

# Engine Dynamics and Torsion Vibration Reduction

## Investigation of various flywheel models

Master's thesis in Applied Mechanics

ANOOP SURYANARAYANA



MASTER'S THESIS IN APPLIED MECHANICS

# Engine Dynamics and Torsion Vibration Reduction

Investigation of various flywheel models

ANOOP SURYANARAYANA

Department of Applied Mechanics  
Division of Dynamics  
CHALMERS UNIVERSITY OF TECHNOLOGY  
Göteborg, Sweden 2015

Engine Dynamics and Torsion Vibration Reduction  
Investigation of various flywheel models  
ANOOP SURYANARAYANA

© ANOOP SURYANARAYANA, 2015

Master's Thesis 2015:20  
ISSN 1652-8557  
Department of Applied Mechanics  
Division of Dynamics  
Chalmers University of Technology  
SE-412 96 Göteborg  
Sweden  
Telephone: + 46 (0)31-772 1000

Cover:  
Centrifugal pendulum absorber model in AVL Excite Timing drive, for more  
information see Chapter 3.3.

Chalmers Reproservice  
Göteborg, Sweden 2015

Engine Dynamics and Torsion Vibration Reduction  
Investigation of various flywheel models  
Master's thesis in Automotive Engineering  
ANOOP SURYANARAYANA  
Department of Applied Mechanics  
Division of Dynamics  
Chalmers University of Technology

## ABSTRACT

Demand for fuel efficient engines and stringent rules on emissions have made automotive industries to down speed or downsize the engines. Down speeding and downsizing will lower the CO<sub>2</sub> emissions but increases vibrations due to instability, which might lead to deleterious consequences like wearing of parts and power losses. This has a huge impact on truck industries making them less durable and less sustainable. Therefore, this thesis focuses on reducing one of these vibrations called torsional vibrations. Reducing torsional vibration will help to achieve higher torques out of multi-cylinder engines at lower speed. Usually internal dampers are used to absorb these torsional vibrations but due to energy dissipations, one has to look for advance flywheels which stores rotational inertia during engine operation that can be used to isolate vibrations with less energy dissipation and transfer power to transmission with less fluctuations. Various concepts to reduce torsional vibrations which are either in research stage or implemented in automotive industry have been investigated in this thesis through modelling and simulation. The work starts with Dual Mass flywheel which is currently being implemented in Volvo Trucks. Alternate concepts includes Powersplit flywheel, Centrifugal Pendulum Vibration Absorber flywheel, and Triple Mass flywheel.

Key words: NVH, Torsional Vibration, Flywheel



# Contents

ABSTRACT.....	I
Contents .....	III
Acknowledgement .....	V
1 Introduction.....	1
1.1 Torsional Vibration .....	1
1.2 Background .....	2
1.3 Goal .....	2
1.4 Limitations and Assumptions.....	3
2 Theory & Modelling Approach .....	5
2.1 Vibration in Internal Combustion engines .....	5
2.2 Systems for absorbing Torsional vibrations.....	5
2.2.1 Conventional dual mass Flywheel .....	5
2.2.2 Centrifugal Pendulum Vibration absorber .....	7
2.2.3 Hydrodynamic Torque Converter .....	7
2.2.4 Planetary gear Dual Mass Flywheel .....	8
2.2.5 Power Split.....	9
2.2.6 Triple Mass Flywheel .....	10
2.2.7 General design steps .....	10
2.3 Formulation of equations of motion for dynamics of the flywheel.....	11
2.4 Method of approach .....	15
3 Modelling and Simulation of different concepts .....	17
3.1 Dual mass flywheel .....	17
3.1.1 Modelling of DMF complete model .....	17
3.1.2 Modelling of DMF in AVL Excite timing drive.....	18
3.2 Power split.....	19
3.2.1 Modelling in Adams .....	19
3.2.2 Alternate model of power split .....	20
3.3 Centrifugal pendulum vibration absorber .....	21
3.3.1 Model in Adams.....	21
3.3.2 Dual mass flywheel with CVPA .....	22
3.3.3 CPVA flywheel Model in AVL Excite.....	23
3.3.4 DMF with CPVA flywheel in AVL Excite.....	24
3.4 Triple Mass flywheel.....	25
3.4.1 Model in Adams.....	25

3.4.2	Model in AVL Excite.....	26
4	Results.....	27
4.1	Dual mass flywheel .....	27
4.2	Power split model.....	30
4.3	Centrifugal pendulum vibration absorber .....	32
4.4	DMF with CPVA .....	33
4.5	Triple Mass flywheel.....	35
4.6	Comparision with single mass Solid flywheel .....	36
5	Conclusion and Future work.....	37
6	References.....	39
7	Appendix.....	41
7.1	CPVA model in AVL Excite.....	41



## Acknowledgement

In the journey of obtaining a Master's degree in Applied Mechanics, this thesis has an important role and it would be incomplete without acknowledging the support from various people for its successful completion. The thesis project has been carried out at the Advanced Technology & Research division of Volvo Group Trucks Technology under the supervision of Arne Andersson. I would like to thank him for giving me the opportunity to explore my ideas and supervising me with his technical guidance. I would also like to extend my gratitude to Per Salomonsson, Tommy Simonsson, and everyone in the department at Volvo GTT/ATR for their unparalleled support with the project.

I would also like to express my deep gratitude to my supervisor and examiner Professor Viktor Berbyuk for suggesting me to Volvo GTT for this thesis and also to Håkan Johansson for his supervision from Department of Applied Mechanics, at Chalmers University of technology. During the project, monthly meeting and regular supervision had been provided by Viktor and Håkan, without their help, advice, expertise and encouragement this thesis would not have happened.

I would also like to thank Rahul Kilpadi and Nandee Mysore for their continuous support with discussions about my project and their opinions to make my models more realistic. I would like to thank Fredrik Sjöqvist for helping me with flywheel data for simulation models. I would also like to thank Martin Svensson, Hans Bondeson, Christer Örtlund and their colleagues at Simulation & NVH division in Powertrain Engineering at Volvo GTT for all their support during the project. I would also thank Laurent Margerie and Sasa Bukovnik from AVL for their support with AVL Excite software. I would also thank MSC Adams support centre for clarifying some doubts with the software.

Finally I would like to thank my family and friends for their moral support during this journey of pursuing my Master's. I would like to thank my father, mother, sister and my dearest love Suchetha for supporting me morally and emotionally during this endeavour. Through their love, support, encouragement and camaraderie I have grown and developed into a better person.

Göteborg, June 2015

ANOOP SURYANARAYANA



# 1 Introduction

Medium to long term developments in Heavy Duty trucks and bus technology will focus on improvements in engine and vehicle efficiency, while meeting increasingly stringent exhaust emissions standards. Ways to increase the fuel conversion efficiency and reduce CO<sub>2</sub> emissions include increased peak cylinder pressure or new engine architectures. This will call for engines with fewer cylinders (downsizing) and running at lower speed (down-speeding). [1]

With the target of high powered and highly efficient engines in future, engines with less number of cylinders compared to current engines will definitely reduce the manufacturing costs. This makes downsizing a challenging task to achieve better driving performance and durability of the vehicle when compared to current vehicles.

## 1.1 Torsional Vibration

Down speeding and downsizing will inevitably increase the excitation of torsional vibrations from the combustion engines. High dynamic fluctuations with high mean torque will create problems associated with Noise, vibration and harshness (NVH) and durability of engine and drivetrain components. This creates a need for improved technology for reduction of torsional vibrations. More efficient engines also create a temperature problem for the after treatment system. One way to address too low exhaust temperature is partial cylinder cut off which also generates torsional vibrations. Figure 1-1 shows the concept of downspeeding by pushing the limits of peak torque to lower speed and downsizing by reducing the number of cylinders in the engine. Figure 1-2 shows the need for vibration reduction with downsizing.

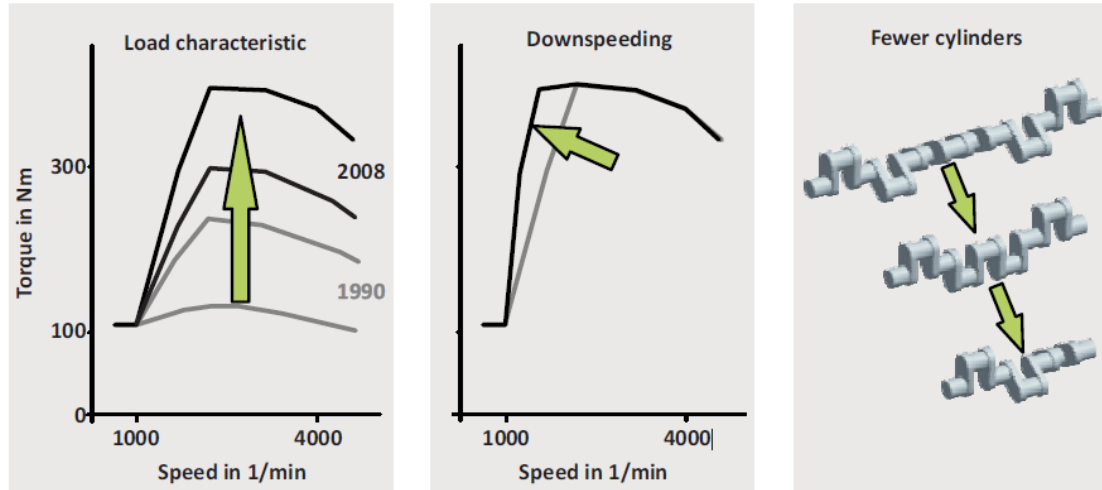


Figure 1-1: Engine development trends effect on engine excitation [1]

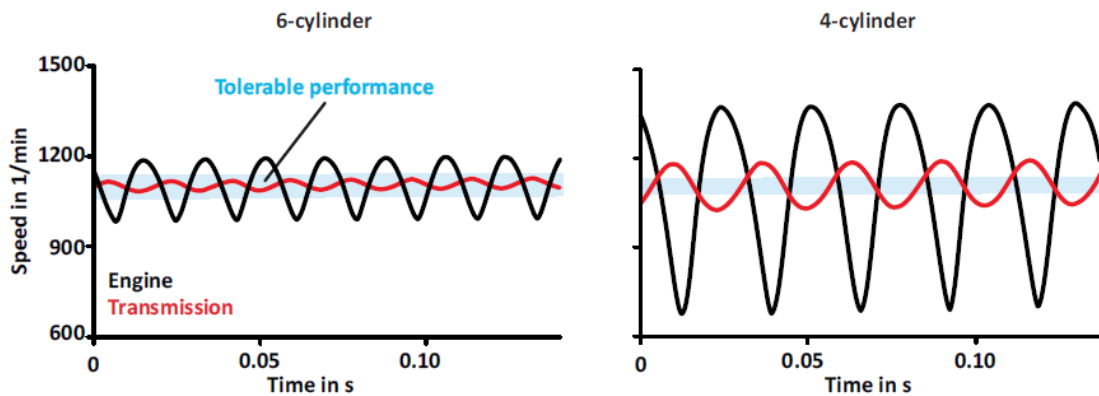


Figure 1-2: Effect on excitation from rotor with downsizing [1]

## 1.2 Background

In trucks, heavy engines with high torque capacities are used. IC engines less than 8 cylinders and engines with odd firing orders have worse torsional pulsing than 8-cylinder engines. When the engine is downsized, balancing of these torsional fluctuations becomes major issue. Usually, these torsional vibrations are damped either using torsional dampers which are attached to crankshaft or by using advanced flywheels. Torsional dampers tend to dissipate energy in the form of heat thus, cooling of these dampers is necessary in order to avoid overheating which might lead to failure.

A flywheel on the other hand, serves the major purpose of storing rotational energy and provides continuous supply of energy during non-operation condition of internal combustion engine. Heavier flywheels have higher rotational inertia, which makes them ideal for reducing torsional fluctuations. But due to increased weight and packaging issues, a heavier flywheel is not practical to use in automobiles such as trucks. Also, large inertias usually require more effort to speed up. This led to invention of advanced flywheels which serves the purpose of storing energy and also to isolate torsional vibrations. Various concepts regarding this development are discussed in further chapters.

## 1.3 Goal

Evaluation different technologies for torsional vibration reduction by creating simulation models. Furthermore, propose a suitable solution that can be adopted for the engine specified by Volvo GTT. The goal formulation can be clarified in the following points:

- To build and validate the model of current vibration absorber investigated by Volvo GTT.
- Evaluation of the model is done by analysing input and output torque and angular velocity fluctuation in the vibration absorber.
- Perform a parametric study on number of cylinders (example 3, 4 and 5 cylinder engines), shape, size and mass of flywheel to create an advanced model.
- Different technologies which are already implemented or a novel method developed during the project.

## 1.4 Limitations and Assumptions

To complete the project within the given time frame, a clear project must be defined with boundaries that are listed:

- Only torsional vibrations will be considered.
- The overall vibration analysis of engine and powertrain will not be performed.
- The simulation data and material properties required for the project is provided by Volvo GTT.
- The project will consider using software available at Volvo GTT such as MSC Adams, AVL Excite, GT power and Matlab.
- Balancing of the engine will not be analyzed in this project.
- Confidentiality of data provided by Volvo GTT will be maintained.
- The project will be done mainly on numerical simulations hence experimentation will not be included in this project.



## **2 Theory & Modelling Approach**

A thorough investigation of the project is done by going through the basic concepts involved, history of research and current advances. This chapter will give a theoretical foundation of the problem to be solved and a brief review of advanced concepts of flywheels which can be used to reduce torsional vibration.

### **2.1 Vibration in Internal Combustion engines**

Most modern Internal Combustion (IC) engines in automobiles implements a 4-stroke cycle. These engines vary from 4-cylinders, 6-cylinders and 8-cylinders depending on power output specification and requirement on the truck. During the operation of a 4-stroke IC engine, torque pulses are generated in power stroke. Thus, power is generated periodically in every second rotation of crankshaft for each cylinder. This creates dynamic forces on the crankshaft which need despite balancing creates vibrations. Twisting forces acting on the crankshaft leads to torsional vibrations which eventually lead to crankshaft failure. If these torsional vibrations are not minimised, they will also damage transmission components. Hence, torsional vibrations are viewed as a critical parameter that needs to be isolated.

### **2.2 Systems for absorbing Torsional vibrations**

Several devices featuring absorber, isolation and damping effects are implemented in automotive powertrains that reduce torsional vibrations. Meingaßner et al. [2] have given a short overview of the functional principles of common torsional vibration reduction systems. The following subsection explains various concepts of vibration absorption that are being published. This includes dual mass flywheel (DMF), Centrifugal pendulum vibration absorber (CPVA), DMF with CPVA, hydrodynamic torque converter, planetary gear DMF, power split flywheel and triple mass flywheel (TMF) that are explained as follows.

#### **2.2.1 Conventional dual mass Flywheel**

This is the most conventional system to reduce torsional vibrations which consists of two flywheels, Primary flywheel and secondary flywheel, which are connected by torsional arc springs. A German company, LuK [4] is the major manufacturer and supplier of Dual Mass flywheel to various automobile companies. Figure 2-1 shows a schematic diagram of dual mass flywheel. The arc spring has two stages stiffness, stage one called soft spring and stage two called hard spring. Figure 2-2 shows the spring stiffness variation with winding angle. The performance of DMF is limited to torsional spring stiffness characteristics. Hence, tuning of the flywheel towards low frequency in order to have a low-pass filtering of dynamics torques with high amplitudes is achieved.

