Maximum Aperture Power Transmission in Lossy Homogeneous Matters

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Abstract—We apply array signal processing techniques to the transmitting aperture field modes in order to determine the optimal field distribution that maximizes the power transmission through lossy media between a transmitting and an ideally receiving antenna aperture. The optimal aperture distribution is then used as a reference field for developing curves applicable to the design of many near-field systems, such as for the detection of foreign objects in lossy matters (e.g. food contamination detectors), wireless charging of batteries of human body implanted devices, and for in-body communication systems.

Index Terms—Near field focusing, array signal processing.

I. INTRODUCTION

FOCUSING the near-field (NF) radiated power from an antenna has been an important research topic for a long time. For instance, in [1], [2], a spherical phase front of the aperture field is used to focus the energy at a certain distance in the Fresnel zone. Sherman [1] has shown that the electric field on the focal plane near the axis of such a focused aperture field exhibits typical far-field properties, e.g., a tapered aperture field will result in decreased sidelobe levels, an increased beamwidth as well as a reduced gain. Conversely, for an inverse taper, Hansen [2] demonstrated that low axial lobes before and after the focal distance are to be expected, at the cost of sidelobe and gain level degradations in the focal plane. Graham [3] shaped the axial pattern of NF focusing antennas in lossless media through far-field pattern synthesis based methods, in which the focal distance is still set by the aperture phase distribution, and where the control over the axial pattern is traded against the transverse pattern quality. Focused apertures have been realized by employing large microstrip arrays or Fresnel zone plate lenses [4], [5]. Recently, Sanghoek et al. [6] proposed a two-port network model to optimize the coupling efficiency between an infinite sheet of magnetic current density and a combination of electric and magnetic dipoles embedded in a planar multilayered medium. It was found that the axial pattern, or forelobes and aftlobes, cannot be defined for a monotonously decreasing NF in a lossy medium, therefore the 3dB NF beam radius becomes a useful parameter for the characterization of NF-focusing antennas [7].

We propose to apply array signal processing techniques to the transmitting aperture field modes in order to determine the optimal field distribution that maximizes the power transmission between a transmitting and an ideally receiving antenna aperture in a lossy homogeneous media. This not only provides us with the fundamental insight on how to achieve the best possible focus in a manner that is more generic than an analytical approach [8], but it also allows us to explore the fundamental limits of NF-focused antennas and constitutes the first step in a design procedure as one can now independently match the aperture fields of our transmitting and receiving antennas to the optimal ones (a concept also used for conjugate-field-matched focal plane array antennas [9]). Optimal design curves will be presented for several representative cases. The herein presented theoretical study excludes the actual design process of specific transmitting and receiving antennas for generality.

II. PROBLEM DESCRIPTION

The NF system in Fig. 1 comprises of a transmitting and a receiving (aperture) antenna embedded in a lossy homogeneous medium whose intrinsic impedance is \( \eta \). It may represent a solid or a flow of liquid matter. The objective is to maximize the power coupling between the transmitting and receiving aperture antennas. In many applications the receiver size is limited, such as in NF power transfer and/or communication systems involving a large antenna outside and an implanted device inside the human body. Although several receiver aperture sizes will be examined, no optimization is done on the receiving antenna as it is assumed to be conjugately impedance and field matched to the receiving aperture fields, thereby harvesting all the power passing through it.

III. OPTIMIZATION PROCEDEURE FORMULATION

The goal is to determine the equivalent aperture current distribution that maximizes the NF coupled power between the source and observation apertures \( S_1 \) and \( S_2 \), respectively, as shown in Fig. 1. Since the fields emanating from the
source aperture $S_1$ are directive in nature, the influence of the ground plane (GP) at the transmit side is neglected. Also, since the medium is very lossy, any backscattered field at the receiving side is assumed to have negligible effect on the transmitting side. Finally, edge diffraction of the GP aperture at the receiving side is neglected. Hence, the NF focusing fields in between $S_1$ and $S_2$ can be regarded as being generated by the unknown source currents $J$ and $M$ that radiate in a homogeneous medium. The design problem at hand is finding $J$ and $M$ such that the power transfer between $S_1$ and $S_2$ is maximized.

