Solar power

Statistical analysis of extinction coefficients

Ingemar Mathiasson

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Department of Energy and Environment

Division of Electric Power Engineering

Chalmers University of Technology

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1. Introduction

This document describes a stochastic method, dealing with the correlation between incoming solar radiation into a photovoltaic silicon solar cell. Of that reason the parameter “extinction coefficients” is used. The parameter provides information about the atmospheric attenuation and is therefore a key parameter for calculation of the effective radiation. The current paper describes the development work of a statistical model with the intent to provide useable values for the extinction coefficient in the wave length band of photovoltaic silicon solar cells. The model is based on measurement data that have been collected in during 2.5 summer months.The model has been applied in a simulation program for statistical analysis of the energy balance in combined electric power grids. The simulation program is described in [1] and [3]. In [2] is documented an earlier version of analysis regarding the extinction coefficient. The document contains data from the performed measurements and a lot of figures, illustrating the extinction coefficient during selected days.

The intent of the model presented in this paper is to provide a tool for statistical analysis of the available solar energy for photovoltaic systems at varying locations, times of the year and cloudiness. The model forms the basis of a software module in the simulation system described in [1] and [3].

2. The extinction coefficient

The transmission through a homogenous part of the atmosphere can be characterized by the Lambert-Beer law according to:

Equ. 1:

$\tau(\lambda) = \exp(-\varepsilon(\lambda) \cdot R)$

Where: $\tau(\lambda)$: Atmospheric transmission
$\varepsilon(\lambda)$: Extinction coefficient
λ: Wave length
R: Transmission distance

The extinction coefficient is composed by two components according to:

Equ. 2:

$\tau(\lambda) = \exp(-\varepsilon(\lambda) \cdot R)$

Where: $\sigma(\lambda)$: Absorption coefficient
$k(\lambda)$: Scattering coefficient

The absorption coefficient is a result of molecular absorption by different gases in the atmosphere, water, carbon dioxide, ozone, nitrous oxide, carbon oxide, and methane and so on.

The scattering coefficient is a result of scattering by aerosols, rain, snow, fog. smoke and so on.

In a homogeneous air mass the extinction coefficient is a function of wave length. Over a wave length band from λ_1 to λ_2 the mean extinction coefficient is:

<u>Equ. 3</u>**:**

$$\varepsilon = \frac{1}{\lambda 2 - \lambda 1} \int_{\lambda 1}^{\lambda 2} \varepsilon(\lambda)\, d\lambda$$

Where: λ_1: Lower wave length limit
λ_2: Upper wave length limit

In an air mass that is not homogeneous, but with different compositions of scattering and absorbing components between air layers, the extinction coefficient is depending on wave length band as well as location in the air mass. The mean extinction coefficient is in this case:

Equ. 4**:**

$$\varepsilon = \frac{1}{(\lambda 2 - \lambda 1)(L2 - L1)} \int_{\lambda 1}^{\lambda 2} \boldsymbol{\varepsilon}(\lambda, \boldsymbol{h}) \mathrm{d}\lambda \int_{L1}^{L2} \boldsymbol{\varepsilon}(\lambda, \boldsymbol{h}) \mathrm{dh}$$

Where: L1: Lower location in the air pass
L2: Upper location in the air pass

It is the extinction coefficient according to Equ. 4 that in the following is intended. It is also assumed, that the percentage of absorbing and scattering sources, that build up the extinction coefficient is the same in all direction of the sky. Of course there are cases where this is not entirely accurate, but it can be considered as a reasonable assumption for the further treatment of the subject. The following definition off extinction coefficient is used:

Equ. 5:

$$\tau = \exp(\varepsilon \times M)$$

Equ. 6:

$$G = G_0 \times \tau$$

Equ. 7:

$$M = h / h_0$$

Equ. 8:

$$M = 1 / \sin \alpha$$

Where:

τ: Atmospheric transmission
ε: Extinction coefficient, in the wave length region of silicon photovoltaic solar cells for an atmospheric depth corresponding to $\alpha = \pi / 2$

M : Relative atmospheric depth (i.e. the distance to pass through the atmosphere by the Sun radiation). It is related to the depth when the Sun is in zenith
h: Atmospheric depth
h_0: Atmospheric depth with $\alpha = \pi /2$
α: Sun altitude above the horizon
G: Irradiance (W/m^2) after the radiation (in the wave length region of silicon photovoltaic solar cells) has passed the atmosphere in question
G_0: "Solar constant = 1367 W/m2 (solar radiance outside the atmosphere)

β: Angle between the surface normal of the measuring surface (solar panel) and the direction to Sun

Cos β is calculated by the expression:

Equ. 9:

$$\cos\beta = \sin\alpha \times \cos\Omega_Z + \cos\alpha \times \sin\Omega_Z \times \cos(\theta - \Omega_S)$$

Where:

Ω_Z : Normal angle of the measuring surface relative to zenith
Ω_S : Normal angle of the measuring surface relative to south
θ : Sun azimuth

α and θ are calculated by using information regarding date, time and geographic location.

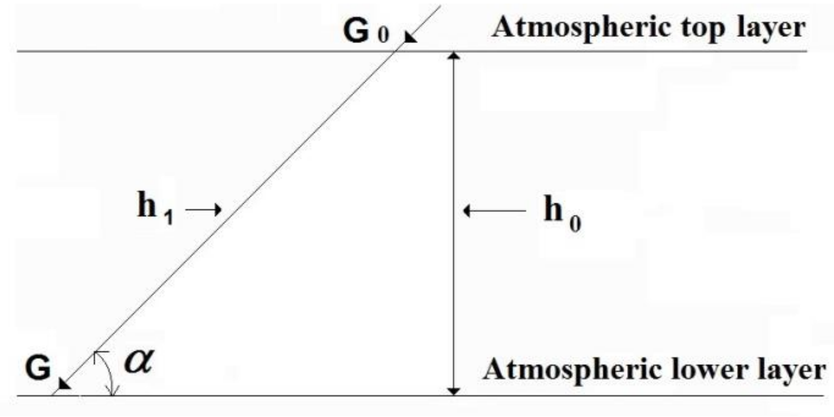

Fig. 1. Radiation G as an effect of atmospheric influence.

The extinction coefficient is dependent on the meteorological conditions regarding:

- Temperature
- Air pressure
- Humidity
- Rain
- Snow
- cloud conditions

In addition to that there is an influence on the extinction coefficient as an effect of parameters not meteorological depended:

- Varying conditions regarding aerosols in the air
- Varying conditions regarding the composition of different molecules in the air

All together there are a lot of parameters that have influences on the extinction in question.

3. Measurements

Measurements to get statistic foundations to make a survey of the extinction coefficient have been performed during the period 21/6 - 7/9 – 2006.

The principle for the measurement arrangement follows by Fig. 2.

There are 3 solar cell panels connected in series by the connection box. Each solar cell panel consists of 72 series connected solar cells. The result of this arrangement is that there are 216 series connected cells at the output of the connection box. The current I_S in Fig. 2 corresponds to the “short circuit current” of the solar cells. As there are so many solar cells that co-operates, two advantages are at hand:

- Small effects regarding the voltage drops in the connection wires
- A good representative value (mean value of a large number cells) regarding the short circuit current in question

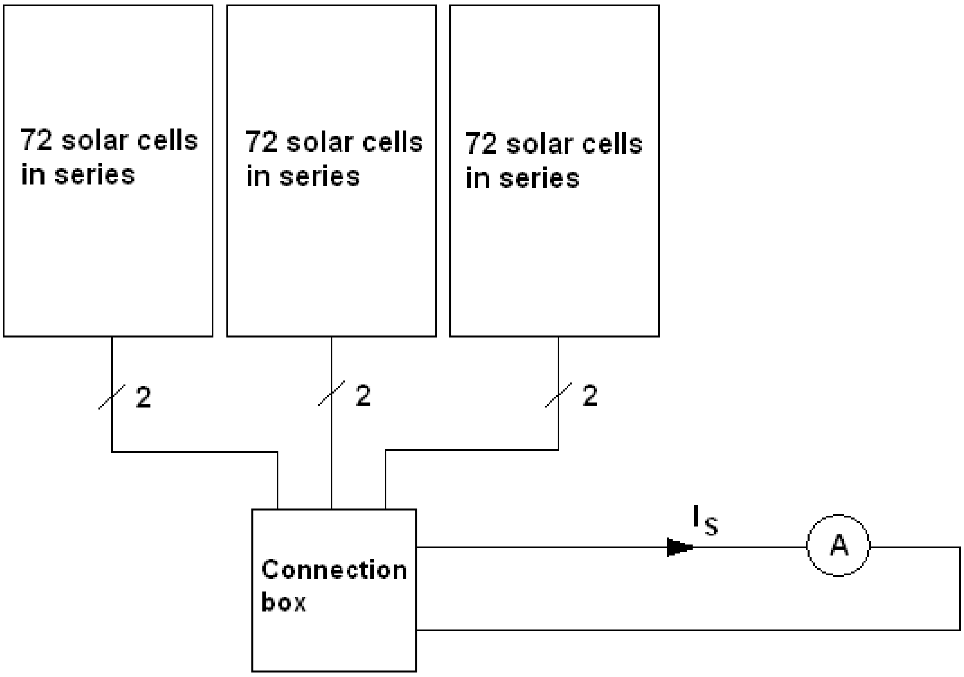

Fig. 2. The principle for the measurement arrangement. The connection box connects the three solar cells panels in series. This results in 216 series connected solar cells.

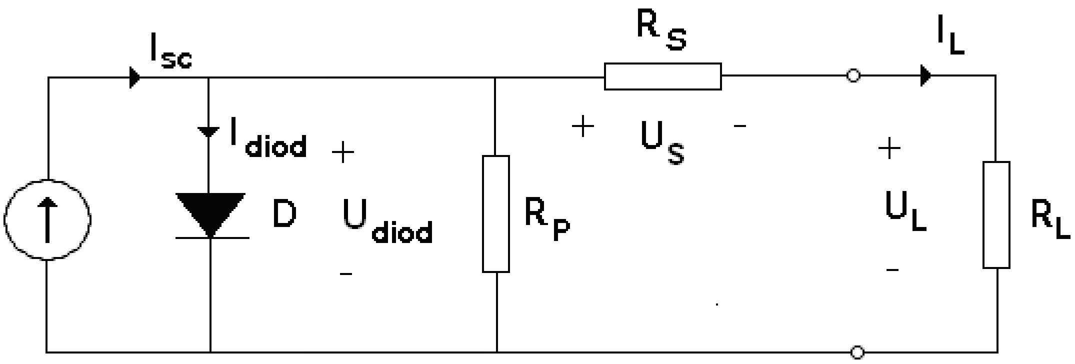

Fig. 3. The equivalent circuit of a solar cell.

The short circuit current is a good measure of the solar irradiance. See Fig. 3 that shows the equivalent circuit of the solar cell. As the resistance R_S in Fig. 3 is quite small (about 15 mΩ) the maximum voltage drop over this resistance (i.e. at short circuit) normally is less than 60 mV. This voltage corresponds to the voltage U_S. A short circuited cell output (i.e. $U_L = 0$) will result in $U_S = U_{diod}$. If U_{diod} is in the region of maximum 60 mV, then the diod current I_{diod}, is very small (in the order of a few mA) compared with the short circuit current (normally in the order of amperes).

The measurement principle was based on measurements of short circuit current from the solar cell. Short circuit current from a solar cell is a good measure of solar irradiance and was measured up during a time period of 71 summer days. The extinction coefficient was then calculated by using (5) – (11).

Equ. 10:

$G_1 = G \times \cos \beta$

Equ. 11:

$I_S = F \times G_1$

Where:

G_1: Effective irradiance (W/m^2) to the solar cell panels
I_S: Measured short circuit
F: Scale factor (Am^2/W)

The analysis takes into account the the contribution from diffuse irradiance. This is done by measuring the short circuit current at time points when no solar radiation hits the solar cells as direct radiation. The corresponding radiance is subtracted from the first measured values used for calculation of the extinction coefficients according to:

Equ. 12:

$G_1 \text{ (used)} = G_1 \text{ (prim)} - G_{diffuse}$

Where:

G_1 (used): Radiance used for calculating the extinction coefficients (W/m^2)
G_1 (prim): Primary radiance when solar radiation hits the solar cells (W/m^2)
$G_{diffuse}$: Calculated diffuse radiance (W/m^2)

4. Result off measurements

Measurements have been done during 71 days. Table II gives the result after calculation of the extinction coefficients (mean values and standard deviations). In order to avoid shadow effects from objects such buildings and trees, the calculations have been limited to the daily time period 9 am to 15 pm. The measurements were done with a sampling frequency of 1 per minute.

Some technical information for used solar cells follows in Table I.

Table I. Technical information for single solar cell.

Material	Polycrystalline silicon
Wavelength region	0.2 μm - 1.15 μm
Short circuit current at solar irradiance 1000 W/m^2 for polycrystalline silicon	35.4 mA/cm^2
Solar cell area	100 cm^2
Short circuit current for solar cell at solar irradiance 1000 W/m^2	100 cm^2 × 35.4 mA/cm^2 = 3.54 A

Table II. Mean values and standard deviations of extinction coefficients during 9 am to 15 pm for 71 days.

