Sensing or Transmission: Causal Cognitive Radio Strategies with Censorship

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Abstract—This paper introduces a novel opportunistic transmission strategy for cognitive radios (CRs). The primary user (PU) is assumed to transmit in a time-slotted manner according to a two-state Markov model, and the CR is either sensing, that is, obtaining a causal, noisy observation of a primary user (PU) state, or transmitting, but not both at the same time. In other words, the CR observations of the PU are censored whenever the CR is transmitting. The objective of the CR transmission strategy is to maximize the utilization ratio (UR), i.e., the relative number of the PU-idle slots that are used by the CR, subject to that the interference ratio (IR), i.e., the relative number of the PU-active slots that are used by the CR, is below a certain level. We introduce an a-posteriori LLR-based CR transmission strategy, called CLAPP, and evaluate this strategy in terms of the achievable UR for different PU model parameters and received signal-to-noise ratios (SNRs). The performance of CLAPP is compared with a simple censored energy detection scheme. Simulation results show that CLAPP has 52% gain in UR over the best censored energy detection scheme for a maximum IR level of 10% and an SNR of $-2$ dB.

Index Terms—Spectrum utilization, interference ratio, spectrum sensing, cognitive radio, hidden Markov model, opportunistic spectrum access, DSA, missing observation, censorship, CLAPP.

I. INTRODUCTION

Proliferation of smartphones and hand-held devices has elevated the demand for high speed wireless services. In the United States alone, as part of the national wireless initiative, there are plans to bring wireless broadband internet access to 98% of the Americans [1]. This desire for high data-rate wireless networking creates huge expectations for more frequency spectrum. However, spectrum is a scarce commodity, which is mostly licensed to certain operators commonly known as primary users (PUs) of the band, who spent a considerable amount of investment in retaining the right to use these bands uninterrupted. Notably, this valuable resource is severely under-utilized [2] and many spectrum holes exist in both the time and spatial domains. Due to their great scope of use and benefits, there is significant interest in techniques providing opportunistic secondary spectrum access [3], [4]; these are collectively termed cognitive radio (CR). CRs adapt to exploit communication opportunities in the spectrum by making use of it without interfering with legitimate users.

An enabler for dynamic spectrum reuse by the CR is agile and reliable spectrum sensing [5], which means to estimate when the PU is not transmitting in the licensed band. There have been several attempts to design spectrum sensing schemes that do not need to have a model for PU transmissions. A straightforward approach is energy detection, which simply means to sum up the energy of received samples and compare with a threshold [6]. However, energy detection performance is limited by the signal-to-noise (SNR) wall, which is the SNR below which robust detection is impossible for the given detector [7], due to the low received power of the PU signal at the CR receiver, as well as uncertainties in signals, noise, and channel, which ultimately result in large sensing delays. In wideband spectrum sensing in particular, the tradeoff between agility and reliability is more noticeable [8], [9]. This creates a demand for a CR which uses all previous observations and makes a transmission decision with the shortest possible delay. Sequential spectrum sensing methods, which collect samples sequentially until one of two thresholds is met, are attractive, since they are on average faster than standard energy detection [6], [10], [11].

In contrast to the models which ignore dependencies between PU transmissions, measurement campaigns and recent studies [12] have shown that hidden Markov models (HMM) fit the PU’s behavior in many different bands. Assuming a Markov model for PU activities provides better reutilization of the spectrum whilst being representative of reality and is used in many CR research papers [13]–[19]. To be able to use HMMs for modeling the PU behavior, the knowledge of model parameters is necessary. The impact of model parameters estimation on the CR performance was investigated in [20], [21], which, overall, appears to be quite promising. Another PU models with finite backlog is considered in [22].

To exploit the Markov model, the Markov decision process [13]–[16] and the partially observed Markov decision process [17], [19] are widely used. Moreover, there exist works in the prediction of the future state of the PU [18], [23]–[25]. In our previous work, we have introduced an optimum causal strategy that utilizes this PU behavior [26], [27] with low complexity. This approach not only considers the PU transmission model but also takes the causality of observations into account and thus provides a better reutilization of the spectrum.

In some CR systems, transmission and reception in the same frequency band at the same time are not possible because at a CR receiver, the signal transmitted by the same CR will...
be much stronger than the received PU signal, which the spectrum sensing mechanism is supposed to detect. To avoid this, the optimization of the sensing time vs. transmission time was considered in [28]–[33]. In [34], the energy consumption of sensing was also considered and attempts were made to minimize it. These methods have considered a fixed length of sensing and transmission. In another interesting publication [35], the probability of detection of the PU was considered as the constraint, under which the secondary rate was optimized. The same authors presented efficient spectrum sensing for CR networks via joint optimization of sensing threshold and duration in [36].

