Wind power

Statistical analysis of wind speed

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Content

1 Introduction

This paper deals with the task, to develop and describe a model for statistical simulation, to be used in combination with the simulation program described in /1/. The work is based on measurements according to /2/. The measurements were performed during a period of totally 5 years, from 2008 to 2012. Measurement location was Hönö outside Göteborg, Sweden.

2 Shortly about the measurements

- Location: Hönö outside Göteborg, Sweden.
- Measurement period: 2008 – 2012.
- Level over sea: 18.5 m.
- Used sample frequency: 10 minutes

3 Analysis.

3.1 Low frequency respectively high frequency wind speed components.

The following analysis divides the wind speed into two components:

- Low frequency component (base component)
- High frequency component (noise component)

The low frequency component intends the mean wind speed during time intervals of about 24 hours. While the high frequency component intends the wind turbulence (noise) that affect the wind speed during time intervals in the region of minutes.

Equ. 1:

$V = V_B + V_N$

Equ. 2:

$V_N = V_B \times C_W$

Where:

V: Total wind speed
V_B: Low frequency component (base component)
V_N: High frequency component (noise component)
C_W: Constant. See section 3.3.

3.2 Low frequency component of wind speed.

3.2.1 Basic principle

Regarding the low frequency component of wind speed, focus has been to study the Weibull distribution function. This is defined according to:

Equ. 3:

$$W(A,C) = \frac{C}{A}\left(\frac{V_B}{A}\right)^{C-1} e^{-\left(\frac{V_B}{A}\right)^C}$$

Where: V_B: Low frequency component of wind speed
A,C: Weibull parameters

The Weibull parameters have been investigated based on the measurement results for the four seasons winter, spring summer and autumn and for the total year (all seasons).

3.2.2 Weibull parameters. Winter.

Fig. 1 - Fig. 6 illustrate comparisons between measurements of wind speed and varied choice of Weibull parameters. The following parameter combination have been tested:

Table I. Choice of Weibull parameters.

A	C
5.9	1.8
5.9	1.9
6.0	1.8
6.0	1.9
6.1	1.8
6.1	1.9

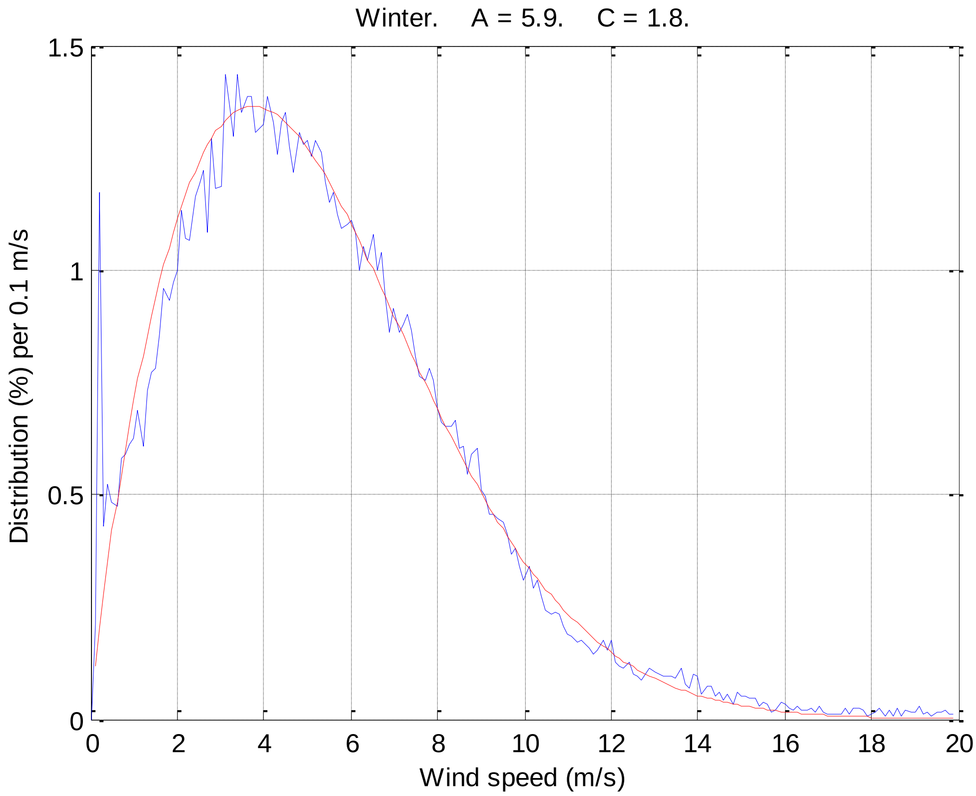

Fig. 1. Comparison between measurement and Weibull distribution with A = 5.9, C = 1.8.

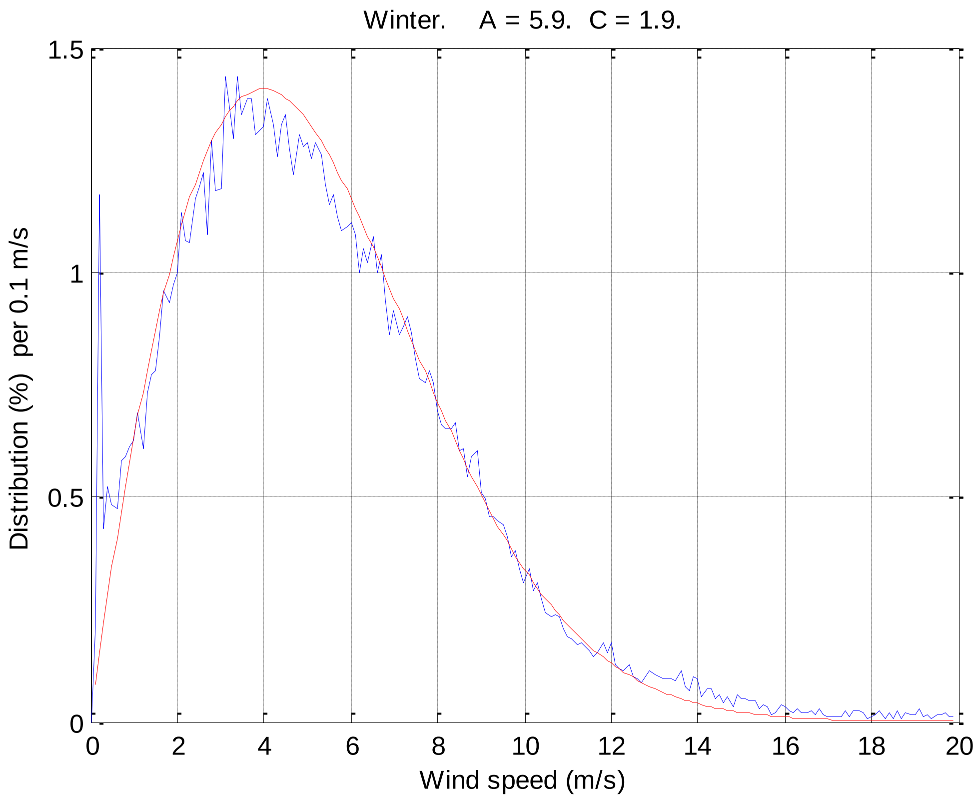

Fig. 2. Comparison between measurement and Weibull distribution with A = 5.9, C = 1.9.

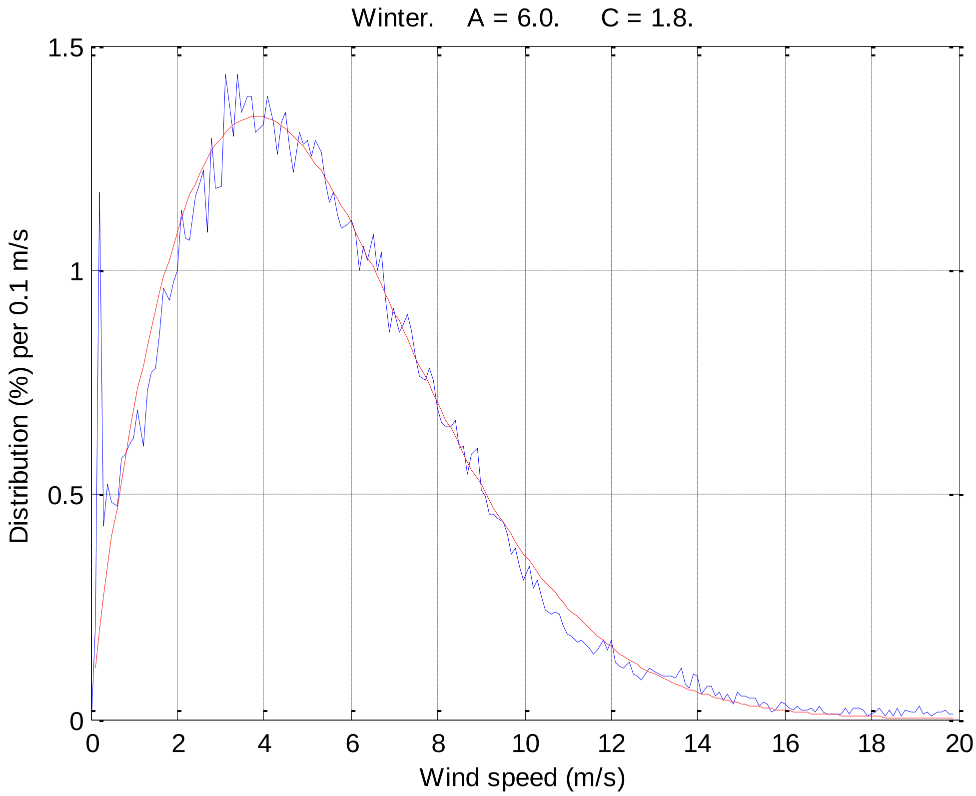

Fig. 3. Comparison between measurement and Weibull distribution with A = 6.0, C = 1.8.

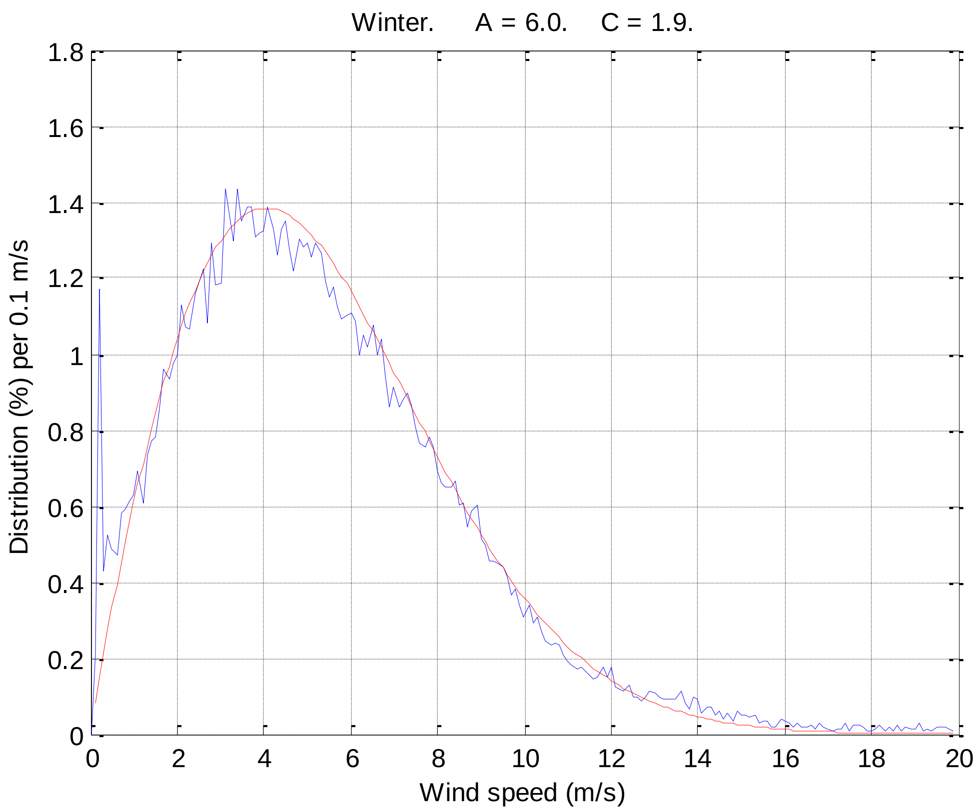

Fig. 4. Comparison between measurement and Weibull distribution with A = 6.0, C = 1.9.

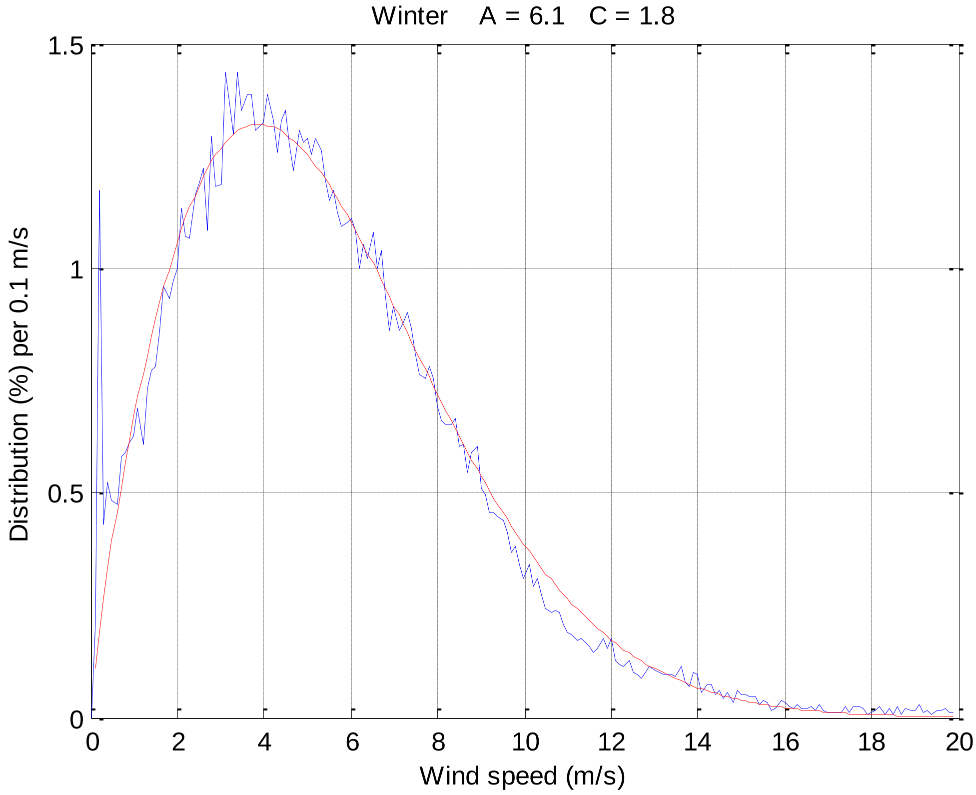

Fig. 5. Comparison between measurement and Weibull distribution with A = 6.1, C = 1.8.

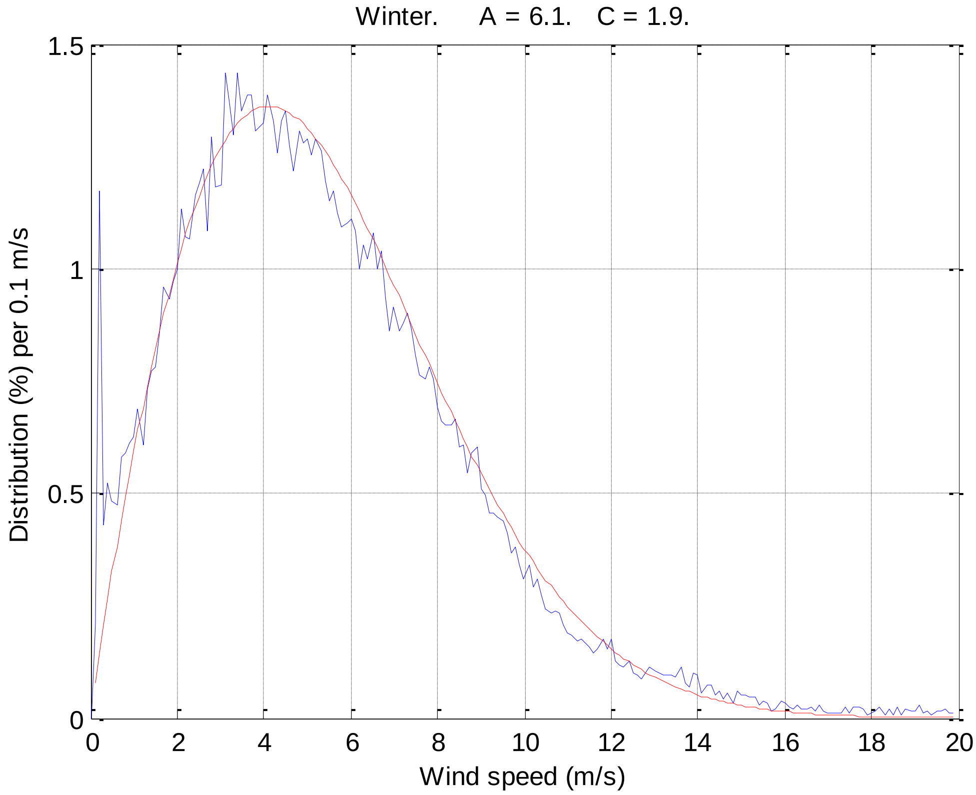

Fig. 6. Comparison between measurement and Weibull distribution with A = 6.1, C = 1.9.

3.2.3 Weibull parameters. Spring.

Fig. 7 Fig. 12 illustrate comparisons between measurements of wind speed and varied choice of Weibull parameters. The following parameter combination have been tested:

Table II. Choice of Weibull parameters.

A	C
5.6	2.0
5.6	2.1
5.7	2.0
5.7	2.1
5.8	2.0
5.8	2.1

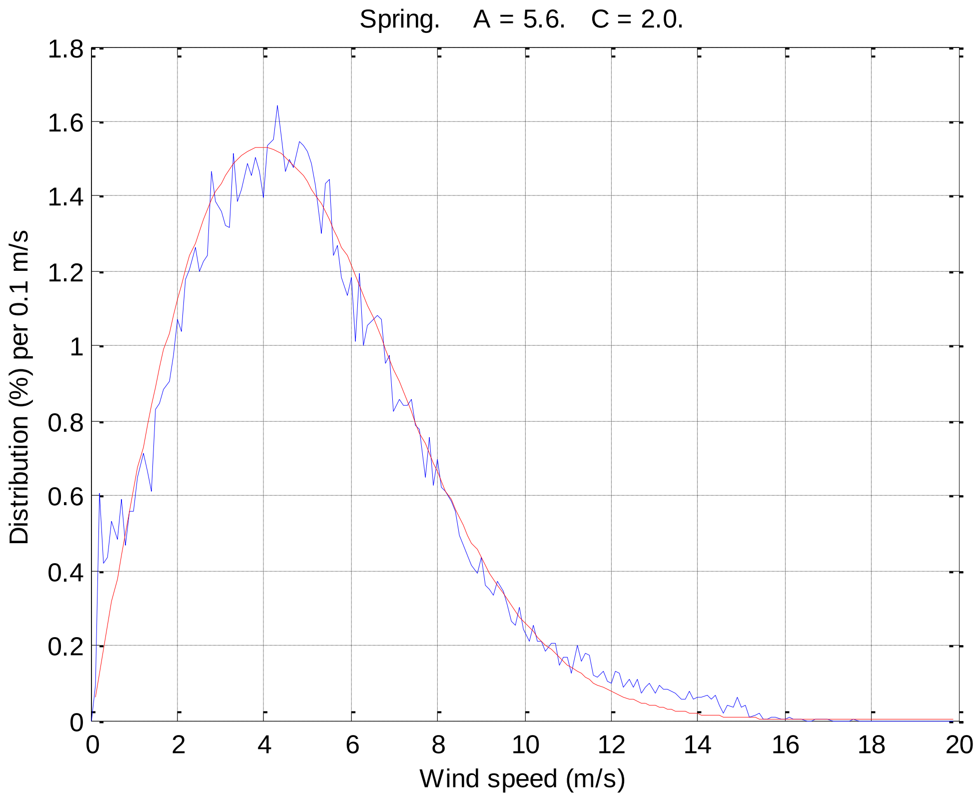

Fig. 7. Comparison between measurement and Weibull distribution with A = 5.6, C = 2.0.

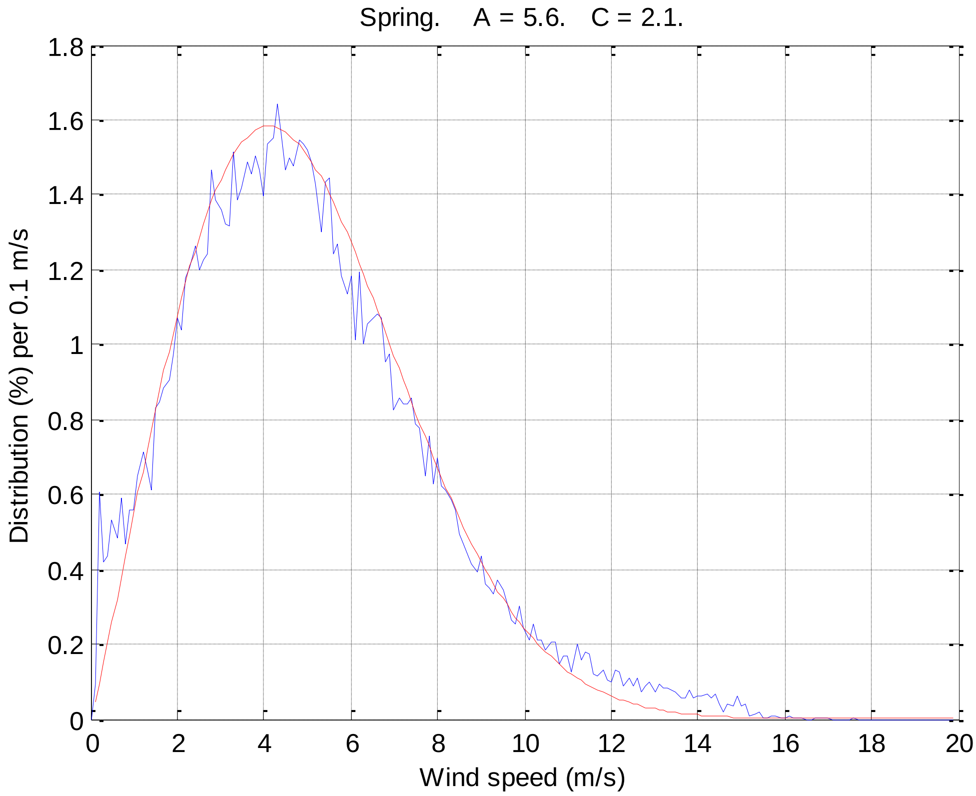

Fig. 8. Comparison between measurement and Weibull distribution with A = 5.6, C = 2.1.

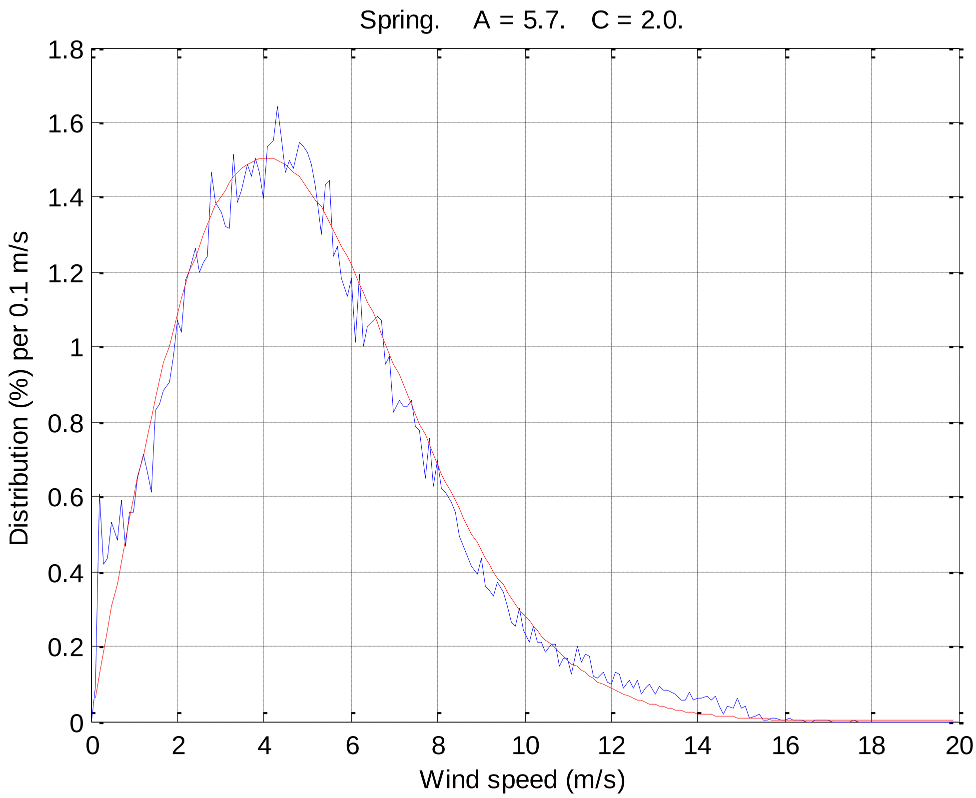

Fig. 9. Comparison between measurement and Weibull distribution with A = 5.7, C = 2.0.

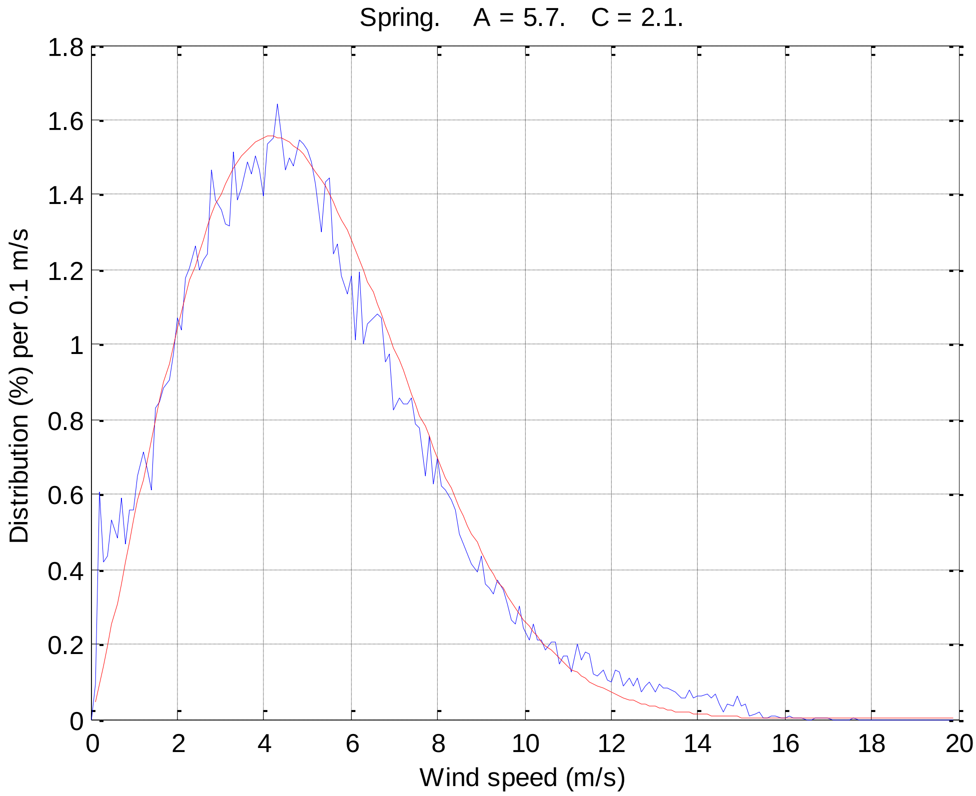

Fig. 10. Comparison between measurement and Weibull distribution with A = 5.7, C = 2.1.

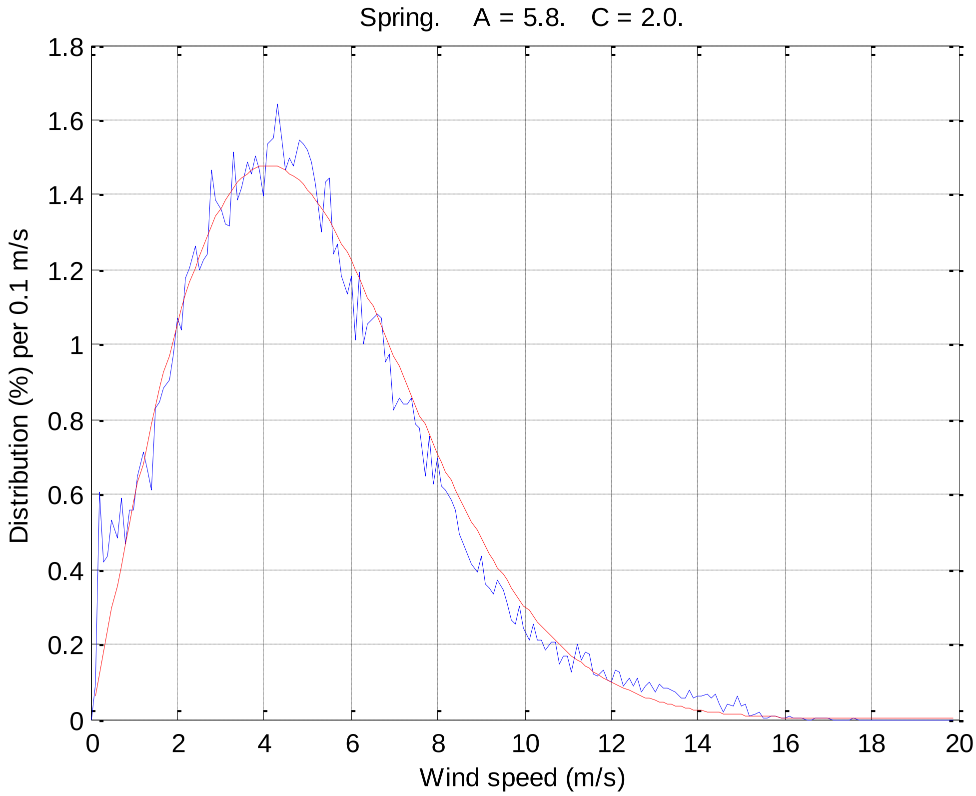

Fig. 11. Comparison between measurement and Weibull distribution with A = 5.8, C = 2.0.

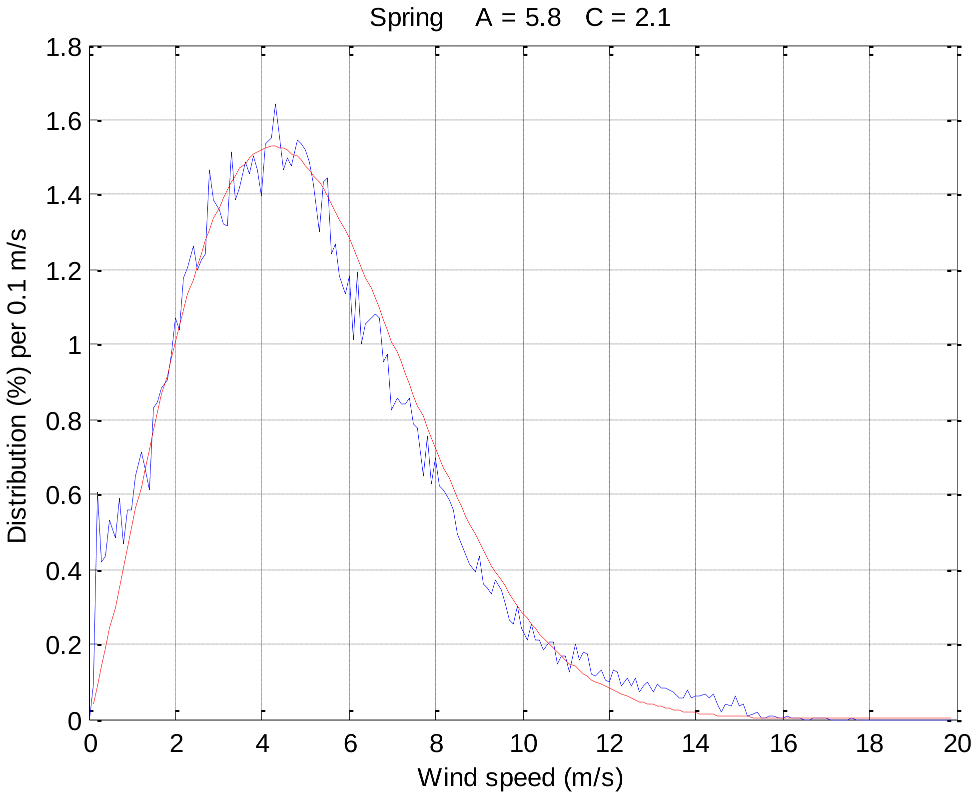

Fig. 12. Comparison between measurement and Weibull distribution with A = 5.8, C = 2.1.

3.2.4 Weibull parameters. Summer.

Fig. 13 - Fig. 18 illustrate comparisons between measurements of wind speed and varied choice of Weibull parameters. The following parameter combination have been tested:

Table III. Choice of Weibull parameters.

A	**C**
5.8	2.0
5.8	2.1
5.9	2.0
5.9	2.1
6.0	2.0
6.0	2.1

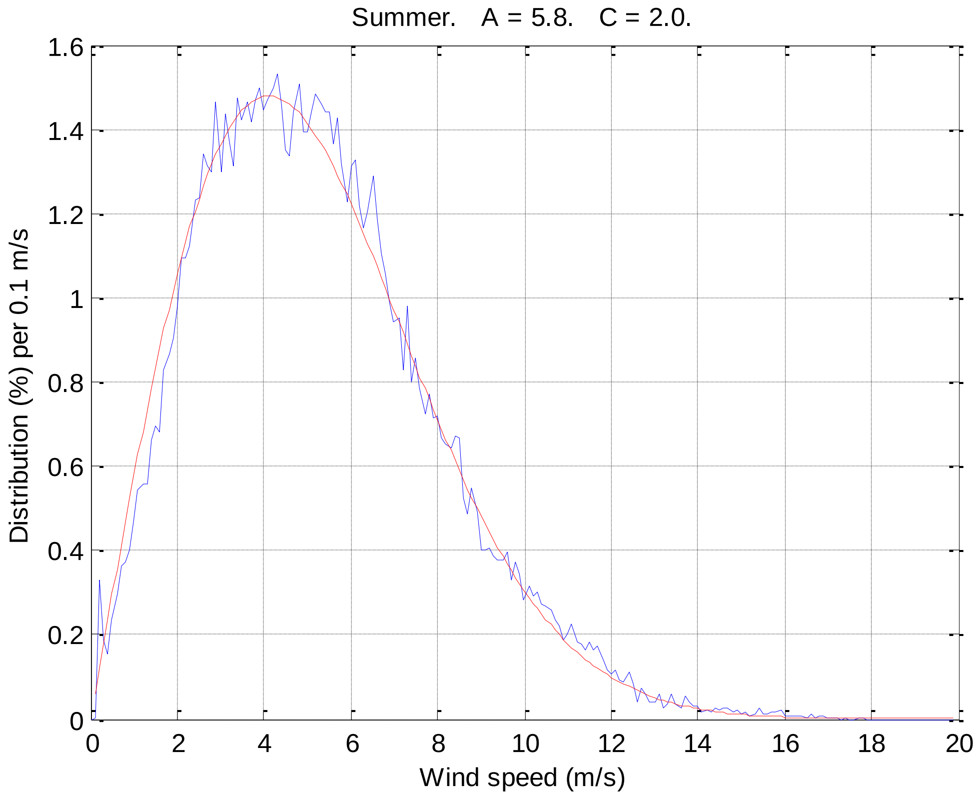

Fig. 13. Comparison between measurement and Weibull distribution with A = 5.8, C = 2.0.

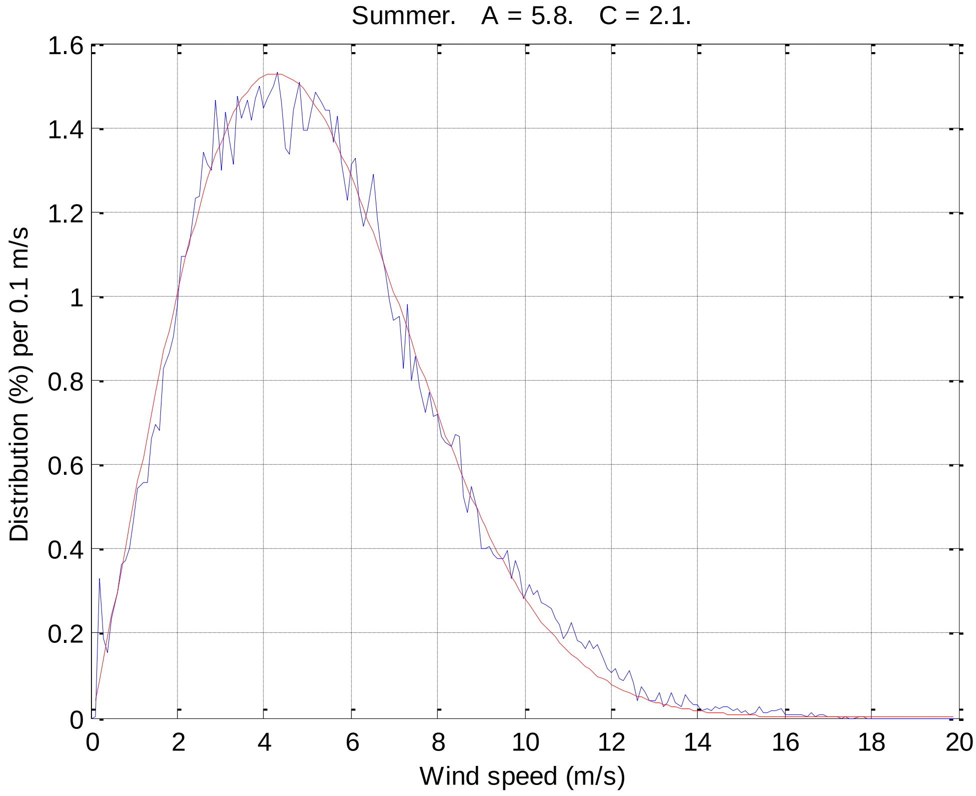

Fig. 14. Comparison between measurement and Weibull distribution with A = 5.8, C = 2.1.

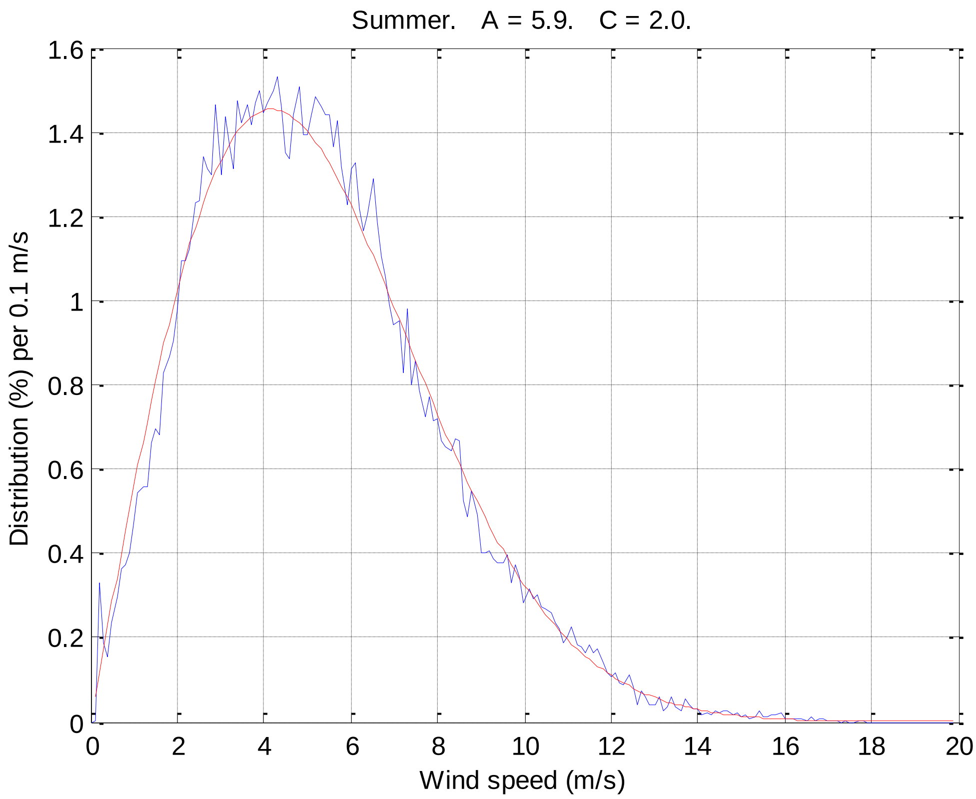

Fig. 15. Comparison between measurement and Weibull distribution with A = 5.9, C = 2.0.

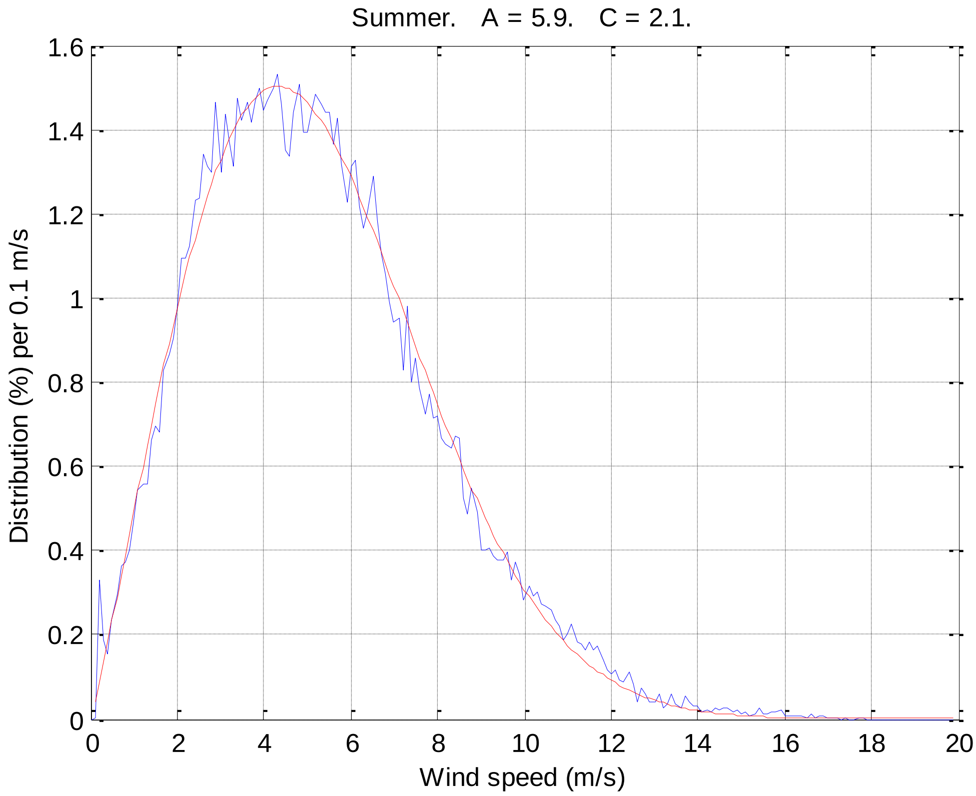

Fig. 16. Comparison between measurement and Weibull distribution with A = 5.9, C = 2.1.

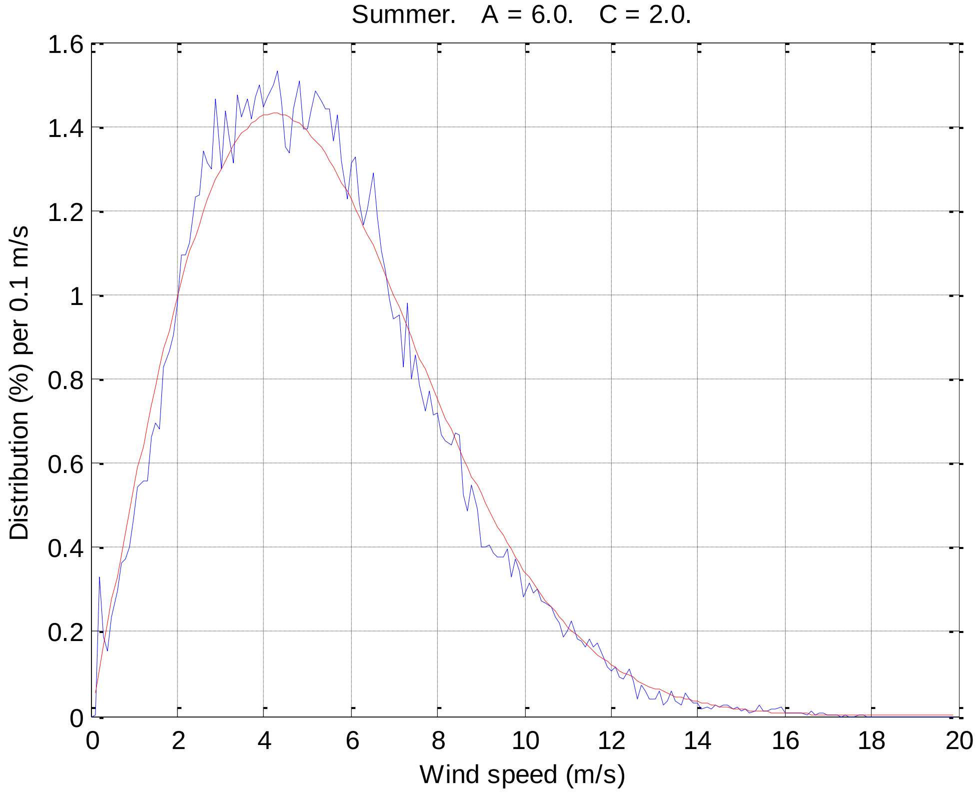

Fig. 17. Comparison between measurement and Weibull distribution with A = 6.0, C = 2.0.

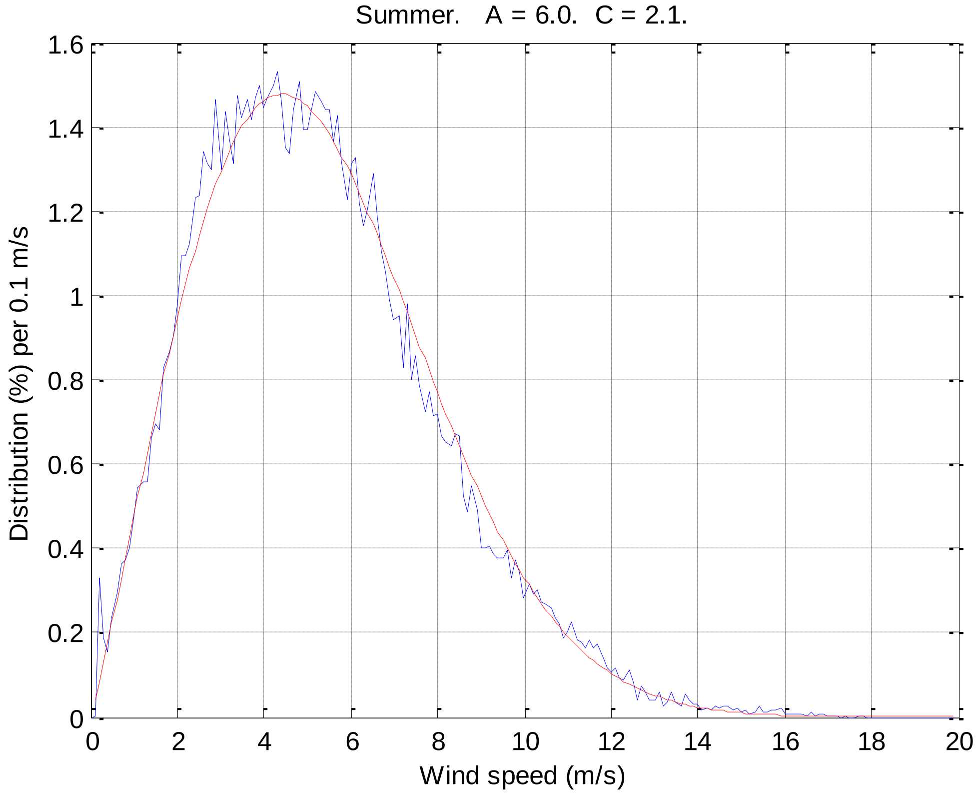

Fig. 18. Comparison between measurement and Weibull distribution with A = 6.0, C = 2.1.

3.2.5 Weibull parameters. Autumn.

Fig. 19 - Fig. 24 illustrate comparisons between measurements of wind speed and varied choice of Weibull parameters. The following parameter combination have been tested:

Table IV. Choice of Weibull parameters.

A	C
7.5	1.9
7.5	2.0
7.6	1.9
7.6	2.0
7.7	1.9
7.7	2.0

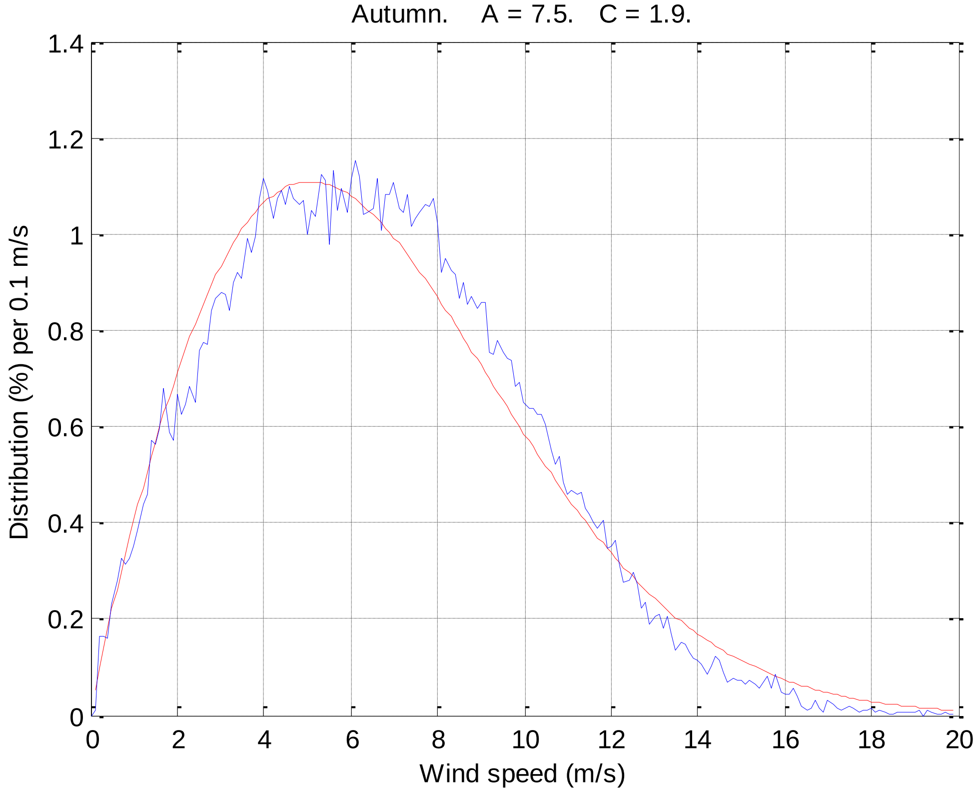

Fig. 19. Comparison between measurement and Weibull distribution with A = 7.5, C = 1.9.

