Streamer branching in air: physical model and simulations in fully 3d spatial domain

Streamer Branching in Air: Physical Model and Simulations in Fully 3D Spatial Domain

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***Abstract*—Fully coupled self-consistent model of streamer branching is presented. The model incorporates inhomogeneities in gas media representing, e.g., large molecular clusters, micro dust particles, etc., which act as centers for localized space charge build-up. The results of the performed simulations show that charge accumulation at such centers leads to their electrostatic interactions with the streamer head and causes its splitting. Quantitative analysis of the dynamics of charge carriers densities, generated space charges and magnitudes of the electric fields associated with the branching process is presented.**

Keywords—streamer propagation, streamer branching, drift-diffusion equation, high voltage, gas insulation

I. INTRODUCTION

It is well-known that development of a streamer, especially for long distances, is associated with its branching into multiple channels. Reasons for this phenomenon are not absolutely clear. Theories have been proposed, see e.g. [1], which consider inherent instabilities and fluctuations of electron densities at the streamer head due to stochastic nature of motion of particles, influence of inhomogeneities, localized charge centers, etc. Early attempts to simulate branching process were based on 2d models with fixed conductivity of streamer channel, e.g. [2]. Among possible reasons for branching, Laplacian instability was considered and treated with 2d axisymmetric approach [3] based on an assumption that the localized charge density at the streamer tip leads to multiple local electron avalanches facilitating branching. Interestingly, no special treatment has been introduced in that model to initiate these instabilities and due to this reason the results were later on interpreted as numerical artifacts [4].

Simulations of branching of streamers in 3d domains have been attempted for liquid dielectrics. Thus, a stochastic model proposed in [5] was based on an assumption of a final conductivity of a streamer channel. It considered discharge progress in fixed steps. A critical electric field was defined on the streamer tip and the channel was extended by adding a new portion of it in a random direction chosen on the basis of a defined 'fluctuation'. This procedure allowed streamer to 'branch' within a small region by permitting its growth in the direction not coinciding with the original direction of the electric field. One should note here that the model used in [5] was not consistent and was based on artificial numerical treatment of the actual phenomenon. More physical approach was used in [6] where charge accumulation centers randomly distributed in oil volume have been introduced. Presence of such centers led to branching of the main streamer channel which occurred due to their electrostatic interaction with the moving streamer head.

In the present study, the model of branching proposed in [6] for streamers in oil is adopted and is modified to account for physical conditions of development of streamers in gas medium. Thus, the charge accumulation centers are introduced as dielectric inhomogeneities appearing due to suspended particulate (micro dust particles, large molecular clusters, etc.). This allows for charge magnitudes to be derived from inherent discharge processes without introducing any artificial approximations as in [6]. It is expected that charge accumulation process on the dielectric inhomogeneities would lead to local field enhancements and provide alternative potential paths for the streamer to propagate and, hence, for branching. The basic approach for the simulation of streamer discharges in air was adopted from the model [7], which was verified against experimental data provided in the literature.

II. STREAMER MODEL EQUATIONS AND PARAMETERS

Various models exist for simulating streamer propagation in air. The most popular and efficient so far is known as drift-diffusion model. It comprises a set of mass conservation equations for charge carriers in gas obtained as zero-moments of the solution of Boltzmann's transport equation [8]. In the present study, three types of species are considered namely electrons, generic positive ions and negative ions. Variations of their densities n, m^{-3}, in time and space due to drift and diffusion in electric field are described by the following equations expressing continuity of their fluxes:

$$\begin{aligned} \partial n_e/\partial t + \nabla \cdot (-n_e \mathbf{w}_e - \mathrm{D}_e \nabla n_e) &= \mathrm{R}_e \\ \partial n_p/\partial t + \nabla \cdot \left(n_p \mathbf{w}_p - \mathrm{D}_p \nabla n_p\right) &= \mathrm{R}_p \\ \partial n_n/\partial t + \nabla \cdot (-n_n \mathbf{w}_n - \mathrm{D}_n \nabla n_n) &= \mathrm{R}_n \end{aligned} \quad (1)$$

Here: subscripts e, p and n indicate electrons, positive and negative ions, respectively; t is time; $\mathbf{w}$ is the drift velocity vector, m/s; D stands for the diffusion coefficient, m^2/s; R represents the sum of rates of generation and losses of the