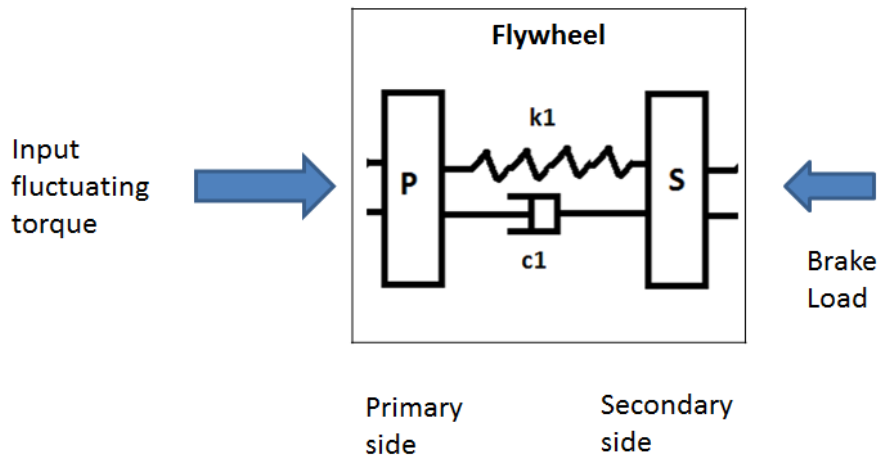


Figure 2-1: Schematic diagram of Dual Mass Flywheel

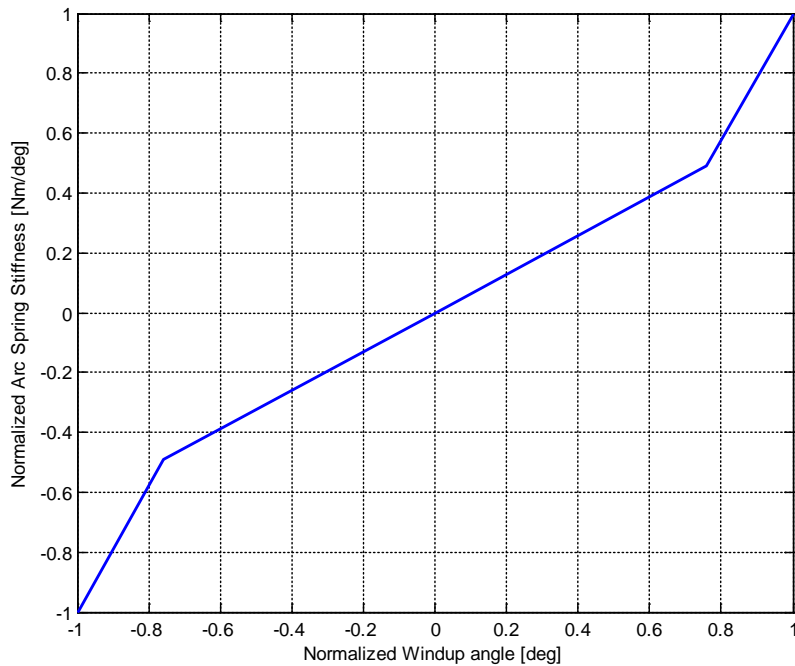


Figure 2-2: Normalized Spring Stiffness vs. Windup angle

Ulf Schaper et al. [3] has worked on modelling and torque estimation of a DMF in MATLAB/SIMULINK and compared the results with test data. The paper presents a conclusion that, DMF is an oscillation dampening device at low engine speeds. The spring model shows a hysteresis effect due to speed-dependent friction. But at higher speeds the DMF becomes unobservable and both flywheels rotate as one unit.



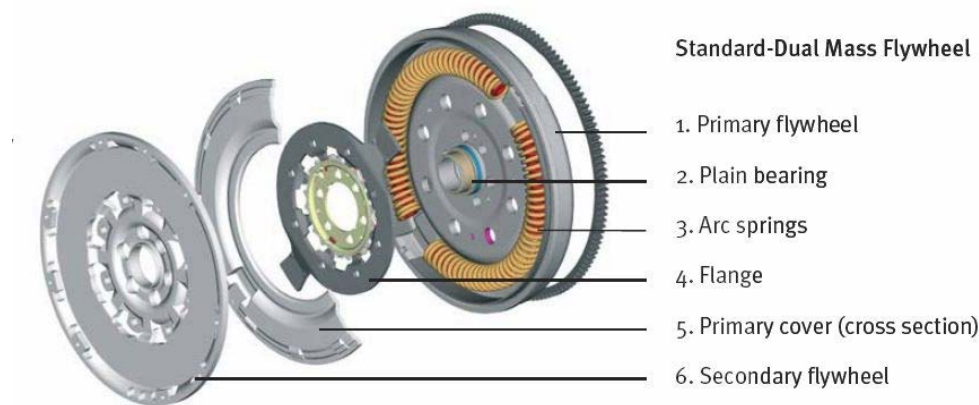


Figure 2-3: Standard-Dual Mass flywheel [4]

### 2.2.2 Centrifugal Pendulum Vibration absorber

This is a type of tuned mass damper where a number of small masses are attached to the main flywheel using roller pins. During high torque fluctuations, the pendulum oscillates to-and-fro applying centrifugal force that leads to increase in inertia of the flywheel. This concept was first patented in 1937 by R. Sazrin and different version was also patented by Chilton in 1938. Anders Wedin [19] made a study on different types of CPVA and created simulation models for Volvo Cars Corporation.



Figure 2-4: DMF with CPVA [5]

The car industry has also adopted an advanced version of flywheel called DMF with CVPA. Schaeffler is the main manufacturer of DMF with CVPA flywheel [5]. This type of flywheel has a primary flywheel attached with CVPA on secondary side using arc spring. The main constraint with this type of flywheel is that manufacturing errors can reduce the efficiency of the pendulum to a large extent. Also, problems related to NVH may increase due to improper tuning of pendulum.

### 2.2.3 Hydrodynamic Torque Converter

These are generally fluid couplings used in Automatic transmissions and Continuous variable transmissions. The fluid coupling plays the main role to transfer torque from

drive side to driven side. Due to fluid coupling, it has a dampening effect which causes energy dissipation. This makes the operation at higher speeds difficult. Figure 2-5 shows the implementation of Hydrodynamic torque converter in an automobile.

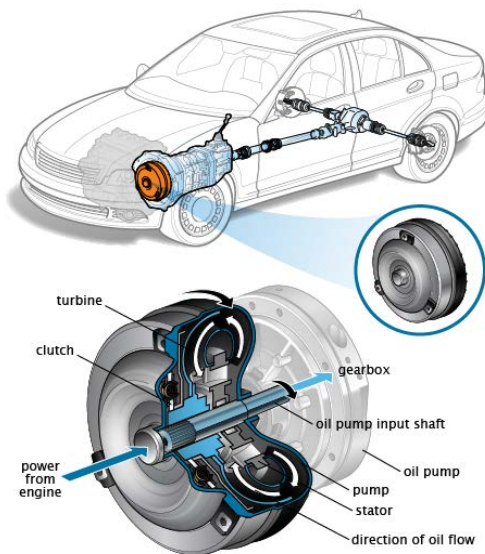


Image courtesy of ClearMechanic.com

Figure 2-5: Hydrodynamic Torque converter in a car engine [6]

## 2.2.4 Planetary gear Dual Mass Flywheel

Just like conventional dual mass flywheel, the primary mass is connected to secondary mass with the help of arc spring and further extended by a set of planetary gears between the two flywheels. This type of flywheel also includes torsional damper [7] which helps to reduce torsional vibration and planetary gear set helps to create anti-resonance in the transfer system behaviour. For vibrational excitations with frequencies close to the anti-resonance frequency a very good vibration reduction can be achieved. For excited vibrations with other frequencies the influence of the DMF transfer behaviour dominates. Drawbacks of this system are not well explained. The main manufacturer and supplier of this type of flywheel is SACHS which is a German brand from ZF.



Figure 2-6: Planetary gear DMF [7]

This concept was further developed by ZF which is called as power split explained in the next section.

### 2.2.5 Power Split

This concept is developed by ZF [8]. The main idea of this concept is that the torque is split into one alternating and one steady part, the upper part travels through arc spring to the ring gear of the primary side. While the lower part through planetary gears. When the torque travels through upper part, the phase changes, and when they are superimposed again with the lower part, the alternating torque cancels and constant torque is obtained. Figure 2-7 shows the concept of power split model and Figure 2-8 explains the rotary configuration of the flywheel.

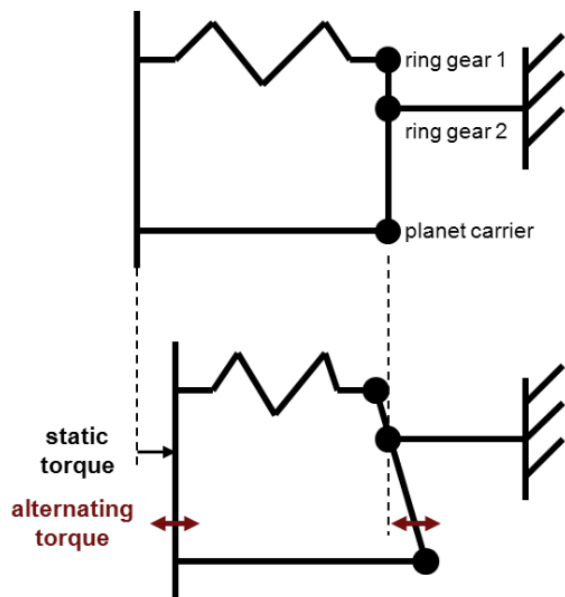


Figure 2-7: Power split concept [8]

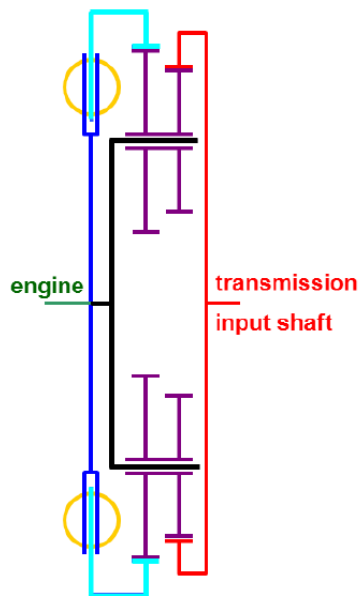


Figure 2-8: Rotary configuration of Power split model [8]

D. Lorenz et al. [8] have published a paper which gives an account that power split flywheel is a better option than compared to next flywheel which is CVPA.

### 2.2.6 Triple Mass Flywheel

In this concept, three flywheels are connected to each other with the help of torsional springs. Primary mass is connected to engine crankshaft; secondary is connected to transmission input shaft. A third mass called intermediate mass is placed in-between primary and secondary flywheel with the help of torsional arc spring. This concept was developed during this thesis, hence it concept has not been published in any research papers. Though this concept was developed during the thesis, later it was found that this had been patented in Korea by Lee Hee Rak and Hur Man Dae in 2008 [9]. A research paper is also published by Shi Wen-ku et al. [10] in 2011, where it is stated in abstract that TMF is better than conventional DMF. However it hasn't been implemented in any automobile. Both patent and the research paper are inaccessible.

### 2.2.7 General design steps

Meingaßner et al. [2] has presented an abstract overview for basic design of torsional vibration reduction system. Figure 2-9 shows the schematic diagram of the flywheel.

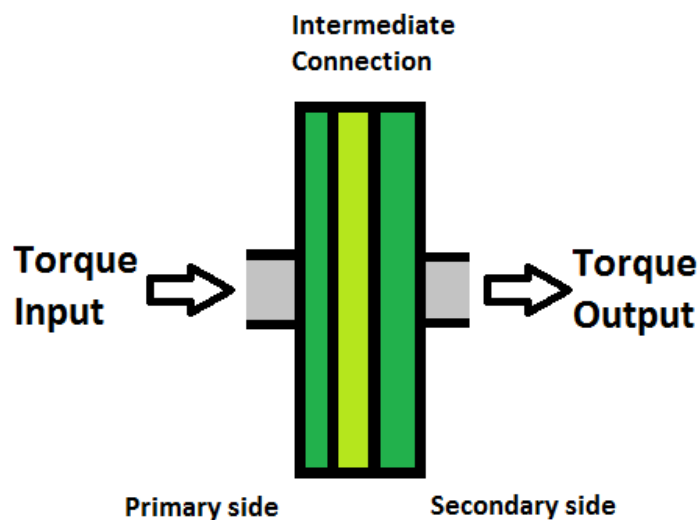


Figure 2-9: Schematic diagram of a flywheel

Following are the basic design steps:

- First step to develop a mechanical concept of the vibration reduction system.
- Depending on the mechanical concept major parameters like masses, mass moments of inertia, spring stiffness and damping parameters are pre-set.
- Integration and functional parameters such as mass, dimensions and flange geometry for the integration in Vehicle/ test rig has to keep in mind while designing.
- Installation and manufacturing aspects is considered in design process itself.
- Iterative process with design parameters and simulation.

## 2.3 Formulation of equations of motion for dynamics of the flywheel

The dynamics of the system is explained in the following subsection by deriving the simple equations of motion (EOM) of the system. However, these equations are not implemented as Adams and AVL excite have in-built algorithms to solve the given engineering model.

### Dual Mass flywheel

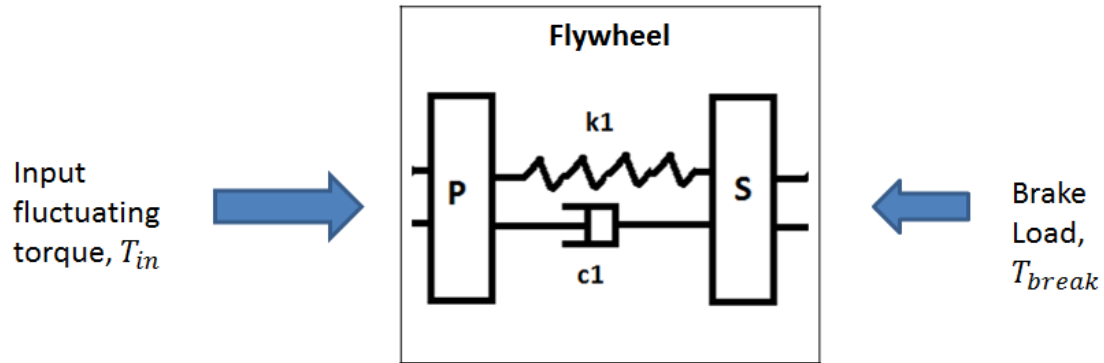


Figure 2-10: Free body diagram of DMF

Figure 2-10 shows free body diagram of a DMF. The equations of motion are given below.

$$J_p \ddot{\theta}_p = T_{in} - k_1(\theta_p - \theta_s) - c_1(\dot{\theta}_p - \dot{\theta}_s) \quad 2.1$$

$$J_s \ddot{\theta}_s = -T_{break} + k_1(\theta_p - \theta_s) + c_1(\dot{\theta}_p - \dot{\theta}_s) \quad 2.2$$

$$\begin{aligned} \dot{\theta}_p &= \omega_p \\ \dot{\theta}_s &= \omega_s \end{aligned} \quad 2.3$$

These equations can be used to obtain the Torque  $T_{break}$  by calculating using the angular velocities of primary and secondary flywheels.

## Power Split flywheel

Free body diagram of power split model is given in Figure 2-11

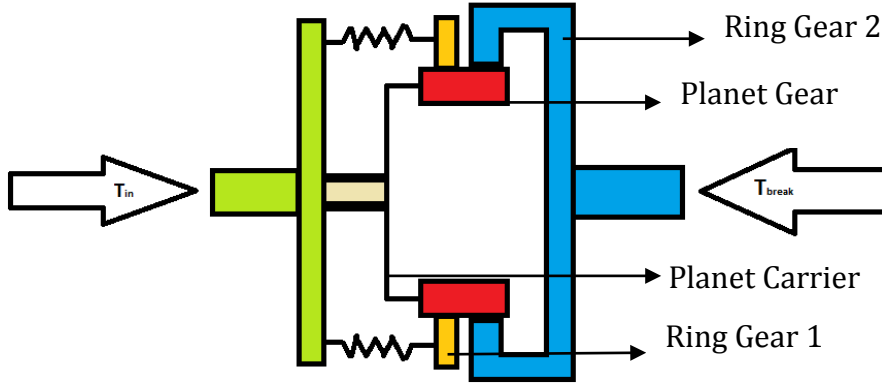


Figure 2-11: Free body diagram of Power split flywheel

Since the torque is split in two path,

$$T_{in} = T_{RG1} + T_{PC} \quad 2.4$$

$T_{RG1}$  is the power transmitted through Ring Gear1 and  $T_{PC}$  is the power transmitted through Planet Carrier.

Path 1: Power through Ring Gear 1

$$J_p \ddot{\theta}_p = T_{in} - k(\theta_p - \theta_{RG1}) - c(\dot{\theta}_p - \dot{\theta}_{RG1}) \quad 2.5$$

$$J_{RG1} \ddot{\theta}_{RG1} = -T_{RG1} + k(\theta_p - \theta_{RG1}) + c(\dot{\theta}_p - \dot{\theta}_{RG1}) \quad 2.6$$

These two equations are solved to obtain  $T_{RG1}$  and angular velocity of Ring Gear 1.

