How Faithful Are N-Body Simulations of Disc Galaxies?—Artificial Suppression of Gaseous Dynamical Instabilities

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Abstract. High-softening two-dimensional models, frequently employed in N-body experiments, do not provide faithful simulations of real galactic discs. A prescription (♠) is given for choosing meaningful values of the softening length. In addition, a local stability criterion (♣) is given for choosing meaningful input values of the Toomre parameter for a given softening length. Such a criterion should also provide a key to a correct interpretation of computational results in terms of real phenomena.

1 Introduction

N-body simulations employing particle-mesh codes have nowadays become a very powerful tool for investigating the dynamics of disc galaxies. In particular, two-dimensional N-body models in which the stars and the cold interstellar gas are treated as two different components have successfully been applied in studies of spiral structure (e.g., Salo, 1991; Thomasson, 1991). A correct interpretation of computational results in terms of real phenomena poses serious problems, also because there are quantities introduced for numerical reasons which do not have clear physical counterparts. One of such artificial quantities is the softening length of the modified (non-Newtonian) gravitational interaction between the computer particles, and its value can critically affect the results of N-body experiments. It is thus of fundamental importance to have a prescription for choosing meaningful values of the softening length. From the stability point of view, it has been suggested that softening introduces a quite reasonable thickness correction for a two-dimensional model and that, even where softening is not introduced directly, a grid has a similar effect (e.g., Sellwood, 1986, 1987; see also Sellwood, 1983 for an extensive discussion of the additional softening introduced by a finite grid size; Byrd *et al.*, 1986). In this paper the analogy between numerical softening and finite-thickness effects is investigated in detail on the basis of a local linear stability analysis, and in particular the question "How faithfully does the softening mimic the thickness of galactic discs?" is addressed. It is found that high-softening two-dimensional models, frequently employed in N-body experiments, do *not* provide faithful simulations of real galactic discs. A prescription (♠) is given for choosing meaningful values of the softening length. Grid effects are also estimated.

Strictly connected with that problem is the choice of meaningful input values of the local stability parameter for a given softening length. In contrast to the softening length, the Toomre parameter is directly related to observable quantities, has a clear physical meaning, and its output values in N-body experiments can be compared to those predicted by theories of spiral structure and secular heating.

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In this paper a local stability criterion (♣) is found in virtue of the descriptive similarity between numerical softening and finite-thickness effects. Such a criterion should indeed provide a key to this problem.

A more thorough discussion is given by Romeo (1993). In this short paper we just focus on a few points.

2 Local Stability

It is convenient to adopt the following scaling and parametrization:

$$\bar{\lambda} \equiv \frac{k_{\rm H}}{|k|}, \quad \text{where} \quad k_{\rm H} \equiv \frac{\kappa^2}{2\pi G \sigma_{\rm H}}; \tag{1}$$

$$\alpha \equiv \frac{\sigma_{\rm C}}{\sigma_{\rm H}}, \quad \beta \equiv \frac{c_{\rm C}^2}{c_{\rm H}^2} \quad (0 < \alpha < +\infty, \ 0 < \beta < 1); \tag{2}$$

$$Q_{\rm H} \equiv \frac{c_{\rm H}\kappa}{\pi G \sigma_{\rm H}} \quad (\textit{local stability parameter}); \tag{3}$$

$$\eta \equiv k_{\rm H} s \quad (0 < \eta < +\infty). \tag{4}$$

In these formulae, k is the local radial wavenumber of the perturbation, κ is the epicyclic frequency, σ_i and c_i ($i = \mathrm{H, C}$) are the unperturbed surface densities and the equivalent planar acoustic speeds of the stars (H) and the cold interstellar gas (C), respectively, s is the softening length of the modified gravitational interaction. The case $\eta = 0$ represents the limit of an unsoftened gravitational interaction. There exists a critical value of the softening length beyond which the model is locally stable even for vanishing $Q_{\rm H}^2$:

$$\text{Stability of Cold Models}: \quad s > s_{\rm crit} = \frac{1}{e}\frac{2\pi G\sigma}{\kappa^2}, \tag{5}$$