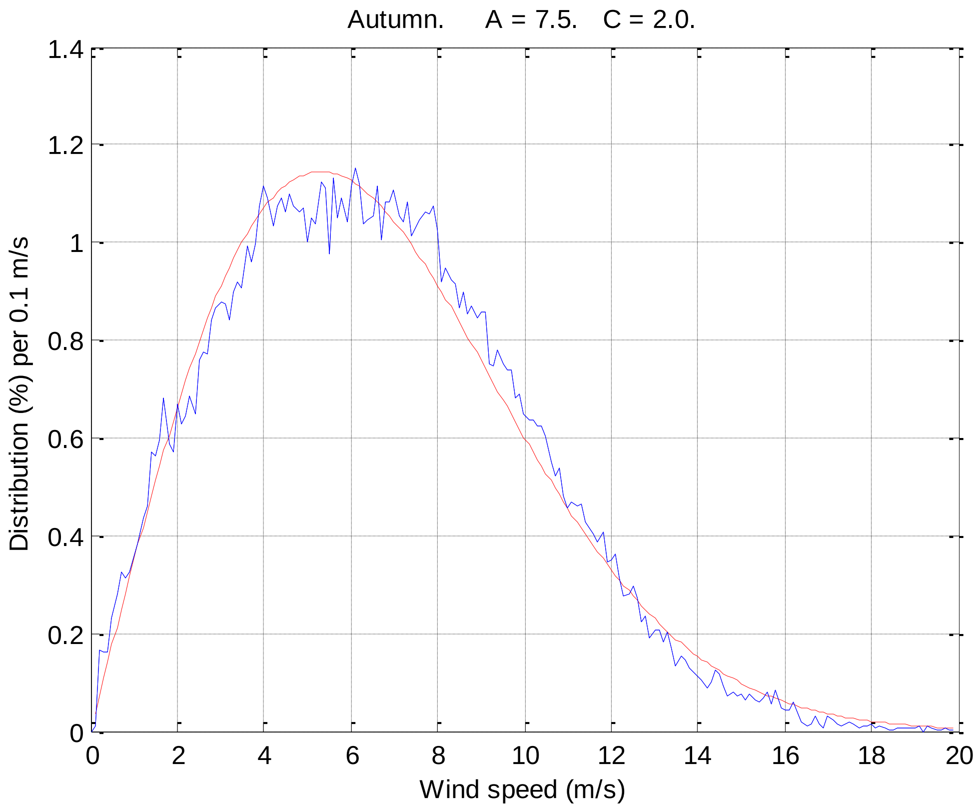

Fig. 20. Comparison between measurement and Weibull distribution with A = 7.5, C = 2.0.

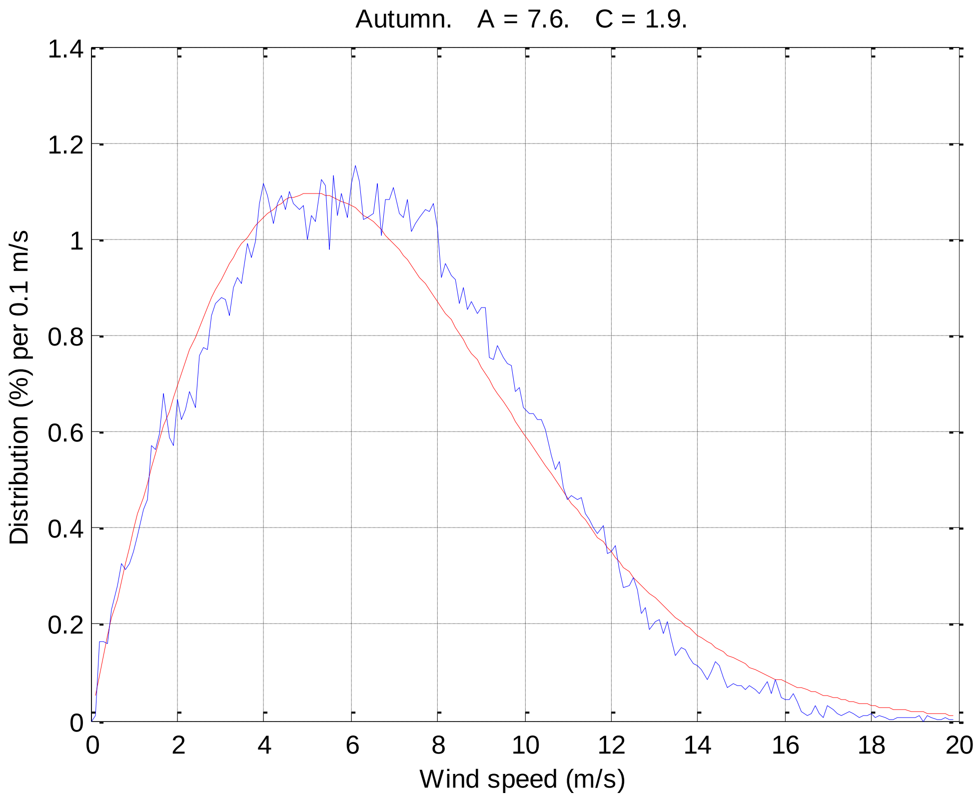

Fig. 21. Comparison between measurement and Weibull distribution with A =7.6, C = 1.9.

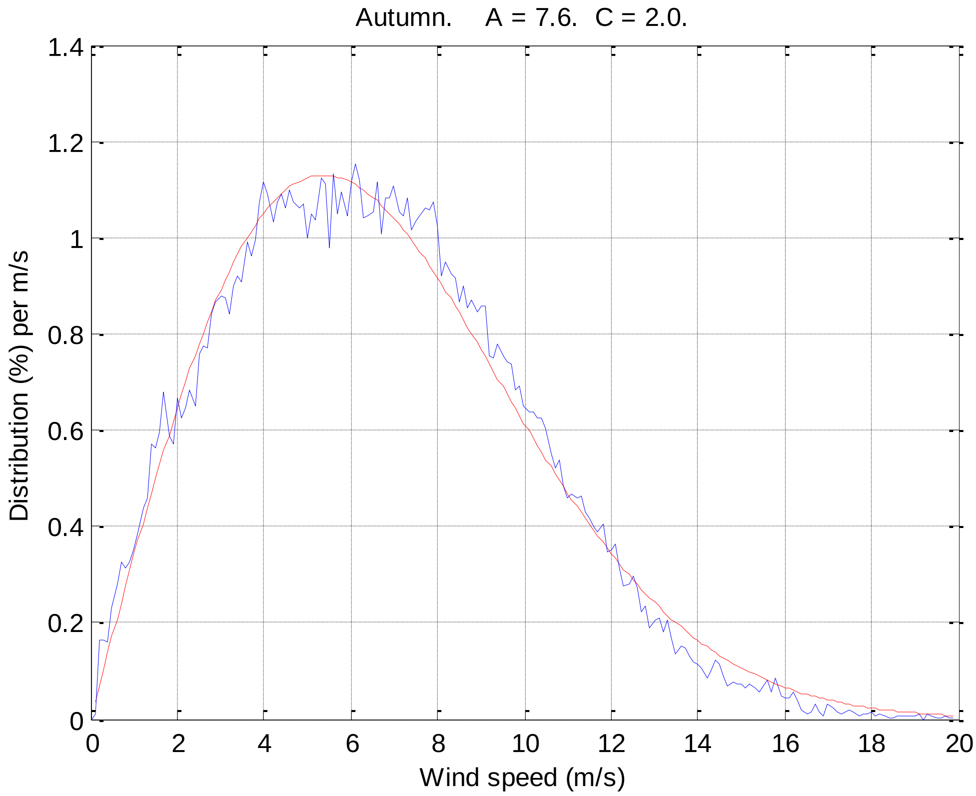

Fig. 22. Comparison between measurement and Weibull distribution with A = 7.6, C = 2.0.

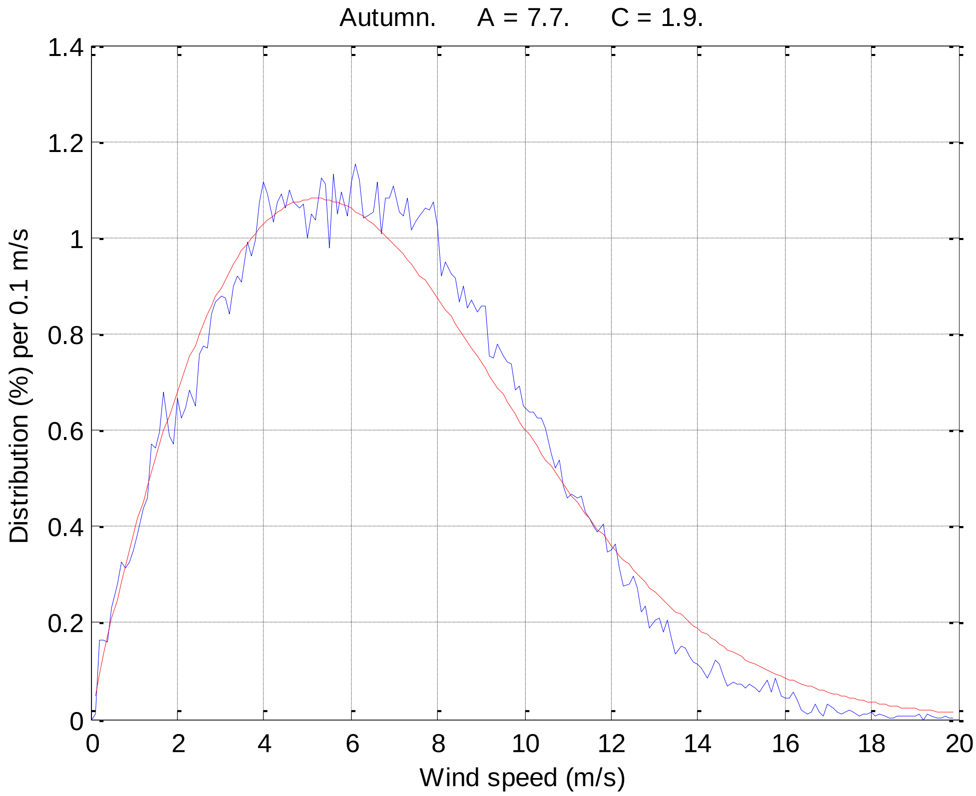

Fig. 23. Comparison between measurement and Weibull distribution with A = 7.7, C = 1.9.

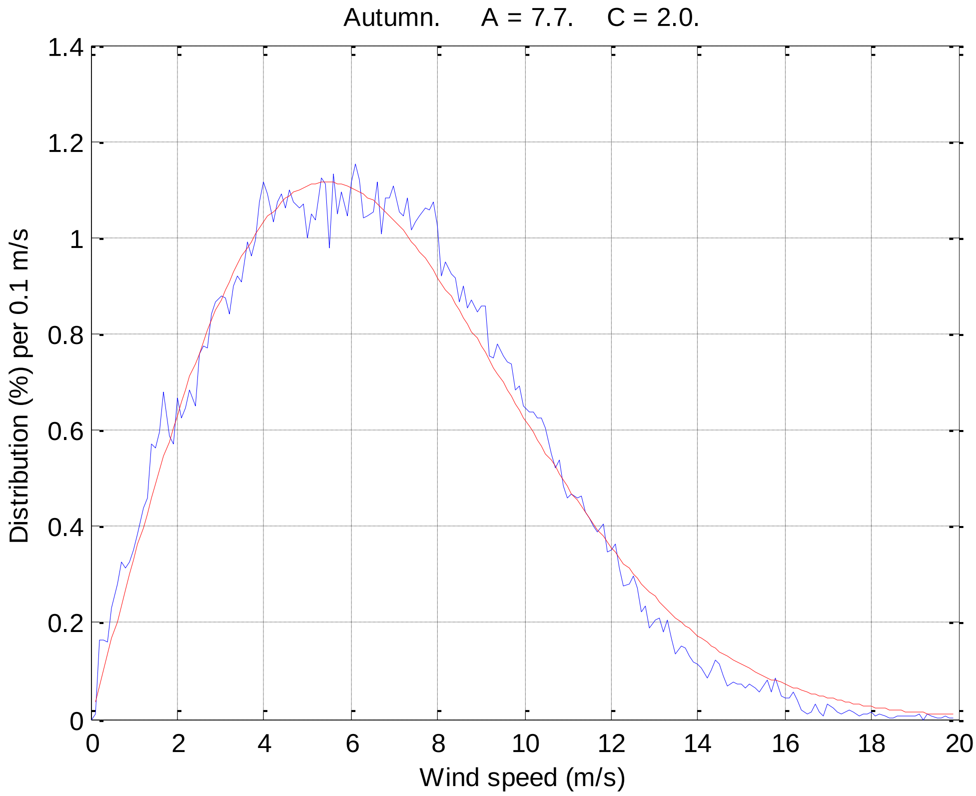

Fig. 24. Comparison between measurement and Weibull distribution with A = 7.7, C = 2.0.

3.2.6 Weibull parameters. Total year (all seasons).

Fig. 25 - Fig. 30 illustrate comparisons between measurements of wind speed and varied choice of Weibull parameters for all seasons. The following parameter combination have been tested:

Table V. Choice of Weibull parameters.

A	**C**
6.1	1.9
6.1	2.0
6.2	1.9
6.2	2.0
6.3	1.9
6.3	2.0

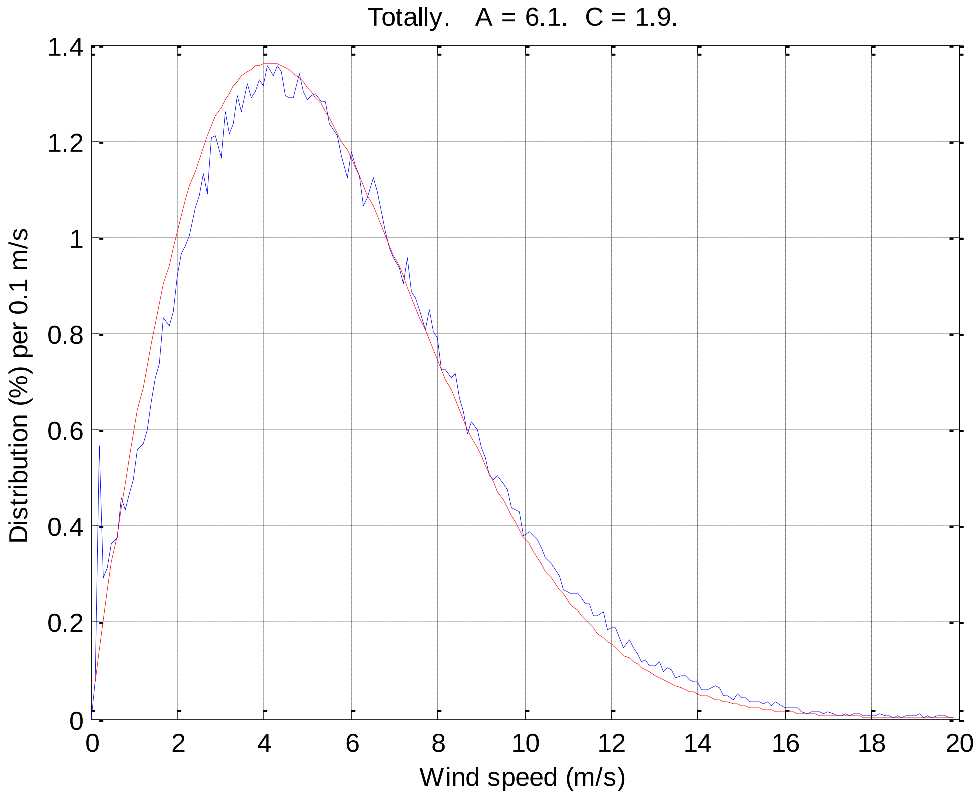

Fig. 25. Comparison between measurement and Weibull distribution with A = 6.1, C = 1.9.

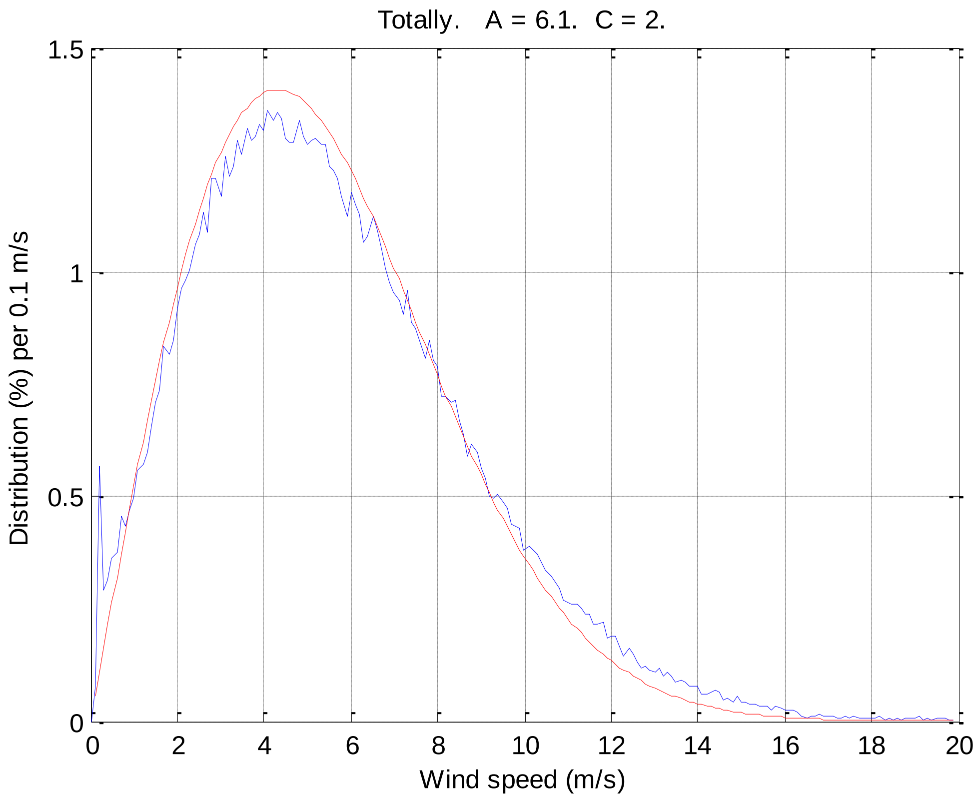

Fig. 26. Comparison between measurement and Weibull distribution with A = 6.1, C = 2.0.

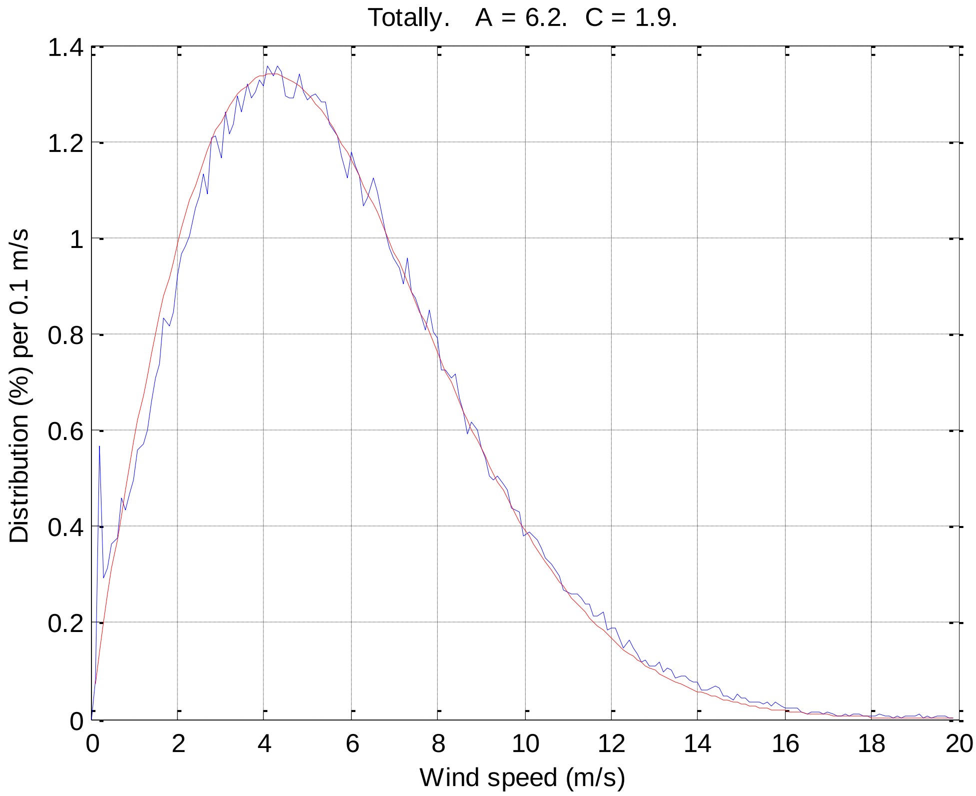

Fig. 27. Comparison between measurement and Weibull distribution with A = 6.2, C = 1.9.

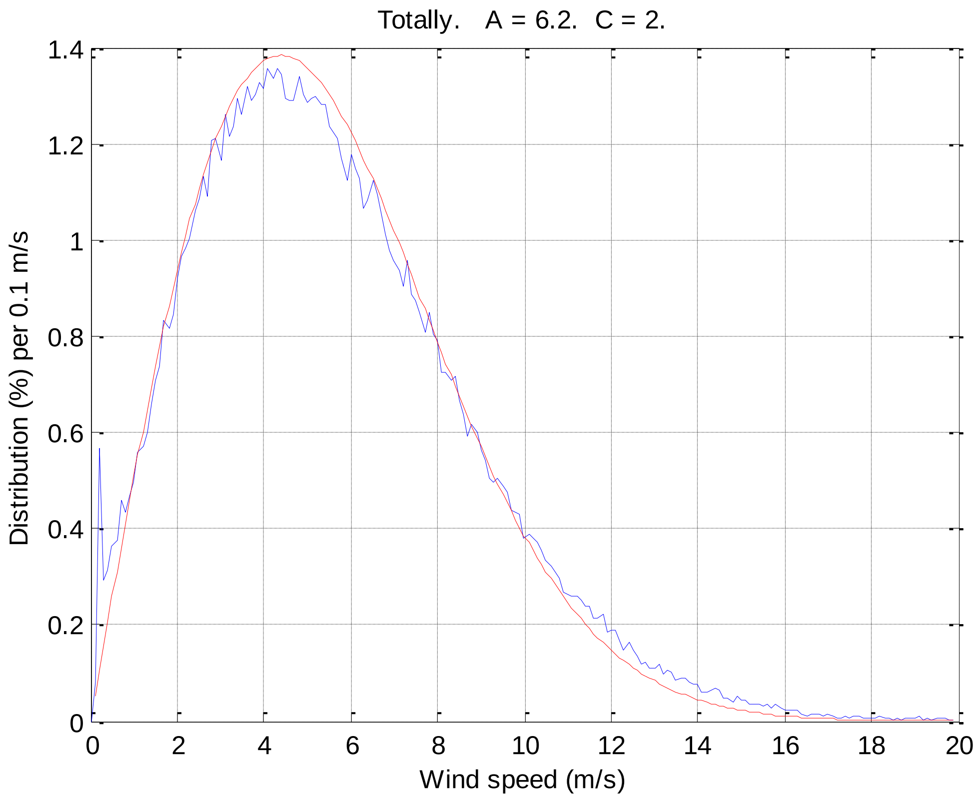

Fig. 28. Comparison between measurement and Weibull distribution with A = 6.2, C = 2.0.

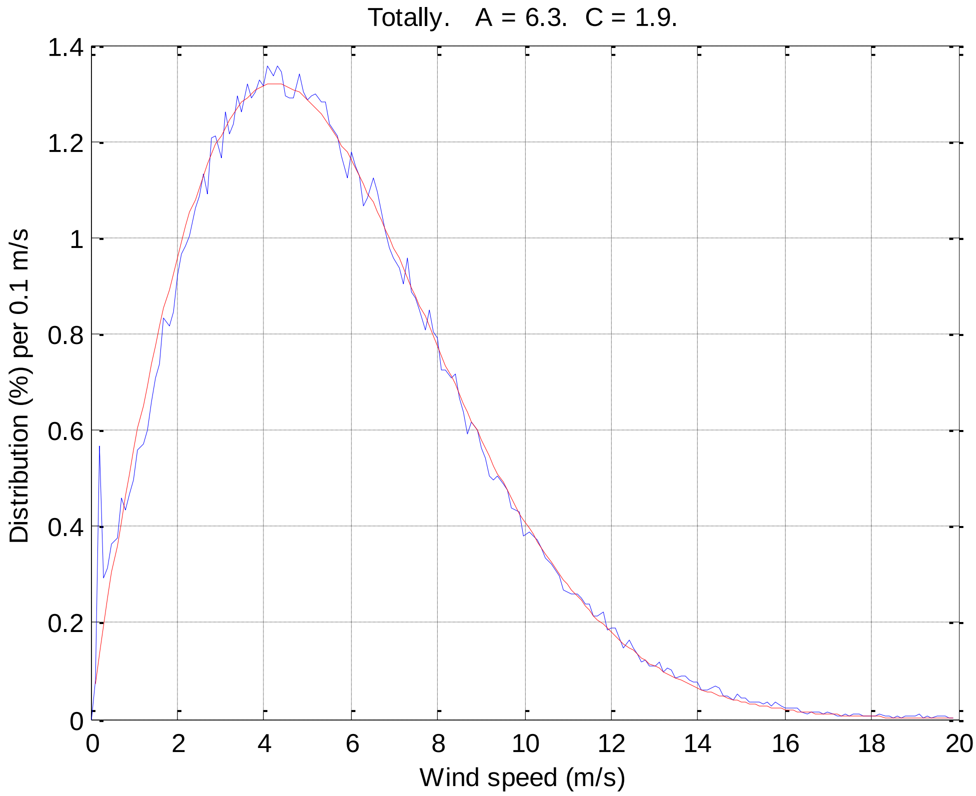

Fig. 29. Comparison between measurement and Weibull distribution with A = 6.3, C = 1.9.

