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ASTRONOMY AND ASTROPHYSICS

How faithful are N-body simulations of disc galaxies?

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Abstract. High-softening two-dimensional models, frequently employed in N-body experiments, do *not* provide faithful simulations of real galactic discs. A prescription [Eqs. (17) and (18)] is given for choosing meaningful values of the softening length in standard regimes of astrophysical interest, when both the stellar and gaseous components are present. In addition, a stability criterion [Eq. (10); see also Eq. (11)] is given for choosing meaningful input values of the Toomre parameter for a given softening length. Such a criterion should also provide a key to the correct interpretation of computational results in terms of real phenomena.

Key words: instabilities – methods: numerical – ISM: kinematics and dynamics – galaxies: ISM – galaxies: kinematics and dynamics – galaxies: spiral

1. Introduction

N-body simulations employing particle-mesh codes have nowadays become a very powerful tool for investigating the dynamics of disc galaxies. In particular, two-dimensional N-body models in which stars and cold interstellar gas are treated as two different components have extensively been applied in studies of spiral structure (e.g., Salo 1991; Thomasson 1991; Combes & Elmegreen 1993; Elmegreen & Thomasson 1993). The correct interpretation of computational results in terms of real phenomena poses serious problems, not least because there are quantities introduced for numerical reasons which do not have clear physical counterparts. One such artificial quantity is the softening length of the modified (non-Newtonian) gravitational interaction between the computer particles, and its value can critically affect the results of N-body experiments. It is thus of fundamental importance to have a prescription for choosing meaningful values of the softening length.

Two basic physical properties are affected by artificial softening: relaxation and stability. Optimal choices of the softening length should altogether satisfy relaxation and stability requirements.

From the relaxation point of view, softening is essential. Two-dimensional discs with Newtonian gravity are in fact always collision-dominated (Rybicki 1972), whereas in real galactic discs relaxation effects become appreciable on secular timescales (see, e.g., Binney & Tremaine 1987). The welcome effect of softening is then to suppress unphysical small-scale fluctuations induced by the artificial geometry, largely responsible for such a rapid relaxation. The two-body relaxation time is directly proportional to the softening length and the number of particles. Thus, by choosing conveniently large values of the softening length, it is possible to simulate collisionless systems with a non-prohibitively large number of computer particles. The relaxation problem was extensively discussed by White (1988), but there are still controversial points favouring the choice of more moderate values of the softening length, whose importance has recently been emphasized by Pfenniger & Friedli (1993).

From the stability point of view, there is a persistent confusion about which values of the softening length are physically consistent, and should therefore be preferred. It is indeed the first objective of our paper to clarify this delicate point. In particular, we draw attention to a fact which so far has only partially been recognized, namely that values of the softening length as large as those commonly chosen alter the stability properties to an *unacceptable* degree. The stability problem is more thoroughly explained in Sect. 2, and is then investigated in detail in Sect. 3 on the basis of a comparative stability analysis between two-dimensional N-body models and real galactic discs.

Strictly connected with that aspect of the stability problem is the choice of meaningful input values of the Toomre parameter for a given softening length. In contrast to the softening length, the Toomre parameter is directly related to observable quantities, has a clear physical meaning, and its output values in N-body experiments can be compared with those predicted by theories of spiral structure and secular heating. A key to this aspect of the stability problem is provided by identifying which values of the Toomre parameter correspond to stable situations for a given softening length. It is indeed the second objective

Send offprint requests to: Alessandro B. Romeo (first address above) of our paper to clarify this other delicate point. The identification of the stability threshold is the first and unavoidable step in studies of spiral structure and secular heating, once physical consistency has been ensured.

2. Overview of the stability problem

From the theoretical point of view, the stability problem was first investigated by Miller (1972) in the case of a softened point-mass potential of the standard (Plummer) type

$$\Phi_s(x,y) = -\frac{Gm}{\sqrt{x^2+y^2+s^2}}\,, \tag{1}$$

where the softening length s is introduced for curing the divergence at short distances. This can be viewed as the Newtonian potential in a plane offset a distance $|z| = s$. Correspondingly, the stability analysis of two-dimensional discs with softened gravity can be carried out as in the case with Newtonian gravity with only minor modifications. Miller (1972) performed a calculation parallel to that of Toomre (1964) to allow for the modified gravitational interaction, and showed that softening has a twofold effect. The direct effect is to weaken the potential perturbation induced by a given surface-density perturbation by a factor

$$\mathscr{S} = \mathrm{e}^{-|k|s}\,, \tag{2}$$

k being the radial wavenumber. Correspondingly, the contribution of self-gravity to the dispersion relation is weakened through a reduction of the active unperturbed surface density by the same factor. This, in turn, has a stabilizing effect. In particular, there exists a critical value of the softening length beyond which even discs supported by rotation alone are stable, in contrast to the case with Newtonian gravity:

STABILITY (COLD DISCS) $\Longleftrightarrow$

$$s > s_{\mathrm{crit}} = \frac{1}{\mathrm{e}}\,\frac{2\pi G\Sigma}{\kappa^2}\,, \tag{3}$$

where Σ is the total unperturbed surface density and κ is the epicyclic frequency. Below such a critical value of the softening length only discs supported by both rotation and sufficient random motions are stable, as in the case with Newtonian gravity but with a lower stability threshold. The dispersion relation of cold discs with softened gravity was also compared with the dispersion relations of kinetic and fluid warm discs with Newtonian gravity (Toomre 1977; Athanassoula 1984), in the spirit of an analogy drawn by Miller (1974) (see also Binney & Tremaine 1987) and further explored by Erickson (1974) and Toomre (1981).

