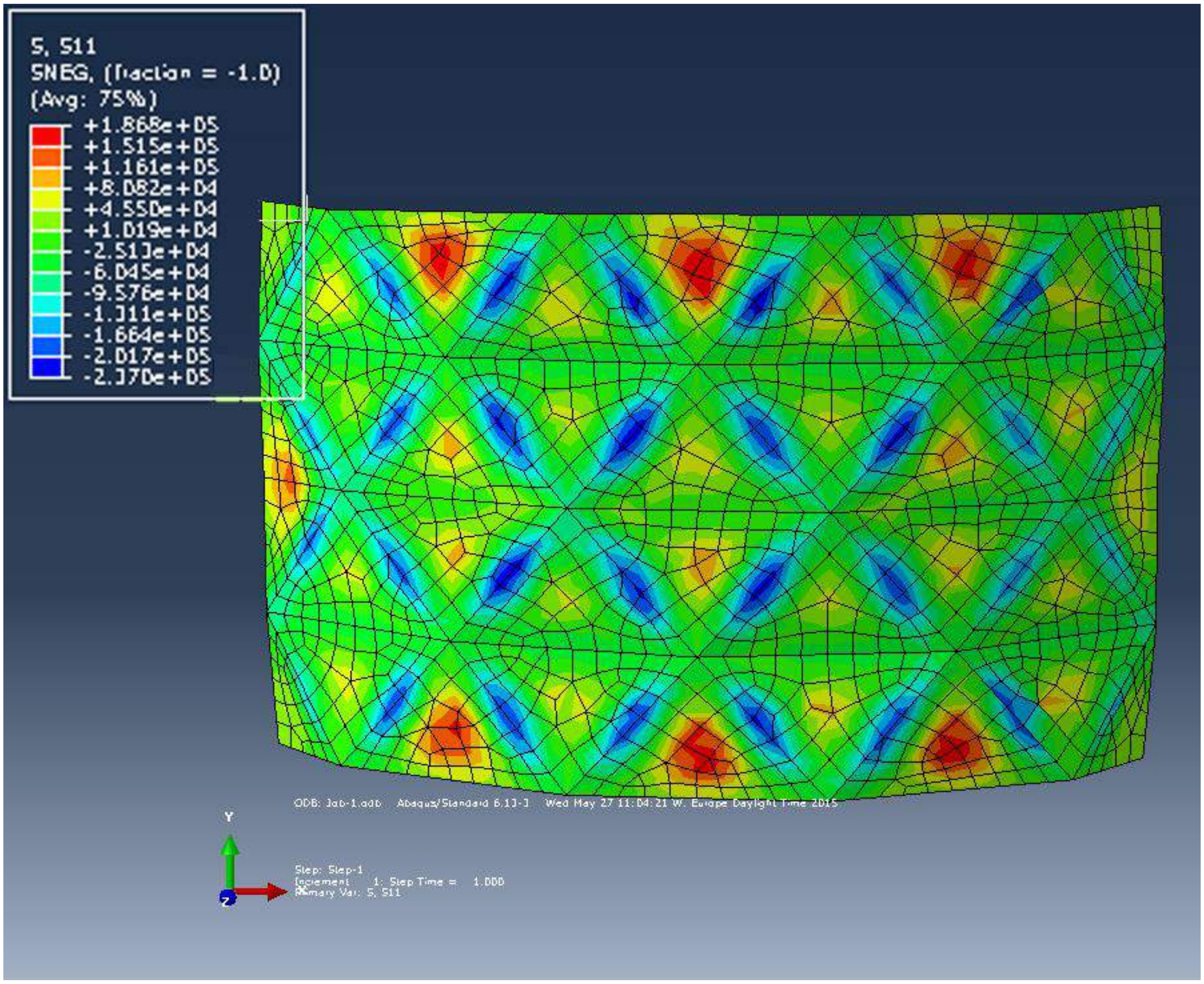

SNEG, (fraction = -1.0)
S, Max. In-Plane Principal
S, Min. In-Plane Principal
S, Out-of-Plane Principal
+1.176e+06
+1.032e+06
+8.884e+05
+7.446e+05
+6.009e+05
+4.571e+05
+3.133e+05
+1.695e+05
+2.566e+04
-1.181e+05
-2.619e+05
-4.057e+05
-5.495e+05
ODB: Job-1.odb Abaqus/Standard 13-3 Thu May 28 17:28:30 W. Europe Daylight Time 2015
Step: Step-1
Increment 1: Step Time = 1.000
Symbol Var: S
Z
Y
X


SNEG, (fraction = -1.0)
S, Max. In-Plane Principal
S, Min. In-Plane Principal
S, Out-of-Plane Principal
+1.176e+06
+1.032e+06
+8.884e+05
+7.446e+05
+6.009e+05
+4.571e+05
+3.133e+05
+1.695e+05
+2.566e+04
-1.181e+05
-2.619e+05
-4.057e+05
-5.495e+05
ODB: Job-1.odb Abaqus/Standard 6.13-1 Thu May 28 17:28:30 W. Europe Daylight Time 2015
Step: Step-1
Increment 1: Step Time = 1.000
Symbol Var: S
Z
X
Y

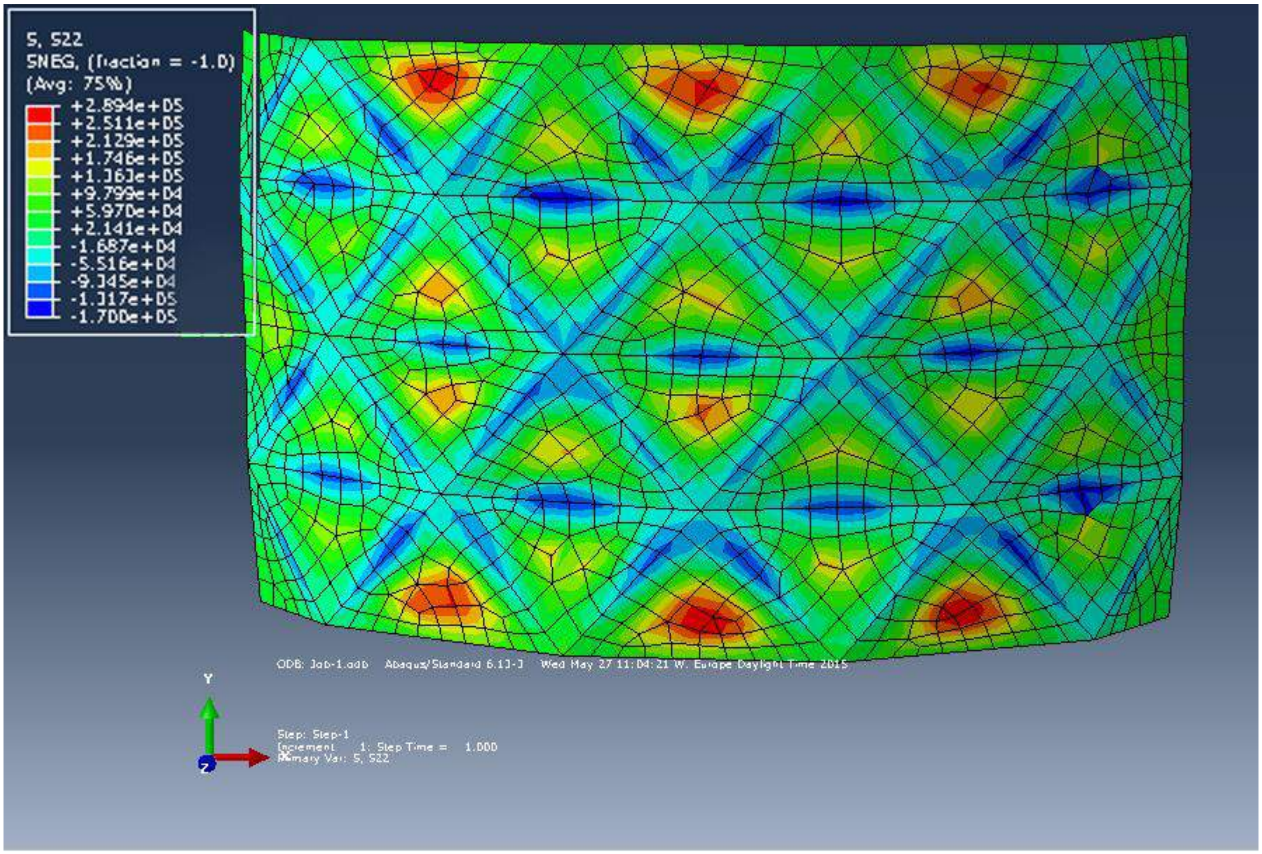
Appendix A5

-FE analysis of the diamond pattern showing S11, S22 and principal stress arrows for hinged and fix connections.

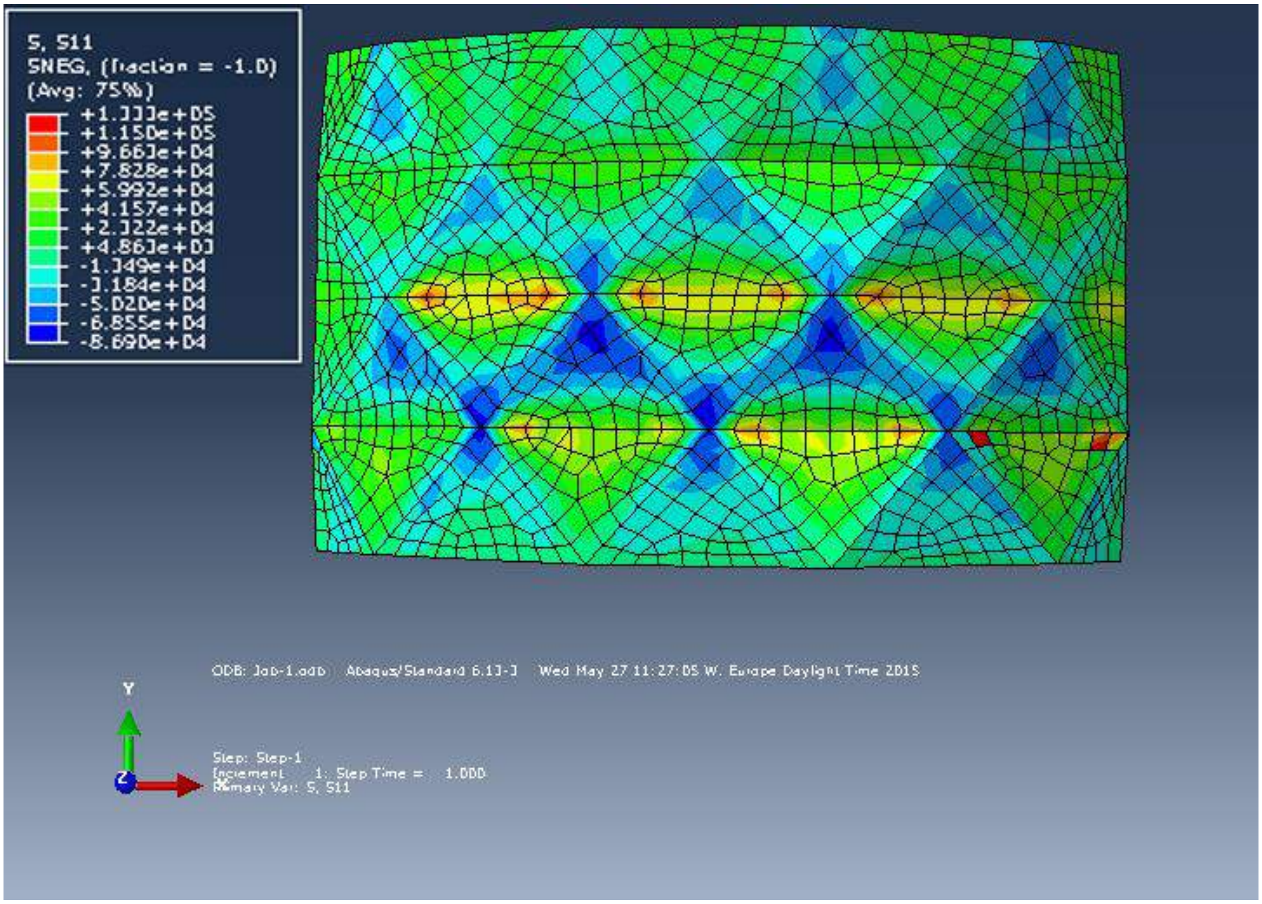
Diamond, fix, S11, load in z-direction:



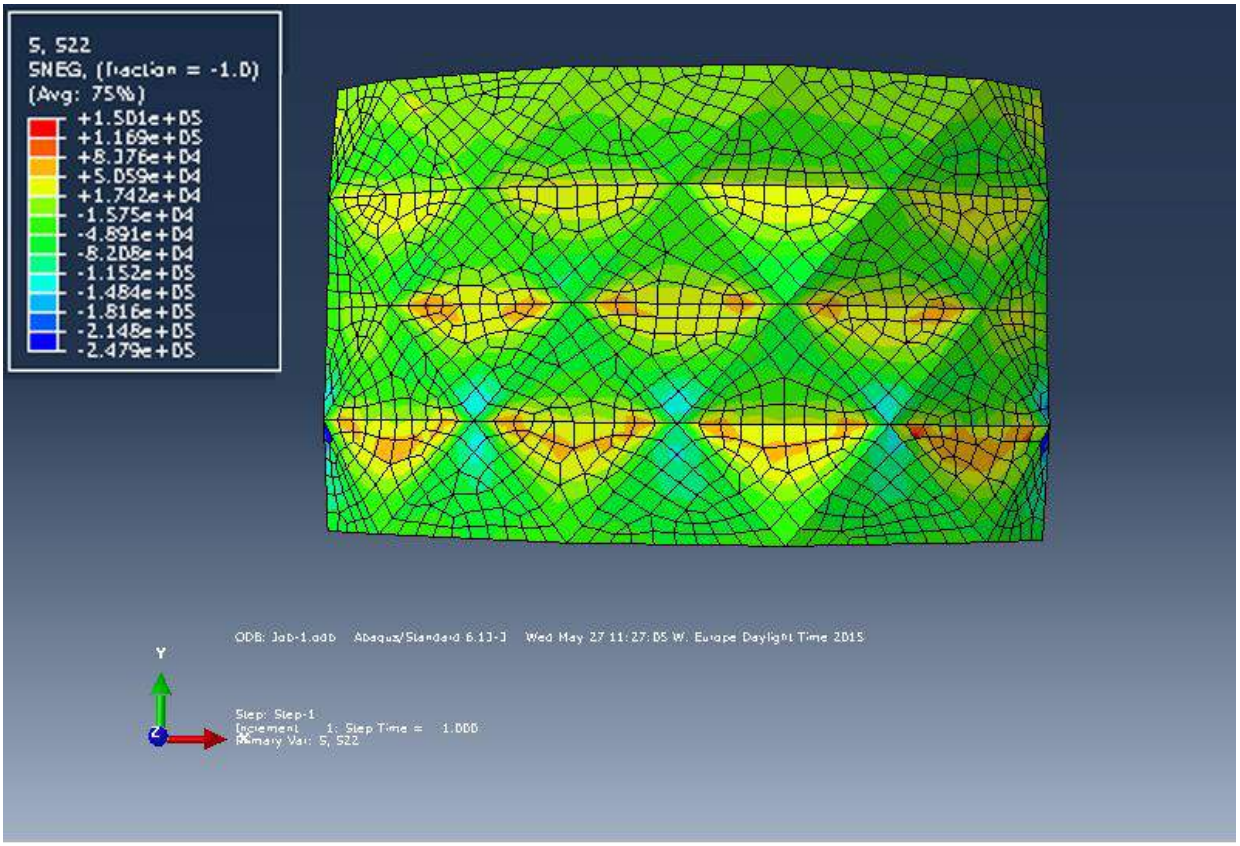
Diamond, fix, S22, load in z-direction:



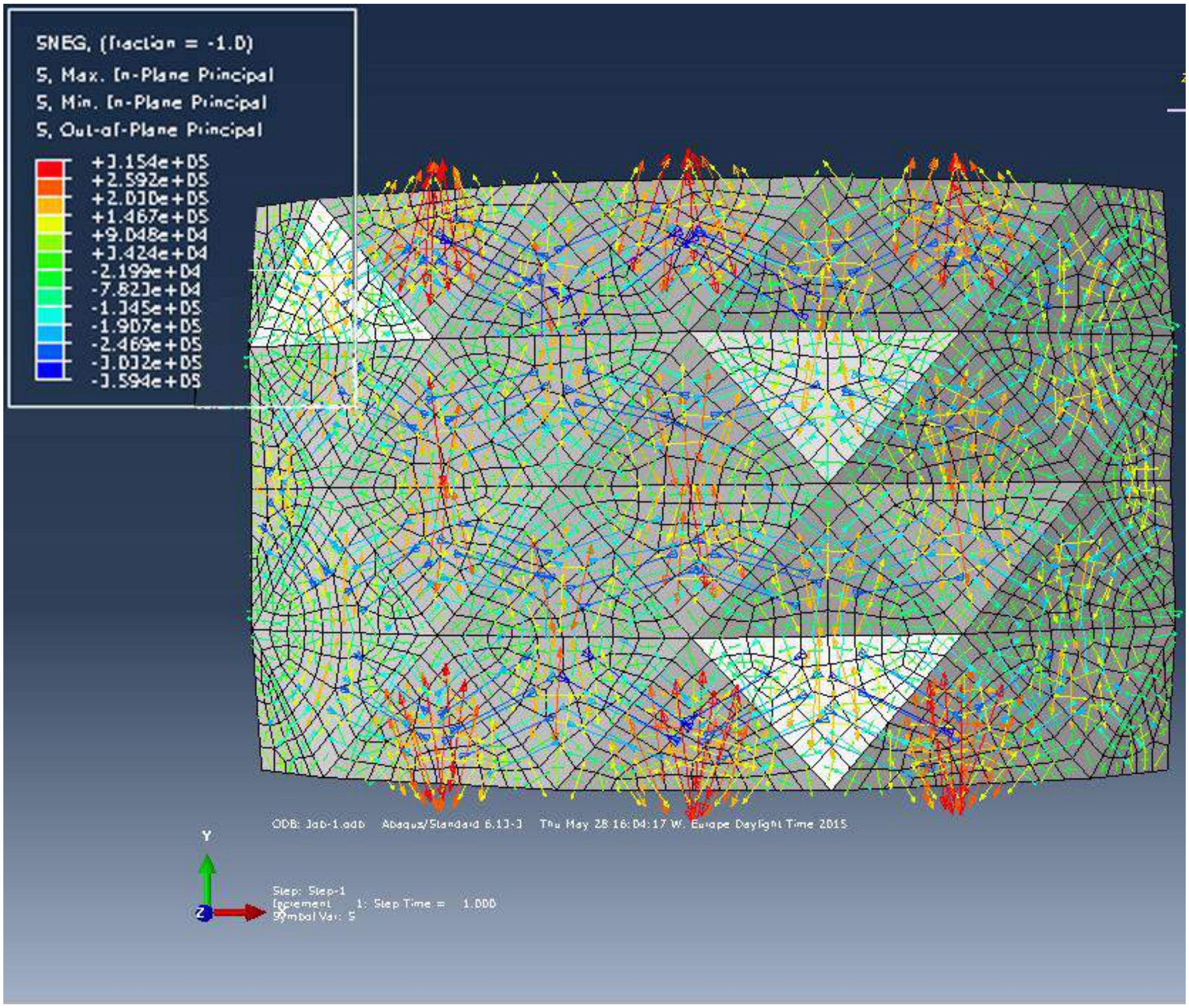
Diamond, fix, S11, load in y-direction:



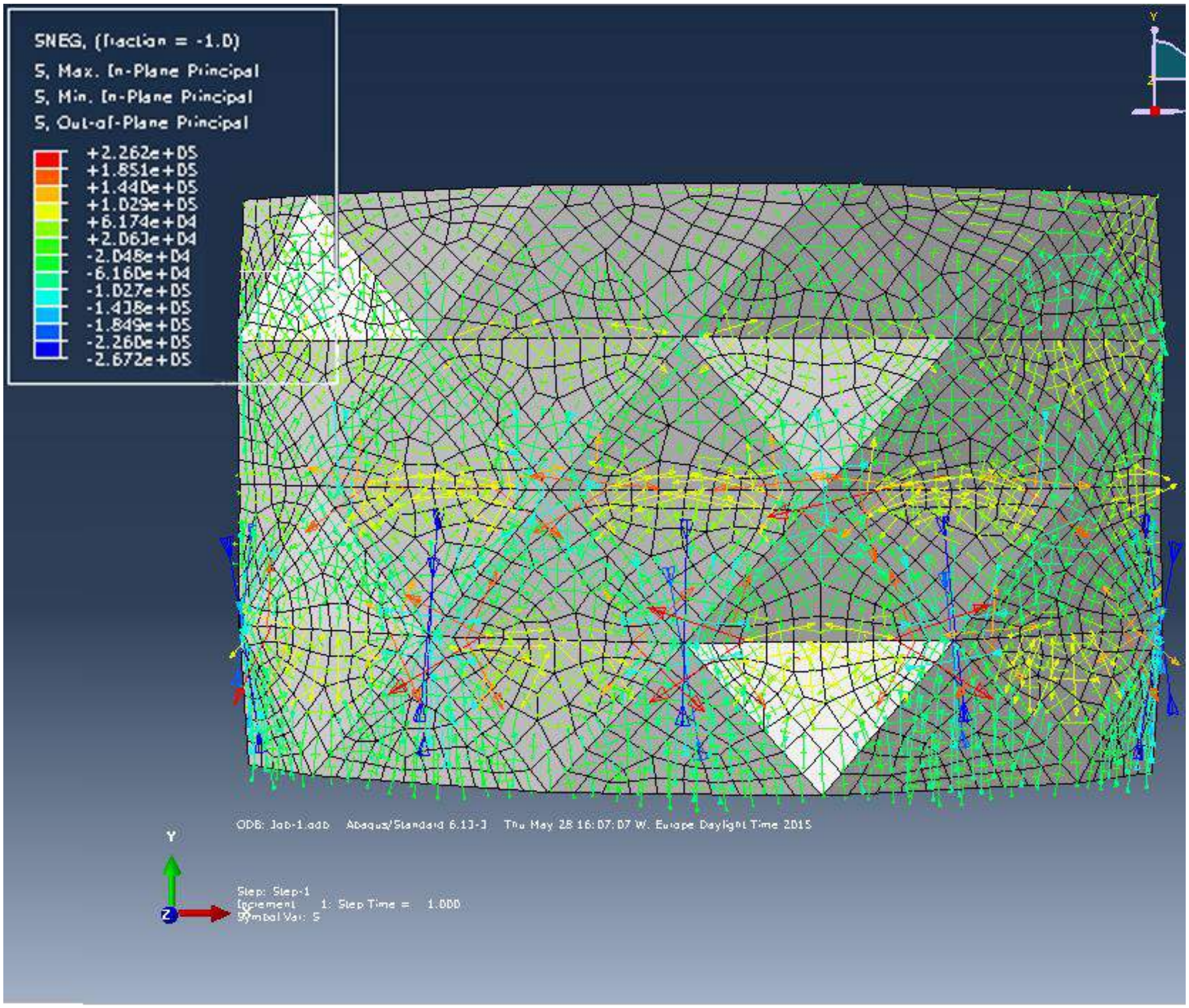
Diamond, fix, S22, load in y-direction:



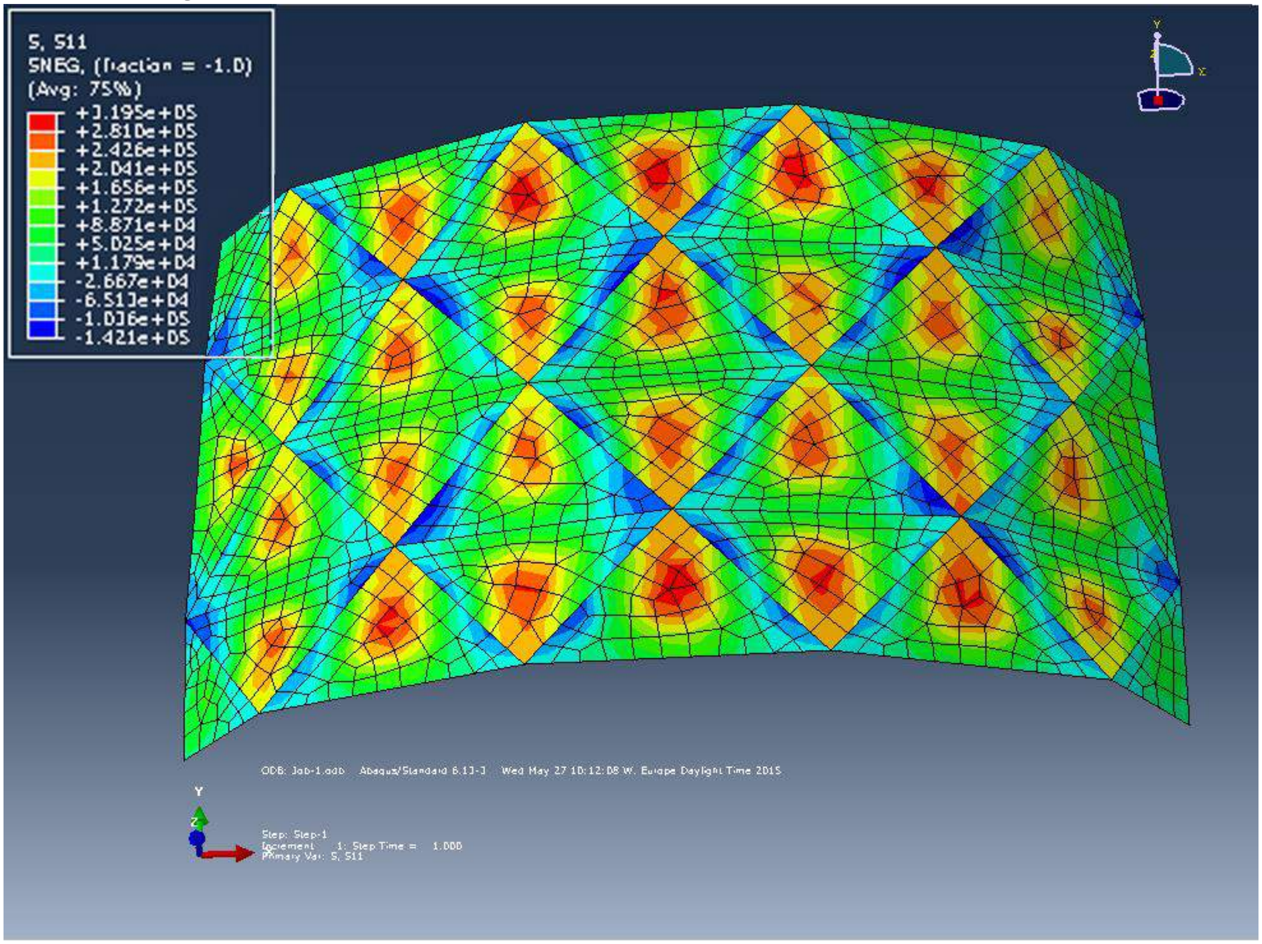
Diamond, fix, principal stress, load in z-direction:



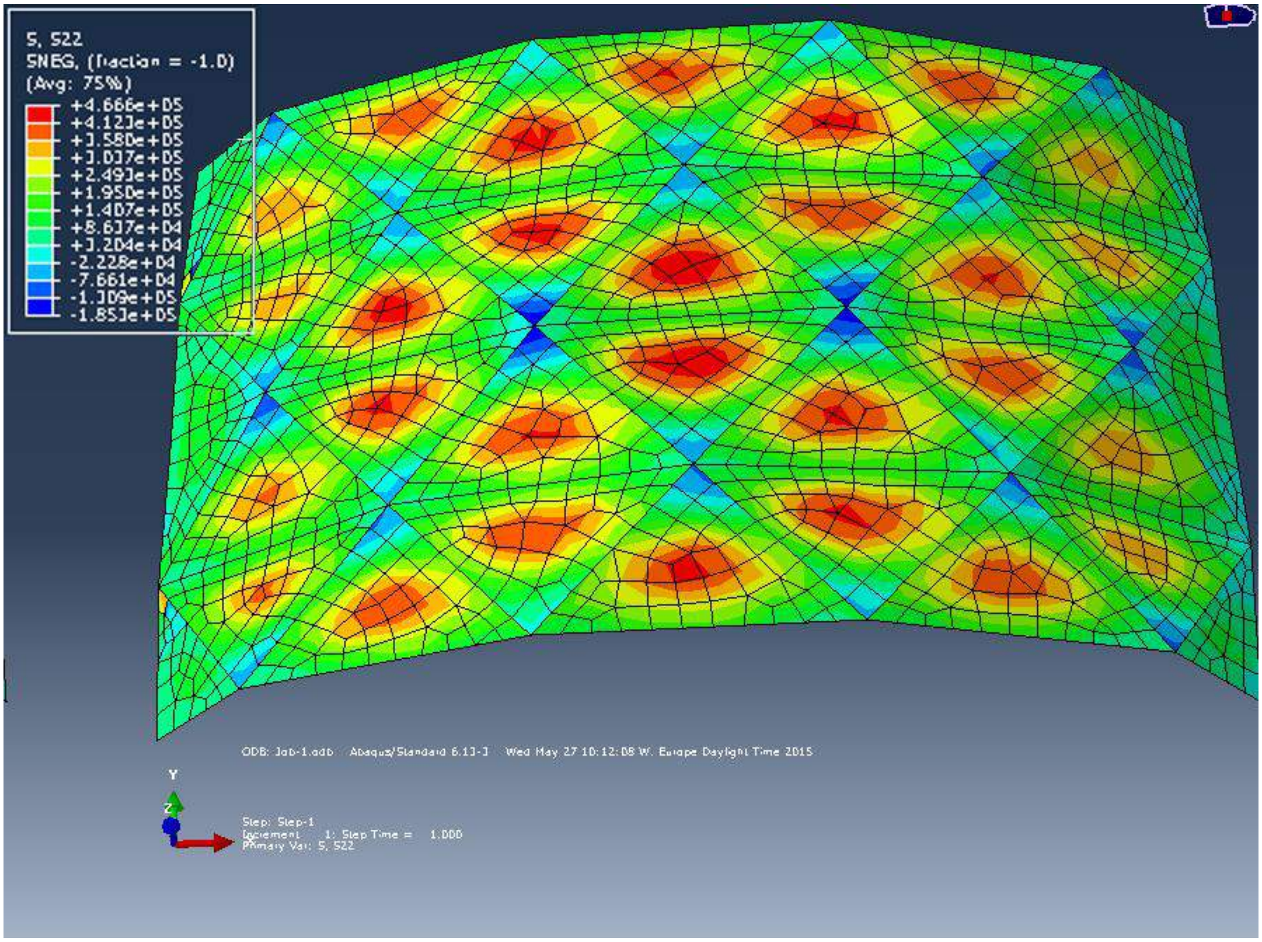
Diamond, fix, principal stress, load in y-direction:



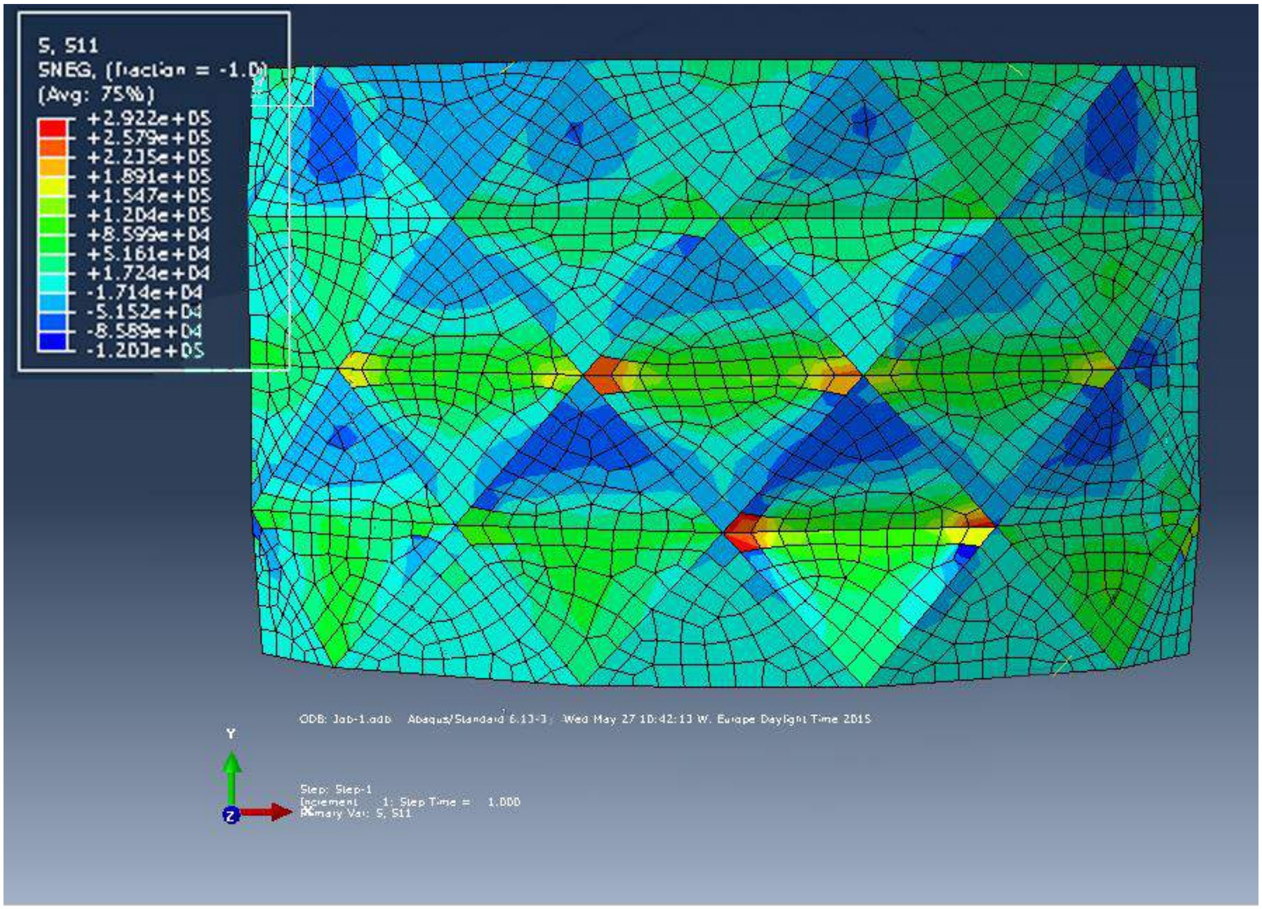
Diamond, hinged, S11, load in z-direction:



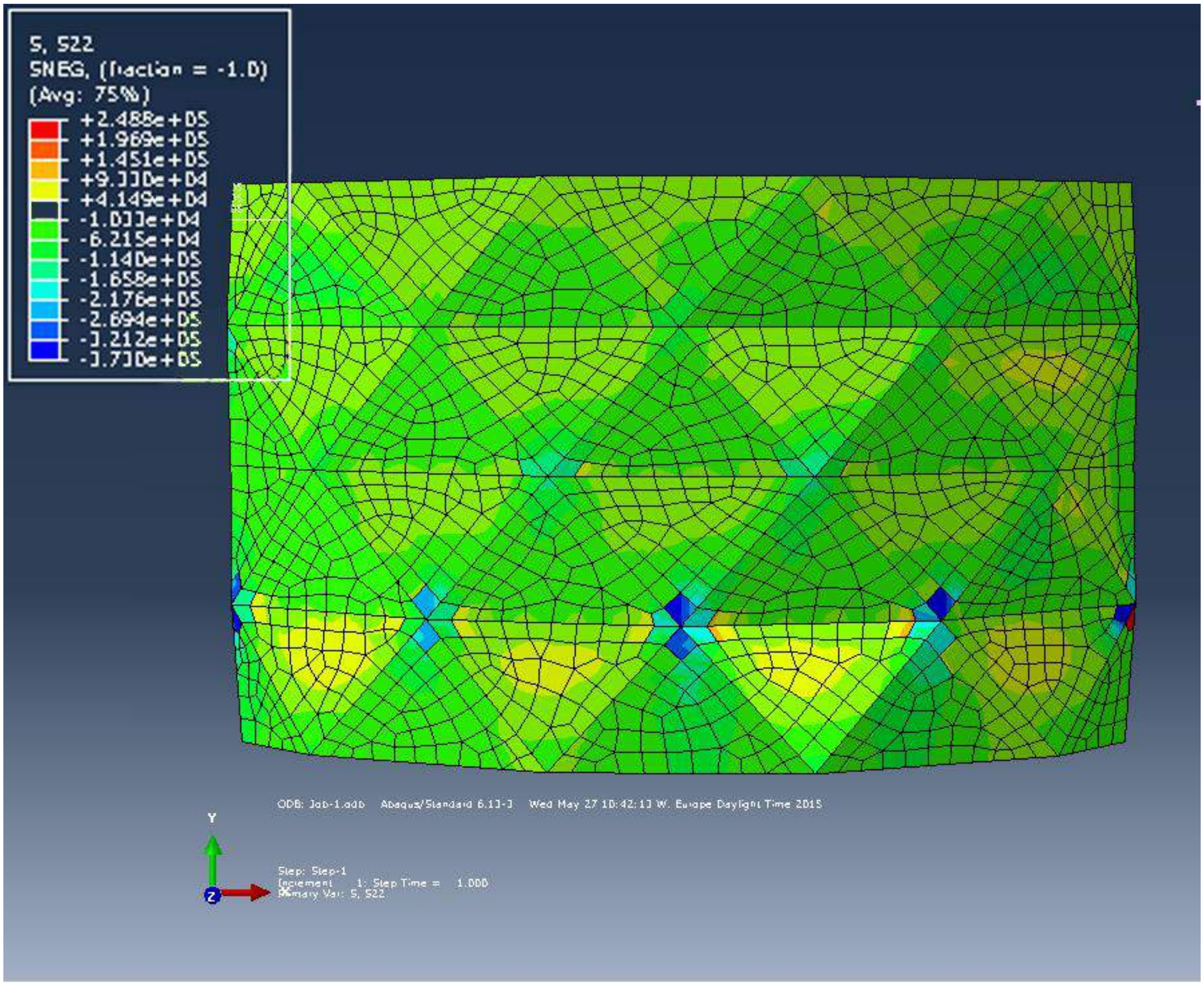
Diamond, hinged, S22, load in z-direction:



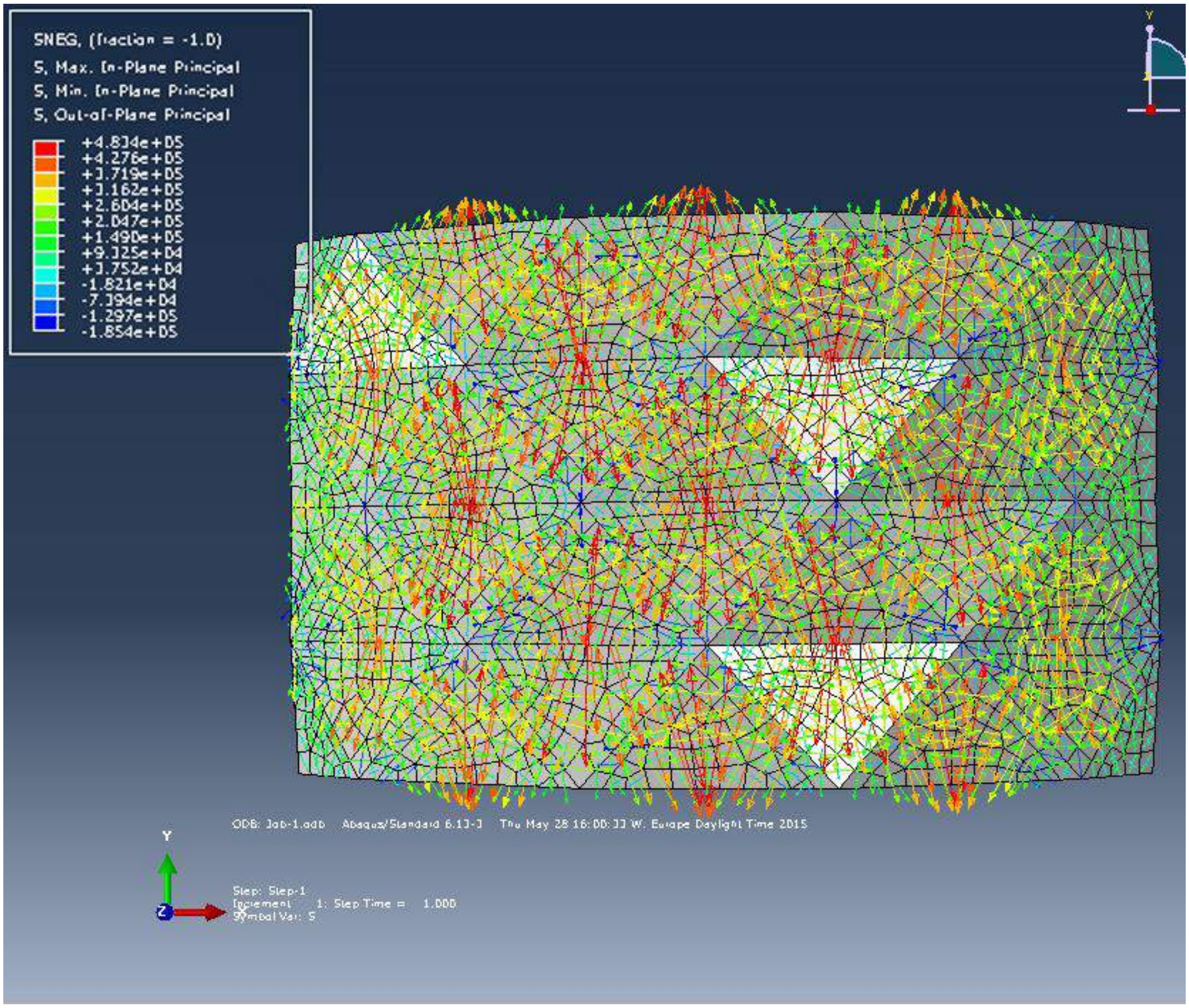
Diamond, hinged, S11, load in y-direction:



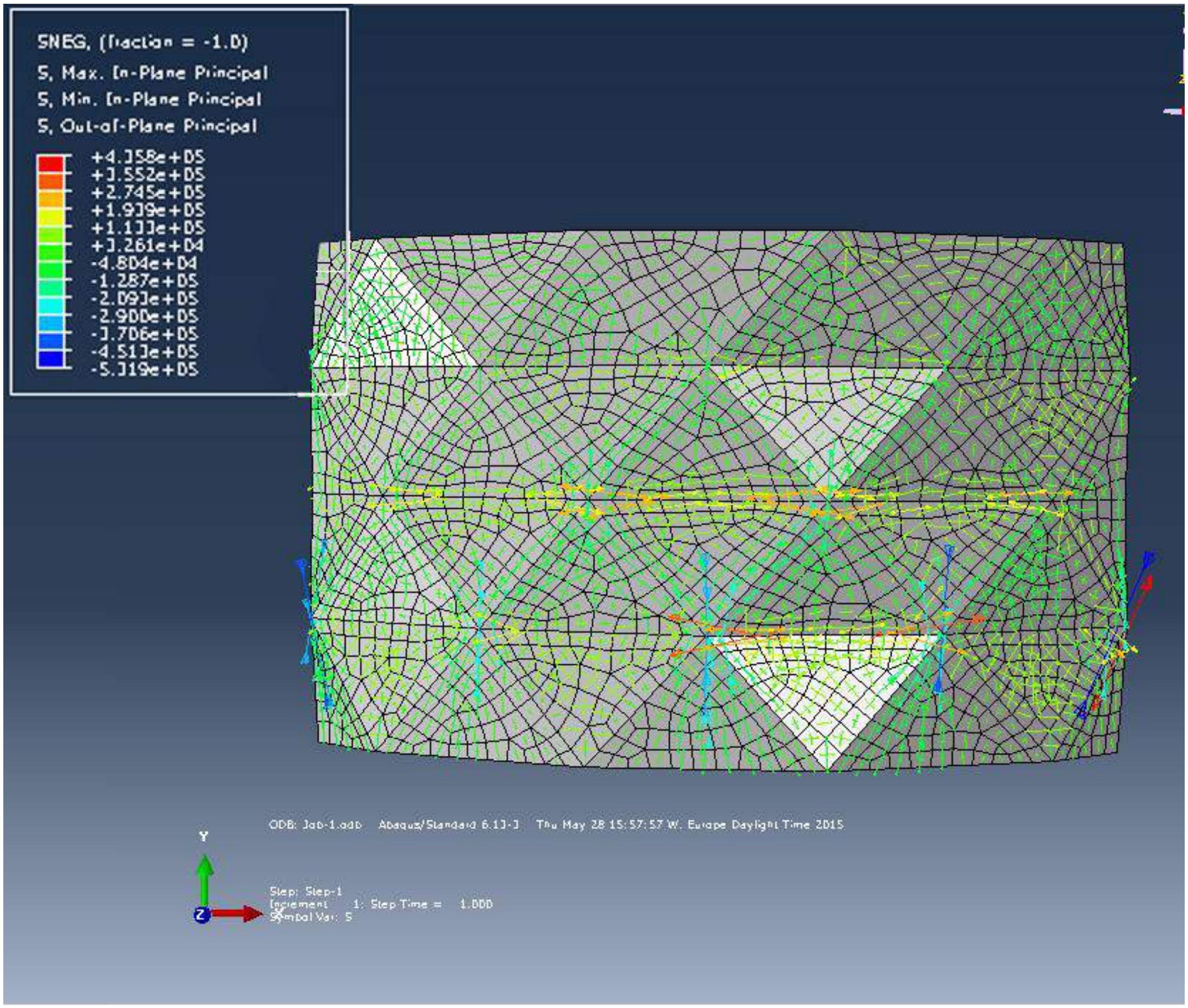
Diamond, hinged, S22, load in y-direction:



Diamond, hinged, principal stress, load in z-direction:



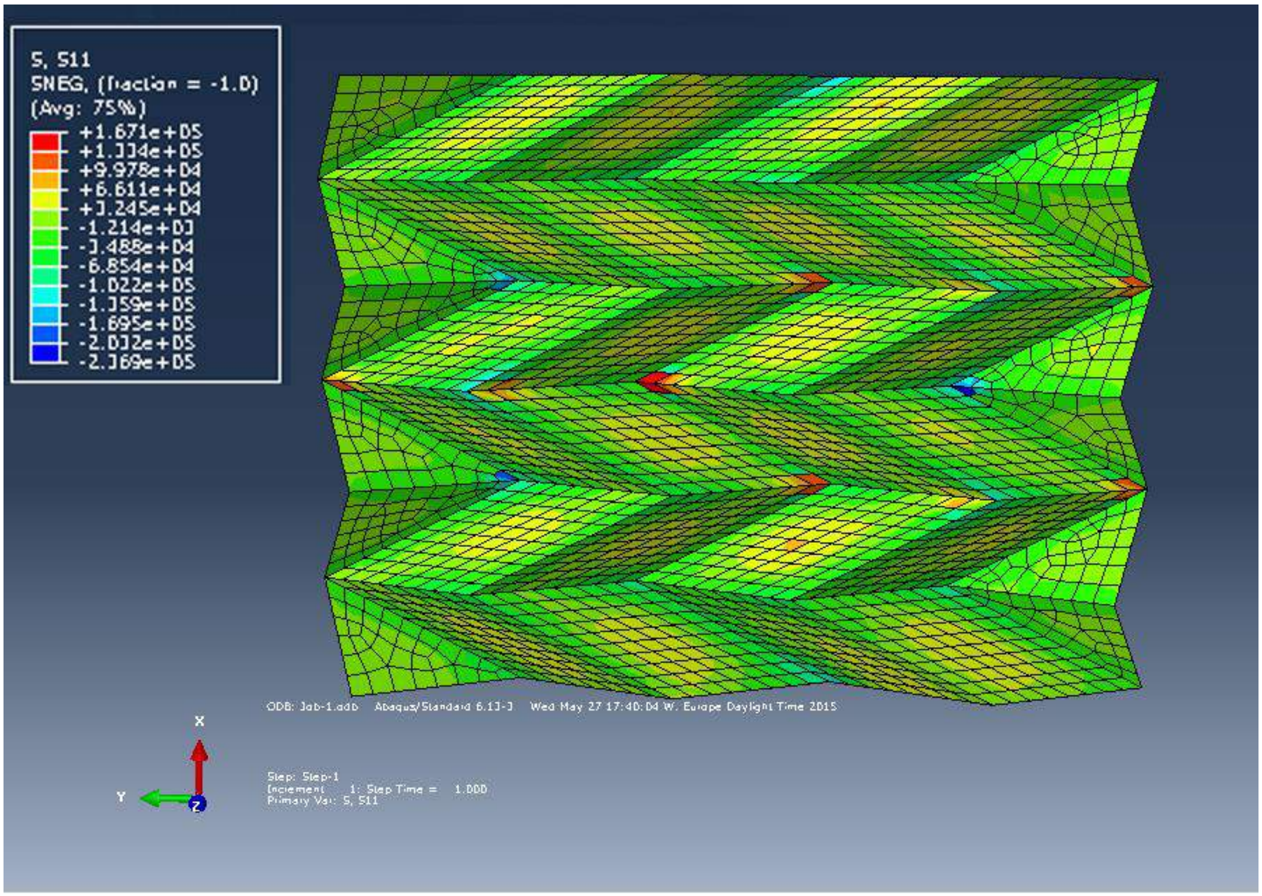
Diamond, hinged, Principal stress, load in y-direction:



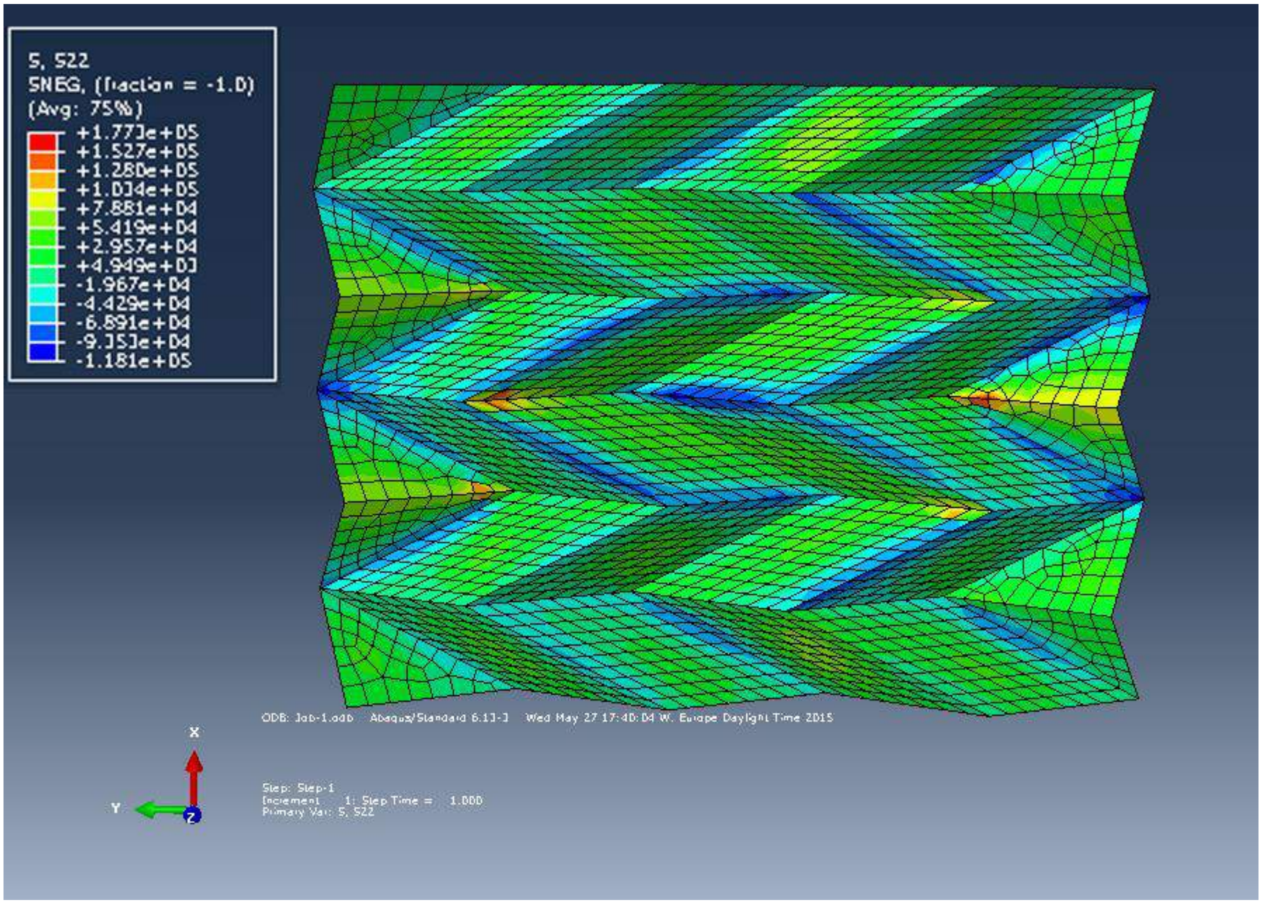
Appendix A6

-FE analysis of the harringbone pattern showing S11, S22 and principal stress arrows for hinged and fix connections.