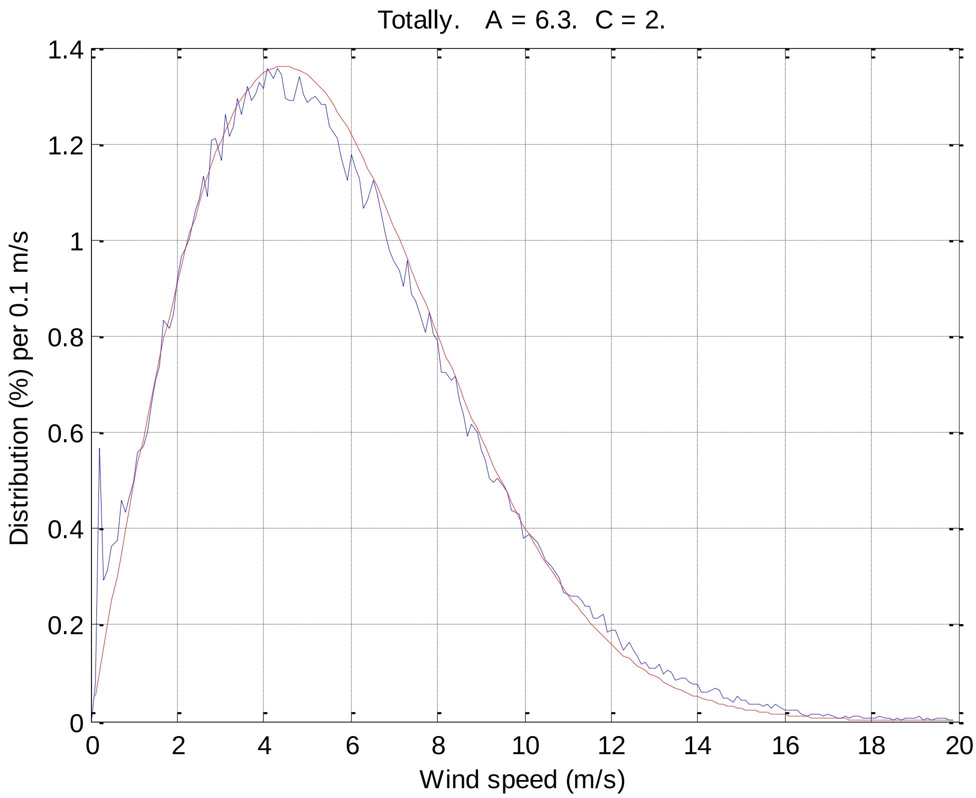

Fig. 30. Comparison between measurement and Weibull distribution with A = 6.3, C = 2.0.

3.2.7 Choice of Weibull parameters

After comparisons of statistical distribution between measured wind speeds and Weibull distribution, the parameters according to Table VI is recommended. These values result in a good agreement to the measured values over the total speed region in question.

Table VI. Weibull parameters for seasons and total year.

Season and Total year	**Weibull parameter**	
	A	**C**
Winter	5.9	1.8
Spring	5.7	2.0
Summer	5.9	2.0
Autumn	7.7	2.0
Total year	6.3	1.9

3.3 **High frequency component of wind speed**

3.3.1 Basic principle

According to section 3, the following relationship is defined:

$V = V_B + V_N$

$V_N = V_B \times C_W = V_B \times (1+C_W)$

Where:

V: Total wind speed
V_B: Low frequency component (base component)
V_N: High frequency component (noise component)
C_W: Constant (positive or negative)

Regarding CW, focus has in this study been to study and use the Laplace distribution function. This function is defined according to:

Equ. 4:

$$p(x) = \frac{1}{2 \cdot \emptyset} \times \exp\left(-\frac{|x-\theta|}{\emptyset}\right)$$

Where. ø and θ are Laplace parameters.

θ is the mean value and is put to zero. Fig. 31 gives an example of a Laplace distribution.

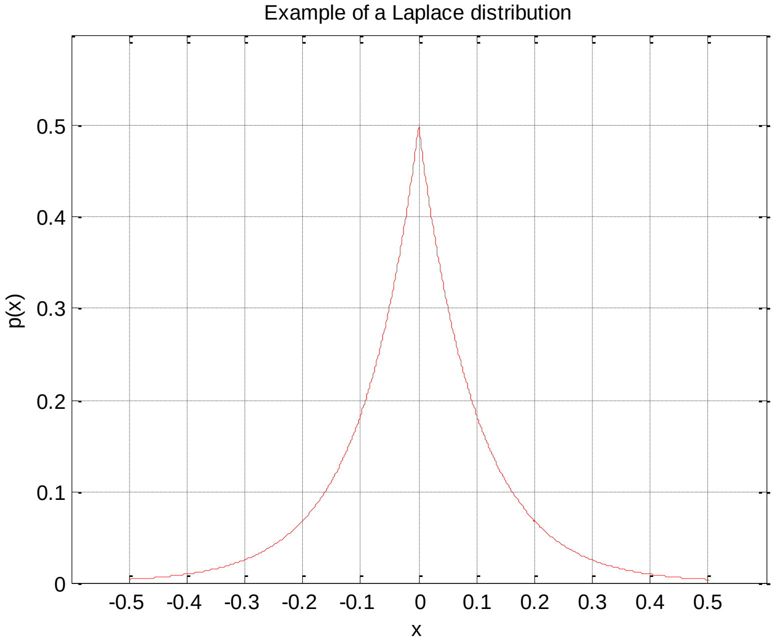

Fig. 31. Example of a Laplace distribution with ø = 0.1 and θ = 0.

The best value of ø, to get an adaptation of the Laplace distribution related to the measured wind speed values is according to:

Equ. 5:

$ø = \sigma(G) / \sqrt{2}$ See e.g. /3/.

Where:

G: Vector with relative gradients of measured wind speeds
σ(g): Standard deviation of vector G

The relative gradients are here defined according to:

Equ. 6:

$$G(n) = \frac{V(n) - V(n+1)}{V(n)}$$

Where:

n: Measure sample. $1 \leq n \leq N - 1$
N: Number of measure samples
V(n): Wind rate measure for sample n

3.3.2 Laplace parameters

The values for seasons and total year (all seasons) regarding σ(g) and ø are presented in Table VII. ø is calculated according to (5) and (6).

Table VII. σ(g) and ø for seasons and total year.

Season and Total year	σ(g)	ø
Winter	0.2398	0.1696
Spring	0.2067	0.1462
Summer	0.1908	0.1349
Autumn	0.1879	0.1329
Total year	0.2062	0.1458

Fig. 32 - Fig. 36 illustrate comparisons between the distribution of measured gradients and the Laplace distribution with ø-values according to Table VII.

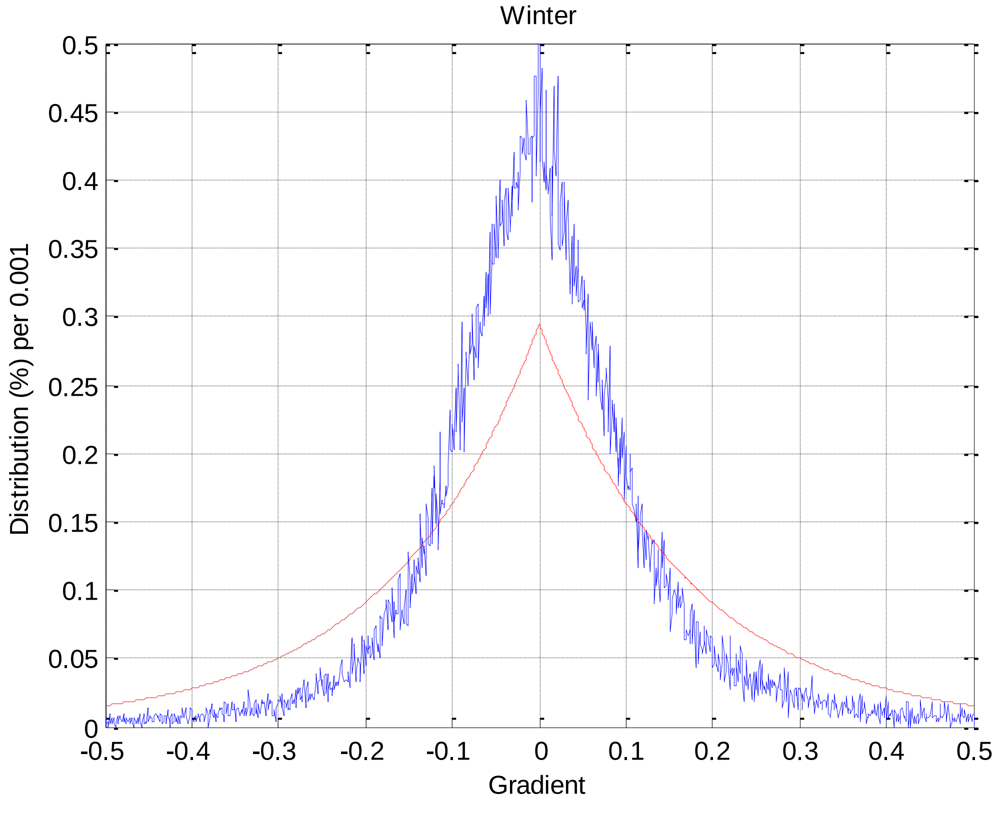

Fig. 32. Comparison between measurement and Laplace distribution with ø = 0.1696.

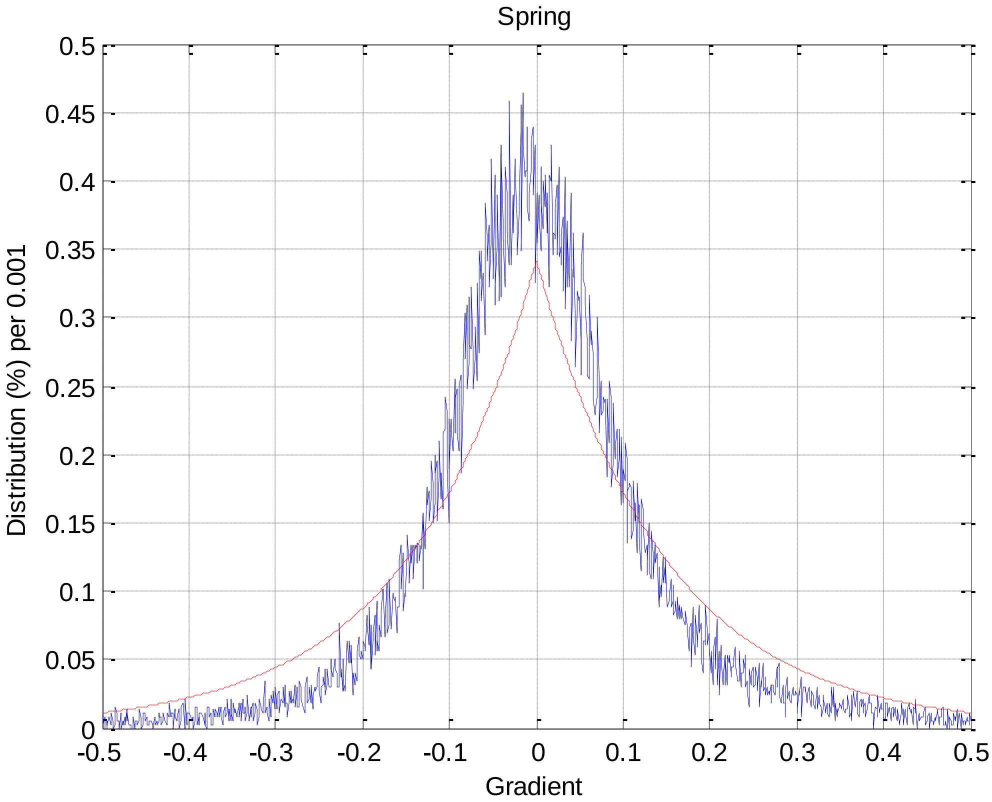

Fig. 33. Comparison between measurement and Laplace distribution with ø = 0.1462.

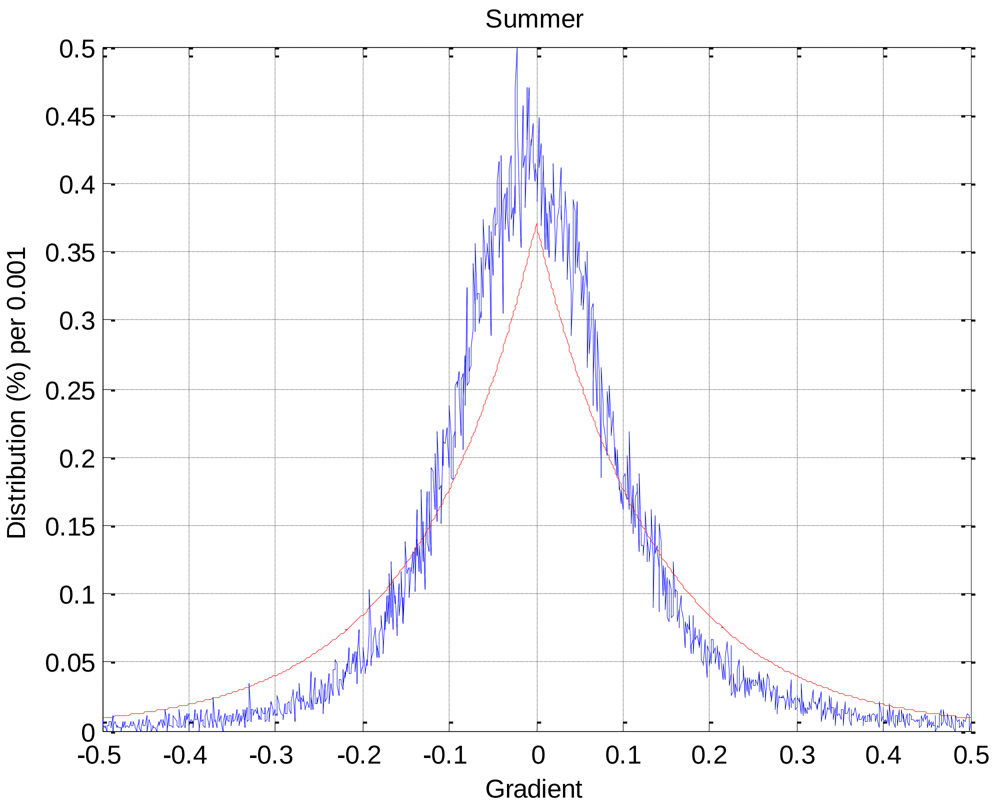

Fig. 34. Comparison between measurement and Laplace distribution with ø = 0.1349.

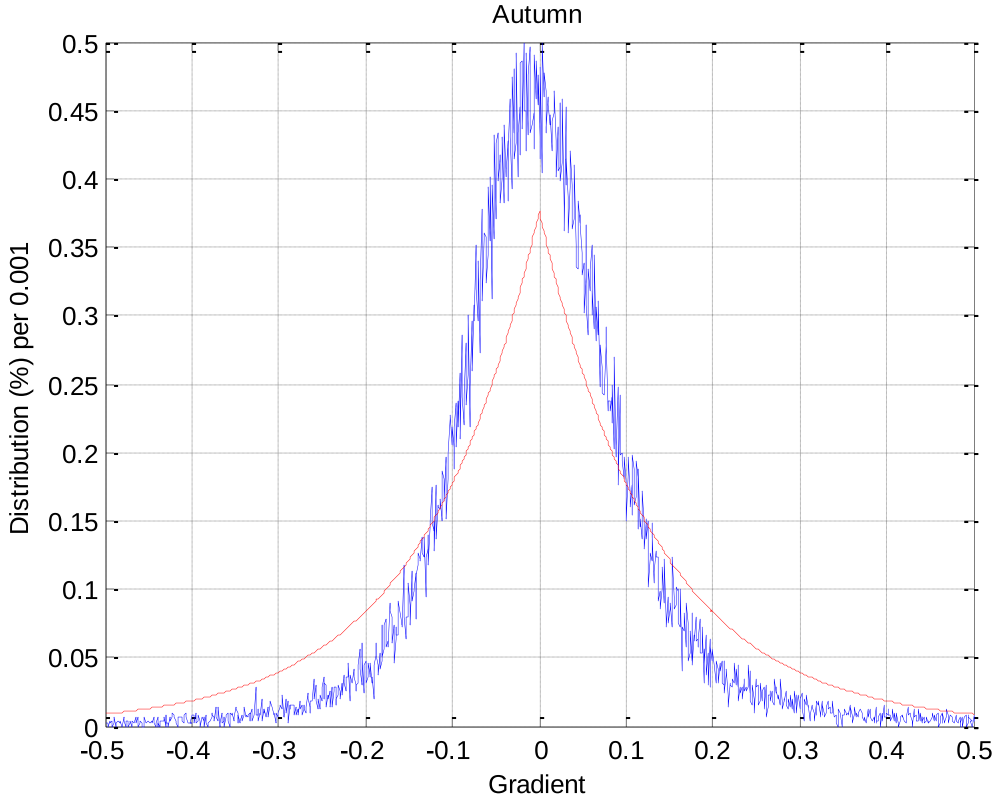

Fig. 35. Comparison between measurement and Laplace distribution with ø = 0.1329.

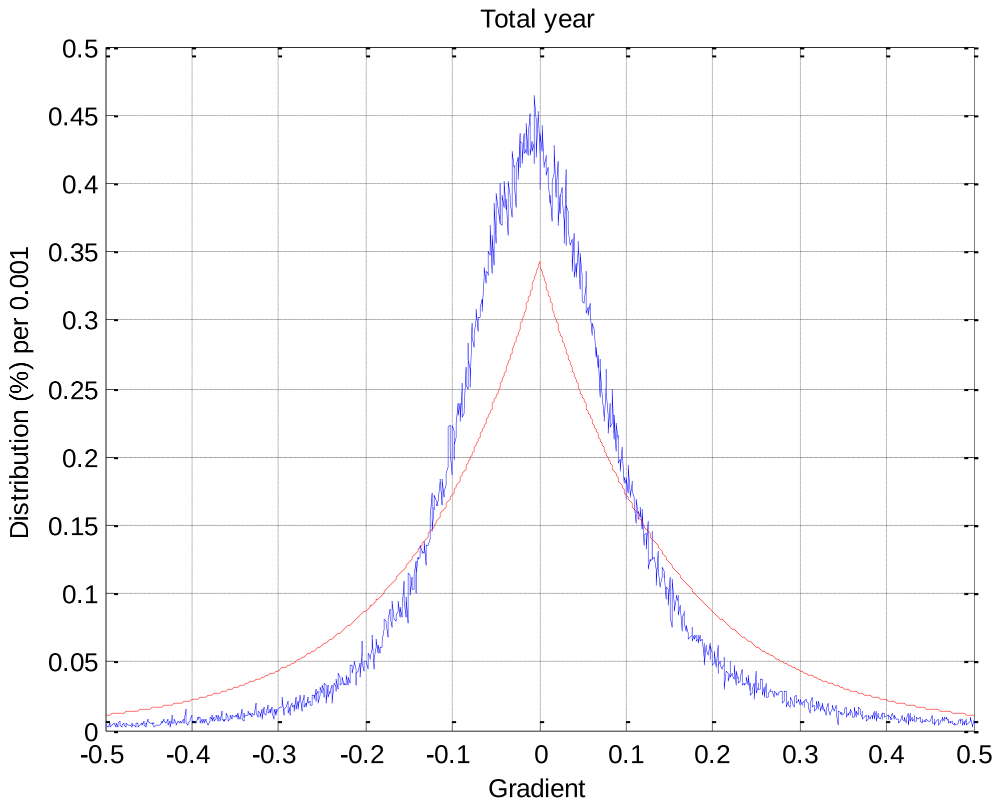

Fig. 36. Comparison between measurement and Laplace distribution with ø = 0.1458.

4 Simulation model

4.1 Input data

The following input parameters are used:

- Time resolution (step time)
- Number of simulation steps
- Number of times to update the low frequency component. See section 4.2.
- Weibull parameters, A and C.
- Laplace parameter ø. (Parameter θ is always zero.)

4.2 Statistical significance and simulation process

To get a good statistical confidence of the simulation result, the low frequency component shall be updated a number of times. According to /1/, this is done 10 times during a total simulation. The process is shown in Fig. 37.

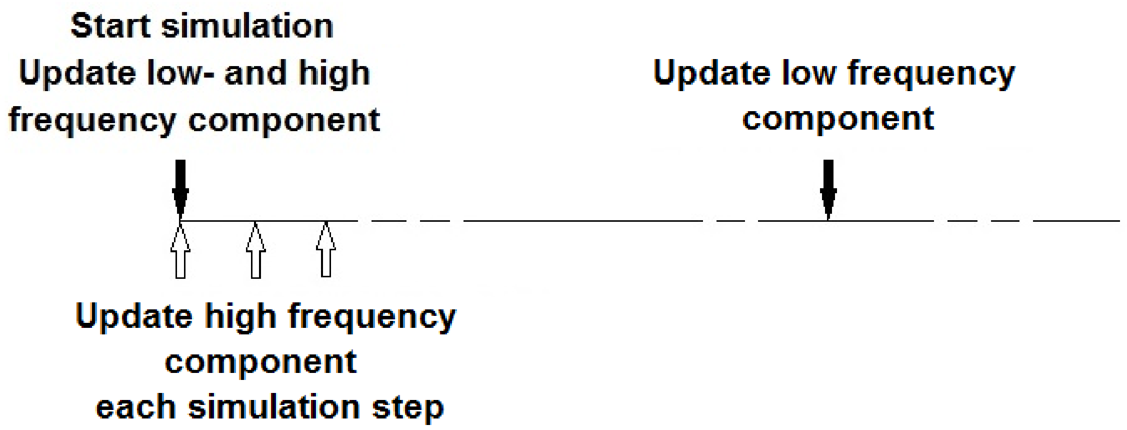

Fig. 37. Simulation process.

4.3 Simulation example

Fig. 38 shows an example of simulated wind speed with the following input parameter values:

- Time resolution: 10 min
- Number of simulation steps: 1440 (equivalent to 10 days)
- Number of times to update the low frequency component: 10
- A = 6.3, C = 1.9
- ø = 0.1458

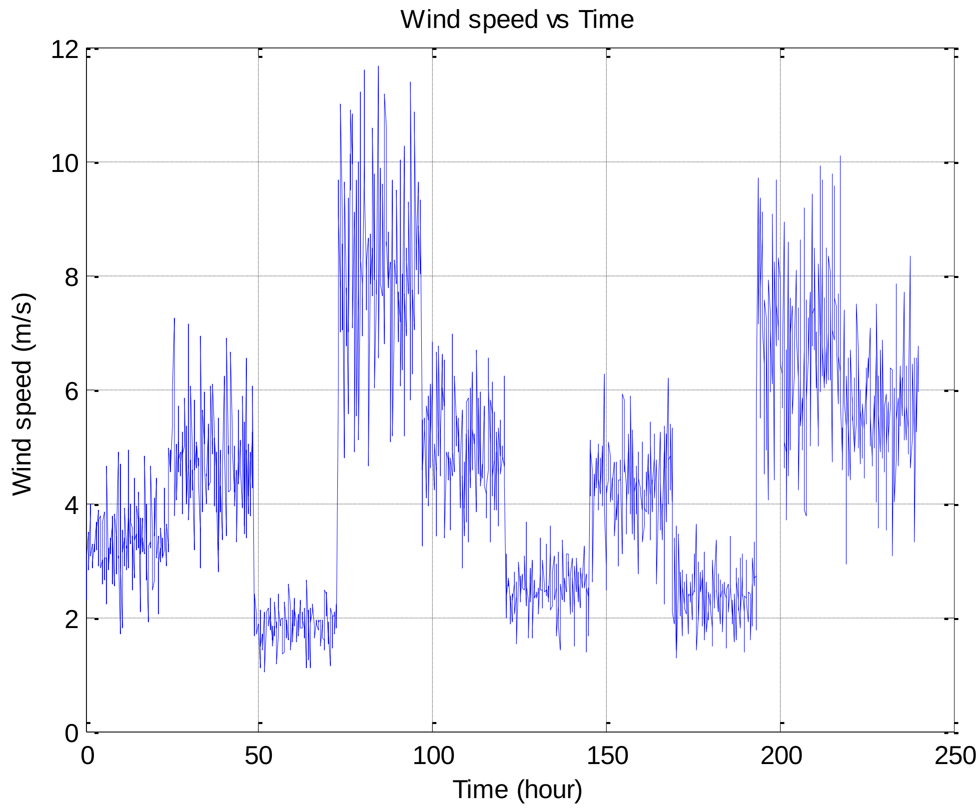

Fig. 38. An example of simulated wind speed.

5 Validation

5.1 Basic principle at validation

In order to test the simulation model, a validation process has been performed. This is based on comparisons with measured wind speeds with corresponding wind speeds, that were obtained at simulations based on the Weibull parameters and Laplace parameters which became the results of the measurement data analysis according to section 3. Comparisons according section 5.1.1.1 - 5.1.1.8 were done. The results are plotted. Generally conditions were:

- Number of simulation cycles = 100
- Time resolution: 10 min
- Number of simulation steps: 1440 (equivalent to 10 days per simulation cycle)
- Number of times to update the low frequency component: 10 (per simulation cycle)

5.1.1.1 Comparison 1

Mean value of simulation result per simulation cycle presented together with mean value of reference measurement.

5.1.1.2 Comparison 2

Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement.

5.1.1.3 Comparison 3

Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement.

5.1.1.4 Comparison 4

Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

5.1.1.5 Comparison 5

The relation according to:
"Mean value of simulation result / Mean value of reference measurement" per simulation cycle.

5.1.1.6 Comparison 6

The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

5.1.1.7 Comparison 7

The relation according to:
"Maximum value of simulation result / Maximum value of reference measurement" per simulation cycle.

5.1.1.8 Comparison 8

The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

5.1.2 Test case 1

- Reference: Total year
- Simulation parameters: A = 6.3, C = 1.9, ø = 0.1458 (according to Table VI and Table VII)

5.1.2.1 Comparison 1

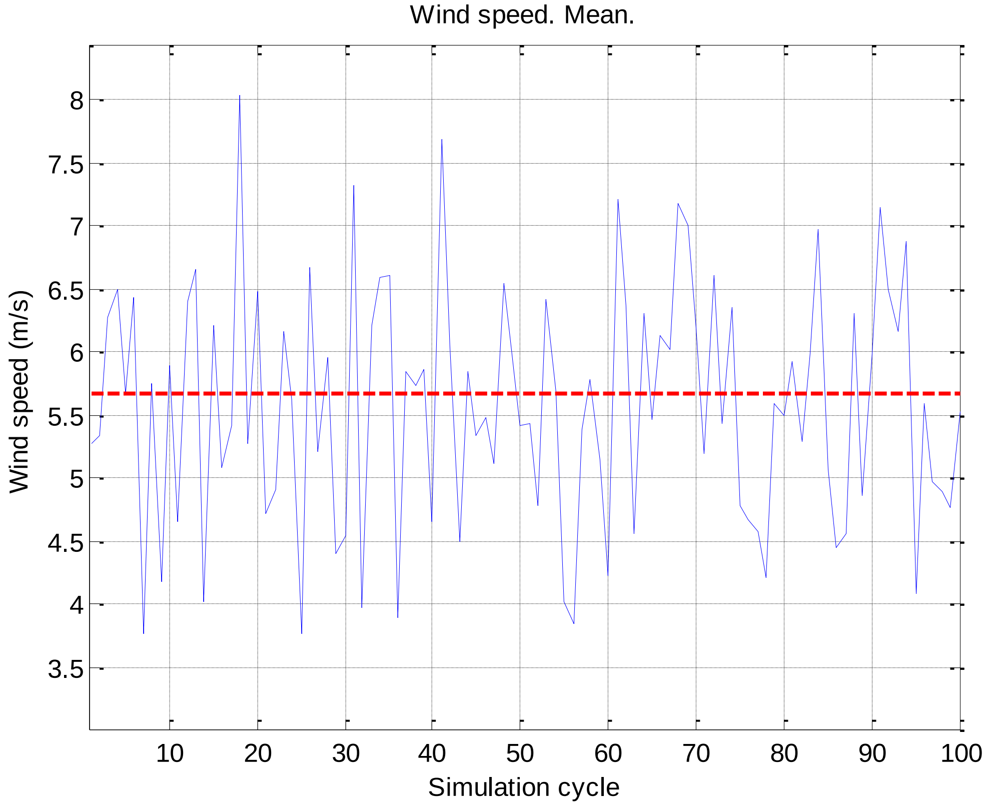

Fig. 39. Mean value of simulation result per simulation cycle presented together with mean value of reference measurement (red dashed line).