Measuring day (number)	Date	Mean value of extinction coefficient 9 am to 15 pm	Standard deviation of extinction coefficient 9 am to 15 pm
1	2006-06-21	1.82	1.12
2	-22	1.58	1.18
3	-23	2.03	1.09
4	-24	0.94	0.70
5	-25	1.98	0.62
6	-26	2.08	0.50
7	-27	2.42	0.49
8	-28	1.96	1.56
9	-29	0.65	0.56
10	-30	0.99	0.79
11	-07-01	0.44	0.04
12	-02	0.46	0.17
13	-03	0.46	0.05
14	-04	0.48	0.05
15	-05	0.50	0.04
16	-06	0.48	0.02
17	-07	2.27	1.54
18	-08	1.96	1.15
19	-09	1.72	1.00
20	-10	1.84	1.06
21	-18	0.42	0.01

22	-19	0.42	0.01
23	-20	0.41	0.12
24	-21	3.85	0.75
25	-22	2.42	1.42
26	-23	1.60	1.10
27	-24	1.99	1.22
28	-25	1.49	0.96
29	-26	0.44	0.02
30	-27	0.53	0.08
31	-28	2.32	1.05
32	-29	0.61	0.28
33	-30	1.02	0.59
34	-31	3.25	0.95
35	-08-01	0.72	0.61
36	-02	1.67	0.85
37	-03	1.70	0.75
38	-04	1.26	0.99
39	-05	0.40	0.12
40	-06	0.39	0.01
41	-07	1.15	0.41
42	-08	0.75	0.60
43	-09	1.05	0.78
44	-10	3.30	0.94
45	-11	0.65	0.38
46	-12	2.18	1.56
47	-13	1.72	0.66

48	-14	3.18	0.81
49	-15	1.55	1.37
50	-16	1.82	1.22
51	-17	1.00	1.00
52	-18	0.57	0.49
53	-19	3.11	0.54
54	-20	2.84	1.36
55	-21	2.68	1.15
56	-22	2.39	1.12
57	-23	0.99	0.78
58	-24	1.80	0.86
59	-25	1.67	1.32
60	-26	1.36	0.65
61	-27	2.09	0.57
62	-28	1.34	0.70
63	-29	1.12	0.61
64	-30	1.58	0.83
65	-31	2.05	1.11
66	-09-01	1.08	0.63
67	-02	1.65	1.08
68	-03	2.41	1.06
69	-04	1.50	1.05
70	-06	1.44	0.29
71	-07	1.37	0.94

Fig. 4 shows a graph of the mean values as function of the measurement days.

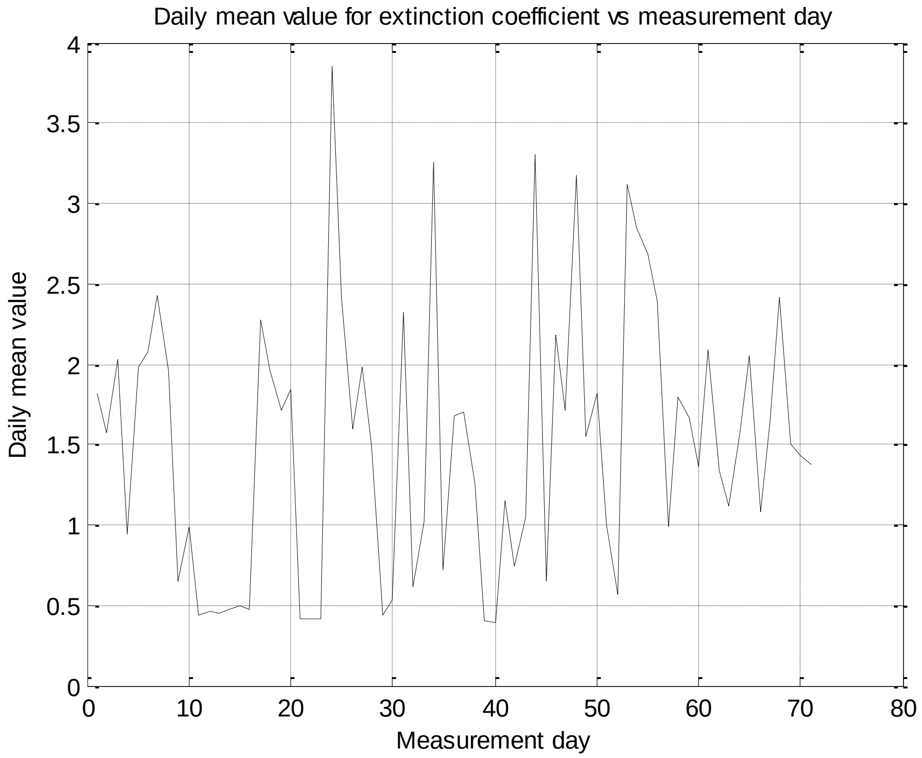

Fig. 4. Mean daily extinctions coefficient for 71 measurement days.

Fig. 4 illustrates the variations of extinction coefficient during the campaign. The values correspond to mean values during each single day between the hours of 9 to 15.

Max mean value for a day (between 9 – 15): 3.85

Min mean value for a day (between 9 – 15): 0.39

Given these min/max-values and a supposed sun altitude above the horizon of e.g. 50°, these results in a variation of incoming solar radiation from 9 W/m^2 to 822 W/m^2. See Equ. 5 - Equ. 8.

Fig. 5 - Fig. 12 illustrate some examples with short circuit currents and corresponding extinction coefficients. The figures show the results for various degree of cloudiness.

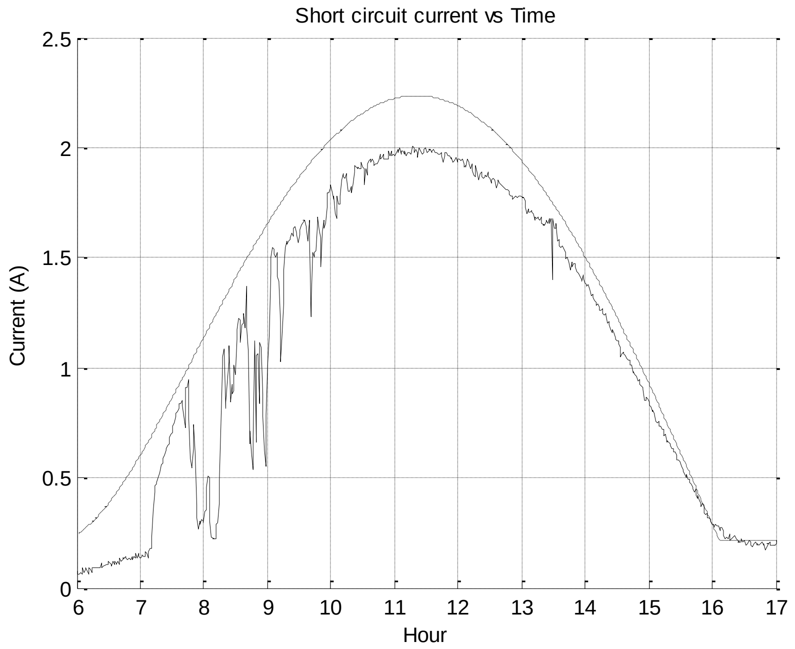

Fig. 5. Measured short-circuit current. Low cloudiness.

