THESIS FOR THE DEGREE OF LICENTIATE OF TECHNOLOGY

Mathematical modelling for optimization of truck tyres selection

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Abstract

This thesis, which consists of an introduction and three appended papers, concerns the optimal selection of tyres for a given vehicle configuration and an operating environment in which the vehicle is to be used. The optimization problem stems from an industrial project performed in cooperation between Chalmers University of Technology and Volvo Group Trucks Technology (GTT). The project began in August 2012 and the expected termination is in January 2016.

We analyze the tyres selection problem from the mathematical optimization point of view. Our aim is to develop a tool for determining an optimal set of tyres for each vehicle and operating environment specification. The overall purpose is to reduce the cost of operations—which is in this case measured by fuel consumption and tyre wear—while preserving the levels of other tyre dependent features such as startability, handling, and ride comfort. We develop a computationally efficient vehicle dynamics model of the vehicle, the tyres, and the operating environment. The tyres are modelled using a surrogate model of the rolling resistance coefficient, i.e., the energy losses caused by the tyre. The development of the surrogate model motivated the development of a methodology for connecting the existing expert knowledge about a certain simulation-based function into its radial basis function interpolation. Suitable solvers for the resulting optimization model with a simulation-based objective function and simulation-based constraints have also been identified by a literature review.

An algorithm for the global optimization of a combinatorial set of problem instances has been developed and tested on a set of test problem instances. This algorithm enables a computationally efficient search for an approximately optimal tyre design for each vehicle configuration and each operating environment specification, in case at least one such design does exist.

Keywords: simulation-based optimization, truck tyres selection, surrogate model, radial basis function interpolation, rolling resistance coefficient, combinatorial set of problem instances, global optimization

Appended papers

Paper I: *Integration of expert knowledge into radial basis surrogates for global optimization*, under revision for publication in Optimization and Engineering (with Peter Lindroth, Ann-Brith Strömberg, and Michael Patriksson)

Paper II: *A joint model of vehicle, tyres, and operation for the optimization of truck tyres*, preprint (with Bengt Jacobson and Peter Lindroth)

Paper III: *Global optimization of a combinatorial set of simulation-based problem instances*, preprint (with Peter Lindroth, Michael Patriksson, and Ann-Brith Strömberg)

Publications written within the TyreOpt project not included in the thesis

1. *An optimization model for truck tyres selection*, Proceedings of the 4th International Conference on Engineering Optimization (EngOpt 2014), Lisbon, Portugal, 8–11 September 2014. (with Peter Lindroth, Ann-Brith Strömberg, and Michael Patriksson)
2. *Optimizing truck tyres—How to improve the realism of simulation-based optimization through physical constraints*, to appear in ORbit: medlemsblad for Dansk Selskab for Operationsanalyse og Svenska OperationsAnalysFöreningen. (with Peter Lindroth, Michael Patriksson, and Ann-Brith Strömberg)

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Zuzana Šabartová
Göteborg, January 2015

Contents

1 Introduction

This thesis constitutes a result of the project *TyreOpt—Fuel consumption reduction by tyre drag optimization*, performed in cooperation between Volvo GTT, the Department of Mathematical Sciences at Chalmers University of Technology and University of Gothenburg, and the Department of Applied Mechanics at Chalmers University of Technology. The project is financed by the Swedish Energy Agency and Volvo GTT.

The purpose of the thesis is to describe and model the practical truck tyres selection problem, introduce the scientific areas utilized in the appended papers, and clearly describe the connections between the real-world problem and the mathematical problems studied. Some well-defined subareas of optimization and the general problem of selecting tyres are more extensively studied in the appended papers.

1.1 Background

In order to improve the truck fuel efficiency various energy losses, such as engine losses, driveline losses, aerodynamic losses, tyre losses, braking losses, stand-still losses, and accessory losses, must be minimized. After engine losses, the tyres and the aerodynamics of the vehicle cause the biggest energy losses (see [52]). Depending on the vehicle type, the operating conditions, and the tyre conditions, eventually 15–30% of the fuel consumption of personal cars are due to the tyre rolling resistance ([6]). For heavy vehicles those losses are even higher, ranging from 15 to 40% ([29]). Accordingly, by decreasing the rolling resistance-induced losses, the vehicle's fuel consumption can be substantially reduced. In this thesis we develop a mathematical optimization model with the objective to minimize the energy losses caused by the truck tyres.

The energy losses caused by the tyres can be represented by a complex function, which is influenced by many parameters, such as inflation pressure, vehicle speed, axle load, tyre type and material, tyre radius and other tyre dimensions, temperature, and tread pattern (see [54]). In this thesis, we identify the most influential parameters and construct a composite function describing how each of the selected parameters influences the energy losses caused by the tyres. This function forms the basis of the optimization model of the truck tyres selection which minimizes the energy losses caused by the tyres summed with the tyre wear while balancing other tyre-dependent features such as handling properties, startability, and ride comfort.

Volvo trucks are used in markets differing in characteristics concerning operating environments and legislations, which has led to a high degree of specialization and truck customization (see [42, Ch. 1]). Therefore, a great variety of truck configurations as well as tyres must be offered. Since there is an enormously large set of combinations of vehicles and tyres offered it is in practice impossible to solve the computationally demanding tyres selection optimization problem for each of these combinations. To overcome these difficulties a specialized optimization strategy has been developed.

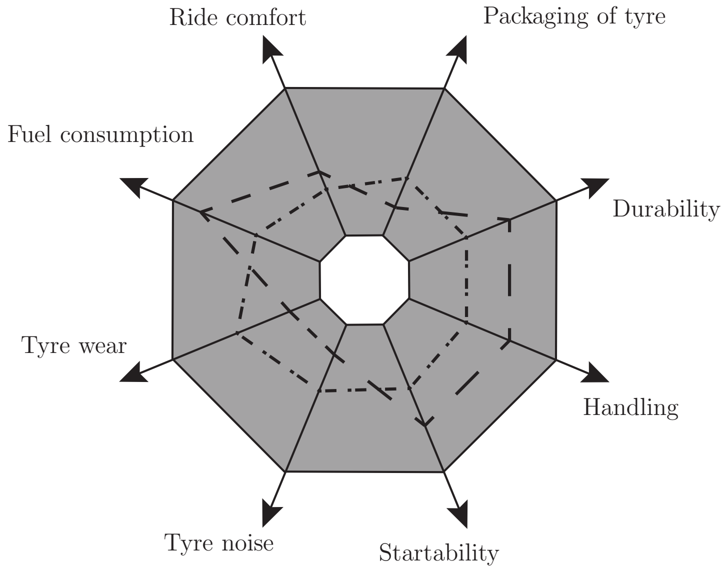

Figure 1: Each existing tyre (the figure illustrates two tyres) can be represented as a polygon intersecting each axis at the numeric measure of the corresponding quality component. The gray region represents possible values of the quality measures. Good values of a quality correspond to values far from the center of the diagram as is indicated by the axes' arrows. The dashed and the dash-dotted polygons are both feasible. The polygon defined by the largest possible values of all the measures corresponding to an ideal tyre is infeasible.