In this paper, we extend the findings in [27] and use HMMs as a tool for better sensing and transmitting in spectrum holes. We introduce a CR strategy which can either transmit or sense, but not both at the same time. In this strategy, the sensing is performed as long as it is necessary and then a transmission lasts as long as it is safe enough to transmit. The PU signal is considered missing, or “censored,” during transmission. Furthermore, we introduce a simple and iterative method to calculate a test statistic from the observations with missing PU samples, where, in contrast to [37], the missing observations are dependent on the previous transmission pattern.

Our main contributions are summarized as follows.

• We propose a realistic model, which considers spectrum sensing with missing observations (due to CR transmission).
• We introduce a method for calculating a-posteriori probabilities log-likelihood ratio (APP-LLR) for the future PU transmission from the observation with censorship, which depends iteratively on previous observations.
• A novel transmission strategy for a CR with censored spectrum observations is established.

The main differences with our previous work in [27] results from the censorship. In particular, since the censorship is dependent on previous transmission decisions, the LLR statistics and its cumulative distribution function (CDF) varies over time. Hence,

• a new method to calculate the LLRs is needed, and
• the calculation of the threshold is quite different, since the empirical CDF of the LLRs cannot be used.

Moreover, this paper analyses the performance of the strategies in certain degenerate cases not found in [27].

II. SYSTEM MODEL

A cognitive communication link consists of a CR transmitter–receiver pair and the channel in between. In this paper, we are evaluating the interaction between a single PU and single CR. The wireless channel in between is assumed to be known.

This section presents the abstract system model as depicted in Fig. 1, which accounts for the PU signal, CR noise, and CR reception with censorship. This model demonstrates the PU activity on top, the CR received signal in the middle, and the CR transmission strategy at the bottom. Whenever the CR decides to transmit, the next PU activity is censored. First, a more general perspective is considered and then a simplified version will be used.

Figure 1. System model; \( q_k = 1 \) indicates a PU transmission in the \( k \)th slot and \( q_k = 0 \) indicates no transmission. The received samples \( r(t T_s) \) are circularly symmetric complex Gaussian random variables, whose variance depends on the PU state. The CR uses the energy detector output to decide whether to transmit \((u_{k+1} = 1)\) or not \((u_{k+1} = 0)\) in slot \( k+1 \). If \( u_{k+1} = 1 \), then the next observation is censored.

A. PU Transmission Model

A cognitive radio system is designed to utilize spectrum vacancies. To take advantage of time–frequency slots which are not used by the PU, the CR must be aware of the PU activity model. The CR estimates this by collecting samples from PU transmissions over a noisy channel.

The PU transmissions are assumed to be slotted, since in most of today’s digital communication systems, transmissions are confined within a packet, frame, or generally some block structure of some minimum frame length \( T_f \). However, the CR will model the PU as having a transmission slot length \( T \), where \( T \ll T_f \). We can think of \( T \) as a tuning parameter, whose effect on the CR performance will be explained below. The PU transmission in the \( k \)th time slot, i.e., for \( t \in [kT, (k+1)T) \), is described by the PU transmission state \( q_k \), where \( q_k = 1 \) and \( q_k = 0 \) indicates transmission and no transmission, respectively. For simplicity, we will assume that the time slots are synchronized to the PU transmissions. This is not a very restrictive assumption, since because \( T \ll T_f \), a synchronization mismatch will only affect a small fraction of the slots (namely those slots in which the PU starts or ends a transmission). The PU transmission state sequence \( q_k \), for \( k = 0, 1, \ldots \), is assumed to follow a two-state Markov model with state transition probabilities

\[
a_{ij} \triangleq \Pr\{q_{k+1} = j \mid q_k = i\}, \quad i, j \in \{0, 1\}, \quad (1)
\]
as depicted in the top part of Fig. 1. We assume that the PU does not remain in the same state forever, i.e., that \( a_{ij} > 0 \) for
i \neq j$. Furthermore, we assume that the Markov chain is in steady state at $k = 0$, which implies that the state probabilities do not depend on $k$, i.e., that
\[
\begin{align*}
\pi_0 & \triangleq \Pr(q_k = 0) = \frac{a_{10}}{a_{01} + a_{10}}, \\
\pi_1 & \triangleq \Pr(q_k = 1) = \frac{a_{01}}{a_{01} + a_{10}}
\end{align*}
\]  
(2)
for $k = 0, 1, \ldots, [38]$. The transition probabilities are assumed to be known or accurately estimated from data, e.g., by using the expectation-maximization algorithm [21], [38].