σ being the total unperturbed surface density. This two-component extension of Miller (1972, 1974) criterion for cold models ($c_i = 0$) is indeed the limiting case of a more general *local stability criterion* for cool models ($c_i > 0$), which can be viewed as the softened two-component extension of Toomre (1964) criterion:

♣ **Local Stability Criterion** ♣

$$\text{Stability of Cool Models}: \quad \boxed{Q_{\rm H}^2} > \bar{Q}^2, \tag{6}$$

$\bar{Q}^2 = \bar{Q}^2(\alpha, \beta, \eta)$ being the global maximum of the marginal stability curve derived by Romeo (1993). In particular, in low-softening standard star-dominated regimes

$$\bar{Q}^2 \approx 1 + 4(\alpha - \eta) \quad [\alpha \ll 1; \ \beta, \eta = O(\alpha)]. \tag{7}$$

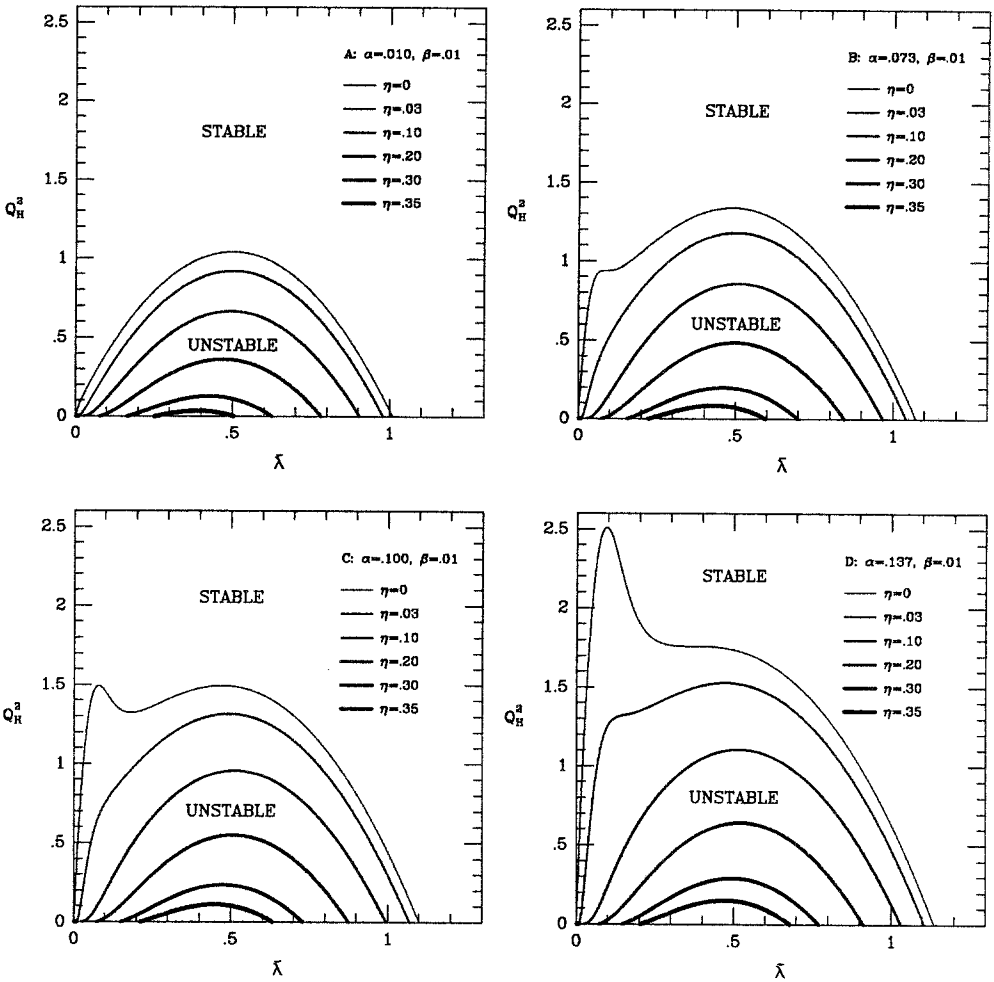

Fig. 1. Two-fluid marginal stability curves in the $(\bar{\lambda}, Q_H^2)$ plane for some values of the local parameters η, α and fixed $\beta = 0.01$. The case $\eta = 0$ represents the limit of an unsoftened gravitational interaction.