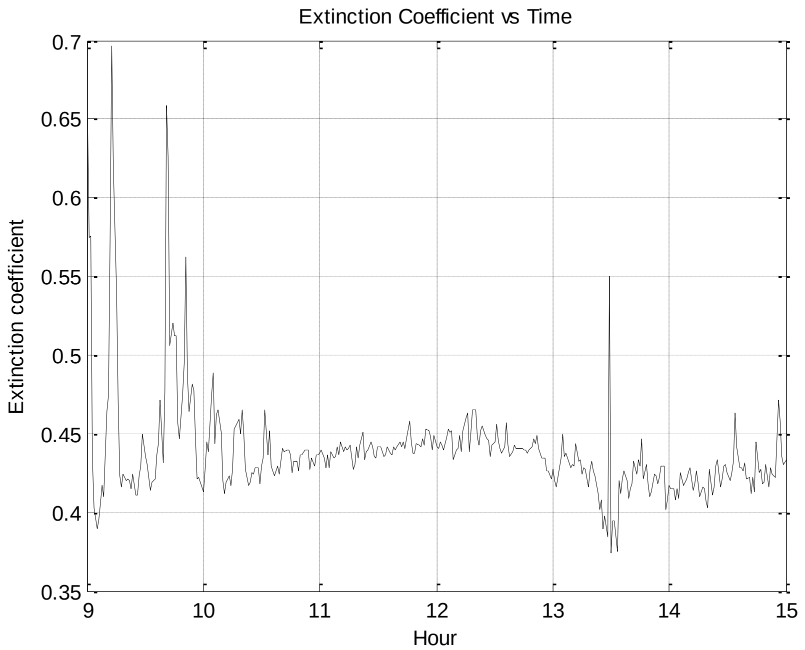

Fig. 6. Extinction coefficient corresponding to Fig. 5.

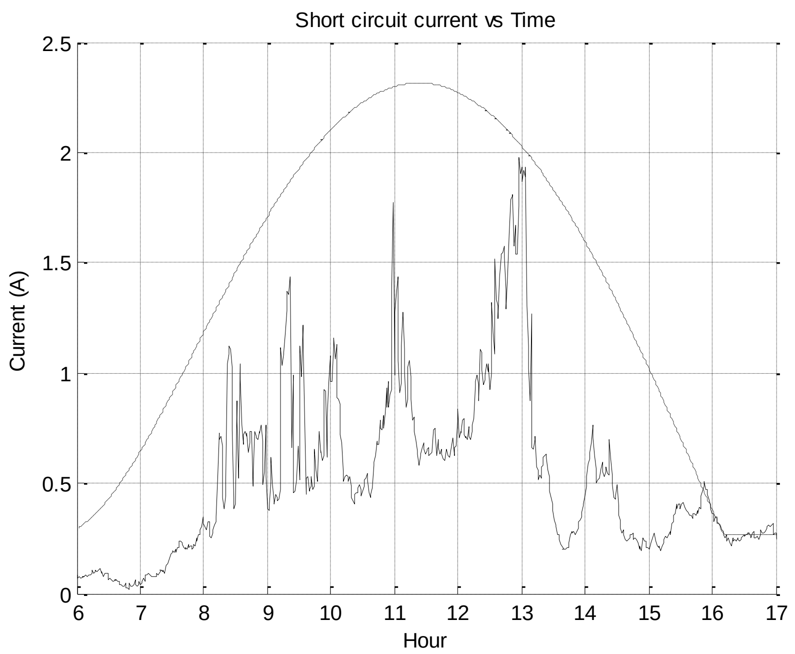

Fig. 7. Measured short-circuit current. Heavy cloudiness.

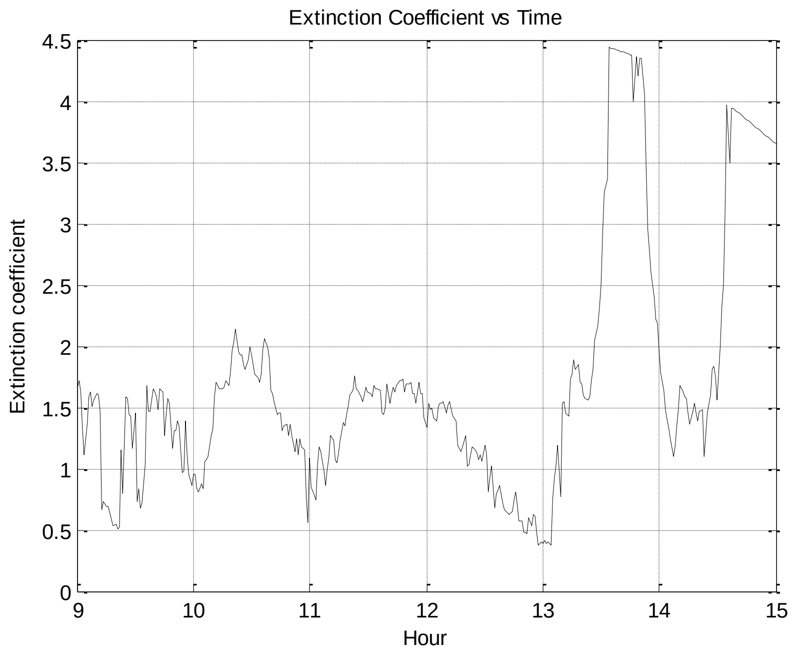

Fig. 8. Extinction coefficient corresponding to Fig. 7.

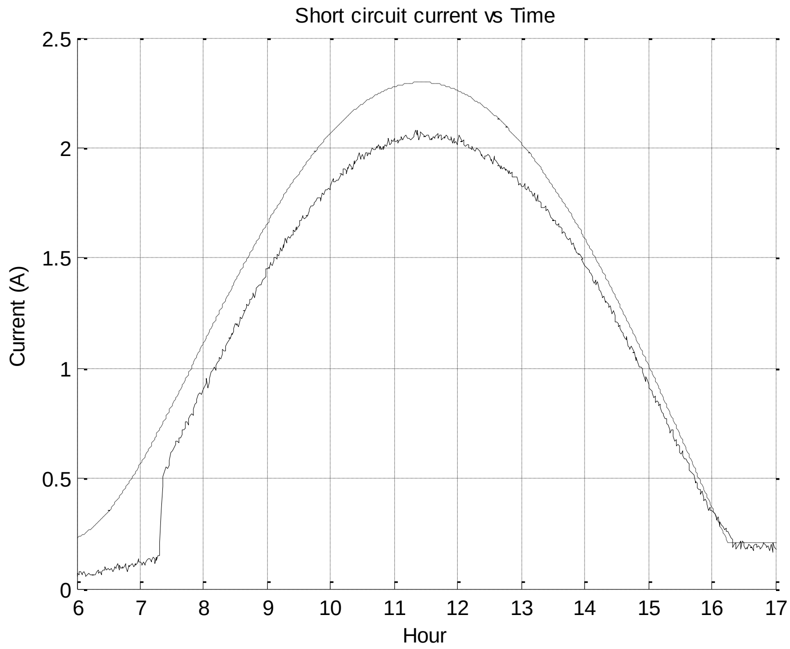

Fig. 9. Measured short-circuit current. No cloudiness.

