Observation of Long-Range Three-Body Coulomb Effects in the Decay of $^{16}$Ne

K. W. Brown,1 R. J. Charity,1 L. G. Sobotka,1 Z. Chajecki,2 L. V. Grigorenko,3,4 I. A. Egorova,5 Yu. L. Parfenova,3,6 M. V. Zhukov,7 S. Bedoor,8 W. W. Buhro,2 J. M. Elson,1 W. G. Lynch,2 J. Manfredi,3 D. G. McNeel,8 W. Reviol,1 R. Shane,2 R. H. Showalter,2 M. B. Tsang,2 J. R. Winkelbauer,2 and A. H. Wuosmaa8

1Departments of Chemistry and Physics, Washington University, Saint Louis, Missouri 63130, USA
2National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
3Flerov Laboratory of Nuclear Reactions, JINR, RU-141980 Dubna, Russia
4Russian Research Center “The Kurchatov Institute”, Kurchatov square 1, RU-123182 Moscow, Russia
5Bogolubov Laboratory of Theoretical Physics, JINR, RU-141980 Dubna, Russia
6Skobeltsyn Institute of Nuclear Physics, Moscow State University, RU-119991 Moscow, Russia
7Fundamental Physics, Chalmers University of Technology, S-41296 Göteborg, Sweden
8Department of Physics, Western Michigan University, Kalamazoo, Michigan 49008, USA

(Received 11 June 2014; published 2 December 2014)

The interaction of an $E/A = 57.6$-MeV $^{17}$Ne beam with a Be target is used to populate levels in $^{16}$Ne following neutron knockout reactions. The decay of $^{16}$Ne into the three-body $^{14}$O + $p + p$ continuum is observed in the High Resolution Array (HiRA). For the first time for a $2p$ emitter, correlations between the momenta of the three decay products are measured with sufficient resolution and statistics to allow for an unambiguous demonstration of their dependence on the long-range nature of the Coulomb interaction. Contrary to previous measurements, our measured limit $\Gamma < 80$ keV for the intrinsic decay width of the ground state is not in contradiction to the small values (of the order of keV) predicted theoretically.

DOI: 10.1103/PhysRevLett.113.232501 PACS numbers: 23.50.+z, 25.10.+s, 27.20.+n

Introduction.—Two-proton ($2p$) radioactivity [1] is the most recently discovered type of radioactive decay. It is a facet of a broader three-body decay phenomenon actively investigated within the last decade [2]. In binary decay, the correlations between the momenta of the two decay products are entirely constrained by energy and momentum conservation. In contrast in three-body decay, the corresponding correlations are also sensitive to the internal nuclear structure of the decaying system and the decay dynamics, providing, in principle, another way to constrain this information from experiment. In $2p$ decay, as the separation between the decay products becomes greater than the range of the nuclear interaction, the subsequent modification of the initial correlations is determined solely by the Coulomb interaction between the decay products. As the range of the Coulomb force is infinite, its long-range contribution to the correlations can be substantial, especially in heavy $2p$ emitters.

Prompt $2p$ decay is a subset of a more general phenomenon of three-body Coulomb decay (TBCD), which exists in mathematical physics (as a formal solution of the $3 \rightarrow 3$ scattering of charged particles), in atomic physics (as a solution of the $e \rightarrow 3e$ process), and in molecular physics (as exotic molecules composed from three charged constituents) [3–8]. The theoretical treatment of TBCD is one of the oldest and most complicated problems in physics because of the difficulty associated with the boundary conditions due the infinite range of the Coulomb force. The exact analytical boundary conditions for this problem are unknown, but different approximations to it have been tried. In nuclear physics, TBCD has not attracted much attention; however, the three-body Coulomb aspect of $2p$ decay will become increasingly important for heavier prospective $2p$ emitters [9].

Detailed experimental studies of the correlations have been made for the lightest $p$-shell $2p$ emitter $^6$Be [10,11], where the Coulomb interactions are minute and their effects are easily masked by the dynamics of the nuclear interactions [12]. The Coulomb effects should be more prominent for the heaviest observed $2p$ emitters; however, these cases are limited by poor statistics; e.g., the latest results for the $pf$-shell $2p$-emitters $^{54}$Zn [13] and $^{45}$Fe [14] are based on just seven and 75 events, respectively. Because of these limitations, previous $2p$ studies dedicated to the long-range treatment of the three-body Coulomb interaction [15] found consistency with the data, but no more.