3 Results

In presenting the results of the local stability analysis performed in this paper, we have considered the standard star-dominated and the peculiar gas-dominated regimes already investigated in the context of thick two-component galactic discs (Romeo, 1990, 1992). The marginal stability curves shown in Fig. 1 should qualitatively be compared to those shown in Fig. 4 of Romeo (1992). It is apparent that, because of the highly stabilizing role of numerical softening, the local linear stability properties of two-dimensional N-body models can indeed be considerably different from those of thick galactic discs, as derived analytically. In particular, *note* the suppression of the gaseous peak in peculiar gas-dominated regimes even for exceedingly low softening. The softening can faithfully mimic the thickness of galactic discs or, more precisely, the effective thickness-scale of the stellar component [defined in Eq. (6) of Romeo (1992)] only in standard star-dominated regimes, provided the softening length is chosen to be very short compared to the characteristic wavelength corresponding to the stellar peak:

♠ **Prescription** ♠

$$\boxed{s} \ll \frac{1}{2}\,\frac{2\pi G\sigma_{\mathrm{H}}}{\kappa^2} \sim 1\ \mathrm{kpc}\,, \tag{8}$$

as a typical value for realistic N-body models of disc galaxies. Softening lengths comparable to the critical value given by the Miller criterion do *not* fulfil this prescription.

4 Discussion

The local stability analysis carried out in Sect. 2 and the results discussed in Sect. 3, strictly speaking, apply to N-body models of disc galaxies in which the mesh size is short compared to the softening length. When this condition is not satisfied grid effects should also be taken into account. The evaluation of such effects is not straightforward for a number of reasons. Mesh-induced anisotropies arise in the inter-particle force. Both the amount of grid noise resulting from such unphysical fluctuations and the form of the mean inter-particle force depend on the characteristics of the N-body code (e.g., mass-assignment and force-interpolation schemes, shape of the Green function, potential-differencing method, grid geometry) (see, e.g., Hockney and Eastwood, 1988; Sellwood, 1987).

When both numerical softening and grid effects are taken into account, the mean inter-particle force can roughly be estimated to be of standard softened type, with an *effective softening length* given by

$$s_{\mathrm{eff}}^2 = s^2 + \Delta^2\,, \tag{9}$$

Δ being the mesh size. This can be deduced for CIC and TSC schemes from Fig. 1a of Efstathiou *et al.* (1985), once a reasonable extrapolation is made to include the presence of numerical softening. Thus, the local stability analysis performed in Sect. 2 and the results presented in Sect. 3 are still expected to apply, provided the numerical softening length is replaced by the effective softening length of the modified gravitational interaction.

5 Concluding Remarks

The suggestion that softening introduces a quite reasonable thickness correction for a two-dimensional model is quantitatively confirmed in low-softening standard star-dominated regimes. A constant effective softening length would then ideally correspond to a constant scaleheight of the stellar component. On the other hand, a realistic simulation of the vertical structure of disc galaxies would in any case require a proper three-dimensional model.

Although the local stability properties of high-softening two-dimensional N-body models are considerably different from those of thick galactic discs, the propagation properties of the spiral waves are still expected to be physically plausible in standard star-dominated regimes, provided the effective softening length is chosen to be shorter than the critical value given by the Miller criterion [cf. the more restrictive prescription (♠)]. In choosing the input values of the local stability parameter as well as in comparing its output values to those predicted by theories of spiral structure and secular heating, it should then be borne in mind that the stability threshold is *not* unity (Toomre 1964 criterion for unsoftened one-component models), but the value given by the local stability criterion (♣) discussed in Sect. 2.

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