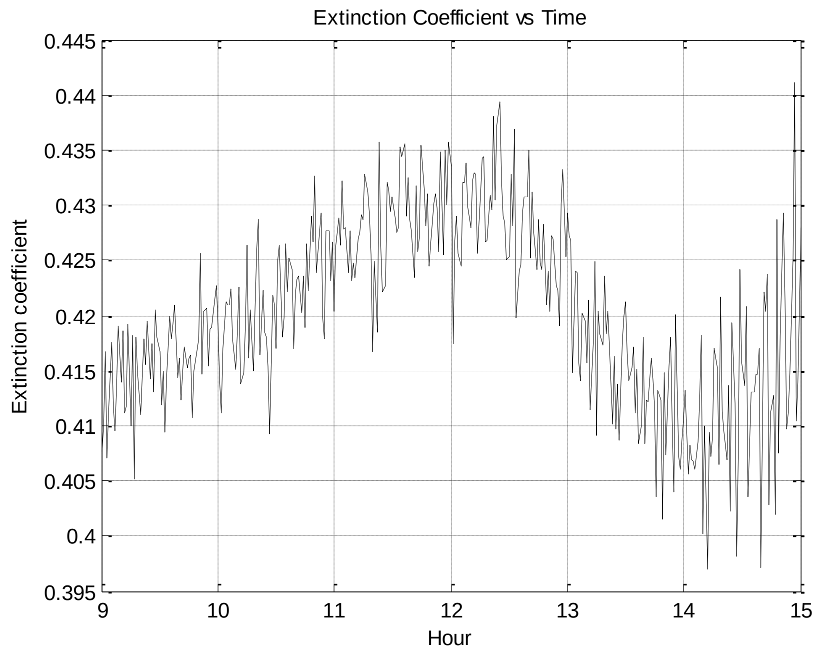

Fig. 10. Extinction coefficient corresponding to Fig. 9.

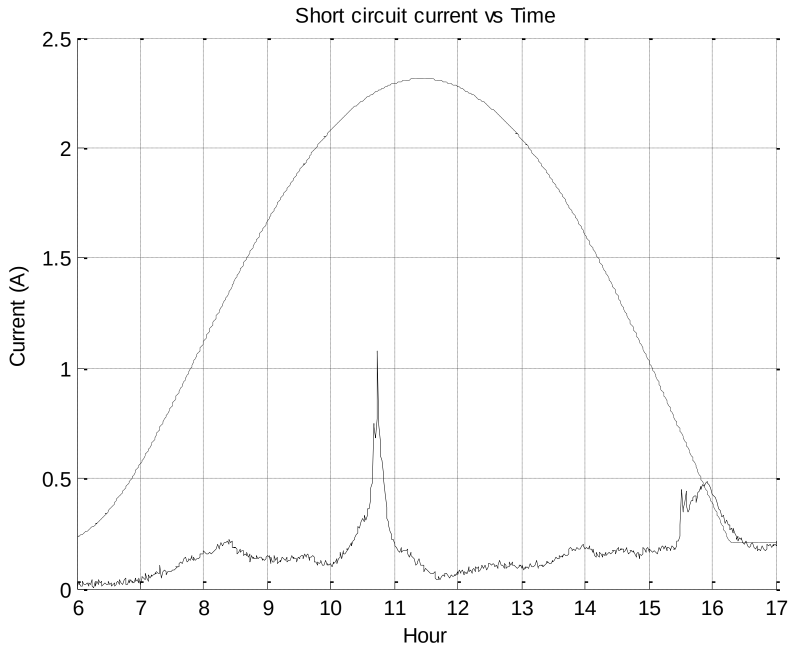

Fig. 11. Measured short-circuit current. Totally cloudiness.

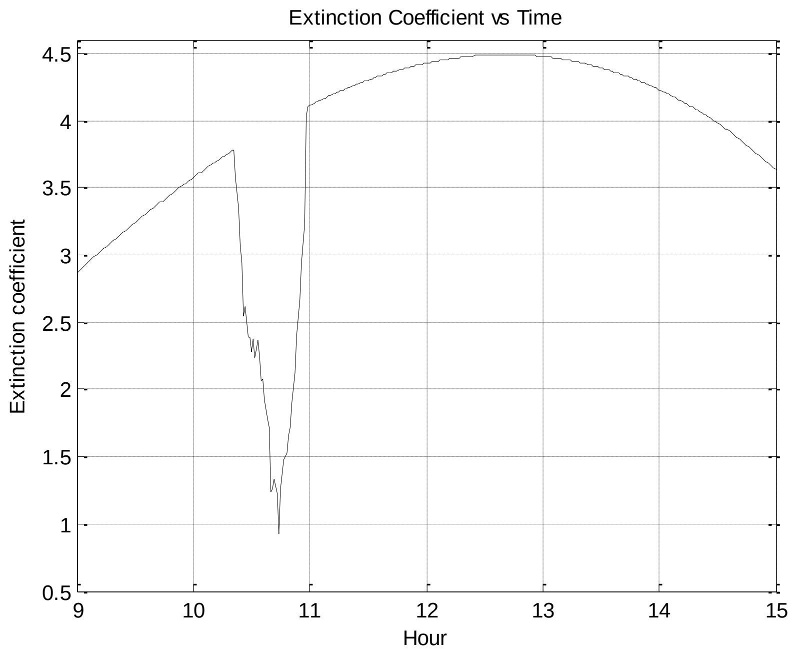

Fig. 12. Extinction coefficient corresponding to Fig. 11

5. Statistical analysis

5.1 Overview

Fig. 13 illustrates the relative distribution for measured extinction coefficients in region 0.31 to 4.5. There is a peak around 0.43. Fig. 14 illustrates this more in detail. The interval between 0.55 – 4.5 is illustrated in Fig. 15. The range up to 0.55 represents conditions when no clouds affect the incoming solar radiation, while the area above represents clouds influence in varying degrees. Two modes have been defined: “Mode 1, meaning “no clouds affect the solar radiation” and “Mode 2, meaning “clouds affect the solar radiation”.

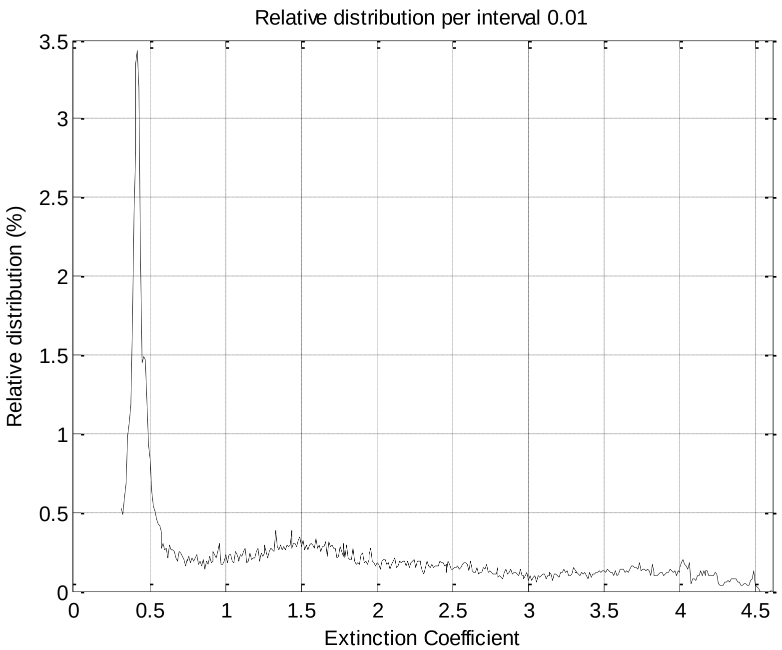

Fig. 13. Relative distribution for extinction coefficient per interval 0.01.

