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IDENTIFICATION OF MATERIAL PARAMETERS OF THIN CURVILINEAR VISCOELASTIC SOLID LAYERS IN SHIPS AND OCEAN STRUCTURES BY SENSING THE BULK ACOUSTIC SIGNALS

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Summary. Ships and other ocean structures have components, which are thin planar or curvilinear viscoelastic solid layers surrounded by air or water. The present work deals with the identification of material parameters of these layers to extend the scope of the real-time structural health monitoring. The work proposes the approach to the parameter identification from passive sensing of acoustic signals resulting from the operational load. The identification is based on the partial integro-differential equation (PIDE) for the non-equilibrium part of the average normal stress. The PIDE is derived in the work. It includes the Boltzmann superposition integral associated with the stress-relaxation function. It is shown that, in the exponential approximation for this function, the PIDE expresses the steady-state solution (with respect to a certain variable) of the corresponding third-order partial differential equation (PDE) of the Zener type. The operators of both the equations are identical. The equations are applicable at all values of the stress-relaxation time. The roots of the characteristic equation of this operator are consistently analyzed, and the acoustic attenuation coefficient for arbitrary high frequencies is indicated.

The approach is exemplified with the identification of the layer-material stress-relaxation time and ratio of the bulk-wave speed to the layer thickness. This identification can be carried out from the acoustic acceleration normal to the layer measured by an acoustic accelerometer attached to the layer surface and is applicable to both planar and curvilinear layers. The identification method presumes the finite-difference calculation of the time derivatives of the

measured acoustic acceleration up to the third order and can be computationally efficient.

1 INTRODUCTION

Ships and other ocean structures have components presenting thin planar or curvilinear layers of solid materials, generally viscoelastic. The examples are skins, bulkheads, and so on. The operational load on these components is contributed by the system operation and the irregular excitations from external environments (e.g., sea waves and winds). The load causes spatiotemporal distributions of various acoustic variables in a thin-layer solid material. These distributions can reach the unforeseen levels corresponding to alterations of the material properties, which can result in failures or damages of the layer, thereby threatening the functionality of the entire system. In order to timely detect and prevent the undesirable events and losses, one uses structural health monitoring.

Many methods of structural health monitoring of systems are based on the measurement of signals by sensors attached to the system. If a tested layer is at equilibrium, there is no acoustic signal in it. In order to make the signal available for sensing, one first generates the signal in the layer by acoustic actuators and then measures the resulting acoustic echo. This technique is known as *active* sensing. If a tested layer is not at equilibrium, there is an acoustic signal distributed in the system due to the aforementioned operational load. This signal can be measured. This sensing is known as *passive*.

One can also try to apply active sensing to test a system, which is not at equilibrium. However, in this case, one needs to separate the activated echo in the measured signal from the operational-load-caused componentwhich is usually unknown. Techniques for this separation have not been developed yet.

The present work considers passive sensing only. Acoustic signals obtained by it provide the data corresponding to the actual operational load. This load can also affect the material parameters of the thin-solid-layer components of marine systems, for example, the mass density, the elastic moduli, and the stress-relaxation time, and result in their variations in space or time. These variations can in particular be caused by damages developed in the layer due to excessive fatigue.

The present work focuses on the identification of the above parameter variations on the basis of the passive-sensing data.

2 A SCALAR ACOUSTIC EQUATION FOR VISCOELASTIC MEDIA

The work assumes that a thin-layer component is isotropic, homogeneous in both space and time at equilibrium, and at a constant temperature.

As is well known, physical quantities at equilibrium are independent of time. The present work only considers materials, which are at equilibrium independent of space as well. Consequently, the equilibrium versions of physical quantities do not depend on space either. These versions are denoted with the sign “overline” applied to the notation of the corresponding quantity (e.g., see the notation “$\bar{s}$” below).

A scalar acoustic equation was derived in [1] in the form of the Stokes-type partial differential equation (PDE) (see [1, (3.1)])

$$\partial^2 \Delta P / \partial t^2 = \bar{s}^2 \nabla^2 (\Delta P + \bar{\theta}\, \partial \Delta P / \partial t) \tag{2.1}$$

for the non-equilibrium part (NEP) ΔP of the average normal stress (ANS) P in a viscoelastic medium. In equation (2.1):

- $\bar{s}$ is the speed of the bulk waves in the medium, which are understood in the present work as the waves corresponding to the bulk modulus $\bar{K}$, more specifically, to *uniform* compressions /rarefactions;
- $\bar{\theta}$ is the stress-relaxation time;
- ∇^2 is the Laplace differential expression with respect to the entries x, y, and z of the space vector.

If the bulk and shear moduli for the medium, $\bar{K}$ and $\bar{G}$, are known, stress-relaxation time $\bar{\theta}$ determines the volume and shear viscosities, $\bar{K}\bar{\theta}$ and $\bar{G}\bar{\theta}$, respectively. Parameters $\bar{s}$ and $\bar{K}$ are coupled with relation (e.g., [1, (2.7)])

$$\bar{s} = \sqrt{\bar{K}/\bar{\rho}} \tag{2.2}$$

where $\bar{\rho}$ is the volumetric mass density of the medium. (Note that, in PDE (2.1), all of the effects associated with the shear modulus are neglected.)