Each tyre possesses a specific quality for each specific customer and operating environment. By assuming that the quality can be measured in quantitative terms and further that the quality can be divided into a number of components, the quality of a tyre can be represented by a polygon, as introduced in [39] and illustrated in Figure 1. We assume that all the customers measure the different quality components equivalently, but prioritize them differently depending on the intended operating environment and the customer's financial strength. Then, there is no reason to offer a tyre possessing worse values than those of any other possible tyre in all components of the quality measure. Formally, we want to identify the tyres which are *Pareto optimal* and keep a limited set approximating the set of Pareto optimal tyres in the tyre database; see Section 3.4 for the definition of Pareto optimality and [44] for a method to approximate the Pareto optimal set. In the image in Figure 1, a *Pareto optimal solution* corresponds to a polygon which is not entirely enclosed in any other feasible polygon (see [42, Sec.1.1]). We wish to identify the tyres which are *not* Pareto optimal for any customer and exclude them from the tyre database. Further, we wish to provide recommendations about Pareto optimal tyres for each customer and each operating environment. The quality measures are differently weighted by different customers; they also differ due to differences in operating environments. Therefore, the Pareto optimal solutions chosen will differ between customers. The phase of the project reported in this thesis focuses on a single-objective optimization problem with the most important qualities (fuel consumption and tyre wear) summed into one objective function, and the rest of the qualities modelled as constraints.

1.2 Purpose and aim

The main goal of the project work behind this thesis and described in [43] is to design a procedure and subsequently a practical tool based on mathematical optimization modelling and computations for identifying an optimal tyre configuration from the tyre database, given the configuration and the operating environment of a vehicle; see Figure 2. Satisfying the main goal will lead to a better energy efficiency of the road transport. Another goal is to be able to satisfy as many customers as possible, while keeping the least possible number of tyres in the database, resulting in a bi-objective problem.

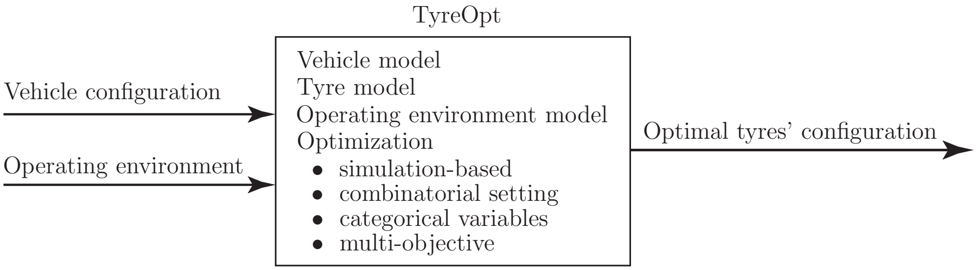

Figure 2: The aim of the TyreOpt project is to find the optimal configuration of tyres for each customer's specification of the vehicle and its operating environment. First, a vehicle model, a tyre model, and a model of the operating environment are established. Then the optimization with respect to the listed attributes (simulation-based, multi-objective optimization problem with categorical variables in a combinatorial setting of vehicles and operating environments) is applied to find the optimal tyres configuration.

The overall goal of the project is to improve the energy efficiency of cargo transportation by minimizing the rolling resistance. As a result both academia and Volvo GTT will gain a better understanding of how the tyres selection can be modelled and optimized. The expected benefit for Volvo GTT will be the ability to offer a better fit of the tyres selected for each vehicle configuration and operating environment specification, and to provide recommendations about suitable tyres to its customers. A certain reduction of costs related to the reduction of the number of tyres in the database should also be achieved. Another goal is to spread the knowledge about mathematical optimization and its profitable utilization within the company, to emphasize what a successful optimization requires (i.e., provide numerical measures of qualities), and give an example of what a production company can gain from the use of mathematical optimization. The expected benefit for academia is insight into the truck tyres selection process. The academia will learn how mathematical optimization can be applied to a real industrial problem and where the biggest differences between real industrial problems and academic ones lie; see Section 4. The expected benefit for the customers is a guidance for searching and choosing among the available tyres which, in this case, reduces the cost of transport operations as measured by fuel consumption.

In this thesis we interpret and model the tyres selection problem mathematically, introduce contributions to a few selected and well defined parts of this complex problem, and suggest paths for moving towards the aims. We develop a mathematical model for the selection of truck tyres. In order to keep the model computationally efficient—as described in Section 4—a lot of information about the tyres, the vehicle configuration, and the operating environment are left out in the model. The missing information has to be taken into consideration for the complete truck tyres selection problem. In Section 4.2 we return to a discussion on the left-out information and how it can be dealt with.

1.3 Previous research

The research presented in this thesis is related to the previous research project *Product Configuration with respect to Multiple Criteria in a Heterogeneous and Dynamic Environment within an Extended Enterprise*, performed at Volvo 3P in cooperation with the Fraunhofer–Chalmers Research Centre for Industrial Mathematics and the Department of Mathematical Sciences at Chalmers University of Technology and the University of Gothenburg, and presented in the thesis [42]. That project reported in [42] takes a mathematical optimization perspective on the product development of platform-based products with a common architecture, enabling shared technology, specifically trucks, that are developed for heterogeneous markets. Certain approaches developed in [42] will be used in our future research, e.g., the approximation of the Pareto optimal set using a reduced set of objective function ([44]), and/or the global descent approach for pure categorical optimization ([42, Paper IV]).

In the literature, there are previous attempts to optimize certain tyre design parameters; see, e.g., [11] in which the cornering stiffness of the tyre is optimized wrt. multiple objectives. The tyres' performance is also tested by tyre suppliers. These tests usually consider only single vehicle specification and a few operating environments. We aim to solve the tyres selection optimization problem—a special case of the products selection problem described in, e.g., [28]—to a much greater extent and higher complexity, i.e., optimize several tyre design parameters simultaneously for all possible combinations of the vehicle configuration and the operating environment.

1.4 Outline

In Section 2 the current structure of the tyres selection in Volvo GTT is described. This is our starting point for determining the framework. Section 3 presents the scientific areas utilized in the thesis. The variables, the objective function, and the constraints used to model the truck tyres selection problem are discussed in Section 4. In Section 5 the main contributions of this thesis as well as some topics for future research are reviewed. Finally, in Section 6, we summarize the appended papers, in which the selected parts of the problem studied are presented.

2 Current status of the tyres selection process at Volvo GTT

At Volvo GTT, a gradually developed sales system with an associated organization is used to select tyres. This system enables the customer—via a dialogue with a sales person—to specify the required vehicle configuration (see Section 2.1) and the intended usage of the vehicle (see Section 2.2) in a systematic way. The output from the sales system—among many outputs allowing for the production of vehicles tailored to their specific purpose—is a set of feasible tyres for the different positions and different seasons to be used for the specified vehicle. The current practice for selecting the tyres' configuration is then usually based on experience and customer input that can be further improved by means of scientific methodologies. An additional factor complicating the selection of tyres is the increasing number of combinations of steered and non-steered axles in future long vehicle combinations, which may have up to 30 tyres, as illustrated in Figure 3.

Figure 3: A long vehicle combination with eleven axles and 26 tyres.

The tyres selection process is to be improved using the results from the research presented in this thesis. We want to find an optimal combination, that is, suggest which tyres from the feasible set of tyres should be used in order to minimize the fuel consumption and the tyre wear, while balancing the other tyre-related features, e.g., the startability and the ride comfort (see Section 3.4).

2.1 Vehicle configuration

At Volvo GTT a truck is specified by its so-called *variants*, each of which belongs to a certain *variant family*. The variant families represent physical choices, such as engine type or frame width, as well as operation-related features, such as the type of roads that the truck is aimed for. The *product type* is defined by a very coarse division of the truck configurations specifying the overall truck type, and which is assumed to be given by the customer's specification. A certain truck configuration is completely defined by its variants; a specific product type defines a subset of each variant family from which the variants defining a configuration for that product type are chosen.

