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Abstract

This paper reports on theoretical investigations of the stochastic properties of the signal series of ionisation chambers, in particular fission chambers. First, a simple and transparent derivation is given of the higher order moments of the random detector signal for incoming pulses with a non-homogeneous Poisson distribution and random pulse heights and arbitrary shape. Exact relationships are derived for the higher order moments of the detector signal, which constitute a generalisation of the so-called higher order Campbelling techniques. The probability distribution of the number of time points when the signal exceeds a certain level is also derived. Then, a few simple pulse shapes and amplitude distributions are selected as idealised models of the detector signals. Assuming that the incoming particles form a homogeneous Poisson process, explicit expressions are given for the higher order moments of the signal and the number of level crossings in a given time interval for the selected pulse shapes.

1 Introduction

The recent interest in new reactor systems and in many other fields drew an increased attention to the development of neutron detectors which are particularly suitable for deployment in such systems, especially to the technical improvement of the fission chambers. The fission chamber is an ionization chamber in which the electron-ion pairs are generated by fission fragments in the gaseous volume of the detector. These electron-ion pairs are collected to form a detector pulse of a certain shape, and the detector signal consists of a random sum of such pulse shapes, induced by the random primary events (impinging neutrons inducing fission).

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Fission chambers have certain advantages over other type of detectors which are suitable for in-core neutron measurements [1,2]. One is that they have a large dynamic range, i.e. they can be used in both low, medium and high neutron fluxes. Unlike other detectors which can operate only either in pulse or current mode, fission chambers can be operated in both. At low neutron intensities, the fissions generate individual current signals, i.e. pulses, that are generally separated and can be counted with a given efficiency. At increasing neutron intensities, the pulses tend to overlap, which finally forms a randomly fluctuating continuous current. The expectation of this fluctuating signal, that is the direct current, is proportional to the neutron flux, which is the main quantity of interest.

Under the conditions that the continuous signal is formed as the superposition of constant pulse shapes induced by independent incoming events, the higher order moments inclusive the semiinvariants of the fluctuating signal are also proportional to the intensity of the primary incoming neutrons, and hence to the neutron flux. This is expressed by the so-called Campbell theorem [3,4], which shows the relationship of the first two moments of the detector signal to the primary event intensity in the form

$$\mathbf{E}\{\eta(t))\} = s_0 \int_{-\infty}^{+\infty} f(t)\, dt \qquad \text{and} \qquad \mathbf{D}^2\{\eta(t)\} = s_0 \int_{-\infty}^{+\infty} f(t)^2\, dt. \quad (1.1)$$

Here the random process $\eta(t)$ represents the fluctuating detector signal, which consists of a random sum of deterministic current signals $f(t)$ created according a homogenous Poisson process with intensity s_0.

Eq. (1.1) shows the second advantage of fission chambers in that instead of the first moment, the variance can also be used to estimate the neutron flux (which is also called "Campbelling techniques" or "Campbell-technique"). Then, even if the detector is sensitive to both neutrons and gamma photons, the gamma photons will produce less charge in the detector per incoming particle than the neutrons (through the fission products), and hence will be represented by a much smaller amplitude of the corresponding current signal shape $f(t)$. Hence the contribution of the unwanted minority component, i.e. that of the gamma detections, can be significantly reduced by the use of Campbelling techniques. One can extend the relationships (1.1) even to higher order moments (semiinvariants), leading to higher order Campbelling techniques [5,6,7], lending the possibility of even further reduction of the contributions from gamma detection and other unwanted components such as alpha particles.

The complete process of neutron detection in a fission chamber, starting with the fission in the fissile deposit of the detector, the slowing down and escape of the fission products, the charge generation, collection and amplification is a complex process. All elements of this process will influence the pulse shape, the statistics of the pulse amplitude and finally the statistics of the detector current fluctuations and hence the form and validity of the Campbelling techniques. Many elements of this process can be simulated and modelled with advanced transport codes for charged particles. Such effects have been investigated by several authors [1,2,8,9]. The emphasis in these works is that by using the basic charge transport equations, to calculate both the mean shape of an individual signal and the mean value of the saturation current.

These questions are not touched on in the present paper, which will concentrate on the understanding of the information content in the temporal randomness of a detector signal composed by a random sum of pulses of a given fixed shape but with a random amplitude distribution. No attempt will be made to derive the signal shapes or the amplitude distributions from calculation from first principles; rather, two physically reasonable shapes and amplitude distributions will be postulated and investigated.

As a first step, the higher order Campbelling techniques will be derived for a non-homogeneous Poisson distribution, with random pulse height of arbitrary distribution and arbitrary signal shape. There exist derivations in the literature of the higher order Campbelling techniques, but these contain unnecessarily complicated and often incorrect calculations. The method used in this work is the backward form of the integral master equation for the probability distribution of the detector signal. Exact relationships will be derived for the higher order moments, which constitute a generalisation of the higher order Campbelling methods. These will, among others, reproduce the results published by Lux and Baranyai in 1982 [5], [6] and by Bärs in 1989 [7] in a simple and transparent way.

In addition to the moments of the signal, the number of cases when the signal is higher than a given level in a given time interval also gives information on the intensity of the primary events, and hence on the neutron flux. Such a measurement on a continuous signal show similarities with measurements in the pulse mode. Because of its information content, the intensity of events when the signal exceeds a certain level is also derived.

Although, as it will be seen, the higher order Campbelling techniques state that all higher order moments are proportional to the intensity of the incoming particles, the proportionality factor is a function of the signal shape and the amplitude distribution of the pulses. Hence, insight can be gained on the performance of the Campbelling techniques if explicit analytical results are available for some concrete characteristic pulse shapes and amplitude distributions. To this end, a few simple pulse shapes and amplitude distributions are selected as idealised models of the detector signals. Assuming that the incoming particles form a homogeneous Poisson process, explicit expressions are given for the higher order moments of the signal and the intensity of level crossings for the selected pulse shapes. The results for the different pulse shapes and amplitude distributions can be compared.

Regarding the analytical work, some of the calculations require extensive derivations. Apart from the derivation of the generalised higher order Campbell relations, which will be given in detail, in the concrete calculations with selected pulse shapes and amplitude distributions, most of the details of the calculations were omitted. Details of the derivations are described in a Chalmers internal report, available electronically [10] where even other pulse shapes are considered.

It can also be mentioned that regarding the assumption of independent primary incoming events, which essential in the derivation of the Campbell theorems, can be relaxed. The treatment used in this paper can be extended to the case when detection events are related to neutrons arising in branching processes in a multiplying medium, and which hence are not independent events. The results of this extension will be published in a later communication [11].

2 General theory

2.1 Signal probability distribution

The objective of this work is to determine the probability distribution function of the sum of random response signals of randomly appearing particles in a simple detector model. We assume that the number of incoming particles within a given time period follows an inhomogeneous Poisson distribution, and that the detector counts all arriving particles. Also, the random response signals related to different particles are considered to be independent and identically distributed. Likewise, the question of correlated detection events, induced by incoming neutrons generated in a branching process, will be treated in a forthcoming publication.