From the experimental point of view, the stability problem has more extensively been investigated (e.g., Miller 1972, 1974, 1978a, b, c; Sellwood 1981, 1983, 1989; Sellwood & Athanassoula 1986; Sellwood & Kahn 1991). In particular, Sellwood (1981) stressed how effective softening can be in suppressing even bar instabilities, and thus how important it is to choose the softening length carefully.

Athanassoula & Sellwood (1986) suggested that the choice of a softening length comparable to the expected disc scale height automatically introduces a quite realistic thickness correction for a two-dimensional model, as can be guessed from Eq. (1) and following physical interpretation (see also Byrd et al. 1986; Sellwood 1986, 1987). The disc scale height h is indeed the natural softening length since most stellar passages are at impact parameter $b \simeq h$ (Binney, private communication).

Our contribution to the understanding of the stability problem is twofold. First, we analyse the stability of two-dimensional N-body models in which stars and cold interstellar gas are treated as two different components. In particular, in Sect. 3.1 we identify the stability threshold and provide a simple approximation to it applicable in standard regimes of astrophysical interest. The consideration of the gaseous component represents an important extension to Miller's (1972) contribution. In fact, the crucial role which cold interstellar gas can play in the dynamics of disc galaxies has progressively been recognized since then (e.g., Romeo 1985, 1990 and references therein; Pfenniger et al. 1994; Pfenniger & Combes 1994), and nowadays a large number of N-body simulations employ such two-component models. Second, we compare the weakening and stabilizing effect of softening with the effect of thickness in real galactic discs. In particular, in Sect. 3.2 we identify the sources of dissimilarity and point out the deep physical inconsistencies which can arise, and in Sect. 4 we discuss their serious implications for studies of spiral structure and secular heating.

3. Comparative stability analysis

3.1. Two-component two-dimensional N-body models

In formulating the stability problem, we assume that the softened point-mass potential is of the standard type. In addition, we adopt the lowest-order WKBJ approach in the framework of the basic two-fluid description, as in the case with Newtonian gravity investigated in a previous paper (Bertin & Romeo 1988, hereafter Paper I). In interpreting the results, one should replace the planar sound speeds of the stellar and gaseous fluids with the radial velocity dispersion of the stellar component and the one-dimensional turbulent velocity dispersion of the gaseous component, respectively, and the coefficient π appearing in the definition of the Toomre parameter of the stellar fluid with 3.36.

It is convenient to scale the radial wavelength of the perturbation in terms of the stellar Toomre wavelength:

$$\bar{\lambda} \equiv \frac{\lambda}{\lambda_{\mathrm{star}}}\,, \quad \text{where} \quad \lambda_{\mathrm{star}} = \frac{2\pi}{k_{\mathrm{star}}} \equiv \frac{4\pi^2 G\Sigma_{\mathrm{star}}}{\kappa^2}\,. \tag{4}$$

The unperturbed quantities which determine the stability properties can be reduced to the following dimensionless parameters:

$$Q_{\mathrm{star}} \equiv \frac{c_{\mathrm{star}}\kappa}{\pi G\Sigma_{\mathrm{star}}} \quad \text{(stellar Toomre parameter)}\,, \tag{5}$$

$$\alpha \equiv \frac{\Sigma_{\mathrm{gas}}}{\Sigma_{\mathrm{star}}} \quad \text{(surface-density ratio)}\,, \tag{6}$$

$$\beta \equiv \frac{c_{\rm gas}^2}{c_{\rm star}^2} \quad \text{(planar-temperature ratio)}\,, \tag{7}$$

$$\eta \equiv s k_{\rm star} \quad \text{(softening parameter)}\,, \tag{8}$$

$c_{\rm star}$ and $c_{\rm gas}$ being the stellar and gaseous planar sound speeds, respectively. The cases already investigated are recovered in the following limits: $Q_{\rm star} \to 0$ (cold case), $\alpha \to 0$ or $\beta \to 1$ (one-component case), $\eta \to 0$ (case with Newtonian gravity).

The physical meaning of $Q_{\rm star}$ is clear and well known: it measures the stability level of galactic discs and, at the same time, serves as a thermometer for the stellar component. In general, the stability threshold $\bar{Q}$ is a complicated function of α, β and η, which reduces to unity in the one-component case with Newtonian gravity. More precisely, $\bar{Q}^2$ is the global maximum of the marginal stability curve, i.e. the dispersion relation for marginally stable perturbations viewed as a curve in the $(\bar{\lambda}, Q_{\rm star}^2)$ plane for given α, β and η:

$$Q_{\rm star}^4\,\beta + Q_{\rm star}^2 \left\{4\bar{\lambda}\left[\bar{\lambda}\,(1+\beta) - {\rm e}^{-\eta/\bar{\lambda}}\,(\alpha+\beta)\right]\right\} + 16\bar{\lambda}^3\left[\bar{\lambda} - {\rm e}^{-\eta/\bar{\lambda}}\,(1+\alpha)\right] = 0\,, \tag{9}$$

where the exponential coefficient is the reduction factor $\mathscr{S}$ expressed in dimensionless form. The region above/below this boundary curve corresponds to stable/unstable perturbations [see Fig. 4 (left) discussed in Sect. 3.2.2]. In particular, values of $Q_{\rm star}^2 > \bar{Q}^2$ correspond to stable situations at any $\bar{\lambda}$:

STABILITY (WARM DISCS) $\Longleftrightarrow$

$$Q_{\rm star} > \bar{Q}(\alpha,\beta,\eta)\,. \tag{10}$$

Even though $\bar{Q}$ can easily be calculated numerically, we provide a simple analytical approximation to it applicable in standard regimes of astrophysical interest:

$$\bar{Q}(\alpha,\beta,\eta) \approx 1 + 2\,(\alpha-\eta) \quad (\alpha \sim \beta \sim \eta \ll 1)\,. \tag{11}$$

This is an extra tool for estimating the stability threshold at a glance. Note for completeness that the Miller stability criterion for cold discs is a limiting case of our stability criterion. In fact, for $\eta = \eta_{\rm crit}$ the marginal stability curve becomes singular and degenerates into the point $(\bar{\lambda}, Q_{\rm star}^2) = (\eta_{\rm crit}, 0)$, and for $\eta > \eta_{\rm crit}$ it is undefined.

3.2. Artificial softening vs physical thickness

The stability of galactic discs, which in reality are three-dimensional systems with Newtonian gravity, has been investigated in a previous paper (Romeo 1992, hereafter Paper II), where particular attention has been devoted to both the effect of thickness and the roles of the stellar and gaseous components. Thickness corrections to the dispersion relation can be expressed in terms of two reduction factors weakening the active unperturbed surface densities of the two components

$$\mathscr{T}_{\rm star} = \frac{1}{1+|k|h_{\rm star}}\,, \tag{12}$$

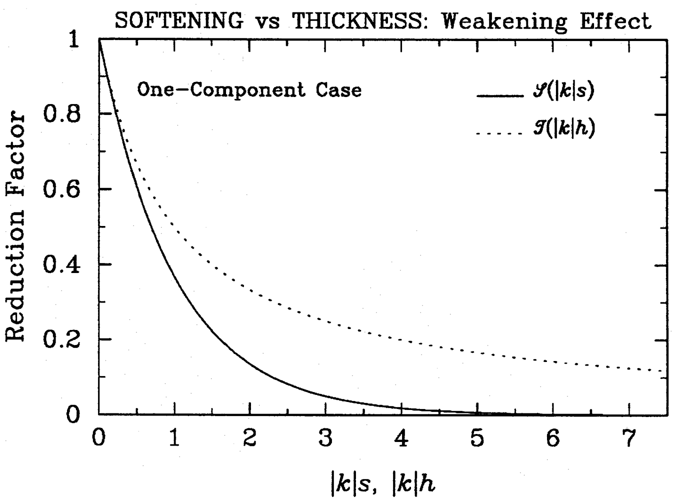

Fig. 1. The reduction factors of two-dimensional discs with softened gravity $\mathscr{S}(|k|s)$ and three-dimensional discs with Newtonian gravity $\mathscr{T}(|k|h)$, where k is the radial wavenumber of the perturbation, s is the softening length and h is the characteristic scale height

$$\mathscr{T}_{\rm gas} = \frac{1}{1+|k|h_{\rm gas}}\,, \tag{13}$$

$h_{\rm star}$ and $h_{\rm gas}$ being the stellar and gaseous characteristic scale heights, respectively. The unperturbed quantities which determine the stability properties can be reduced to the dimensionless parameters $Q_{\rm star}$, α, β and

$$\delta_{\rm star} \equiv \frac{c_{z\,\rm star}^2}{c_{\rm star}^2} \quad \text{(stellar 'temperature anisotropy')}\,, \tag{14}$$

$c_{z\,\rm star}$ being the stellar vertical velocity dispersion. Note that $\delta_{\rm gas} = 1$ since the gaseous component is collisional. The two-dimensional case investigated in Paper I is recovered in the limits $\delta_{\rm star}, \delta_{\rm gas} \to 0$.

The sources of dissimilarity between two-component two-dimensional N-body models and real galactic discs can be identified with:

1. the different wavenumber-dependence of the reduction factors $\mathscr{S}$ and $\mathscr{T}$;
2. the different number of 'softening lengths' involved in the two cases, when both the stellar and gaseous components are present: one artificial, s, and two natural, $h_{\rm star}$ and $h_{\rm gas}$.

These two sources of dissimilarity are examined in Sects. 3.2.1 and 3.2.2, respectively, together with their direct consequences.

3.2.1. One-component case

The weakening effects of artificial softening and physical thickness are compared in Fig. 1. Softening weakens very effectively all perturbations on scales less than s. This is convenient because in a three-dimensional disc vertical velocity dispersion weakens effectively all perturbations on scales less than h. So to a first approximation we can ideally choose $s = h$ and obtain a reasonable response even at zero vertical velocity dispersion. On

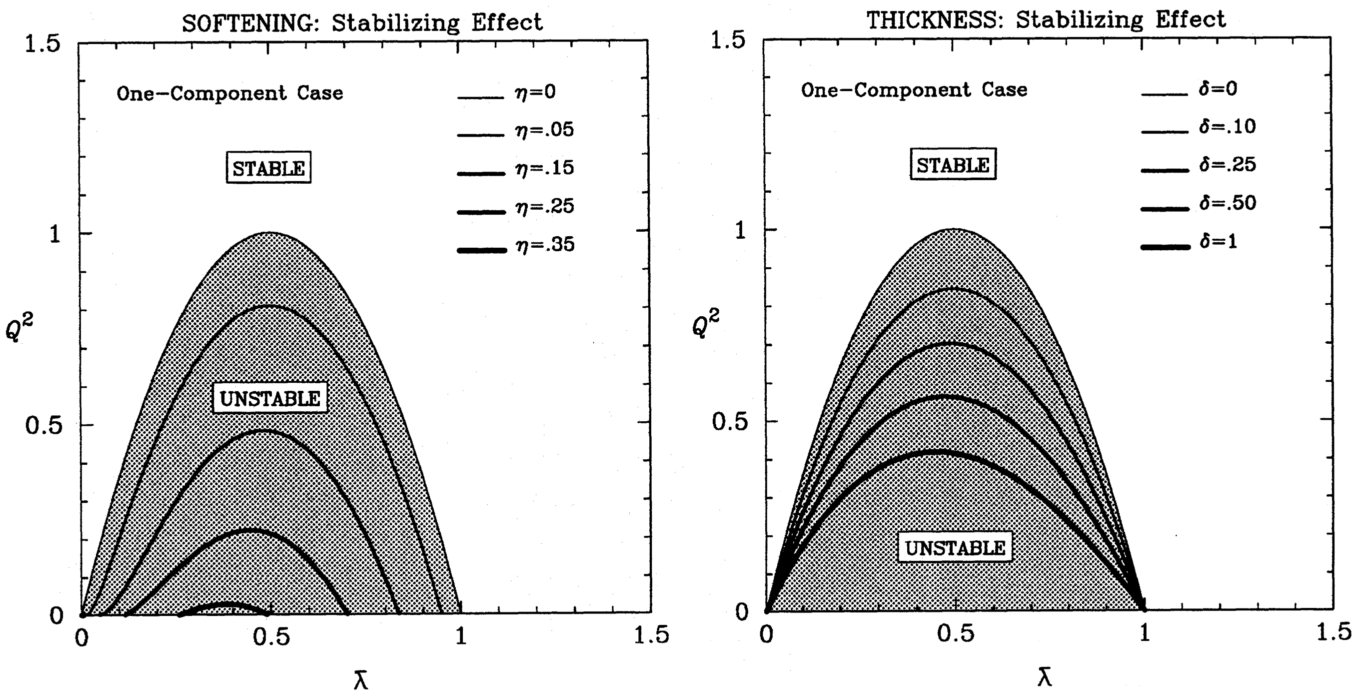

Fig. 2. Marginal stability curves of two-dimensional discs with softened gravity (left) and three-dimensional discs with Newtonian gravity (right), where $\bar{\lambda}$ is the radial wavelength of the perturbation scaled in terms of the Toomre wavelength and Q is the Toomre parameter. In addition, η is the softening parameter and δ is the temperature anisotropy. The stability and instability regions are shown in the two-dimensional case with Newtonian gravity

the other hand, the weakening effect is similar in the two cases only on significantly larger scales, where the reduction factors $\mathscr{S}$ and $\mathscr{T}$ have approximately the same linear k-dependence. So we obtain a realistic response if and only if we choose, at the same time, values of s 'safely' smaller than the inverse of the typical radial wavenumber, in the sense specified by Eqs. (17) and (18) derived below. The less this condition is satisfied, the more the dispersion (i.e., stability and propagation) properties are altered.

The stabilizing effects of artificial softening and physical thickness are shown in Fig. 2, and are then compared in Fig. 3.

Figure 2 shows that in general, as a direct consequence of the first source of dissimilarity, the stabilizing effect is qualitatively different in the two cases. The stability properties are characterized by the stability threshold $\bar{Q}$ and, in addition, by the radial wavelength which requires the highest level of random motions for being stabilized, $\bar{\lambda}_{\max}$, the shortest and longest unstable radial wavelengths in the absence of random motions, $\bar{\lambda}_{01}$ and $\bar{\lambda}_{02}$, respectively. These characteristic radial wavelengths correspond to the global maximum, the first and second zeros of the marginal stability curve, respectively. Indeed, $\bar{\lambda}_{\max}$ is also the typical radial wavelength.

Figure 3 shows the degree to which $\bar{Q}$, $\bar{\lambda}_{\max}$, $\bar{\lambda}_{01}$ and $\bar{\lambda}_{02}$ are different in the two cases and, as a result of the comparison, the ranges of approximate physical consistency and inconsistency. Before discussing this figure, let us introduce two thickness-related quantities. One is the characteristic scale height expressed in units of the inverse of the Toomre wavenumber, as the softening length:

$$\zeta \equiv hk_{\mathrm{T}} \quad \text{(thickness parameter)}\,. \tag{15}$$

The other is the temperature anisotropy corresponding to the stability threshold for a given ζ:

$$\delta(\zeta) = \zeta \left[\frac{1}{2}\,\bar{Q}^2(\zeta)\right]^{-1}\,. \tag{16}$$

The natural limitation $\delta(\zeta) < 1$ is imposed in Fig. 3 for identifying which values of ζ are of astrophysical interest. As a result, $\zeta \lesssim \frac{1}{5}$, where the upper bound corresponds to the isotropic case. We are now ready to discuss Fig. 3.