Equation (2.1) was obtained under the conditions listed in [1, Table 1] and is applicable to gases, liquids, and solid including viscoelastic media. A necessary condition for the applicability of linear quasi-equilibrium continuum-mechanics models such as (2.1) is relation

$$|\Delta p| / \bar{K} << 1 . \tag{2.3}$$

where Δp is the NEP of the pressure p in the medium.

In the derivation of PDE (2.1), equations

$$\Delta P = \Delta p + \bar{\theta}\, \partial \Delta p / \partial t, \tag{2.4}$$

$$\partial^2 \Delta p / \partial t^2 = \bar{s}^2 \nabla^2 \Delta P . \tag{2.5}$$

were also derived (see [1, (3.2) and (A.3.6)]). It was shown that the last term on the right-hand side of (2.1) is due to the last term on the right-hand side of (2.4).

The present work generalizes (2.4) to the form, which includes the normalized stress-relation function (NSRF) discussed in Appendix. The generalization can be obtained from Remark A.3. Indeed, in the present case where the elastic-stress NEP $\Delta \sigma_e$ and total-stress NEP $\Delta \sigma$ are specified with expression (A.5) and expression

$$\Delta \sigma = \Delta P, \tag{2.6}$$

relations (A.11) and (A.13) are specified to

$$\Delta P = \Delta p + \int_0^\infty \Gamma(\psi)\, [\partial \Delta p(t - \psi) / \partial t]\, \partial \psi, \tag{2.7}$$

$$\bar{\theta}\, \partial \Delta P / \partial t + \Delta P = \Delta p + 2 \bar{\theta}\, \partial \Delta p / \partial t \tag{2.8}$$

where the latter equation is only applicable under the condition that (see (A.12))

$$\Delta P = \Delta p + \int_0^\infty \exp(-\psi/\bar{\theta})\, [\partial \Delta p(t - \psi) / \partial t]\, d\psi . \tag{2.9}$$

Applying operation $\partial^2/\partial t^2$ to (2.7)–(2.9) and substituting (2.5) into the resulting equalities, one obtains

$$\partial^2\Delta P/\partial t^2 = \bar{s}^2\,\nabla^2\Big\{\Delta P + \textstyle\int_0^\infty \Gamma(\psi)\,[\partial\Delta P(t-\psi)/\partial t]\,\partial\psi\Big\}, \tag{2.10}$$

$$\bar{\theta}\,\partial^3\Delta P/\partial t^3 + \partial^2\Delta P/\partial t^2 = \bar{s}^2\,\nabla^2(\Delta P + 2\bar{\theta}\,\partial\Delta P/\partial t), \tag{2.11}$$

$$\partial^2\Delta P/\partial t^2 = \bar{s}^2\,\nabla^2\Big\{\Delta P + \textstyle\int_0^\infty \exp\big(-\psi/\bar{\theta}\big)\,[\partial\Delta P(t-\psi)/\partial t]\,\partial\psi\Big\}, \tag{2.12}$$

respectively. Equation (2.10) presents an acoustic partial integro-differential equation (PIDE) for NEP ΔP of the ANS. It includes NSRF $\Gamma(\psi)$. On the strength of (A.7), the small-$\bar{\theta}$ or, equivalently, the Kelvin–Voigt particular cases of expression (2.7) and PIDE (2.10) are reduced to expression (2.4) and Stokes-type wave PDE (2.1). Consequently, (2.7) and (2.10) generalize (2.4) and (2.1) from small $\bar{\theta}$ to arbitrary $\bar{\theta}$ and, thus, present acoustic model for a fairly large set of viscoelastic media. The other versions of the small-$\bar{\theta}$ particular case of continuum-mechanics /acoustic PDEs can be found in [2, (6.15)] and [3, §34].

Remark 2.1. One can check that PIDE (2.12) expresses the steady-state solution of PDE (2.11) regarded as ordinary differential equation (ODE)

$$\bar{\theta}\,\partial(\partial^2\Delta P/\partial t^2 - \bar{s}^2\,\nabla^2\Delta P)/\partial t + (\partial^2\Delta P/\partial t^2 - \bar{s}^2\,\nabla^2\Delta P) = \bar{s}^2\,\bar{\theta}\,\nabla^2(\partial\Delta P/\partial t)$$

for variable $\partial^2\Delta P/\partial t^2 - \bar{s}^2\,\nabla^2\Delta P$. Thus, equality (2.12) shows in what specific sense PDE (2.11) should be considered. A steady-state solution of an asymptotically stable ODE in Euclidian space or a function Banach space is, loosely speaking, its solution with an initial condition in the limit case as the initial time point tends to $-\infty$ (e.g., see [4] for the details). ☐

At $\bar{\theta} = 0$, both PIDE (2.12) and PDE (2.11) are reduced to the common wave equation, i.e., $\partial^2\Delta P/\partial t^2 = \bar{s}^2\,\nabla^2\Delta P$. For the case where $\bar{\theta} > 0$, one can prove the following theorem.