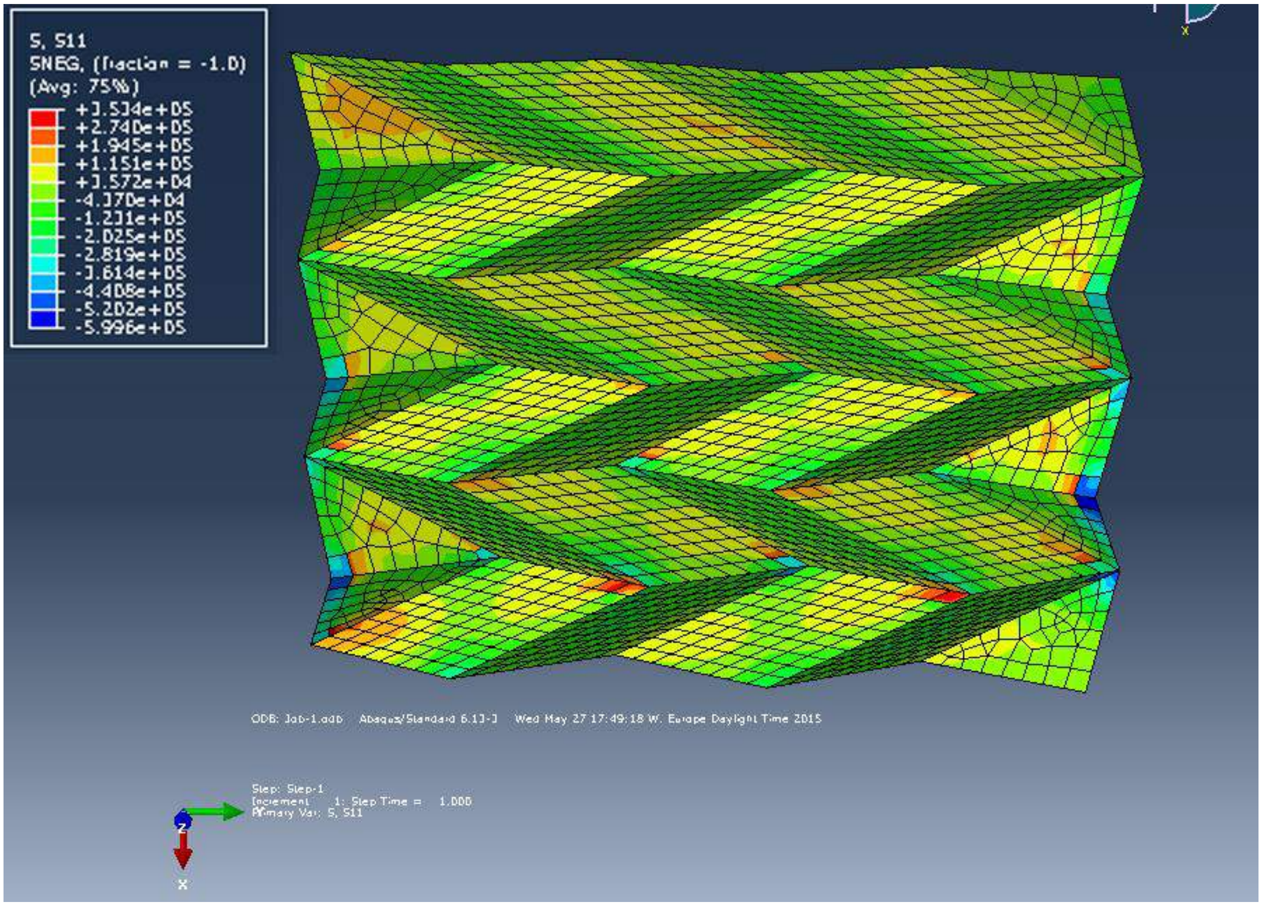
Herrringbone, fix, S11, load in z-direction:



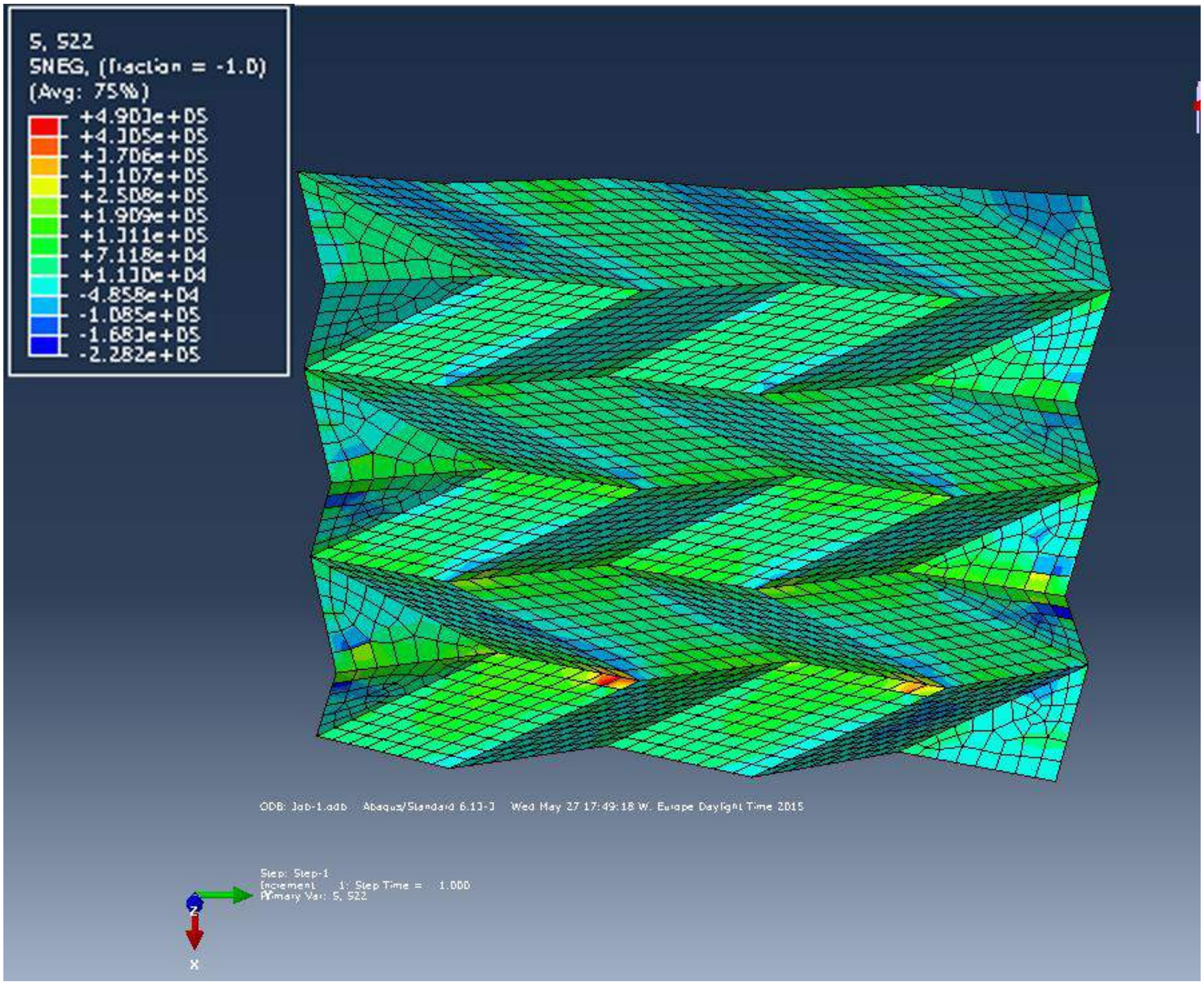
Herrringbone, fix, S22, load in z-direction:



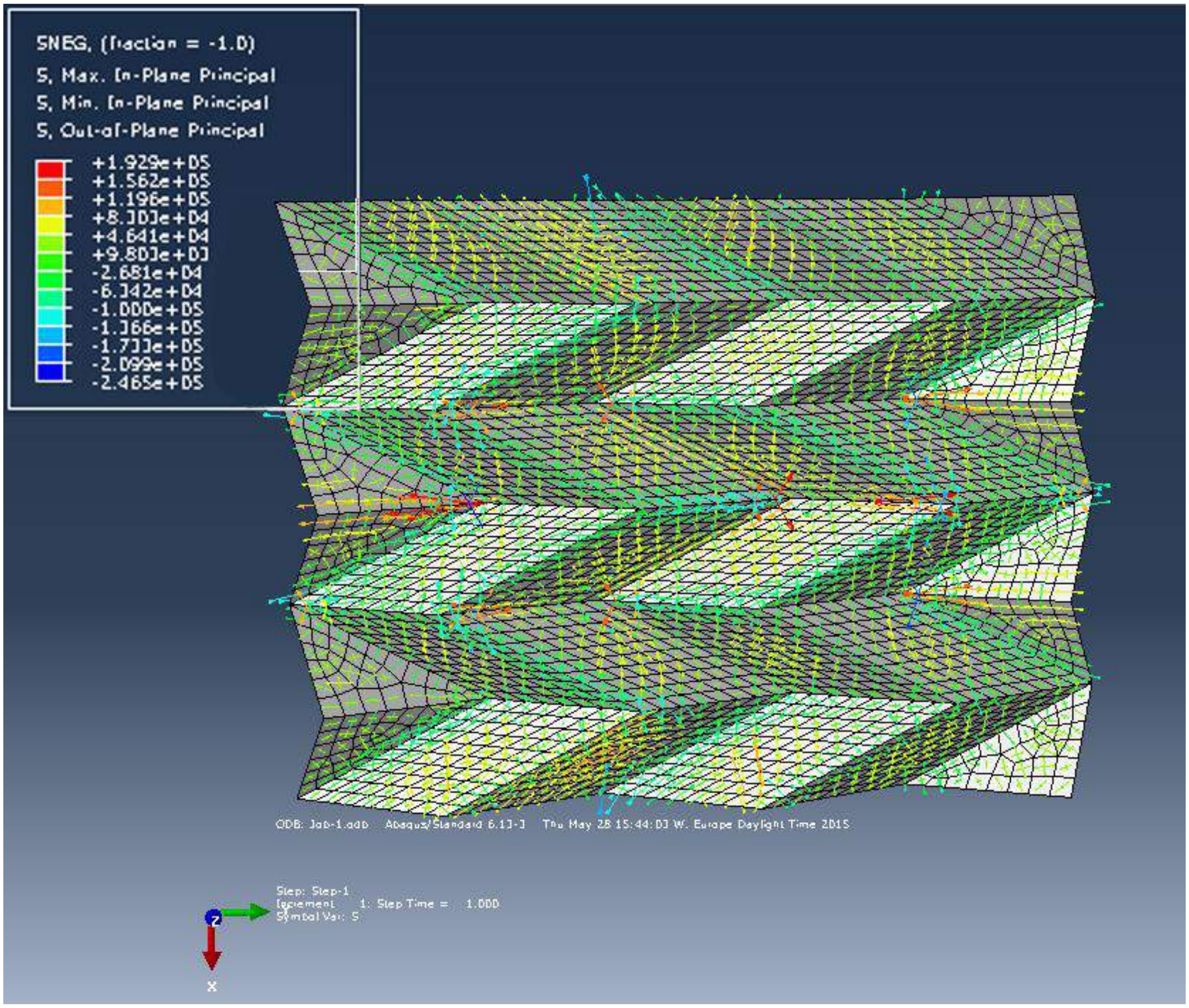
Herrringbone, fix, S11, load in x-direction:



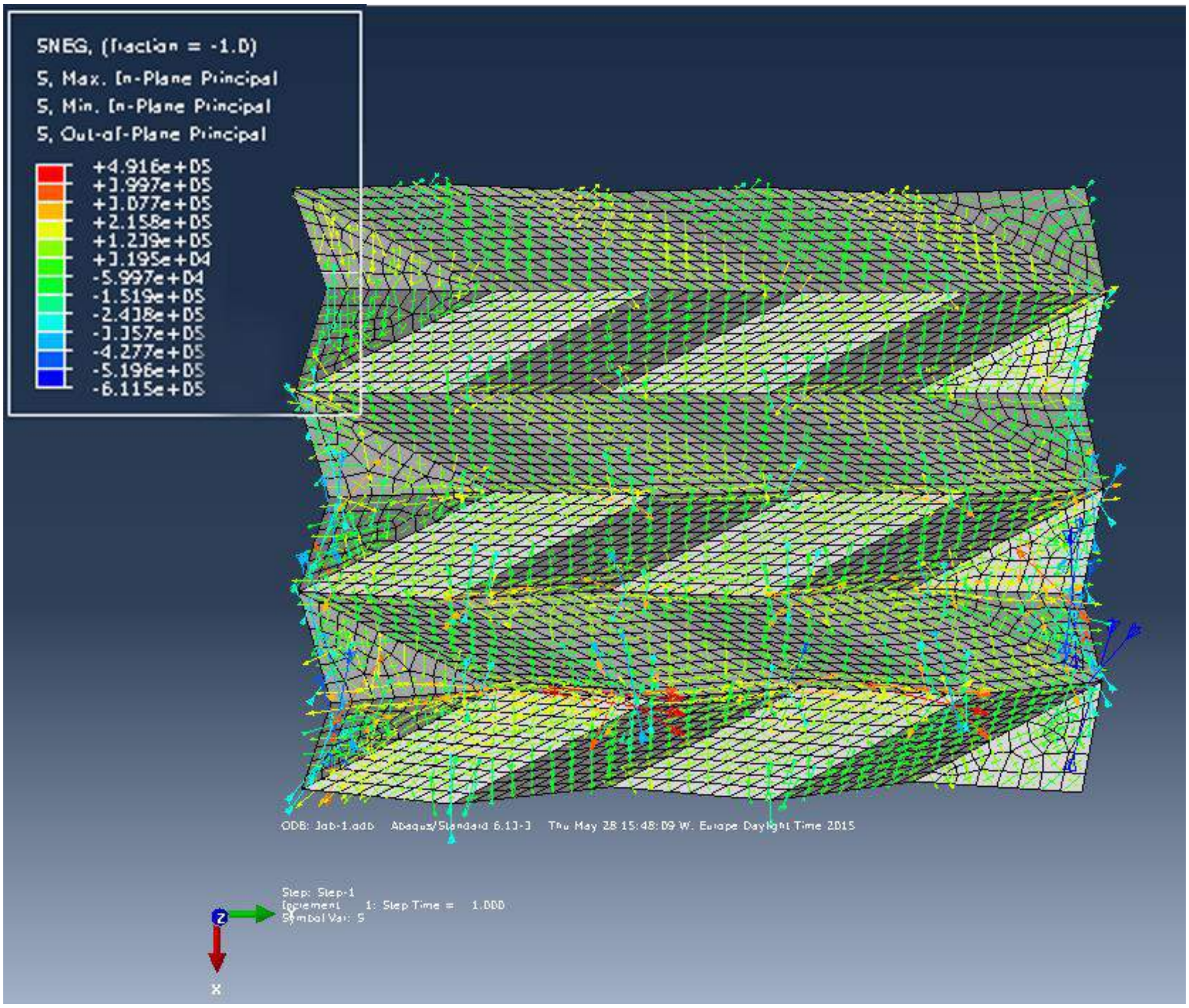
Herrringbone, fix, S22, load in x-direction:



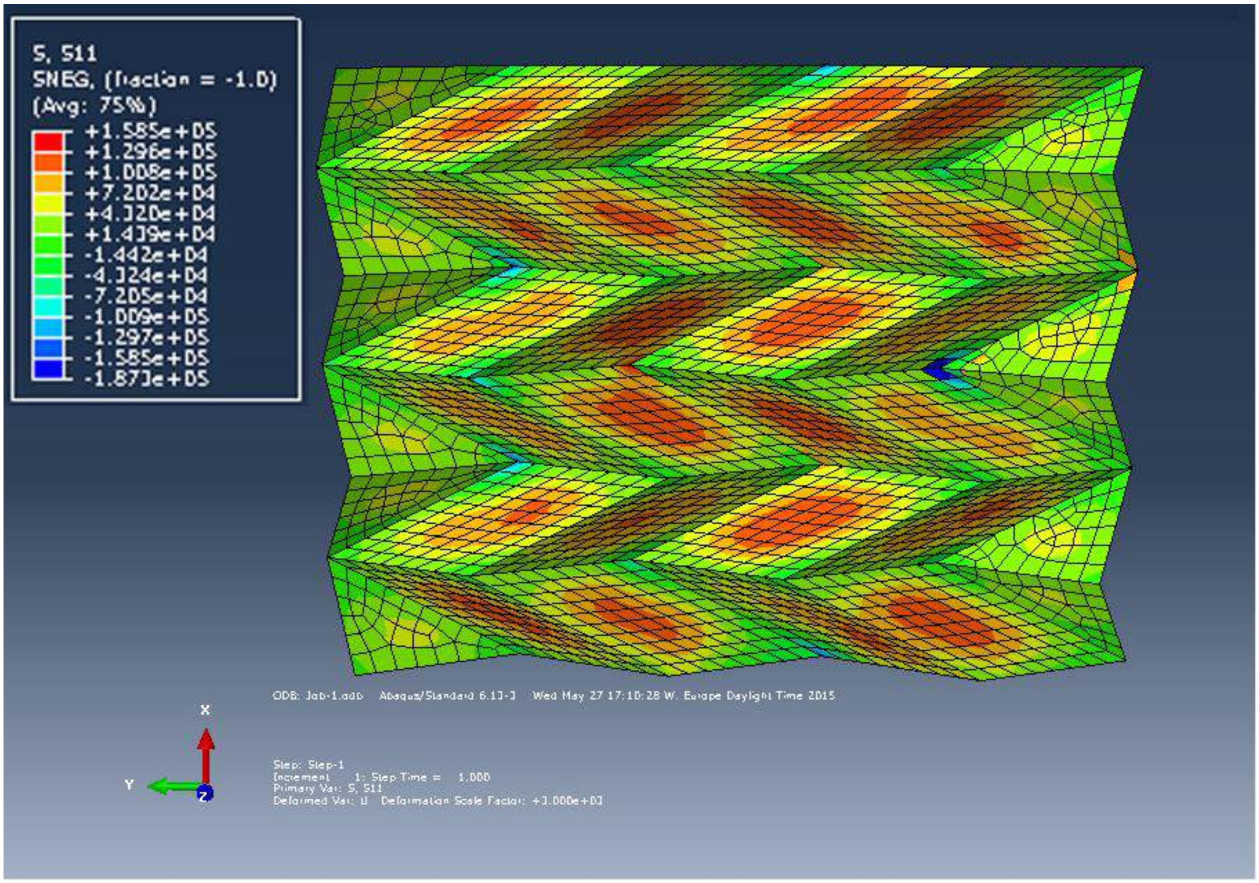
Herrringbone, fix, principal stresses, load in z-direction:



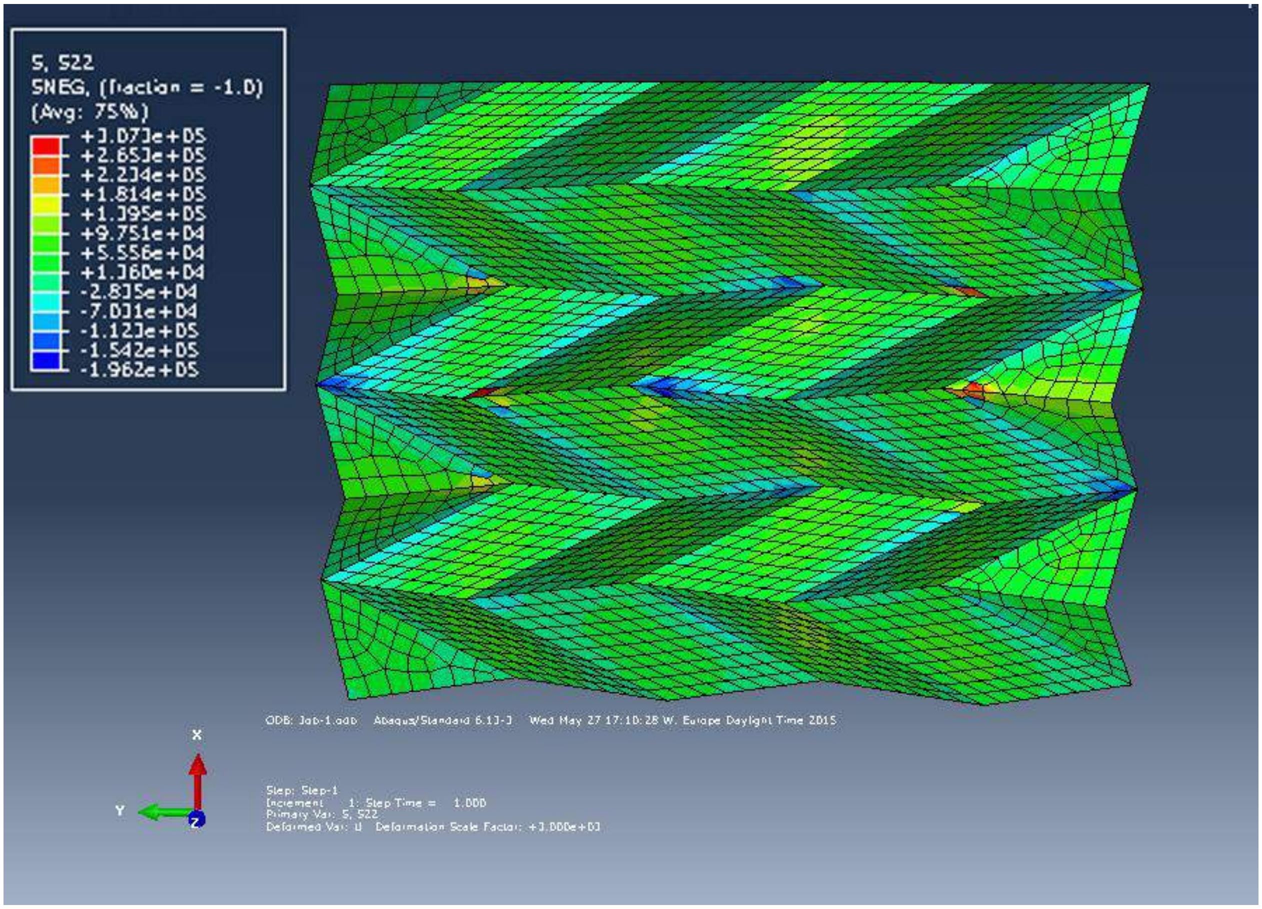
Herrringbone, fix, principal stresses, load in x-direction:



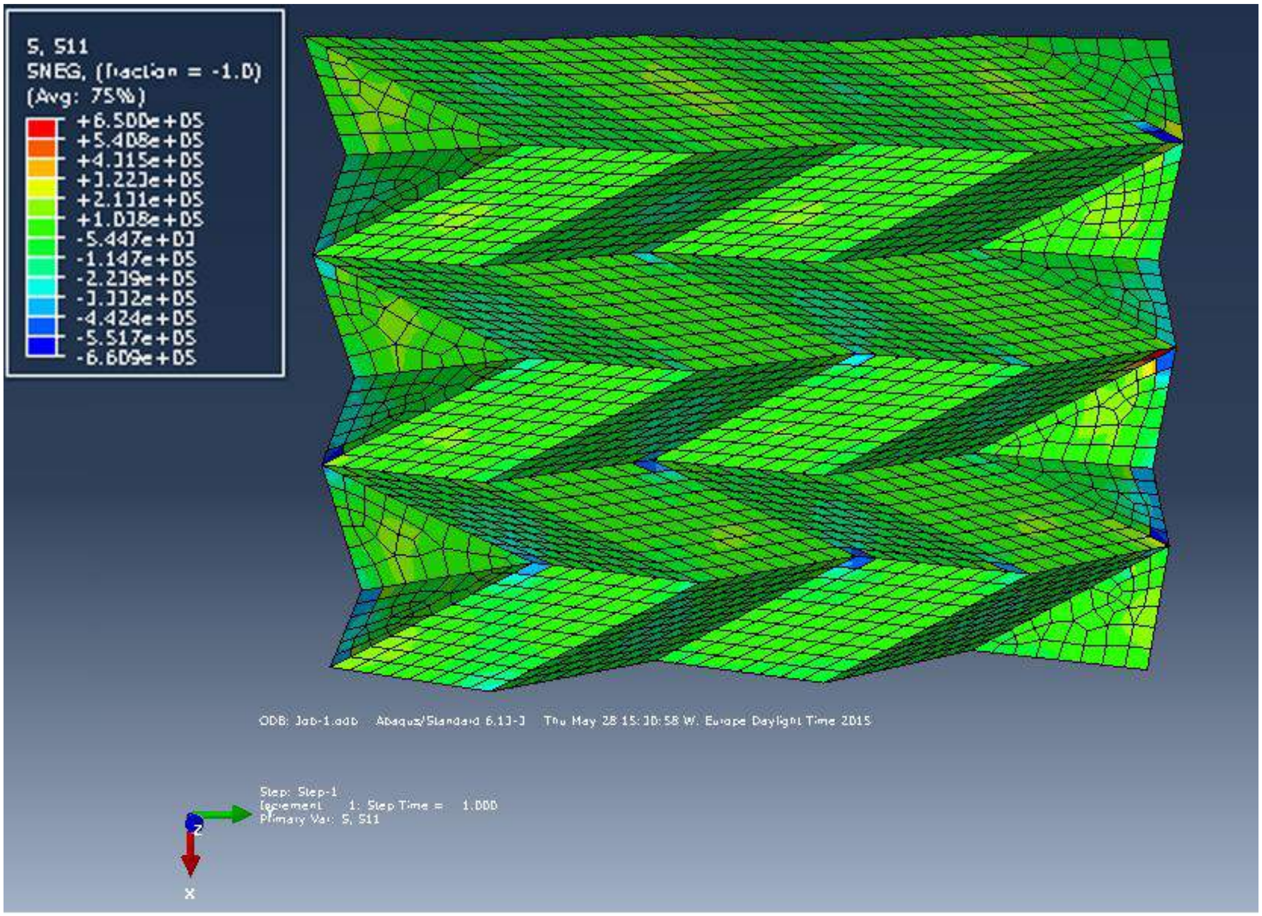
Herrringbone, hinged, S11, load in z-direction:



Herrringbone, hinged, S22, load in z-direction:



Herrringbone, hinged, S11, load in x-direction:



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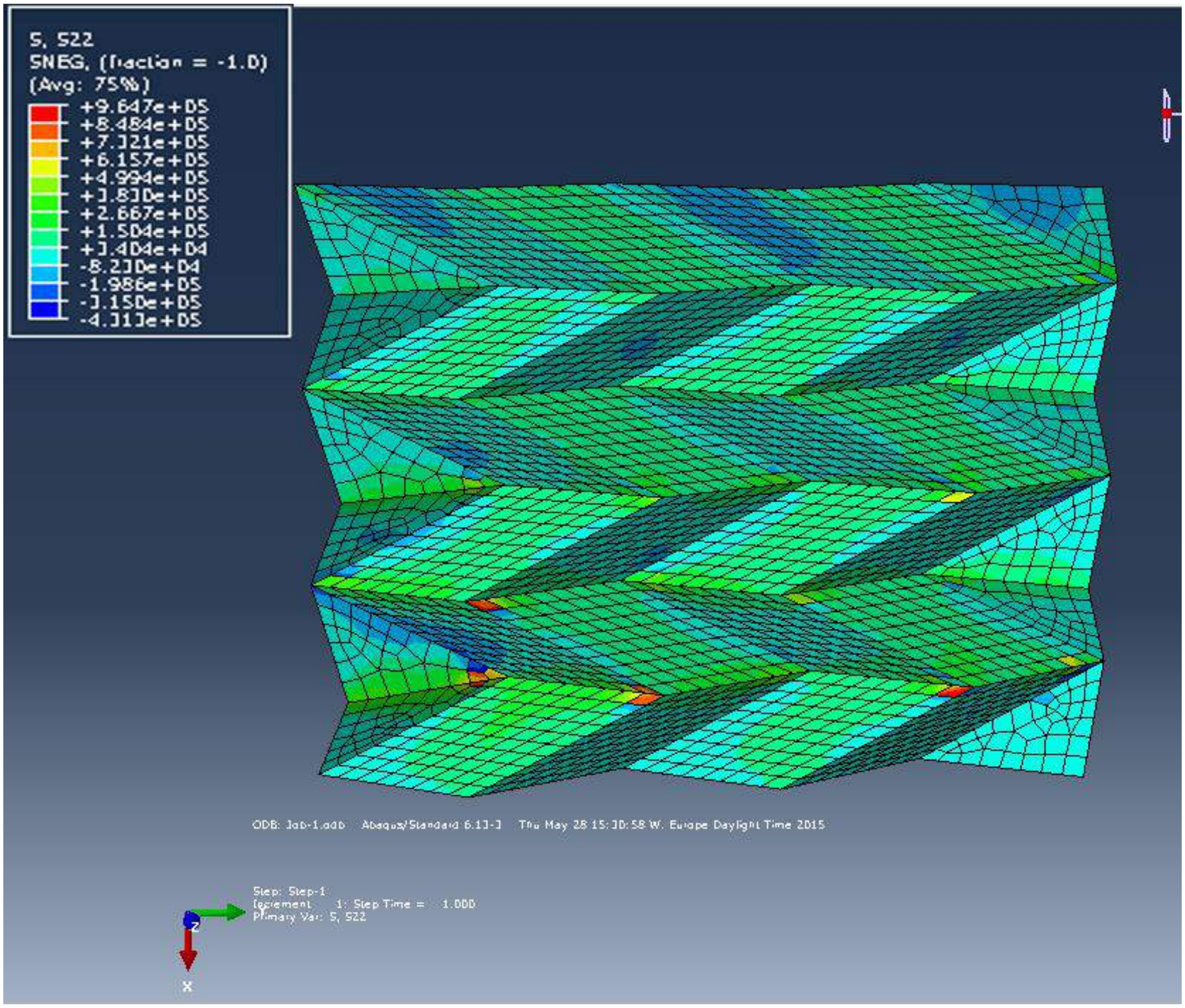
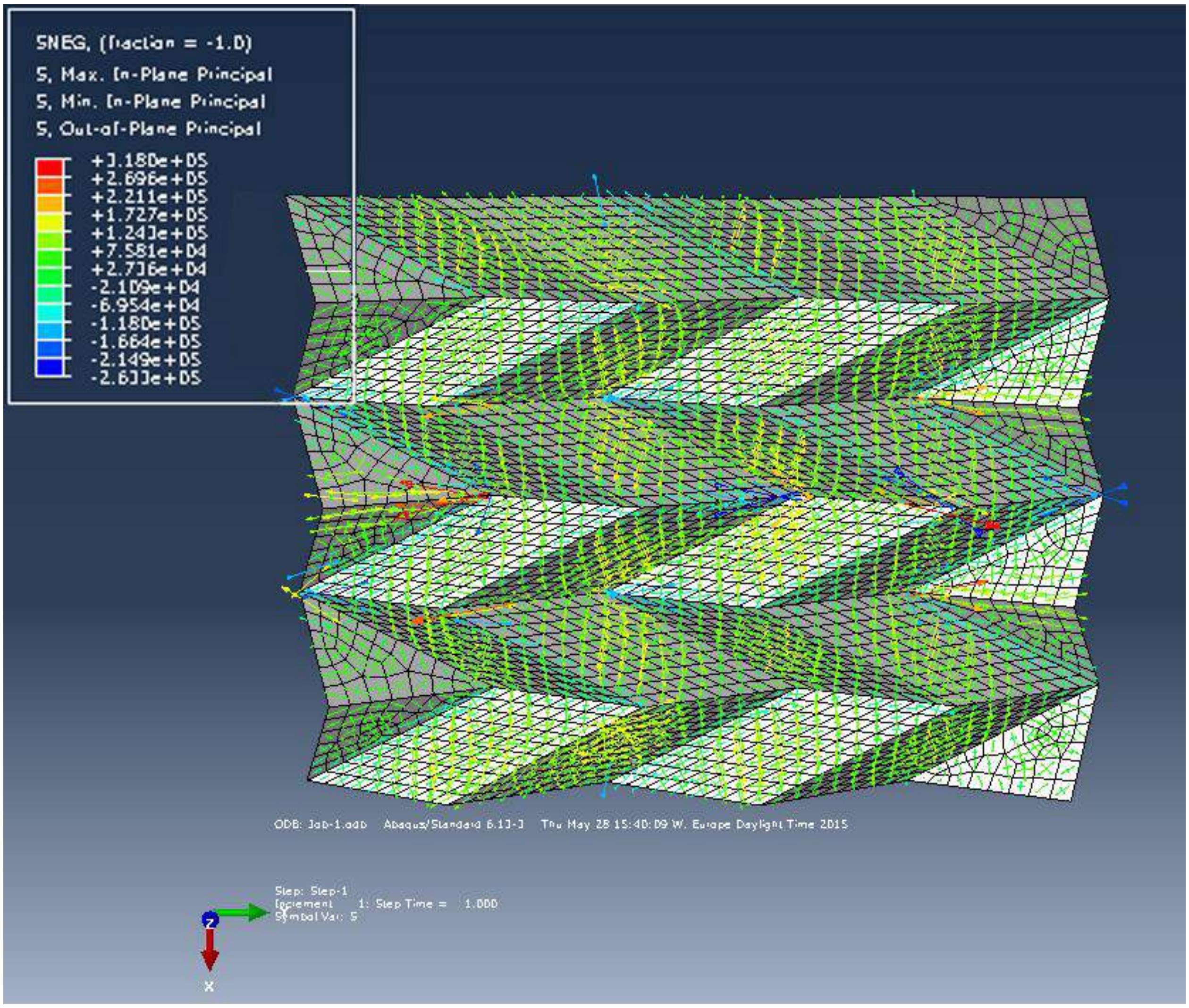


Herrringbone, hinged, principal stresses, load in z-direction:



Herrringbone, hinged, principal stresses, load in x-direction: