

High-energy gamma-ray beams from nonlinear Thomson and Compton scattering in the ultra-intense regime

Christopher Harvey^{*a}, Mattias Marklund^{a,b}, Erik Wallin^b

^aDepartment of Applied Physics, Chalmers University of Technology, SE-41296 Gothenburg, Sweden; ^bDepartment of Physics, Umeå University, SE-90187 Umeå, Sweden

ABSTRACT

We consider the Thomson and Compton scattering of high-energy electrons in an intense laser pulse. Our simulations show that energy losses due to radiation reaction cause the emitted radiation to be spread over a broader angular range than the case without these losses included. We explain this in terms of the effect of these energy losses on the particle dynamics. Finally, at ultra-high intensities, i.e. fields with a dimensionless parameter $a_0 \sim 200$, the energy of the emission spectrum is significantly reduced by radiation reaction and also the classical and QED results begin to differ. This is found to be due to the classical theory overestimating the energy loss of the electrons. Such findings are relevant to radiation source development involving the next generation of high-intensity laser facilities.

Keywords: nonlinear Compton scattering, ultra-intense lasers, strong field QED, radiation reaction

1. INTRODUCTION

In recent decades there has been a steady increase in the powers and intensities of state of the art laser facilities. Over the next few years a number of new facilities are expected to come online, including the UK's upgraded Vulcan 20PW laser, the European Extreme Light Infrastructure facility (ELI) and the Russian ExaWatt Centre for Extreme Light Science (XCELS). It is anticipated that these will achieve peak intensities of the order of 10^{23} - 10^{25} W/cm², which raises the prospect of the generation of ultra-high energy, tunable γ -ray sources via the Thomson/Compton scattering of electrons in such fields. Since well-collimated, high-energy electron bunches are themselves now routinely available via laser-plasma wakefield acceleration¹, it will be possible to generate the γ -ray beams using an all optical setup. Such radiation sources are important for fundamental research², as well as more practical applications including cancer radiotherapy³ and the radiography of dense objects⁴. Recent experiments in this area^{5,6} have been pushing the limits of peak energies and brilliances. However, as we approach the ultra-high intensity regime we will need to consider additional physical effects such as radiation reaction (RR) and strong field quantum electrodynamics (QED). In these proceedings we will present the results of simulations showing how energy losses due to RR and QED photon emission changes the properties of the emission spectra in high-intensity fields. We will then explain these changes in terms of the particle dynamics.

2. BACKGROUND

Setup

We consider the case of high-energy electrons in a head-on collision with an intense laser pulse. As they pass through the pulse the electrons will radiate due to Thomson/Compton scattering, and we will model this both classically and via strong field QED. We will then analyze the resulting emission spectra and determine how the spectral properties are affected by classical RR and QED effects. In order to quantify what we mean by 'intense' we introduce the dimensionless measure of intensity $a_0 = eE/\omega mc$, where e is the electron charge and m the mass, E the magnitude of the peak electric field, ω the laser frequency and c the speed of light. In this work we will consider a_0 over the range 100 to 200, corresponding to peak intensities of the order 10^{22} - 10^{23} W/cm².

Classical dynamics

The radiation emissions from an electron in an intense field can become so strong that the resulting energy loss begins to effect the particle dynamics^{7,8}. Thus we need an equation of motion that incorporates this RR effect. Obtaining the 'correct' equation is a problem that has troubled physicists for decades. The most common approach is the Lorentz-

Abraham-Dirac equation^{9,10,11}, obtained by solving the coupled Lorentz and Maxwell's equations. However, as a result of treating the particle as a point charge, this equation has a number of undesirable (and unphysical) properties, such as pre-acceleration and runaway solutions. While there are numerous alternative equations in the literature, we will here adopt the perturbative approximation introduced by Landau and Lifshitz¹². This equation has recently been shown¹³ to be consistent with QED to the order of the fine structure constant α . There are, however, numerous alternatives in the literature¹⁴. The LL equation takes the form

$$\begin{aligned} \frac{d\mathbf{p}}{dt} = & -(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\gamma \left[\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{E} + \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{B} \right] \\ & + \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right) [(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \times \mathbf{B} + (\mathbf{v} \cdot \mathbf{E})\mathbf{E}] - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\gamma^2 [(\mathbf{E} + \mathbf{v} \times \mathbf{B})^2 - (\mathbf{v} \cdot \mathbf{E})^2]\mathbf{v} \end{aligned} \quad (1)$$

where \mathbf{p} is the electron momentum, $r_e=e^2/mc^2$ is the classical electron radius, $\lambda=2\pi c/\omega$ is the laser wavelength and we have normalized the \mathbf{E} and \mathbf{B} fields in terms of a_0 . It can be seen that the equation consists of the Lorentz force together with some correction terms that take into account the energy loss due to RR.

Throughout this work we will use the single particle code SIMLA¹⁵ which, in its classical mode, determines the motion of the electrons via equation (1). The resulting emission spectra are calculated via a novel Monte Carlo scheme¹⁶. The method is simple and computationally efficient. For an ultra-relativistic particle in an external, homogenous magnetic field, the spectrum of the emitted radiation can be expressed as¹⁷

$$\frac{\partial I}{\partial \omega} = \frac{\sqrt{3}}{2\pi} \frac{e^3 H}{mc^2} F\left(\frac{\omega}{\omega_c}\right) \quad (2)$$

where $F(\xi) = \xi \int_{\xi}^{\infty} K_{5/3}(\xi) d\xi$. In this case the acceleration and velocity of the particle are perpendicular, and we call the radiation *synchrotron radiation*. Since the particle in our simulation is ultra-relativistic, the radiation due to transverse acceleration is dominant, as this is a factor γ^2 larger than that due to longitudinal acceleration¹⁷. In the method¹⁶ we calculate the *effective magnetic field*, H_{eff} , acting on the particle at each timestep. This is the magnetic field which would cause the same acceleration as the electric and magnetic fields together. This enables us to use the expression for synchrotron radiation. The typical frequency of synchrotron emission is calculated and a Monte-Carlo method is used to sample from the spectra. The direction of the emission is taken to be that of the particle, a good approximation for the ultra-relativistic case¹⁷.

Including QED effects

As the intensity of the incoming laser increases, a classical description of the electron motion will gradually become less applicable and we will need to take quantum effects into account. We can measure the importance of quantum effects by considering the ‘quantum efficiency parameter’, $\chi_e = \hbar \sqrt{(\mathbf{E} + \mathbf{p} \times \mathbf{B})^2 - (\mathbf{p} \cdot \mathbf{E})^2} / m^2 c^3 \approx \gamma E / E_S$, where $E_S=1.3 \times 10^{16}$ V/cm is the QED ‘critical’ field (‘Sauter-Schwinger’ field^{18,19,20}). This parameter is equal to the work done by the laser field on the particle over the distance of a Compton wavelength. Thus when $\chi_e \sim 1$ quantum effects will start to dominate, with processes such as vacuum pair production occurring. It can be seen that we can increase the importance of quantum effects either by increasing the laser intensity or by increasing the γ -factor of the incoming particle. In the current discussions we will remain in the regime where $a_0 \gg 1$, $\chi_e < 1$, such that quantum effects will play a role in the Compton scattering, but at the same time we can neglect other phenomena, such as pair production.

Since we are considering the case of an intense field, it is reasonable to assume that the effects of beam depletion due to the scattering will be negligible. Thus we adopt a Furry picture²¹, adding a classical background field to the QED action²². In general the electron will radiate multiple times as it interacts with the laser field. In the high intensity regime the multiphoton emission processes are dominated by the incoherent sum of tree level processes²³. Additionally, in the limit $a_0 \gg 1$ the size of the radiation formation length is of the order $\lambda/a_0 \ll \lambda$ - much shorter than the laser wavelength. Therefore the laser can be treated as locally constant over the photon formation period, allowing the probability of each subsequent photon emission to be determined by the differential rate for a constant, crossed field²⁴

$$d\Gamma = \frac{\alpha m}{\sqrt{3\pi}\chi_e} \left[\left(1 - \eta + \frac{1}{1 - \eta} \right) K_{2/3}(\tilde{\chi}) - \int_{\tilde{\chi}}^{\infty} dx K_{1/3}(x) \right] d\chi_\gamma \quad (3)$$

where K_ν is the modified Bessel function of order ν , $\eta = \chi_\gamma / \chi_e$, $\tilde{\chi} \equiv 2\eta / (3\chi_e(1 - \eta))$ and, analogously to the quantum efficiency parameter χ_e for the electron, we have introduced an equivalent invariant for the emitted photon χ_γ defined in terms of the photon momentum.