The total received power $P_{\text{out}}$ at $S_2$ is

$$P_{\text{out}} = \frac{1}{2} \Re \left\{ \int_{S_2} [E \times H^*] \cdot \hat{n}_2 \, dS \right\}$$

(1)

where the $E$- and $H$-fields are generated by $J$ and $M$ as

$$E(J, M) = E_J(J) + E_M(M)$$

$$H(J, M) = H_J(J) + H_M(M).$$

(2a) (2b)

Next, $J$ and $M$ are expanded in $N$ basis functions as

$$J = \sum_{n=1}^{N} j_n f_n(r), \quad \text{and} \quad M = \sum_{n=1}^{N} m_n g_n(r)$$

(3)

where the unknown expansion coefficient vectors $j = [j_1, j_2, \ldots, j_N]^T$ and $m = [m_1, m_2, \ldots, m_N]^T$ are yet to be determined. Substituting (3) in (2), and then in (1), and by introducing the complex power between basis functions as $P_{pq} = \int_{S_2} [E_A(f_p) \times H_B(g_q)] \cdot \hat{n}_2 \, dS$, where $\{A, B\} \in \{J, M\}$, yields for the total output power

$$P_{\text{out}} = \frac{1}{2} \Re \left\{ \sum_{p=1}^{N} \sum_{q=1}^{N} j_p^* P_{pq} j_q^* + j_p^* P_{pq} m_q^* + \ldots + m_p^* P_{pq}^* j_q^* + m_p^* P_{pq}^* m_q^* \right\}$$

(4)

or, in matrix-vector form,

$$P_{\text{out}} = \frac{1}{2} \Re \left\{ \begin{bmatrix} j^T \cr m^T \end{bmatrix}^H \begin{bmatrix} P_{JJ} & P_{JM}^* \\ P_{MJ} & P_{MM} \end{bmatrix} \begin{bmatrix} j \cr m \end{bmatrix} \right\}$$

(5)

$$= \frac{1}{2} \Re \{w^H P_{\text{out}} w\}$$

(6)

where we have taken the conjugate of the Poynting vector, and where we have introduced

$$w = \begin{bmatrix} j \\ m \end{bmatrix}, \quad \text{and} \quad P_{\text{out}} = \begin{bmatrix} P_{JJ} & P_{JM}^* \\ P_{MJ} & P_{MM} \end{bmatrix}.$$  

(7)

The input power $P_{in}$ at the aperture is computed through the aperture equivalent currents, i.e., $E_a = \hat{n}_1 \times M$ and $H_a = J \times \hat{n}_1$, so that the Poynting vector integral reduces to

$$P_{\text{in}} = \frac{1}{2} \Re \left\{ \int_{S_1} [J^* \times M] \cdot \hat{n}_1 \, dS \right\}$$

(8)

Next, by substituting (3) in (8), $P_{\text{in}}$ becomes

$$P_{\text{in}} = \frac{1}{2} \Re \{J^H A m\}$$

(9)

where $A_{pq}$ are the complex powers radiated by current basis functions, given by

$$A_{pq} = \int_{S_1} [f_p^* \times g_q] \cdot \hat{n}_1 \, dS.$$  

(10)

By padding $A$ with zeros and forming $P_{in}$ as

$$P_{in} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

the input power $P_{\text{in}}$ is given as $P_{\text{in}} = \frac{1}{2} \Re \{w^H P_{in} w\}$, so that the power transfer ratio $P_t$ can be written as

$$P_t = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\Re \{w^H P_{\text{out}} w\}}{\Re \{w^H P_{in} w\}} = \frac{w^H P_{\text{out}} P_{in}^H w}{w^H P_{in} + P_{\text{in}}^H w}.$$  

(11)

The objective is to determine $w$ that maximizes (12) at a stationary point, i.e., $\nabla_{\hat{w}} P_t(w, w^H) = 0$, which leads to the generalized eigenvalue equation [10, Sec. 10.2]

$$[P_{\text{in}} + P_{\text{in}}^H]w = P_t \hat{w}.$$  

(13)

Hence, maximizing (12) is equivalent to finding the largest positive eigenvalue $P_t$ in (13). Once the optimal $w = w_{\text{opt}}$ is found, the currents $J$ and $M$ can be computed through (3).

IV. NUMERICAL RESULTS

A. Basis Function Choice and Optimization Results

We employ a rectangular grid of pulse basis function currents and assume an $x$-polarized electric aperture field. The basis function currents are modeled as

$$f_n(r) = \hat{x} \Pi(r_n) \quad \text{and} \quad g_n(r) = \hat{y} \Pi(r_n)$$

(14)

where $\Pi(r_n)$ is the support of the $n$th pulse basis function of size $(0.1 \lambda)^2$ with centroid $r_n$.

In the following, muscle tissue ($\epsilon_r = 57$, $\sigma = 1.2 \, \text{S/m}$) and/or fat ($\epsilon_r = 4.6$, $\sigma = 0.02 \, \text{S/m}$) are used as the lossy medium at $1 \, \text{GHz}$ [11].