The present work fills a gap between these previous studies by measuring correlations in the $2p$ ground-state (g.s.) decay of the $sd$-shell nucleus $^{16}$Ne where the Coulombic effects appear to be strong enough to be observable. Known experimentally for several decades [16], $^{16}$Ne has remained poorly investigated with just a few experimental studies [17–20]. However, interest has returned recently with the decay of $^{16}$Ne measured in relativistic neutron-knockout reactions from a $^{17}$Ne beam [21,22]. We study the same reaction, but at an “intermediate” beam
energy, and obtain data with better resolution and smaller statistical uncertainty. Combined with state-of-the-art calculations, we find unambiguous evidence for the role of the long-range Coulomb interactions in the measured correlations.

Apart from the Coulomb interactions, predicted correlations show sensitivity to the initial 2p configuration and nuclear final-state interactions that are also evident in 2n decay [23–25]. While there are indications of such sensitivities in 2p data [2], the long-range Coulomb interactions must first be determined accurately before the effects of structure and nuclear final-state interactions can be better probed and properly accounted for.

Experiment.—A primary beam of $E/A = 170$-MeV $^{20}$Ne, extracted from the Coupled Cyclotron Facility at the National Superconducting Cyclotron Laboratory at Michigan State University, bombarded a $^9$Be target. The A1900 separator was used to select a secondary $^{17}$Ne beam with a momentum acceptance of $+\pm 1.0\%$, an intensity of $\sim 1.5 \times 10^5$ s$^{-1}$, and a purity of $11\%$ (the largest component was $^{15}$O). This secondary beam impinged on a 1-mm-thick $^9$Be target with an average of $E/A = 57.6$ MeV in the target’s center.

$^{16}$Ne decay products were detected in the High Resolution Array (HiRA) [26] in an arrangement of 14 $\Delta E - E$ [Si-CsI(Tl)] telescopes subtending zenith angles from $2^\circ$ to $13.9^\circ$[10,27]. Energy calibrations were achieved using beams of 55 and 75 MeV protons and $\rho_{\text{out}}$ out to distances $\rho_{\text{ext}} \gg \rho_{\text{max}}$. The asymptotic momentum distributions are reconstructed from the set of trajectories after the radial convergence is achieved. The accuracy of this approach has been tested in calculations with simplified three-body Hamiltonians allowing exact semianalytical solutions [15].

Excitation spectrum.—The spectrum of the total decay energy $E_T$ constructed from the invariant mass of detected $^{14}$O + $p + p$ events is shown in Fig. 1. Because of a low-energy tail in the response function of the Si $\Delta E$ detectors, there is leaking of a few $^{15}$O ions into the $^{14}$O gate in the $\Delta E - E$ spectrum. However, this contamination can be accurately modeled by taking detected $^{15}$O + $p + p$ events and analyzing them as $^{14}$O + $p + p$. The resulting background spectrum (dashed histogram) was normalized to the $\sim 1$-MeV peak associated with the second excited state of $^{14}$Ne, where $r_\text{c}$ is the radius vector of the removed neutron. The $^{17}$Ne$_{g.s.}$ WF $\Psi^{17}\text{Ne}$ was obtained in a three-body model of $^{15}$O + $p + p$ and broadly tested against various observables [30]. Similar ideas had been applied to different reactions populating the three-body continuum of $^9$Be [10–12]. The $^{14}$O-$p$ potential sets were taken from Ref. [28], which are consistent with a more recent experiment [31], providing $1/2^-$ and $5/2^+$ states at $E_r = 1.45$ and 2.8 MeV, respectively, and consistent with the experimental properties of these states in both $^{13}$F and $^{15}$C. We used the potential of Ref. [32] for the $p-p$ channel.