The SIMLA code works along the same lines as a number of QED particle in cell codes that have recently appeared in the literature²⁵. It implements a classical particle pusher that propagates electrons through the background field according to the Lorentz force equation. After every time step dt the code calls a statistical event generator to determine via equation (3) if a photon has been emitted due to Compton scattering and, if so, its energy. When a photon is emitted the electron momentum is updated (i.e. recoiled) by subtracting the photon momentum. For simplicity, it is assumed that the photon is emitted in the electron's direction of motion. In reality the emissions will be in a cone pointing in the direction of motion^{17,26} with an opening angle of $1/\gamma$. Since typically $\gamma \gg 1$, this is a reasonable approximation. For more details see the references^{15,27}.

3. PARTICLE DYNAMICS AND EMISSION SPECTRA

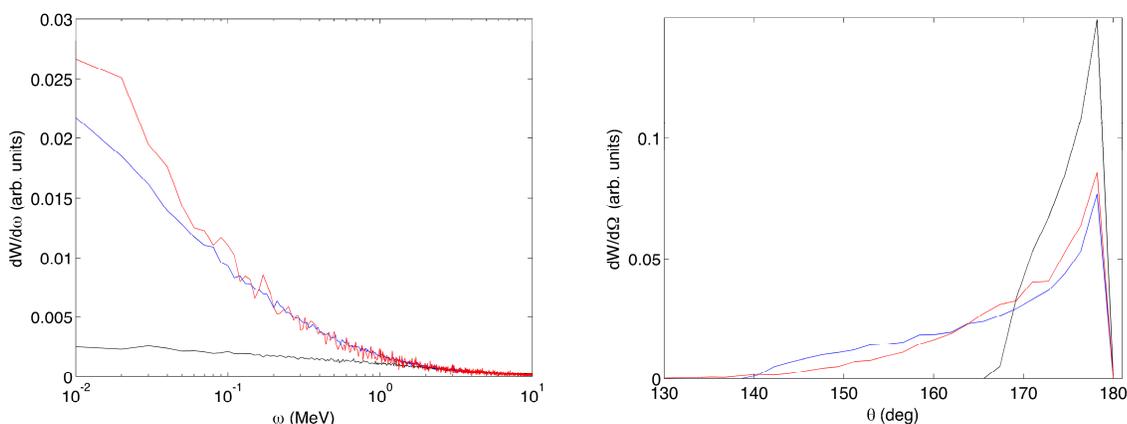


Figure 1. Emission spectra for an electron with $\gamma_0=500$ in collision with a laser pulse of intensity $a_0=100$, wavelength $\lambda=0.8\mu\text{m}$ and duration 30fs. Black line: Lorentz force. Blue line: Landau Lifshitz. Red line: QED.

We begin by considering the case of an electron with an initial $\gamma_0=500$ brought into collision with a linearly polarized plane wave of peak intensity $a_0=100$, wavelength $\lambda=0.8\mu\text{m}$, and with a Gaussian time envelope of duration 30fs (FWHM). The electron trajectory and resulting emission spectra are calculated in three ways: via the Lorentz force equation (i.e. without taking into account RR effects), via the LL equation (including RR effects) and via the stochastic QED numerical routines. (Since the QED routines are statistical in nature, we run the code 500 times for the same electron and find the average.) The frequency and angular emission spectra for each are plotted in Figure 1. It can be seen that the LL and QED spectra are in close agreement with each other, but the Lorentz force spectra differs from them significantly. The reason for this is found to be as follows. The parameters in this example are high enough for the electron to lose a significant portion of its energy due to RR as it passes through the laser pulse. It has been shown that in the high intensity regime, $a_0 \gg 1$, RR can cause the electron to travel in a backwards direction during part of the laser cycle²⁸ when $2\gamma > a_0$. (In fact, at very high intensities the backwards motion can last for more than just a part of the cycle, causing the electron to become reflected and carried along in the direction of the laser pulse⁷.) The radiation emissions will be in the electron's instantaneous direction of motion, which will be continuously changing since the particle will move in a figure-of-eight orbit. However, the emissions will occur predominantly at times when the acceleration is greatest. It is found that the peak angular emissions are related to the ratio of the electron momentum to the laser momentum^{26,29}. In the laboratory frame, for the case $2\gamma \gg a_0$ the emissions will be predominantly in the backward