The basic quantity, the "building brick" of the stochastic model of the detector signal is the current (or voltage) pulse form generated by each incoming particle arriving to the detector. This pulse shape can be considered as the response function of the detector. As mentioned, due to the statistical properties of the generation of such a pulse from the underlying physical processes, which also contain random elements, this response pulse form cannot be given by a deterministic function $f(t)$. In the general case it is described by a function $\varphi(\boldsymbol{\xi}, t)$ which depends on the possible realisations of a random variable $\boldsymbol{\xi}$. The continuously arriving particles generate the detector current as the aggregate of such response function current signals, each related to a different realisation of $\boldsymbol{\xi}$.

In the treatment that follows we will restrict the study to cases in which the dependence of $\varphi(\boldsymbol{\xi}, t)$ on its arguments is factorised into a form $\varphi(\boldsymbol{\xi}, t) = a(\boldsymbol{\xi})\, f(t)$ where a is the random amplitude of the pulse and $f(t)$ is the pulse shape. Although this assumption restricts somewhat the generality of the description, it will lead to a formalism which, for several basic signal shapes $f(t)$ is amenable to an analytical treatment, while still representing a realistic model of the detector signal. For signal shapes that are constant or monotonically decreasing for $t > 0$ (square and exponential) $\boldsymbol{\xi}$ will be the (random) initial value of the response signal. For other, non-monotonically varying signal shape it can be identified with a given parameter of the signal pulse. We assume that $\boldsymbol{\xi} \in \Re$, where $\Re$ is the set of real numbers, and it has a finite expected value and variance.

The derivation of the main quantity of interest, the probability density of the stochastic signal $\eta(t)$ at time t, given that at time $t = 0$ it was zero, needs the following definitions and considerations. We assume that the sequence of particle arrivals constitutes an inhomogeneous Poisson process. In this case the probability that no particle arrives at the detector during the time interval $[t_0, t],\ t \geq t_0$ is given by

$$T(t_0, t) = \exp\left\{-\int_{t_0}^{t} s_0(t')\, dt'\right\}. \tag{2.1}$$

Here $s_0(t)$ is the intensity of the particle arrivals at time t. Each particle will induce a pulse with shape $f(t)$ and a realisation x of the random amplitude $\boldsymbol{\xi}$. The cumulative probability distribution and the probability density of the amplitude distribution are described by the functions $W(x)$ and $w(x)$, respectively, as

$$\mathcal{P}\left\{\boldsymbol{\xi} \leq x\right\} \equiv W(x) = \int_{-\infty}^{x} w(x')\, dx' \tag{2.2}$$

The probability that the value of the response at time t after the arrival of *one single* particle is not greater than y is given by the degenerate distribution function

$$H(y, t) = \int_{-\infty}^{+\infty} \Delta[y - \varphi(x, t)]\, w(x)\, dx. \tag{2.3}$$

where $\Delta(x)$ is the unit step function. The probability density function $h(y, t)$ of $H(y, t)$ is given as

$$h(y, t) = \int_{-\infty}^{+\infty} \delta\left[y - \varphi(x, t)\right]\, w(x)\, dx. \tag{2.4}$$

Denoting the sum of the signals at $t \geq t_0$ as $\eta(t)$, we shall seek the probability of the event that $\eta(t)$ is less than or equal to y with the condition that its value was zero at t_0, i.e. the quantity

$$\mathcal{P}\left\{\eta(t) \leq y | \eta(t_0) = 0\right\} = P(y, t|0, t_0) = \int_{-\infty}^{y} p(y', t|0, t_0)\, dy'. \tag{2.5}$$

Straightforward considerations yield the following backward-type integral Chapman-Kolmogorov equation for the probability density function $p(y, t|0, t_0)$:

$$p(y, t|0, t_0) = T(t_0, t)\,\delta(y) + \int_{t_0}^{t} T(t_0, t') s_0(t') \int_{-\infty}^{y} h(y', t-t')\, p(y-y', t|0, t')\; dy'\; dt'. \tag{2.6}$$

The r.h.s. of Eq. (2.6) consists of the sum of the probabilities of the mutually exclusive events that there will not, or there will be a first detection sometime between t_0 and t, respectively. The product of the probabilities in the last integral of the second term is due to the fact that the contribution to the total signal at terminal time t from the first detection at time t' is independent from that of the subsequent detections between t' and t.

Equation (2.6) shows that the probability density of the detector signal induced by the aggregate of pulses, generated by the sequence of incoming particles with an inhomogeneous Poisson distribution ("source induced distribution"), is expressed by the probability density of the detector signal induced by one single particle ("single-particle induced distribution"), in form of a convolution. In this respect it is a complete analogue of the backward master equation of neutron multiplication, connecting the source induced distribution with the single-particle distribution (the "Sevast'yanov formula" [12]; in many papers it is called the "Bartlett formula"). The arguments used in the derivation are also essentially the same in both cases.

As is known, in the case of the "Sevast'yanov formula", turning the probability balance equation for the discrete random variable representing the neutron number into an equation for the probability generation functions of the source and single-particle induced distributions, the arising equation can be solved by quadrature. This means that the sought source-induced generating function can be expressed as an exponential integral of the single-particle induced generating function and the source intensity. Hence, once the single-particle generation function, or at least its moments are known, the source-induced distribution (or its moments) can be obtained by integration, without solving any further equations.

It will be seen that the same is true for the master equation (2.6), if the characteristic functions of the source-induced and the single-particle induced distributions are introduced. One difference compared to the neutron number distribution is that for this latter, the single-particle induced distribution is not given in advance, rather it is obtained as the solution of the master equation of the branching process. However, the moments of the single induced distribution have a very simple form, and moreover the first moment serves as the Green's function of the higher order moments. For the detector signal in our case, the single-particle induced distribution will be given in advance and hence known (through the model signal shapes and their amplitude distributions that we shall define); in turn these will be more complicated than in the case of the neutron transport and the arising integrals will be considerably more complicated to perform.

For continuous random variables that can take also negative values (which we shall assume for generality in this Section), the characteristic functions are defined by the Fourier transform in the signal value variable. Hence, the characteristic functions π and χ of p and h, respectively, are defined as

$$\pi(\omega, t|0, t_0) = \int_{-\infty}^{+\infty} e^{\imath\omega y}\, p(y, t|0, t_0)\, dy, \tag{2.7}$$

and

$$\chi(\omega, t) = \int_{-\infty}^{+\infty} e^{\imath\omega y}\, h(y, t)\, dy =$$
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{\imath\omega y}\, \delta\left[y - \varphi(x, t)\right]\, dy\, w(x)\, dx = \int_{-\infty}^{+\infty} e^{\imath\omega\varphi(x,t)}\, w(x)\, dx. \tag{2.8}$$

Then, from Eq. (2.6) one obtains

$$\pi(\omega, t|0, t_0) = T(t_0, t) + \int_{t_0}^{t} T(t_0, t') s_0(t')\, \chi(\omega, t - t')\, \pi(\omega, t|0, t')\, dt'. \tag{2.9}$$

From this integral equation it is seen that

$$\lim_{t\downarrow t_0} \pi(\omega, t|0, t_0) = 1. \tag{2.10}$$

Derivation w.r.t. t_0 leads to the differential equation

$$\frac{\partial \pi(\omega, t|0, t_0)}{\partial t_0} = s_0(t_0)\, \pi(\omega, t|0, t_0)\, [1 - \chi(\omega, t - t_0)] \tag{2.11}$$

Accounting for the initial condition (2.10), the solution of (2.11) can be written as

$$\pi(\omega, t|0, t_0) = \exp\left\{-\int_0^{t-t_0} s_0(t - t')\, [1 - \chi(\omega, t')]\, dt'\right\} \tag{2.12}$$

where $\chi(\omega, t')$ is defined in (2.8). Eq. (2.12) can be considered to be the characteristic function of the generalized non-homogeneous Poisson-process. Again, a close analogue is seen with the similar expression for the neutron multiplication process.