Path 2: Power through Planet Carrier

As the planet carrier is directly coupled to primary flywheel, the angular velocity of Primary flywheel will be equal to Planet carrier. From the gear relations, angular velocity of planet gear is calculated as

$$\omega_p = g_{RP} \cdot \omega_{RG1} - (g_{RP} - 1) \omega_c \quad 2.7$$

The torque transfer is calculated as

$$T_p = \frac{-T_{RG1} + T_{loss}}{g_{RP}} \quad 2.8$$

With  $T_{loss} = 0$  in ideal case

The power is then transferred to Ring Gear 2 depending on the gear ratio  $g_{RP2}$

$$T_{RG2} = -g_{RP2} T_p \quad 2.9$$

## Centrifugal Pendulum Vibration Absorber flywheel

In this concept, the total inertia of the flywheel is a function of time and velocity i.e, as angular velocity varies; the centrifugal force of oscillating pendulum counter balances the fluctuating torque. The concept was first patented by Sarazin in 1937 which includes a compact design pendulum with rollers. The concept was further developed and patented by Kutzbach, Carter and Salomon. In this section, EOM (given in [33]) of Sarazin model will be analysed which consists of pendulum mass that acts as absorber mass connected to bifilar rotor using rollers as shown in Figure 2-12.

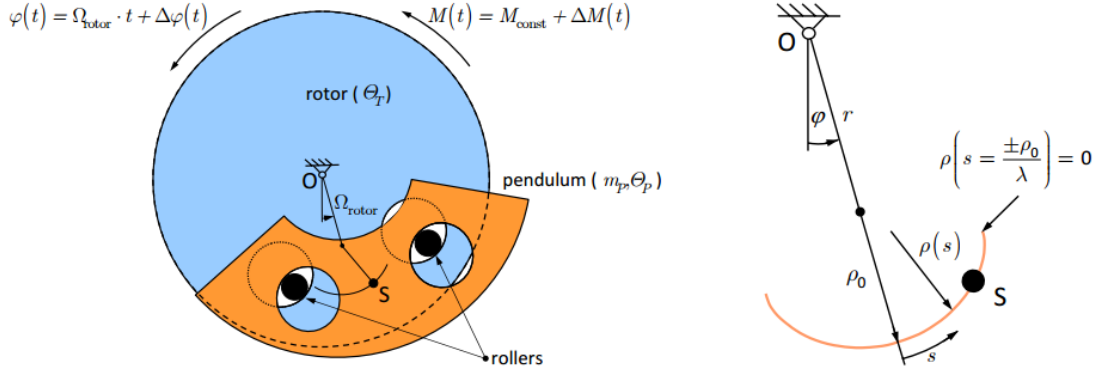


Figure 2-12: Centrifugal Pendulum Vibration absorber design by Sarazin

The rollers are free to roll in the grooved shape relative to rotor and pendulum. The linearized absorber frequency  $f_{abs}$  is calculated as a function of rotor speed  $\Omega_{rotor}$  as

$$f_{abs} = n\Omega_{rotor} \quad 2.10$$

Where  $n = \sqrt{\frac{r}{\rho_0}}$  is a tuning order calculated from geometrical parameters as shown in Figure 2-12. The tuning order relates to centrifugal field which happens by the stiffness of the system.

Moment  $M(t)$  drives the rotor includes  $M_{const}$  (constant torque) and  $\Delta M(t)$  (fluctuating torque of certain order) is obtained from GT power in our case.

Denman [32] has derieved EOM for a CPVA with epicycloidal pendulum path by neglecting roller masses and damping with only one pendulum, using Lagrange equation of second kind that results in

$$\begin{bmatrix} \Theta_T + \Theta_p + m_p(c^2 - n^2s^2) & m_p\chi \\ m_p\chi & m_p \end{bmatrix} \begin{pmatrix} \ddot{\phi} \\ \ddot{s} \end{pmatrix} + \begin{pmatrix} 2\dot{\phi}\dot{s}m_p n^2s + \dot{s}^2 \left( -m_p n^2(1 + n^2) \frac{s}{\chi} \right) \\ \dot{\phi}^2 m_p n^2s \end{pmatrix} = \begin{pmatrix} M(t) \\ 0 \end{pmatrix} \quad 2.11$$

$$\text{With } \chi = \sqrt{c^2 - n^2(1 + n^2)s^2} \quad \text{and} \quad c = \rho_0 + r \quad 2.12$$

## Triple Mass Flywheel

This method of approach has the following freebody diagram shown in Figure 2-13

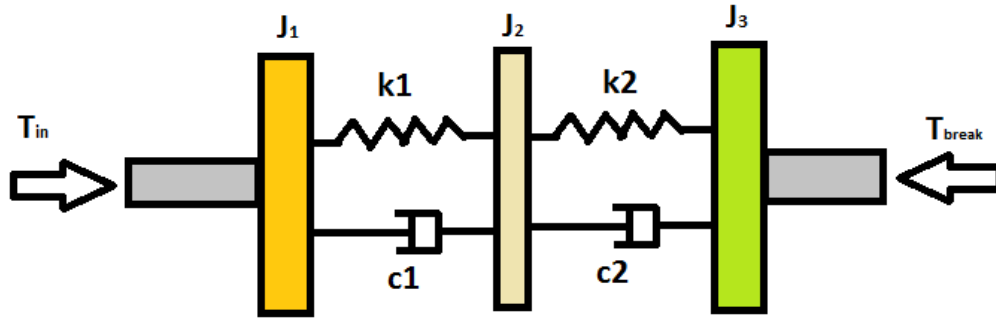


Figure 2-13: Free body diagram of TMF

Using Newton method, the Equation of motion are as follows

$$J_1 \ddot{\theta}_1 = T_{in} - k_1(\theta_1 - \theta_2) - c_1(\dot{\theta}_1 - \dot{\theta}_2) \quad 2.13$$

$$J_2 \ddot{\theta}_2 = k_1(\theta_1 - \theta_2) + c_1(\dot{\theta}_1 - \dot{\theta}_2) - k_2(\theta_2 - \theta_3) - c_2(\dot{\theta}_2 - \dot{\theta}_3) \quad 2.14$$

$$J_3 \ddot{\theta}_3 = -T_{break} + k_2(\theta_2 - \theta_3) - c_2(\dot{\theta}_2 - \dot{\theta}_3) \quad 2.15$$

Using these equation,  $T_{break}$ , angular velocity of intermediate and angular velocity of secondary mass can be calculated.



## 2.4 Method of approach

The simulation model of DMF is made using the general purpose multibody dynamics simulation software MSC ADAMS [11]. A simple rigid body model of DMF with mass and inertias of the real flywheel used in Volvo Trucks. The arc spring stiffness is also obtained from the existing flywheel. The torque curves are taken from GT power [13] model of Volvo MD13 euro DST engine. This is a 4-stroke, 6 cylinder engine with 1572 [Nm] torque at 800 [rpm]. The reason for choosing this rpm is to extract maximum torque with minimum rpm from the given engine. At low rpm and high torque torsional vibration is a major issue. Figure 2-14 shows the Torque vs Crank angle obtained from GT power model.

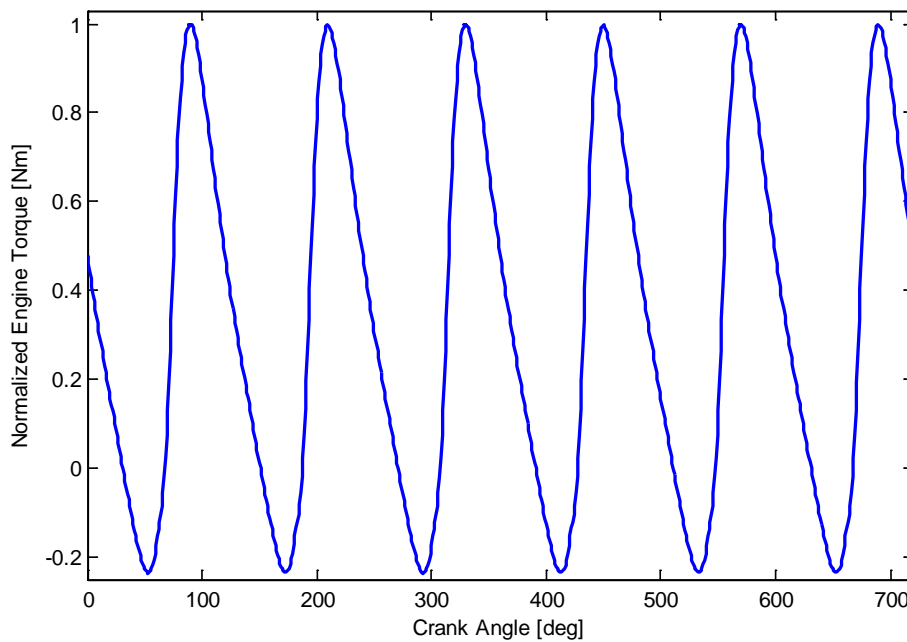


Figure 2-14: Normalized Torque vs Crank angle

As Adams performs calculations in time domain, the torque has to be plotted against time which is calculated by Equation 2.16.

$$t = \frac{\theta}{720} * 2 * \left(\frac{rpm}{60}\right)^{-1} \text{ [s]} \quad 2.16$$

Therefore, if the speed is 800 [rpm], total time for one cycle is 0.15[s] in a 4-stroke engine. The torque plotted against time is shown in Figure 2-15

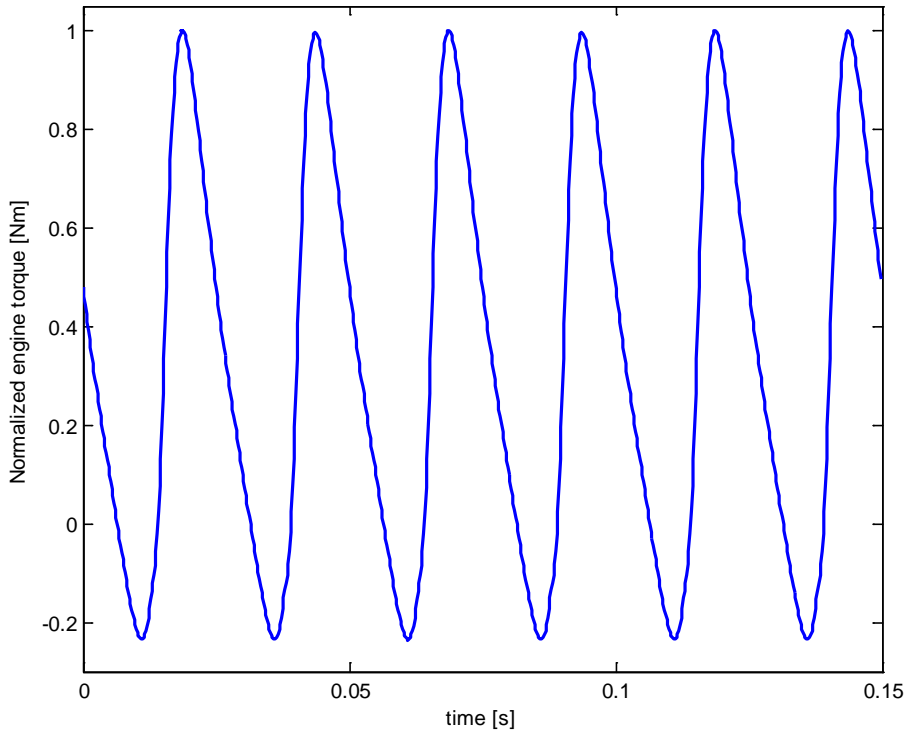


Figure 2-15: Normalized Torque vs Time

As the torque is a harmonic function, Fourier series is used to form an equation for Torque with respect to time which is given below. Matlab command cftool is used to formulate this equation.

$$T(t) = T_o + T \sin(n\omega t) \text{ [Nm]} \quad 2.17$$

Where  $T_o$  is a constant torque,  $n$  is excitation order,  $\omega$  is angular velocity in [rad/s] and  $t$  is time in seconds. This is the torque load applied from the engine side and an excitation load will be applied from the transmission side to hold the flywheel at 800 [rpm] assuming the gear ratio is 1.

This simulation will be used to evaluate the performance of the flywheel by comparing the Torque, Angular Velocity and Angular Acceleration at points of observation before and after the flywheel.

In the later part of the simulation, complete upstream model and downstream model has been added to simulate torsional vibrations reduction model.

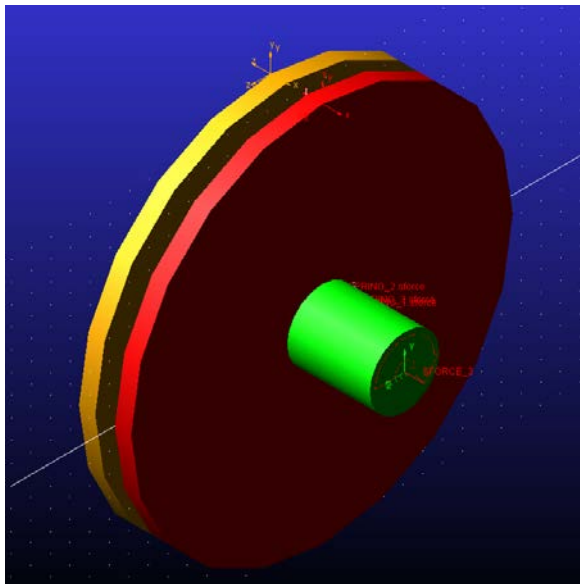
The CPVA flywheel has also been modelled and simulated in AVL Excite which will be useful to Simulation and NVH group at Volvo GTT for further analysis. AVL Excite is a powertrain oriented rigid and flexible multi-body dynamic analysis software that provides advanced techniques to calculate the dynamics, strengths, vibration and acoustics of combustion engines, transmissions and conventional or electrified powertrains and drivelines [12]. It has various tools for different purposes, and in this project AVL Excite Timing Drive is used which is a flexible and modular system for multi-body analysis in time domain.

### 3 Modelling and Simulation of different concepts

To understand the working of the flywheel in the process of reduction of torsional vibrations, a detail mathematical model is made which consists of simple rigid bodies and their interaction in 3-dimensional space. A free body diagram shown in the Figure 2-9 shows the forces acting on each component. The effect of gravity is neglected in all the models. Thesien T. [40] mentions the effects of gravity for CPVA in 1<sup>st</sup> order and 2<sup>nd</sup> order while gravity effect is neglected for 3<sup>rd</sup> engine order models of CPVA. Therefore, it can be neglected for simplicity. The 3D models are made in global x, y and z coordinate system. x-direction is used as axis of rotation in all the simulation models in both MSC Adams and AVL Excite.

#### 3.1 Dual mass flywheel

The dual mass flywheel model consists of two flywheels with mass and inertias as given below is modelled in Adams. Torsional spring is used to connect these two bodies with stiffness shown in Figure 2-2.

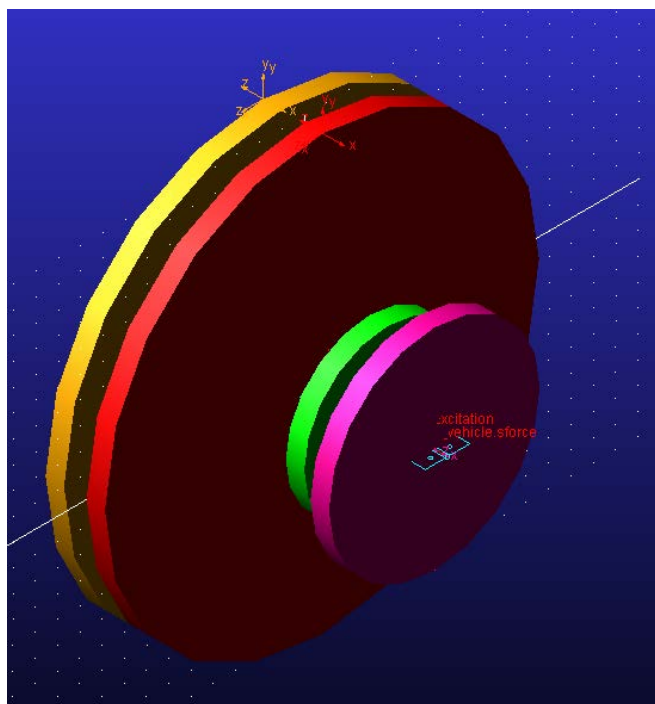


*Figure 3-1: Dual mass flywheel model in MSC Adams*

The input torque from the engine crankshaft which is called brake torque is applied on the primary side of the flywheel. An output shaft which is directly connected to secondary flywheel will be the point of application of excitation from transmission side assuming that gear ratio is 1. This done by using step function in MSC Adams which holds the angular velocity at 800 rpm.