The three-body Coulomb treatment in our model consists of two steps. (i) We are able to impose approximate boundary conditions of TBCD on the hypersphere of very large $\rho_{\text{max}} \lesssim 4000$ fm) hyperradius by diagonalizing the Coulomb interaction on the finite hyperspherical basis [33]. Within this limitation the procedure is exact; however, it breaks down at larger hyperradii as the accessible basis size becomes insufficient. (ii) Classical trajectories are generated by a MC procedure at the hyperradius $\rho_{\text{max}}$ and propagated out to distances $\rho_{\text{ext}} \gg \rho_{\text{max}}$. The asymptotic momentum distributions are reconstructed from the set of trajectories after the radial convergence is achieved. The accuracy of this approach has been tested in calculations with simplified three-body Hamiltonians allowing exact semianalytical solutions [15].

where $\Phi_q$ is the source function of the $^{15}$O core of $^{17}$Ne$_{g.s.}$, $\Phi_q = \int d^3r e^{ik \cdot r} \langle \Psi^{15}\text{O} | \Psi^{17}\text{Ne} \rangle$. 

FIG. 1 (color online). Experimental spectrum of $^{16}$Ne decay energy $E_T$ reconstructed from detected $^{14}$O + $p + p$ events. The dashed curves are predictions (without detector resolution) for the indicated $^{16}$Ne states. The inset compares the contamination-subtracted data to the simulation of the g.s. peak for $\Gamma = 0$, $f_{\text{stat}} = 0.95$, where the dotted line is the fitted background.
The broad structure at $E_T \sim 5.0(5)$ MeV is well described as a $1^-$ “soft” excitation, which is not a resonance but a continuum mode, sensitive to the reaction mechanism [11]. In the mirror $^{16}$C system, there are also $J = 2^{(=)}, 3^{(=)}$, and $4^+$ contributions in this energy range, but for neutron knockout from $p_{1/2}$, $p_{3/2}$, and $s_{1/2}$ orbitals in $^{17}$Ne, we should only expect strong population for $0^+$, $2^+$, and $1^-$ configurations. We will concentrate on the g.s. for the remainder of this work ($1.27 < E_T < 1.72$ MeV) and all subsequent figures will show contamination-subtracted data.

Three-body energy-angular correlations.—The final state of a three-body decay can be completely described by two parameters [29]: an energy parameter $\epsilon$ and an angle $\theta_k$ between the Jacobi momenta $\mathbf{k}_x$, $\mathbf{k}_y$ with

$$\epsilon = E_x/E_T, \quad \cos(\theta_k) = (\mathbf{k}_x \cdot \mathbf{k}_y)/(k_x k_y),$$

$$\mathbf{k}_x = \frac{A_2 k_1 - A_1 k_2}{A_1 + A_2}, \quad \mathbf{k}_y = \frac{A_3 (k_1 + k_2) - (A_1 + A_2) k_3}{A_1 + A_2 + A_3},$$

$$E_T = E_x + E_y = k_x^2/2M_x + k_y^2/2M_y,$$  \hfill (3)

where $M_x$ and $M_y$ are the reduced masses of the $X$ and $Y$ subsystems. With the assignment $k_3 \rightarrow k_{1/3}$, the correlations are obtained in the “$T$” Jacobi system where $\epsilon$ describes the relative energy $E_{pp}$ in the $p+p$ channel. For $k_3 \rightarrow k_p$, the correlations are obtained in one of the “$Y$” Jacobi systems where $\epsilon$ describes the relative energy $E_{core+p}$ in the $^{14}$O+$p$ channel.

The experimental and predicted (MC simulations) energy-angular distributions, in both Jacobi representations are compared in Fig. 2 and found to be similar. More detailed comparisons will be made with the projected energy distributions.

The convergence of three-body calculations is quite slow for some observables [29,34]. Figure 3 demonstrates the convergence, with increasing $K_{\text{max}}$ (maximum principle quantum number of the hyperspherical harmonic method) for two observables for which the slowest convergence is expected. This work provides considerable improvement compared to the calculations of Ref. [28], which were limited by $K_{\text{max}} = 20$.