Theorem 2.1. Let $\bar{s}^2 \neq 0$ and $\bar{\theta} > 0$ be arbitrary fixed. Let also the Laplace differential expression in (2.11) and (2.12), more precisely, $\bar{s}^2\,\nabla^2$, be endowed with the boundary condition such that they together present the Laplace operator in the corresponding L^2 function Banach space ([5, § 21]). As is well known, all eigenvalues of this operator are nonnegative.

Then the linear operators of equations (2.12) and (2.11) are the same if and only if the Laplace operator does not have zero eigenvalue, and if it does not, the real parts of all eigenvalues of the operators is greater than $-1/\theta$. Moreover, at any eigenvalue of the Laplace operator, say, $-\omega^2$ where $\omega \neq 0$, the following assertions are also valid, thereby indicating asymptotic stability.

(i) The characteristic equation of the linear operator in PIDE (2.12) or PDE (2.11) is

$$\lambda^2 + \bar{\theta}\,\omega^2\,\lambda/(1 + \bar{\theta}\,\lambda) + \omega^2 = 0. \tag{2.13}$$

(ii) Characteristic equation (2.13) has one real root, λ_R, and a pair of the complex conjugate roots, $\lambda_C = \mathrm{Re}\,\lambda_C \pm \iota\,\mathrm{Im}\,\lambda_C$ where ι is the imaginary unit, i.e., $\iota^2 = -1$.

(iii) $\lambda_R + 2\,\mathrm{Re}\,\lambda_C = -1/\bar{\theta}$.

(iv) $\lim_{|\omega|\to 0}\lambda_R = -1/\bar{\theta}$, $\lim_{|\omega|\to\infty}\lambda_R = -1/(2\bar{\theta})$, and λ_R strictly monotonically increases as $|\omega|$ increases from zero to infinity.

(v) $\lim_{|\omega|\to 0} \operatorname{Re}\lambda_C = 0$, $\lim_{|\omega|\to\infty} \operatorname{Re}\lambda_C = -1/(4\bar{\theta})$, and $\operatorname{Re}\lambda_C$ strictly monotonically decreases as $|\omega|$ increases from zero to infinity.
(vi) Asymptotic representation for $\operatorname{Re}\lambda_C$ in the limit case as $|\omega|\to 0$ is $\operatorname{Re}\lambda_C = -\bar{\theta}\,\omega^2/2$, and $\operatorname{Re}\lambda_C$, as a function of ω^2, is convex.
(vii) $|\operatorname{Im}\lambda_C| = \sqrt{-\omega^2/(2\bar{\theta}\operatorname{Re}\lambda_C) - (\operatorname{Re}\lambda_C + 1/\bar{\theta})^2}$.
(viii) Asymptotic representations for $|\operatorname{Im}\lambda_C|$ in the limit cases as $|\omega|\to 0$ and $|\omega|\to\infty$ are $|\operatorname{Im}\lambda_C| = |\omega|$ and $|\operatorname{Im}\lambda_C| = \sqrt{2}\,|\omega|$, respectively. □

We note three main implications of Theorem 2.1. First, it reveals in what specific sense equations (2.12) and (2.11) are equivalent and establishes asymptotic stability of them. The complex root in assertion (ii) of the theorem shows that both the equations are wave equations.

Next, the complete analytical dependences of λ_R and $\operatorname{Re}\lambda_C$ on $\bar{\theta}^2\omega^2$ can be obtained by means of assertion (iii) and the well-known method of S. del Ferro and N. F. Tartaglia for solving cubic equations, which is nowadays known as the Cardano solution (e.g., [6, 1.8-3, 1.8-4]). The result for $\operatorname{Re}\lambda_C$ provides a new, unavailable in theoretical physics before insight in the acoustic-wave attenuation coefficient $\alpha = -\operatorname{Re}\lambda_C/s_*$ where s_* is the speed of the longitudinal, transverse, or bulk acoustic waves. This result generalizes the small-$\bar{\theta}\omega$ particular case (e.g., [3, (35.3), (35.4)], [7, pp. 198, 203]) to arbitrary $\bar{\theta}\omega$. In order to illustrate the matter, we indicate the simplest approximation for $\operatorname{Re}\lambda_C$, which allows for all of the features indicated in assertions (v) and (vi), namely

$$\operatorname{Re}\lambda_C = -\bar{\theta}\,\omega^2/[2(1+2\bar{\theta}^2\omega^2)]. \tag{2.14}$$

There are in-depth discussions on a possibility of a unified expression for attenuation coefficient as a function of $|\omega|$ (e.g., [7, Chapter 17]). The above outcomes on $\operatorname{Re}\lambda_C$, in particular, (2.14) contribute to the finalizing this discussion. The expressions for $|\operatorname{Im}\lambda_C|$ and λ_R resulting from (2.14) can readily be obtained with the help of assertions (vii) and (iii), respectively.

Finally, the aforementioned dependences and assertions of Theorem 2.1 provide the characteristic times $-1/\lambda_R$, $-1/\operatorname{Re}\lambda_C$, and $\pi/|\operatorname{Im}\lambda_C|$ of PDE (2.11) (or PIDE (2.12)), which reveal the time scale of it. This picture in particular allows to evaluate the relevance of the time step in passive sensing of acoustic signals, which corresponds to exact or approximate solutions of equation (2.11) (e.g., the time step between points $t_1,\dots,t_N$ discussed in Section 4).

Wave PDE (2.11) is a wave PDE of the Zener type (e.g., [2, (6.40)]). However, profound differences between PDE (2.11) and Zener's PDE originates from the comparison summarized in Table A.1. Compared to PIDE (2.12), a significant advantage of PDE (2.11) is that it does not include the NSRF explicitly and depends on the stress-relaxation time $\bar{\theta}$ linearly.

Applying operation $\partial/\partial t$ to (2.8) and substituting (2.5) into the resulting equality, one obtains $\partial\Delta p/\partial t = \bar{\theta}\,\partial^2\Delta P/\partial t^2 + \partial\Delta P/\partial t - 2\bar{\theta}\,\bar{s}^2\,\nabla^2\Delta P$ that, after combination with (2.8), leads to

$$\Delta p = \Delta P - \bar{\theta}\,\partial\Delta P/\partial t - 2\bar{\theta}^2\partial^2\Delta P/\partial t^2 + 4\bar{\theta}^2\bar{s}^2\,\nabla^2\Delta P. \tag{2.15}$$

that generalizes equality [1, (3.3)]. Expression (2.15) and inequality (2.3) endow PDE (2.11) with, so to say, the self-testing capabilities. Indeed, after a solution of (2.11) is obtained in the space-time domain of interest, one calculates term (2.15) and tests inequality (2.3) in this domain.

If it is valid, PDE (2.11) is applicable and the determined solution can be accepted. Otherwise, the solution must not be accepted.

3 EQUATION UNDERLYING THE IDENTIFICATION METHOD

Without a loss of generality, one assumes that an accelerometer is located at spatial point $(0,0,0)$ and considers the planar disk in the layer component of a marine system, which is normal to the x-axis, with the center at point $(y,z)=(0,0)$ and radius $R>0$. Then the measured acoustic acceleration normal to the (y,z)-plane is expressed as follows

$$a(t) = -\bar{\rho}^{-1}\,\partial\Delta P(0,0,0,t)/\partial x. \tag{3.1}$$

Denoting the thickness of the layer with $h>0$, one regards the inner and outer surfaces of the layer located at $x=0$ and $x=h$, respectively. These settings endow the PDE for the above disk, (2.11), with the corresponding geometric parameters as follows

$$\bar{\theta}\,\partial^3\Delta P/\partial t^3 + \partial^2\Delta P/\partial t^2 = \bar{s}^2\nabla^2(\Delta P + 2\bar{\theta}\,\partial\Delta P/\partial t),\quad 0\le x\le h,\quad 0\le r\le R, \tag{3.2}$$

where $r=(y^2+z^2)^{1/2}$. The disk of the thickness h and radius R is a cylinder. Using the well-known form of the Laplace differential expression in the cylindrical coordinates (e.g., [8, (96) of Chapter XXX]), one rewrites PDE (3.2) as

$$\bar{\theta}\,\partial^3\Delta P/\partial t^3 + \partial^2\Delta P/\partial t^2 = \bar{s}^2\left\{\frac{\partial^2}{\partial x^2} + \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right]\right\}(\Delta P + 2\bar{\theta}\,\partial\Delta P/\partial t),$$
$$0\le x\le h,\quad 0\le r\le R, \tag{3.3}$$

where φ is the angle variable in the cylindrical coordinates.

Remark 3.1. The term in the brackets in (3.3) emphasizes the fact that the (x,t)-dependent stress $\Delta P|_{(y,z)=(0,0)}$ is contributed with the stress distribution in the (r,φ)-coordinates. □

Passing in PDE (3.3) to dimensionless variables $\chi = x/h$ and $\zeta = r/R$, one obtains

$$\bar{\theta}\,\partial^3\Delta P/\partial t^3 + \partial^2\Delta P/\partial t^2 = (\bar{s}/h)^2$$
$$\times\left\{\frac{\partial^2}{\partial \chi^2} + \left(\frac{h}{R}\right)^2\left[\frac{\partial^2}{\partial \zeta^2} + \frac{1}{\zeta}\frac{\partial}{\partial \zeta} + \frac{1}{\zeta^2}\frac{\partial^2}{\partial \varphi^2}\right]\right\}(\Delta P + 2\bar{\theta}\,\partial\Delta P/\partial t),\quad 0\le\chi\le 1,\quad 0\le\zeta\le 1. \tag{3.4}$$

We assume that the disk of thickness h and radius R in the layer is, firstly, *thin* in the sense that inequality

$$(h/R)^2 << 1 \tag{3.5}$$

holds and, secondly, *planar* in the sense that the curvilinear-layer region corresponding to the disk is well approximated with the plane presumed by Cartesian coordinates in PDE (3.4). This assumption, which is, for brevity, termed the *thin-planar-disk* (TPD) approximation. In this approximation, equation (3.4) with small parameter $\varepsilon = (h/R)^2$ is a singularly perturbed PDE. Theory and methods for PDEs of this type are well-known (e.g., [9]). In this treatment, the small parameter is in many cases introduced by means of geometric reasoning similar to (3.5).