- If we choose $\eta = \zeta \lesssim \frac{1}{5}$, the stabilizing effect is indeed similar in the two cases. The deviations are at most linear, and correspond to appreciable but modest alterations in the stability properties. In particular, note the low sensitivity of $\bar{\lambda}_{\max}$, which remains practically constant. Of course, the smaller the value of η compared with $\bar{\lambda}_{\max} \simeq \frac{1}{2}$, the better the agreement between the characteristic functions in the two cases.
- If we choose $\frac{1}{5} \lesssim \eta < \eta_{\mathrm{crit}} = \frac{1}{e}$, the highly stabilizing effect of artificial softening has no physical counterpart, and the alterations in the stability properties are *significant*. More precisely, the fact itself that $\bar{Q}$ is well below the natural bound $\bar{Q} \approx \frac{3}{5}$ does not result in physical inconsistencies, if we choose input values of Q corresponding to realistic

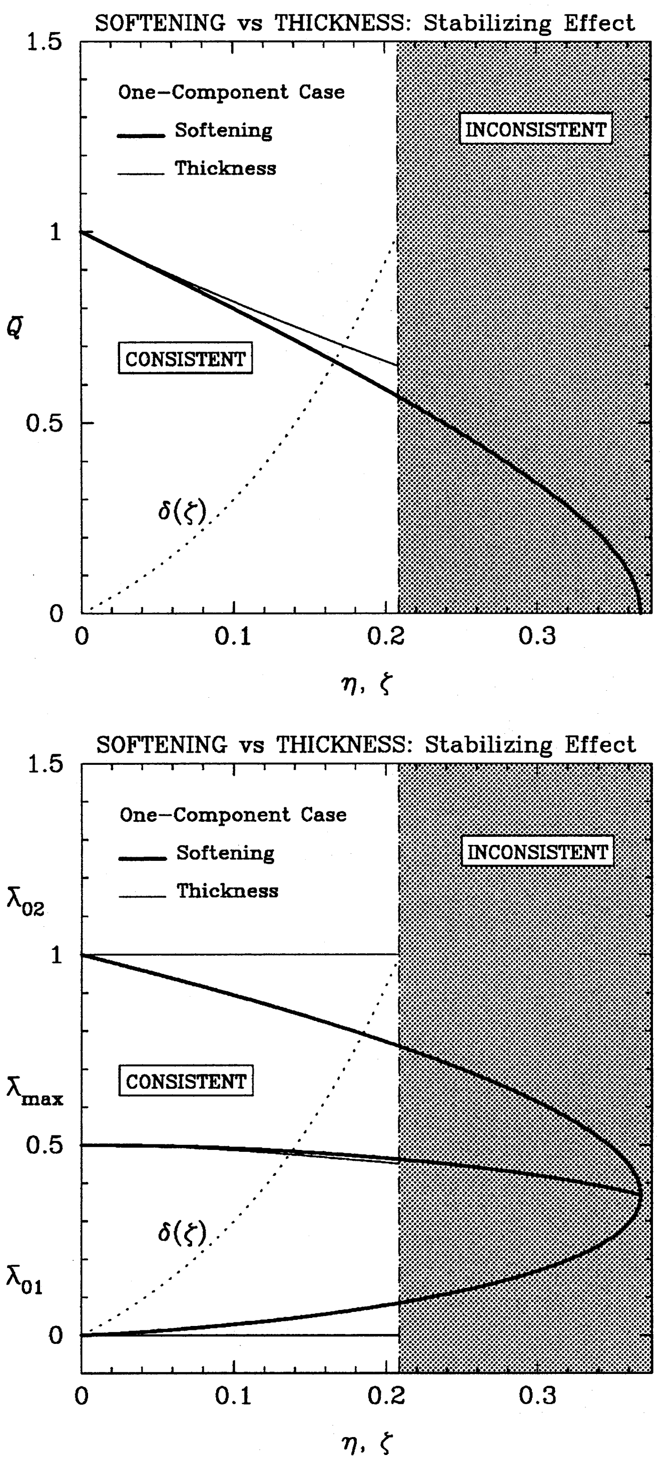

Fig. 3. The stability thresholds of two-dimensional discs with softened gravity $\bar{Q}(\eta)$ and three-dimensional discs with Newtonian gravity $\bar{Q}(\zeta)$ (top) and the characteristic radial wavelengths $\bar{\lambda}_{max}$, $\bar{\lambda}_{01}$ and $\bar{\lambda}_{02}$ in the two cases (bottom), where η is the softening parameter and ζ is the thickness parameter. In addition, $\delta(\zeta)$ is the temperature anisotropy corresponding to the stability threshold for a given ζ. The ranges of approximate physical consistency and inconsistency are shown

stability levels, since $\bar{\lambda}_{max}$ is still reasonably close to the natural $\bar{\lambda}_{max} \simeq \frac{1}{2}$. Physical inconsistencies do instead result from the fact that $\bar{\lambda}_{01}$ and $\bar{\lambda}_{02}$ are now 'dangerously' close to $\bar{\lambda}_{max}$, which gives an irremediably bad representation of the stability properties especially at short $\bar{\lambda}$. Of course, the closer the value of η is to η_{crit}, the deeper the physical inconsistencies are.