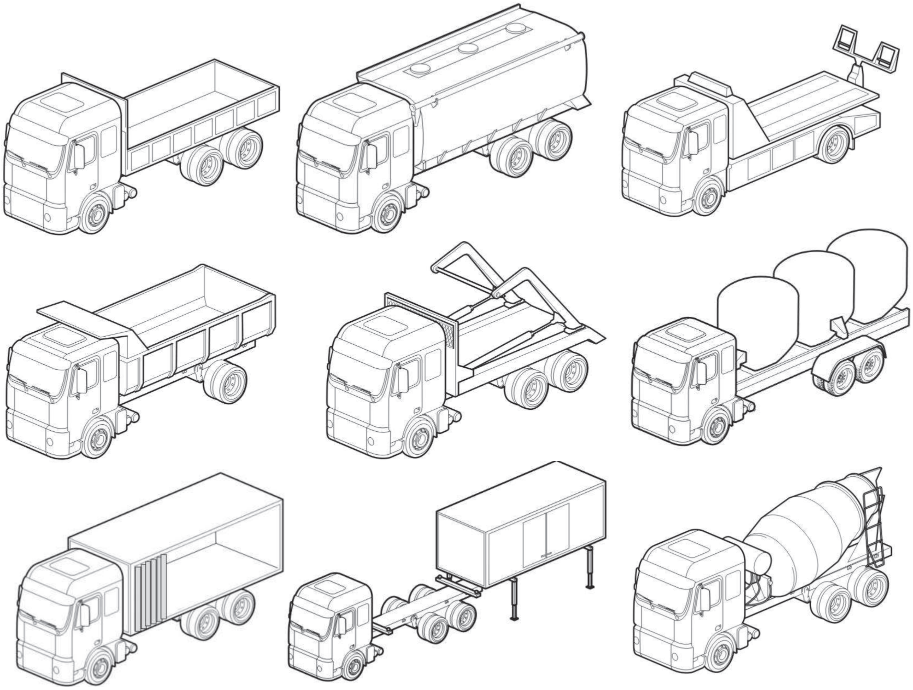

Figure 4: A selection of vehicle configurations, illustrating the variety of vehicles produced by Volvo GTT. The product type specifies, e.g., the axle configuration, hence, the upper- and right-most configuration does not belong to the same product type as the lower- and right-most configuration.

A typical product comprises has about 500 valid variant families, each containing two or more variants. Thus, the number of possible configurations is huge, even though not all variants can be combined due to different kinds of documented *restrictions*; see Figure 4 for illustrations of some truck configurations. The sales system contains a subset of the complete set of variant families from which the actual variant is chosen by the customer.

2.2 Discretization of the operating environment

Once a specific truck is defined, it is also necessary to introduce an environment in which it should operate. The operating environment is characterized by many parameters, e.g., topography, road conditions, and curve density. The properties of the operating environment are continuous over the driving cycle and hard to measure. However, well-defined discretizations of these properties were introduced in [16] in order to design vehicles adapted to the actual customers rather than to the worst-case conditions. The Global Transport Application (GTA) defines a number of vehicle-independent parameters that specify differences in driving and transport conditions for vehicle operations worldwide (see [16]). Among the GTA parameters defining a certain transport application are the operating cycle (divided into the four well-defined classes *Stop&Go*, *Local*, *Regional*, and *Long Distance*), the road condition (divided into *Smooth*, *Rough*, *Very Rough*, and *Cross Country*), and the topography (divided into *Flat*, *Predominantly Flat*, *Hilly*, and *Very Hilly*). Some of the GTA parameters, as, e.g., road conditions, are considered as variant families in the product structure as well as in the sales system. The operating environment is completely defined by 15 GTA parameters, each containing three or more classes. Thus, the number of classes of operating environments is also very high.

The vehicle configuration and the discretization of the operating environment are considered as inputs to the tyres selection problem. Given these, we need to find the most important parameters describing the tyres.

2.3 Tyre specification

An extensive database of tyres, which can be used by the trucks produced, was developed in the cooperation between Volvo GTT and its tyre suppliers; see Figure 5 for an illustration of differences between tyres. The database contains a lot of information about each tyre (e.g., rolling resistance coefficient class, noise class, load capacity, axle load at each given pressure value, width, diameter, sidewall height, weight, brand name, and recommended applications); additional information can be requested if needed. The most important tyres parameters will be used to define the decision variables in the optimization model developed for solving the tyres selection problem.

Figure 5: Three tyres available in Volvo GTT's tyre database.

2.4 Complexity of the tyres selection process

A complex system is formed by a set of interconnected components. To understand the behaviour of a complex system one must understand the behaviour of its components and their interactions (see [5, Ch. 0]). To solve the tyre selection problem we need to model a complex system consisting of the three interacting systems of vehicle, tyres, and operating environment, as illustrated in Figure 6 (compare with the schema of TyreOpt project in Figure 2).

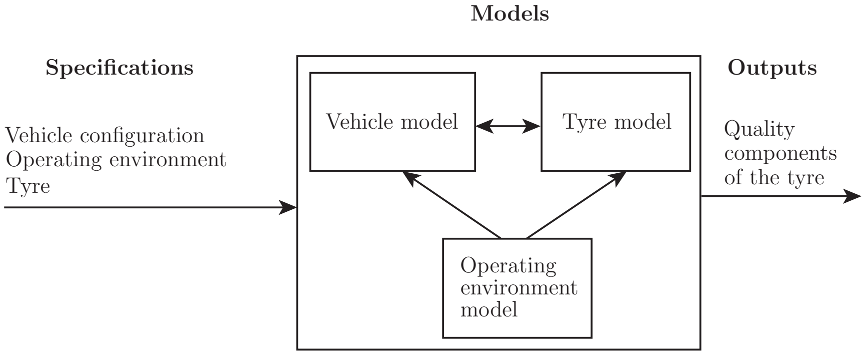

Figure 6: A complex system that needs to be modelled in order to solve the tyres selection problem. In order to compute the values of the quality components of a tyre, specifications of the vehicle configuration and the operating environment must be input.

Complexity can, according to [5, Ch.0], be defined as the amount of information needed in order to describe the complex system. The truck is a complex system in itself. The complexity of the truck and the operating environment model varies with the scale in which the truck is viewed. The tyre model has to be complex enough to differentiate between different tyre designs. Therefore solving the tyres selection problem for each customer is time consuming. However, the main complexity lies in the large number ($\sim 10^{120}$, according to [69]) of vehicle configurations manufactured by Volvo GTT. It is not possible to test such a huge amount of combinations for all the ingoing parameters that would be needed to find the respective optimal tyre configurations. Therefore, we need to develop a method to solve the tyres selection problem efficiently in a combinatorial setting of the vehicles and the operating environments.

Our aim is to solve the computationally expensive truck tyres selection problem only for *strategic vehicle specifications* (SVS) and then to assemble the optimal tyres' configurations for each other combination of vehicle and operating environment in a computationally efficient way. The concept of SVS was introduced to systematize and simplify the production and development processes for trucks at Volvo GTT (see [42]).

3 Scientific areas concerned

Since this thesis is driven by an industrial application, it has to utilize theory from several separate, although interconnected scientific areas. The most important areas for our application are *Engineering design* (see Section 3.1) and several subdisciplines of optimization such as *Global optimization* (see Section 3.2), *Simulation-based optimization* (see Section 3.3), *Multi-objective optimization* (see Section 3.4), and *Semi-infinite programming* (see Section 3.5). The extents of the overviews in the following subsections are biased towards their utilization in the appended papers.

3.1 Engineering design

Computationally expensive design problems are becoming common in manufacturing industries. The design of complex systems usually involves multiple disciplines and computation-intensive processes such as finite element analysis (FEA) for the product simulation. Therefore, the engineering design problems usually cannot be solved by an exact analysis. One needs to start with an approximate assessment and set up idealized simplifications, and then construct models to represent the real physical conditions. These simple models with a limited validity range are called *surrogate models* or *metamodels*. Metamodelling techniques in engineering design are surveyed in [18] and [70]. Having a surrogate model, common optimization methods can be applied to search for an approximation of an optimal design. The process towards a final design usually involves gradual enhancements of the accuracy of the surrogate model utilizing design space exploration (e.g., [34]) and various model validation techniques (see [58]).