If the particle arrivals correspond to a homogeneous Poisson process with constant intensity s_0, then the characteristic function $\pi(\omega, +\infty|0, -\infty)$ is given by

$$\pi(\omega, +\infty|0, -\infty) = \pi_{st}(\omega) = \exp\left\{-s_0 \int_{-\infty}^{+\infty} [1 - \chi(\omega, t)]\, dt\right\} \tag{2.13}$$

if the integral on the r.h.s. exists. Hence there exists an *asymptotically stationary signal level* $\eta^{(st)}$, with the probability density function

$$\pi_{st}(y) = \mathcal{L}^{-1}\left\{\pi_{st}(\omega)\right\}. \tag{2.14}$$

For the subsequent calculations it is also practical to introduce the logarithm $\gamma_{st}(\omega)$ of the characteristic function (2.13), i.e.

$$\gamma_{st}(\omega) = \ln \pi_{st}(\omega) = s_0 \int_{-\infty}^{+\infty} [\chi(\omega, t) - 1] \; dt. \tag{2.15}$$

2.2 Expectation, variance and cumulants

From (2.15) the expectation of the stationary signal $\eta^{(st)}$ can be easily calculated as

$$\mathbf{E}\left\{\eta^{(st)}\right\} = i_1^{(st)} = \frac{1}{\imath}\left[\frac{d\gamma_{st}(\omega)}{d\omega}\right]_{\omega=0} = s_0 \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\varphi(x,t)\, w(x)\; dx\right]\; dt, \tag{2.16}$$

and its variance as

$$\mathbf{D}^2\left\{\eta^{(st)}\right\} = \sigma_{st}^2 = -\left[\frac{d^2\gamma_{st}(\omega)}{d\omega^2}\right]_{\omega=0} = s_0 \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\varphi(x,t)^2\, w(x)\; dx\right]\; dt. \tag{2.17}$$

If the value of the signal amplitude $\boldsymbol{\xi}$ is always unity, that is $w(x) = \delta(x-1)$, then (2.16) and (2.17) revert to the expressions of the Campbell's theorem, (1.1), in the form

$$\mathbf{E}\left\{\eta^{(st)}\right\} = s_0 \int_{-\infty}^{+\infty} f(t)\; dt \tag{2.18}$$

and

$$\mathbf{D}^2\left\{\eta^{(st)}\right\} = s_0 \int_{-\infty}^{+\infty} f(t)^2\; dt \tag{2.19}$$

with $f(t) = \varphi(1,t)$. After proper calibration, both of these forms are suitable to determine the particle intensity s_0.

As is known [13], the *cumulants* or *semiinvariants* $\kappa_n^{(st)}$ of $\eta^{(st)}$, which can be expressed by the moments of $\eta^{(st)}$, can be derived from the logarithmic characteristic function $\gamma_{st}(\omega)$ through the formula

$$\kappa_n^{(st)} = \frac{1}{\imath^n}\left[\frac{d^n\gamma_{st}(\omega)}{d\omega^n}\right]_{\omega=0} = s_0 \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\varphi(x,t)^n\, w(x)\; dx\right]\; dt.$$

This way one immediately arrives at the results referred to in the literature as *higher order Campbell techniques* [5], [6]. It is readily seen that all cumulants are linearly proportional to the intensity s_0. A few cumulants are given below for illustration.

$$\kappa_1^{(st)} = i_1^{(st)},$$
$$\kappa_2^{(st)} = i_2^{(st)} - \left(i_1^{(st)}\right)^2,$$
$$\kappa_3^{(st)} = i_3^{(st)} - 3\, i_2^{(st)}\, i_1^{(st)} + \left(i_1^{(st)}\right)^3,$$
$$\kappa_4^{(st)} = i_4^{(st)} - 4\, i_3^{(st)}\, i_1^{(st)} - 3\, \left(i_2^{(st)}\right)^2 + 12\, i_2^{(st)}\, \left(i_1^{(st)}\right)^2 - 6\, \left(i_1^{(st)}\right)^4,$$

where

$$i_n^{(st)} = \mathbf{E}\Big\{\left(\eta^{(st)}\right)^n\Big\}, \qquad n = 1, 2, \ldots \quad .$$

2.3 Definition of the selected pulse shapes

In the rest of the paper we will only deal with processes of the form $\varphi(x,t) = x\, f(t)$ where $f(t)$ is a deterministic signal function. We will also assume that the realizations x of the random variable $\boldsymbol{\xi}$, as well as the signal function $f(t)$ itself take only non-negative real values. The signal forms considered all start with a jump, i.e. at time $t = 0$, $f(t)$ jumps from zero to its maximum (equal to x), whereafter $f(t)$ will either monotonically decrease or remain constant during a period, after which it jumps back to zero. For the total detector signal, this means that the arrival of a particle to the detector incurs a jump of the signal level with the value x.

Since the detector signals are now non-negative, the Fourier transforms of the previous formulae will be replaced by Laplace transforms. Thus we shall consider the *Laplace-transform* of the density function $p(y,t|0,t_0)$, defined in (2.5) as

$$\widetilde{p}(s,t|0,t_0) = \int_0^\infty e^{-sy}\, p(y,t|0,t_0)\, dy \tag{2.20}$$

as the characteristic function. Introducing the transforms

$$\widetilde{h}(s,t) = \int_0^\infty e^{-sy}\, h(y,t)\, dy \qquad \text{and} \qquad \widetilde{w}(s) = \int_0^\infty e^{-sx}\, w(x)\, dx, \tag{2.21}$$

from (2.12) we obtain

$$\widetilde{p}(s,t|0,t_0) = \exp\left\{-\int_0^{t-t_0} s_0(t-t')\left[1-\widetilde{h}(s,t')\right]\, dt'\right\}, \tag{2.22}$$

where $\widetilde{h}(s,t')$ is defined as

$$\widetilde{h}(s,t') = \int_0^\infty \exp\left\{-s\, f(t')\, x)\right\}\, w(x)\, dx = \widetilde{w}[sf(t')]. \tag{2.23}$$

To simplify the further considerations let us choose

$$s_0(t-t') = s_0 \qquad \text{and} \qquad t_0 = 0$$

and use the notation

$$\widetilde{p}(s,t|0,0)=\widetilde{p}(s,t). \tag{2.24}$$

From equation (2.22) one immediately obtains

$$\widetilde{p}(s,t)=\exp\left\{-s_0\int_0^t\left[1-\widetilde{h}(s,t')\right]\,dt'\right\}=\exp\left\{-s_0\int_0^t\left\{1-\widetilde{w}[sf(t')]\right\}\,dt'\right\}. \tag{2.25}$$