- If we choose $\eta \geq \eta_{crit} = \frac{1}{e}$, the stabilizing effect of softening is so high as to suppress all instabilities artificially, and the alterations in the stability properties are *considerable* and unacceptable, at least in regimes of normal spiral structure. In particular, the fact that the stability level is no longer actively controlled by Q has serious dynamical implications, whose discussion is postponed to Sect. 4.

The results of this comparative stability analysis can be summarized in the form of a prescription for choosing meaningful values of s:

PHYSICAL CONSISTENCY $\Longleftrightarrow$

$$s < \frac{\varepsilon}{2} \frac{2\pi G \Sigma_{star}}{\kappa^2} \quad (\varepsilon \ll 1)\,, \tag{17}$$

where the 'safety' threshold is

$$\varepsilon \sim \frac{2}{5}\,. \tag{18}$$

We mention in advance that this criterion of approximate physical consistency is indeed more general and applicable in standard regimes of astrophysical interest, when both the stellar and gaseous components are present.

3.2.2. Two-component case

When both the components are present, each component has its natural softening length h_{star} and $h_{gas} < h_{star}$. If the softening length is ideally set to h_{star}, small-scale perturbations of the gaseous component will be over-weakened. On the other hand, if we ideally set the softening length to h_{gas} we will over-estimate the responsiveness of the stellar component. So, apparently, we cannot obtain realistic responses of both the components at zero vertical velocity dispersions. Fortunately, that is not always true and, indeed, no significant physical inconsistencies arise from this second source of dissimilarity in rather general situations, as is explained below.

Figure 4 shows the stabilizing effects of artificial softening and physical thickness in standard star-dominated regimes (represented by the case **A**) and peculiar gas-dominated regimes (represented by the case **B**). These and other regimes of astrophysical interest have been identified in Paper I, as a result of a thorough inspection of the (β, α) plane (see in particular Fig. 4), and have further been examined in Paper II. (The terminology adopted in the present paper is slightly different.)

- In standard star-dominated regimes (roughly corresponding to the region $\frac{1}{5} \lesssim \beta^{1/2} \ll 1$, $\alpha \ll 1$ or the region $\beta^{1/2} \lesssim \frac{1}{5}$, $\alpha \lesssim 5\beta$), the stability properties are analogous to those of