3.2 Global optimization

Consider the optimization problem to

$$\begin{aligned} &\text{minimize} \;\; f(\mathbf{x}), \\ &\text{subject to} \;\; \mathbf{x} \in X, \end{aligned} \tag{1}$$

with the decision variables $\mathbf{x} \in \mathbb{R}^m$. The goal is to find an optimal solution $\mathbf{x}^* \in X$ that minimizes the *objective function* $f : \mathbb{R}^m \to \mathbb{R}$ over a *feasible set* $X \subset \mathbb{R}^m$. The feasible set X is typically determined by a number of *equality* and/or *inequality constraints* involving the decision variables. According to [7], a point $\mathbf{x}^* \in X$ is a *global minimum* of f over X if it holds that

$$f(\mathbf{x}^*) \leq f(\mathbf{x}), \quad \mathbf{x} \in X. \tag{2}$$

A point $\mathbf{x}^* \in X$ is a *local minimum* of f over X if there exists a neighborhood $\mathcal{N}(\mathbf{x}^*)$ of $\mathbf{x}^*$ such that

$$f(\mathbf{x}^*) \leq f(\mathbf{x}), \quad \mathbf{x} \in \mathcal{N}(\mathbf{x}^*) \cap X, \tag{3}$$

where the definition of $\mathcal{N}(\mathbf{x}^*)$ varies with the definition of X (cf. [32]). When $X \subset \mathbb{R}^m$ is considered $\mathcal{N}(\mathbf{x}^*)$ is an open Euclidean ball centered at $\mathbf{x}^*$ and with a bounded radius.

Assuming that the function f is convex on the convex set X, the fundamental theorem of global optimality (e.g., [7, Theorem 3.4.2]) implies that any local minimum of f over X is also a global minimum. Therefore, when a problem fulfills these convexity assumptions, it is enough to apply a local optimization algorithm to find a global optimum of (1). Local optimization algorithms typically possess a lower computational complexity than their global analogues.

In nonconvex optimization problems we have to expect multiple local minima that differ from the set of global minima. The discipline concerning the search for a global minimum is called *global optimization*; see [31] for an introduction to global optimization.

Some nonconvex optimization problems have properties which make the optimization easier, e.g., minimization of a concave function over a nonempty compact polyhedral set where the global minimum is attained at an extreme extreme point ([7, Theorem 3.4.7]), many integer programs (for example, binary programs with a linear objective function and a convex feasible set fulfilling certain assumptions, see [66]) which can be reformulated as convex programs in continuous variables, or optimization of Lipschitz functions with a known Lipschitz constant; see [62, Chs. 2 and 5].

For a general global optimization problem, where the evaluation of the objective function is sufficiently cheap, it is possible to use a method that switches between *local* and *global* phases. During the global phase all of the feasible region is explored, while the local phase is restricted to explore a local portion of the feasible region. The aim of the local phase is to refine the current solution. The local exploration is performed by sampling more observations in a neighborhood of the current point, with the aim to find an improved solution in terms of a lower objective function value. Examples of local phases are standard local searches performed by means of local optimization methods. In contrast, the aim of the global phase is to explore the search domain. The global phase usually generates points without restricting to the neighborhoods of past solutions. Examples of global phases are the random generation of feasible points or the choice of points which maximizes some measure of their distance from all previously sample points. See [45] for an overview of methods for global optimization.

One of the most popular methods for solving general nonconvex optimization problems with a sufficiently cheap objective function evaluations is the DIRECT algorithm ([37]). The method does not provide any convergence guarantee or any error measure unless some strong assumptions are imposed on the problem. The convergence proof for this algorithm is based on showing that the subdivision procedure used in the global phase to divide the feasible set into hyper-rectangles, in centers of which the objective function is evaluated, will eventually generate a dense set of observations in the feasible set. This type of convergence property is common for all global optimization methods which do not use any prior information on the structure of the problem ([45]) (the TOMLAB ([30]) solver *glbFast* is based on the DIRECT algorithm). Alternatively, nonconvex optimization problems can be solved to global optimality using the algorithm BARON, which is described in [65] and which is based on the branch-and-reduce approach introduced in [64].

The discipline concerning the search for a global minimum when dealing with very expensive objective (or constraint) functions is called *simulation-based optimization*, which is discussed next.

3.3 Simulation-based optimization

The assumption in simulation-based optimization is that the objective function f in (1) is not directly available, but must be estimated through a simulation, which implies the absence of analytic derivatives; see [13] for an overview of algorithms for derivative-free optimization. Computer simulations are extensively used as models of real systems in order to evaluate output responses. Applications of simulation-based optimization are found in engineering design ([25]), manufacturing analysis ([68]), portfolio selection ([63]), biomedicine ([33]), etc. Simulation-based optimization integrates optimization techniques with simulation analysis.

Optimization problems including simulation-based functions cannot in practice be solved by algorithms requiring many function evaluations, such as, e.g., *direct search methods* ([40]) or algorithms inspired by physics and/or natural selection (e.g., *genetic algorithms* [49]). Instead, we need to consider global optimization algorithms, in which a surrogate model, that mimics the behaviour of the expensive function as closely as possible while being computationally cheap to evaluate, is constructed; this surrogate model is then optimized. These algorithms are denoted *response surface methods* and are reviewed in [35]. The response surface methods construct the surrogate model of the simulation-based function iteratively; see Algorithm 1.

Algorithm 1 General response surface optimization method

0: Create an initial set of sample points and evaluate the simulation-based function on this set.
1: Construct a surrogate model of the simulation-based function using the evaluated points.
2: Select and evaluate a new sample point, balancing local and global searches, to refine the surrogate model.
3: Go to step 1 unless a stopping criterion is met.
4: Solve the simulation-based optimization problem where the objective function is replaced by the surrogate model constructed.

The initial set of sample points at step 0 is created by some design of experiments technique, such as the latin hypercubes, introduced in [47]. The strategies to select a new point to evaluate in step 2 differ between specific algorithms. The strategy must balance local and global searches so that the information in the surrogate model is utilized, but also so that no part of the feasible set is left unexplored. The stopping criterion in step 3 varies between optimization problems (e.g., that a maximum number of function evaluations have been calculated, or that a certain quality measure of the model has to be attained wrt. some model validation technique, such as the cross-validation studied in [48]). The resulting surrogate model is then optimized by a global optimization solver and the optimal solution found approximates the optimal solution of the underlying expensive function.

In this thesis we deal with surrogate models constructed as radial basis function (RBF) interpolations (see [71, Chs. 1, 6, and 11]) of sample points, which often yield good global representations of the expensive function ([10]) and has a closed form expression. But the surrogate model can also be obtained as a linear or quadratic approximation (see [4]), Kriging approximation (see [67]), a general regression function (see [8]), or some other kind of interpolation (see [53]). The response surface methods utilizing RBF interpolation are described in [22, 34].

Each evaluation of the true simulation-based function is time-consuming, and often there exists some expert knowledge about the true function. Therefore, in **Paper I** we have developed a methodology to incorporate the existing expert knowledge into the RBF interpolation based on sample points, in order to improve the accuracy of the surrogate model.

Since the complexity of solving one simulation-based optimization problem is high and our application requires the solution of a lot of the problem instances, in **Paper III** we have developed a computationally efficient optimization algorithm for combinatorial set of simulation-based optimization problem instances which reduces the variable space and also the number of simulation-based optimization problem instances that have to be solved to optimality.