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charge carriers. The latter accounts for main processes in air discharges. Thus, the rate of electron impact ionization of molecules is represented as $R_{ion} = \alpha\, \mathrm{n_e}|\mathbf{w_e}|$, where α, $\mathrm{m^{-1}}$, is the Townsend's ionization coefficient. The rate of capturing of free electrons by electronegative molecules in air (electron attachment) leading to formation of negative ions is defined as $\mathrm{R_{att}} = \eta\, \mathrm{n_e}|\mathbf{w_e}|$, where η, $\mathrm{m^{-1}}$, is the attachment coefficient. Electron-ion and ion-ion recombination processes are represented by corresponding rates $\mathrm{R_{xy}} = \beta_{xy}\, \mathrm{n_x n_y}$, where β_{xy} is recombination coefficient and subscripts x and y indicate participating carries (can be *ep* or *pn*). The other processes included in the model are detachment of electrons from negative ions with the rate $R_{det} = \mathrm{k}_{det}\, \mathrm{n_e n_n}$ (k_{det} is the corresponding coefficient) and photoionization of molecules characterized by its rate R_{ph}. Thus, the source terms in (1) can be expanded to

$$\begin{aligned} R_e &= R_{ion} + R_{det} + R_0 + R_{ph} - R_{att} - R_{ep} \\ R_p &= R_{ion} + R_0 + R_{ph} - R_{ep} - R_{pn} \\ R_n &= R_{att} - R_{det} - R_{pn} \end{aligned} \quad (2)$$

Photoionization is important process supporting streamer propagation by creating a localized source of electrons just in front of the streamer head. It is commonly accepted today [9]. that photoionization in air is due to photons generated by quenching of exited states of nitrogen molecules which are capable to ionize oxygen molecules (the energy of irradiated photons is higher than the ionizations threshold of O_2). The local photoionization rate at a given point in discharge volume is a sum total of all the volumetric photoionization events by photons travelling through the medium characterized by certain absorption coefficient. An integral method for calculating photoionization rate has been proposed in [9]. This approach, however, is extremely computationally expensive. Recently, highly efficient differential method based on solution of Helmholtz equations was suggested [10]. According to that, the photoionization rate is given as a sum (3) of solutions of (4)

$$R_{ph} = \sum_i R_{ph}^i \quad (3)$$

$$\nabla^2 R_{ph}^i(\mathbf{r}) - \left(\lambda_i \mathrm{p_{O_2}}\right)^2 R_{ph}^i(\mathbf{r}) = -\mathrm{A}_i \mathrm{p_{O_2}}^2\, \mathrm{I}(\mathbf{r}) \quad (4)$$

where $\mathrm{p_{O_2}}$ is partial pressure of oxygen and $\mathrm{I}(\mathbf{r})$ is the photons generation rate. Index *i* represents the number of components contributing to the total photoionization rate. In the present study, the three-term approximation is chosen following [10]. The generation rate of photons I(r) is given by equation (5), where $\mathrm{p_q}$ represents the quenching pressure (equal to 60 Torr) and ξ is the efficiency of the process:

$$\mathrm{I}(\mathbf{r}) = \mathrm{p_q} \xi R_{ion} / (\mathrm{p_{O_2}} + \mathrm{p_q}) \quad (5)$$

The drift-diffusion equations (1) are coupled with Poisson's equation for the electric potential V

$$\nabla \cdot (-\epsilon_0 \epsilon_\mathrm{r} \nabla \mathrm{V}) = \mathrm{q}_e \left(\mathrm{n_p} - \mathrm{n_e} - \mathrm{n_n}\right) \quad (6)$$

which allows for obtaining space charge controlled electric field $-\nabla \mathrm{V} = \mathbf{E}$ at each time moment. In (6), ϵ_0 is permittivity of vacuum, ϵ_r is relative permittivity, $\mathrm{q_e}$ is electron charge, and the right hand side represent the net space charge density.