Small parameter $\varepsilon = (h/R)^2$ in (3.4) enables one to neglect the term in the brackets. Then the passing back from dimensionless variable χ to spatial variable x, one obtains

$$\bar{\theta}\,\partial^3\Pi/\partial t^3 + \partial^2\Pi/\partial t^2 = \bar{s}^2\left[\partial^2\Pi/\partial x^2 + 2\bar{\theta}\,\partial(\partial^2\Pi/\partial x^2)/\partial t\right], \quad 0 \le x \le h, \quad (3.6)$$

where $\Pi = \Pi(x,t)$ is the (r,φ)-independent approximation for the function indicated in Remark 3.1. The corresponding version of (3.1) is

$$a(t) = -\bar{\rho}^{-1}\,\partial\Pi(0,t)/\partial x. \quad (3.7)$$

Comparison of (3.6) with (3.2) shows that the TPD approximation allows reduction of the equation in the three spatial coordinates to the equation in one spatial coordinate and application of the present approach to the thin-layer components of marine systems, no matter if they are planar or curvilinear.

The boundary condition for PDE (3.6) at the air/water-solid interfaces are written as

$$\Pi(0,t) = \Pi(h,t) = 0. \quad (3.8)$$

Notably, in view of (3.8), the versions of PDE (3.6) at $x=0$ and $x=h$ are

$$\partial^2\Pi(0,t)/\partial x^2 + 2\bar{\theta}\,\partial[\partial^2\Pi(0,t)/\partial x^2]/\partial t = 0, \quad (3.9)$$

$$\partial^2\Pi(h,t)/\partial x^2 + 2\bar{\theta}\,\partial[\partial^2\Pi(h,t)/\partial x^2]/\partial t = 0. \quad (3.10)$$

Differentiating (3.6) with respect to x, substituting value $x=0$ into the resulting equality, and combining with (3.7), one obtains

$$\bar{\rho}\left(\bar{\theta}\,d^3a(t)/dt^3 + d^2a(t)/dt^2\right) + \bar{s}^2\left[\partial^3\Pi(0,t)/\partial x^3 + 2\bar{\theta}\,d(\partial^3\Pi(0,t)/\partial x^3)/dt\right] = 0. \quad (3.11)$$

The thin-layer parameters such as $\bar{\theta}$, $\bar{s}$, h, or $\bar{\rho}$ are important for structural health. For example, the last term on the right-hand side of (2.10) and hence of (2.12) or (2.1) is due to the internal friction in the material. The intensity of this friction is directly proportional to stress-relaxation time $\bar{\theta}$ (see (A.6)). The last term on the right-hand side of (2.1) shows that explicitly. The higher the internal friction is, the more prone to structural damages the material is. Moreover, as is noted in Section 1, equilibrium values of the layer parameters at a fixed spatial point generally become dependent on the time due to the resulting non-equilibrium effects of the operational load.

Equation (3.11) enables identification of the some of the aforementioned parameters of the layer. The identification method is considered in the next section.

4 IDENTIFICATION OF THE STRESS-RELAXATION TIME AND THE RATIO OF THE BULK-WAVE SPEED TO THE LAYER THICKNESS

Equality (3.11) can be used for identification of the layer-material parameters provided that one estimates the term in the brackets by means of available acceleration $a(t)$ or the parameters to be identified. In order to do that, it can be sufficient to approximate $\Pi(x,t)$ with the fourth-order polynomial, i.e.

$$\Pi(x,t) \approx [b_2(t)x^2 + b_1(t)x + b_0(t)](x^2 - hx), \qquad 0 \le x \le h, \quad (4.1)$$

which has two t-independent roots $x=0$ and $x=h$ according to (3.8), and a pair of real or complex conjugate roots that need not be t-independent. Indeed, combining (4.1), (3.7), (3.9), and