Mean value for all simulated mean values = 5.6 m/s.

Reference value = 5.7 m/s.

5.1.2.2 Comparison 2

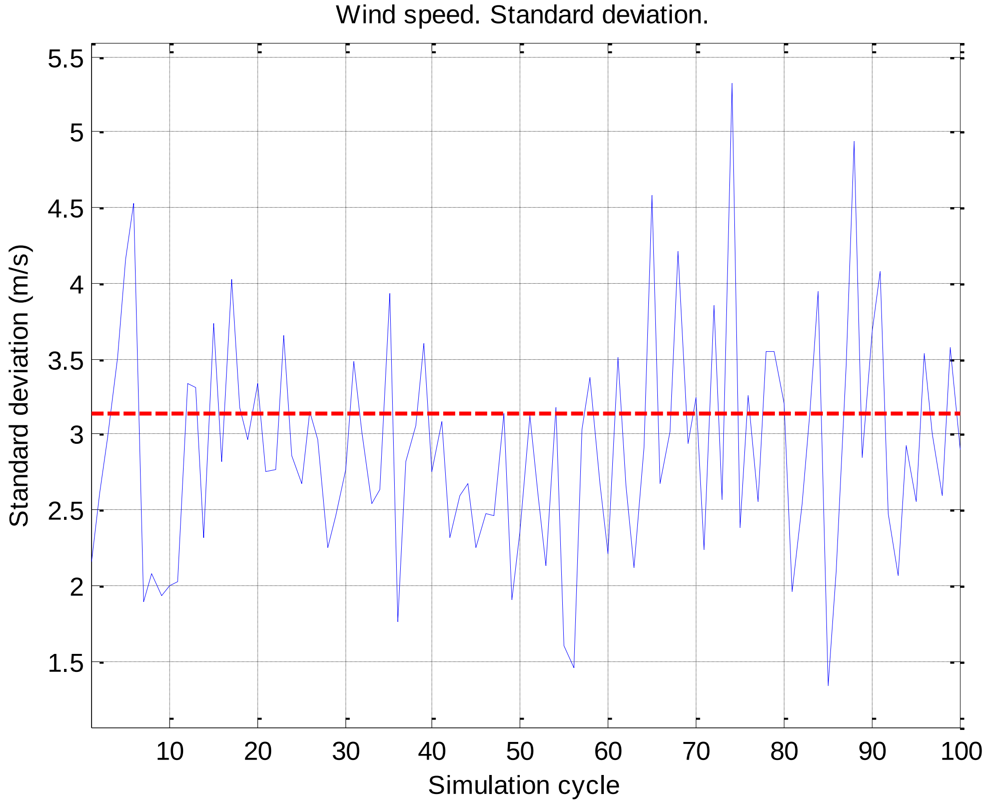

Fig. 40. Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement (red dashed line).

Mean value for all simulated standard deviations = 2.9 m/s.

Reference value = 3.1 m/s.

5.1.2.3 Comparison 3

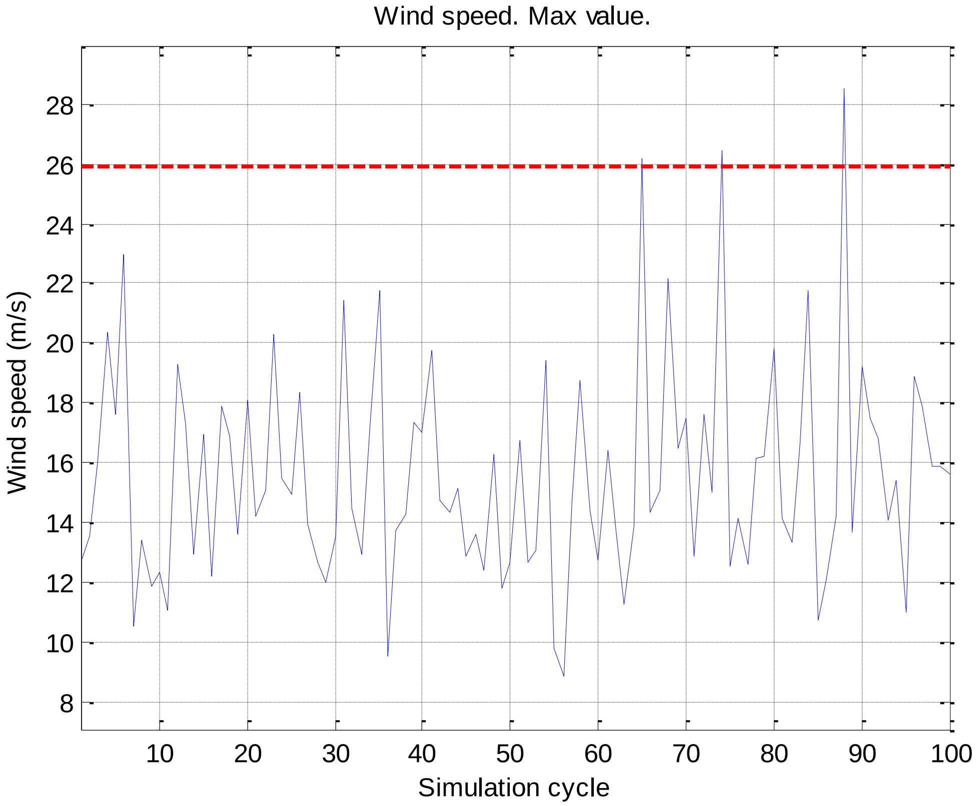

Fig. 41. Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement (red dashed line).

Mean value for all simulated max values = 15.6 m/s.

Reference value = 25.9 m/s.

5.1.2.4 Comparison 4

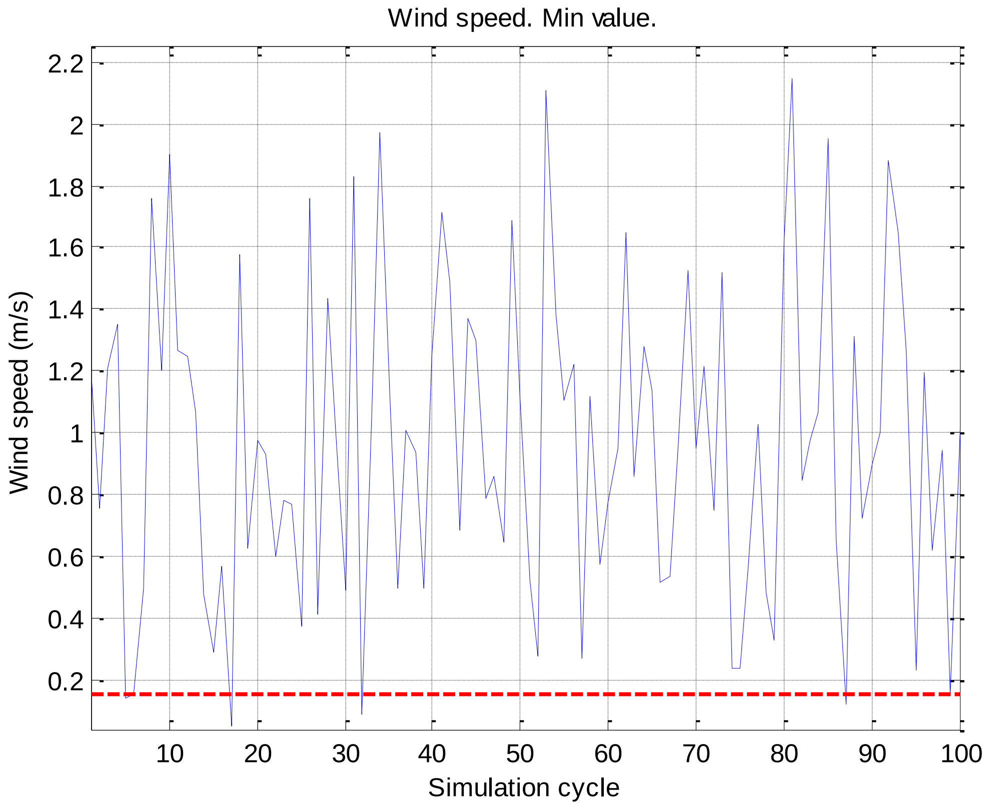

Fig. 42. Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

Mean value for all simulated max values = 1.0 m/s.

Reference value = 0.2 m/s.

5.1.2.5 Comparison 5

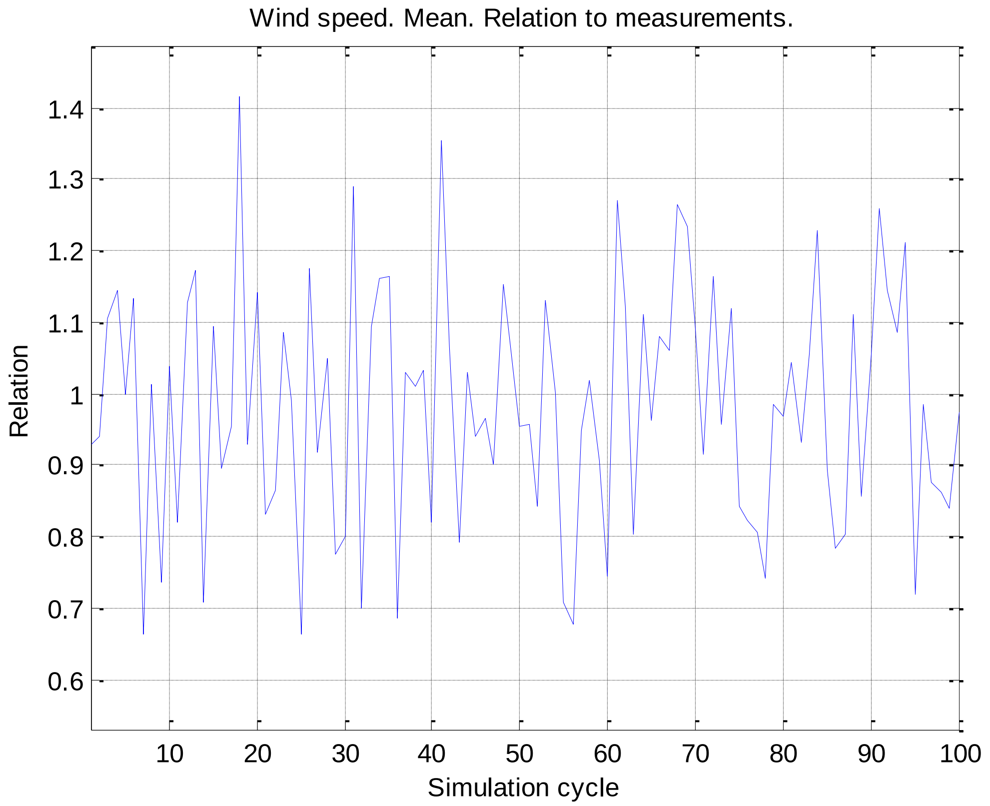

Fig. 43. The relation according to:
"Mean value of simulation result / Mean value of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.2.6 Comparison 6

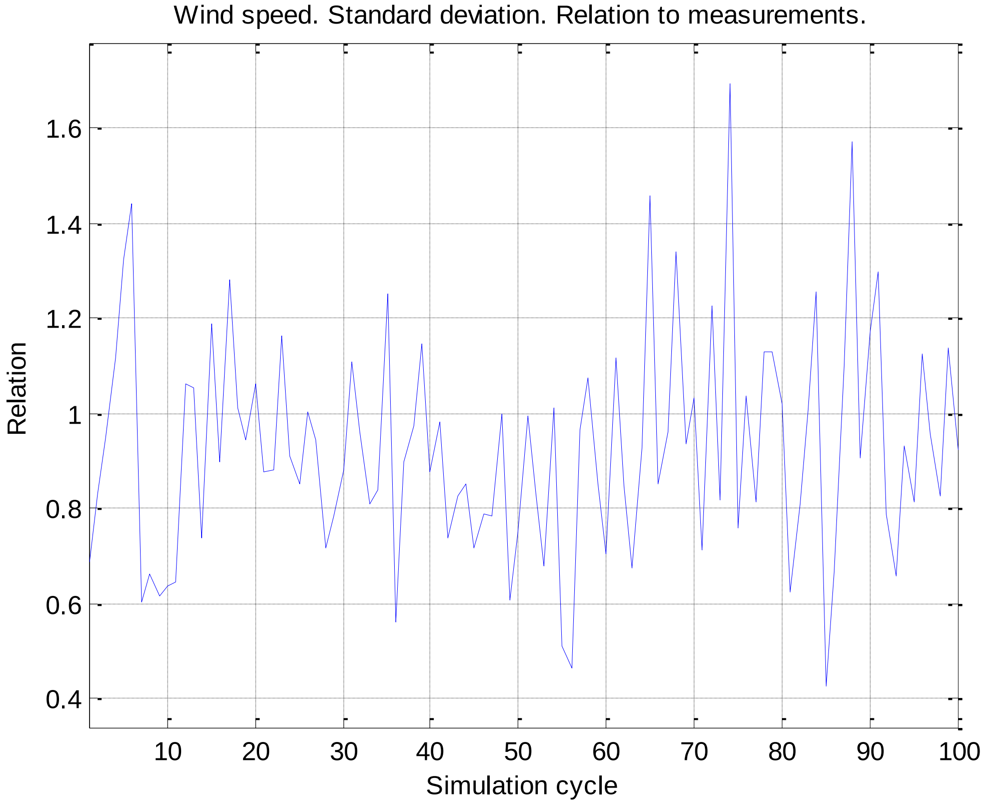

Fig. 44. The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

Mean value over all cycles = 0.9.

5.1.2.7 Comparison 7

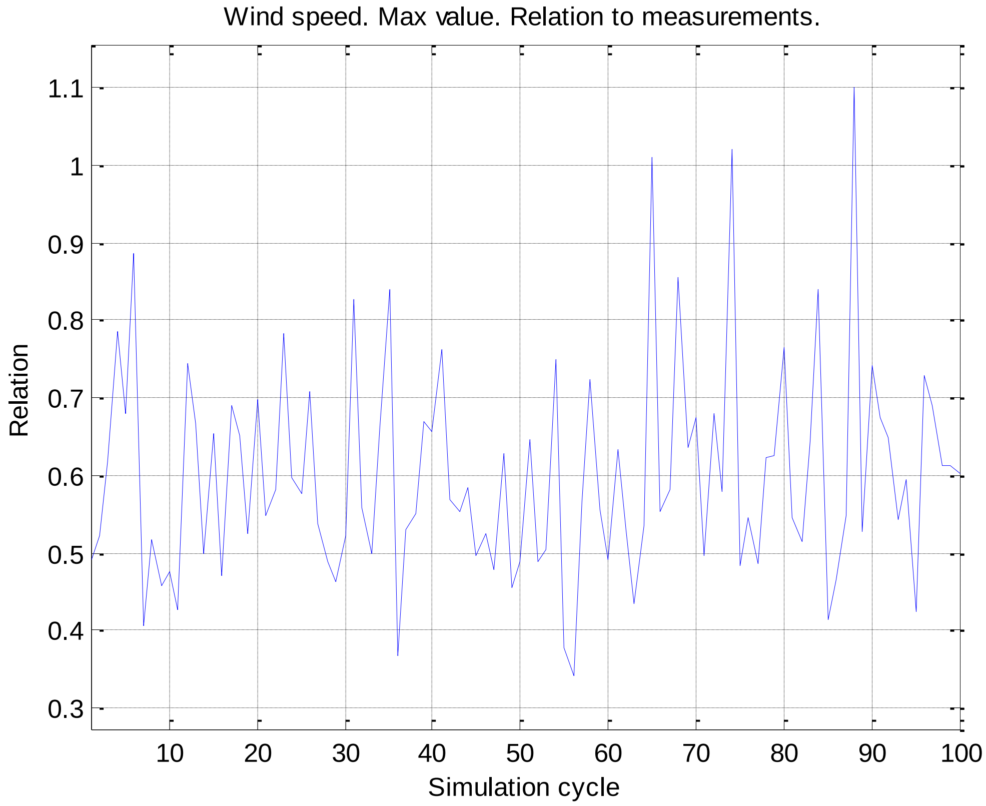

Fig. 45. The relation according to:
"Maximum value of simulation result / Maximum value of reference measurement" per simulation cycle.

Mean value for all relations = 0.6.

5.1.2.8 Comparison 8

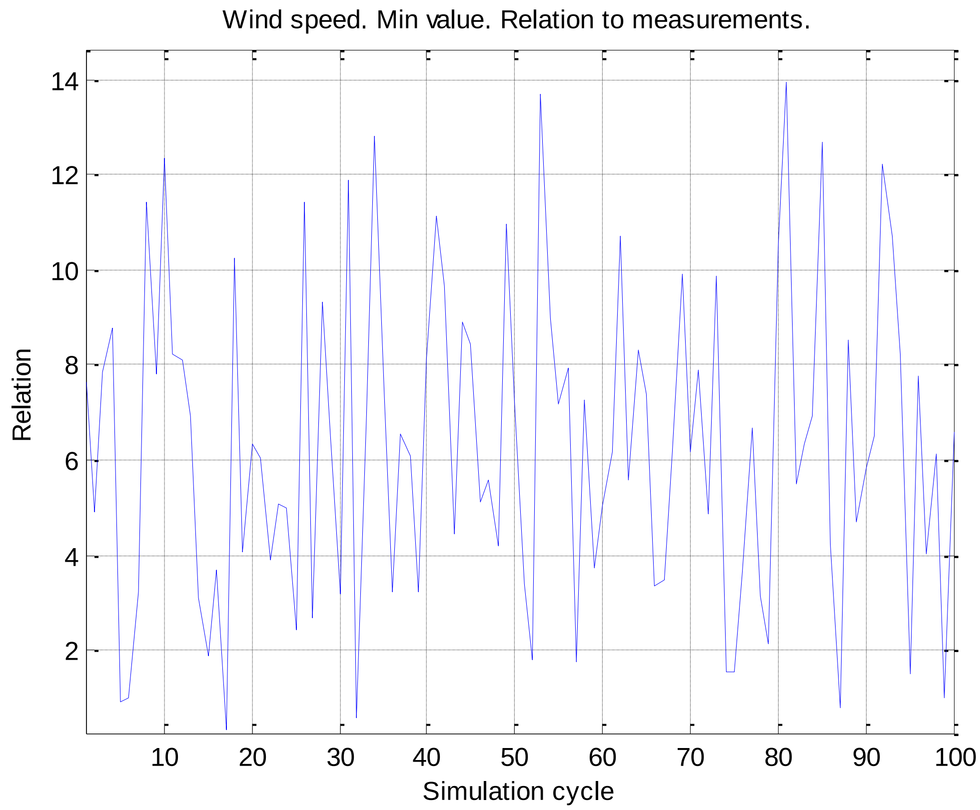

Fig. 46. The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

Mean value for all relations = 6.3.

5.1.3 Test case 2

- Reference: Winter
- Simulation parameters: A = 5.9, C = 1.8, ø = 0.1696 (according to Table VI and Table VII)

5.1.3.1 Comparison 1

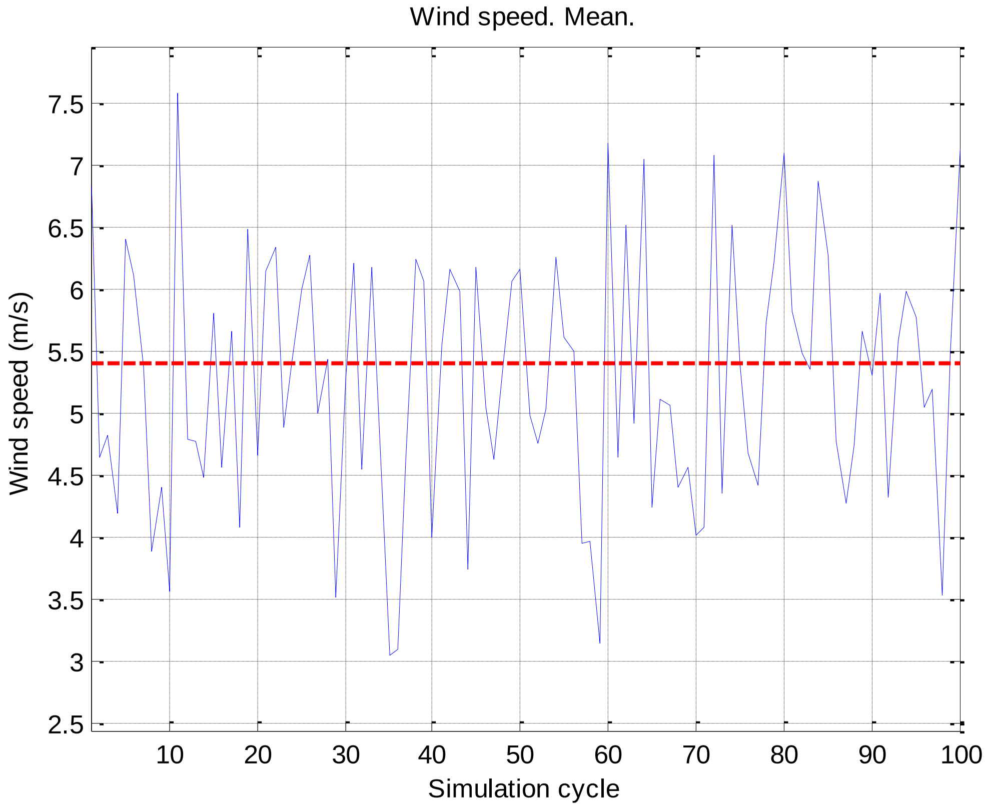

Fig. 47. Mean value of simulation result per simulation cycle presented together with mean value of reference measurement (red dashed line).

Mean value for all simulated mean values = 5.3 m/s.

Reference value = 5.4 m/s.

5.1.3.2 Comparison 2

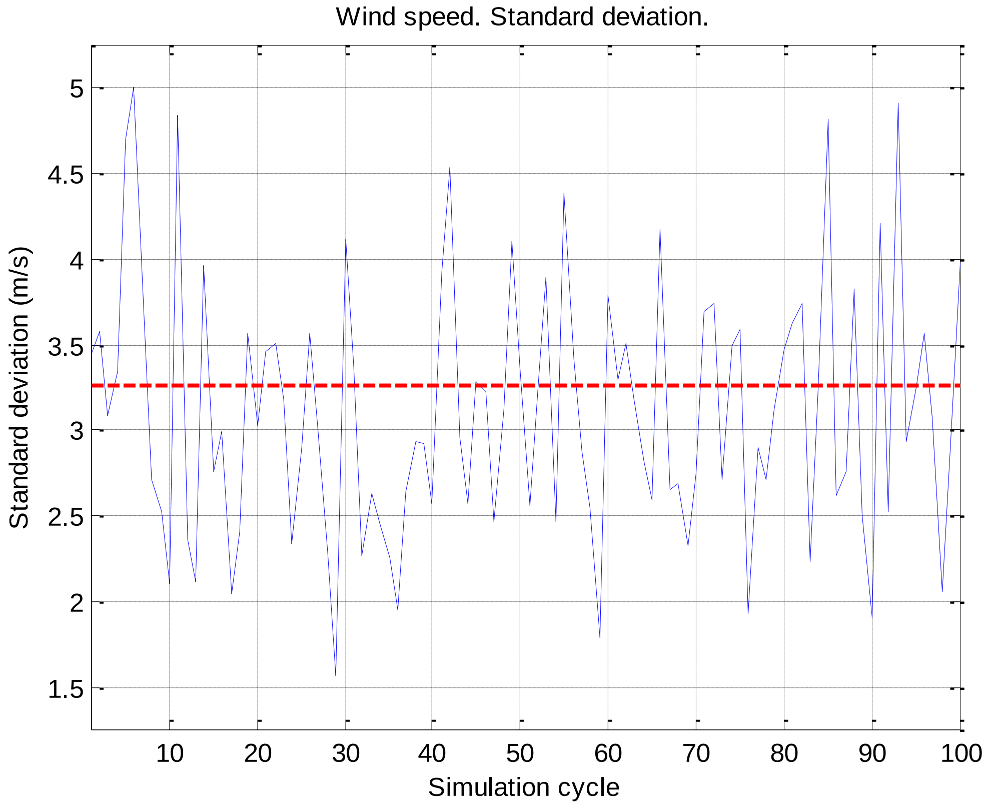

Fig. 48. Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement (red dashed line).

Mean value for all simulated standard deviations = 3.1 m/s.

Reference value = 3.3 m/s.

5.1.3.3 Comparison 3

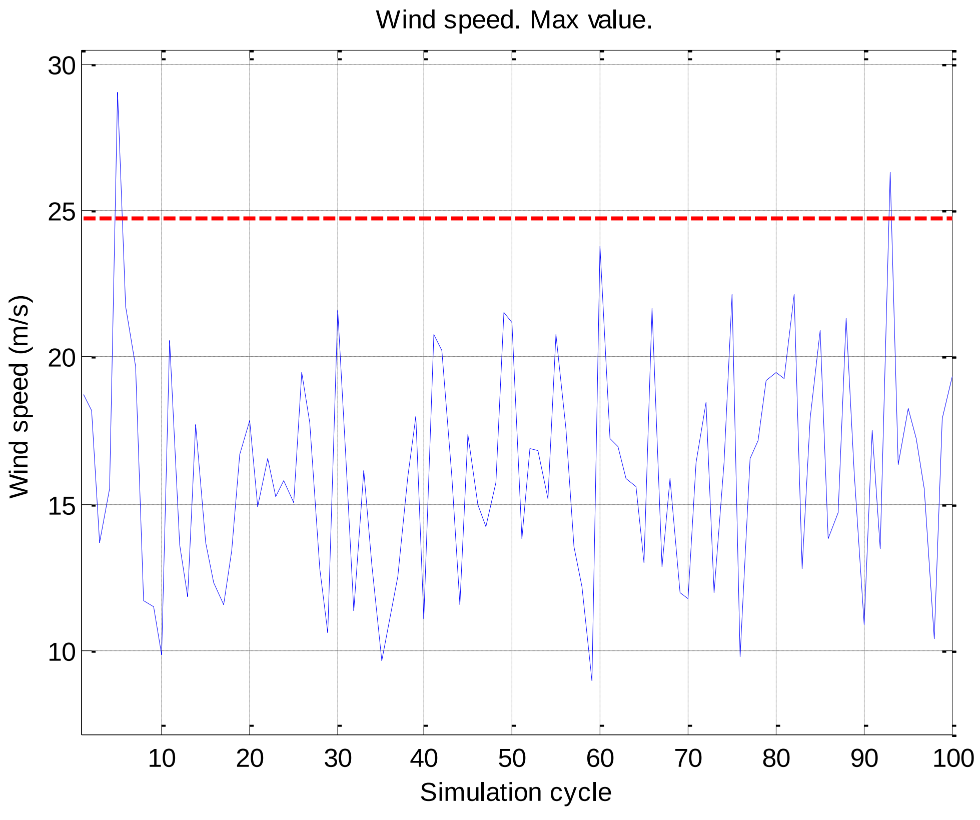

Fig. 49. Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement (red dashed line).

Mean value for all simulated max values = 16.1 m/s.

Reference value = 24.8 m/s.

5.1.3.4 Comparison 4

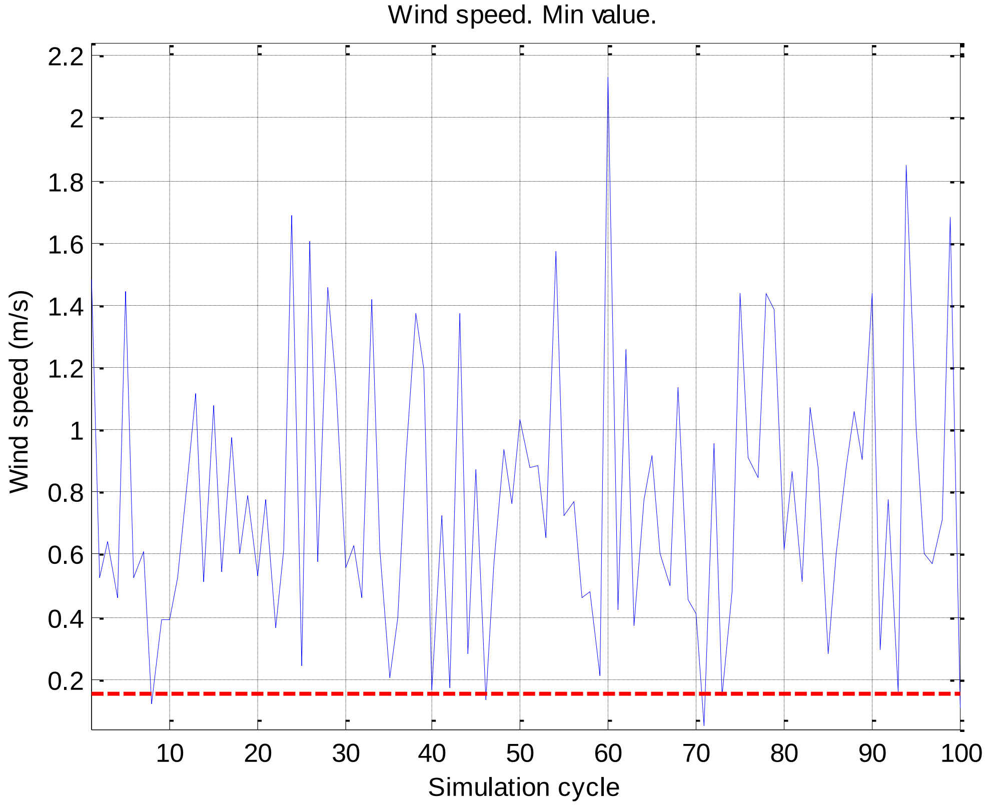

Fig. 50. Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

Mean value for all simulated max values = 0.8 m/s.

Reference value = 0.2 m/s.

5.1.3.5 Comparison 5

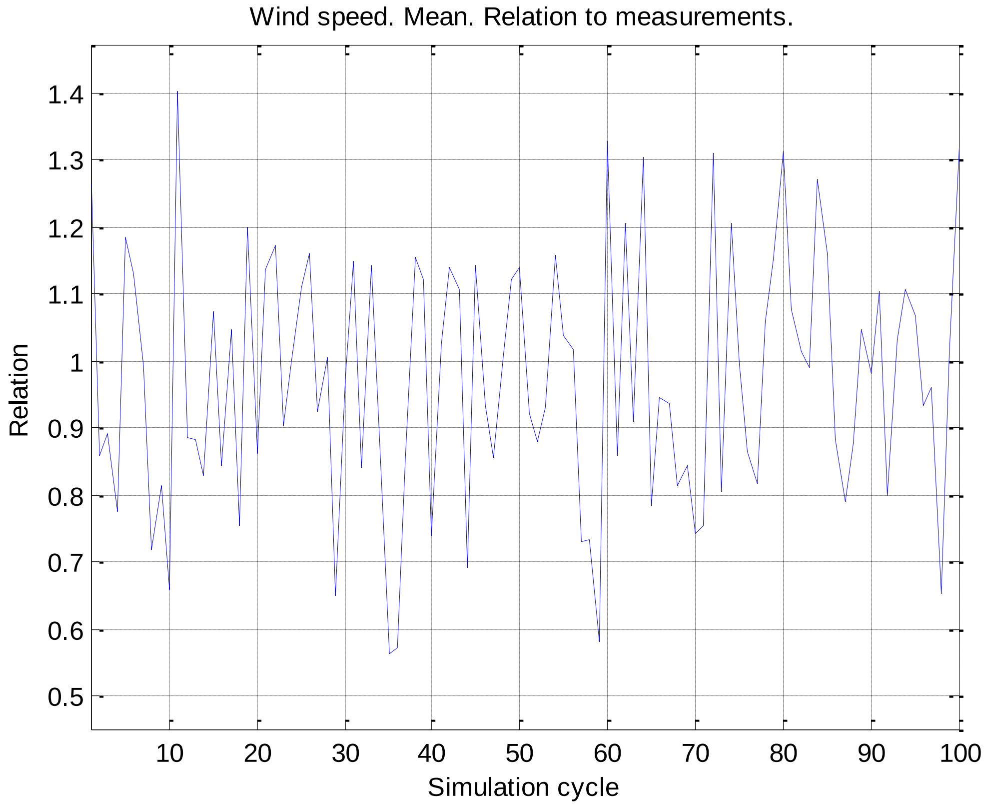

Fig. 51. The relation according to:
"Mean value of simulation result / Mean value of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.3.6 Comparison 6

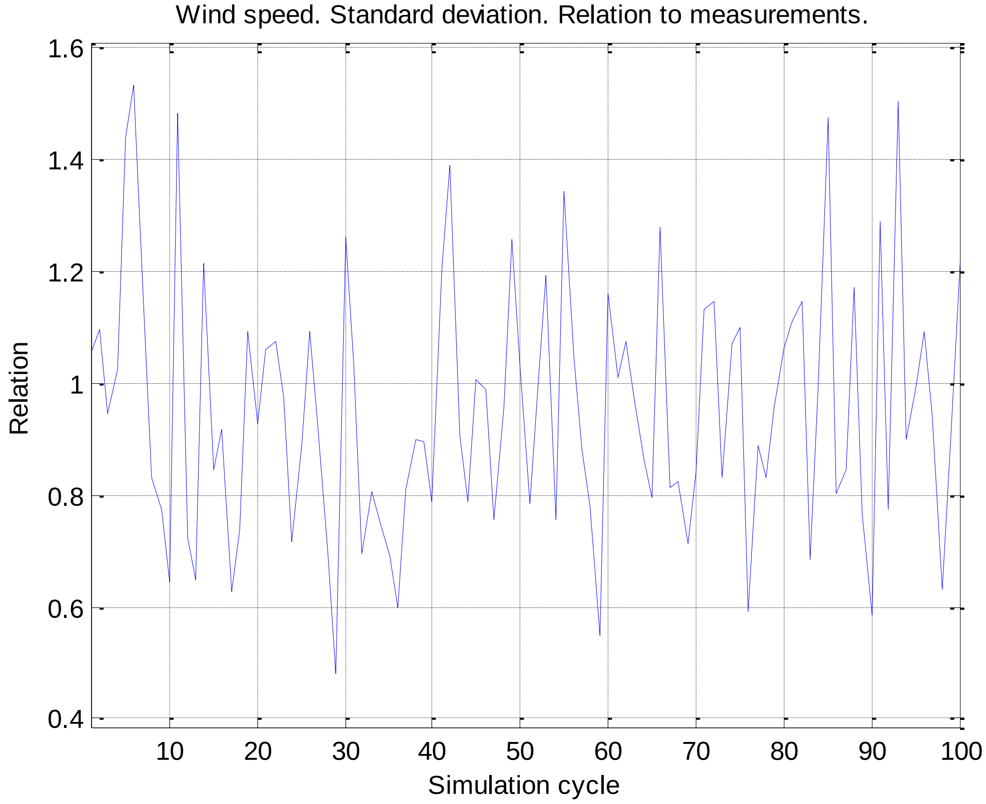

Fig. 52. The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.3.7 Comparison 7

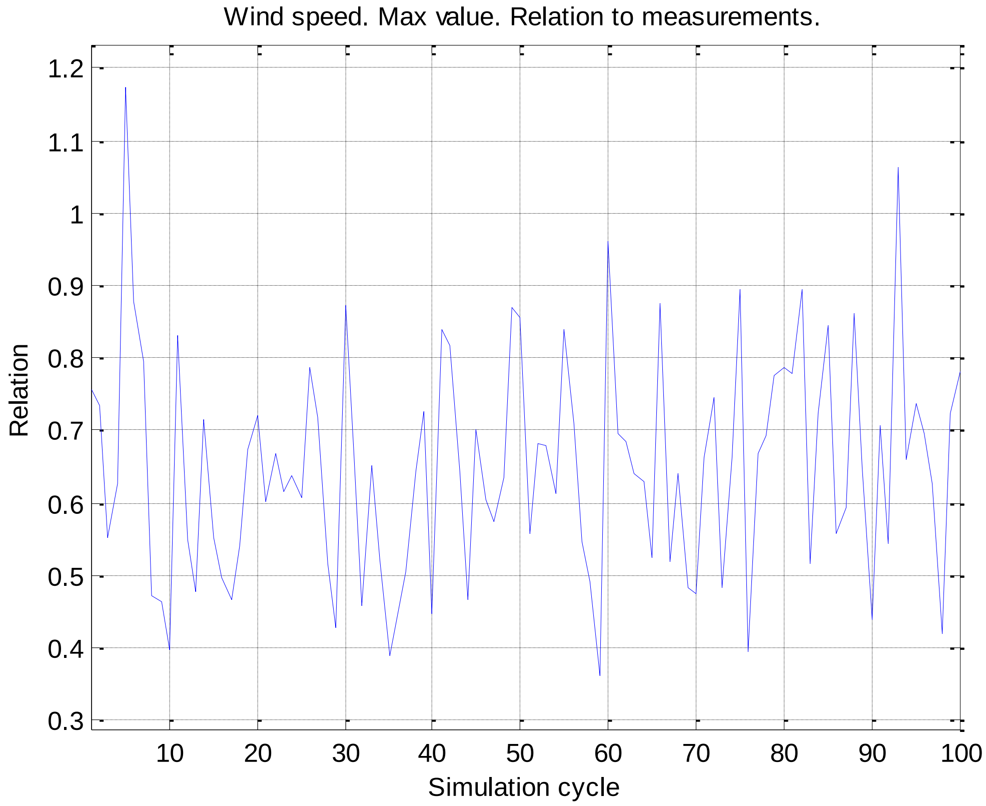

Fig. 53. The relation according to:
"Maximum value of simulation result / Maximum value of reference measurement" per simulation cycle.

Mean value for over all cycles = 0.6.

5.1.3.8 Comparison 8

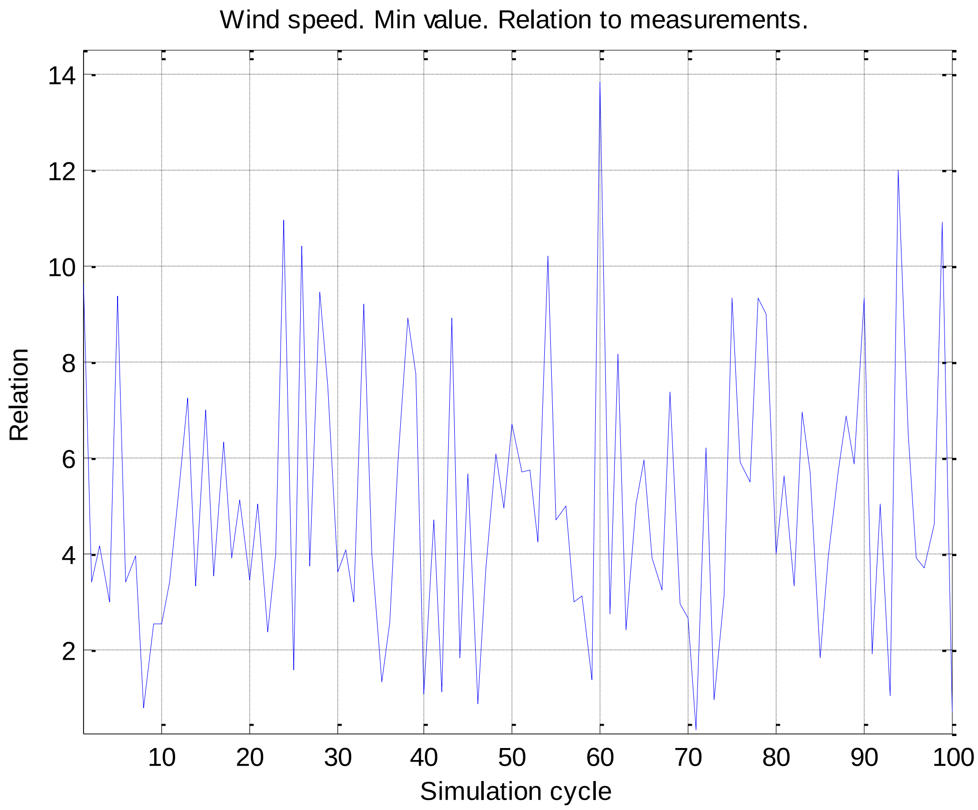

Fig. 54. The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

Mean value for all relations = 5.0.

5.1.4 Test case 3

- Reference: Spring
- Simulation parameters: A = 5.7, C = 2.0, ø = 0.1462 (according to Table VI and Table VII)

5.1.4.1 Comparison 1

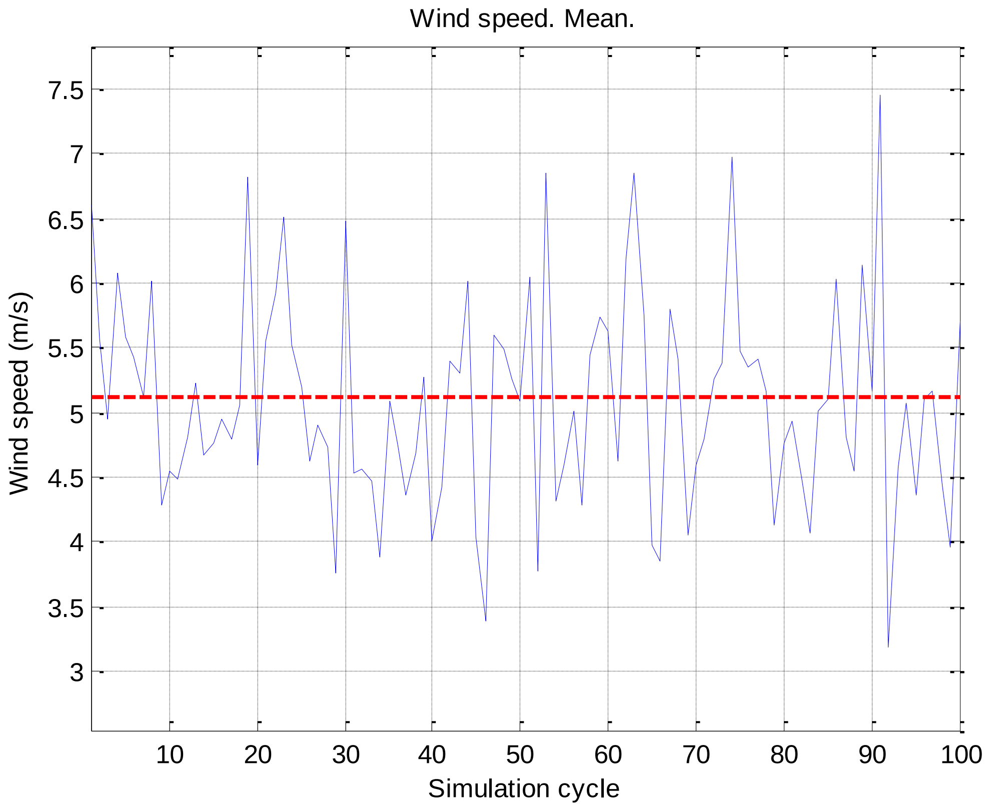

Fig. 55. Mean value of simulation result per simulation cycle presented together with mean value of reference measurement (red dashed line).

Mean value for all simulated mean values = 5.1m/s.

Reference value = 5.1 m/s.

5.1.4.2 Comparison 2

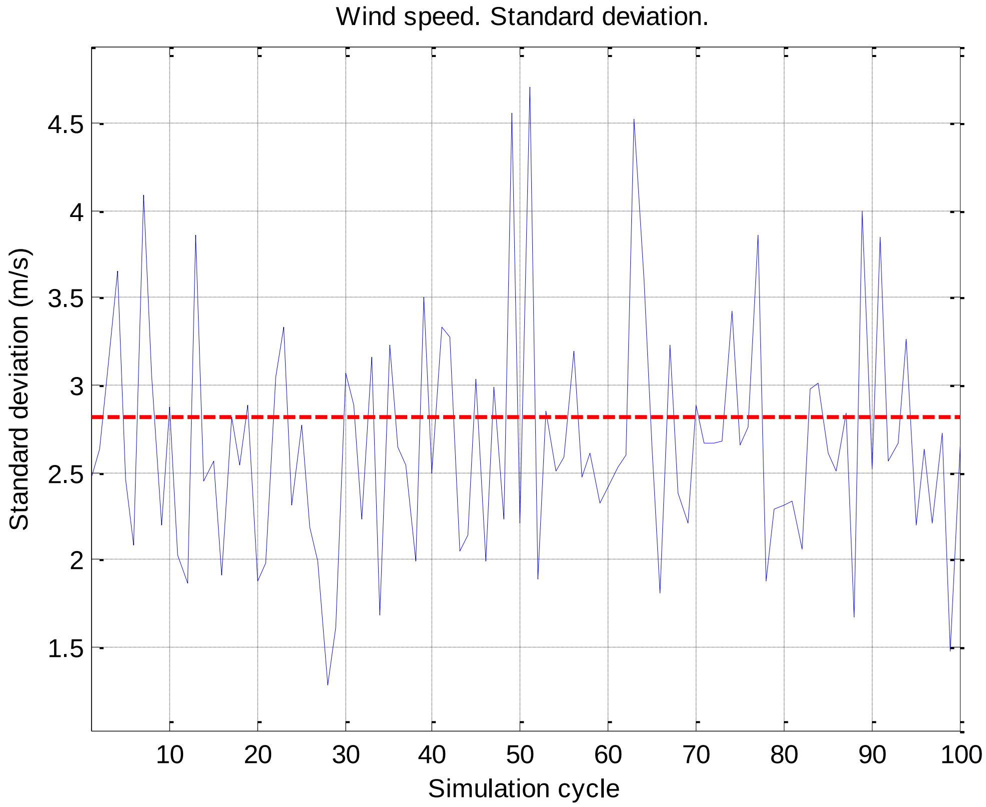

Fig. 56. Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement (red dashed line).

Mean value for all simulated standard deviations = 2.7 m/s.

Reference value = 2.8 m/s.

5.1.4.3 Comparison 3

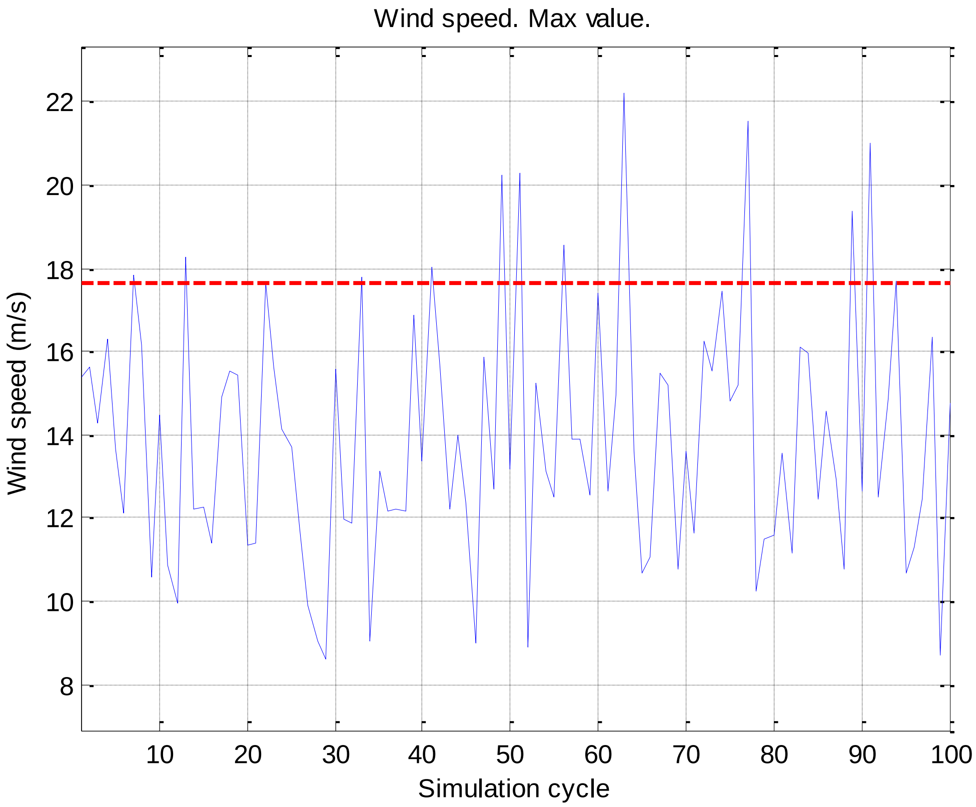

Fig. 57. Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement (red dashed line).

Mean value for all simulated max values = 13.9 m/s.

Reference value = 17.7 m/s.

5.1.4.4 Comparison 4

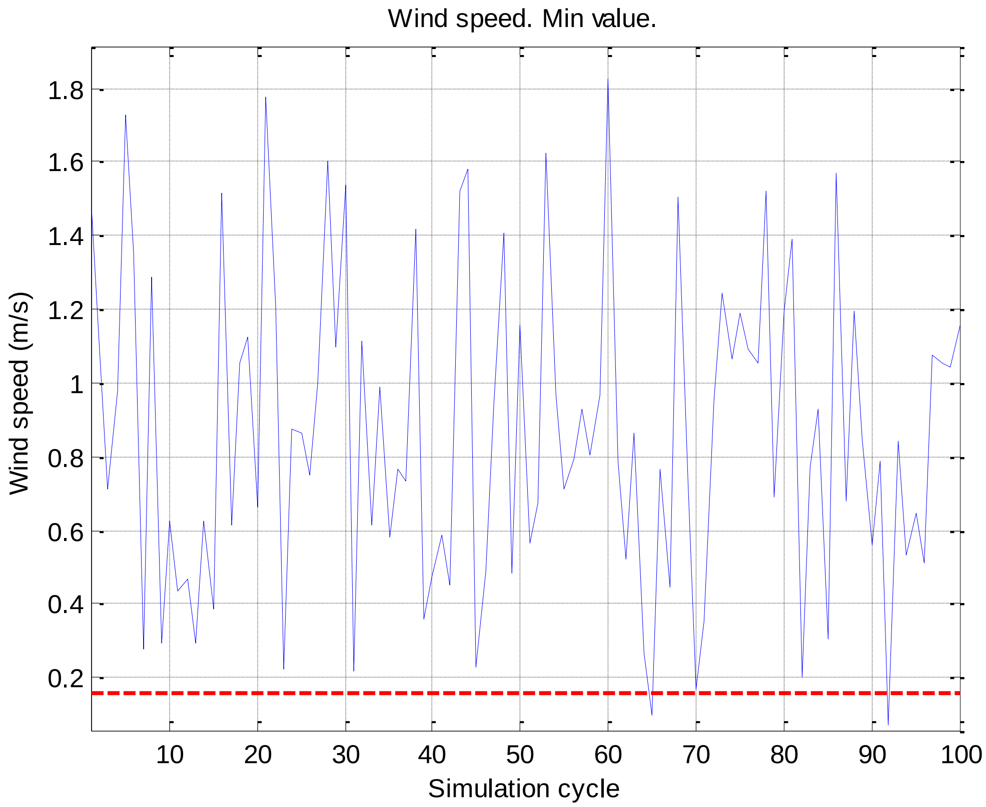

Fig. 58. Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

Mean value for all simulated max values = 0.9 m/s.

Reference value = 0.2 m/s.