TABLE 1 PARAMETERS USED IN THE SIMULATIONS

Parameter	Value
μ_p , $\mathrm{m^2/Vs}$	$2 \cdot 10^{-4}$
D_p , $\mathrm{m^2/s}$	$5.05 \cdot 10^{-6}$
μ_n , $\mathrm{m^2/Vs}$	$2.2 \cdot 10^{-4}$
D_n , $\mathrm{m^2/s}$	$5.56 \cdot 10^{-6}$
β_{ep} , $\mathrm{m^3/s}$	$5 \cdot 10^{-14}$
β_{pn} , $\mathrm{m^3/s}$	$2.07 \cdot 10^{-12}$
R_0 , $\mathrm{m^{-3}s^{-1}}$	$1.7 \cdot 10^{9}$
k_{det} , $\mathrm{m^3/s}$	10^{-18}

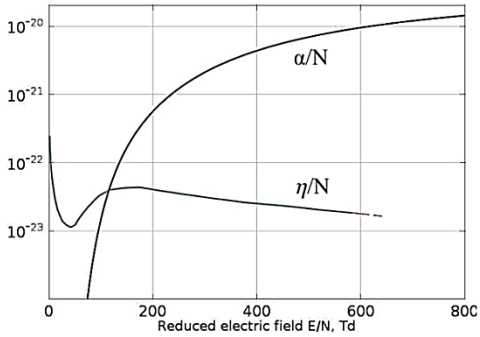

Fig. 1 Reduced ionization and attachment coefficients, $\mathrm{m^2}$, as functions of the reduced electric field.

Model parameters used in the simulations are adopted from [7, 8, 11] and are listed in Table1. Dependences of the reduced ionization α/N and attachment η/N coefficients on reduced electric field strength *E*/*N* (*N* if the gas density, $\mathrm{m^{-3}}$, for the given pressure and temperature) are shown in Figure 1. The drift velocity and diffusion coefficient of electrons are approximated as

$$\mathrm{w_e} = 3.2 \cdot 10^3 (E/N)^{0.8} \quad (7)$$

$$\mathrm{D_e} = 7 \cdot 10^{-2} + 8\, (E/N)^{0.8} \quad (8)$$

Note that the reduced field in (7)-(8) is in Td.

III. MODEL IMPLEMENTATION

Solving drift-diffusion equations (1) numerically is challenging due to the extremely large drift terms as compared with the diffusion terms. Moreover, the positivity of the solution is not guaranteed when utilizing conventional algorithms for resolving matrix problem arising from e.g. finite element discretization. To overcome these, the equations are transformed into a logarithmic form using substitution of type $n_l = \ln(n)$ that yields

$$\begin{aligned} e^{n_{el}}\, \partial n_{el}/\partial t + \nabla \cdot (-e^{n_{el}}\, \mathbf{w_e} - \mathrm{D_e} e^{n_{el}} \nabla\, n_{el}) &= \mathrm{R_e} \\ e^{n_{pl}}\, \partial n_{pl}/\partial t + \nabla \cdot \left(e^{n_{pl}}\, \mathbf{w_p} - \mathrm{D_p} e^{n_{pl}} \nabla\, n_{pl}\right) &= \mathrm{R_p} \\ e^{n_{nl}}\, \partial n_{nl}/\partial t + \nabla \cdot (-e^{n_{nl}}\, \mathbf{w_n} - \mathrm{D_n} e^{n_{nl}} \nabla\, n_{nl}) &= \mathrm{R_n} \end{aligned} \quad (9)$$

The set of equations (9) was implemented in Comsol Multiphysics software using customized weak formulation

which was stabilized by introducing a test function from the scheme of streamline diffusion with shock capturing [12]. Since the processes at the streamer head are deterministic for its development, proper space resolution of this region is crucial. In the present study, adaptive mesh refinement was applied that allowed for resolving regions with sharp gradients of concentrations and fields close to streamer head. During solution process, the mesh was iteratively refined in each time step by splitting the base mesh. This allows for obtaining solution within reasonable time by utilizing coarse mesh away from the streamer head and thus controlling the number of degrees of freedom in the problem. The electrode boundaries were resolved with finer static mesh to accurately capture flux transfer. An implicit time stepping technique based on backward differential formula was used. The dependent variables were segregated in two groups and iteratively converged for each time step by continuous Jacobian update.

IV. Simulations of Positive Streamer Branching

Fully 3d simulations were performed for a needle-plane electrode arrangement. The electrodes were immersed in air and separated by 1 mm distance. The needle tip radius was 0.5 mm. A step potential of 2 kV was applied to the needle electrode while the plane electrode was grounded. The boundary conditions for the charge carriers were specified as outward or zero fluxes depending on signs of their charge and polarity of the electrodes.