Since the PU is assumed to have a minimum transmission slot $T_F$, a two-state Markov model with slot length $T \ll T_F$ can only approximate the true PU behavior. The reason for selecting $T \ll T_F$ is to improve the CR agility, i.e., its ability to quickly sense changes in the PU state and to mitigate the impact of synchronization errors. However, we cannot choose $T$ to be too small, since the resulting Markov model will lose in accuracy and the SNR will be reduced (as explained below). Finally, we note that as a consequence of choosing $T \ll T_F$, the probability that the PU switches states is small, i.e., we can safely assume that $a_{01} \ll a_{00}$ and $a_{10} \ll a_{11}$.

Another factor in modeling the PU-CR interaction is the channel in between. Wireless channels are normally considered as random fading processes such as Rayleigh, Rician, Nakagami, etc. [39], [40]. Another approach to modeling the fading process is to include the fading in the PU transmission model. Thus, whenever the channel is in a deep fade, it is assumed that there is no PU transmission, no matter what the real state of the PU is. Conversely, in case of no deep fade, the standard PU transmission model is used. Thus, a simple two-state Markov model can approximate a wide range of PU transmissions, PU network activities, and even fading channels.

**B. Signal and Noise Model**

We model the PU-CR channel as an additive white Gaussian noise (AWGN) channel. The complex envelope of the CR received signal, low-pass filtered to the PU signal bandwidth $W$, is
\[
r(t) = \begin{cases} n(t), & q_k = 0 \\ s(t) + n(t), & q_k = 1 \end{cases}, \quad t \in [kT, (k+1)T),
\]
where $n(t)$ is the filtered AWGN channel noise and the contribution from the PU transmitted signal, $s(t)$, is modeled as a circularly symmetric complex Gaussian random process with bandwidth $W$. This PU signal model is common in the literature [41] [4], and is reasonable for many combinations of PU signal formats and channels (fading as well as nonfading). If we select the sample interval $T_s$ such that $T_s \gg 1/W$, then the samples in the $k$th slot can be approximated as independent, identically distributed (i.i.d.) complex Gaussian random variables
\[
r(kT + iT_s) = n(kT + iT_s) + s(kT + iT_s)
\sim \begin{cases} \mathcal{CN}(0, \sigma_0^2), & q_k = 0 \\ \mathcal{CN}(0, \sigma_i^2), & q_k = 1 \end{cases}, \quad i = 0, 1, \ldots, K_{\text{max}},
\]  
(3)
where $\mathcal{CN}(\mu, \sigma^2)$ denotes a circularly symmetric, complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. $K_{\text{max}} = \lfloor T/T_s \rfloor$ with $\lfloor x \rfloor$ being the largest integer not greater than $x$, $\sigma_0^2$ is the noise variance, and $\sigma_i^2 = \sigma_0^2 + \sigma_i^2$ where $\sigma_i^2$ is the signal power. We define the signal-to-noise-ratio as $\text{SNR} \triangleq \sigma_i^2/\sigma_0^2$. For ease of presentation, we fixed $\sigma_0^2 = 2$ in the figures and simulations.

Since we do not have knowledge of the PU signal phase, the CR uses an energy detector front-end to form the statistics
\[
y_k \triangleq \sum_{i=0}^{K-1} \left| r(kT + iT_s) \right|^2,
\]  
(4)
where $K \leq K_{\text{max}}$. Hence, since $y_k$ is the sum of the squared magnitude of $K$ i.i.d. complex Gaussian random variables, or equivalently the sum of $2K$ squared real-valued Gaussian random variables, $y_k$ is proportional to a standard Chi-squared random variable with $2K$ degrees of freedom. To be precise, $y_k/(\sigma_i^2/2) \sim \chi^2_{2K}$ if $q_k = 0$ and $y_k/(\sigma_i^2/2) \sim \chi^2_{2K}$ if $q_k = 1$, where $\chi^2_N$ denotes a standard Chi-squared random variable with $N$ degrees of freedom. The tuning parameter $K$ essentially determines the SNR in $y_k$. Since $K$ must be no greater than $K_{\text{max}} = \lfloor T/T_s \rfloor$, we see that reducing $T$ will eventually limit the maximum SNR.

**C. CR model**

In this paper, it is assumed that the CR always has information to send, i.e., it has a full buffer, and will seek to reuse the spectrum whenever it is available. However, spectrum sensing cannot always be performed, as the CR is not able to observe the spectrum during its transmission periods. This limitation arises from the fact that, in practice, a transmission from a CR transmitter will saturate its receiver and, thus, it will be extremely difficult to sense at the same time in the same frequency band. The CR strategy decides to transmit or sense in each time slot. At time slot $k$, a transmission decision for the next time slot is represented by $u_{k+1} = 1$ and a sensing decision is denoted by $u_{k+1} = 0$. In this paper, we assume that the transmission strategy has access to the spectrum’s energy $y_k$ only when $u_k = 0$. In other words, the CR will observe the list $y_k'$,
\[
y_k' = \begin{cases} y_{k-1}', & \text{if } u_k = 1 \\ [y_{k-1}', y_k], & \text{if } u_k = 0 \end{cases}, \quad k = 1, 2, \ldots,
\]  
(5)
where $y_0' = [\ ]$, i.e., the empty list. Obviously, the length of $y_k'$ is smaller than or equal to $k$.