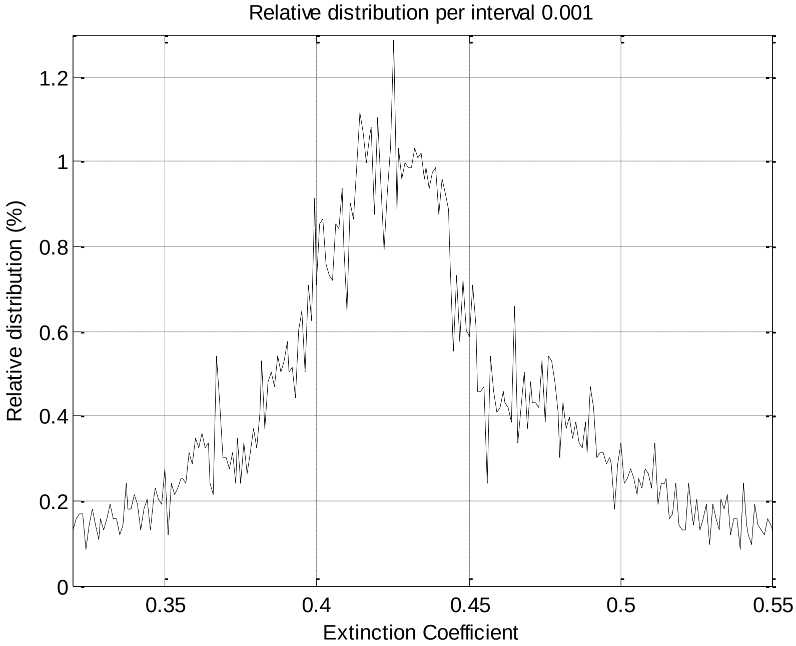

Fig. 14. Relative distribution for extinction coefficient per interval 0.001. Interval 0.31 - 0.55.

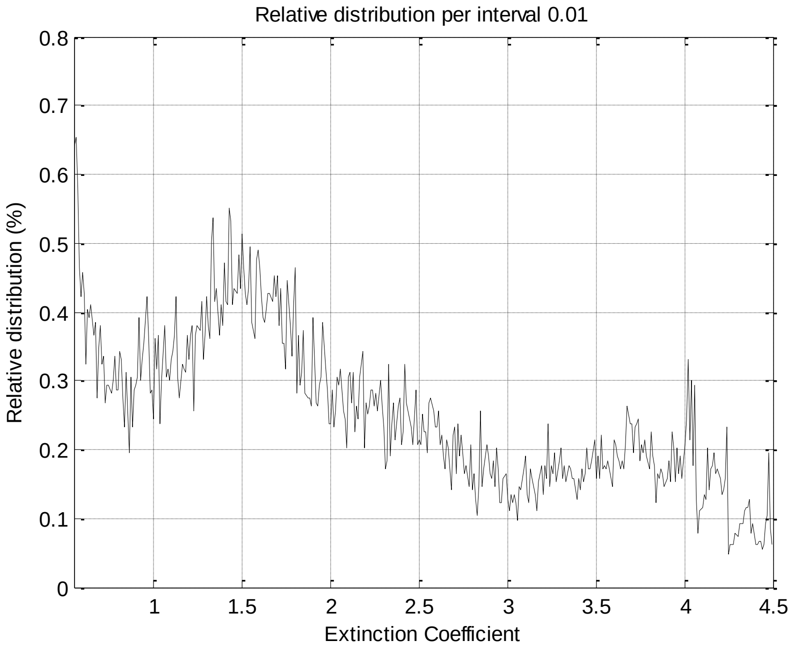

Fig. 15. Relative distribution for extinction coefficient per interval 0.01. Interval 0.55 – 4.5.

5.2 Mode 1. No clouds affect the solar radiation.

In order to get a statistical description of the distribution for the extinction coefficient, the Gaussian distribution has been found to be of special interest. Fig. 16 shows the relative distribution of extinction coefficients compared with a perfect Gaussian distribution. The μ-value is the mean between 0.31 – 0.55. The standard deviation σ, is 0.048. This standard deviation implies that the band limits 0.31 and 0.55 correspond to distances 2.5×σ relative to μ. This means that 98.76 % of the area is within the band limits.

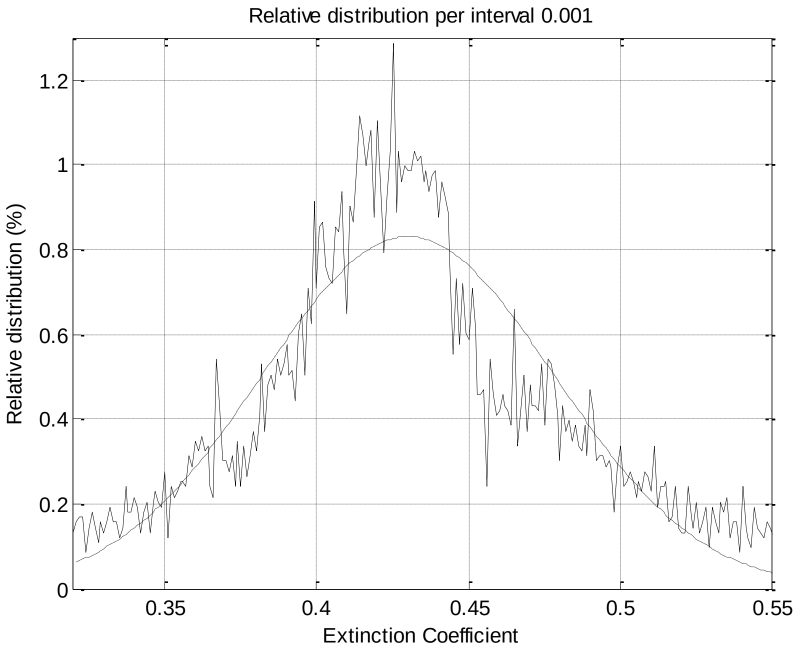

Fig. 16. Measured values compared with a perfect Gaussian distribution.
Solid curve: measured values. Dashed curve: Gaussian distribution.