direction ($\theta=180^\circ$), for the case $2\gamma \sim a_0$ the peak emissions will be in the perpendicular direction ($\theta=90^\circ$), and when $2\gamma \ll a_0$ the emissions will move towards the forward direction (relative to the laser axis). However, in the case we are considering here it is important to remember that the electron momentum will be continuously decreasing due to RR losses. Normally the total spectrum will be dominated by the emissions from the region around the most intense part of the pulse. Thus when considering the direction of the emissions we must remember that the γ -factor will be lower than its initial value when the electron reaches this region. A lower γ -factor will result in more emissions directed away from the backscattering direction, $\theta=180^\circ$, which is precisely what we see in the right-hand panel of Figure 1. Note also that there is very little difference between the LL and QED spectra. This is to be expected since the quantum efficiency parameter is fairly low for this example, $\chi \sim 0.1$.

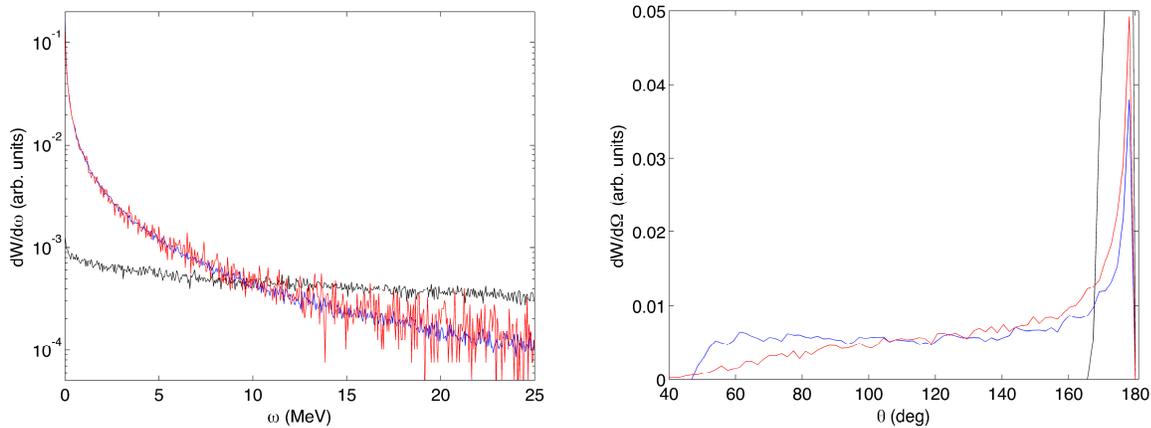


Figure 2. Emission spectra for an electron with $\gamma_0=1000$ in collision with a laser pulse of intensity $a_0=200$, wavelength $\lambda=0.8\mu\text{m}$ and duration 30fs. Black line: Lorentz force. Blue line: Landau Lifshitz. Red line: QED.