Our work considers an optimization problem (1) with a computationally expensive objective function f subject to computationally expensive inequality constraints determining the feasible set X. This kind of optimization problem can (in principle) be solved by the ConstrLMSRBF algorithm described in [61] or by some recent implementations of the NOMAD software, based on the MADS algorithm; see [15]. ConstrLMSRBF is a response surface method which builds RBF-based surrogate models of the objective and constraint functions in each iteration and uses these models to guide the selection of the next point, in which the functions will be evaluated. The MADS algorithm searches sequentially a set of points, called mesh, around the current point. If a point in the mesh improves the objective function, this point becomes the current point. The algorithms suitable for solving the considered optimization problem are discussed in Section 4.3 in details.

3.4 Multi-objective optimization

The quality of a tyre can be measured by a number of objective functions. The discipline concerning the search for an optimum when dealing with more than one objective function is called *multi-objective* optimization, which is discussed next.

Consider the optimization problem to

$$\begin{aligned} &\text{minimize } \{f_1(\mathbf{x}), \ldots, f_K(\mathbf{x})\}, \\ &\text{subject to } \mathbf{x} \in X, \end{aligned} \tag{4}$$

with K (≥ 2) possibly conflicting objective functions $f_k : \mathbb{R}^m \to \mathbb{R}, k = 1, \ldots, K$, that are considered simultaneously. The problem (4) is a so-called *multi-objective optimization problem* (see [50]). To avoid the trivial cases, when the objective functions are not in conflict, we assume that there exists no single solution $\mathbf{x} \in X$ that is optimal wrt. all K objective functions.

The problem (4) is not generally well-defined because there is no generally valid total ordering among the vectors $(f_1(\mathbf{x}), \ldots, f_K(\mathbf{x}))$, $\mathbf{x} \in X$. Therefore, we need to introduce a definition of optimality for multi-objective optimization problems. The predominant concept in defining an optimal solution for multi-objective optimization problem is that of *Pareto optimality* (see [55] and [50]).

Definition 1. (Pareto optimality). A point $\mathbf{x}^* \in X$ is *Pareto optimal* in the multi-objective optimization problem (4) if and only if there does not exist any point $\mathbf{x} \in X$ such that $f_k(\mathbf{x}) \leq f_k(\mathbf{x}^*)$, $k \in \{1, \ldots, K\}$, and $f_l(\mathbf{x}) < f_l(\mathbf{x}^*)$ for at least one $l \in \{1, \ldots, K\}$.

For any given continuous optimization problem, there may be an infinite number of Pareto optimal points constituting the Pareto optimal set, which is usually of a lower dimension than the decision variable space; see Figure 7. There exist different approaches for finding approximations of the Pareto optimal set in the literature; a review can be found in [46]. The Pareto optimal set is typically found by solving single objective problems created from (4) through a scalarization technique, e.g., a weighting method or an ε-constraint method, (cf. [17, Chs. 3–4]).

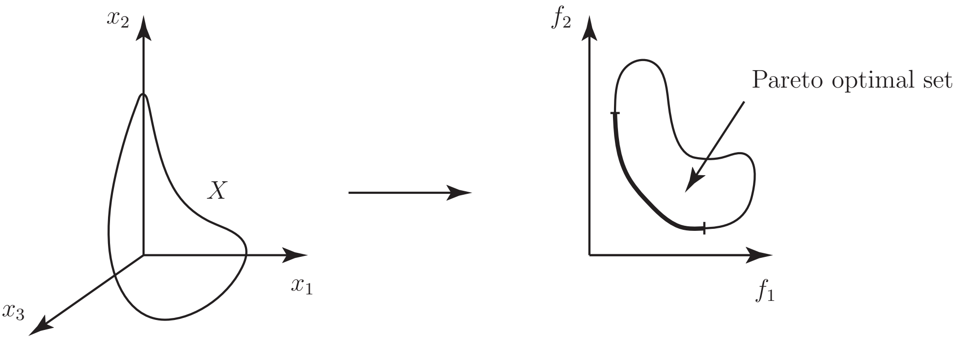

Figure 7: Illustration of the Pareto optimal set for a biobjective optimization problem with the feasible set $X \subset \mathbb{R}^3$.

Mathematically, every Pareto optimal point is an equally acceptable solution of the multi-objective optimization problem (4). However, it is generally desirable to obtain one single point as a solution. Therefore, we need a *decision maker* to select one solution out of the set of Pareto optimal solutions. The decision maker is a person (or a group of persons) with insight into the real problem, who can express preference relations among different solutions, and is usually also responsible for the final solution ([50]). Solving a multi-objective optimization problem calls for the co-operation between the decision maker and an *analyst* who is responsible for the mathematical side of the solution process.

The multi-objective optimization methods are usually divided into four classes based on the availability of the decision maker. If *no decision maker* is available, then a neutral compromise Pareto optimal solution has to be selected. The method is denoted *a priori* if the decision maker sets hopes and the solution that is closest to these hopes is found. It may however be hard to express the preferences without knowing the problem well. In *a posteriori* methods a representation of the Pareto optimal set is found before the decision maker selects the preferred solution. These methods are typically computationally demanding. The last class of the methods contains *interactive methods*, in which the iterative search through the Pareto optimal set is directed by the decision maker. The decision maker needs time and interest for co-operation and cannot be overloaded with information by the analyst. A review of the methods available can be found in [50].

3.5 Semi-infinite programming

Semi-infinite programming (SIP) problems include finitely many variables and infinitely many constraints. SIP problems arise in approximation theory, optimal control, and other engineering applications (see [20]), where the restrictions on the state or the control of the system are considered during a continuos period of time or at every point in a geometric region. The class of semi-infinite optimization problems is described in [27], including optimality conditions for general nonlinear SIP problems as well as a procedure for reducing the SIP problem to an optimization problem with only finitely many constraints. Surveys of algorithms for solving SIP problems are given in [20, 56], covering also theoretical analysis of algorithms, and in [26], focusing on implementation of the algorithms.

General semi-infinite optimization problems cannot be practically solved without a discretization in which the objective function is minimized subject to only a finite subset of the infinite set of constraints; the procedure is possibly repeated for an enlarged set when a higher precision is requested.

Alternatively, an optimum of a general (possibly nonconvex) semi-infinite programs with continuous objective and constraints functions can be found by the algorithm presented in [51]. The algorithm is based on a restriction of the right-hand side of the infinitely many constraints, which causes a shrinkage of the feasible set. The restriction is reduced in the course of the algorithm until it terminates finitely with a guaranteed feasible point, and a certificate of local optimality. In **Paper I** we have employed a modified version of Remez algorithm (introduced in [59]) to solve the infinitely constrained problem resulting from requiring non-negativity of a surrogate function for all feasible variable values.

4 Optimization of truck tyres selection

Building an optimization model, which typically starts with a quantification of the real-world description of an optimization problem, is the first and yet critical step for solving each optimization problem. The resulting optimization model directly affects the choice of the optimization algorithm that will be used to solve the problem, the feasibility, cost, and effectiveness of the optimization.

Optimization in industry is an iterative process, where, at each iteration, a model of the true problem is formulated and an optimal solution is found. Then the outcome is evaluated by experts, with a typical conclusion that some parts of the model are missing or need adjusting. Then the model is adjusted and a new iteration takes place. The usage of optimization in industry helps to illuminate and understand the problem, and to create basic data for decision-making. The important ingredients of an optimization problem are

- the objective, which measures the quality of a solution and is to be minimized or maximized,
- the variables, which are to be selected within some given intervals, and
- the constraints, which correspond to the design or feature requirements on a solution to be acceptable.

The aim of this section is to describe these three components for the tyre optimization/selection problem (Section 4.1), explain the vehicle models created in order to evaluate the objective function and constraints (Section 4.2), and list available solvers for the tyre optimization problem (Section 4.3).