##### 3.1.1 Modelling of DMF complete model

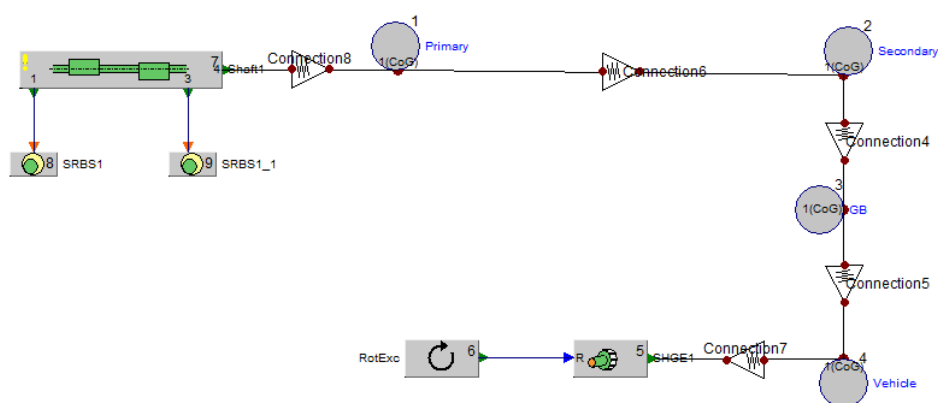
After the flywheel an additional mass is added this resembles the transmission/gear box with its simplified Inertia and stiffness. The gear ratio is assumed as 1; hence the engine and transmission are running at same speed. After the gear box, another rigid mass is added which represents vehicle inertia and simplified shaft stiffness is also assumed to model between them. Break load is applied from the vehicle side with the help of step function in Adams [11] to evaluate the complete model.



*Figure 3-2: Dual mass flywheel with downstream model*

### 3.1.2 Modelling of DMF in AVL Excite timing drive

The drive train model is prepared with shaft elements representing engine crankshaft and brake torque obtained from GT power is applied to this element. To the shaft a mass element with inertia equal to that of primary flywheel is attached. Another mass representing secondary flywheel is added with inertias defined accordingly. These two masses can be connected with pre-defined DMF spring module or a simple rotational spring with given spring stiffness characteristics. Similar to previous model Gear box and vehicle are added as generic masses with given inertias and stiffness. The complete model is shown in Figure 3-3.



*Figure 3-3: Dual mass flywheel in AVL Excite TD*

## 3.2 Power split.

The power split model is modelled close to the concept presented by D. Lorenz et al [8] with ZF Group as explained in section 2.2.5

### 3.2.1 Modelling in Adams

In order to evaluate this concept, a rough modelling of the flywheel is done in CATIA V5 and then it is imported to MSC Adams with the help of an intermediate software called CAD exchanger [20] to convert .stp (step format) file to .x\_t (parasolid format) file. A simple gear profile is chosen from GRABcad website [21] with gear ratio between ring gear and planetary gear equal to 4. Addition of shafts and other required modifications are done in CATIA and then transferred to Adams. Appropriate mass and inertias are chosen such that the total weight of the system remains close to Dual Mass Flywheel. Since these are rigid bodies, it is assumed that the bodies does not deform on the application of loads. Next revolute connection is applied so that the body has only rotational degree of freedom. Contact between ring gear and planet gear is also defined with the help of Hertzian theory in order to have a gear mesh interaction. The spring stiffness is chosen same as the one used in DMF model. Torques is applied and the model is simulated. The results are discussed in next section. The contact stiffness is calculated as

$$k_c = 2aE^* \quad \text{with } E^* = \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{-1} \quad 3.1$$

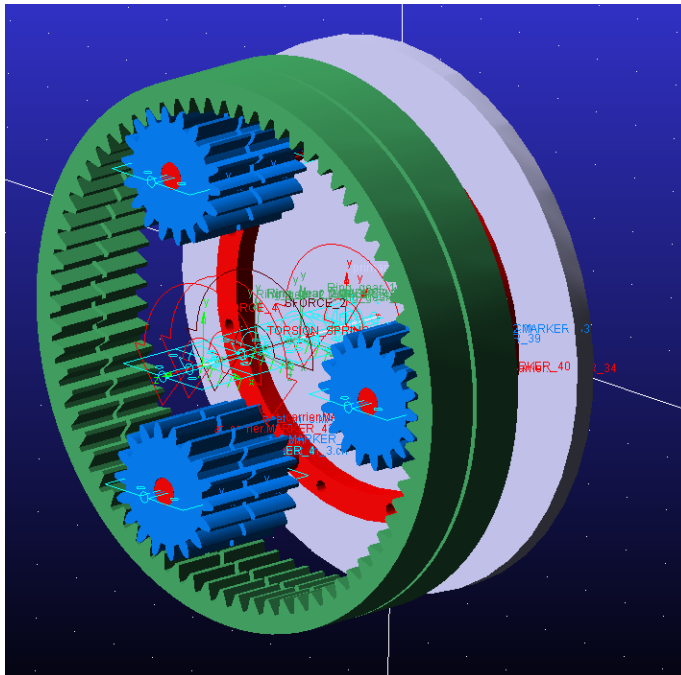
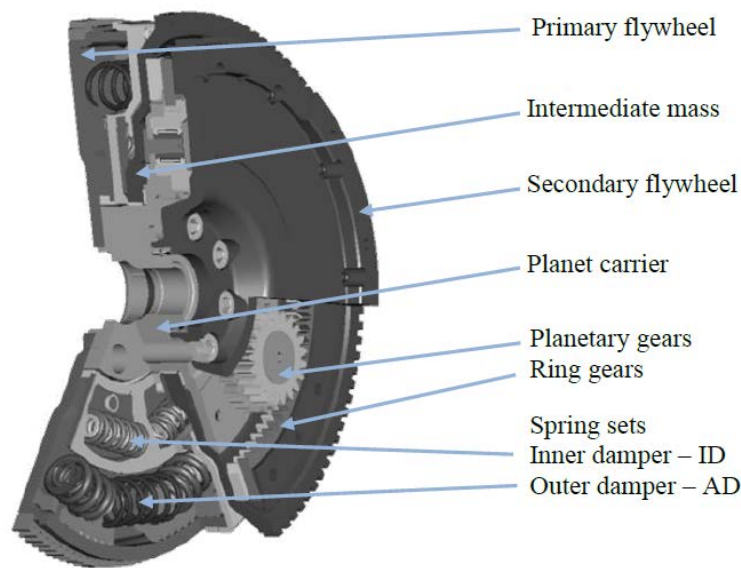


Figure 3-4: Power split flywheel model in Adams

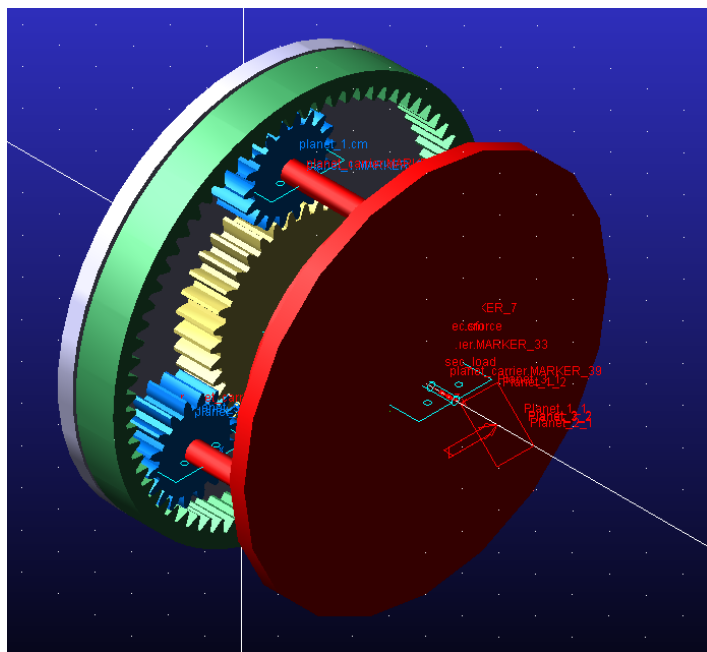


*Figure 3-5: Cut section of power split flywheel*

The Figure 3-4 shows the Adams model which is close to the one presented in paper Figure 3-5 by ZF.

### **3.2.2 Alternate model of power split**

In this alternate model, same CAD is used but an additional sun gear is added. The primary flywheel is connected directly to Ring Gear through torsional spring and the second path is through Sun gear which is connected directly to primary flywheel. The Ring gear and Sun gear are then meshed to planet gears which will make planet carrier to rotate. Hence, input shaft of transmission is connected to planet carrier. Figure 3-6 shows Adams model of this concept. Even in this case, the assumptions remain same and mass of the flywheel is made close to DMF model.



*Figure 3-6: Alternate model of power split model*

### 3.3 Centrifugal pendulum vibration absorber

This concept works on the principle of tuned mass damper. Widely used in car industries, with LuK Group as the main manufacturer. Modelling of this concept is done in both Adams and AVL Excite. Adams required a CAD model with appropriate dimensions; while in AVL Excite simple modules of generic mass elements with given mass and rotational inertia is defined. The roller interaction between pendulum and flywheel was based on mathematical relations defined with respect to position of pendulum on flywheel and their movement in the defined grooved path. Following section explains the modelling technique of both methods. Due to unavailability of CAD model from supplier, the flywheel was measured and modelled approximately in CATIA V5.

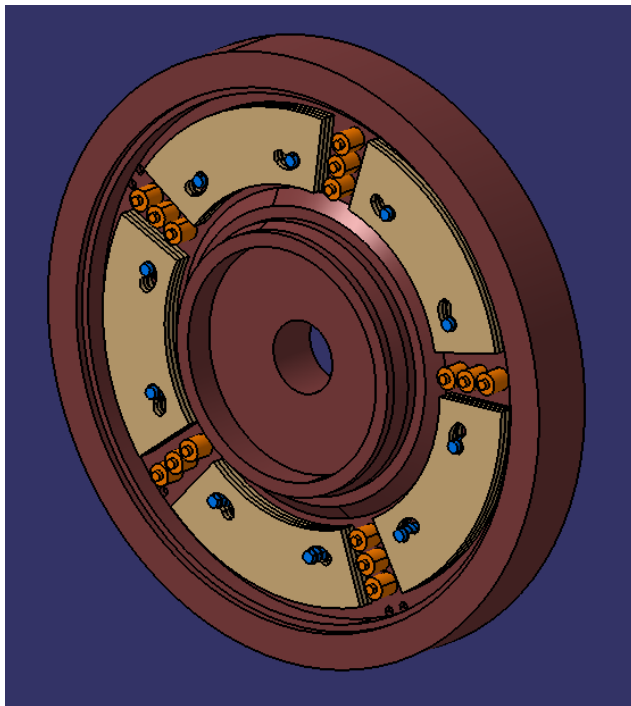


Figure 3-7: CPVA model in CATIA V5

#### 3.3.1 Model in Adams

The model from CATIA is imported into Adams and parts are defined with respective masses and inertia values. Revolute joint is applied to the flywheel to have rotational degree of freedom. Contacts are defined for solid interaction of rollers and pendula. The roller and pendulum have local coordinate system with two translational degrees of freedom and one rotational degree of freedom. The total number of degrees of freedom (DOF) of the system is 46. Transformation of coordinate system from global to local is done internally in the software. Torque is applied on the engine side and transmission load is applied on the opposite side just as in previous case. Figure 3-8 shows Adams model of CPVA flywheel.



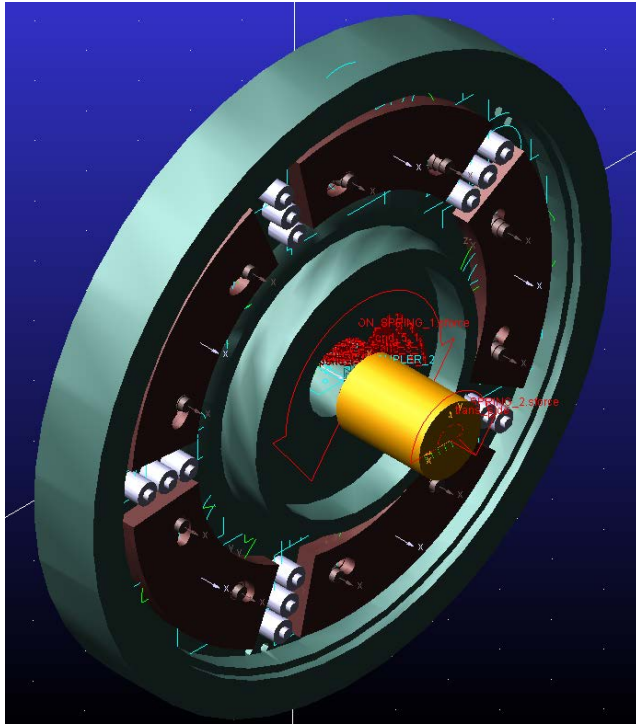


Figure 3-8: CPVA flywheel in Adams

### 3.3.2 Dual mass flywheel with CVPA

In this concept, the secondary flywheel in DMF is replaced with CVPA. Hence, an additional mass which acts as primary flywheel is added on the engine side. The primary flywheel is then connected to pendulum flywheel placed on secondary side with torsional arc spring. The spring stiffness is assumed to be same as the one used in DMF model. The total mass of this flywheel is kept same as DMF to have a good comparison. Adams model of DMF with CVPA is shown in Figure 3-9.

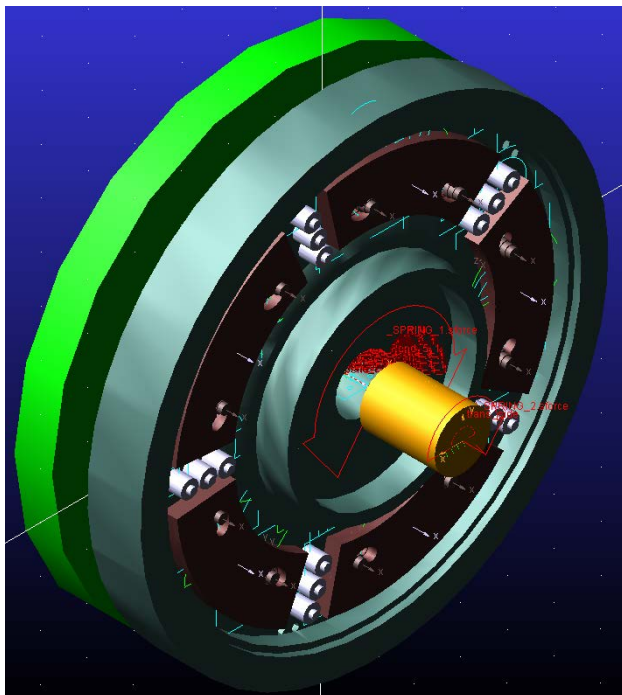


Figure 3-9: DMF with CVPA flywheel in Adams



### 3.3.3 CPVA flywheel Model in AVL Excite

This concept is built mathematically with the help of modules and connections predefined in the software. The engine side is modelled with the help of shaft macro on which the engine brake torque is applied as load. This shaft is connected to a generic mass element which is considered as the main flywheel. The flywheel is free to rotate only in x-direction in 3D coordinate system. As this CPVA flywheel consists of five pendula, five generic masses are used with two free translational DOF and one rotational DOF in local coordinate system. The pendulum is attached to flywheel with the help of rollers placed in epicyclic grooves as shown in Figure 3-10. The rollers are also defined with the help of generic mass element and connected to both flywheel and pendulum. Gear box and vehicle are added as generic masses with given inertias and stiffnesses. The AVL Excite model is shown in Figure 3-10 and Figure 3-11.