5.1.4.5 Comparison 5

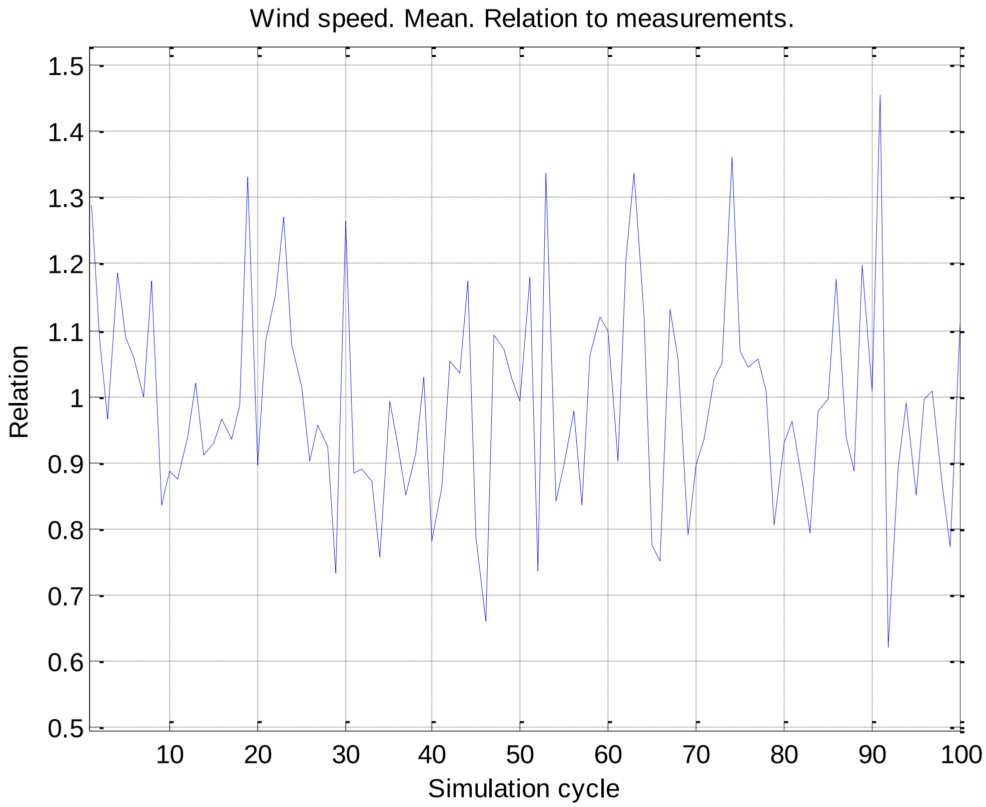

Fig. 59. The relation according to:
"Mean value of simulation result / Mean value of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.4.6 Comparison 6

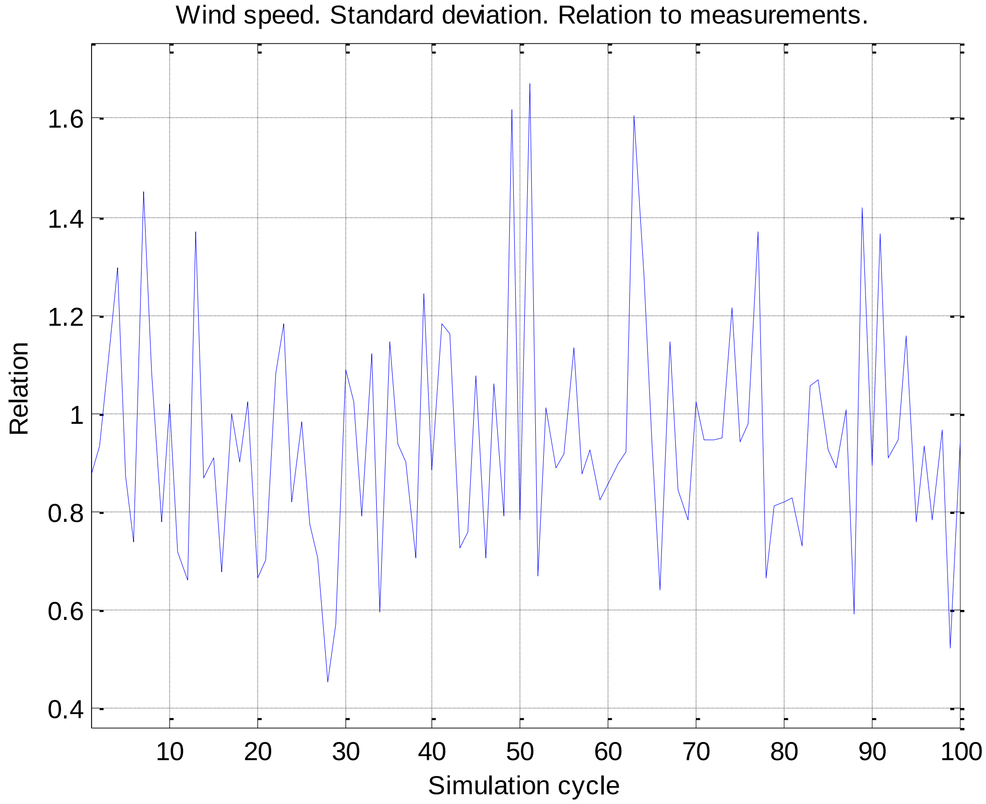

Fig. 60. The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

Mean value over all cycles = 0.9.

5.1.4.7 Comparison 7

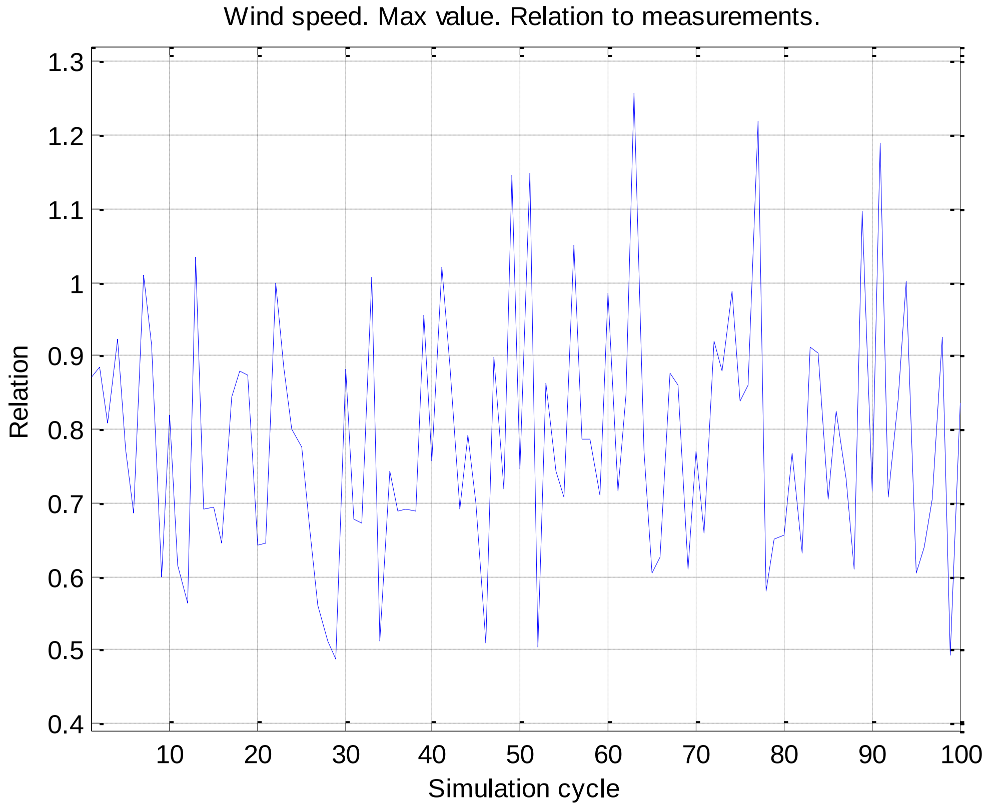

Fig. 61. The relation according to:
“Maximum value of simulation result / Maximum value of reference measurement” per simulation cycle.

Mean value over all cycles = 0.8.

5.1.4.8 Comparison 8

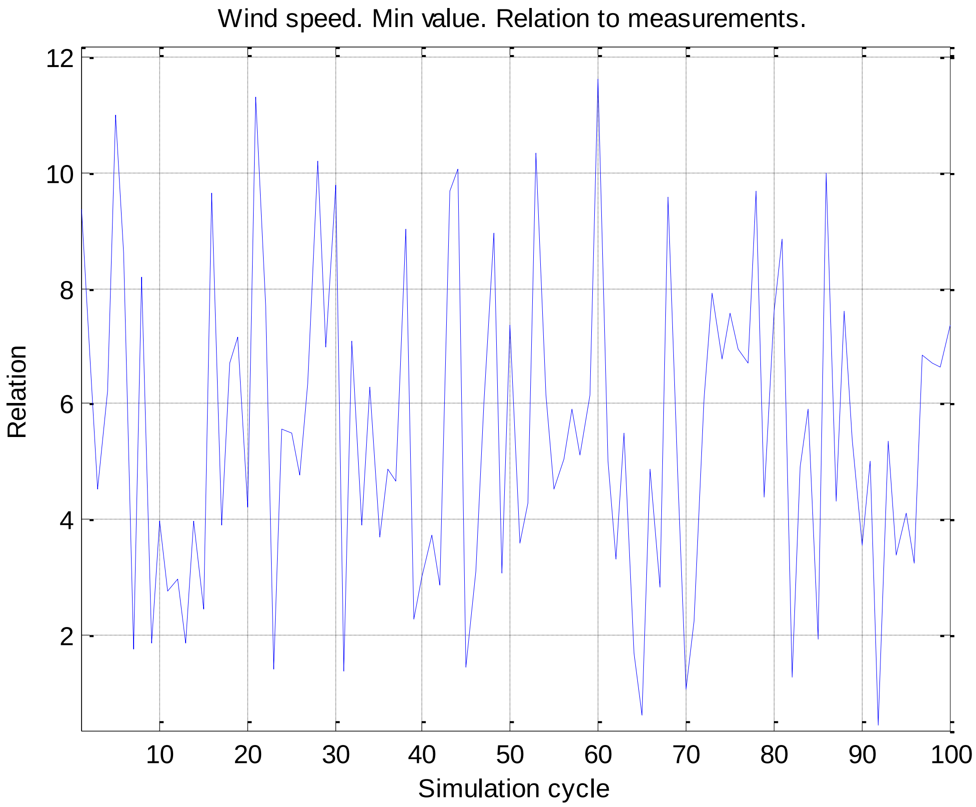

Fig. 62. The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

Mean value for all relations = 5.5.

5.1.5 Test case 4

- Reference: Summer
- Simulation parameters: A = 5.9, C = 2.0, ø = 0.1349 (according to Table VI and Table VII)

5.1.5.1 Comparison 1

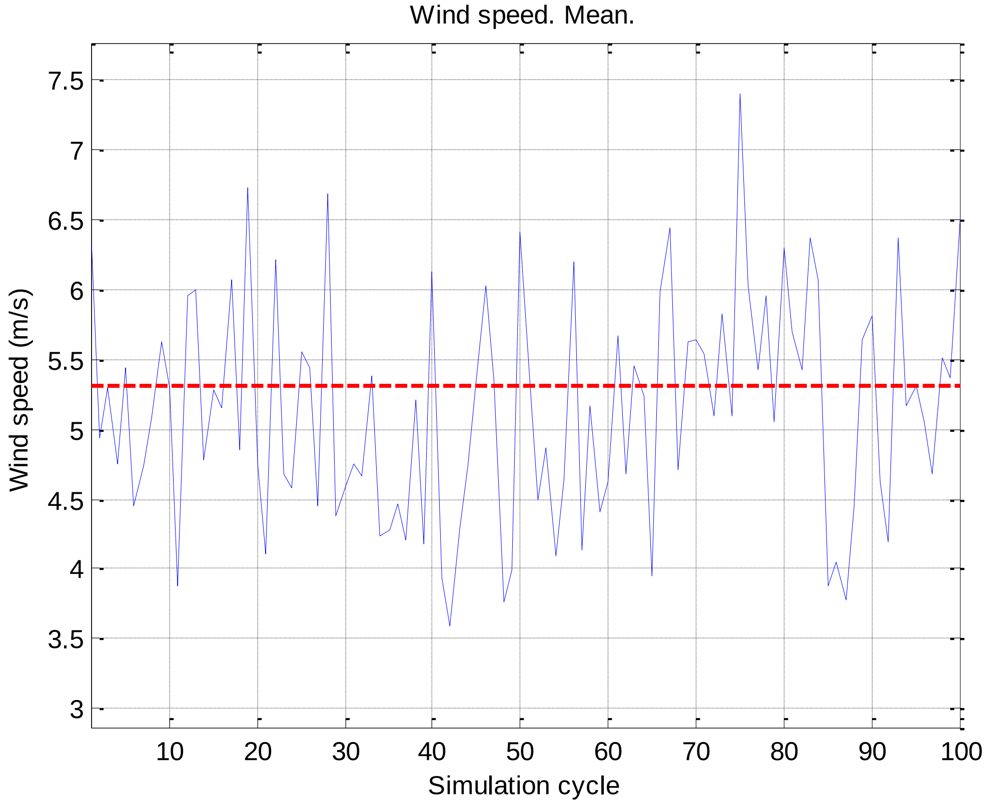

Fig. 63. Mean value of simulation result per simulation cycle presented together with mean value of reference measurement (red dashed line).

Mean value for all simulated mean values = 5.1 m/s.

Reference value = 5.3 m/s.

5.1.5.2 Comparison 2

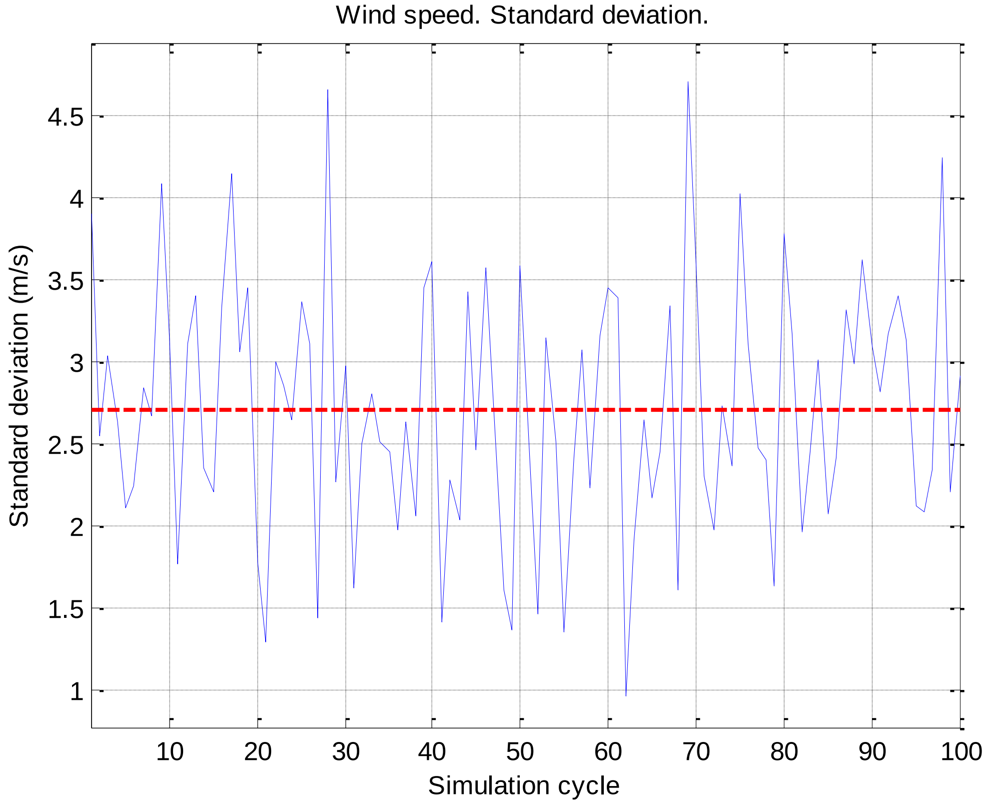

Fig. 64. Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement (red dashed line).

Mean value for all simulated standard deviations = 2.7 m/s.

Reference value = 2.7 m/s.

5.1.5.3 Comparison 3

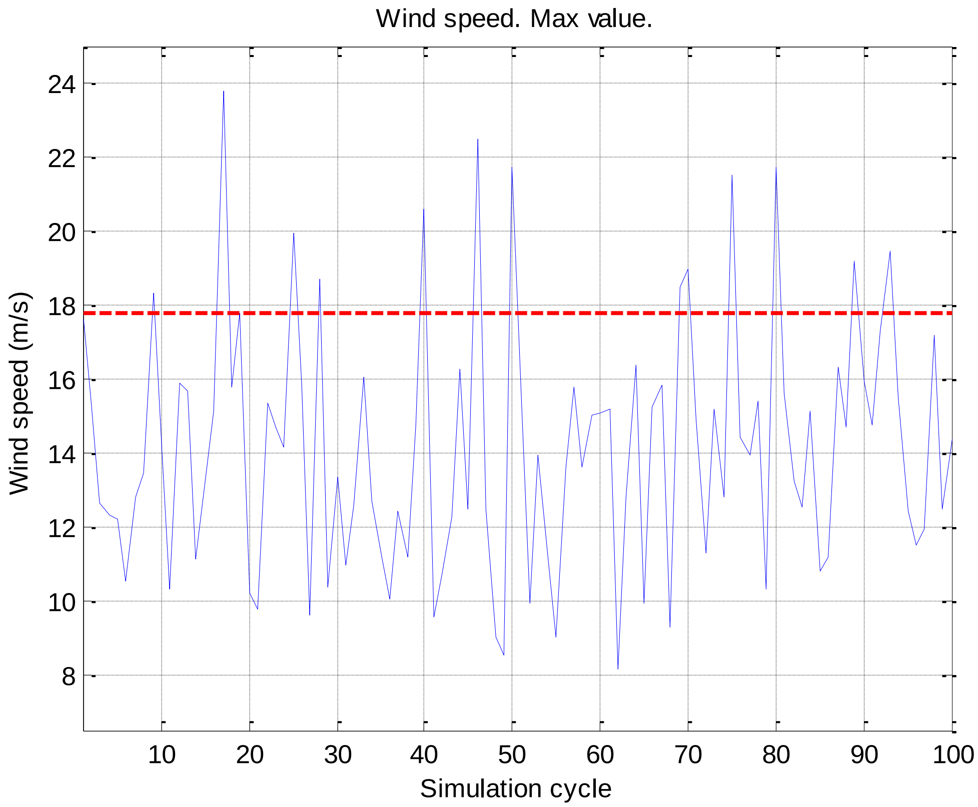

Fig. 65. Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement (red dashed line).

Mean value for all simulated max values = 14.2 m/s.

Reference value = 17.8 m/s.

5.1.5.4 Comparison 4

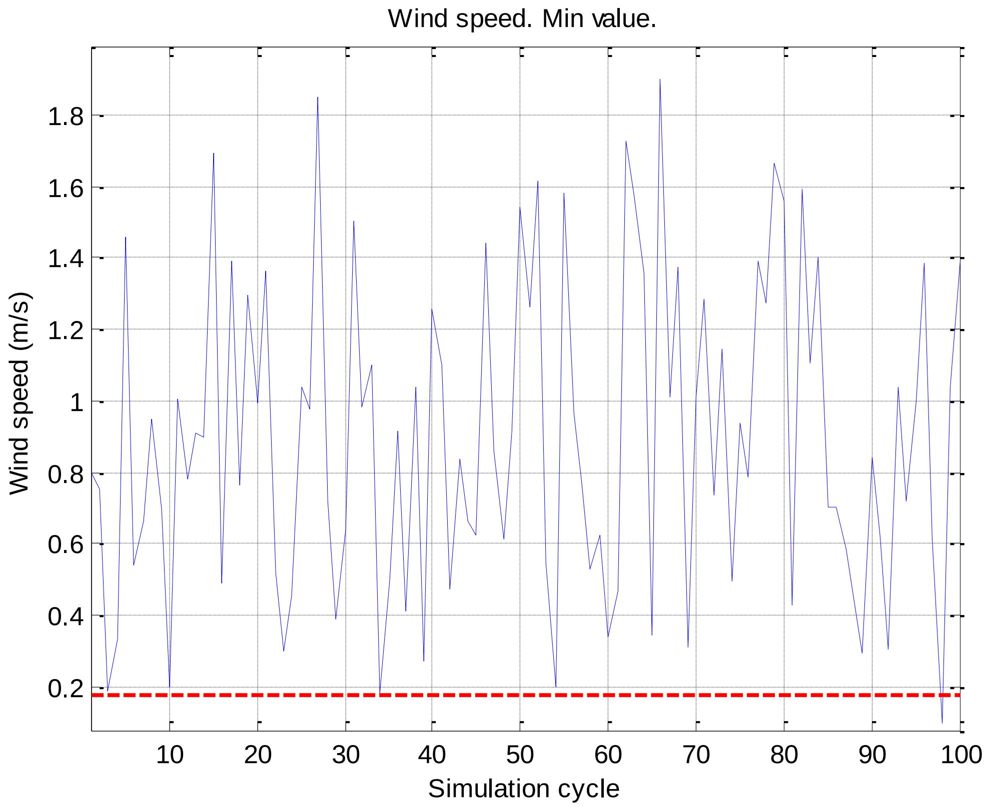

Fig. 66. Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

Mean value for all simulated max values = 0.9 m/s.

Reference value = 0.2 m/s.

5.1.5.5 Comparison 5

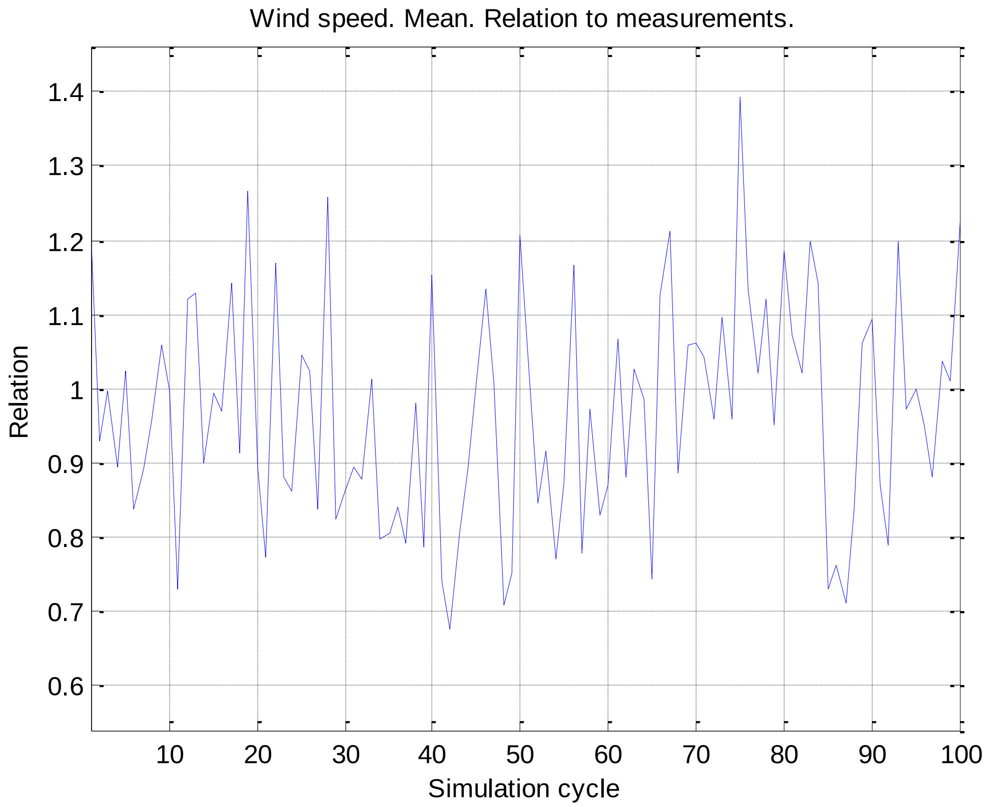

Fig. 67. The relation according to:
“Mean value of simulation result / Mean value of reference measurement” per simulation cycle.

Mean value over all cycles = 1.0.