So, the actual observation, which is used by the strategy to make the next transmission decision, is dependent on the previous transmission decisions.

**D. Definition of a CR transmission strategy**

To be able to judge different cognitive radio transmission strategies with censorship, first we need to establish a proper mathematical definition for such a strategy. Our goal is to design the best CR transmission strategy, with the output $u_{k+1}$, where $u_{k+1} = 0$ and $u_{k+1} = 1$ represent no transmission and
transmission, respectively, in slot $k+1$ using the observations until time $k$, $y'_k$ as defined in (5).

Now, we formally define the CR transmission strategy as a series of functions $f_k(\cdot)$, which produce the transmissions decisions. In other words, a CR transmission strategy is

$$\mathbf{\mathcal{S}} = (f_0, f_1, f_2, \ldots),$$

and

$$u_{k+1} = f_k(y'_k, a_{01}, a_{10}, \sigma^2_0, \sigma^2_1).$$

Later, for ease of notation, we will omit the PU model information $(a_{01}, a_{10}, \sigma^2_0, \sigma^2_1)$ and simply denote the decision functions by $f_k(y'_k)$. This formal definition of a strategy does however not offer a practically implementable CR algorithm, due to the prohibitive complexity of storing and processing the full history $y'_k$. In Sec. III, we will develop a recursive algorithm, which avoids storing $y'_k$.

E. Problem Statement

In wireless communications, cognitive radios are employed to reuse idle spectrum slots by utilizing the spectrum sensing information, whenever possible. The CR has access to observations from the spectrum to decide whether to transmit or not. However, due to the uncertainties in the channel, the noise, and the PU future states, the CR will create unintentional interference for the PU. Interference will happen whenever the CR transmits in a vacant time slot, and this is measured by the interference ratio (IR) $\eta$, quantified by the interference ratio (IR) $\rho$, defined as [26], [27]

$$\rho \triangleq \Pr\{u_{k+1} = 1|q_{k+1} = 1\},$$

where we have implicitly assumed that $k$ is large enough for the initial transient to have passed and that the system is in steady state, in the sense that $\Pr\{u_{k+1}|q_{k+1}\}$ does not depend on $k$. A CR is supposed not to interfere with the PU more than a specific limit $\rho_{\text{max}}$.

As explained in Sec. II-C, a CR strategy considers observations when sensing is allowed and decides about the next transmission. Utilization of the spectrum occurs whenever the CR transmits in a vacant time slot, and this is measured by the spectral utilization ratio (UR), defined as [26], [27]

$$\eta \triangleq \Pr\{u_{k+1} = 1|q_{k+1} = 0\},$$

where we have again assumed that $k$ is sufficiently large.

Algorithm 1 Baseline strategy

Input: Sense/transmit frame length $n$ and threshold $\theta_c$
Output: Transmission decisions $u_1, u_2, u_3, \ldots$
1: Initialize $k \leftarrow 1$
2: loop
3: Let $u_k \leftarrow 0$ and take an energy sample $y_k$ for slot $k$
4: if $y_k \leq \theta_c$ then
5: Let $u_{k+1} \leftarrow 1, \ldots, u_{k+n-1} \leftarrow 1$, i.e., transmit in $n-1$ consecutive slots
6: $k \leftarrow k + n$
7: else
8: $k \leftarrow k + 1$
9: end if
10: end loop

F. Baseline strategy

The baseline strategy, which is using energy detection and censoring, is explained in Algorithm 1. In this strategy there are two parameters to optimize: $\theta_c$ and $n$. These two design parameters should be chosen to maximize $\eta$, subject to the condition $\rho \leq \rho_{\text{max}}$. Moreover, we know that UR and IR are increasing functions of $\theta_c$ [26], [27]. Thus, to find optimum parameters for the baseline strategy, we first fix $n$ and find the threshold such that $\rho = \rho_{\text{max}}$ through bisection search. Then we repeat this for different $n$ to maximize the UR. For different $a_{01}, a_{10}, \sigma^2_0, \sigma^2_1$, and $\rho_{\text{max}}$, different sets of $\theta_c$ and $n$ must be chosen.