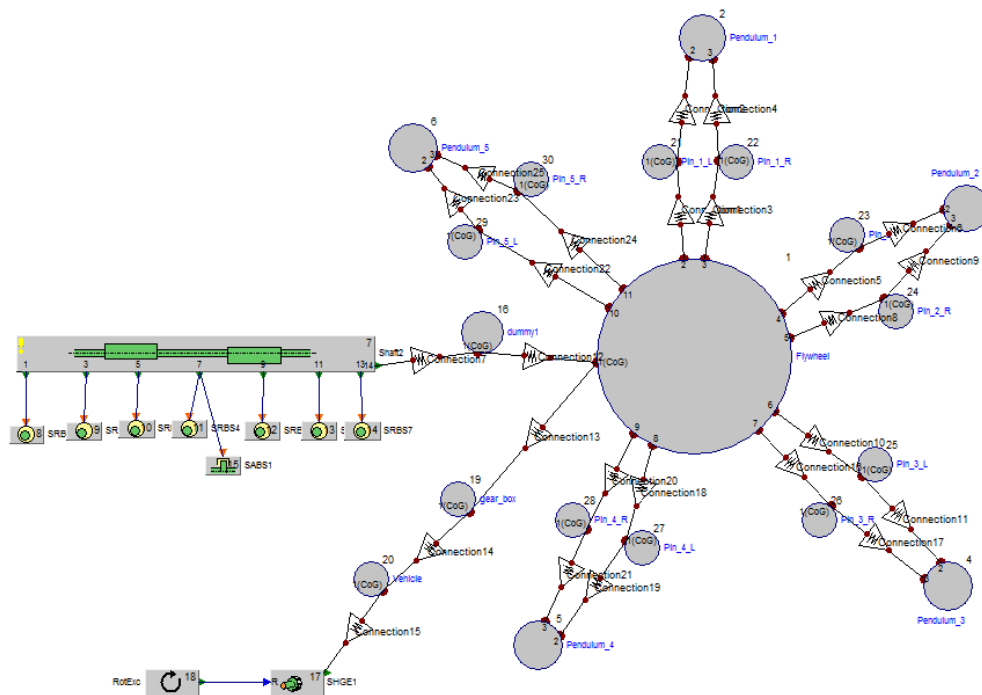


Figure 3-10: CPVA model in AVL Excite TD

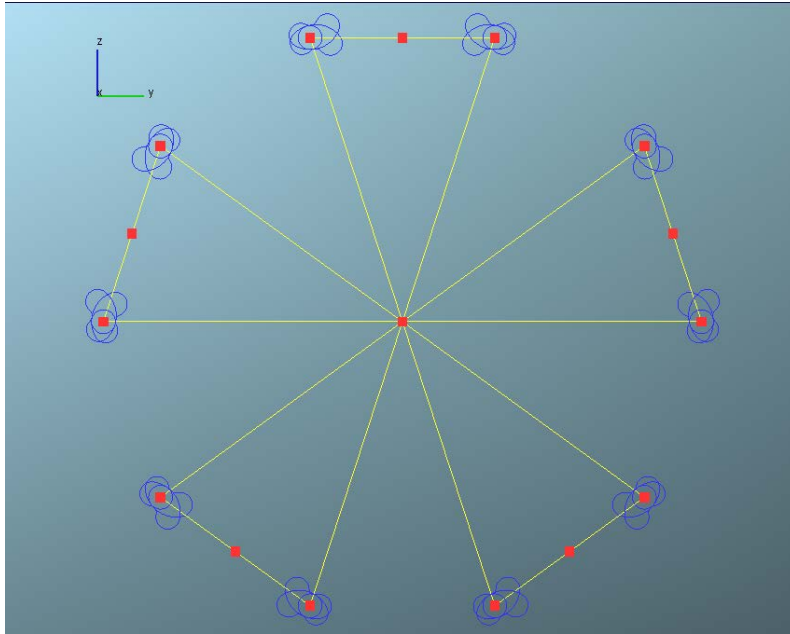


Figure 3-11: Contour of grooves made on flywheel and pendulum in CVPA model

The positioning of these pendulums is defined with parameters consisting of trigonometric functions (Appendix 7.1). This will allow a parametric study on the effect of position of pendulum on torsional vibration reduction.

### 3.3.4 DMF with CPVA flywheel in AVL Excite

A primary flywheel is added in between engine crankshaft and CPVA flywheel. This primary flywheel is connected to CPVA through torsional arc springs with stiffness same as initial DMF model. Gear box and vehicle are added as generic masses with given inertias and stiffnesses. The Figure 3-12 shows DMF with CPVA model in AVL excite.

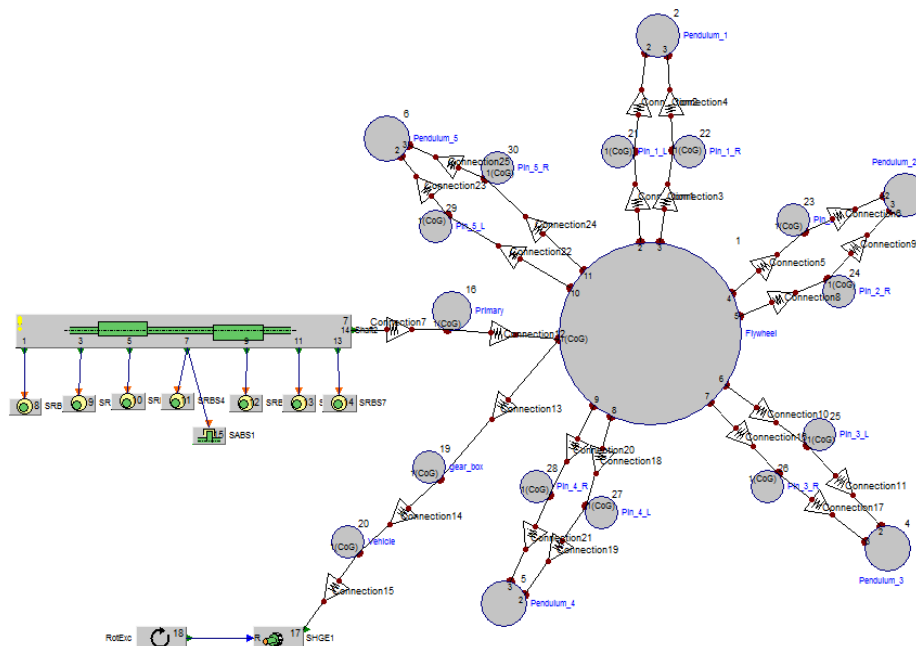


Figure 3-12: DMF with CPVA in AVL Excite TD

### 3.4 Triple Mass flywheel

Developed from dual mass flywheel, in this model three masses are used. Primary flywheel connected to engine crankshaft, secondary flywheel connected on transmission side. Hence, a possible design is developed which can be further modified and studied.

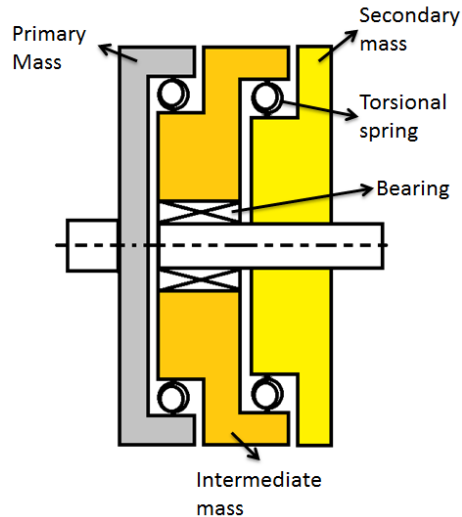


Figure 3-13: Possible design for Triple mass flywheel

#### 3.4.1 Model in Adams

The modelling of this concept remains same as for dual mass flywheel. And additional flywheel is added to the same model and assuming that overall mass remains same as dual mass flywheel, the secondary mass is split into two parts. The two torsional springs used between these masses will be same as the one used in dual mass flywheel. Figure 3-14 shows triple mass flywheel in Adams.

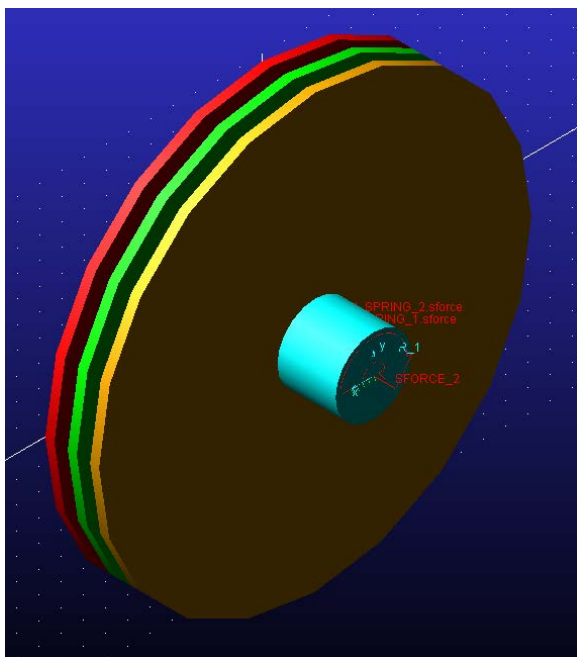


Figure 3-14: Triple mass flywheel model in Adams

### 3.4.2 Model in AVL Excite

Modelling in AVL excite will be useful to parameterize and compare the results with Adams model. Similar modelling approach with three generic masses as flywheel will be defined and connected with rotational spring. The stiffness will be same as in previous case.

Figure 3-15 shows the triple mass flywheel in AVL Excite

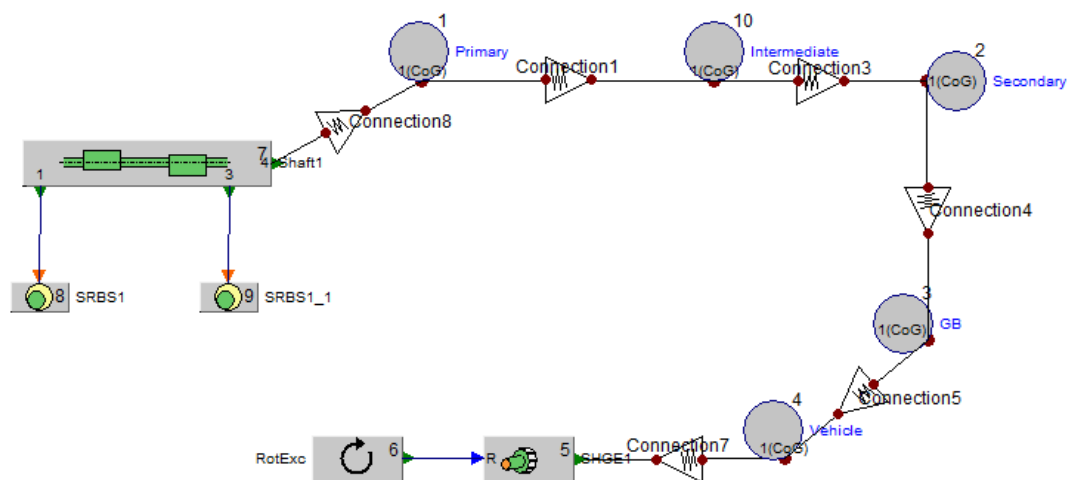


Figure 3-15: Triple mass flywheel model in AVL Excite TD

## 4 Results

The simulation is run for 600 cycles at 800 rpm. Break Torque from the engine is compared with the torque at input shaft of transmission. This will give a measure of input torque into the system and reduced torque fluctuations transferred to transmission by the flywheel. Power Split model and CPVA model in Adams became computationally very huge and difficult to solve, and hence there were simplified without downstream data.

### 4.1 Dual mass flywheel

The following figure shows the comparison of torque of the system at a point where the system stabilizes in Adams and AVL Excite.