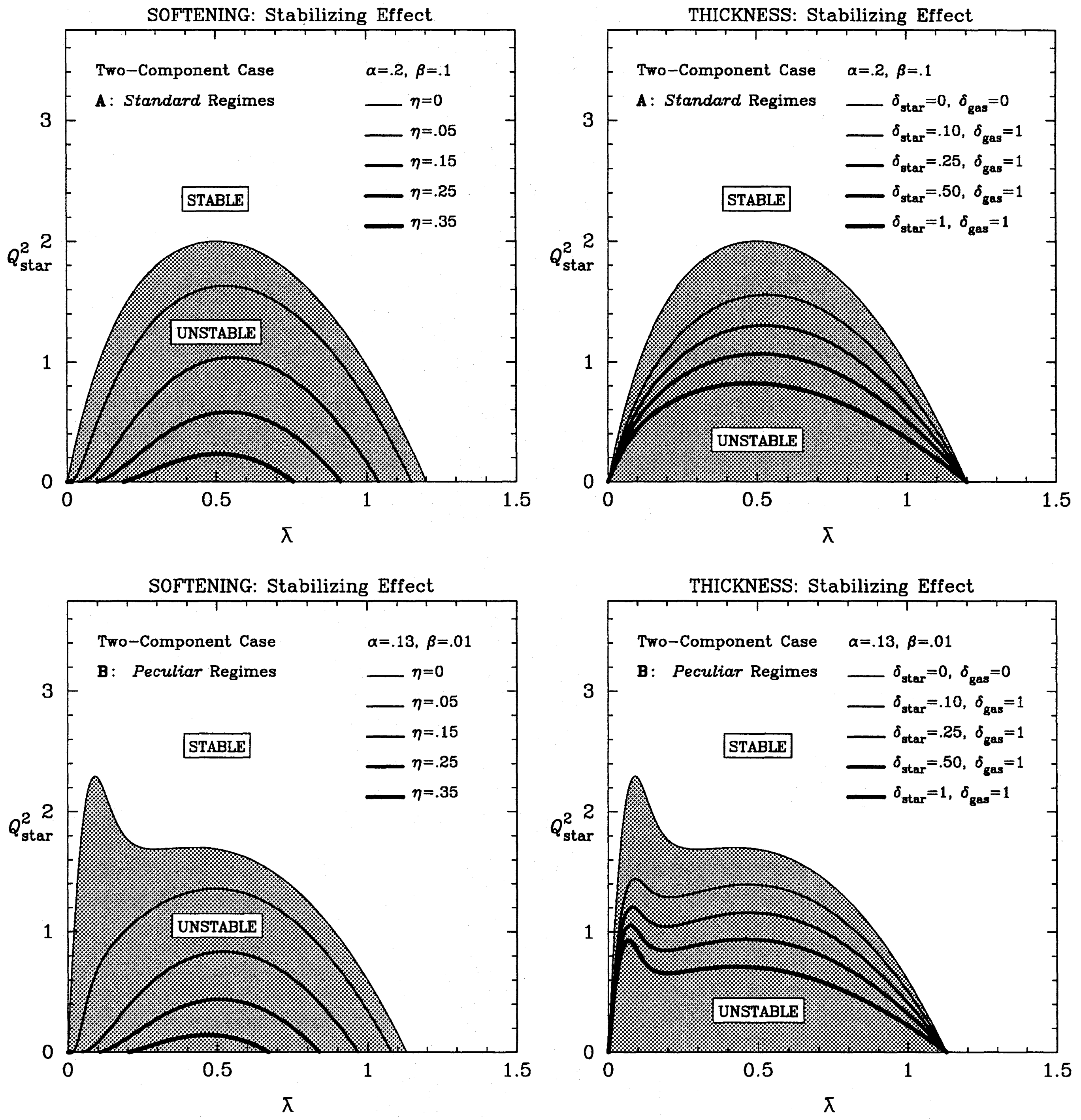

Fig. 4. Marginal stability curves of two-component two-dimensional N-body models (left) and real galactic discs (right) in standard star-dominated regimes (top) and peculiar gas-dominated regimes (bottom), where $\bar{\lambda}$ is the radial wavelength of the perturbation scaled in terms of the stellar Toomre wavelength and Q_{star} is the stellar Toomre parameter. In addition, α is the surface-density ratio, β is the planar-temperature ratio, η is the softening parameter, δ_{star} and δ_{gas} are the stellar and gaseous temperature anisotropies, respectively. The stability and instability regions are shown in the two-dimensional case with Newtonian gravity

purely stellar discs. In particular, the typical radial wavelength is practically the same. In addition, the natural upper bound of the stellar characteristic scale height is appreciably larger, but still significantly smaller than the critical value of the softening length. This means that the second source of dissimilarity has no significant direct consequences in such regimes, and approximate physical consistency can be ensured by following the same prescription given in the one-component case.

- In peculiar regimes [roughly corresponding to the region $\beta^{1/2} \lesssim \frac{1}{5}$, $5\beta \lesssim \alpha \lesssim \beta^{1/2}$ (star-dominated) or the region $\beta^{1/2} \lesssim \frac{1}{5}$, $\beta^{1/2} \lesssim \alpha \lesssim \frac{1}{5}$ (gas-dominated)], the responses of the two components decouple and peak at two typical radial wavelengths, roughly one half the stellar and gaseous Toomre wavelengths; and the instabilities which first appear, as $Q_{\rm star}$ drops below the stability threshold, are those of the dominant component. All gaseous dynamical instabilities are artificially suppressed by softening, except for prohibitively small values of η, and such peculiar regimes are *unfaithfully* turned into standard star-dominated regimes.
- In non-peculiar gas-dominated regimes (roughly corresponding to the region $\beta^{1/2} \lesssim \frac{1}{5}$, $\frac{1}{5} \lesssim \alpha < 1$), the responses of the two components are again coupled, and the stability properties are analogous to those of purely gaseous discs in the neighbourhood of the typical radial wavelength. Even though in principle rough physical consistency might be ensured (cf. standard star-dominated regimes), in practice the resulting condition would be *prohibitively* demanding (cf. peculiar regimes).
- In the other non-standard regimes of astrophysical interest ($\beta, \alpha < 1$), the responses of the two components are coupled and comparable, but the physical inconsistencies arising from the second source of dissimilarity are not so significant as in peculiar regimes, if we set $s = h_{\rm star} < s_{\rm crit}$ and choose input values of $Q_{\rm star}$ corresponding to realistic stability levels.

4. Conclusions

Two questions which naturally arise in N-body simulations of discs galaxies are:

1. Which choices of the softening length s are physically consistent from the stability point of view?
2. Which choices of the stellar Toomre parameter $Q_{\rm star}$ are meaningful for a given s?

Our paper aims at clarifying these points in the case of two-dimensional simulations, when both the stellar and gaseous components are present.

1. The solution to the first aspect of the stability problem is provided in the form of a prescription for choosing meaningful values of s in standard regimes of astrophysical interest [Eqs. (17) and (18)]. If we follow such a prescription, the stability properties are similar to those of real galactic discs, and s has a natural physical counterpart: the stellar characteristic scale height $h_{\rm star}$. One should be *cautious* about choosing larger values of s, because then artificial softening alters the stability properties to a significant degree and, even worse, can affect the results critically by suppressing large-scale spiral instabilities. Non-standard regimes of astrophysical interest, in which the role of the gaseous component is particularly important, can*not* be simulated faithfully (see Sect. 3.2.2).
2. The solution to the second aspect of the stability problem requires two keys.
   (a) One key consists in identifying which values of $Q_{\rm star}$ correspond to stable situations for a given s, and is provided in our paper [Eq. (10); see also Eq. (11)].
   (b) Once the stability threshold $\bar{Q}$ is known, the other key consists in identifying which values of the effective stability parameter $Q_{\rm eff} = Q_{\rm star}/\bar{Q}$ correspond to realistic stability levels. This issue, whose importance has recently been re-emphasized by Bertin et al. (1989a, b) and Romeo (1990), is still under debate.

