Effects of Three-Nucleon Forces and Two-Body Currents on Gamow-Teller Strengths

Physical Review Letters (ISSN: 0031-9007)

Citation for the published paper:
Ekström, A. ; Jansen, G. ; Wendt, K. (2014) "Effects of Three-Nucleon Forces and Two-Body Currents on Gamow-Teller Strengths". Physical Review Letters, vol. 113(26), pp. 262504.

http://dx.doi.org/10.1103/PhysRevLett.113.262504 arXiv:1406.4696v1 [nucl-th] 18 Jun 2014

Effects of three-nucleon forces and two-body currents on Gamow-Teller strengths

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We optimize chiral interactions at next-to-next-to leading order to observables in two- and three-nucleon systems, and compute Gamow-Teller transitions in ^{14}C and ^{22,24}O using consistent two-body currents. We compute spectra of the daughter nuclei ^{14}N and ^{22,24}F via an isospin-breaking coupled-cluster technique, with several predictions. The two-body currents reduce the Ikeda sum rule, corresponding to a quenching factor $q^2 \approx 0.84 - 0.92$ of the axial-vector coupling. The half life of ^{14}C depends on the energy of the first excited 1^+ state, the three-nucleon force, and the two-body current.

PACS numbers: 23.40.-s, 24.10.Cn, 21.10.-k, 21.30.-x

Introduction. – β decay is one of the most interesting processes and most useful tools in nuclear physics. On the one hand, searches for neutrino-less double-β decay probe physics beyond the standard model and basic properties of the neutrino, see Avignone et al. [1] for a recent review. If neutrinoless double-β decay is observed, an accurate nuclear-physics matrix element is needed to extract neutrino masses from the life time. On the other hand, β decay of rare isotopes populates states in exotic nuclei and thereby serves as a spectroscopic tool [2, 3]. The theoretical calculation of electroweak transition matrix elements in atomic nuclei is a challenging task, because it requires an accurate description of the structure of the mother and daughter nuclei, and an employment of a transition operator that is consistent with the Hamiltonian.

For the transition operator, the focus is on the role of meson-exchange currents [4] and two-body currents (2BCs) from chiral effective field theory (χEFT). Two-body currents are related to three-nucleon forces (3NFs) [5, 6] because the low energy constants (LECs) of the latter constrain the former within χEFT. Consistency of Hamiltonians and currents is one of the hallmarks of an EFT [7], and 2BCs are applied in electromagnetic processes of light nuclei, see Gazit et al. [8], Grießhammer et al. [9], and Pastore et al. [10]. For weak decays, only the calculation of triton β decay [8], the related μ decay on ^{3}He and the deuteron [11], and proton-proton fusion [12], exhibits the required consistency, while the very recent calculation of the neutral-current response in ^{12}C employs phenomenological 3NFs together with chiral 2BCs [13].

The one-body operator $g_A \sum_{i=1}^{A} \sigma_i \tau_i^{\pm}$ induces Gamow-Teller transitions. Here g_A is the axial-vector coupling, σ denotes the spin, and $\tau^{\pm}$ changes the isospin. Gamow-Teller strength functions [14, 15] are of particular interest also because of their astrophysical relevance [16]. Charge-exchange measurements on ^{90}Zr and other medium mass nuclei have suggested that the total strength for β decay is quenched by a factor of $q^2 \approx 0.88 - 0.92$ [17–20] when compared to the Ikeda sum rule [21]. Similarly, shell-model calculations [22, 23] suggest that g_A needs to be quenched by a factor $q \approx 0.75$ to match data. It is not clear whether renormalizations (including 2BCs) of the employed Gamow-Teller operator, missing correlations in the nuclear wave functions, or model-space truncations are the cause of this quenching.

Recent calculations [24–26] show that chiral 2BCs yield an effective quenching of g_A. However, the Hamiltonians employed in these works are not consistent with the currents (and they contain no 3NFs), and/or the 2BCs are approximated by averaging the second nucleon over the Fermi sea of symmetric nuclear matter. The recent studies [27, 28] of electroweak transitions in light nuclei employ 3NFs but lack 2BCs. This gives urgency for a calculation of weak decays that employs 3NFs and consistent 2BCs.

In this Letter, we address the quenching of g_A and employ 3NFs together with consistent 2BCs for the computation of β decays and the Ikeda sum rule. We study the β decays of ^{14}C and ^{22,24}O with interactions and currents from χEFT at next-to-next-to leading order (NNLO) for cutoffs $\Lambda_\chi = 450, 500, 550$ MeV. For the states of the daughter nuclei, we generalize a coupled-cluster technique and compute them as isospin-breaking excitations of the mother nuclei. We present predictions and spin assignments for the exotic isotopes ^{22,24}F, and revisit the anomalously long half life of ^{14}C [28–30].

Hamiltonian and model space. – The chiral nucleon-nucleon (NN) interactions are optimized to the proton-proton and the proton-neutron scattering data for laboratory scattering energies below 125 MeV, and to deuteron observables. The χ^2/datum varies between 1.33 for $\Lambda_\chi = 450$ MeV and 1.18 for $\Lambda_\chi = 550$ MeV. The χ^2-

optimization employs the algorithm POUNDERS [31]. Table I shows the parameters of the NN interaction for the cutoff $\Lambda_\chi = 500$ MeV; the parameters for the other cutoffs are supplementary material. The parameters displayed in Table I are close to those of the chiral interaction NNLO$_{\mathrm{opt}}$ [32].

LEC	value	LEC	value	LEC	value
c_1	-0.91940746	c_3	-3.88983848	c_4	4.30736747
$\tilde{C}^{pp}_{^1S_0}$	-0.15136364	$\tilde{C}^{np}_{^1S_0}$	-0.15215263	$\tilde{C}^{nn}_{^1S_0}$	-0.15180482
$C_{^1S_0}$	2.40431235	$C_{^3S_1}$	0.92793712	$\tilde{C}_{^3S_1}$	-0.15848125
$C_{^1P_1}$	0.41482908	$C_{^3P_0}$	1.26578978	$C_{^3P_1}$	-0.77998484
$C_{^3S_1-^3D_1}$	0.61855040	$C_{^3P_2}$	-0.67347042		

TABLE I. Pion-nucleon LECs c_i and partial-wave contact LECs (C, $\tilde{C}$) for the chiral NN interaction at NNLO using $\Lambda_\chi = 500$ MeV and $\Lambda_{\mathrm{SFR}} = 700$ MeV [33]. The c_i, $\tilde{C}_i$, and C_i have units of GeV^{-1}, 10^4 GeV^{-2}, and 10^4 GeV^{-4}, respectively.

The 3NF is regularized with nonlocal cutoffs [34, 35] (to mitigate the convergence problems documented by Hagen et al. [36] for local cutoffs). Following Gazit et al. [8], we optimize the two LECs (c_D and c_E) of the 3NF to the ground-state energies of $A = 3$ nuclei and the triton lifetime. Figure 1 shows the reduced transition matrix element $\langle E_1^A \rangle = \langle {}^3\mathrm{He} || E_1^A || {}^3\mathrm{H} \rangle$ as a function of c_D. Here E_1^A is the $J = 1$ electric multipole of the weak axial vector current at NNLO [8]. The leading-order (LO) contribution to E_1^A is proportional to the one-body Gamow-Teller operator, $E_1^A|_{\mathrm{LO}} = i g_A (6\pi)^{-1/2} \sum_{i=1}^{A} \sigma_i \tau_i^{\pm}$. For the current we use the empirical value $g_A = 1.2695(29)$. The 2BCs enter at NNLO and depend on the LECs c_D, c_3, c_4 of the chiral interaction [37, 38]. The triton half-life yields an empirical value for $\langle E_1^A \rangle_{\mathrm{emp}}$, which constrains c_D and c_E. For the three different chiral cutoffs $\Lambda_\chi = 450, 500, 550$ the sets of (c_D, c_E) that reproduce the triton half-life and the $A = 3$ binding energies are $(0.0004, -0.4231), (0.0431, -0.5013), (0.1488, -0.7475)$, respectively. The vertical bands in Fig. 1 give the range of c_D that reproduce $\langle E_1^A \rangle_{\mathrm{emp}}$ within the experimental uncertainty.

We employ an $N = 12$ model space consisting of $N + 1$ oscillator shells with frequency $\hbar\Omega = 22$ MeV. The 3NFs use an energy cutoff of $E_{3\max} = N\hbar\Omega$, i.e. the sum of the excitation energies of three nucleons does not exceed $E_{3\max}$. We employ the intrinsic Hamiltonian

$$H = T - T_{\mathrm{cm}} + V_{NN} + V_{3\mathrm{NF}} \tag{1}$$

to mitigate any spurious center-of-mass excitations [39, 40]. Here, T and T_{cm} are the kinetic energy and the kinetic energy of the center-of-mass, while V_{NN} and $V_{3\mathrm{NF}}$ are the chiral NN interaction and 3NF, respectively.

We perform a Hartree-Fock (HF) calculation and compute the normal-ordered Hamiltonian H_{N} with respect to the resulting reference state $|\mathrm{HF}\rangle$. We truncate H_{N} at the normal-ordered two-body level, and note that 3NFs contribute to the vacuum energy, and the normal-ordered one-body and two-body terms. This approximation is accurate in light and medium-mass nuclei [41, 42].

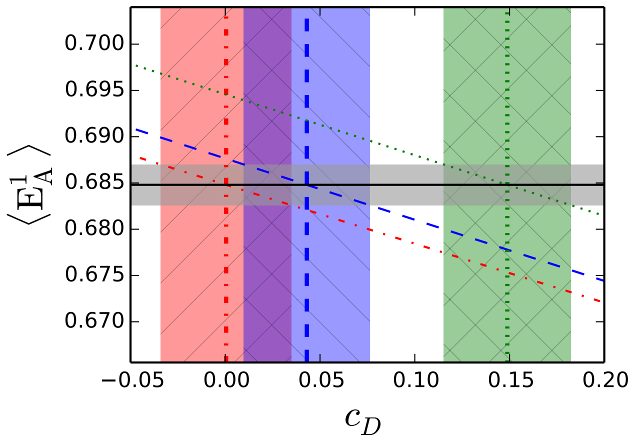

FIG. 1. (Color online) The quantity related to the triton half life $\langle E_1^A \rangle$ as a function c_D for chiral cutoffs $\Lambda_\chi = 450, 500, 550$ MeV (red dashed-dotted, blue dashed, green dotted, respectively) with corresponding error bands. The different lines was determined by a fit of c_D and c_E to $A = 3$ binding energies.

Formalism. – We compute the closed-subshell mother nuclei ^{14}C and ^{22,24}O with the coupled-cluster method [43–50]. The similarity-transformed Hamiltonian

$$\overline{H} \equiv e^{-T} H_{\mathrm{N}} e^{T} \tag{2}$$

employs the cluster amplitudes

$$T = \sum_{ia} t_i^a N_a^\dagger N_i + \frac{1}{4} \sum_{ijab} t_{ij}^{ab} N_a^\dagger N_b^\dagger N_j N_i \tag{3}$$

that create 1-particle – 1-hole (1p-1h) and 2-particle – 2-hole (2p-2h) excitations. Here, i, j denote occupied orbitals of the HF reference while a, b denote orbitals of the valence space. The operators $N_q^\dagger$ and N_q create and annihilate a nucleon in orbital q, respectively. It is understood that the cluster amplitudes T do not change the number of protons and neutrons, i.e. they conserve the z-component T_z of isospin. We note that $|\mathrm{HF}\rangle$ is the right ground state of the non-Hermitian Hamiltonian $\overline{H}$. Its left ground state is not $\langle \mathrm{HF}|$ but $\langle \Lambda | = \langle \mathrm{HF}|(1 + \Lambda)$, with Λ being a linear combination of 1p-1h and 2p-2h de-excitation operators [49, 50].

The daughter nuclei ^{14}N and ^{22,24}F are computed via a novel generalization of the coupled-cluster equation-of-motion approach [51–53]. We view the states of the daughter nuclei as isospin-breaking excitations $|R\rangle \equiv R|\mathrm{HF}\rangle$ of the coupled-cluster ground state, with

$$R \equiv \sum_{ia} r_i^a p_a^\dagger n_i + \frac{1}{4} \sum_{ijab} r_{ij}^{ab} p_a^\dagger N_b^\dagger N_j n_i \; . \tag{4}$$

Here, $p_q^\dagger$ and p_q ($n_q^\dagger$ and n_q) create and annihilate a proton (neutron) in orbital q. The combination $N_q^\dagger N_s$ either involves neutrons $N_q^\dagger N_s = n_q^\dagger n_s$ or protons $N_q^\dagger N_s = p_q^\dagger p_s$. We note that R lowers the isospin component T_z of the HF reference by one unit and keeps the mass number unchanged.

The states of the daughter nucleus result from solving the eigenvalue problem $\overline{H} R_\alpha |\mathrm{HF}\rangle = \omega_\alpha R_\alpha |\mathrm{HF}\rangle$. Here, ω_α is the excitation energy with respect to the HF reference, and R_α denotes a set of amplitudes $R_\alpha = (r_i^a(\alpha), r_{ij}^{ab}(\alpha))$. We recall that the similarity-transformed Hamiltonian in Eq. (2) is not Hermitian. Therefore, we also introduce the left-acting de-excitation operator

$$L \equiv \sum_{ia} l_a^i n_i^\dagger p_a + \frac{1}{4} \sum_{ijab} l_{ab}^{ij} n_i^\dagger N_j^\dagger N_b p_a \ , \tag{5}$$

and solve the left eigenvalue problem $\langle \mathrm{HF}| L_\beta \overline{H} = \omega_\beta \langle \mathrm{HF}| L_\beta$. The left and right eigenvectors are bi-orthogonal, i.e. $\langle \mathrm{HF}| L_\alpha R_\beta |\mathrm{HF}\rangle = \sum_{ia} l_a^i(\alpha) r_i^a(\beta) + \frac{1}{4}\sum_{ijab} l_{ab}^{ij}(\alpha) r_{ij}^{ab}(\beta) = \delta_{\alpha\beta}$.

The operators R and L in Eqs. (4) and (5) excite states in the daughter nucleus that results from β^- decay. If instead we were interested in β^+ decay, we would employ $R^\dagger$ and $L^\dagger$, and solve the corresponding eigenvalue problems. Our approach allows us to compute excited states in the daughter nucleus that are dominated by isospin-breaking 1p-1h excitations of the closed-shell reference $|\mathrm{HF}\rangle$ (with 2p-2h excitations being smaller corrections).

Results. – The spectra for ^{14}N and ^{22,24}F are shown in Fig. 2 for $\Lambda_\chi = 500$ MeV and compared to data. Errorbars from variation of the chiral cutoff Λ_χ are shown for selected states. The odd-odd daughter nuclei ^{14}N and ^{22,24}F exhibit a higher level density than their mother nuclei. Overall, 3NFs increase the level densities slightly and yield a slightly improved comparison to experiment. For the neutron-rich isotopes of fluorine we make several predictions and spin assignments. In these isotopes, our spectra compare also well to shell-model calculations by Brown and Richter [54]. The ground state energies of the mother nuclei (obtained at $N = 12, \hbar\Omega = 22$ MeV and $\Lambda_\chi = 500$ MeV) are -74.4 MeV, -104.6 MeV, and -105.7 MeV for ^{14}C, and ^{22,24}O, respectively. Thus, these nuclei are significantly underbound compared to experiment. Our calculations employ the same nucleon mass for protons and neutrons, and we find the ground-state energies of the daughter nuclei are 0.54 MeV, -2.62 MeV, and -6.55 MeV with respect to their corresponding mother nuclei, and in fair agreement with experiment.

Within the coupled-cluster framework we compute the total strengths

$$S_+ = \langle \Lambda | \overline{\hat{O}_{\mathrm{GT}}} \cdot \overline{\hat{O}^\dagger_{\mathrm{GT}}} |\mathrm{HF}\rangle \ , \ S_- = \langle \Lambda | \overline{\hat{O}^\dagger_{\mathrm{GT}}} \cdot \overline{\hat{O}_{\mathrm{GT}}} |\mathrm{HF}\rangle$$

for $\beta^\pm$ decays. Here $\overline{\hat{O}_{\mathrm{GT}}}$ is the similarity-transformed Gamow-Teller operator

$$\hat{O}_{\mathrm{GT}} \equiv \hat{O}^{(1)}_{\mathrm{GT}} + \hat{O}^{(2)}_{\mathrm{GT}} \equiv g_A^{-1} \sqrt{3\pi} E_1^A \ . \tag{6}$$

The one-body operator is $\hat{O}^{(1)}_{\mathrm{GT}} = g_A^{-1}\sqrt{3\pi} E_1^A|_{\mathrm{LO}}$, and the two-body operator $\hat{O}^{(2)}_{\mathrm{GT}}$ is from the 2BC at NNLO [37, 38].

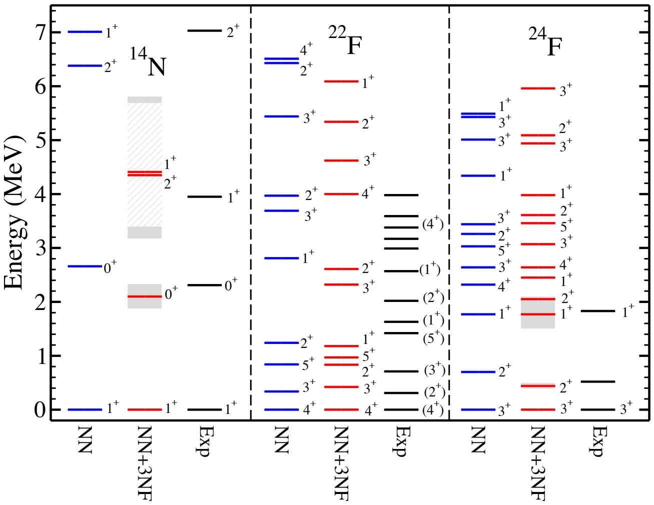

FIG. 2. (Color online) Spectra of the odd-odd daughter nuclei ^{14}N and ^{22,24}F resulting from the NN interaction with chiral cutoff $\Lambda_\chi = 500$ MeV (blue), the NN interaction and 3NF at NNLO with chiral cutoff $\Lambda_\chi = 500$ MeV (red), compared to experiment (black). Errorbars from variation of the chiral cutoff $\Lambda_\chi = 450$ to 550 MeV are shown for the $0^+, 2^+, 1^+$, and the $2^+, 1^+$ excited states in ^{14}N and ^{24}F, respectively. The band with diagonal gray lines in ^{14}N is for the 1^+ excited state.

The Ikeda sum rule is $S_- - S_+ = 3(N - Z)$ for $\hat{O}_{\mathrm{GT}} = \hat{O}^{(1)}_{\mathrm{GT}}$. This identity served as a check of our calculations. Our interest, of course, is in the contribution of the 2BC operator $\hat{O}^{(2)}_{\mathrm{GT}}$ to the total β decay strengths $S_\pm$. We considered two approximations of this two-body operator. In the normal-ordered one-body approximation (NO1B), the second fermion of the 2BC is summed over the occupied states of the HF reference. In the second approximation we add the leading order (LO) contribution of the similarity transformed two-body operator, $\overline{\hat{O}^{(2)}_{\mathrm{GT}}} \approx \hat{O}^{(2)}_{\mathrm{GT}}$ to the NO1B contribution. We will see below that this LO contribution is a smaller correction to the NO1B contribution for the nuclei we study.

Figure 3 shows the quenching factor $q^2 = (S_- - S_+)/[3(N-Z)]$ for ^{14}C, and ^{22,24}O. For the cutoff $\Lambda_\chi = 500$ MeV we vary c_D between -0.9 and 0.9 and fix c_E such that the binding energies of the $A = 3$ nuclei are reproduced. The ground-state energies and excited states in ^{14}C and ^{22,24}F are insensitive to this variation. Thus, the dependence of $(S_- - S_+)/[3(N-Z)]$ on c_D is due to 2BCs. The dotted lines show the NO1B result. Thus, a major part of the quenching results from the NO1B approximation. The sensitivity of our results to the chiral cutoffs ($\Lambda_\chi = 450, 500, 550$ MeV) is shown as the gray band for values of c_D and c_E that reproduce the triton half-life. The quenching factor depends on the nucleus, with $q^2 \approx 0.84 - 0.92$ due to 2BCs for the studied nuclei. We recall that $q^2 \approx 0.88 - 0.92$, extracted from experiments on ^{90}Zr [18–20], are within our error band. We also computed the low-lying strengths for β^- decay, and found that only 70% - 80% of the total strength $S_\pm$ is exhausted below 10 MeV of excitation energy.

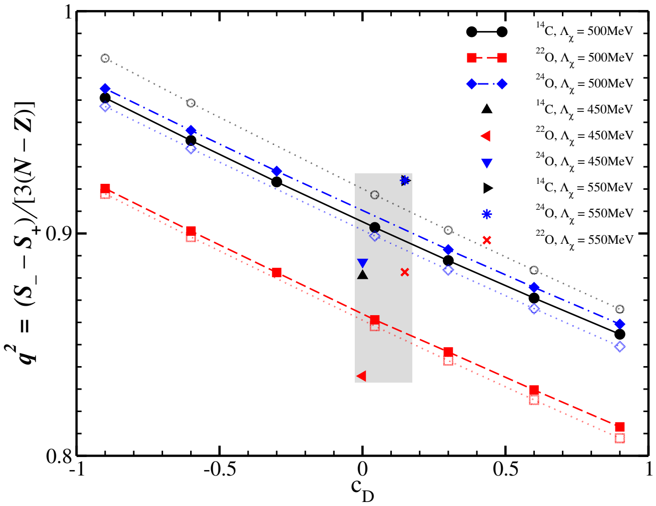

FIG. 3. (Color online) The quenching factor q^2 for ^{14}C (black line), ^{22}O (red dashed line), and ^{24}O (blue dashed-dotted line) for different c_D values. The calculations used NN and 3NF with consistent 2BCs. The gray area marks the region of c_D that yields the triton half life and shows the cutoff dependence. The dotted lines show the NO1B result.

Let us finally turn to the β^- decay of ^{14}C. The long half life of this decay, about 5700 a, is used in carbon dating of organic material. This half life is anomalously long in the sense that it exceeds the half lives of neighboring β unstable nuclei by many orders of magnitude. Recently, several studies attributed the long half life of ^{14}C to 3NFs [28–30], while the experiment points to a complicated strength function [55]. What do 2BCs contribute to this picture? To address this question, we compute the matrix element $\langle E_1^A \rangle \equiv \langle ^{14}\mathrm{N}|E_1^A|^{14}\mathrm{C}\rangle$ that governs the β^- decay of ^{14}C to the ground state of ^{14}N, with c_D and c_E from the triton life time. Figure 4 shows the various contributions to the matrix element. In agreement with Maris et al. [28] and Holt et al. [30], 3NFs reduce the matrix element significantly in size, and our result is similar in magnitude as reported by Maris et al. [28]. However, 2BCs counter this reduction to some extent, with the NO1B approximation and the LO approximation both giving significant contributions. Our results for $\langle E_1^A \rangle$ from 2BCs and 3NFs are between 5×10^{-3} and 2×10^{-2}. This is more than an order of magnitude larger than the empirical value $\langle E_1^A \rangle_{\mathrm{emp}} \approx 6 \times 10^{-4}$ extracted from the 5700 a half life of ^{14}C.

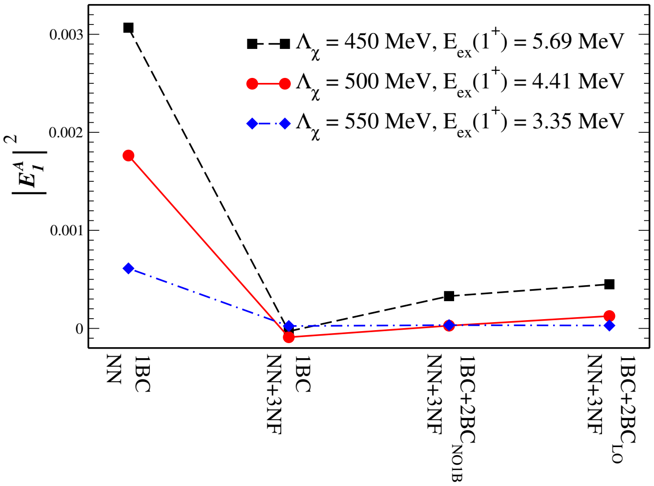

FIG. 4. (Color online) The squared transition matrix element for β^- decay of ^{14}C from increasingly sophisticated calculations (from left to right). NN, 1BC: NN interactions and one-body currents (1BC) only. NN + 3NF, 1BC: addition of 3NF. NN + 3NF, 1BC + 2BC$_{\mathrm{NO1B}}$: addition of 2BC in the NO1B approximation. NN + 3NF, 1BC + 2BC$_{\mathrm{LO}}$: addition of leading-order 2BC.

We also find that the matrix element $\langle E_1^A \rangle$ depends on the energy of the first excited 1^+ state in ^{14}N. For the three different cutoffs $\Lambda_\chi = 450, 500, 550$ MeV this excited 1^+ state is at 5.69, 4.41, 3.35 MeV, respectively (compared to 3.95 MeV from experiment). As the value of $\langle E_1^A \rangle$ decreases strongly with decreasing excitation energy, a correct description of this state is important for the half-life in ^{14}C.

Summary. – We studied β^- decays of ^{14}C, and ^{22,24}O. Due to 2BCs we found a quenching factor $q^2 \approx 0.84 - 0.92$ from the difference in total β decay strengths $S_- - S_+$ when compared to the Ikeda sum rule value $3(N - Z)$. To carry out this study, we optimized interactions from χEFT at NNLO to scattering observables for chiral cutoffs $\Lambda_\chi = 450, 500, 550$ MeV. We developed a novel coupled-cluster technique for the computation of spectra in the daughter nuclei and made several predictions and spin assignments in the exotic neutron-rich isotopes of fluorine. We find that 3NFs increase the level density in the daughter nuclei and thereby improve the comparison to data. The anomalously long half life for the β^- decay of ^{14}C depends in a complicated way on 3NFs and 2BCs. While the former increase the theoretical half life, the latter somewhat counter this effect. Taken together, the inclusion of 3NFs and 2BCs yield an increase in the computed half life.

We thank D. J. Dean, J. Engel, Y. Fujita, K. Hebeler, M. Hjorth-Jensen, M. Sasano, T. Uesaka, and A. Signoracci for useful discussions. We also thank E. Epelbaum for providing us with nonlocal 3NF matrix elements. This work was supported by the Office of Nuclear

Physics, U.S. Department of Energy (Oak Ridge National Laboratory), under DE-FG02-96ER40963 (University of Tennessee), DE-SC0008499 (NUCLEI SciDAC collaboration), NERRSC Grant No. 491045-2011, the Field Work Proposal ERKBP57 at Oak Ridge National Laboratory, and by the National Research Council and by the Nuclear Science and Engineering Research Council of Canada. DG's work is supported by BMBF ARCHES. Computer time was provided by the Innovative and Novel Computational Impact on Theory and Experiment (INCITE) program. This research used resources of the Oak Ridge Leadership Computing Facility located in the Oak Ridge National Laboratory, which is supported by the Office of Science of the Department of Energy under Contract No. DE-AC05-00OR22725, and used computational resources of the National Center for Computational Sciences, the National Institute for Computational Sciences, and the Notur project in Norway.

Supplementary material. – The LECs for the NNLO interactions with cutoffs $\Lambda = 450, 550$ MeV can be found in Tables II-III.

LEC	value	LEC	value	LEC	value
c_1	-0.91029482	c_3	-3.88068766	c_4	4.67092062
$\tilde{C}^{pp}_{^1S_0}$	-0.15203546	$\tilde{C}^{np}_{^1S_0}$	-0.15282740	$\tilde{C}^{nn}_{^1S_0}$	-0.15247258
$C_{^1S_0}$	2.43109829	$C_{^3S_1}$	0.98757436	$\tilde{C}_{^3S_1}$	-0.16953957
$C_{^1P_1}$	0.46691821	$C_{^3P_0}$	1.21516744	$C_{^3P_1}$	-0.85034985
$C_{^3S_1-^3D_1}$	0.68142133	$C_{^3P_2}$	-0.67318268		

TABLE II. Pion-nucleon LECs c_i and partial-wave contact LECs (C, $\tilde{C}$) for the chiral NN interaction at NNLO using $\Lambda_\chi = 450$ MeV and $\Lambda_{\rm SFR} = 700$ MeV [33]. The c_i, $\tilde{C}_i$, and C_i have units of GeV^{-1}, 10^4 GeV^{-2}, and 10^4 GeV^{-4}, respectively.

LEC	value	LEC	value	LEC	value
c_1	-0.90630268	c_3	-3.89738533	c_4	3.90628243
$\tilde{C}^{pp}_{^1S_0}$	-0.15067278	$\tilde{C}^{np}_{^1S_0}$	-0.15162371	$\tilde{C}^{nn}_{^1S_0}$	-0.15121579
$C_{^1S_0}$	2.38965389	$C_{^3S_1}$	0.83899578	$\tilde{C}_{^3S_1}$	-0.14677863
$C_{^1P_1}$	0.38612051	$C_{^3P_0}$	1.32532984	$C_{^3P_1}$	-0.68424744
$C_{^3S_1-^3D_1}$	0.56266120	$C_{^3P_2}$	-0.67444090		

TABLE III. Pion-nucleon LECs c_i and partial-wave contact LECs (C, $\tilde{C}$) for the chiral NN interaction at NNLO using $\Lambda_\chi = 550$ MeV and $\Lambda_{\rm SFR} = 700$ MeV [33]. The c_i, $\tilde{C}_i$, and C_i have units of GeV^{-1}, 10^4 GeV^{-2}, and 10^4 GeV^{-4}, respectively.

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