However, this strategy has some limitations. By design, the baseline CR transmits for $n-1$ consecutive slots before sensing for at least one slot. Hence, even if the CR transmits as often as it can, i.e., for $n-1$ out of $n$ slots, the UR and IR are upper-bounded: $\eta, \rho \leq (n-1)/n = 1 - 1/n$. Clearly, we can remove this problem by increasing $n$. However, as $n$ increases, we need to decrease the threshold $\theta_c$ to ensure that $\rho \leq \rho_{\text{max}}$, and this will lead to fewer transmissions and a reduced $\eta$. Intuitively, we therefore expect that there exists a finite optimum $n$ for each combination of PU parameters $(a_{01}, a_{10}, \sigma^2_0, \sigma^2_1)$. This intuition is verified by the numerical results in Sec. IV-B. Hence, the complexity in finding the optimum $n$ for the baseline strategy is not excessive, and is anyways not important, since we are not suggesting to use the baseline strategy in practice.

III. Censored APP-LLR based Cognitive Transmission Strategy

In this section, we introduce a new strategy which observes spectrum energy samples through censorship by its own transmissions. In our previous paper [27], we have shown that the APP-LLR transmission strategy is optimum in an uncensored scenario (i.e., when simultaneous sensing and transmission are allowed). Now, since we are dealing with censored observations, a straightforward direction for designing a CR strategy is to extend the non-censored APP-LLR (NCLAPP) strategy in [27] to the censored APP-LLR (CLAPP) strategy described below. CLAPP (as well as NCLAPP) has the advantages that

- it captures all information about the previous observations recursively,
- it includes the PU Markov model in its decisions,
it predicts the next state of the PU.
At the same time, NCLAPP is very simple to implement.

\textbf{A. Introduction to the CLAPP strategy}

In our previous contribution [27], the LLRs were calculated based on the forward variables \(\alpha_k(j)\) of all observations. Specifically, NCLAPP calculates the LLRs as [26, Eqs. (19)–(21)]

\[ z_k \triangleq \log \frac{\Pr \{ q_{k+1} = 1|y_k \}}{\Pr \{ q_{k+1} = 0|y_k \}} = \log \frac{a_{01}\alpha_k(0) + a_{11}\alpha_k(1)}{a_{00}\alpha_k(0) + a_{10}\alpha_k(1)}, \tag{10} \]

where \(y_k \triangleq [y_1, y_2, \ldots, y_k]\) and \(\alpha_k(j) \triangleq \Pr \{ q_k = j|y_k \}, \ j \in \{0,1\}\).

The CLAPP algorithm is explained in Algorithm 2. In CLAPP, we have access only to the censored observations \(y'_{k}\). Thus, we have to calculate the LLRs based on \(y'_{k}\). We first define the censored APP LLRs as

\[ z'_k \triangleq \log \frac{\Pr \{ q_{k+1} = 1|y'_k \}}{\Pr \{ q_{k+1} = 0|y'_k \}} \tag{11} \]

and then express \(z'_k\) in terms of the joint distributions instead of the conditionals as

\[ z'_k = \log \frac{p(q_{k+1} = 1,y'_k)}{p(q_{k+1} = 0,y'_k)} = \log \frac{\gamma_k(1)}{\gamma_k(0)}, \tag{12} \]

where \(\gamma_k(j) \triangleq p(q_{k+1} = j,y'_k)\). These censored LLRs capture all the information needed for making decisions based on the censored observations. Clearly, a decision rule of the general form (7) requires enormous amounts of memory for storing all previous censored observations \(y'_{k}\). However, Algorithm 2 has the advantage of requiring only the latest observation for making decisions, which is very suitable for real world implementations.

One simple approach in NCLAPP, which was proven to be optimal in terms of \(\eta\) in [27], is the comparison of LLRs with a fixed threshold. In CLAPP, we also implement the same approach and compare the censored LLRs with a fixed threshold. The strategy \(u_{k+1} = f_k(y'_k)\) is thus

\[ f_k(y'_k) = \begin{cases} 1, & \text{if } z'_k \leq \theta_l \\ 0, & \text{if } z'_k > \theta_l \end{cases}, \tag{13} \]

where \(\theta_l\) is the threshold found using the bisection search described below. Fig. 2 demonstrates the performance (IR and UR) of CLAPP versus the threshold. As expected, UR and IR approach zero for very low thresholds. For higher thresholds, both UR and IR approach one (which eventually violates the IR requirement). As we can see, there is a smooth, monotonic transition for \(\rho\) and \(\eta\) from zero to one as the threshold increases; the smoothness property holds in general, except in a degenerate situation which is explained in Section III-C. This smooth transition enables us to compute the threshold with a bisection search. This search method is quite fast and determines a threshold with a resulting IR close to \(\rho_{\text{max}}\) in a training period for which energy samples and corresponding PU states \(q_k\) are known. The CR can also compute the IR as a function of the threshold without help from the PU by, e.g., simulating the PU activity and the resulting observations (which is possible given the system parameters).

The behavior of CLAPP for a PU with high activity level \((\pi_1 > \pi_0)\) and long transmission bursts (i.e., periods for which \(q_k = 1\)) and a less active PU \((\pi_1 < \pi_0)\) with short transmission bursts is shown in Figs. 3 and 4, respectively. As expected, for the PU in Fig. 3, fewer observations are censored and the CLAPP LLR follows closely the NCLAPP LLR at the end of the transmission burst. During the period when \(q_k = 0\), some observations should still be made to detect a change of PU state. As seen from the plots, the CLAPP LLR increases as a function of time when the CR is transmitting and will eventually reach the threshold at which the CR ceases transmission and senses the channel. The LLR is a measure of how the CR perceives the risk that the PU is transmitting. Without observations, the risk increases until it reaches the threshold, and CLAPP decides to sense the channel to make sure it is on the safe side.

In the next section, we show that CLAPP is a reasonable choice when our observations are suffering from censorship.
Even though we cannot prove that CLAPP is the optimum censored strategy, we will show that the new method of calculating the censored LLRs will capture the information needed to make a decision based on the censored sequence of observations.

B. Validity and Derivation of CLAPP strategy

During transmission, the observation for that slot will be missing. Censorship, due to the transmissions, must be reflected in the calculation of LLRs. To simplify the analysis and implementation of CLAPP, the threshold $\theta_l$ for decision-making on whether to sense or transmit is time-invariant.

In Fig. 5 we depict the relationship between PU state ($q_k$), received energy ($y_k$), observed energy at CR after censorship ($y'_k$), and the CR transmission decision ($u_k$) which causes the censorship. If the CR decides to transmit ($u_k = 1$), the switch in Fig. 1 will be open and the energy will not be observed at the CR.

If the CR decides to sense, the received energy sample $y_k$, conditioned on $q_k = j$ and normalized with $\sigma_j^2/2$, is a standard Chi-square random variable with $2K$ degrees of freedom. Hence, the conditional pdf for $y_k$ is

$$b_j(y_k) \triangleq p(y_k|q_k = j) = \begin{cases} \frac{1}{\sigma_j^{2K}(K-1)!} y_k^{K-1} e^{-y_k/\sigma_j^2}, & \text{if } y_k \geq 0, \\ 0, & \text{if } y_k < 0. \end{cases}$$

(14)

In the following theorem, we presented an iterative method for calculating the joint distribution of the censored observations and the future PU state, known as $\gamma_k(j)$.

**Theorem 1:** For a given sequence of observation $y'_k$ censored by $u_k = f_{k-1}(y'_{k-1})$, a sequence of transmission decision functions $f_k$, system model parameters $a_{01}, a_{10}, \sigma_0^2, \sigma_j^2$, distributions of $y_k$ under noise only and signal plus noise, $b_0$ and $b_1$, respectively, $\gamma_k(j) \triangleq p(q_{k+1} = j, y'_k)$ can be calculated recursively as $\gamma_0(j) = \pi_j$ and

$$\gamma_k(j) = \begin{cases} \sum_{i=0}^{1} \gamma_{k-1}(i) a_{ij}, & \text{if } f_{k-1}(y'_{k-1}) = 1, \\ \sum_{i=0}^{1} \gamma_{k-1}(i) a_{ij} b_i(y_k), & \text{if } f_{k-1}(y'_{k-1}) = 0. \end{cases}$$

(15)
no censoring. However, since the censored observation vector \( y'_k \) is dependent on the previous decisions \( u_1, u_2, \ldots, u_{k-1} \), it is rather difficult to prove that CLAPP is an optimum strategy for all sequences of observations. Indeed, it is not even straightforward to define optimality in a CR system with censorship dependent on previous decisions.

There are cases in which the transition probabilities are time-varying and hence this simple model does not hold. However, a more general variant of Markov models, namely semi-Markov models, can be employed [42]. All derivations presented in the rest of this paper can be generalized using semi-Markov models, which is outside the scope of this paper.

C. CLAPP limitation

Both CLAPP and NCLAPP are based on the assumption that the PU is following a Markov model. However, in the special case when \( a_01 + a_{10} = 1 \), the next state of the PU is independent of all previous observations. In this case, since \( a_01 = a_{11} \) and \( a_{10} = a_{00} \), the future state of the PU \( q_{k+1} \) is independent of the current state \( q_k \). Furthermore, since \( y_k, y_{k-1}, \ldots, y_1 \) are functions of \( q_k, q_{k-1}, \ldots, q_1 \) and the noise, \( q_{k+1} \) is independent of all observations \( y_k \). Thus, any causal CR, operating in the presence of such PUs, cannot perform better than a randomized transmission scheme that ignores the observations and transmits with probability \( \rho_{\text{max}} \). This is also in accordance with [27, Eq. (8)], where an upper bound of \( \eta \) for any causal CR is specified. For \( a_01 + a_{10} = 1 \), the upper bound of \( \eta \) is \( \rho_{\text{max}} \). For the case when \( a_01 + a_{10} \) is close to one, intuitively we expect that CLAPP and NCLAPP lose their ability to predict the PU states. Indeed, simulation results presented in Sec. IV-B show that the problem of unpredictable PU states kicks in not only when \( a_01 + a_{10} = 1 \), but also when \( a_01 + a_{10} \) is close to one.

This is, however, not a serious limitation, since as mentioned in Sec. II-A, we are mainly interested in the case \( a_01 \ll 1/2 \) and \( a_{10} \ll 1/2 \).

IV. PERFORMANCE EVALUATION AND RESULTS

In this section, we compare CLAPP with optimized censored energy detection (baseline) and NCLAPP (no censorship). All of the comparisons are performed with the same PU model, the same level of maximum interference \( \rho_{\text{max}} \), and even the same samples to ensure fairness. To find thresholds in the censored methods, we use a simple bisection search to obtain an IR as close as possible to \( \rho_{\text{max}} \), within a certain small tolerance, but no more than \( \rho_{\text{max}} \). The rest of this section discusses the evaluation setup by which these CRs are assessed. It then presents some results and comparisons.

A. Evaluation Setup

In simulating the performance of a CR transmission strategy, the ratio of received primary signal power (at the CR receiver) to the CR receiver noise power is important. In this simulation, \( K \), which is another design parameter, is selected to be 5. This parameter plays a role for the SNR scaling. The higher the \( K \), the higher the SNR, which is translated into better CR performance. However, higher \( K \) means more delay in the decision making. This can in turn reduce the performance of the CR. The other factor which is important in evaluating CRs is the maximum allowable IR, \( \rho_{\text{max}} \). This parameter is normally decided by regulatory authorities like the Federal Communications Commission (FCC). In practice, \( \rho_{\text{max}} \) must be small and we have chosen it to be 10% as suggested in [32].

We are interested in examining the impact of an active PU with long transmission bursts (\( \pi_1 > \pi_0 \) and \( 1/\pi_{10} \) large) and an infrequently active PU with short transmission bursts (\( \pi_1 < \pi_0 \) and \( 1/\pi_{10} \) small). We further want to observe what happens when \( a_01 + a_{10} \) is close to one. Thus, we have simulated the cases when \( (a_01, a_{10}) = (0.10, 0.01) \Rightarrow \pi_1 = 0.91, 1/\pi_{10} = 100 \) slots, \( (a_01, a_{10}) = (0.01, 0.10) \Rightarrow \pi_1 = 0.091, 1/\pi_{10} = 10 \) slots, and \( (a_01, a_{10}) = (0.45, 0.30) \). To find the threshold, \( 10^6 \) simulated slots are used. To evaluate the performance, another \( 10^6 \) slots are simulated.

B. Results

The UR of the different CRs are plotted versus SNR in Figs. 6 and 7 for different PU parameters. Fig. 6 depicts UR vs. SNR for an active PU with long transmission bursts. UR is an increasing function of SNR, as expected. For very low SNR, there is little information in the observations. Thus, no strategy can perform better than a random transmission, i.e., when the CR transmits with probability \( \rho_{\text{max}} \), regardless of the observations, which results in \( \eta = \rho_{\text{max}} \). However, at SNRs as low as \(-10\) dB, the impact of including the PU model knowledge in transmission decisions is apparent. At the SNR of \(-2\) dB, CLAPP has 52% gain over the best censored energy detection (the one with \( n = 5 \)). In the high SNR region, such as \( 13\) dB, this gain over the censored baseline with \( n = 5 \) reduces to 32% and over the best censored baseline with \( n = 12 \) to 10%. This UR gain is due to utilizing PU model knowledge and memory in the system to predict the future state of the PU. Censorship costs some utilization gain for the CLAPP and the baseline strategies compared with NCLAPP.

In Fig. 7, we have evaluated UR vs. SNR for a less active PU with short transmission bursts. The same general trend is visible as in Fig. 6. CLAPP has 47% UR gain over the best censored baseline with \( n = 3 \), at a low SNR of \(-2\) dB.

At high SNR, the IR is dominated by the time from a transition from \( q_k = 0 \) to 1 until the CR notices this transition and stops transmitting. If the PU is expected to transmit for a...
long time, the delay between sensing times can be relatively large without violating the constraint \( \rho \leq \rho_{\text{max}} \). As mentioned before, the expected duration of a PU transmission is \( 1/a_{10} \). This value is less in Fig. 7 than in Fig. 6, and hence, the CR needs to sense more often to maintain the same \( \rho_{\text{max}} \). This is the reason why the censored strategies, CLAPP and baseline, yield a lower UR at high SNR in Fig. 7 than in Fig. 6. The uncensored strategy NCLAPP, on the other hand, experiences a higher UR in Fig. 7 than in Fig. 6, because the UR in this case approaches \( 1 - a_{01} \) at high SNR (the expected duration of a period of PU silence is \( 1/a_{01} \) time slots, and the CR transmits during all except the first of these slots).