5.3 Mode 2. Clouds affect the solar radiation.

In order to get a statistical description of the distribution for the extinction coefficient, the trapezoidal distribution has been found to be of special interest. Fig. 17 shows the relative distribution of extinction coefficients compared with a trapezoidal distribution. The trapezoidal distribution with associated parameters is illustrated in Fig. 18. The location of the trapezoidal line between p(A) and p(B) is a good adaptation to the measured values.

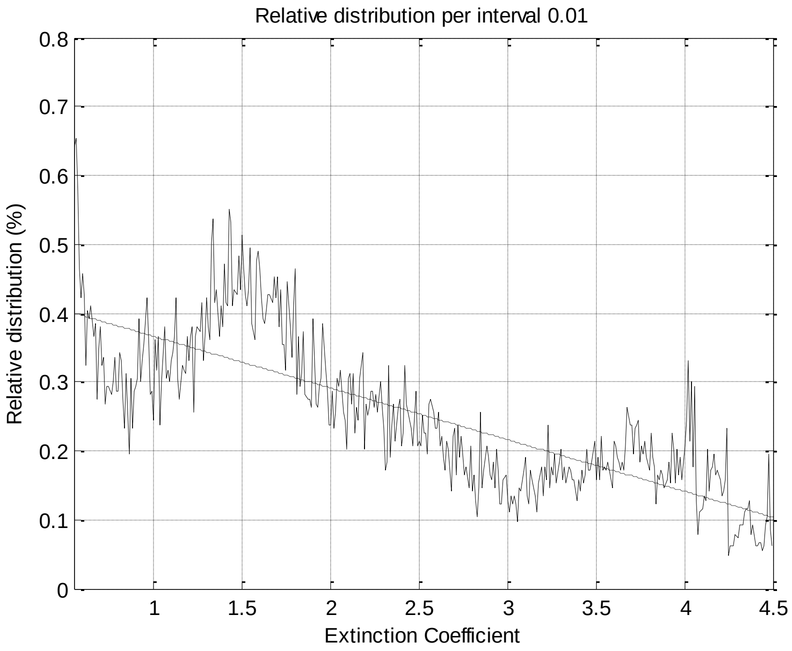

Fig. 17. Measured values compared with a trapezoidal distribution.
Solid curve: measured values. Dashed curve: trapezoid distribution.

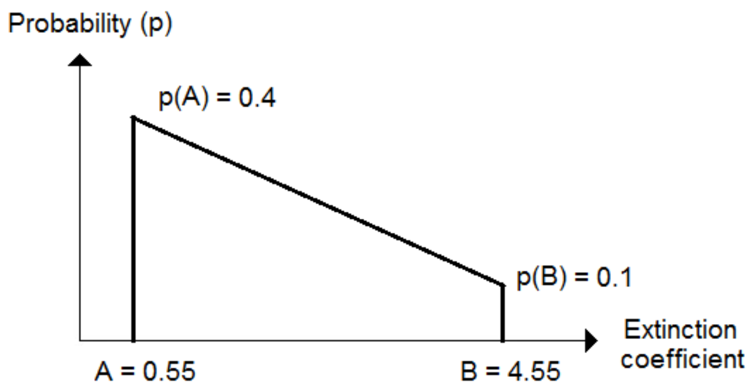

Fig. 18. The Trapezoidal distribution with associated parameters.

5.4 Mode 1 and Mode 2 in combination

During the simulation, it is appropriate to use a combination of Mode 1 and Mode 2. This is done as follows:

- Simulation according mod 1 during time interval t1
- Simulation according mod 2 during time interval t2

Equ. 13:

$$t1 / t2 = \frac{100}{p(cloud)} - 1$$

Where:

t1: Simulation time according to mode 1
t2: Simulation time according to mode 2
p(cloud): Expected cloudiness in percentage

5.5 Statistical Confidence

To get a good statistical confidence of the simulation result, mode 1 and mode 2 shall be updated a number of times. According to [3] this is done 10 times during a total simulation.

5.6 Noise regarding extinction coefficient.

The measured extinction coefficients indicate variations between each measurement sample. This is a result of small variations in atmospheric attenuation of solar radiation. Fig. 19 illustrates the distribution of gradients between all samples during the measurement period. The y-axis gives the density (%). The x-axis represents the corresponding relative difference between samples.

In order to get a statistical description of this noise, the Laplace distribution has been found to be of special interest. Fig. 20 shows a comparison between measured relative differences and a Laplace distribution. The Laplace distribution is defined according to:

Equ. 14:

$$p(x) = \frac{1}{2 \cdot \emptyset} \times \exp\left(- \frac{|x - \theta|}{\emptyset}\right)$$

The parameters have been chosen to:

$\emptyset = 0.0631$

$\theta = 0$

The value of ø corresponds to:

Equ. 15:

$ø = \sigma / \sqrt{2}$

Where: σ: standard deviation of measured relative differences.

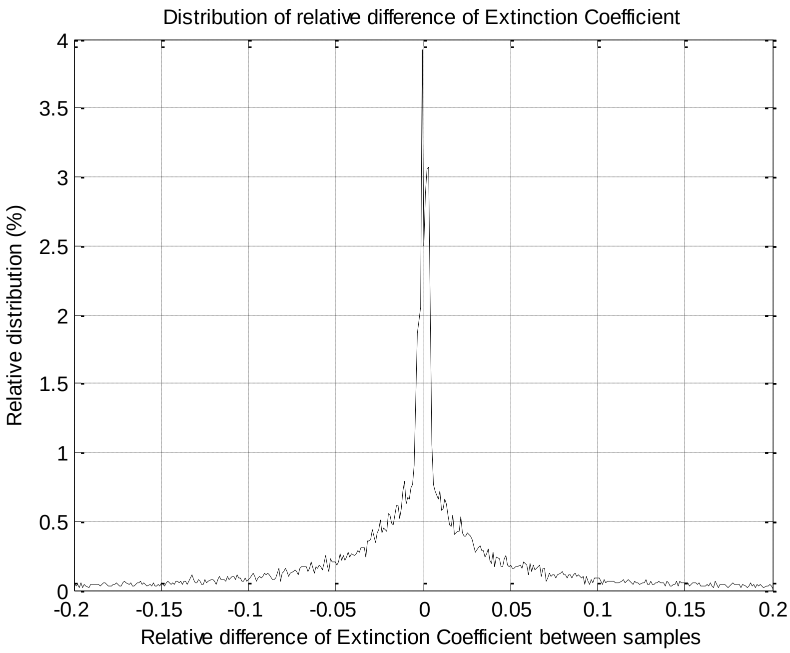

Fig. 19. Distribution of relative difference of Extinction Coefficient between samples.