As in the previous case, the Laplace transform of the stationary case are defined by the Laplace-transform

$$\lim_{t\to\infty}\widetilde{p}(s,t)=\widetilde{p}_{st}(s)=$$

$$\exp\left\{-s_0\int_0^\infty\left[1-\widetilde{h}(s,t)\right]\,dt\right\}=\exp\left\{-s_0\int_0^\infty\left\{1-\widetilde{w}[sf(t)]\right\}\,dt\right\} \tag{2.26}$$

provided that the integrals exist. From this it follows that there exists an asymptotically stationary signal level $\eta^{(st)}$ with the density function

$$p_{st}(y)=\mathcal{L}^{-1}\left\{\widetilde{p}_{st}(s)\right\}. \tag{2.27}$$

In this case too, for the determination of the cumulants it is practical to use the logarithm of the Laplace-transform $\widetilde{p}_{st}(s)$ of the density function $p_{st}(y)$:

$$\widetilde{g}_{st}(s)=\ln\widetilde{p}_{st}(s)=s_0\int_0^\infty\left[\widetilde{h}(s,t)-1\right]\,dt, \tag{2.28}$$

where

$$\widetilde{h}(s,t)=\widetilde{w}[sf(t)].$$

2.4 Level crossing intensity

The intensity $n_{st}(V)$ of the events when the signal level jumps above a certain level V from a value $y\le V$ is a measurable quantity. The level V is usually called the *threshold*. Since it also contains information about the intensity of the incoming particle events, it is interesting to derive expressions for it. In the stationary case, it can be written in the following form:

$$n_{st}(V)=s_0\int_0^V p_{st}(y)\,\left[1-W(V-y)\right]\,dy, \tag{2.29}$$

whose Laplace-transform is given as

$$\widetilde{n}_{st}(s)=s_0\,\frac{1-\widetilde{w}(s)}{s}\,\widetilde{p}_{st}(s). \tag{2.30}$$

The expression for $n_{st}(V)$ in (2.29) consists of the product of the intensity of the particle arrivals and the probability that the signal level y is under the threshold V *and* the amplitude of the induced pulse is larger than $V - y$. Since these two latter events are independent, their probabilities are multiplied, and one has to integrate for all values of y between zero and the threshold.

It can be expected that $n_{st}(V)$ is small for high threshold values, since the density function $p_{st}(y)$ is close to zero for such cases. For small threshold values, the behaviour of $n_{st}(V)$ will sensitively depend on both the intensity of the incoming particles, as well as on the signal shape. For high intensities, $n_{st}(V)$ will be small for low threshold values. In most cases the intensity has a maximum at a threshold value V_{max}. The knowledge of this quantity can be useful when trying to eliminate by thresholding the background noise which contaminates the useful signal. The reduction of this component can only be achieved by using a threshold greater than the threshold V_{max}, corresponding to the maximum of the intensity $n_{st}(V)$.

In many cases the value of the jump $\boldsymbol{\xi}$ can be assumed to be constant, i.e.

$$\mathcal{P}\left\{\boldsymbol{\xi} \leq x\right\} = W(x) = \Delta\left(x - \frac{1}{\mu}\right), \tag{2.31}$$

hence one has

$$n_{st}(V) = s_0 \left[P_{st}(V) - P_{st}(V - 1/\mu)\right], \tag{2.32}$$

since

$$\int_0^V p_{st}(y)\,\Delta\left(V - y - \frac{1}{\mu}\right)\,dy = \int_0^{V-1/\mu} p_{st}(y)\,dy = P_{st}(V - 1/\mu).$$

It will be seen that in the case of a constant jump, the determination of the density function $p_{st}(y)$ from the Laplace-transform $\widetilde{p}_{st}(s)$ is not an easy task. The problems encountered will be shown for the pulse shape $f(t) = e^{-\alpha t}$.

In order to study the characteristics of the detector signal functions in more detail, in the next section we perform detailed calculations for a few selected pulse shapes $f(t)$.

3 Results for concrete cases

3.1 Rectangular pulses

In this case the particles, arriving at the detector according to a Poisson process, generate a signal with a constant width T_0 and random height $\boldsymbol{\xi}$. Two different realisations of such a pulse are shown in Fig. 1. For simplicity, an exponential distribution of the amplitudes will be assumed, i.e.

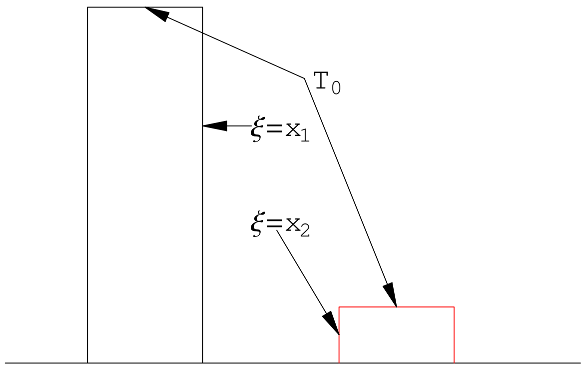

Figure 1. Illustration of the rectangular pulses with random heights.

$$\mathcal{P}\{\boldsymbol{\xi} \le x\} = W(x) = 1 - e^{-\mu x}. \tag{3.1}$$

where $1/\mu$ is expectation of the starting amplitude of a single signal. Since

$$f(t) = \Delta(T_0 - t), \tag{3.2}$$

from (2.25) one obtains

$$\widetilde{p}(s,t) = \exp\left\{-s_0 \int_0^t \frac{s\,\Delta(T_0 - t')}{s\,\Delta(T_0 - t') + \mu}\right\}\, dt'. \tag{3.3}$$

The integral in (3.3) can be performed analytically, leading to

$$\widetilde{p}(s,t) = \begin{cases} \exp\left\{s_0\, t\left(\frac{\mu}{s+\mu} - 1\right)\right\}, & \text{if } t \le T_0, \\ \\ \exp\left\{s_0\, T_0\left(\frac{\mu}{s+\mu} - 1\right)\right\}, & \text{if } t > T_0. \end{cases} \tag{3.4}$$

The resulting expressions can be inverted analytically, yielding the result as

$$p(y,t) = \begin{cases} e^{-s_0 t}\, e^{-\mu y}\left[\delta(y) + \sqrt{\frac{s_0 t \mu}{y}}\, I_1\left(2\sqrt{s_0 t \mu y}\right)\right], & \text{if } t \le T_0, \\ \\ e^{-s_0 T_0}\, e^{-\mu y}\left[\delta(y) + \sqrt{\frac{s_0 T_0 \mu}{y}}\, I_1\left(2\sqrt{s_0 T_0 \mu y}\right)\right], & \text{if } t > T_0, \end{cases} \tag{3.5}$$

where $I_1(x)$ is the modified Bessel function of order one. It is worth noting that $p(y,t)$ converges rather fast to the asymptotically stationary density function $p_{st}(y)$ with increasing t. As the second part of (3.5) shows, the sum of the individual signals of particles arriving according to a homogeneous Poisson process with intensity s_0 has a stationary distribution already for $t > T_0$.

From the Laplace transform (3.4) we can get immediately the expected value of the sum of detector pulses at time t as

$$\mathbf{E}\left\{\eta(t)\right\} = -\left[\frac{\partial \ln \tilde{p}(s,t)}{\partial s}\right]_{s=0} = \frac{s_0}{\mu}\left[t\,\Delta(T_0 - t) + T_0\,\Delta(t - T_0)\right] \tag{3.6}$$

and its variance as

$$\mathbf{D}^2\left\{\eta(t)\right\} = \left[\frac{\partial^2 \ln \tilde{p}(s,t)}{\partial s^2}\right]_{s=0} = 2\,\frac{s_0}{\mu^2}\left[t\,\Delta(T_0 - t) + T_0\,\Delta(t - T_0)\right]. \tag{3.7}$$

It is also worth noting that the *Fano factor* (variance to mean) for this case is equal to

$$\mathcal{F} = \frac{2}{\mu}, \tag{3.8}$$

where $1/\mu$ is the expected value of the pulse jump. It is seen that the Fano factor does not depend on time.