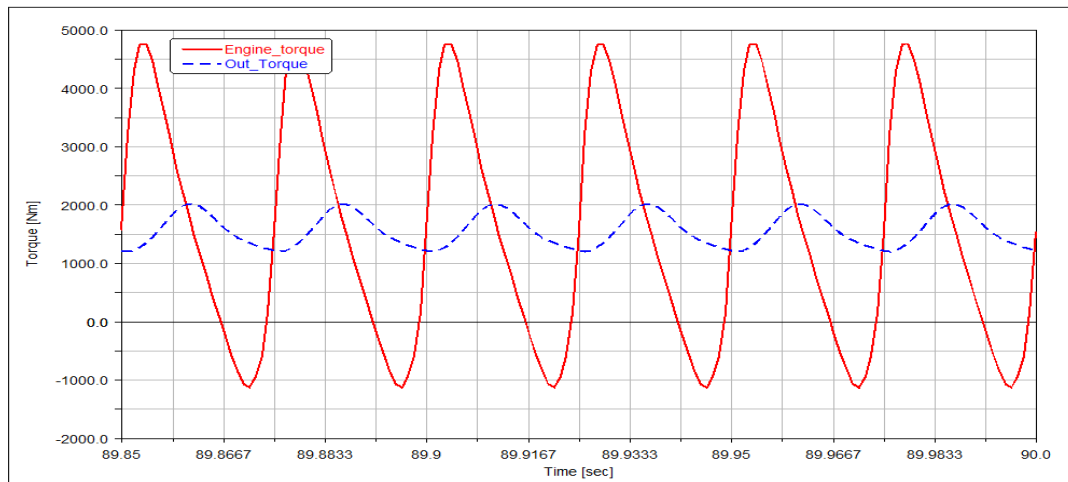


Figure 4-1: Comparison of Torque in Adams

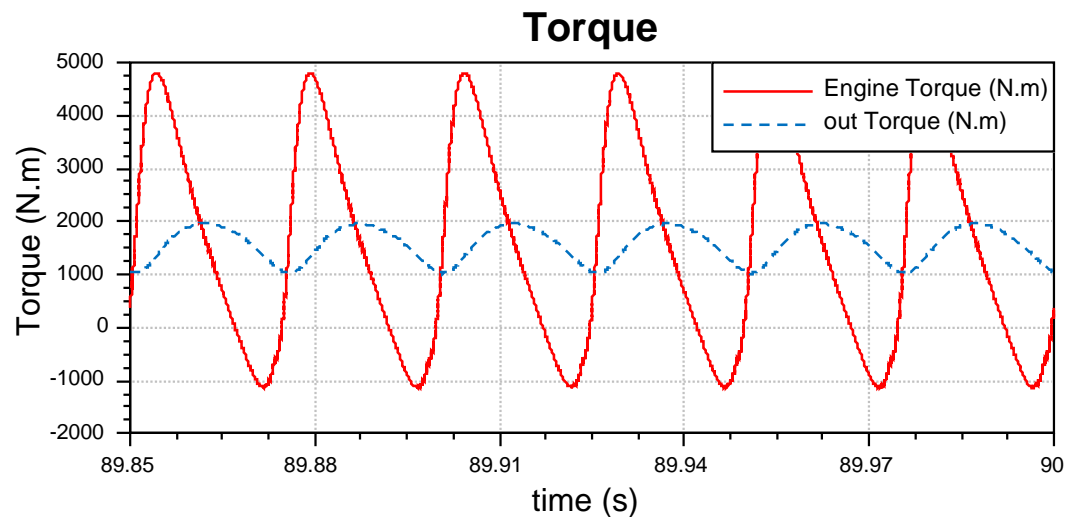


Figure 4-2: Comparison of Torque in AVL Excite TD

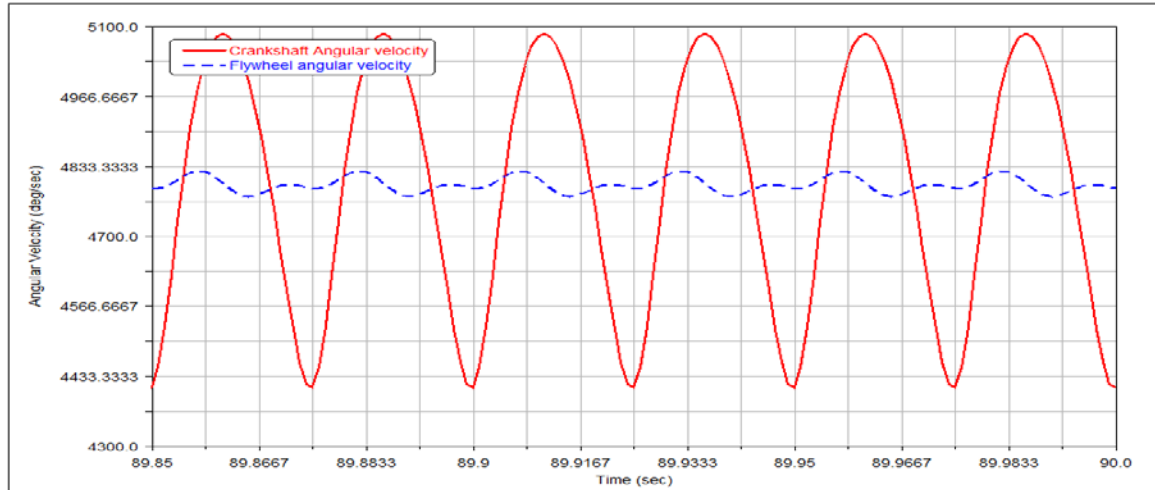


Figure 4-3: Plot of Angular velocity in Adams

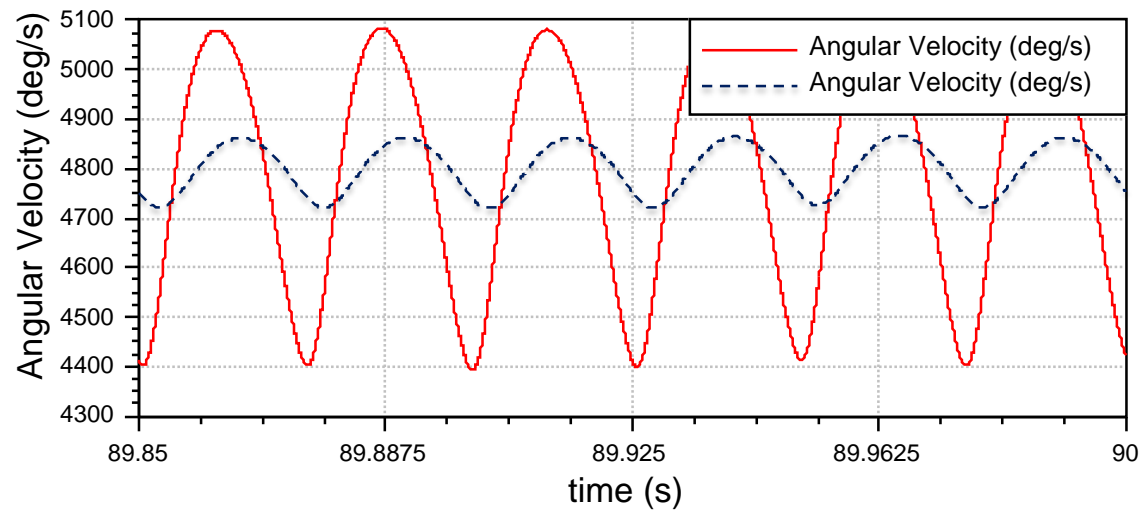


Figure 4-4: Plot of Angular velocity in AVL Excite TD

From the above plot it is observed that the torque curve is similar in both software, however the angular velocity plot is quite different. This might be due inefficient application of excitation load on the wheel in Adams model which AVL is designed for transmission analysis hence excitation is accurately defined.

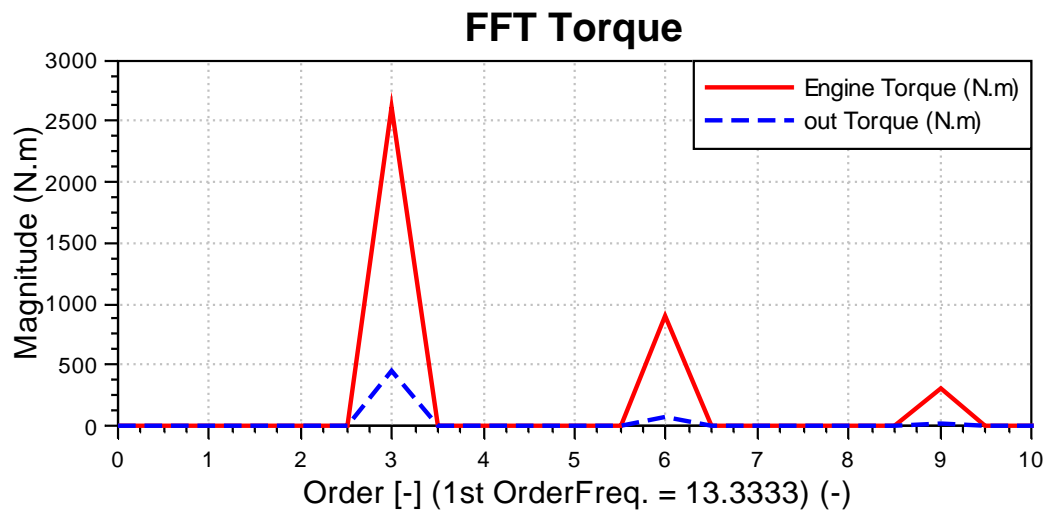


Figure 4-5: FFT diagram of Torque

From the Fast Fourier transform (FFT) diagram it can be seen that the magnitude of 3<sup>rd</sup> order and 6<sup>th</sup> order frequencies are reduced. If the magnitude of these frequencies is not reduced then there is a possibility of resonance at higher rpm which might damage the parts.

## 4.2 Power split model

Due to modelling constraints in AVL excite, this concept is only analysed in Adams. Figure 4-6 shows the torque measurement on the either side of the flywheel. The torque that is entering the transmission has spikes which show gear rattling between the planetary gear and ring gear. Also the average torque has reduced which shows loss of energy due to rattling.

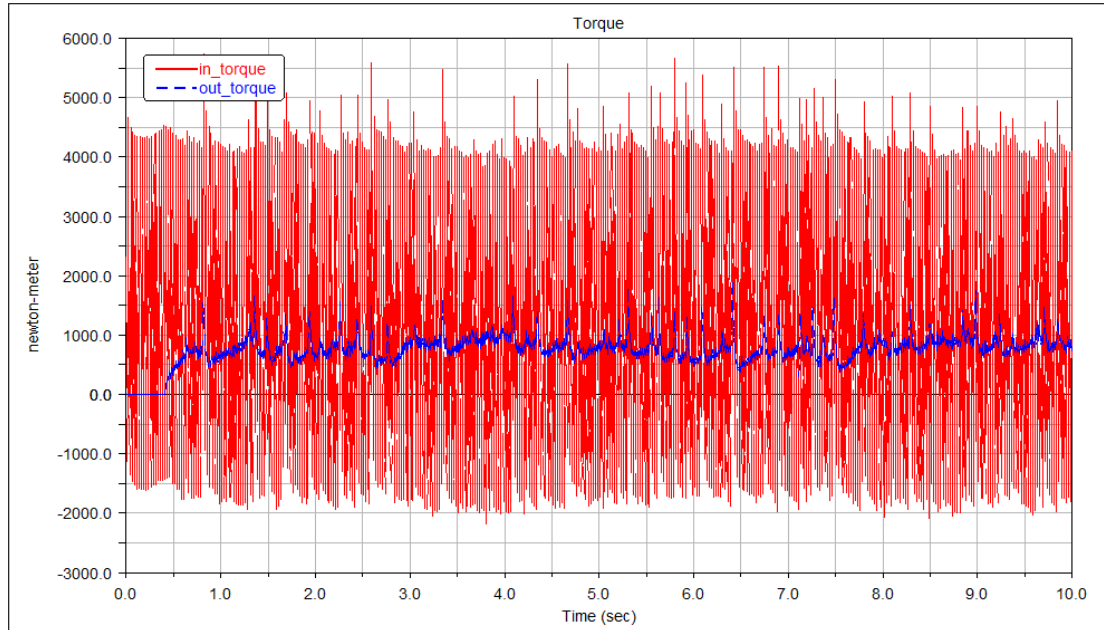


Figure 4-6: Torque comparison in Power split model in Adams

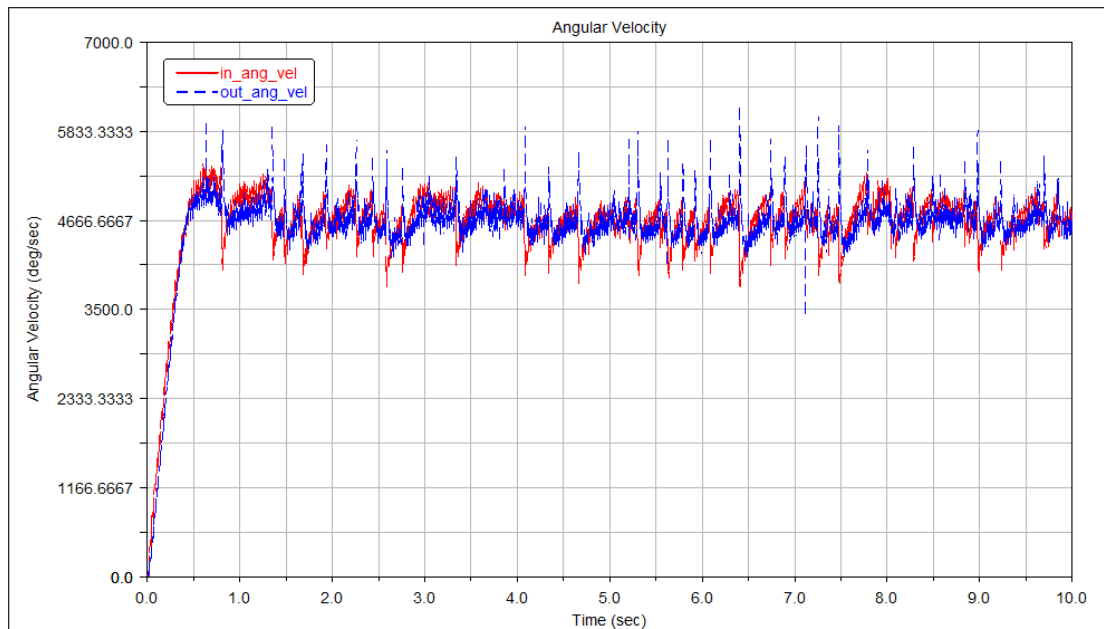


Figure 4-7: Velocity plot in Adams



### Alternate model of Power split flywheel

Figure 4-8 shows the torque curve for alternate power split flywheel explained in section 3.2.2. The performance seems to be better compared to previous model but presence of few spikes shows the occurrence of gear rattle during the operation.

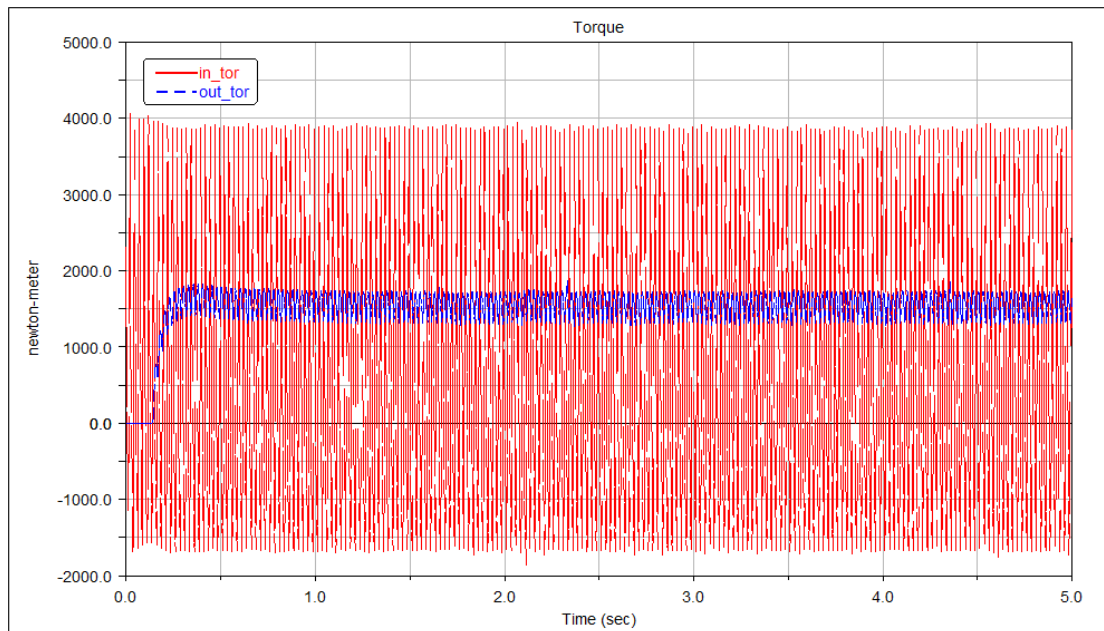


Figure 4-8: Torque curves plotted in Adams

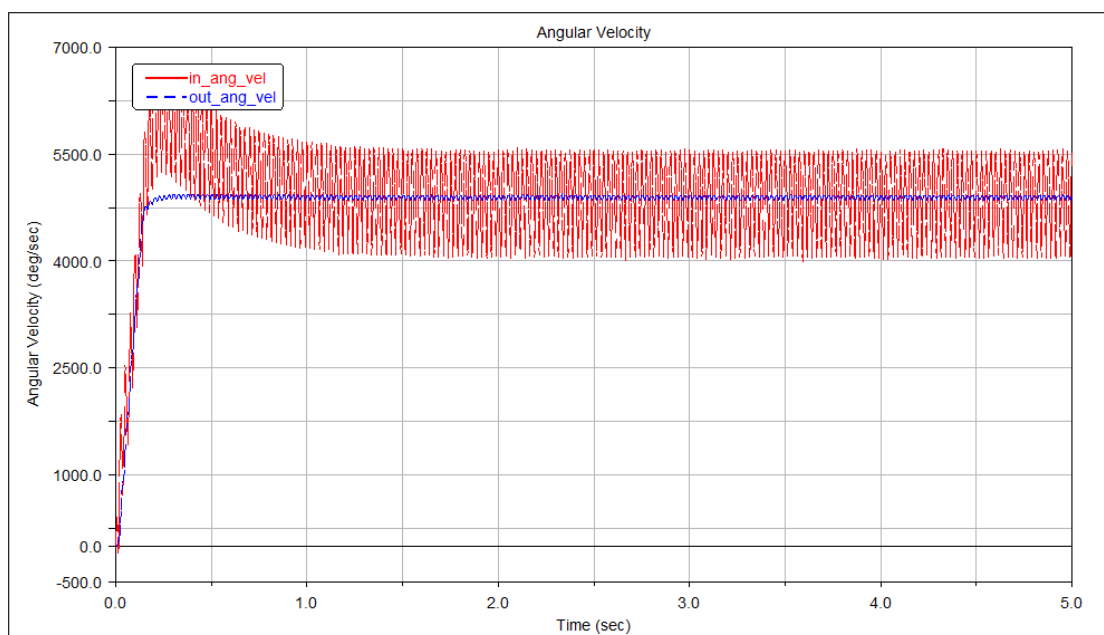


Figure 4-9: Angular velocity plot in Adams

### 4.3 Centrifugal pendulum vibration absorber

The following figure shows the comparison of torque of the system at a point where the system stabilizes in Adams and AVL Excite.

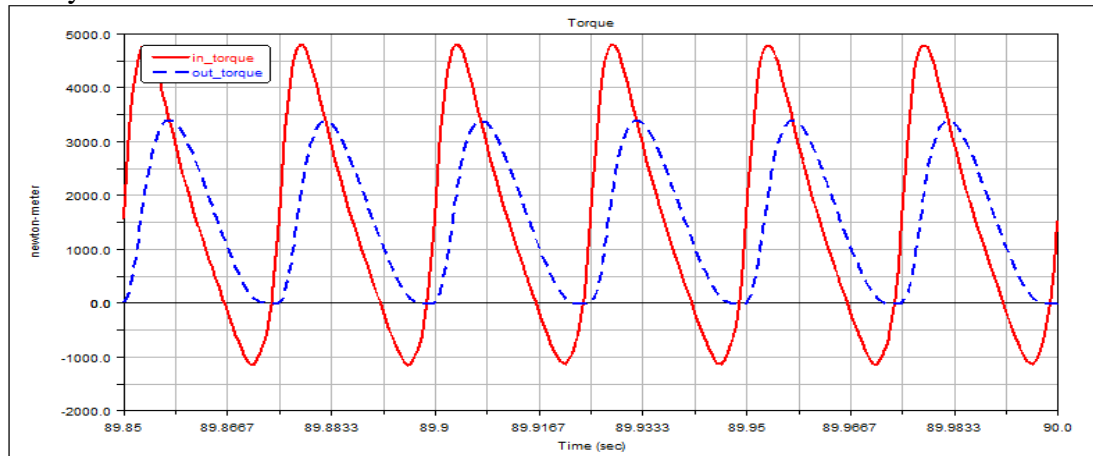


Figure 4-10: Plot of torque in Adams

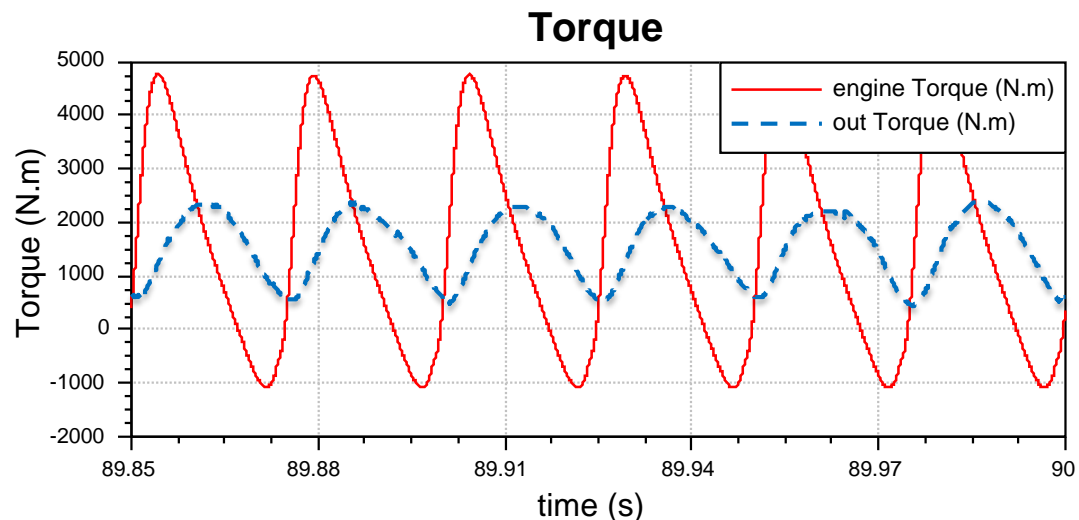


Figure 4-11: Plot of Angular velocity in AVL Excite

FFT diagram shown in Figure 4-12 clearly shows the reduction of magnitude of torque in 3<sup>rd</sup> order and 6<sup>th</sup> order frequencies.