As an example, the normalized amplitude and phase of the optimized electric currents for a pair of $2\lambda \times 2\lambda$ apertures with $4\lambda$ spacing in muscle tissue are plotted in Fig. 2. The magnetic currents are not shown since the solution turns out to resemble a Huygen’s source. More specifically, $M(r) \approx \eta \hat{n}_1 \times J(r)$. As expected, the source current distribution has rotational symmetry around the axis of both apertures; henceforth, all current distributions will be plotted along the $x$-axis only.

B. Changes in Transmitter Aperture Size

A uniform field aperture provides the highest directivity in the far-field in a lossless medium (not considering super-directivity) and is therefore chosen as a suitable reference field distribution to compare the $P_t$ of the optimized aperture field relative with a uniform one of the same size. As is evident from Fig. 3(a), the difference in $P_t$ between the optimal and uniform aperture field is negligible when both of the apertures are small, and it is only beyond a certain size that the optimal aperture field demonstrates a (small) advantage over the uniform one.

1Although $P_{\text{in}} + P_{\text{in}}^H$ is generally an indefinite matrix, it was numerically verified that by employing Huygen’s source basis functions (constrained to positive input power), the same optimal $P_t$ value is achieved.
As for the phase, an increase in aperture size results in a larger phase lead taper towards the edges to constructively interfere the fields as the NF beam is focused. The optimal reference aperture field may be realized through a single (lens) antenna and/or an antenna array.

### C. Transmitter and Receiver Aperture Distance Variations

To examine the effect of the spacing $d$ between the two apertures, Fig. 5 shows a comparison of the $P_{tr}$ values between the optimal and uniform apertures, but now for the larger distance $d = 8\lambda$ ($R = 1\lambda,$ muscle tissue). Upon comparing this with Fig. 3(b), one can readily observe that: (i) $P_{tr}$ reduces for increasing $d$, which is expected due to the field attenuation in the lossy medium; (ii) for transmitting apertures up to almost $3\lambda \times 3\lambda$, the optimal and uniform apertures provide almost the same $P_{tr}$, as compared to $2\lambda \times 2\lambda$ for $d = 4\lambda$ in Fig. 3(b), and; (iii) the size at which the uniform aperture provides its maximum $P_{tr}$ increases with $d$.

### D. Changes in Receiver Aperture Size

Fig. 6 shows the $P_{tr}$ value for the uniform and optimal aperture field cases ($d = 4\lambda$, muscle tissue) as a function of the transmitting aperture size for various different receiver aperture sizes. It is observed that the $P_{tr}$ advantage of the optimal aperture field over the uniform one becomes higher for smaller receiver apertures, however, it should also be noted that a larger $P_{tr}$ advantage occurs at a lower $P_{tr}$ level. Since this affects the achievable signal-to-noise ratio and therefore the attainable receiving sensitivity, it should be taken into account.
whilst selecting a proper size for the receiver antenna; a small receiver antenna simplifies the system design and the optimal aperture can provide a large $P_{tr}$ advantage, but the received signal level should not be lower than the sensitivity of the electronics in order for the whole system to work.

E. Design Curves

Plots that show the optimal transmitting aperture size along with the achievable $P_{tr}$ advantage relative to the uniform aperture field case are indispensable during NF system design phases. In the following, we consider the case where the receiver aperture size $R$ and aperture separation distance $d$ are constrained (i.e., fixed), while the best possible transmitting aperture is synthesized. For the purpose of developing design curves, $P_{tr}$ for the optimal aperture as a function of the transmitter size $T$ is computed, up to a point where $P_{tr}$ increases less than 0.1dB per 0.2λ increase in $T$ – we call this point $T_{opt}$. Accordingly, $P_{tr}$ is computed for the uniform aperture field and its maximum is found. The $P_{tr}$ advantage $PA$ is then defined as the difference between the so-determined maximum values (cf. also Fig. 7). This, in turn, allows us to plot $T_{opt}$ and $PA$ as a function of $d$ for various $R$.

Fig. 8(a) and (b) show $T_{opt}$ and $PA$ as a function of $d$ for various values of $R$ both in muscle tissue and fat. As an example, assume that the objective is to design a transmitter aperture and optimal reference fields, prior to fine-tuning the system. The effects of the transmitter and receiver aperture sizes, as well as the spacing between both apertures on the power coupling have been investigated and practical design curves for two representative media are presented.

**REFERENCES**