For the illustration of the signal level crossing in the stationary case, we shall calculate the intensity $n_{st}(V)$ of particle arrivals which induce a jump of the signal from a level $y \leq V$ to a signal level higher than V. Fig. 2. shows a

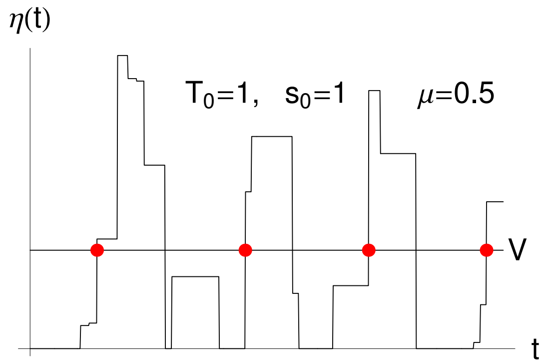

Figure 2. A possible realization of the sum of signals in an arbitrary time interval.

possible realization of the sum of pulses within a stationary time interval. The red dots mark the particles which induce the jump of the signal level from a state $y \leq V$ to above the threshold V. By using Eq. (2.29) we obtain

$$n_{st}(V) = s_0 \int_{+0}^{V} e^{-\mu(V-y)}\, p_{st}(y)\, dy, \tag{3.9}$$

where

$$p_{st}(y) = e^{-s_0 T_0}\, e^{-\mu y}\left[\delta(y) + \sqrt{\frac{s_0 T_0 \mu}{y}}\, I_1\left(2\sqrt{s_0 T_0 \mu y}\right)\right].$$

From this it follows that

$$n_{st}(V) = s_0\, e^{-s_0 T_0}\, e^{-\mu V} \int_0^V \left[\delta(y) + \sqrt{\frac{s_0 T_0 \mu}{y}}\, I_1\left(2\sqrt{s_0 T_0 \mu y}\right)\right] dy =$$

$$s_0\, e^{-s_0 T_0}\, e^{-\mu V}\, I_0\left(2\sqrt{s_0 T_0 \mu\, V}\right), \qquad (3.10)$$

where $I_0(x)$ is the modified Bessel function of order zero.

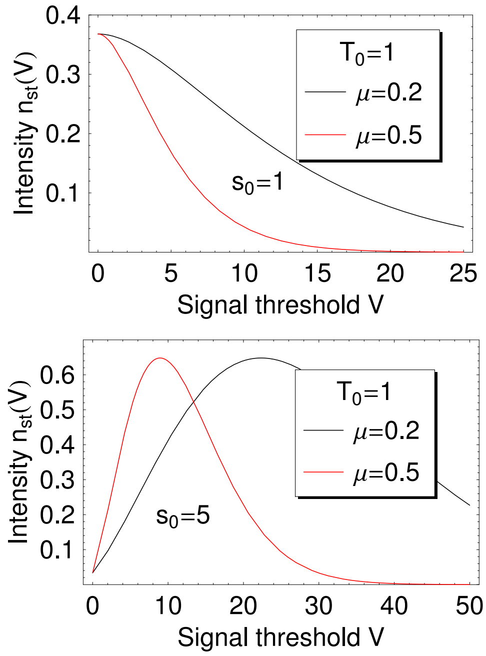

Figure 3. Dependence of the intensity $n_{st}(V)$ on the signal threshold V at two different input intensities s_0.

The dependence of the intensity $n_{st}(V)$ on the threshold V is shown in Fig. 3 for two different values of μ and at two different input intensities s_0. It is seen how the values of μ and s_0 influence the dependence of the output intensity $n_{st}(V)$ on the threshold value V. In order to show the influence of the input intensity s_0 on the output intensity $n_{st}(V)$, in Fig. 4 the dependence of $n_{st}(V)$ on s_0 is plotted for two threshold values V. One concludes that the mean amplitude $1/\mu$ of the rectangular signal must be chosen very carefully.

3.2 Exponential pulses

3.2.1 Random amplitude

We will treat now the case when the pulses have an exponential decay shape,

$$f(t) = e^{-\alpha t}, \qquad (3.11)$$

the initial values of which being the realizations of the random variable $\boldsymbol{\xi}$. For simplicity, assume again exponential distribution of the amplitudes as in (3.1). Fig. 5. shows two possible pulses with $\alpha = 2$.

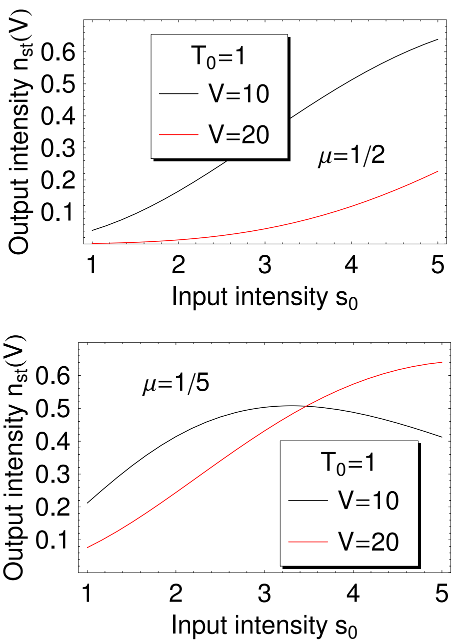

Figure 4. Dependence of the output intensity $n_{st}(V)$ on the input intensity s_0 for two different values of μ and at two different threshold values V.

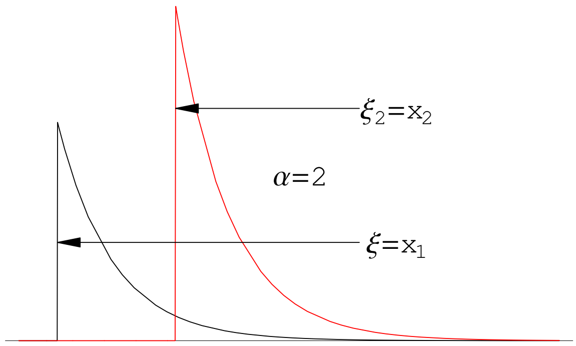

Figure 5. Exponential pulses generated by incoming particles.

The probability density of the signal induced by one particle is given as

$$h(y,t) = \int_0^\infty \delta\left(y - xe^{-\alpha t}\right)\, w(x)\, dx \tag{3.12}$$

whose Laplace transform is obtained (for details, see [10]) as

$$\widetilde{h}(s,t) = \frac{\mu}{\mu + s\, e^{-\alpha t}}. \tag{3.13}$$

Using (2.25) yields the Laplace transform of the density function $p(y,t)$ as

$$\widetilde{p}(s,t) = \exp\left\{-s_0 \int_0^t \left[1 - \frac{\mu}{\mu + s\, e^{-\alpha v}}\right] dv\right\} = \exp\left\{-s_0 \int_0^t \frac{s\, e^{-\alpha v}}{\mu + s\, e^{-\alpha v}}\, dv\right\}. \tag{3.14}$$

which leads to

$$\widetilde{p}(s,t) = \exp\left\{\frac{s_0}{\alpha}\ \ln\ \frac{\mu + s\,e^{-\alpha 1t}}{\mu + s}\right\} = \left(\frac{\mu + s\,e^{-\alpha t}}{\mu + s}\right)^{s_0/\alpha}. \tag{3.15}$$

From (3.15) it is also obvious that a *stationary density function* exists with the Laplace transform

$$\lim_{t\to\infty} \widetilde{p}(s,t) = \widetilde{p}_{st}(s) = \left(\frac{\mu}{\mu + s}\right)^{s_0/\alpha}. \tag{3.16}$$

Eq. (3.15) can be rewritten as

$$\widetilde{p}(s,t) = \left[1 - \left(1 - e^{-\alpha t}\right)\ \frac{s}{s+\mu}\right]^{q}, \tag{3.17}$$

where

$$q = \frac{s_0}{\alpha} > 0. \tag{3.18}$$

The inversion of (3.17) will depend on the fact whether q is an integer number or not. Details of the extensive calculations will be omitted here, these can be found in [10].