(3.10), one obtains $\partial^3\Pi(0,t)/\partial x^3 + 2\bar{\theta}\, d[\partial^3\Pi(0,t)/\partial x^3]/dt = \bar{\rho}(12/h^2)[2\bar{\theta}\; da(t)/dt + a(t)]$ that, after substitution into (3.11), results in

$$\bar{\theta}\; d^3a(t)/dt^3 + d^2a(t)/dt^2 + 12w[2\bar{\theta}\; da(t)/dt + a(t)] = 0 \qquad (4.2)$$

where

$$w = (\bar{s}/h)^2. \qquad (4.3)$$

Acceleration $a(t)$ is usually measured in a set of successive time points, say, $t_1 < t_2 < t_3 < \ldots$ with constant time step $\Delta t = t_i - t_{i-1}$. This provides corresponding values $a_1, a_2, a_3, \ldots$ of $a(t)$. Consequently, equation (4.2) can be regarded as a series of equations

$$\bar{\theta}\; a_i^{(3)} + a_i^{(2)} + 12\, w\, (2\bar{\theta}\; a_i^{(1)} + a_i) = 0, \qquad i = 1,2,3,\ldots, \qquad (4.4)$$

where $a_i^{(l)}$, $l = 1,2,3$, is the finite-difference (FD) approximation for $d^{(l)}a(t)/dt^{(l)}$ at point t_i.

There are different ways to identify parameters $\bar{\theta}$ and w from (4.4). The simplest one is based on the equations at two successive time points, t_i and t_{i+1}. This results in relations

$$w_{i.i+1} = -\frac{\bar{\theta}_{i.i+1} a_i^{(3)} + a_i^{(2)}}{12\,(2\bar{\theta}_{i.i+1} a_i^{(1)} + a_i)} = -\frac{\bar{\theta}_{i.i+1} a_{i+1}^{(3)} + a_{i+1}^{(2)}}{12\,(2\bar{\theta}_{i.i+1} a_{i+1}^{(1)} + a_{i+1})}, \qquad (4.5)$$

$$(2\bar{\theta}_{i.i+1} a_i^{(1)} + a_i)\,(\bar{\theta}_{i.i+1} a_{i+1}^{(3)} + a_{i+1}^{(2)}) = (\bar{\theta}_{i.i+1} a_i^{(3)} + a_i^{(2)})\,(2\bar{\theta}_{i.i+1} a_{i+1}^{(1)} + a_{i+1}). \qquad (4.6)$$

If quadratic equation (4.6) has the unique non-negative solution $\bar{\theta}_{i.i+1}$ such that $w_{i.i+1}$ determined with (4.5) is finite and positive, then $\bar{\theta}_{i.i+1}$ and $w_{i.i+1}$ are the identified values of parameters $\bar{\theta}$ and w, which are t-independent in interval $[t_i, t_{i+1}]$. Applying this procedure to the intervals between any two neighboring time points, one obtains the piecewise-constant t-dependent approximations for the parameters. These t-dependences include the influence of the operational load upon the parameters.

In view of the approximate nature of the aforementioned FD formulas, the t-dependences can be rather irregular. Consequently, they, in general, need to be "smoothed" in order to provide the component, which is caused by the operational load rather than the quantitative FD errors. The "smoothing" method can be a topic for future research. Another topic for future research is calibration of the proposed identification method with respect to the related experimental data.

Importantly, if w is identified and any of the two parameters on the right-hand side of (4.3) is available, then the other one can be identified as well. Also note that knowledge of $\bar{s}$ allows evaluation of mass density $\bar{\rho}$ by means of (2.2) where $\bar{K}$ is generally a function of $\bar{\rho}$.

Quantity ω^2 used in the analysis of the roots of characteristic equation (2.13) (see Theorem 2.1 and the discussion below it) is, in terms of (4.2), expressed as follows

$$\omega^2 = 12w. \qquad (4.7)$$

Thus, if $\bar{\theta}$ and w are identified, then one can by means of (4.7) apply the aforementioned analysis to the time scales of the behavior described with (4.2). They can in particular be used to evaluate the relevance of time step Δt (as was already mentioned in Section 2).

The above identification method based on relations (4.4) can computationally efficient and suitable for implementation in the real-time mode. In particular, it can be used in conjunction with the corresponding sensing system outlined below.

The marine-system curvilinear thin-layer components can be regarded as the surfaces where the domains of low curvatures are viewed as planar disks. The radiuses of these disks should be sufficiently small in order to reasonably approximate the corresponding domains of the actual, curvilinear surface with planar disks but sufficiently big in order to allow inequality (3.5) for each disk to hold. At the center of each disk, one attaches a wireless acoustic accelerometer. The accelerometer network can wirelessly be controlled by a personal computer (PC) and a gateway in the real-time mode. Each of the accelerometers measures and transmits to the PC the time development of the acoustic acceleration at the point of its locations, thereby forming the time-varying "landscape" of the data over the entire surface of the thin-layer component. These data are transformed into the space-time heterogeneous values of the material parameters by the above identification method.

These settings can be implemented already today, by means of the wireless systems available from a number of the related sources. One of them is [10]. The wireless accelerometer network can be endowed with acoustic-energy harvesters for better energy efficiency and less costly maintenance.

5 CONCLUSIONS

Ships and other ocean structures have components, which are thin planar or curvilinear viscoelastic solid layers surrounded by air or water. The present work deals with the identification of material parameters of these layers to extend the scope of the real-time structural health monitoring. The work proposes the approach to the parameter identification from passive sensing of acoustic signals resulting from the operational load.