4.1 Mathematical models

As mentioned in Section 1, a quality of each tyre can be measured by a number of quality components implying that the tyres selection is a multi-objective optimization problem. However, this thesis focuses on a single-objective optimization problem with the most important qualities (fuel consumption and tyre wear) summed into one objective function, and the rest of the qualities modelled as constraints. The multi-objective aspects of the tyres selection problem will be analyzed in our future research.

We want to find an optimal combination of tyres, i.e., suggest which tyres from the feasible set (here characterized by Volvo GTT's tyre database) should preferably be used on each individual axle in order to optimize the fuel consumption and the tyre wear, while balancing the other tyre-related constraints. We may also wish to find the optimal set of tyres for each customer, regardless of whether the optimal tyres exist in the database, hence, subsequently being able to put requirements on and/or provide recommendations to the tyre suppliers. Therefore, two optimization problems differing in the definition of feasible set need to be solved in order to meet the goals of the project:

- Find optimal values of the tyre design parameters (such as tyre radius, tyre width, and inflation pressure). The feasible set is continuous.
- Select the best possible set of tyres from those available in Volvo GTT's tyre database, where each tyre's functional parameter is given (e.g., the rolling resistance coefficient). The feasible set is discrete.

A remark is that the second tyres selection optimization problem cannot be solved through complete enumeration, since there is a huge number of tyres, there are many axles on the truck combination, and the vehicle simulations are costly.

Variables

In this thesis we assume that the tyre design variables are continuous, i.e., variables whose values lie in intervals on the real line, and we aim for their optimal values. Many optimization models, e.g., routing problems, require the use of discrete or integer variables, representing on/off decisions or indivisible quantities. Engineering design optimization problems often also involve a certain type of discrete decision variables that cannot be naturally ordered. These variables are denoted *categorical*. Our future research will focus also on the selection of tyres available in the database, i.e., tyres will be represented by categorical variables.

The categorical variables are discrete and can be given numerical values. The values themselves do not have to have a physical meaning. A typical use of a categorical variable is to model the selection of an optimal technical solution. In real design optimization problems the choices can be made from specified lists of alternatives. Such alternatives can be modelled using categorical variables. A certain alternative represents values for each of a number of parameters, i.e., a point in a multi-dimensional parameter space. A survey on approaches for handling categorical variables in optimization problems is found in [42, Paper II], which also introduces an algorithm for pure categorical optimization problems.

The decision variables, which describe the tyres and enable the formulation and solution of the tyre design problem and the tyres selection problem, need to be identified. We typically wish to keep the number of variables at a minimum in order to obtain a solvable optimization problem, but sufficiently many variables have to be considered in order to solve the intended problem.

The tyre specification used in the TyreOpt project is based on a literature survey on tyre models available identifying which parameters are most often used to describe it. Our conclusion is that the rolling resistance is a natural starting point for describing the tyre in the optimization model. In turn, see [54], the rolling resistance of a free rolling tyre is determined as a function of the vehicle speed, the tyre radius, the load on the tyre, the inflation pressure, the tyre stiffness, and the thread pattern. The tyre stiffness is influenced by the width and the material as well as the construction technology of the tyre (radial tyre or bias tyre). The thread pattern can be described by the number of grooves and their depth.

The FEA truck tyre model described in [2] and utilized in this thesis allows for investigations of the influence of the tyres' inflation pressure, width, diameter, groove depth, vehicle speed, and load on the tyres on the energy losses caused by the tyre. So far our optimization model contains the four tyre design variables (see Figure 8)

- inflation pressure,
- tyre width,
- tyre diameter, and
- groove depth.

The tyre design variables are denoted **x** further. The rest of the parameters describing the tyres are assumed to be fixed at their nominal values so far. This tyre specification allows to differentiate between tyres but does not appear to represent the influence of all the desired tyre design aspects. Therefore, we intend to use the material specification of the tyre's thread as an additional tyre design variable influencing the energy losses caused by the tyres whenever allowed by the FEA truck tyre model. Information about vehicle speed and the load on the tyre stems from the vehicle configuration and operating environment specification. We intend to include the influence of the applied torque on the energy losses as an additional vehicle parameter whenever allowed by the FEA model.

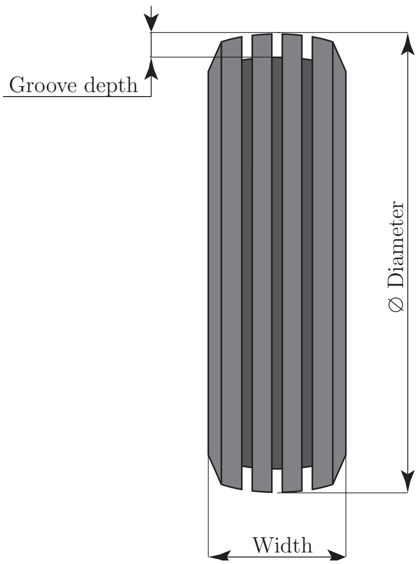

Figure 8: The considered tyre's dimensions: tyre diameter, tyre width, and groove depth.

Objective function and constraints

Engineers often use a combination of experience, computer simulations, and testing to decide which technical solutions are good and which are not. In optimization, we assume that there are real-valued functions (possibly outputs from black-box simulations) assigning a quality measure to each single technical design. It is often a hard and complex task to define an objective to be minimized, measuring the qualities of the technical solutions as functions of the design variables. Further, it is difficult to keep the total computer work at a reasonable level. To get a good solution in the end, the accuracy of each model must be weighted against the simulation time required.

We create and solve a model of the real problem. The process has been started with a small set of reasonable functions; then we can modify and complement this set based on the results from using these functions. We use surrogate models to get reasonable computation times. The surrogate models are constructed such that their corresponding decisions are as similar as possible to those that would have been the result of using the original functions.

As the full name of the project *TyreOpt—fuel consumption by tyre drag optimization* suggests the fuel efficiency of a vehicle is improved while minimizing the energy losses caused by the tyres, i.e., the rolling resistance. The function describing the rolling resistance can be represented by a complex function of a large number of variables affecting each other and of which only limited knowledge exists.

The rolling resistance is determined by the rolling resistance coefficient (RRC). Ali et al. ([2]) identified that the RRC is influenced mainly by the tyre inflation pressure, the tyre width, the tyre diameter, the groove depth, the vehicle speed, and the vertical load on the corresponding axle. A model for the RRC is hence represented by a six-dimensional function.

We have constructed a surrogate model of the RRC function based on sample points simulated by the FEA truck tyre model developed in [2]. We have chosen an RBF-based interpolation for this purpose and connected it with the existing expert knowledge about the RRC, i.e., we require a surrogate model which is non-negative and smooth; details are described in **Paper I**.

The rolling resistance is one aspect in the tyres selection; it significantly influences several tyre related objectives and constraints. Nevertheless, there are also important criteria, not related to the rolling resistance, when selecting tyres. Therefore, we need to identify the most important objectives and constraints for the optimization problem describing the tyres selection.

The most important aspect with regard to the cost of operation when considering the tyres selection is the vehicle's fuel consumption. The fuel is burned inside the engine cylinders to produce power, which is used to overcome all the resistance forces acting on the truck. Therefore, almost all components of a vehicle—including the tyres—influence the overall fuel cost (see [21]). The tyre wear, which determines the tyre cost per mileage, also forms a significant part of the operation cost and is mainly influenced by the tyres selection. The tyre wear is the result of abrasive processes brought about by the forces, which act on the vehicle during service (see [41]). We have chosen

- the fuel cost and
- the tyre cost

to form the objective function to be minimized in the tyres selection optimization problem.

Apart from the objective function, suitable constraints should be chosen in order to meet the requirements on the vehicle configuration wrt. safety, comfort, and transport efficiency. We have decided to consider one constraint corresponding to each basic direction standardly used in vehicle dynamics [12], i.e.,

- startability for the longitudinal dynamics,
- handling quality for the lateral dynamics, and
- ride comfort for the vertical dynamics.