In Fig. 8, UR is plotted as a function of IR for different SNRs and PU Markov parameters. The left column is for relatively low SNR (0 dB) and the right column for relatively high SNR (10 dB). The top row plots are for a PU with relatively long transmission bursts \( (1/a_{10} = 100) \) slots), the middle row is for shorter transmission bursts \( (1/a_{10} = 10) \) slots), and the bottom row is for a rather unpredictable PU \( (a_{10} + a_{01} = 0.75); \text{ recall that the PU is completely unpredictable when } a_{10} + a_{01} = 1 \). As expected, all transmission strategies perform similarly for the latter case. Otherwise, CLAPP gives the largest gains compared to the baseline methods for low SNR (left column), and the loss for CLAPP versus NCLAPP is smallest for long PU transmission bursts (middle row). However, in all cases, CLAPP performs better than all baseline methods, especially for low IRs, which is the more practically relevant region. As explained earlier, the UR and IR for the baseline method is upper-bounded by \( (n - 1)/n \). Hence, the UR versus IR curve for the baseline methods is only defined for \( 0 \leq \rho \leq (n - 1)/n \).

V. Conclusion

In this paper, we have introduced a cognitive radio framework which either senses the spectrum or transmits in it, in the presence of a Markovian PU. To capture all the effects that the CR will experience, the PU system is modeled as a hidden Markov model whose continuous-amplitude outputs \( r(iT_s) \) are censored by CR transmission decisions. The performance of each transmission strategy is judged by the maximum achievable UR, under the constraint that IR does not exceed a fixed constant \( \rho_{\text{max}} \).

A new LLR-based CR strategy, called CLAPP, has a substantial UR gain over the optimized baseline method. The gains are more pronounced for low to moderate SNRs. For high SNRs, the gains are smaller, and for very low SNRs, all methods perform similarly, which is expected since the received signal contains very little information about the PU. However, in all cases, the CLAPP UR is an upper bound to the baseline method UR.

The loss in UR for CLAPP versus an (idealized) CR that can sense and transmit at the same time is more significant for PUs with transmission bursts that are short relative to the CR slot time. We can, however, partially compensate for this loss by reducing the CR slot time.

The same is true for an unpredictable PU (i.e., when knowledge of the current PU state implies little knowledge about future states).


Appendix

Proof of Theorem 1

By forming the joint distribution of \( p(y'_k, q_{k+1}, q_k) \), factorizing it according to the Markov chain presented in Fig. 5, marginalizing with respect to \( q_k \), and utilizing (5), we obtain
Figure 8. UR vs. IR for SNR = 0 dB (left column) and 10 dB (right column) (black solid is CLAPP, black dashed is NCLAPP and the colored curves with numbers are baselines).
for $k = 1, 2, \ldots$ if $f_k(y'_{k-1}) = 1$

$$\gamma_k(j) \triangleq p(q_{k+1} = j, y'_k)$$

$$= \sum_{i=0}^{1} p(y'_k, q_k = i, q_{k+1} = j)$$

$$= \sum_{i=0}^{1} p(y'_{k-1}, q_k = i, q_{k+1} = j)$$

$$= \sum_{i=0}^{1} p(q_k = i, y'_{k-1}) Pr\{q_{k+1} = j | q_k = i, y'_{k-1}\}$$

$$= \sum_{i=0}^{1} p(q_k = i, y'_{k-1}) Pr\{q_{k+1} = j | q_k = i\}$$

$$= \sum_{i=0}^{1} \gamma_{k-1}(i) a_{ij},$$

and if $f_k(y'_{k-1}) = 0$

$$\gamma_k(j) \triangleq p(q_{k+1} = j, y'_k)$$

$$= \sum_{i=0}^{1} p(y'_k, q_k = i, q_{k+1} = j)$$

$$= \sum_{i=0}^{1} p(y'_{k-1}, y_k, q_k = i, q_{k+1} = j)$$

$$= \sum_{i=0}^{1} p(q_k = i, y'_{k-1}) Pr\{q_{k+1} = j | q_k = i, y'_{k-1}\}$$

$$= \gamma_k(j) Pr\{q_{k+1} = j | q_k = i\}$$

$$= \gamma_k(j) p(y_k | q_k = i)$$

$$= \sum_{i=0}^{1} \gamma_{k-1}(i) b_{ij}(y_k).$$

The recursion is initiated by $\gamma_0(j) = p(q_1 = j, y'_0) = Pr\{q_1 = j\} = \pi_j$, since $y'_0 = [\; ]$, i.e., the empty list. Hence, the theorem is proven.

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