   Both these keys are also necessary for the correct interpretation of computational results in terms of real phenomena, but the identification of $\bar{Q}$ itself is sufficient for comparing the output values of $Q_{\rm star}$ with those predicted by theories of spiral structure and disc heating.

The comparative stability analysis carried out in our paper provides a further cautionary message about the choice of s. As s rises beyond the critical value $s_{\rm crit}$ [Eq. (3)], the stability level is no longer actively controlled by $Q_{\rm eff}$. This fact has serious dynamical implications: it *artificially* precludes the possibility of simulating regimes of normal spiral structure, which require fine-tuned choices of the stability level. Other discussions of related computational issues have been given by Lin & Bertin (1985) and Bertin et al. (1989a, b).

When applying the results of our paper to concrete cases, we should recall that in particle-mesh codes the grid implicitly contributes to softening the mean inter-particle force (see, e.g., Hockney & Eastwood 1981 and in particular Efstathiou et al. 1985). Grid softening is potentially important especially in the case of polar grids, where the mesh size Δ increases linearly with the radius R. The combination of gravity softening and grid softening results in an effective softening length $s_{\rm eff} > s$, which depends on the characteristics of the code and cannot be evaluated straightforwardly. In order to minimize the inaccuracy in the determination of the stability level, it is advisable to choose $\Delta(R) < s$ throughout the range of astrophysical interest.

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References

Athanassoula E., 1984, Phys. Rep. 114, 319

Athanassoula E., Sellwood J.A., 1986, MNRAS 221, 213
Bertin G., Romeo A.B., 1988, A&A 195, 105 (Paper I)
Bertin G., Lin C.C., Lowe S.A., Thurstans R.P., 1989a, ApJ 338, 78
Bertin G., Lin C.C., Lowe S.A., Thurstans R.P., 1989b, ApJ 338, 104
Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton University Press, Princeton
Byrd G.G., Valtonen M.J., Sundelius B., Valtaoja L., 1986, A&A 166, 75
Combes F., Elmegreen B.G., 1993, A&A 271, 391
Efstathiou G., Davis M., Frenk C.S., White S.D.M., 1985, ApJS 57, 241
Elmegreen B.G., Thomasson M., 1993, A&A 272, 37
Erickson S.A., 1974, Vibrations and Instabilities of a Disk Galaxy with Modified Gravity. PhD thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
Hockney R.W., Eastwood J.W., 1981, Computer Simulation Using Particles. McGraw-Hill, New York
Lin C.C., Bertin G., 1985, Formation and Maintenance of Spiral Structure in Galaxies. In: van Woerden H., Allen R.J., Burton W.B. (eds.) Proc. IAU Symp. 106, The Milky Way Galaxy. Reidel, Dordrecht, p. 513
Miller R.H., 1972, Numerical Experiments in Collisionless Systems. In: Lecar M. (ed.) Proc. IAU Colloq. 10, Gravitational N-Body Problem. Reidel, Dordrecht, p. 213
Miller R.H., 1974, ApJ 190, 539
Miller R.H., 1978a, ApJ 223, 811
Miller R.H., 1978b, ApJ 224, 32
Miller R.H., 1978c, ApJ 226, 81
Pfenniger D., Combes F., 1994, A&A (in press)
Pfenniger D., Friedli D., 1993, A&A 270, 561
Pfenniger D., Combes F., Martinet L., 1994, A&A (in press)
Romeo A.B., 1985, Stabilità di Sistemi Stellari in Presenza di Gas: Fenomeni Collettivi in Sistemi Autogravitanti a Due Componenti. Tesi di Laurea, University of Pisa and Scuola Normale Superiore, Pisa, Italy
Romeo A.B., 1990, Stability and Secular Heating of Galactic Discs. PhD thesis, SISSA, Trieste, Italy
Romeo A.B., 1992, MNRAS 256, 307 (Paper II)
Rybicki G.B., 1972, Relaxation Times in Strictly Disk Systems. In: Lecar M. (ed.) Proc. IAU Colloq. 10, Gravitational N-Body Problem. Reidel, Dordrecht, p. 22
Salo H., 1991, A&A 243, 118
Sellwood J.A., 1981, A&A 99, 362
Sellwood J.A., 1983, J. Comput. Phys. 50, 337
Sellwood J.A., 1986, Disc Galaxy Dynamics on the Computer. In: Hut P., McMillan S.L.W. (eds.) The Use of Supercomputers in Stellar Dynamics. Springer-Verlag, Berlin, p. 5
Sellwood J.A., 1987, ARA&A 25, 151
Sellwood J.A., 1989, MNRAS 238, 115
Sellwood J.A., Athanassoula E., 1986, MNRAS 221, 195 (Corrigendum in MNRAS 229, 707)
Sellwood J.A., Kahn F.D., 1991, MNRAS 250, 278
Thomasson M., 1991, Simulations of Spiral Galaxies. PhD thesis, Chalmers University of Technology, Göteborg, Sweden
Toomre A., 1964, ApJ 139, 1217
Toomre A., 1977, ARA&A 15, 437
Toomre A., 1981, What Amplifies the Spirals? In: Fall S.M., Lynden-Bell D. (eds.) The Structure and Evolution of Normal Galaxies. Cambridge University Press, Cambridge, p. 111
White R.L., 1988, ApJ 330, 26

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