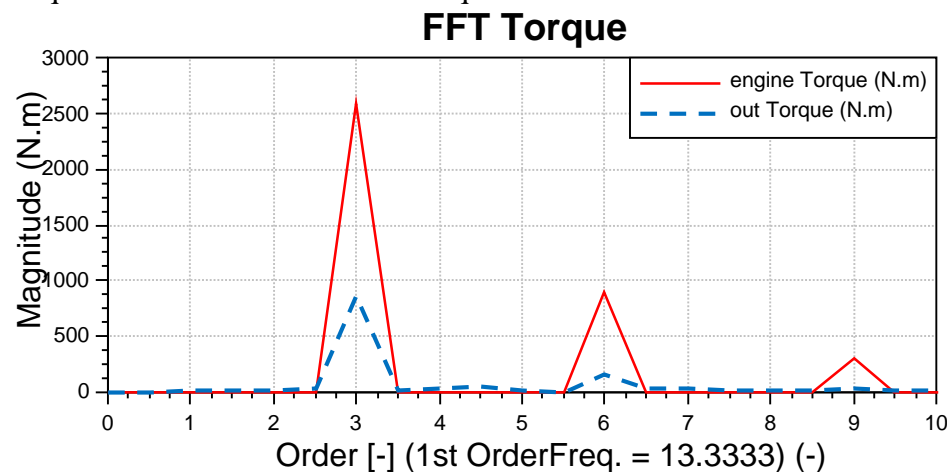


Figure 4-12: FFT diagram of Torque

## 4.4 DMF with CPVA

The following figure shows the comparison of torque of the system at a point where the system stabilizes in Adams and AVL Excite.

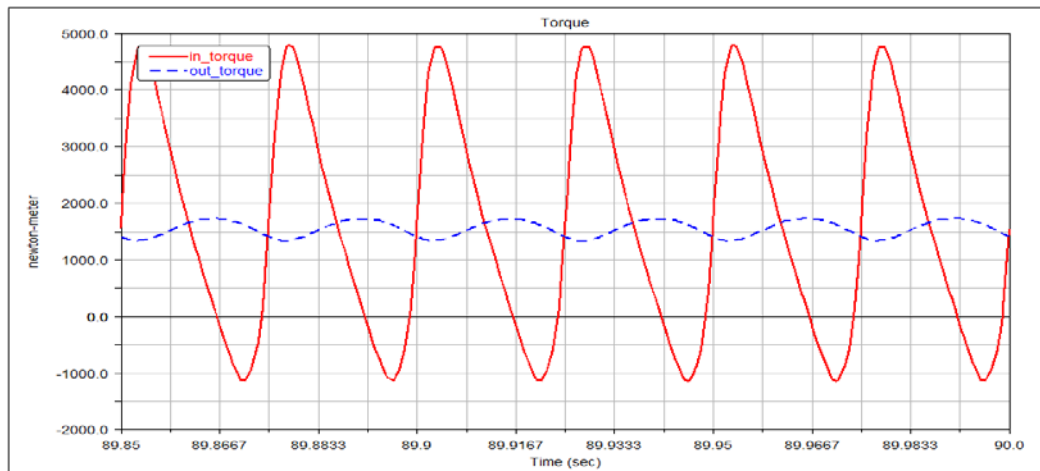


Figure 4-13: Plot of Torque in Adams

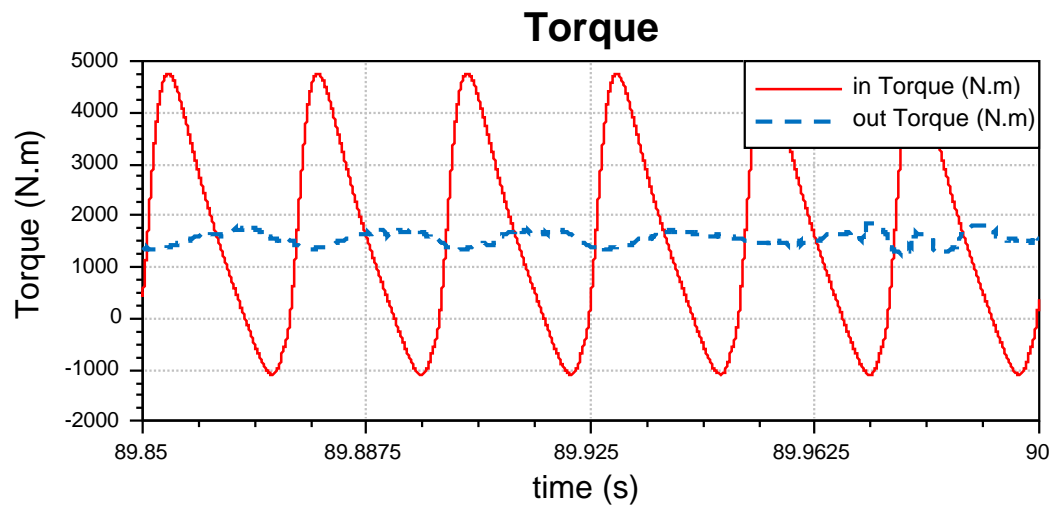


Figure 4-14: Plot of Torque in AVL Excite

The AVL Excite torque plot in Figure 4-14 shows a disturbance between time period 89.9625 [s] and 90 [s]. This could be due to rough movement of pendulum to extreme end that causes the roller to hit the ends of the groove.

FFT diagram shown in Figure 4-15 clearly shows the reduction of magnitude of torque in 3<sup>rd</sup> order and 6<sup>th</sup> order frequencies.

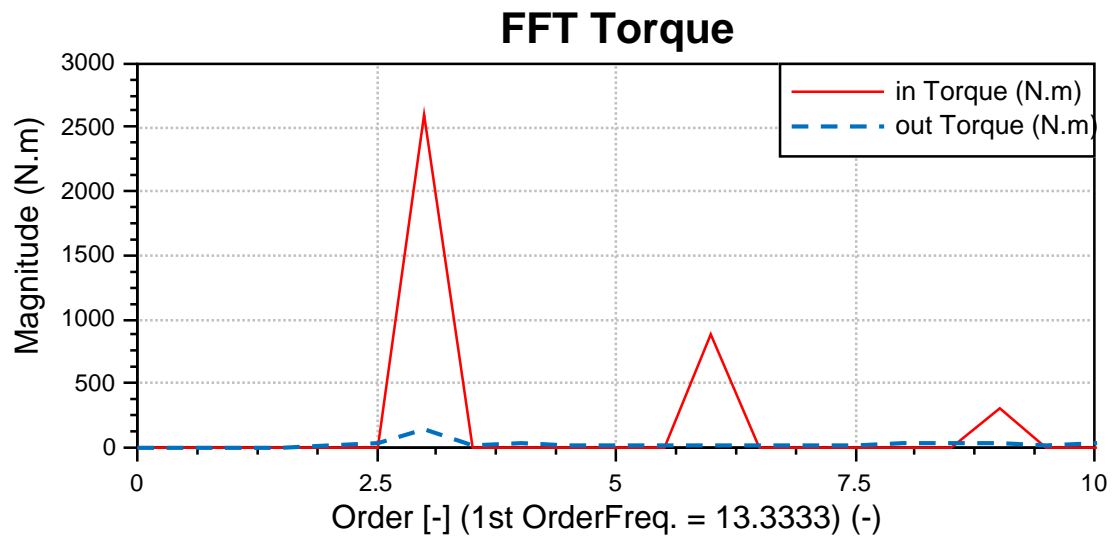


Figure 4-15: FFT diagram of Torque

## 4.5 Triple Mass flywheel

The following figure shows the comparison of torque of the system at a point where the system stabilizes in Adams and AVL Excite.

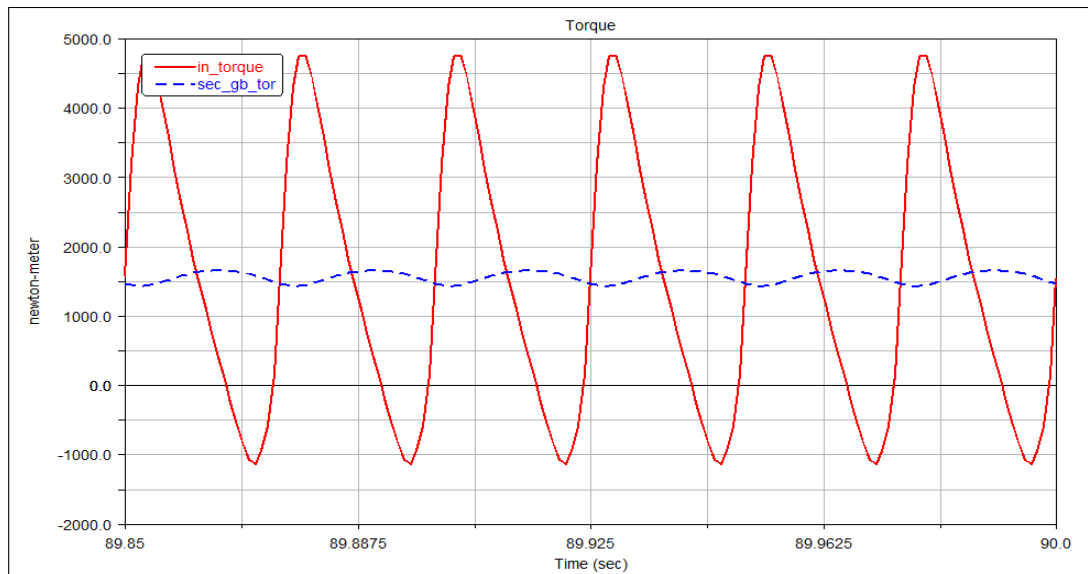


Figure 4-16: Plot of Torque in Adams

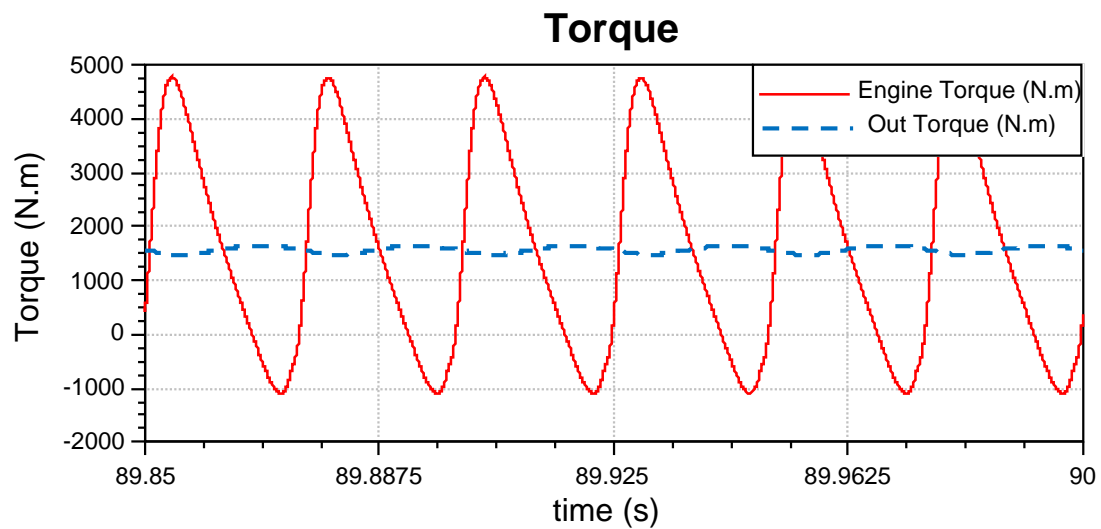


Figure 4-17: Plot of Torque in AVL Excite

FFT diagram shown in Figure 4-18 shows the reduction of magnitude of torque in 3<sup>rd</sup> order and 6<sup>th</sup> order frequencies.

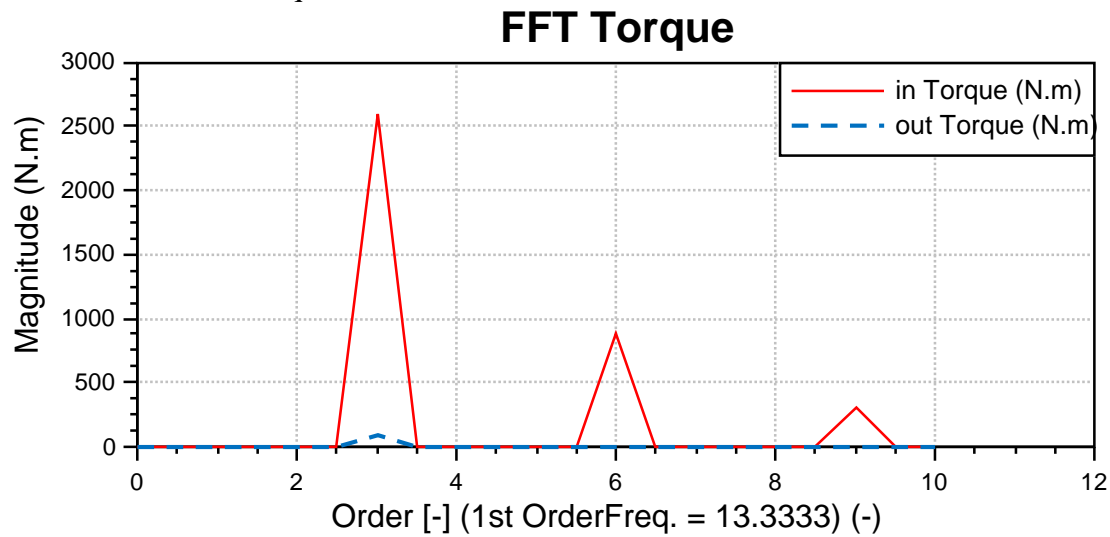


Figure 4-18: FFT diagram of Torque

#### 4.6 Comparison with single mass Solid flywheel

Comparison of FFT diagram of all the models with a solid flywheel is shown in Figure 4-19. The total mass of all the system has been maintained constant in all the cases. It is seen that 3<sup>rd</sup> order torque fluctuation is reduced in all the concepts while 6<sup>th</sup> order which can be critical at higher speeds has been reduced to a great extent in DMF with CPVA model and TMF model.