The identification is based on the partial integro-differential equation (PIDE) for the non-equilibrium part of the average normal stress. The PIDE is derived in the work (Section 2 and Appendix). It includes the Boltzmann superposition integral associated with the stress-relaxation function. It is shown that, in the exponential approximation for this function, the PIDE expresses the steady-state solution (with respect to a certain variable) of the corresponding third-order partial differential equation (PDE) of the Zener type but the operators of both the equations are identical. These equations applicable at all values of the stress-relaxation time.

It is proven that, at any nonzero frequency, the characteristic equation related to the aforementioned operator has a real root and a pair of complex conjugate roots. The asymptotes of these roots in the limit cases of zero and infinite frequencies are obtained. In the case of the real part of the complex root (which determines the acoustic-attenuation coefficient), the zero-frequency asymptote coincides with the one well known in theoretical physics, whereas the infinite-frequency asymptote provides an extra insight unavailable before.

The approach is exemplified with the identification of the layer-material stress-relaxation time and the ratio of the bulk-wave speed to the layer thickness. This identification can be carried out from the acoustic acceleration normal to the layer measured by an acoustic accelerometer attached to the layer surface and is applicable to both planar and curvilinear layers (Section 3). The developed identification method presumes the FD calculation of the time derivatives of the measured acoustic acceleration up to the third order and can be computationally efficient (Section 4).

The two topics for future research are suggested (see Section 4).

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APPENDIX. GENERAL DYNAMIC SCALAR STRESS-STRAIN RELATIONSHIP

One of the most general scalar stress-strain relationships in theory of viscoelastic linear isotropic solids is (e.g., [11, (10b) on p. 18 and the discussion on p. 18], [12, (32)], [13, (8.6)])

$$\Delta\sigma(t) = \Delta\sigma_e(t) + \Delta\sigma_v(t), \qquad (A.1)$$

where $\Delta\sigma(t)$, $\Delta\sigma_e(t)$, and $\Delta\sigma_v(t)$ are the NEPs of the total, elastic, and viscous stresses, respectively. The viscous-stress NEP $\Delta\sigma_v(t)$ is expressed with the Boltzmann superposition integral (e.g., [11, Section 6 of Chapter II], [13, (8.6)]) as follows

$$\Delta\sigma_v(t) = \int_{-\infty}^{t} \Gamma(t-s)\,[d\Delta\sigma_e(s)/dt]\,ds = \int_0^{\infty} \Gamma(\psi)\,[d\Delta\sigma_e(t-\psi)/dt]\,d\psi, \qquad (A.2)$$

where $\Gamma(\cdot)$ is the NSRF. It is a function of the time separation $\psi = t-s \ge 0$ at all s and t such that $s \le t$. In view of (A.2), expression (A.1) is equivalent to

$$\Delta\sigma(t) = \Delta\sigma_e(t) + \int_{-\infty}^{t} \Gamma(t-s)\,[d\Delta\sigma_e(s)/dt]\,ds = \Delta\sigma_e(t) + \int_0^{\infty} \Gamma(\psi)\,[d\Delta\sigma_e(t-\psi)/dt]\,d\psi. \qquad (A.3)$$

Remark A.1. One can briefly term a continuum medium, which is described by a linear model with the time-independent parameters, the time-invariant linear medium. Theories of time-invariant linear solids and fluids are based on Lagrangian and Eulerian approaches to modeling the space-time phenomena (e.g., [14, Sections 2.1 and 2.2]). They are formally different but equivalent in the sense of mechanics. This equivalence contributes to the fact that the approaches share many theoretical-physics relations. A number of these relations are discussed in particular in [1, Appendixes A and B; e.g., see Remarks A.1.1 and A.2.1]. Also note that expressions (A.1) and (A.2) are equally applicable to time-invariant linear solids and fluids. □

An example of the relation specific to solids is the Hook law discussed in Remark A.2.

Remark A.2. Elastic stress in a time-invariant linear solid is described with the Hook law,

$$\Delta\sigma_e(t) = E\,\Delta\grave{\varepsilon}(t), \qquad (A.4)$$

where $\Delta\grave{\varepsilon}(t)$ is the NEP of the scalar strain and E is the related elastic modulus of the medium. If $\Delta\grave{\varepsilon}(t)$ and $\Delta\sigma_e(t)$ represent tensile, shear, or normal strains and stresses, then E is the Young modulus, shear modulus, or bulk modulus $\overline{K}$, respectively. In the latter case, it is well known (e.g., [2, (1.38), (1.13)]) that the normal-stress NEP $\Delta\sigma_e(t)$ is the NEP of the pressure, i.e.,

$$\Delta\sigma_e(t) = \Delta p, \qquad (A.5)$$

the normal-strain NEP $\Delta\grave{\varepsilon}(t)$ is the dilation with the sign "minus", and $E = \overline{K}$. □