$^{16}$Ne g.s. width.—The theoretical difficulty of reproducing the large experimental g.s. widths measured for $^{12}$O and $^{16}$Ne has been pointed out many times in the last 24 years [$^{28,35-38}$]. For $^{12}$O, this issue was resolved when a new measurement [39] gave a small upper bound. For $^{16}$Ne, previous measurements of $\Gamma = 200(100)$ keV [17], 110(40) keV [18], and 82(15) keV [22] are large compared to the theoretical predictions, e.g., 0.8 keV in Ref. [28].

The experimental resolution is dominated by the effects of multiple scattering and energy loss in the target. The magnitudes of these effects were fine tuned in the MC simulations by reproducing the experimental $^{15}$O+$p+p$ invariant-mass peak associated with the narrow (predicted lifetime of the 1.4 fs [40]) second excited state in $^{17}$Ne by scaling the target thickness from its known value by a factor $f_{\text{tar}}$. The best fit is obtained with $f_{\text{tar}} = 0.95$ with $3\sigma$ limits of 0.91 and 1.00. With $f_{\text{tar}}$, we find that the simulated shape of the $^{16}$Ne g.s. peak for $\Gamma = 0$ is consistent with the data [Fig. 1, inset]. To obtain a limit for $\Gamma$, we used a Breit-Wigner line

![FIG. 2 (color online). Energy-angular correlations for $^{16}$Ne g.s. Experimental and predicted (MC simulations) correlations for Jacobi T and Y systems are compared.](image1)

![FIG. 3 (color online). The convergence of the predicted (a) decay width and (b) energy distribution in the T system on $K_{\text{max}}$ (maximum principal quantum number of hyperspherical harmonics method). The asymptotic decay width of $^{16}$Ne assuming exponential $K_{\text{max}}$ convergence is given in (a) by the dashed line.](image2)
shape in our simulations and find a $3\sigma$ upper limit of $\Gamma < 80$ keV with $f_{\text{cut}} = 0.91$. This limit is the first experimental result consistent with theoretical predictions of a small width [in the keV range, see, e.g., Fig. 3(a)]. However, our limit is still considerably larger than the predictions and, on the other hand, it is still consistent with two of the previous experiments so even higher resolution measurements are needed to fully resolve this issue.

**Evolution of energy distribution between core and proton.**—To investigate the long-range nature of TBCD, we studied the effect of terminating the Coulomb interaction at some hyperradius $\rho_{\text{cut}}$. The energy distribution in the Y Jacobi system is largely sensitive to just the TBCD and the global properties of the system ($E_T$, charges, separation energies) [2]. This makes it most suitable for studying the $\rho_{\text{cut}}$ dependence [Fig. 4(a)]. Note the arbitrary normalization of the theoretical curves, while the MC results are always normalized to the integral of the data. The comparison with the data in Fig. 4(b) demonstrates consistency with the theoretical calculations only if the considered range of the Coulomb interaction far exceeds $10^3$ fm ($\rho_{\text{cut}} = 10^5$ fm guarantees full convergence). This conclusion is only possible due to the high quality of the present data. In contrast in Ref. [22], where the experimental width of the g.s. peak is almost twice as large and its integrated yield is $\sim 3$ times smaller, the corresponding $\epsilon$ distribution is broader with a FWHM of 0.41 compared to our value of 0.33. This difference is similar to that obtained over the range of $\rho_{\text{cut}}$ considered in Fig. 4(a), demonstrating the need for high resolution to isolate these effects.

Our conclusions on TBCD are dependent on the stability of the predicted correlations to the other inputs of the calculations. Figure 4(d) demonstrates the excellent stability of the core-$p$ energy distribution over a broad range ($\pm 200$ keV) of $E_T$ centered around $E_T = 1.476$ MeV. Indeed, in this range we have a maximum in the width for this distribution. This maximum is expected as, below this range, the width must approach zero in the limit of $E_T \rightarrow 0$ [1] and, above this range, we expect the width to have a minimum at $E_T \sim 2E_r$, $\sim 2.9$ MeV, where $E_r$ is the $^{15}\text{F}_{g.s.} \rightarrow \text{core} + p$ decay energy. The predictions of such a “narrowing” of the width at $E_T \sim 2E_r$ [2] were recently proven experimentally [10]. The curve for $E_T = 1.976$ MeV is also provided in Fig. 4(d) to show that a really large change in energy is required to produce a significant modification of the $\epsilon$ distribution.