5.1.5.6 Comparison 6

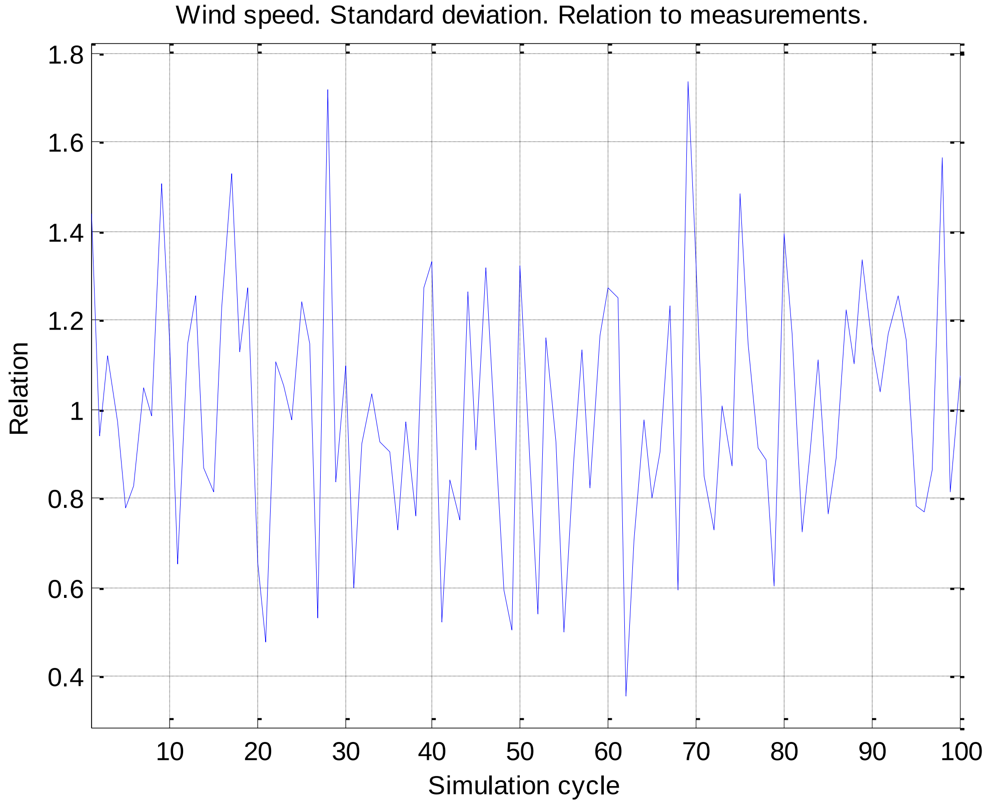

Fig. 68. The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.5.7 Comparison 7

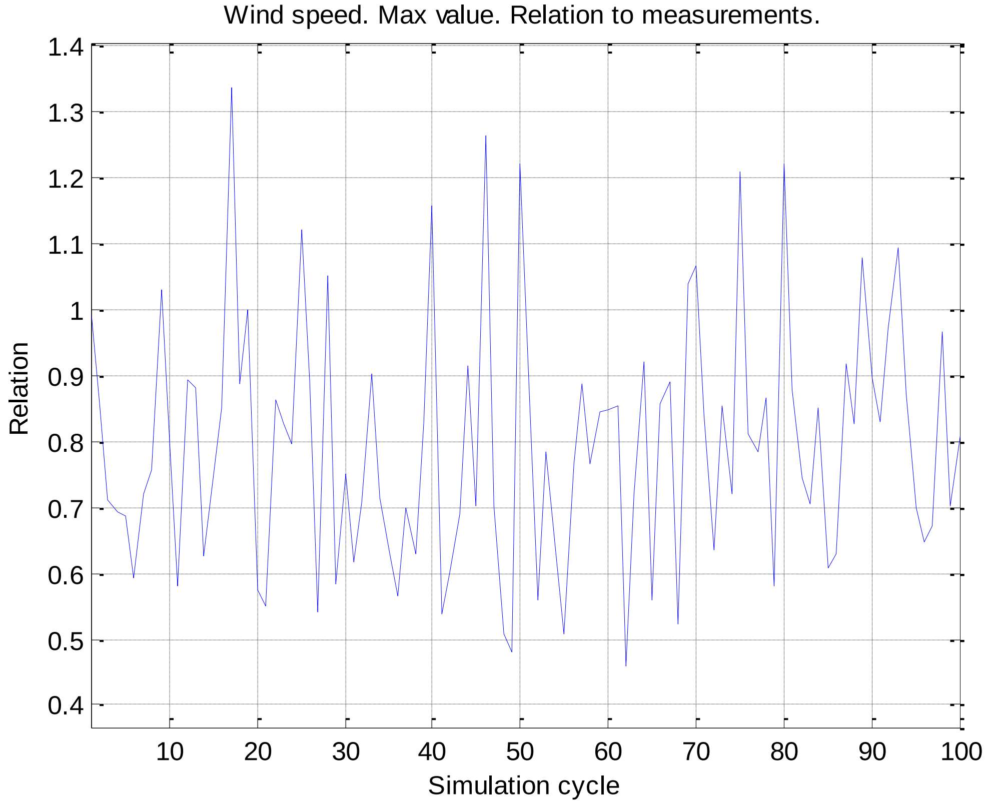

Fig. 69. The relation according to:
"Maximum value of simulation result / Maximum value of reference measurement" per simulation cycle.

Mean value for all relations = 0.8.

5.1.5.8 Comparison 8

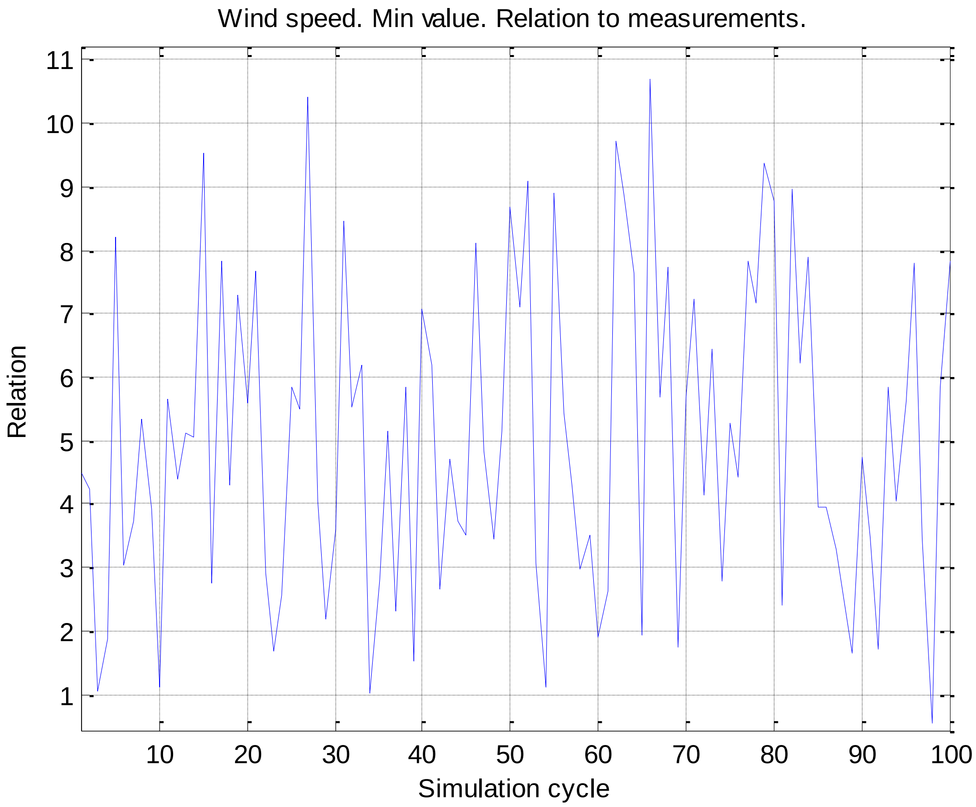

Fig. 70. The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

Mean value for all relations = 5.0.

5.1.6 Test case 5

- Reference: Autumn
- Simulation parameters: A = 7.7, C = 2.0, ø = 0.1329 (according to Table VI and Table VII)

5.1.6.1 Comparison 1

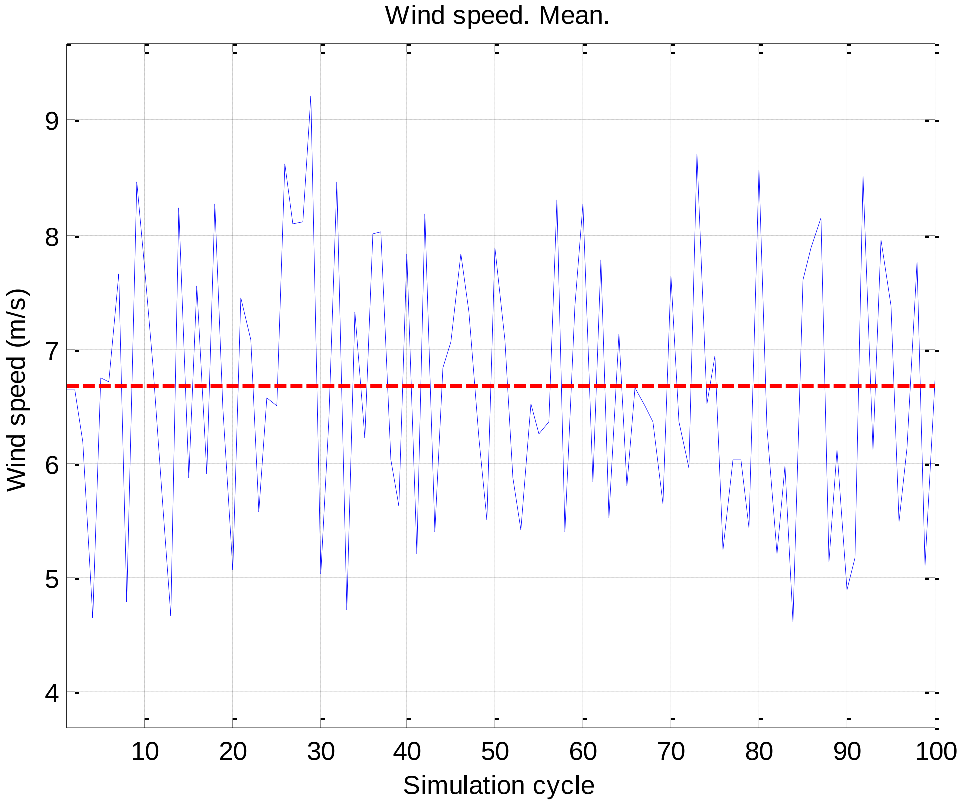

Fig. 71. Mean value of simulation result per simulation cycle presented together with mean value of reference measurement (red dashed line).

Mean value for all simulated mean values = 6.7 m/s.

Reference value = 6.7 m/s.

5.1.6.2 Comparison 2

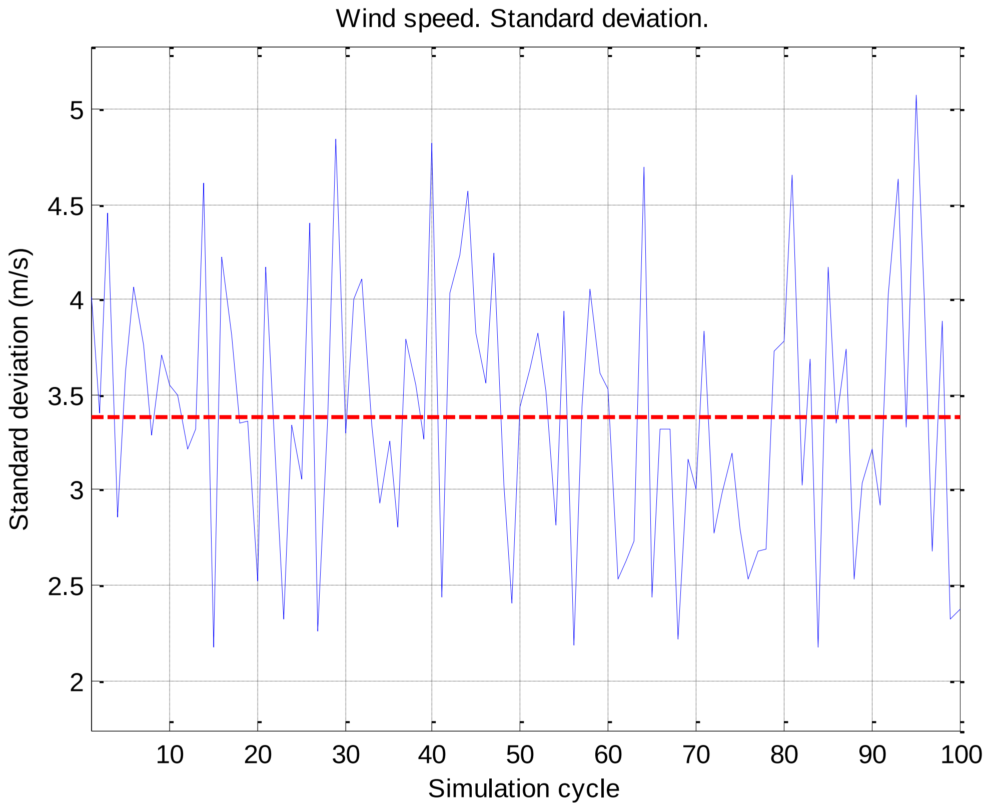

Fig. 72. Standard deviation of simulation result per simulation cycle presented together with standard deviation of reference measurement (red dashed line).

Mean value for all simulated standard deviations = 3.4 m/s.

Reference value = 3.4 m/s.

5.1.6.3 Comparison 3

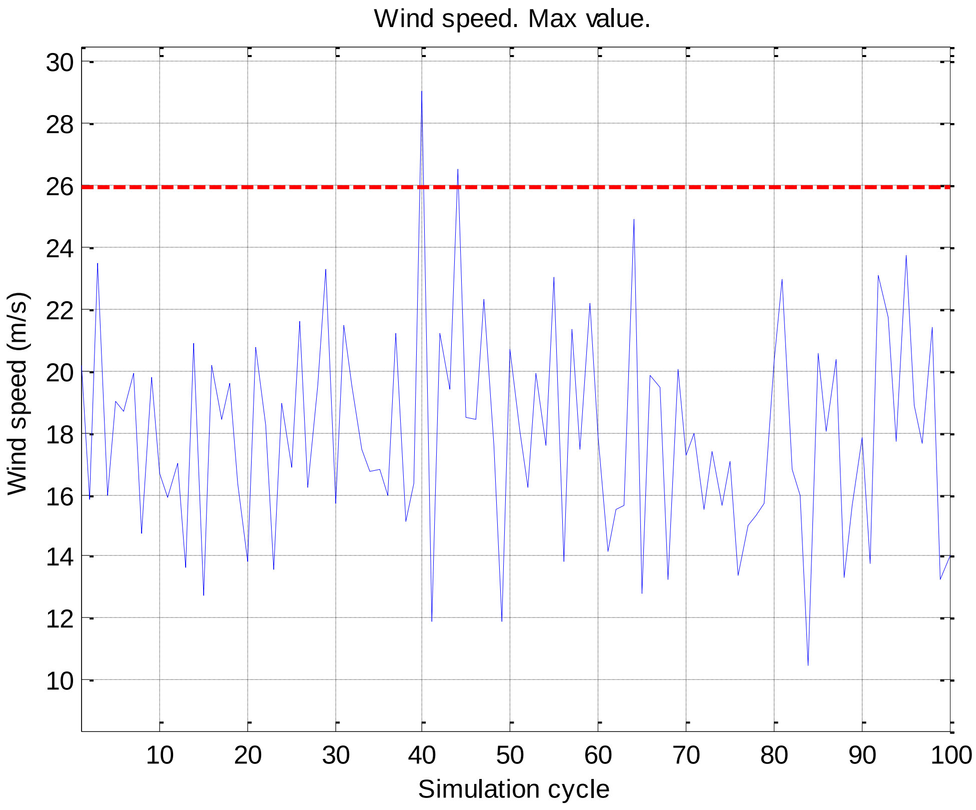

Fig. 73. Maximum value of simulation result per simulation cycle presented together with maximum value of reference measurement (red dashed line).

Mean value for all simulated max values = 18.0 m/s.

Reference value = 25.9 m/s.

5.1.6.4 Comparison 4

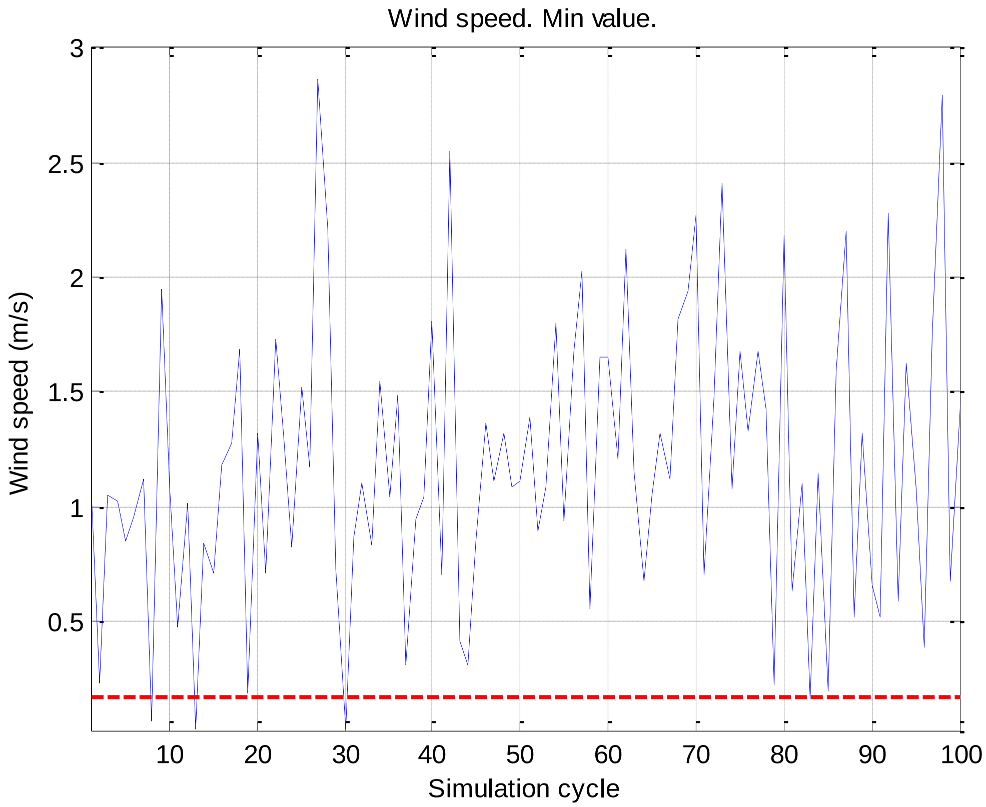

Fig. 74. Minimum value of simulation result per simulation cycle presented together with minimum value of reference measurement.

Mean value for all simulated max values = 1.2 m/s.

Reference value = 0.2 m/s.

5.1.6.5 Comparison 5

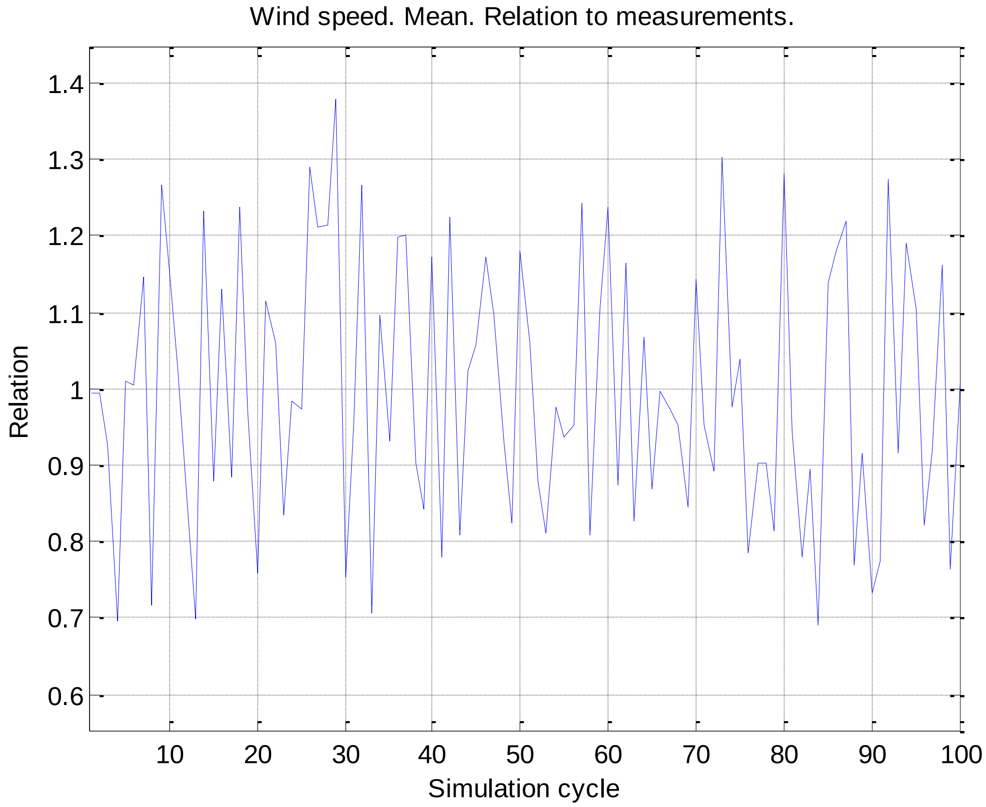

Fig. 75. The relation according to:
"Mean value of simulation result / Mean value of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.6.6 Comparison 6

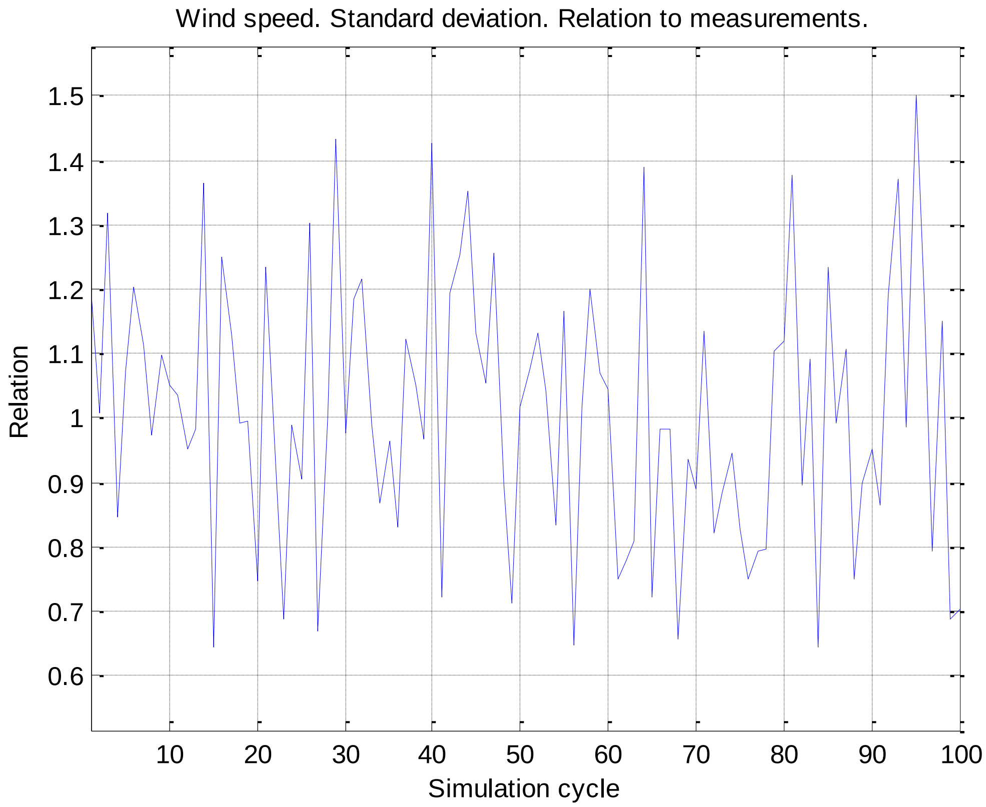

Fig. 76. The relation according to:
"Standard deviation of simulation result / Standard deviation of reference measurement" per simulation cycle.

Mean value over all cycles = 1.0.

5.1.6.7 Comparison 7

Fig. 77. The relation according to:
"Maximum value of simulation result / Maximum value of reference measurement" per simulation cycle.

Mean value over all cycles = 0.7.

5.1.6.8 Comparison 8

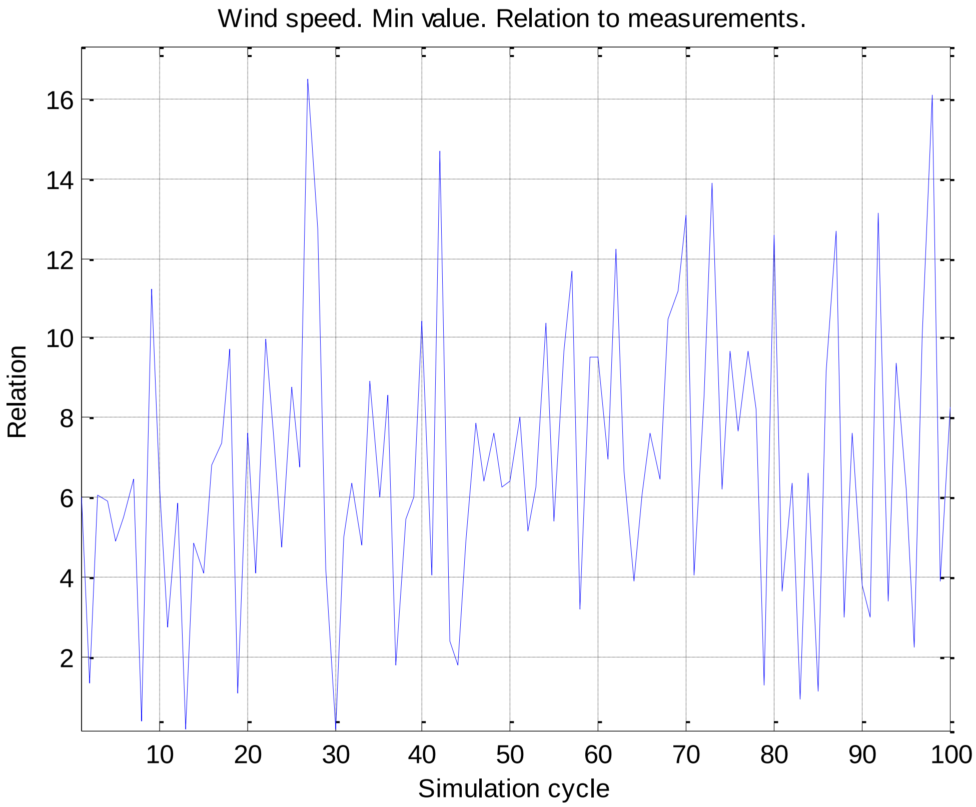

Fig. 78. The relation according to:
"Minimum value of simulation result / Minimum value of reference measurement" per simulation cycle.

Mean value over all cycles = 6.8.

5.2 Summary of validation

The validation shows the following

- The correlation between measured and simulated results are not good regarding maximum or minimum values. These extremes are in the intended application not important. The lack of accuracy in respect of these cases, can therefore be accepted.
- The correlation between measured and simulated results are very good regarding mean values and standard deviations. See Table VIII.

Table VIII. Comparison between measurement and simulation regarding mean value and standard deviation.

Case	**Mean over all cycles regarding the relation: "***Mean value of simulation result / Mean value of reference measurement***"**	**Mean over all cycles regarding the relation: "***Standard deviation of simulation result / Standard deviation of reference measurement***"**
Total year	1	0.9
Winter	1	1
Spring	1	0.9
Summer	1	1
Autumn	1	1

6 Conclusion

This paper presents a model for statistical simulation regarding wind speed. It has been developed to be used as a module in the system that is described in /1/. It is a result of measurements performed during a time period of five years. The investigation is focused on five special cases:

- Winter
- Spring
- Summer
- Autumn
- Total year

The model is validated and meets needed specifications on accuracy.

7 References

/1/ Mathiasson I. "Simulation of Autonomous Electrical Power Systems". Chalmers University of Technology, 2015.

/2/ Koushik Madapati, Magnus Ellsén."Hönö Wind Resource Assessment. A statistical report on Chalmers wind turbine station data". Chalmers University of Technology, 2014.

/3/ Norman L. Johnson, Samuel Kotz. "Continuous univariate distributions 2", John Wiley & Sons, 1970.