The aforementioned term "normalized" with respect to NSRF $\Gamma(\cdot)$ means that $\Gamma(0) = 1$. The other main properties of the NSRF are $\Gamma(\psi) > 0$ and $d\Gamma(\psi)/d\psi < 0$ at all $\psi \ge 0$, as well as the fact that integral $\int_0^{\infty} \Gamma(\psi)\,d\psi$ exists and is finite. The latter property in particular implies that $\lim_{\psi\to\infty}\Gamma(\psi) = 0$. The mentioned integral also determines the stress-relaxation time, $\overline{\theta}$,

$$\overline{\theta} = \int_0^\infty \Gamma(\psi)\, d\psi. \tag{A.6}$$

The asymptotic representation

$$\int_0^\infty \Gamma(\psi)\,[d\Delta\sigma_e(t-\psi)/dt]\, d\psi = \overline{\theta}\,[d\Delta\sigma_e(t)/dt], \quad \text{in the limit case as } \overline{\theta}\downarrow 0, \tag{A.7}$$

is well known. Under condition (A.4), it reduces expression (A.3) to the Kelvin–Voigt model (e.g., [2, (6.8)]).

The simplest form of function Γ is

$$\Gamma(\psi) = \exp(-\psi/\overline{\theta}), \qquad \psi \geq 0. \tag{A.8}$$

It possesses all of the aforementioned properties of the NSRF $\Gamma(\psi)$. In general, the form of the ψ-dependence of the NSRF is more complicated that the one shown in (A.8), and the NSRF depends on various parameters. Stress-relaxation time (A.6) is one of them. There are a number of empirical or semi-empirical models for NSRF $\Gamma(\psi)$ (e.g., [15]). However, the consistent theoretical-physics model for it and the corresponding list of its parameters remain unknown. In connection with this problem, one can note that approximation (A.8) describes the NSRF in terms of a single parameter, stress-relaxation time $\overline{\theta}$ (see (A.6)). Moreover, expression (A.8) enables other simplifications. Indeed, it reduces (A.3) to

$$\Delta\sigma(t) = \Delta\sigma_e(t) + \int_0^\infty \exp(-\psi/\overline{\theta})\,[d\Delta\sigma_e(t-\psi)/dt]\, d\psi. \tag{A.9}$$

Differentiating (A.9) with respect to t and combining the resulting relation with the mentioned equality, one obtains

$$\overline{\theta}\, d\Delta\sigma(t)/dt + \Delta\sigma(t) = \Delta\sigma_e(t) + 2\,\overline{\theta}\, d\Delta\sigma_e(t)/dt. \tag{A.10}$$

This means that (A.9) implies (A.10). However, in general, (A.10) does not imply (A.9). Consequently, in applications of (A.10), condition (A.9) cannot be omitted.

Notably, in contrast to (A.9), equation (A.10) does not apply the integral where the NSRF plays a role of the memory function. Also, equation (A.10) under condition (A.4) is an equation of the Zener type (e.g., [2, (6.25)]), which nowadays is also known as standard linear solid model

Table A.1. Differences between the Zener equation (e.g., [2, (6.25)]) and equation (A.10). (see Remark A.1 on the term "time-invariant linear")

PROPERTY	THE ZENER EQUATION	EQUATION (A.10)
Origin	assumed, as a heuristic combination of terms	consistently derived from (A.3) under condition (A.8)
Is condition (A.4) presumed?	yes	no
Materials which the equation describes	time-invariant linear solids (see Remark A.2)	time-invariant linear solids and fluids
Relation to the Boltzmann superposition integral (see any of the integrals in (A.3))	unknown	explicit due to a use of (A.3)
Number of the relaxation terms	two	two
Number of the relaxation-time parameters	two	one

(e.g., http://en.wikipedia.org/wiki/Standard_linear_solid_model; last modified on the 23rd of January, 2014). However, there are profound differences between (A.10) and Zener's equation. They are summarized in Table A.1.

Remark A.3. If $\Delta\sigma_e(t)$ and $\Delta\sigma(t)$ depend not only on t but also on the t -independent variables, equations (A.3), (A.9), and (A.10) are specified to

$$\Delta\sigma(t) = \Delta\sigma_e(t) + \int_0^\infty \Gamma(\psi)\,[\partial\Delta\sigma_e(t-\psi)/\partial t]\,\partial w, \tag{A.11}$$

$$\Delta\sigma(t) = \Delta\sigma_e(t) + \int_0^\infty \exp(-\psi/\bar{\theta})\,[\partial\Delta\sigma_e(t-\psi)/\partial t]\,d\psi \tag{A.12}$$

$$\bar{\theta}\,\partial\Delta\sigma/\partial t + \Delta\sigma = \Delta\sigma_e + 2\bar{\theta}\,\partial\Delta\sigma_e/\partial t \tag{A.13}$$

at all values of the t -independent variables. Note that equation (A.13) is only applicable under condition (A.12) as follows from the text on (A.9) below (A.10). ☐

Equations (A.11)–(A.13) are used in Section 2.

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