The other important stability issue is with respect to the properties of $^{15}\text{F}_{g.s.}$ for which there is no agreement on its centroid $E_r$ and width [41]. Figure 4(c) shows predicted $\epsilon$ distributions based on four different $^{14}\text{O} + p$ interactions, which give the indicated $^{15}\text{F}_{g.s.}$ properties. Even if we use the data from Ref. [42], which differ the most from the other results ($E_r \sim 1.23$ MeV instead of $E_r \sim 1.4$–1.5 MeV), no drastic effect is seen.

**The evolution of the energy distribution between the two protons with $\rho_{\text{cut}}$.**—In Fig. 4(e) this distribution has greater sensitivity to the initial $2p$ configuration of the decaying system [2]. In addition, the spin-singlet interaction in the $p$-$p$ channel provides the virtual state (“diproton”), which also can affect the long-range behavior of the correlations (see Refs. [23–25] for the corresponding effects in $2n$ decay). The theoretical prediction for $\rho_{\text{cut}} = 10^2$ fm in Fig. 4(f) reproduces experimental data quite well; however, the sensitivity to $\rho_{\text{cut}}$ is diminished compared to the core-$p$ energy distribution.

**Limits on classical motion.**—In our model the very long distances are achieved by classical extrapolation. This approximation has been studied using calculations with simplified Hamiltonians where it was demonstrated that the classical extrapolation provides stable results if the starting distance $\rho_{\text{max}}$ exceeds some hundreds of fermis for $E_T \sim 1$ MeV [15], (e.g., $\sim 300$ fm for $^{19}\text{Mg}_{g.s.}$ decay where $E_T = 0.75$ MeV). At such distances, the ratio of the Coulomb potential to the kinetic energy of fragments is of

![FIG. 4](color online). Panels (a)–(d) show energy distributions in the Jacobi Y system where (a) gives the sensitivity of the predictions to $\rho_{\text{cut}}$, (c) to the $^{15}\text{F}_{g.s.}$ properties, and (d) to the decay energy $E_T$. Panels (e) and (f) show energy distributions in the Jacobi T system where (e) gives the sensitivity to $\rho_{\text{cut}}$. The theoretical predictions, after the detector bias is included via the MC simulations, are compared to the experimental data in (b) and (f) for the Y and T systems, respectively. The normalization of the theoretical curves is arbitrary, while the MC results are normalized to the integral of the data.
beyond 10−3. The body Coulomb interaction is considered out to distances far above 200 fm.

FIG. 5 (color online). The core-proton relative-energy distribution (Y system) obtained by classical extrapolation started from different $\rho_{\text{max}}$ values.

the order of $10^{-2} - 10^{-3}$. Figure 5 shows that for $^{16}\text{Ne}_{\text{e.s.}}$ the predictions are consistent with the data only if the conversion from quantum to classical dynamics is made at or above 200 fm.

Conclusions.—The continuum of $^{16}\text{Ne}$ has been studied both experimentally and theoretically with emphasis on the ground state, which decays by prompt two-proton emission. The measured decay correlations in this work were found to require a theoretical treatment in which the three-body Coulomb interaction is considered out to distances far beyond $10^3$ fm. Our theoretical treatment is now validated for use in interpreting the results of future studies of heavier two-proton decay with particular emphasis on extracting nuclear-structure information from correlation observables.

We extract a limit of $\Gamma < 80$ keV for the intrinsic decay width of the ground state, and while this is not inconsistent with some of the previous measurements, it is the first measurement consistent with the theoretical predictions. All conclusions of this work were only possible due to the high statistics and fidelity of the present measurements.

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Awards No. DE-FG02-87ER-40316 and No. DE-FG02-04ER41320 and the National Science Foundation under Grants No. PHY-1102511 and No. PHY-9977707. I. A. E. is supported by the Helmholtz Association under Grant Agreement No. IK-RU-002 via FAIR-Russia Research Center and L. V. G. by the RFBR Foundation under Grants No. PHY-1102511 and No. PHY-192501. K. W. B. is supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1143954.