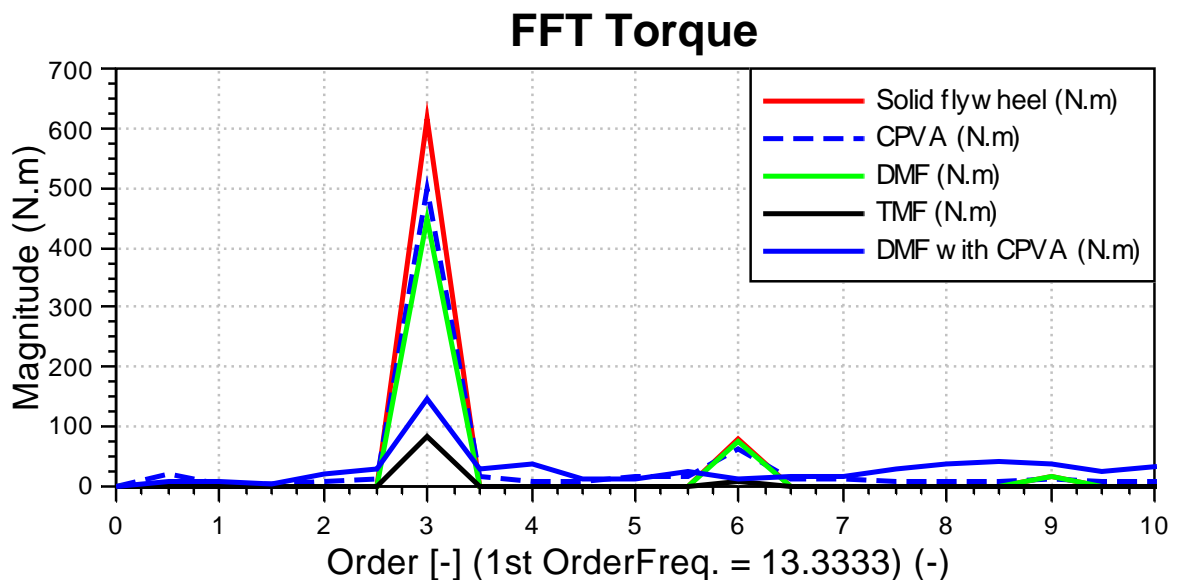


Figure 4-19: Comparison of FFT plot of Torque for different models

## 5 Conclusion and Future work

### Dual Mass Flywheel

Dual mass flywheel is the current flywheel used in Volvo trucks. The models of dual mass flywheel in MSC Adams and AVL excite have similar torque output. However, the angular velocity plots are slightly different. This is due to the difference in algorithm used in defining the break load. In AVL Excite only the speed is defined as a parameter for excitation, while in Adams a function called STEP function is used which is defined in Adams software [11]. The models could not be validated with experimental results due to unavailability.

Parametrization of spring characteristics in dual mass flywheel was done in early study. However, variation of spring characteristics did not alter the results significantly. Hence, it was concluded that the spring is already optimised by the vendor (LuK [4]) and no further investigations are done for the current model of dual mass flywheel. Vibration reduction is observed with increase in inertia of flywheel masses but due to packaging constraints there is a limit on the total mass of the flywheel.

### Centrifugal pendulum vibration absorber

CPVA model in AVL Excite shows a good improvement when compared to solid flywheel of similar mass. This model has been given to simulation group in Volvo GTT. This model will be used for following further investigations that includes changing the absorber path, distance of centre of mass of pendulum and mass of pendulum. Effect of gravity and tuning the pendulum for two different orders can also be in future.

### Triple mass flywheel

In DMF, the presence of weak arc spring made it possible to reduce the rigidity of the flywheel and thus reducing the torsional fluctuation. With another set of arc spring in triple mass flywheel, the rigidity of the secondary part of the flywheel is further broken down. Hence, triple mass flywheel is less complex and has better results when compared to other concepts. Triple mass flywheel seems promising and should be further analysed by spring parameterization and changing mass ratios of the three flywheels. Thus, an optimised model can be made according to flywheel design parameters, torque and speed of the engine.

### Power split flywheel

The paper presented on power split flywheel by Lorenz et al. [8] claims that power split is better alternative than pendulum flywheel but there is no mention of gear rattle which is seen in the velocity plots. Use of torsional dampers might have absorbed gear rattle. But using internal torsional dampers will cause energy dissipation and hence, cooling of flywheel at high speed has to be taken care. The modelling of power split flywheel can be continued further by varying gear ratios, spring stiffness and gear ratios. Further, modelling in AVL Excite will be useful for to compare and investigate the model.

Furthermore, use of advanced flywheel seems very promising for reducing torsional vibrations in down-sized engine. Due to time constraint one goal point is missed, i.e., analysis on 3-cylinders, 4-cylinders and 5-cylinders IC engines. This study will be helpful to understand the adaptability of these flywheels.

And finally, to extend the knowledge further, a complete 3-D FFT plot of torque magnitude vs. vibration orders at different engine speed can be made which will be useful to analyse different flywheel models for a given speed range.



## 6 References

- [1] Kroll J., Kooy A., Seebacher R. (2010): "Torsional vibration damping for future engines". *9<sup>th</sup> Schaeffler Symposium*, 2010.
- [2] Meingaßner, G., Pflaum, H., Stahl K. (2014): "Innovative Torsional Vibration Reduction Devices - Vehicle-Related Design and Component Strength Analysis". *SAE Int. J. Passeng. Cars - Mech. Syst.* 7(4):2014, doi:10.4271/2014-01-2862.
- [3] Schaper U., Sawodny O., Mahl T., Blessing U. (2009): "Modeling and torque estimation of an automotive Dual Mass Flywheel". *American Control Conference*, June 2009.
- [4] LuK, February 2015, URL <http://www.luk.de/content.luk.de/en/>
- [5] Schaeffler, February 2015, URL <http://www.schaeffler.de/>
- [6] Torque Converter, accessed February 2015, URL <http://repairpal.com/torque-converter>
- [7] ZF Friedrichshafen AG, February 2015, URL <http://www.zf.com/>
- [8] Lorenz, D., Fischer, M., Orlamünder, A., Power Split as Torsional Damping system, *Update and advanced functionalities*, ZF Friedrichshafen AG.
- [9] Rak, L. H., Dae, H. M. (2008): "Triple mass flywheel for vibration damping, capable of reducing torsional vibration by changing an amplitude of a vibration into a state before idling of an engine". *Korea Patent 1020080008179*, January 2008, Publication 1020080089152
- [10] Wen-ku, S., De-lei, M. (2011): "Study on Vibration Damping Characteristics of Triple Mass Flywheel Torsional Damper", *China Academic Journal Electronic Publishing House*, {Jilin University}, {China}
- [11] MSC Adams User guide, accessed on March 2015, URL <http://www.mscsoftware.com/product/adams>
- [12] AVL Excite User Guide, accessed on May 2015, URL <https://www.avl.com/excite>
- [13] GT Power User guide, accessed on February 2015, URL <https://www.gtisoft.com/>
- [14] CATIA V5 user documentation, accessed on March 2015
- [15] A. Fidlin and R. Seebacher. (2006): "DMF simulations techniques – Finding the needle in the haystack". *8<sup>th</sup> LuK Symposium*, pages 55–71, 2006.
- [16] MATLAB documentation, Version R2012b, <http://se.mathworks.com/help/matlab/>
- [17] Monroe R., Shaw S. (2013): "Nonlinear Transient Dynamics of Pendulum Torsional Vibration Absorbers—Part I: Theory". *Journal of Vibration and Acoustics*, February 2013, Vol. 135 / 011017-1
- [18] Monroe R., Shaw S. (2013): "Nonlinear Transient Dynamics of Pendulum Torsional Vibration Absorbers— Part II: Experimental Results". *Journal of Vibration and Acoustics*, February 2013, Vol. 135 / 011018-1
- [19] Wedin A. (2011): "Reduction of Vibrations in Engines using Centrifugal Pendulum Vibration Absorbers". Master Thesis, Chalmers University of technology. 2011
- [20] CAD Exchanger, accessed March 2015, URL <http://www.cadexchanger.com/>
- [21] GRABcad, accessed March 2015, URL <https://grabcad.com/>
- [22] Tuplin W.A. (1934): *Torsional vibration: elementary theory and design calculations*, Wiley, New York, xviii, 320 s. : ill.

- [23] Berbyuk V. (2014): *Structural dynamics control*, 2<sup>nd</sup> edition. Chalmers University of Technology, Göteborg, Sweden.
- [24] Denbratt I. (1999): *Piston engine mechanics*. Chalmers university of technology
- [25] A. Boström. (2009): *Rigid Body Dynamics*. Chalmers University of Technology, October 2009.
- [26] Steinel, K. (2000): "Clutch Tuning to Optimize Noise and Vibration Behavior in Trucks and Buses", *SAE Technical Paper* 2000-01-3292, 2000, doi:10.4271/2000-01-3292.
- [27] Steinel, K. and Tebbe, G. (2004): "New Torsional Damper Concept to Reduce Idle Rattle in Truck Transmissions," *SAE Technical Paper* 2004-01-2722, 2004, doi:10.4271/2004-01-2722.
- [28] Jianjun, H., Datong, Q., Yusheng, Z., and Yonggang, L. (2009): "Study on Natural Torsional Vibration Characteristics of Dual Mass - Flywheel Radial Spring Type Torsional Vibration Damper," *SAE Technical Paper* 2009-01-2062, 2009, doi:10.4271/2009-01-2062.
- [29] Nerubenko, G. and Nerubenko, C. (2014): "Flywheel with tuned damping," *SAE Technical Paper* 2014-01-1685, 2014, doi:10.4271/2014-01-1685.
- [30] Faust H. (2014): "Solving the Powertrain Puzzle", *10<sup>th</sup> Schaeffler symposium*, 2014, VII, 521 p.
- [31] Cavina, N. and Serra, G. (2004): "Analysis of a Dual Mass Flywheel System for Engine Control Applications," *SAE Technical Paper* 2004-01-3016, 2004, doi:10.4271/2004-01-3016.
- [32] Denman H.H. (1992): "Tautochronic bifilar pendulum torsion absorbers for reciprocating engines", *Journal of Sound and Vibration* 159 (2) (1992) 251–277.
- [33] Pfabe, M. and Woernle, C. (2009): "Reduction of Periodic Torsional Vibration using Centrifugal Pendulum Vibration Absorbers". *Proc. Appl. Math. Mech.*, 9: 285–286.
- [34] Mayet J., Ulbrich H. (2014): "Tautochronic centrifugal pendulum vibration absorbers: General design and analysis", *Journal of Sound and Vibration*, Volume 333, February 2014, pages 711-729
- [35] Mayet J., Rixen D., Ulbrich H. (2013): "Experimental Investigation of Centrifugal pendulum vibration absorbers", *11<sup>th</sup> International conference on Vibration Problems*, Libson, Portugal, September 2013
- [36] Xia, Y., Pang, J., Hu, C., Zhou, C. et al. (2014): "Multi-Body Dynamic Analysis of Driveline Torsional Vibration for an RWD Vehicle," *SAE Technical Paper* 2014-01-2064, 2014, doi:10.4271/2014-01-2064.
- [37] Bighal P. (2012): "High Efficiency Heavy Duty Truck Engine", Master Thesis, Chalmers University of Technology, 2012
- [38] Zink M., Hausner M. (2009): "The Centrifugal Pendulum-type Absorber", *ATZ Worldwide*, Volume 111, Issue 7-8, pp 42-47
- [39] Stahl K., Pflaum H., Meingaßner, G., Ulbrich H., Mayer J. (2013): "Planetary Centrifugal Pendulum Absorber (pCPA) – New type of Centrifugal Pendulum Absorber for Applications in Highly Downsized Hybrid and Range Extender Combustion Engines". *Conference on Future Automotive Technology*, DOI 10.1007/978-3-658-01141-3\_5.
- [40] Theisen T. (2011): "Gravity's effect on Centrifugal Pendulum Vibration Absorbers", Master Thesis, Michigan State University.

## 7 Appendix

### 7.1 CPVA model in AVL Excite

#### Equations for parameterizing pendulums for the flywheel

Distance of centre of mass of pendulum,  $R$  [mm]

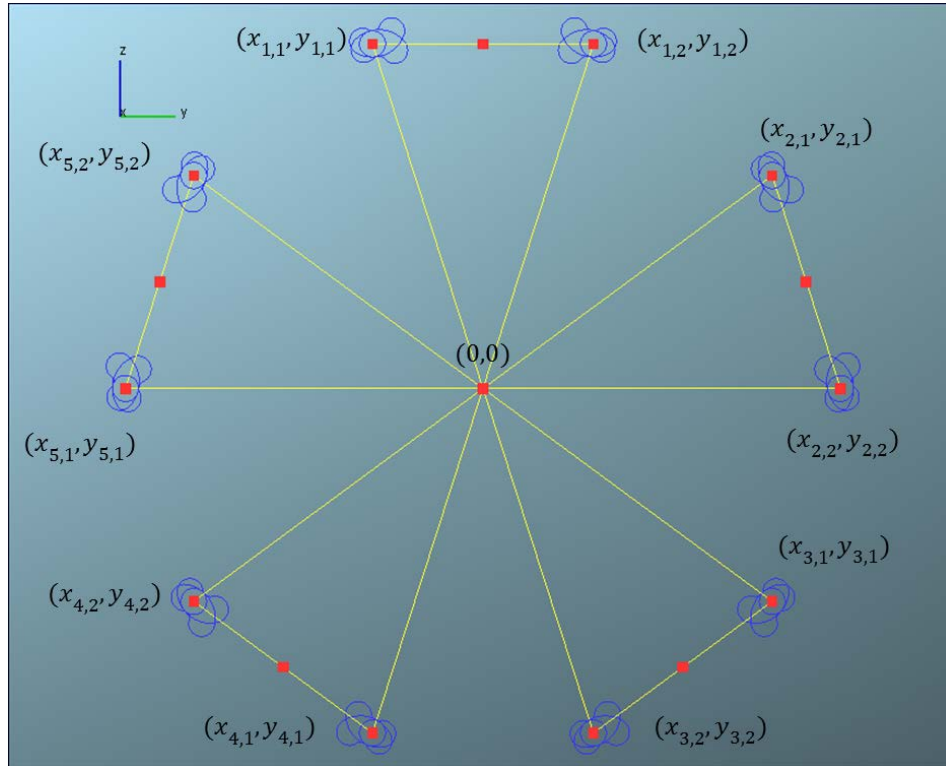


Figure 7-1: CPVA model with positions of link points

Table 7-1: Link point and formula to calculate the position

Link Point	Position [mm]
$x_{1,1}$	$\frac{-R}{\sqrt{1 + \tan^2\left(\frac{-72\pi}{180}\right)}}$
$y_{1,1}$	$-x_{1,1}\tan\left(\frac{-72\pi}{180}\right)$
$x_{1,2}$	$\frac{R}{\sqrt{1 + \tan^2\left(\frac{72\pi}{180}\right)}}$
$y_{1,2}$	$x_{1,2}\tan\left(\frac{72\pi}{180}\right)$

$x_{2,1}$	$\frac{R}{\sqrt{1 + \tan\left(\frac{36\pi}{180}\right)^2}}$
$y_{2,1}$	$x_{2,1}\tan\left(\frac{36\pi}{180}\right)$
$x_{2,2}$	$\frac{R}{\sqrt{1 + \tan\left(\frac{0\pi}{180}\right)^2}}$
$y_{2,2}$	$x_{2,2}\tan\left(\frac{36\pi}{180}\right)$
$x_{3,1}$	$\frac{R}{\sqrt{1 + \tan\left(\frac{-36\pi}{180}\right)^2}}$
$y_{3,1}$	$x_{3,1}\tan\left(\frac{-36\pi}{180}\right)$
$x_{3,2}$	$\frac{R}{\sqrt{1 + \tan\left(\frac{-72\pi}{180}\right)^2}}$
$y_{3,2}$	$x_{3,2}\tan\left(\frac{-72\pi}{180}\right)$
$x_{4,1}$	$\frac{-R}{\sqrt{1 + \tan\left(\frac{72\pi}{180}\right)^2}}$
$y_{4,1}$	$x_{4,1}\tan\left(\frac{72\pi}{180}\right)$
$x_{4,2}$	$\frac{-R}{\sqrt{1 + \tan\left(\frac{36\pi}{180}\right)^2}}$
$y_{4,2}$	$x_{4,2}\tan\left(\frac{36\pi}{180}\right)$
$x_{5,1}$	$\frac{-R}{\sqrt{1 + \tan\left(\frac{0\pi}{180}\right)^2}}$
$y_{5,1}$	$x_{5,1}\tan\left(\frac{0\pi}{180}\right)$
$x_{5,2}$	$\frac{-R}{\sqrt{1 + \tan\left(\frac{-36\pi}{180}\right)^2}}$
$y_{5,2}$	$x_{5,2}\tan\left(\frac{-36\pi}{180}\right)$