Startability is one of the most critical longitudinal performance indicators, which indicates the ability to start from stand-still and maintain a steady forward motion on a specified grade when operating at a maximum laden mass. *Handling* generally refers to the response of the truck to the driver's operation of steering wheel. *Ride comfort* is an important measure of the vehicle performance; the selection of the wrong tyre can completely change the ride comfort in the cab from smooth to harsh.

The optimization model of the tyres selection problem approved by Volvo GTT is to minimize the described objective function (the sum of the fuel and tyre costs) subject to the described constraints (startability, ride comfort, and handling):

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_{\text{fuelcost}}(\mathbf{x}, \mathbf{p}) + f_{\text{tyrecost}}(\mathbf{x}, \mathbf{p}), && \text{(5a)} \\ \text{subject to} \quad & g_{\text{startability}}(\mathbf{x}, \mathbf{p}) \leq 0, && \text{(5b)} \\ & g_{\text{ridecomfort}}(\mathbf{x}, \mathbf{p}) \leq 0, && \text{(5c)} \\ & g_{\text{handling}}(\mathbf{x}, \mathbf{p}) \leq 0, && \text{(5d)} \end{aligned}$$

where the vector $\mathbf{x}$ is defined by the tyre design variables and the vector $\mathbf{p}$ represents the operating parameters, the parameters characterizing the surface, on which the tyre is running, and the vehicle.

Several other tyre related objectives and constraints (e.g., durability, noise, and low speed swept path) are omitted in the optimization model (5) due to the requirement to keep the model solvable by existing optimization solvers.

4.2 Vehicle dynamics models

To model the selected objective functions and constraints a vehicle dynamics model has to be developed. This so-called joint model consists of three main components:

- vehicle,
- tyres, and
- operating environment.

The model was established with the aim to be computationally efficient, but complete enough to reflect the influence of the design and selection of tyres on the cost of operation. The structure of the joint vehicle, tyres, and operating environment model is illustrated in Figure 9, which is a detailed version of Figure 6, and is described below.

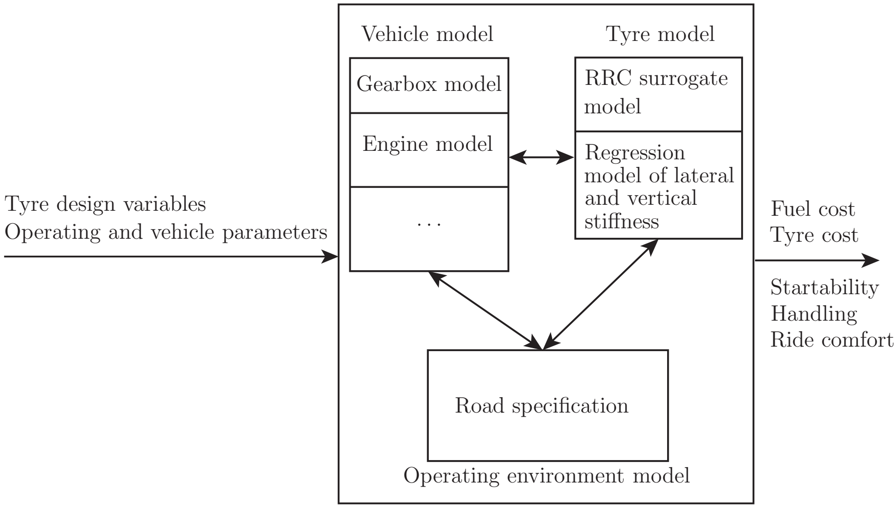

Figure 9: Joint vehicle, tyres, and operating environment model.

A rigid truck has been modelled in order to determine the functions describing the objective function and the constraints of the optimization problem (5). All truck data stems from real Volvo trucks.

The tyres are modelled using the surrogate model of the RRC, as described in Section 4.1, and regression models of the lateral and vertical stiffnesses of the tyre. All parameters values were set as close to real values as possible.

The model for the operating environment consists of sample roads available in Volvo GTT's databases. The roads are specified by the road height and the speed limit, as functions of the longitudinal position on the road. With these data, the road inclination and the speed profile were calculated as functions of time, prior to the running of the joint model.

The joint model of the vehicle, the tyres, and the operating environment is a simplification of reality. For example, in the tyre model it is assumed that the material properties of the tyre are fixed. We intend, as a part of our future research, to implement the influence of the Mooney Rivlin coefficient of the thread material, standardly used to model rubber materials (see [9]), into the model, in order to increase the accuracy of the estimation of the objective and the constraints functions. We assume that there is no torque applied on the tyres. The applied torque is also aimed to be added to the tyre model in future research to make the results more realistic. Since the lateral dynamics of the truck is not modelled at all, we cannot evaluate the performance of the vehicle in cornering manoeuvres. The lateral dynamics is aimed to be modelled as a part of our future research. The specification of the operating environment also needs to be extended to consider transport mission, vehicle utilization, and road conditions.

To populate the joint model, values of forty parameters are needed for the vehicle model, values of ten parameters for the tyre model and the discretized speed and height profile of the road, and values of five road parameters for the operating environment model. Below, we briefly describe the principle behind the modelling of the objectives and the constraints in the optimization model (5). See [12] and **Paper II** for details.

Fuel consumption

A basic inverse dynamic vehicle model was used to calculate the fuel consumption. This model consists of a vehicle block, a gearbox block, an engine block, and a fuel tank block created with the aid of Simulink ([14]), using the QSS toolbox ([23]). The computed fuel consumption is then recalculated to the fuel cost per distance unit.

Tyre wear

A simple tyre wear model, introduced in [19] and adapted in [12], is used to provide an estimation of the overall tyre wear, subsequently recomputed to the tyre cost per distance unit. The wear is influenced by the rolling resistance and includes a tyre surface temperature calculation.

Startability

The startability model imposes equilibrium equations in terms of the total resistive force and the total tractive force to calculate the maximum slope angle that the truck can handle, i.e., startability. There is presently no significant influence from the tyre design variables on the startability. A development towards calculating the road friction coefficient will be done as a part of future research. The road friction is typically varying with road conditions and the tyre design.

Handling

The tyre understeer gradient is calculated in order to predict the handling performance. The handling model in [12] has been extended through a regression model of the lateral stiffness of the tyre to include the influence of the tyre inflation pressure and the spindle load.

Ride comfort

The ride comfort is modelled using a half vehicle model of vertical dynamics ([24, Ch. 10]). The ride comfort model in [12] has been extended by a regression model of the vertical stiffness of the tyre to include the influence of the tyre width, the tyre inflation pressure, and the spindle load.

4.3 Optimization solvers

The solution of the simulation-based optimization problems considered in this thesis requires a global optimization algorithm which keeps the function evaluations at a minimum, is derivative-free, and is able to handle simulation-based constraints.

The optimization problem (5) possesses a simulation-based objective function and constraints and at most ten continuous variables. The variety of algorithms implemented in software solvers for this kind of optimization problems is not extensive. Below, we survey the solvers rbfSolve, EGO, NOMAD, and ConstrLMSRBF, which are among the most frequently used solvers for simulation-based optimization. The performance of each of these four solvers, when applied to the specific optimization problem (5), will be tested in our future research.

The solver *ConstrLMSRBF* presented in [60] is based on a stochastic derivative-free RBF method for the optimization of expensive black-box objective functions subject to black-box inequality constraints. The algorithm uses RBF surrogate models of the objective and constraint functions in each iteration to guide the selection of the next iterate, in which the objective and constraint functions will be evaluated. The algorithm outperformed several alternative methods, including a MADS algorithm ([1]), a genetic algorithm, a pattern search algorithm, and COBYLA ([57]) when tested on 14 problem instances, as presented in [60]. The test instances include four engineering design problems and the MOPTA08 benchmark problem developed in [36] involving 2–124 decision variables and 2–68 inequality constraints.