If q is not an integer and the following inequality holds:

$$\left(1 - e^{-\alpha t}\right)\left|\frac{s}{s+\mu}\right| < q,$$

then, (3.17) can be written in the form

$$\widetilde{p}(s,t) = 1 + \sum_{k=0}^{\infty} (-1)^k\ \frac{q(q-1)\cdots(q-k+1)}{k!}\ \left(1 - e^{-\alpha t}\right)^k\ \left(\frac{s}{s+\mu}\right)^k. \tag{3.19}$$

After considerable algebra, the inversion of (3.19), i..e that of (3.17), is obtained as

$$\begin{aligned} p(y,t) = \delta(y)\left[1 + \sum_{k=1}^{\infty} \frac{\Gamma(-s_0/\alpha + k)}{\Gamma(-s_0/\alpha)\ \Gamma(k+1)}\ (1 - e^{-\alpha t})^k\right] - \\ \mu\, e^{-\mu y}\ \sum_{k=1}^{\infty} \frac{\Gamma(-s_0/\alpha) + k)}{\Gamma(-s_0/\alpha)\ \Gamma(k+1)}\ L_{k-1}^{(1)}(\mu y)\ (1 - e^{-\alpha t})^k, \end{aligned} \tag{3.20}$$

where $L_{k-1}^{(1)}(\mu y)$ is the generalised Laguerre polynomial. Fig. 6. shows the dependence of the density function $p(y,t)$ on the parameter αt for $q = s_0/\alpha = 0.8$, for three different signal levels. It is interesting to note that the density function becomes constant relatively fast; in the present case the stationary behaviour is already reached for $\alpha t \approx 5$.

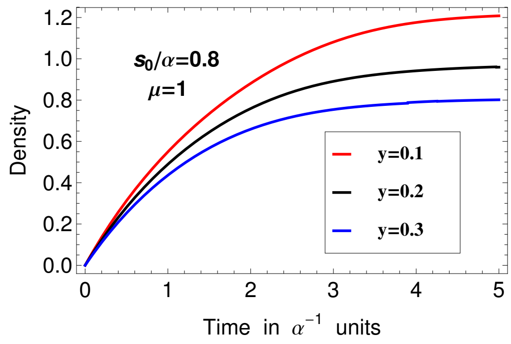

Figure 6. Dependence of the density function $p(y,t)$ on the time parameter αt at three signal levels and at $q = s_0/\alpha = 0.8$.

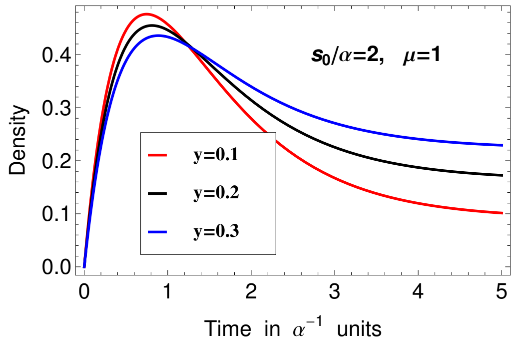

Figure 7. Dependence of the density function $p(y,t)$ on the time parameter αt at three signal levels for $q = s_0/\alpha = 2$.

If $q = s_0/\alpha$ is a positive integer, expression (3.19) cannot be used. One has instead to return to Eq. (3.17) and use the rearrangement

$$\widetilde{p}(s,t) = 1 - \sum_{k=1}^{q} \binom{q}{k} (-1)^{k-1} \left(1 - e^{-\alpha t}\right)^k \frac{s^k}{(s+\mu)^k}. \tag{3.21}$$

From this one obtains

$$p(y,t) = \delta(y) \left[1 - \sum_{k=1}^{q} \binom{q}{k} (-1)^{k-1} (1 - e^{-\alpha t})^k \right] +$$

$$\mu\, e^{-\mu y} \sum_{k=1}^{q} \binom{q}{k} (-1)^{k-1} L_{k-1}^{(1)}(\mu y)\, (1 - e^{-\alpha t})^k, \tag{3.22}$$

which does not contain singular Gamma functions. Fig. 7. shows the dependence of the density function $p(y,t)$ on the parameter αt for $q = s_0/\alpha = 2$, for three different signal levels. One finds again that the density function is close to stationary already at $\alpha t \approx 5$.

We will now investigate the properties of the *stationary* signal sequence. The Laplace transform (3.16) of the density function $p_{st}(y)$ can easily be inverted.

One obtains

$$p_{st}(y) = \frac{(\mu y)^{\frac{s_0}{\alpha}-1}}{\Gamma\left(s_0/\alpha\right)}\, e^{-\mu y}\, \mu, \tag{3.23}$$

which shows that the stationary distribution of the sum of exponential pulses is given by

$$P_{st}(y) = \int_0^y p_{st}(y')\, dy' = \frac{\Gamma\left(s_0/\alpha\right) - \Gamma\left(\mu y, s_0/\alpha\right)}{\Gamma\left(s_0/\alpha\right)}, \tag{3.24}$$

in which

$$\Gamma\left(\mu y, s_0/\alpha\right) = \int_{\mu y}^{\infty} \frac{v^{\frac{s_0}{\alpha}-1}}{\Gamma\left(s_0/\alpha\right)}\, e^{-v}\, dv$$

is the incomplete Gamma function.

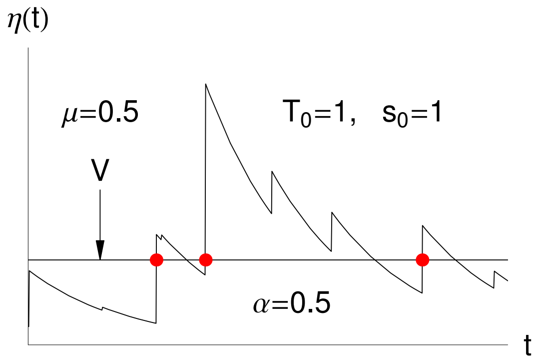

Figure 8. A possible realization of the sum of signals in stationary case in an arbitrary time interval.