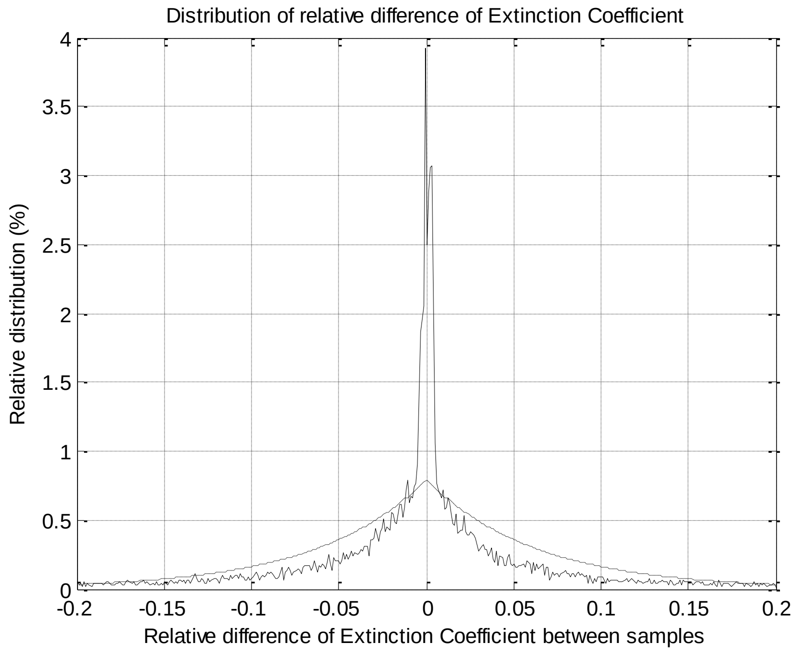

Fig. 20. Measured values compared with a Laplace distribution.
Solid curve: measured values. Dashed curve: Laplace distribution.

5.7 Conclusion of the statistics in simulations

The simulation routines for the extinction coefficient are shown in Fig. 21.

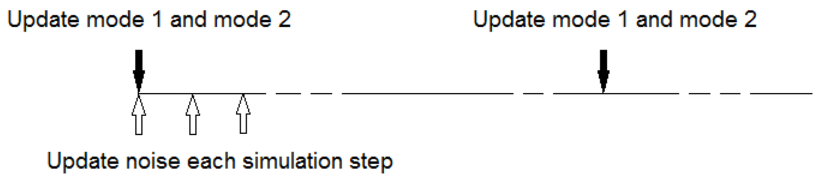

Fig. 21. Simulation routines for the extinction coefficient.

The routines are as follows:

- Updating of mode 1 and mode 2. See 5.2 and 5.3. This is done 10 times during a simulation process, with equal intervals. See 5.5.
- The combination between mode 1 and mode 2 is depending on the expected cloudiness. See 5.4.
- Random drawing regarding noise each simulation step. See 5.6.

6. Example

Fig. 22 and Fig. 23 illustrate the result of a simulation, based on a solar power farm. The simulation conditions are according to Table II.

Table III. Conditions for the exemplified simulation.

Parameter	**Input**
Location	Göteborg, Sweden. Longitude = 11.968° Latitude = 57.710°
Equivalent start date for simulation	June 21
Equivalent stop date for simulation	July 01
Time resolution	10 minutes
Cloudiness	70%
Total effective solar cell area	3000 m^2
Solar cell surface normal. Direction relative south.	0°
Solar cell surface normal. Direction relative zenith.	0°
Solar farm efficiency	14%

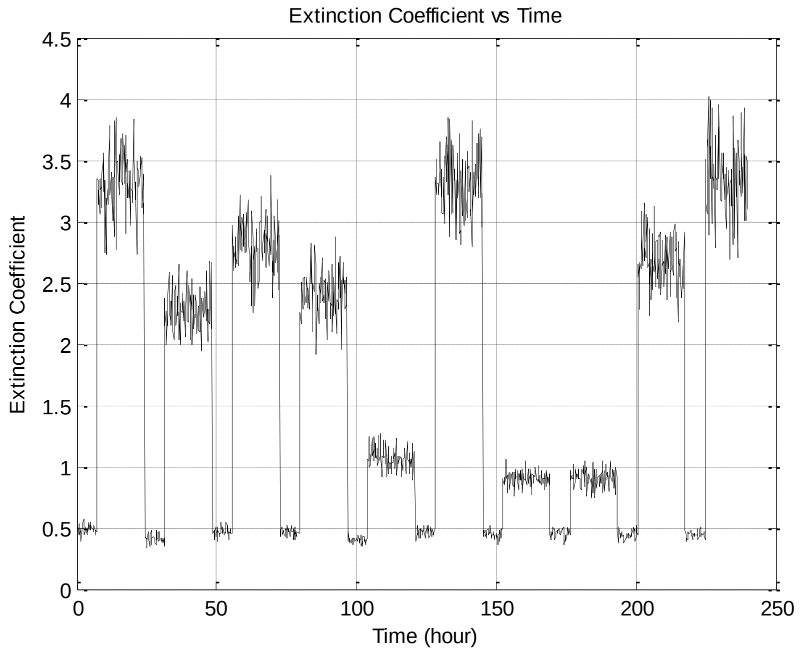

Fig. 22. Simulated extinction coefficients.

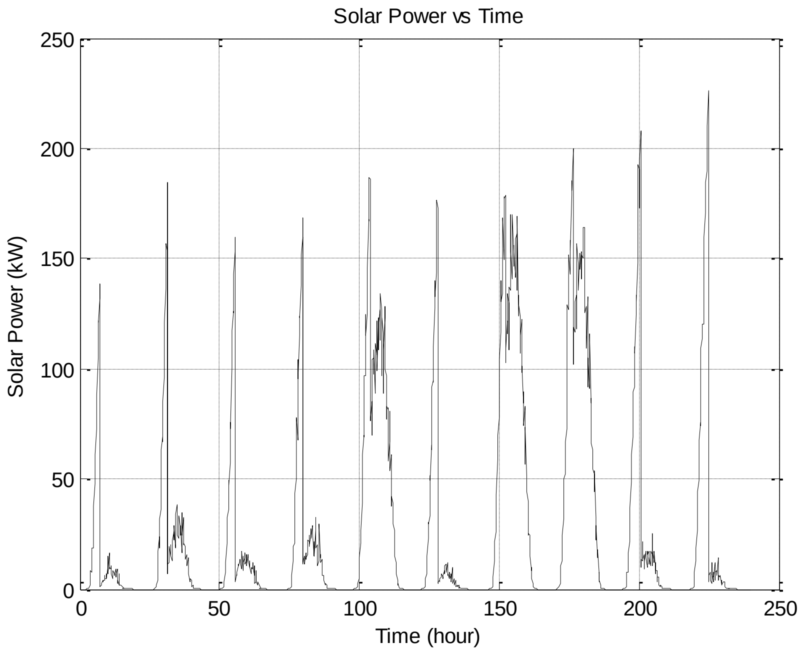

Fig. 23. Generated power from the solar power farm.

7. References

[1] Analysis of Combined Power Systems.General description of software. Chalmers University of Technology, 2006, Ingemar Mathiasson.

[2] Stochastic modeling of Extinction coefficients for solar power applications. Chalmers University of Technology, 2007, Ingemar Mathiasson.

[3] Mathiasson I. “Simulation of Autonomous Electric Power Systems”. Chalmers University of Technology, 2015.