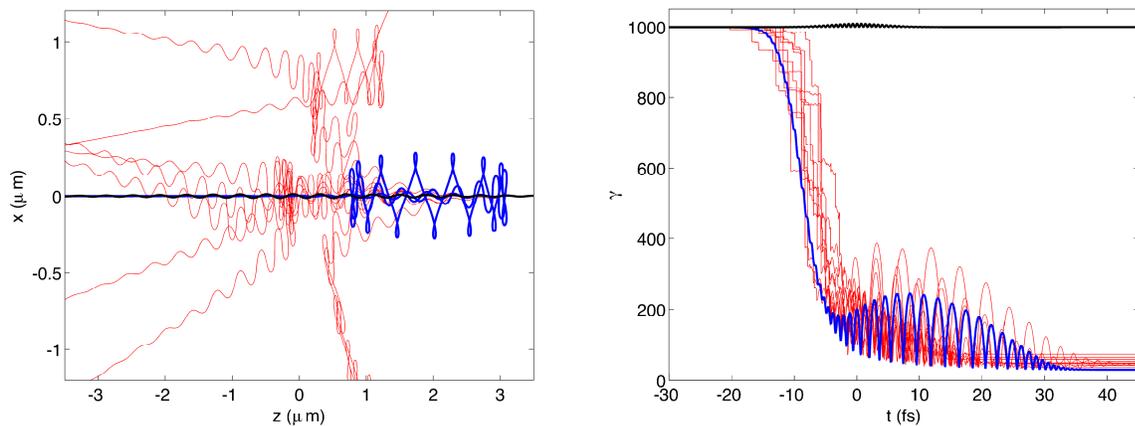


Figure 3. Plots showing the trajectories (left panel) and γ -factors (right panel) for electrons of initial $\gamma_0=1000$ in collision with a laser pulse of intensity $a_0=200$, wavelength $\lambda=0.8\mu\text{m}$ and duration 30fs. Black line: Lorentz force. Blue line: Landau Lifshitz. Red lines: sample of 10 QED trajectories.

Next we will investigate how the situation changes if we increase the laser intensity. In Figure 2 we show the three emission spectra once again, this time for an electron with an initial $\gamma_0=1000$ brought into collision with a linearly polarized plane wave of peak intensity $a_0=200$, wavelength $\lambda=0.8\mu\text{m}$ and duration 30fs (FWHM). It can be seen from the left panel that there is now a strong decrease high-energy emissions (i.e. those above 10MeV) due to radiation losses. The relationship between the angular spectra emitted from the electrons where radiative energy losses are included (LL and QED) and the spectrum from the electron without including energy losses (Lorentz) is similar to the previous case, with the emissions from the Lorentz particle being confined to a much smaller angular range. However, also observe that at smaller angles the LL and QED spectra no longer agree with each other: the QED spectrum now dies off faster than

the LL spectrum. This is not completely unexpected since the quantum efficiency parameter typically rises to about 0.4 for these parameters.

To understand more fully why the LL and QED spectra differ we consider the particle dynamics. In the right-hand panel of Figure 3 we show the γ -factor as a function of time for the electron according to the three different models. In the case of the QED simulations we show a sample for 10 electrons since, being statistical in nature, the precise dynamics will be different each time. (To calculate the QED emission spectra we ran the code 500 times and found the average.) It can be seen that the classical LL expression overestimates the energy loss of the electron compared to the average of the QED runs^{30,31}. This is important for two reasons. Firstly, the fact that the classical γ -factor is lower than that calculated at respective times using the QED routines means that the typical radiation emissions at a given time will be at a smaller angle than those from the QED electrons. Secondly, the system is strongly nonlinear and any small differences in energy loss at a given time will lead to different particle dynamics at later times. This can be seen in the left panel of Figure 3, where it can be seen that the extra energy loss of the classical particle causes it to be reflected and carried along with the pulse for a few cycles. The QED electrons on the other hand are not so strongly reflected since they lose less energy. Observe also that the QED electrons spread out in transverse space, whereas the LL electron remains centered on the laser axis. This is due to the discrete nature of the QED photon emissions and has been proposed as an observable signature of QED effects in its own right³².

4. CONCLUSIONS

In these proceedings we have reviewed the properties of the emitted radiation spectra resulting from the Thomson/Compton scattering of high-energy electrons in an intense laser pulse. It is shown that at high intensities, i.e. $a_0 \sim 100$, the energy losses due to RR cause the electron trajectories to change, resulting in an emitted beam of γ -rays that is spread over a broader angular range than would be the case without these losses. At very high intensities, e.g. $a_0 \sim 200$, the energy of the γ -rays is reduced due to RR losses and quantum effects start to become important. We find that, because the classical theory overestimates the radiated energy loss, the classical emissions sweep a broader angle than those found using a numerical QED method. This suggests that taking into account QED effects is particularly important when modelling Compton scattering experiments involving the new generation of ultra-intense facilities. Additionally, such effects could be actively looked for as signatures of strong-field QED in intense laser fields.

5. ACKNOWLEDGEMENTS

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