Fig. 8 shows a possible realization of the exponential pulse train in the stationary state. The red dots mark the particles which induce a jump of the signal level to above the threshold V from a level under V. By using (2.30) we obtain the Laplace transform

$$\tilde{n}_{st}(s) = s_0\, \frac{1-\tilde{w}(s)}{s}\, \tilde{p}_{st}(s) = s_0\, \frac{1}{\mu+s} \left(\frac{\mu}{\mu+s}\right)^{s_0/\alpha}, \tag{3.25}$$

whose inverse is given as

$$n_{st}(V) = s_0\, \frac{(\mu V)^{\frac{s_0}{\alpha}}}{\Gamma\left(s_0/\alpha+1\right)}\, e^{-\mu V}. \tag{3.26}$$

Fig. 9 shows the dependence of the intensity $n_{st}(V)$ on the threshold V for two different values of the parameter μ. It is seen that the intensity $n_{st}(V)$ has a distinct maximum at the threshold

$$V_{max} = \frac{s_0}{\alpha\mu}.$$

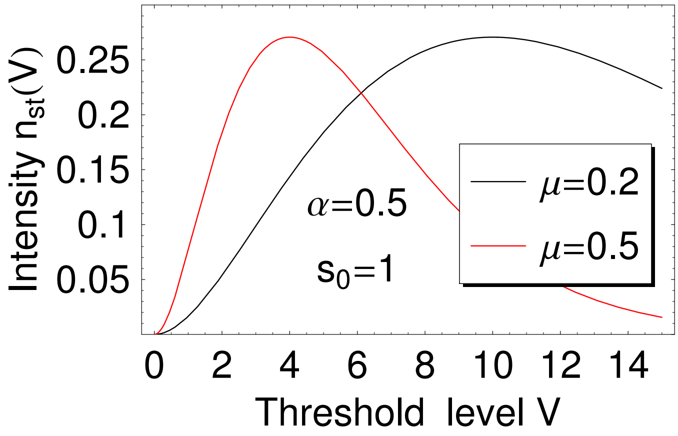

Figure 9. Dependence of the intensity $n_{st}(V)$ on the signal threshold V.

The knowledge of this maximum could be important for the design of the detector electronics.

The expectation and the variance, important for practical applications, will now be calculated for both the non-stationary and the stationary case. From the logarithm of the Laplace transform (3.15) one obtains

$$-\left[\frac{d\ln\widetilde{p}(s,t)}{ds}\right]_{s=0} = \mathbf{E}\left\{\eta(t)\right\} = \begin{cases} \frac{s_0}{\alpha\mu}\left(1-e^{-\alpha t}\right), & \text{if } t \leq \infty, \\ \\ \frac{s_0}{\alpha\mu}, & \text{if } t = \infty, \end{cases} \tag{3.27}$$

and

$$\left[\frac{d^2\ln\widetilde{p}(s,t)}{ds^2}\right]_{s=0} = \mathbf{D}^2\left\{\eta(t)\right\} = \begin{cases} \frac{s_0}{\alpha\mu^2}\left(1-e^{-2\alpha t}\right), & \text{if } t \leq \infty, \\ \\ \frac{s_0}{\alpha\mu^2}, & \text{if } t = \infty. \end{cases} \tag{3.28}$$

For illustration, the Fano factor for this case is also given. It reads as

$$\mathcal{F} = \begin{cases} \frac{1}{\mu}\left(1+e^{-\alpha t}\right), & \text{if } t \leq \infty, \\ \\ \frac{1}{\mu}, & \text{if } t = \infty. \end{cases} \tag{3.29}$$

3.2.2 Constant amplitude

For more insight, the special case of constant pulse amplitudes, i.e. when the amplitude probability density function is given as $w(x) = \delta(x - x_0)$, will be calculated. From (3.12) one has

$$h(y,t) = \delta\left[y - x_0\, e^{-\alpha t}\right], \tag{3.30}$$

whose Laplace transform is obtained as

$$\tilde{h}(s,t) = \exp\left[-s\, x_0\, e^{-\alpha t}\right]. \tag{3.31}$$

Substituting into (2.26) yields

$$\tilde{p}_{st}(s) = \exp\left\{-s_0 \int_0^\infty \left[1 - \exp\left[-s\, x_0\, e^{-\alpha t}\right]\right]\, dt\right\}, \tag{3.32}$$

from which one obtains

$$\tilde{g}_{st}(s) = \ln \tilde{p}_{st}(s) = s_0 \int_0^\infty \left[\exp\left(-s\, x_0\, e^{-\alpha t}\right) - 1\right]\, dt. \tag{3.33}$$

From (3.33), one can immediately determine the expectation and the variance of $\eta^{(st)}$. One obtains

$$\mathbf{E}\left\{\eta^{(st)}\right\} = s_0\, \frac{x_0}{\alpha} \qquad \text{and} \qquad \mathbf{D}^2\left\{\eta^{(st)}\right\} = s_0\, \frac{x_0^2}{2\,\alpha}, \tag{3.34}$$

whereas the Fano factor equals $\mathcal{F} = x_0/2$.

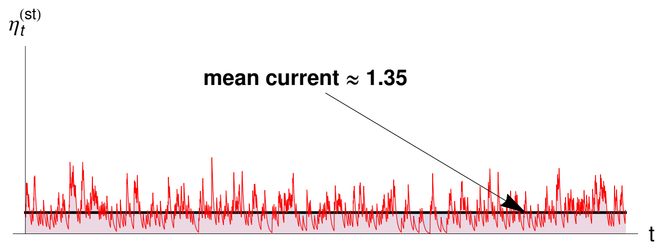

Figure 10. Simulation of the stationary current $\eta_t^{(st)}$ of the ionization chamber as a function of time, with exponentially decaying pulses of unit initial value (cf. Fig. 8).

Fig. 10. shows the time dependence of the stationary current $\eta_t^{(st)}$ of the ionisation chamber, when the current is formed by exponentially decaying pulses of unit initial value. Except for the level crossing and the random amplitude of the pulses, this Figure is analogous to Fig. 8, but shows a much longer time period of the process. For this case one has

$$\mathbf{E}\left\{\eta_t^{(st)}\right\} \approx 1.35 \qquad \text{and} \qquad \mathbf{D}^2\left\{\eta_t^{(st)}\right\} \approx 0.58.$$

The inversion of (3.32) is a formidable task, and one way of performing it will be only outlined here. For the details we refer to [10]. With a change of variables the integration can be performed and (3.32) can be written as

$$\widetilde{p}_{st}(s) = \exp\left[-\frac{s_0}{\alpha}\left(C + \ln x_0\, s + \int_{x_0}^{\infty} \frac{e^{-s\,u}}{u}\, du\right)\right] = \frac{\exp\left(-\frac{s_0}{\alpha}\int_{x_0}^{\infty}\frac{e^{-s\,u}}{u}\, du\right)}{\left(x_0\, e^{C}\, s\right)^{s_0/\alpha}}. \tag{3.35}$$

Eq. (3.35) is written in the following, absolute convergent, infinite series:

$$\widetilde{p}_{st}(s) = \frac{1}{\left(x_0\, e^{C}\right)^{s_0/\alpha}} \left\{\frac{1}{s^{s_0/\alpha}} + \sum_{n=1}^{\infty} (-1)^n \frac{1}{n!}\left(\frac{s_0}{\alpha}\right)^n \frac{1}{s^{s_0/\alpha}}\left(\int_{x_0}^{\infty}\frac{e^{-s\,u}}{u}\, du\right)^n\right\}. \tag{3.36}$$