RbfSolve solves box-bounded global optimization problems with additional linear and nonlinear constraints using an RBF interpolation algorithm. The solver is not designed to take computationally expensive simulation-based constraints explicitly into account. Therefore, such constraints have to be introduced as penalty terms with suitable penalty parameters to the objective function. The solver is based on the same algorithm as ConstrLMSRBF.

NOMAD is a software application for simulation-based optimization of black-box functions ([3]). It is based on the MADS algorithm [1]; recent implementations allow for an implicit use of simulation-based constraints.

EGO solves costly box-bounded global optimization problems with additional linear and nonlinear constraints. Its main idea is to first fit a response surface to the data collected by evaluating the objective function at a few points. Then, it balances between the search for the point at which the surface is minimized, and an improvement of the approximation by sampling points at which the prediction error is expected to be high; see [38]. EGO cannot handle simulation-based constraints explicitly.

The solvers listed will be tested on the tyre design optimization problem and the results will be reported as a part of future research.

5 Conclusions

We now summarize the results of the research behind this thesis, make conclusions, and suggest suitable future work in order to meet the goals of the research project.

5.1 Main contributions

A survey on how the rolling resistance coefficient as well as other tyre related objectives and constraints depend on different tyre, vehicle, and operating environment parameters has been performed. Then, the most important tyre parameters were selected as variables for the tyre design and tyres selection optimization problems. A computationally efficient model of the rolling resistance coefficient has been created, and a methodology for implementing the expert knowledge into the radial basis interpolation has been developed and tested.

The most important objective and constraints functions for the tyres selection problem were selected and the optimization model was created. A survey on simulation models of the tyre related functions was performed and computationally efficient vehicle dynamics models of the objective and constraints functions were constructed.

The available solvers suitable for solving the tyre design problem obtained were identified by a literature review and described.

An algorithm for the global optimization of a combinatorial set of problem instances was developed and tested on a set of problems. This algorithm is able to find the optimal tyre design for each customer in a computationally efficient way.

In the next section we propose some areas of future research that we think will lead to meeting the goals of the TyreOpt research project.

5.2 Future research

It has been identified that the influence of the tyre material and the torque applied on the tyres is missing in the computationally efficient model of the rolling resistance. The missing data will be collected and the model will be improved. A development towards calculating the road friction coefficient has to be done. A high-fidelity model of the vehicle, the tyres, and the operating environment will also be constructed to verify the results from our optimization.

The performance of the available solvers suitable for solving the tyre design problem will be tested as a part of future research. The best solver will be selected and used for our future development.

We are currently characterizing the tyres by using categorical variables in the optimization problem formulated, thus allowing for the solution of the tyres selection problem. The algorithm for global optimization of a combinatorial set of problem instances then has to be modified accordingly. Further, the results obtained and the algorithms developed have to be collected in a tool which should then be incorporated in the truck sales tool.

6 Summary of appended papers

This section summarizes the results gained in the appended papers.

6.1 Paper I: Integration of expert knowledge into radial basis surrogates for global optimization

(coauthored with Peter Lindroth, Ann-Brith Strömberg, and Michael Patriksson)

The main motivation for the research presented in this article was provided by the TyreOpt project, in which we need to explicitly express and optimize a complex simulation-based function describing the rolling resistance coefficient (RRC) of a truck tyre. Many optimization algorithms for solving simulation-based optimization problems are based on a computationally efficient surrogate model of the computationally expensive objective function. It is seldom the case that all important characteristics—referred to as expert knowledge in this paper—such as non-negative values, of the complex function, are automatically inherited by the surrogate model.

We consider several types of expert knowledge about the reality behind a simulation-based function. We show that the utilization of expert knowledge when developing an RBF interpolation of an unknown function is possible and is often also computationally cheaper than performing additional costly simulations of the unknown function. We have demonstrated that the RBF interpolation can be reformulated as a tractable optimization problem, allowing for the utilization of constraints stemming from expert knowledge. We have also developed a methodology for accomplishing this. The methodology is illustrated on simple example functions and then applied to a function describing the RRC of truck tyres. Numerical results show that the utilization of the expert knowledge typically leads to an increase of the goodness of fit in comparison with an interpolation of the sample points.

This paper is under revision for publication in *Optimization and Engineering*. The results presented in this paper have also been presented at the SOAK/NOS6 conference on mathematical optimization and operations research, Gothenburg, Sweden, in October 2013 (by Zuzana Šabartová), at the SIAM Conference on Optimization, San Diego, CA, USA, in May 2014 (by Zuzana Šabartová), and at the 4th International Conference on Engineering Optimization (EngOpt2014), Lisbon, Portugal in September 2014 (by Zuzana Šabartová).

6.2 Paper II: A joint model of vehicle, tyres, and operation for the optimization of truck tyres

(coauthored with Bengt Jacobson and Peter Lindroth)

This paper introduces the tyres selection optimization problem, aimed to be solved in the TyreOpt project. It focuses mainly on an improvement of the joint vehicle, tyres, and operating environment model developed in the MSc thesis [12]. The tyres are modelled using regression models of the lateral and the vertical stiffness of the tyre and a surrogate model of the RRC. The surrogate model is based on a RBF interpolation of sample points evaluated using the FEA truck tyre model developed in [2] and expert knowledge about the RRC. A rigid truck has been modelled in order to explicitly describe the objective function and the constraints in the tyre design optimization problem. The model of the operating environment consists of sample roads that are available in Volvo GTT databases. The joint model is validated through investigations of how certain functional properties, such as the RRC, vary with certain tyre design parameters, such as the tyre radius and the inflation pressure, as well as with some operating parameters, such as the tyre vertical force.

The paper is to be submitted to the proceedings of the 4th International Tyre Colloquium: Tyre Models for Vehicle Dynamics Analysis, Guildford, UK in April 2015.

6.3 Paper III: Global optimization of a combinatorial set of simulation-based problem instances

(coauthored with Peter Lindroth, Michael Patriksson, and Ann-Brith Strömberg)

This paper analyzes a special case of a mathematical optimization problem where it is a priori known that its simulation-based objective function is influenced more by its so-called variables than by its parameters. We aim to solve this optimization problem for a large number of parameter settings in a computationally efficient way.

This paper introduces a global optimization algorithm for solving the large set of similar simulation-based problems. The algorithm initially finds optimal solutions for a selection of parameter settings using surrogate models of the objective function over the variable space. Subsequently, the approximate optimal solution for any other parameter setting is found by weighting the surrogate models assembled.

The main motivation for the research presented stems from the TyreOpt project, where we aim to find an optimal tyre configuration for each vehicle and environment combination in order to minimize the energy losses caused by the tyre. The need to solve this simulation-based optimization problem efficiently in the combinatorial setting of vehicles and environments led to the development of the algorithm presented. The numerical tests of the algorithm's performance on a set of global optimization problems, differing in both dimension and difficulty, show that the algorithm outperforms a naive approach based on a surrogate model of the objective function over the complete space of variables and parameters. The methodology developed can be used to efficiently solve the tyre design problem with continuous variables for each customer and many other practical problems, such as the design of a freight aircraft to be utilized for several types of transport missions or the optimization of charge for melting wrt. the quality of various products and which is desirable to be optimized in real time. For a direct application to the tyres selection problem the approach has to be extended in order to handle categorical variables.

The main ideas presented in this paper have been presented at the 4th International Conference on Engineering Optimization (EngOpt2014), Lisbon, Portugal in September 2014 (by Zuzana Šabartová).

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