Small spherical solid domains were introduced at some distance away from the path of a single channel streamer discharge (symmetry axis of the system). The surfaces of these spherical domains allowed for charge accumulation which was modeled as a discontinuity in local displacement field

$$-(D_{1n} - D_{2n}) = \rho_s \tag{9}$$

where $D_{1n,2n}$ are normal components of the displacement field in air and inside the inclusion, respectively, and ρ_s is the surface charge density. The accumulated charges were obtained from normal components of incoming fluxes of the volume charges as

$$d\rho_s/dt = \sum_n J_n \tag{10}$$

Here, J_n is the normal flux of charged species on the surface based on their sign.

In order to insure the validity of the model and to detect any possible numerical artifact, the simulation was first run without including the inhomogeneities. The simulations yielded a single streamer channel propagating from the needle to the plane electrode similarly to that observed in 2d axisymmetric domain. The calculated electron density in the streamer channel (behind its front) was ~10^{14} cm^{-3} and the peak of the electric field strength associated with the head was ~100 kV/cm. These values are comparable with data provided in other publications, see e.g. [11]. The obtained results confirmed that the dependent variables are properly resolved in the 3d simulations so that any instability due to the numerical discretization is avoided.

With the inclusion of the inhomogeneities, splitting of the streamer is observed. As shown in Fig. 2a, after streamer inception at the needle, the channel moves away from the axis

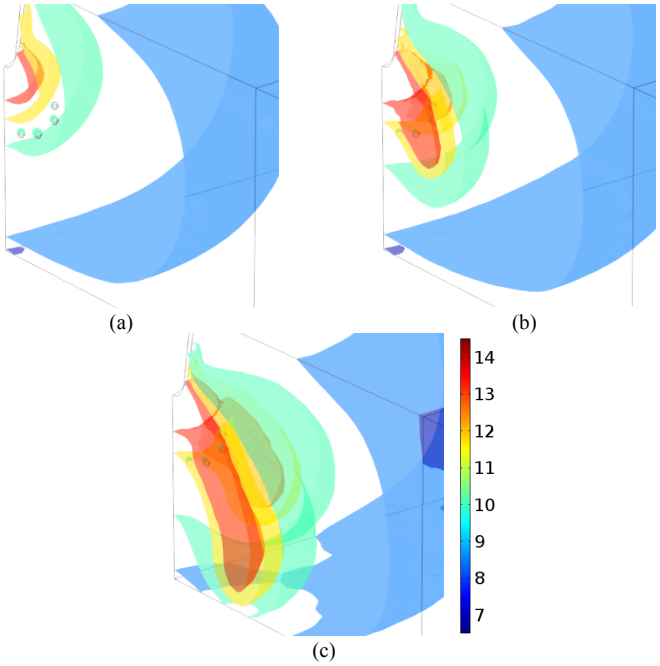

Fig. 2 Iso-surface plot of the log of electron density (cm^{-3}) for instants (a) 1 ns, (b) 3 ns, and (c) 6.4 ns after voltage application.

of the system propagating towards the counter electrode. This trajectory is due to the fact that the electric field is enhanced in gas at the inclusions that causes local generation of charges. The charge movement and accumulation causes a local potential to be seen by the primary single channel discharge and the ionizing front is split into multiple channels. This can be seen clearly with the iso-surface plot of electron density in Fig. 2b. The splitting of the channel also has an influence on propagating velocity causing it to be lower than that of the single channel. The bridging of gap by the streamer branch can be seen in Fig. 2c.

The plot of the electric field strength superimposed on the electron density iso-surface plot is shown in Fig. 3. As seen, strong fields appear on the edges of the electronic cloud right at the inception stage and the branching is initiated at early stages of streamer development. The magnitudes of the electric fields at the heads of different branches remain practically constant during their propagation. Moreover, the magnitudes are similar to that obtained for the single channel (from 3d model without the inclusions). This can be observed in Figs. 4 and 5, where distributions of the electrons density and field strength along the axis of the branch shown in Fig. 2c are presented. As seen, the magnitudes of the electron density are ~10^{14} cm^{-3} and the field at the head is ~80 kV/cm that are typical values for single streamers. In addition, the average propagation velocity ~10^7 cm/s calculated from the positions of field peaks is in agreement with known data for positive streamers in air.