After a considerable algebra it can be shown that the inverse Laplace transform of (3.36) can be given by formula

$$p_{st}(y) =$$

$$\frac{1}{\left(x_0\, e^{C}\right)^{s_0/\alpha}\,\Gamma(s_0/\alpha)} \left\{y^{s_0/\alpha-1} + \sum_{n=1}^{[y/x_0]} \frac{(-1)^n\,(s_0/\alpha)^n}{n!}\int_{n\,x_0}^{y} (y-u)^{s_0/\alpha-1}\, h_n(u)\, du\right\}, \tag{3.37}$$

where $[y/x_0]$ is the largest integer less or equal to y/x_0. The function $h_n(u)$ is defined by the recursive equation

$$h_n(u) = \int_{x_0}^{u-(n-1)\,x_0} \frac{h_{n-1}(u-v)}{v}\, dv \tag{3.38}$$

with the starting function

$$h_1(u) = \frac{1}{u}\,\Delta(u - x_0).$$

The next term is still simple to calculate with the result

$$h_2(u) = \int_{x_0}^{u-x_0} \frac{dv}{(u-v)\,v} = 2\,\frac{\ln\left(\frac{u}{x_0} - 1\right)}{u}\,\Delta(u - 2\,x_0), \tag{3.39}$$

However, calculation of the third term already shows that the complexity of the expressions increase drastically with the order of the terms:

$$\begin{aligned} h_3(u) = \frac{1}{6\,u}\Big\{\pi^2 + 24\,\ln\frac{x_0}{u-2\,x_0}\,\ln\frac{x_0}{u-x_0} - \\ 12\,\mathrm{PolyLog}\left[2, 1 + \frac{x_0}{x_0-u}\right] + \\ 12\,\mathrm{PolyLog}\left[2, \frac{x_0}{u-x_0}\right] + 12\,\mathrm{PolyLog}\left[2, 2 - \frac{u}{x_0}\right]\Big\}\,\Delta(u - 3\,x_0), \end{aligned} \tag{3.40}$$

where

$$\text{PolyLog}(n, x) = \text{Li}_n(x) = \sum_{k=1}^{\infty} x^k/k^n$$

is the Jonquièrés function.

The subsequent terms are still possible to calculate with *Mathematica*, but they are prohibitively long to reproduce in print.

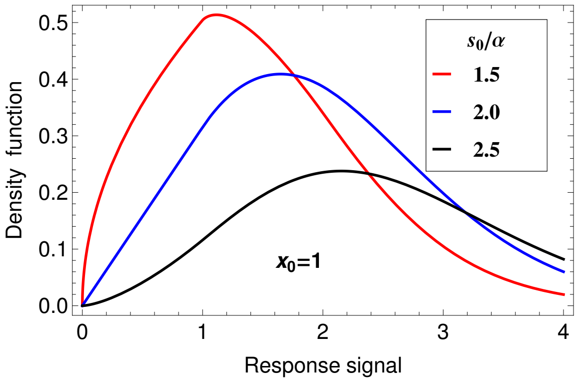

Figure 11. Probability density function of the stationary current $\eta_t^{(st)}$ of the ionization chamber with exponentially decaying pulses of constant $x_0 = 1$ amplitude with three different s_0/α parameter values.

By using the *Mathematica code* the dependence of the stationary density function $p_{st}(x)$ on x has been calculated in the case of constant $x_0 = 1$ amplitude for three different s_0/α parameter values. In Figure 11 one can see that the value of the ratio s_0/α sensitively influences the shape of $p_{st}(x)$. This sensitivity has to be taken into the count by the construction of the detector electronics.

4 Conclusions

In this report the theory of the Campbell method and that of the so-called higher order Campbelling techniques was revisited, as applied to the signals of ionisation chambers, with the fission chamber as the main interest of applications. First, a compact derivation was given of the higher order Campbell relationships for the case of an incoming particle sequence with a non-homogeneous Poisson distribution and with arbitrary signal amplitude distributions and signal shapes. The derivation is based on the integral form of the backward equation and it shows considerable resemblance to a similar formula in neutron branching processes, connecting the source-induced and single-particle induced distributions. For pulse forms which commence with a jump from zero to a maximum value after which they decay monotonically or remain constant for some time, a formula was given for the intensities of crossing a threshold level from below.

Concrete analytical solutions were given for the full probability distribution of the signal amplitudes, the first two moments and the level crossing intensities for two selected signal forms; a square and an exponential pulse form. In both cases an exponential amplitude distribution was assumed; for the exponential pulse, also the case of constant amplitude was considered.

In addition to giving insight, these results can also serve for benchmarking of Monte-Carlo codes. The explicit results presented in this report can serve to benchmark the accuracy of the higher order moments of the Monte Carlo simulation results. After benchmarking against the theoretical results presented in this papers, such codes can be used to calculate also other pulse forms (triangular, gamma-distribution-form etc.) which are not amenable for analytical solutions.

It is a pre-requisite of applying the Campbell method that the particle arrivals to the detector constitute a Poisson process, and their responses are independent. In reality these conditions are usually not fulfilled. It appears therefore interesting to extend the theoretical model calculate the distribution function of the detector signal if the individual responses are not independent. This would give a possibility to determine higher order moments of the statistics of the neutron arrival times to the detector, to be used for diagnostic purposes. A further open question is whether interaction between the charges generated by the ionising particles, in the present case by the fission products, can be modelled by response functions, characterized by independent probability distributions.

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References

[1] P. Filliatre, C. Jammes, B. Geslot, L. Buiron, In vessel neutron instrumentation for sodium-cooled fast reactors: Type, lifetime and location, Annals of Nuclear

Energy 37 (11) (2010) 1435–1442.

[2] P. Filliatre, L. Oriol, C. Jammes, L. Vermeeren, Reasons why plutonium 242 is the best fission chamber deposit to monitor the fast component of a high neutron flux, *Nucl. Instr. Meth.* **A 593** (2008) 510.

[3] A. Papoulis, Probability, Random Variables and Stochastic Processes, 3rd Edition, McGraw-Hill, Inc, New York, 1991.

[4] H. L. Pécseli, Fluctuations in Physical Systems, Cambridge University Press, Cambridge, 2000.

[5] I. Lux, A. Baranyai, Higher order campbell techniques for neutron flux measurement i. theory, *Nucl. Instr. Meth.* **202** (1982) 469–475.

[6] I. Lux, A. Baranyai, Higher order campbell techniques for neutron flux measurement ii. correlated campbelling, *Nucl. Instr. Meth.* **202** (1982) 477–480.

[7] B. Bärs, Variance and higher order signal moments in neutron flux measurements, *Nucl. Instr. Meth.* **A 275** (1989) 403–410.

[8] S. Chabod, G. Fioni, A. Letourneau, F. Marie, Variance and higher order signal moments in neutron flux measurements, *Nucl. Instr. Meth.* **A 566** (2006) 633.

[9] E. W. Pontes, A. Ferreira, *IEEE Transactions on Nuclear Science.* **53** (2006) 1292.

[10] L. Pál, I. Pázsit, Comments on the stochastic characteristics of fission chamber signals, Tech. Rep. CTH-NT-284, Chalmers University of Technology (2013).

[11] L. Pál, I. Pázsit, Ionization chamber signals in a fissile medium, To be submitted (2014).

[12] I. Pázsit, L. Pál, Neutron Fluctuations: a Treatise on the Physics of Branching Processes, 1st Edition, Elsevier, New York, 2008.

[13] L. Pál, *A valószínűségszámítás és a statisztika alapjai, I., II.*, Akadémiai Kiadó, Budapest, 1995.