It was observed in the simulations that initiation of branching process was possible if the inhomogeneities were located in close vicinity to the streamer channel. Being placed far away from it, charge accumulation on the inclusions had little influence on the local electric field distribution. It was also notable that the branching was sensitive to instabilities in

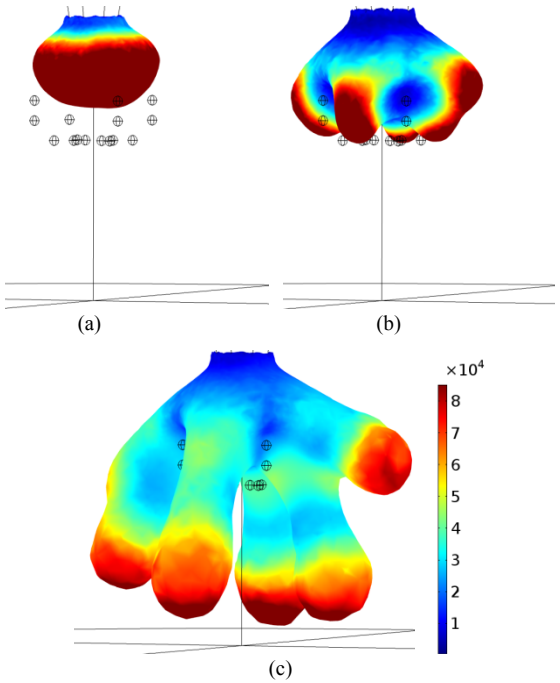

Fig. 3 Electric field superimposed on iso-surface of log of electron density(13 cm^{-3}) plotted at (a) 1 ns (b) 3 ns (c) 6.4 ns

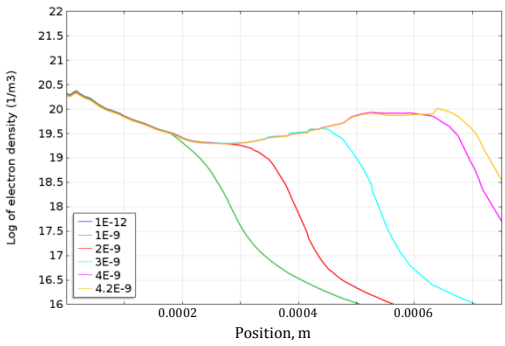

Fig. 4 Distribution of the log of electrons density along the axis of the branch in Fig. 1c.

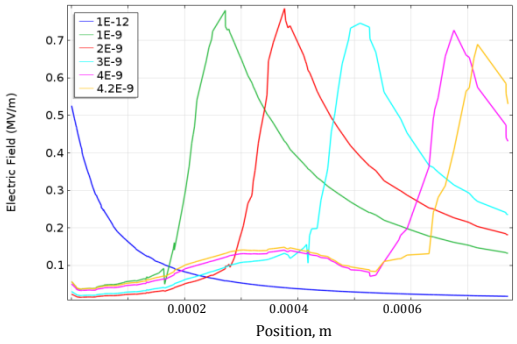

Fig. 5 Distribution of the electric field strength along the axis of the branch in Fig. 1c.

the simulation model induced, e.g., by poor mesh resolution. Thus, it was observed that with a small streamer head and, correspondingly, very narrow photoionization region in front of the streamer, the splitting of the channel occurred even without using inclusions as charge accumulation centers if the mesh was not fine enough to resolve the head. Hence, the adaptive mesh refinement strategy is extremely important for efficient modeling.

V. Conclusion

The deterministic fully 3d model of propagation of a streamer discharge in atmospheric pressure air has been presented. The branching of the streamer channel was implemented in the model by introducing distributed inclusions which mimic real micro dust particles or large molecular clusters in air. The charge accumulation on the inclusions surfaces lead to localized charge centers providing field enhancements and thus leading to splitting of the discharge channel. The model was carefully checked against numerical instabilities to avoid any non-physical reasons for streamer branching. It was demonstrated by the simulations that the developed 3d model provide reasonable results, which are in agreement with known data, and thus allows for studying streamer branching characteristics quantitatively.

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